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Abstract

We study the dynamic interplay between monetary policy and market power in a decentralized monetary economy. Building on Choi and Rocheteau (2024), our key innovation is to model rent seeking as a process that takes time, allowing market power to evolve gradually. Our model predicts that a gradual reduction in the nominal interest rate causes a simultaneous increase in rent-seeking effort and producers' market power, consistent with the stylized correlation observed in the US over the last few decades. Producer entry can however reverse this relation in the short run, and neutralize it in the long run. Indeterminacy and hysteresis emerge when consumers benefit from valuable outside options, with short-run monetary policy shocks potentially locking the economy into high- or low-market-power equilibria in the long run.

JEL Classification: D82, D83, E40, E50

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1 Introduction

This paper explores the relationship between monetary policy and market power in a dynamic, micro-founded model of decentralized trade. Our theory is motivated by recent evidence on the negative correlation between market power and nominal interest rates over the last four decades. Over this time period, several measures point at a gradual increase of firm market power in the US. For example, between 1980 and 2015, the average price-over-marginal-cost markup increased from 1.05 to over 1.2 when calculated from aggregated data (Eggertsson et al. 2021) and from 1.2 to 1.5 when using firm-level data (De Loecker et al. 2020). During the same period, the federal funds rate fell from 16% in 1981 to practically zero in the 2009-2015 period. Year-over-year inflation dropped from well over 10% in 1981 to an average of less than 2% in the decade prior to the Covid-19 pandemic. Empirical evidence also points at an increase in sales concentration during that same period. The Herfindahl-Hirschman Index (HHI), defined as the sum of the squares of the sales shares of firms within an industry, increased more than 400% in the retail industry between 1981 and 2012, and approximately 50% in the service industry (Autor et al. 2020).[1]

Our work builds on Choi and Rocheteau (2024, CR24 hereafter), who lay strategic foundations for market power in monetary economies. Following the New Monetarist tradition, surveyed in Lagos et al. (2017), the model explicitly formalizes how trading activities take place, how payments and pricing occur, and how consumers and producers interact with one another strategically. More precisely, consumers and producers interact bilaterally, and a need for a medium of exchanges emerges endogenously. Trades sizes and prices are determined through a bilateral negotiation subject to liquidity constraints. Relative to the standard New Monetarist literature, two additional ingredients allow CR24 to study the endogenous formation of market power: (1) the addition of meaningful outside options for consumers in pairwise meetings, and (2) the assumption that producers can engage in rent-seeking effort to adjust their bargaining power as desired prior to a pairwise meeting. CR24 highlight three main channels through which monetary policy impacts market power. Specifically, changes in the nominal interest rate affect market power by affecting consumers' liquidity constraints, the value of their outside options, and producers' rent-seeking effort. They also demonstrate that there can exist multiple steady-state equilibria due to strategic complementarities in the amount of the rent-seeking effort exerted by producers, and that monetary policy can engineer determinacy.

We adapt the CR24 framework to study how monetary policy and market power evolve dynamically. Specifically, our main methodological innovation is to model the acquisition of monopoly power as a slow-moving process, where producers must exert effort over time to try and develop their ability to extract rents. For simplicity, we assume that producers can behave either as perfect competitors or as perfect monopolists in pairwise meetings. The former are unable to appropriate rents, while the latter are able to extract all of the rents. Conditional on a producer exerting a flow effort e, he acquires perfect monopoly power at a

[1] Industry concentration is calculated for each four-digit industry code, and then averaged across all industries within each sector.

Poisson rate e. In other words, the average duration of effort until acquiring the ability to extract rents perfectly is $1/e$. The type of effort-intensive activities that this formalization encompasses includes the development of promotional campaigns geared towards brand loyalty, advanced pricing models to enhance price differentiation, as well as providing training for retail staff. While producers engage in these activities at all times, it is only over time that doing so eventually grants them with the ability to extract more and more rents. We believe this assumption is realistic: new entrants to a market have little market power, and will only garner market power over time. In addition, this assumption makes the study of non-stationary equilibra meaningful—market power, which we proxy by the share of perfect monopolists among producers, becomes a state variable that evolves gradually, rather than adjusting instantaneously as in CR24.

We can then address the following questions: Can a gradual decline in the nominal interest rate generate a simultaneous, gradual rise in market power? Do outside options, rent-seeking effort, and real activity display qualitatively different dynamics in the short and long run following a monetary policy shock? How do initial conditions impact equilibrium determination in the presence of multiple steady-state equilibria? In that setting, how do coordination and self-fulfilling beliefs affect the response of market power to a monetary policy shock?

We highlight that studying transition dynamics, as opposed to comparing a sequence of steady states, offers many novel insights as to the response of market power to a shock. For example, we will show that under some parametrizations, the equilibrium system displays hysteresis. That is, the equilibrium is path-dependent, and differs depending on the initial condition.[2]

Our assumption that producers effectively enjoy a bargaining power of either zero or one offers several advantages. In theory, the rent-seeking process could function as a ladder, where producers could keep exerting effort to acquire a gradually higher ability to extract rents. However, focusing on the extreme cases where producers extract either none or all of the rents allows us to dispense with the choice of a specific bargaining solution, such as Nash bargaining or Kalai (proportional) bargaining, to determine the terms of trade in pairwise meetings. This is appealing because a specific bargaining solution can have a significant impact on the size of rents and how they are shared in decentralized monetary economies.[3] In our setting, terms of trade are determined very simply: (1) they are Pareto efficient, (2) rents go entirely to consumers when she is matched with a perfect competitor, and entirely to producers when she is matched with a perfect monopolist. In addition, only modeling perfect monopolists and perfect competitors implies that the share of perfect monopolists among all producers is mechanically equal to the share of aggregate rents that producers appropriate in the market. Hence, the share of perfect monopolists also directly reflects the aggregate market power of producers, and this will be the measure we focus on for most of our analysis. Lastly, another appealing feature of our formalization is that it generates an endogenous distribution of producer

[2] While such results are novel in relation to market power in monetary economies, Wang (2024) highlights similar mechanics in the context of payment acceptability in a similar type of monetary economy. Dynamics are guided by the producers' time-intensive investment effort to acquire the technology required to accept a certain means of payment.

[3] See Hu and Rocheteau (2020) and Rocheteau et al. (2021) for more details.

types despite producers being ex-ante homogeneous. Even if all producers make the same rent-seeking effort choice, their idiosyncratic transitions to perfect monopolists creates a non-degenerate distribution of producer types. In addition to providing some more realism, this allows us to consider alternative measures of market power, such as HHI.

To better understand the link between monetary policy and market power, and the contribution of each of the three main mechanisms outlined earlier, we proceed step-by-step. We attempt to keep the model tractable so as to characterize equilibrium paths analytically whenever possible.

As a first step, we study equilibrium dynamics when outside options are muted, so that only the rent-seeking and payment constraint channels are active. This corresponds to the standard New Monetarist model augmented with rent-seeking behavior. Equilibrium dynamics follow from two straightforward relationships between rent-seeking and the measure of perfect monopolists. The first is positive: as producers exert more effort to gain monopoly power, the measure of perfect monopolists increases. The second is negative: as the measure of perfect monopolists rises, the benefits of holding real balances decline because consumers can extract rents only when matched with a perfect competitor. Reduced money holdings diminish trade quantities and lower rents, reducing producers' incentives for rent-seeking. These relationships mirror CR24's mechanisms, though we model the share of monopolists while they model producers' bargaining power. We analytically demonstrate that there is a unique equilibrium path for the measure of perfect monopolists from any initial condition.

We then analyze monetary policy's impact on market power, proxied by the share of perfect monopolists, in both the long run and during the transition. The policymaker's tool is the growth rate of the money supply, which targets the nominal interest rate, interpreted as the return on illiquid assets and the opportunity cost of holding money. Following a permanent interest rate reduction, rent-seeking effort rises immediately, and the share of monopolists gradually increases to a higher steady state. Lower nominal rates reduce the cost of holding real balances, relaxing payment constraints, increasing rents, and incentivizing rent-seeking. Alternatively, a permanent increase in the target real balances slowly raises the share of perfect monopolists in parallel to a gradual decrease in the nominal interest rate, aligning with the stylized facts motivating this paper. Our results also have normative implications, providing an additional dimension to the well-studied question of the welfare cost of inflation. In this example, as inflation reduces, wasteful rent-seeking activity rises.

Second, we examine equilibria when the outside option channel is active, consistent with decentralized market models (Rubinstein and Wolinsky, 1985; Gale, 1986a, b), enhanced with monetary exchange and rent-seeking effort. The equilibrium system exhibits strategic complementarities: as the share of monopolists increases, consumers' outside options diminish, reducing rents and producers' incentives for rent-seeking. These complementarities can lead to multiple steady states: e.g., one with few monopolists and strong outside options, and another with many monopolists and weak outside options. Unlike CR24, we explore the implications of these strategic complementarities for equilibrium dynamics, yielding two main insights: (1) equilibrium paths are indeterminate for some initial conditions, implying that identical economies may converge to different steady states, affecting real activity and welfare; (2) the system exhibits hysteresis, where different initial conditions may lead to distinct outcomes both in the short- and long-run. In the presence of multiple steady states, outside options become consequential for the response of the economy to monetary policy. For example, a policy-induced shift to a new steady state may be irreversible due to path dependence, effectively locking the economy into a specific state with high or low market power.

Third, we investigate market concentration by introducing producer entry, which endogenizes matching rates. New entrants, who incur a fixed cost, start as perfect competitors. Again, there can be multiple equilibria. The underlying strategic complementarities, however, are different: as real balances go up, rents go up and producers are incentivized to enter. As more producers enter, consumers can trade faster and are, in turn, incentivized to carry more real balances. This type of multiplicity is common in search models with entry (including CR24). In this environment, a permanent increase in aggregate real balances target boosts entry on impact, reducing the share of perfect monopolists, but rent-seeking gradually restores the original share in the long run. The nominal rate drops on impact and converges to a lower steady state. Thus, entry dynamics can invert the short-run impact of monetary policy on market power, and constrain producers' long-run ability to acquire monopoly power following an expansionary policy shock.

Lastly, we consider the dynamics of alternative measures of market power aside from the share of aggregate rents appropriated by producers. We show that two commonly-used indicators of market power—markups and HHI—can respond in opposite directions to a monetary policy shock that shifts aggregate real balances, both at the steady-state and along the transition. This divergence arises because these measures capture different channels through which monetary policy influences market power. While the dynamics of markups are influenced solely by aggregate real balances and the measure of monopolists, the dynamics of HHI also reflect the composition effects between perfect monopolists and perfect competitors.

1.1 Literature

Like CR24, our environment is a continuous-time version of Lagos and Wright (2005) and Rocheteau and Wright (2005), presented in Choi and Rocheteau (2021). We refer the reader to the comprehensive literature review in CR24 regarding: the body of evidence documenting the rise in market power through markups, profits, or market concentration, some of which we referenced earlier in the introduction; the theoretical literature that laid the foundations underlying our formalization of trade in a decentralized market with pairwise meetings and the bilateral determination of the terms of trade, also surveyed in Osborne and Rubinstein (1990); the New Monetarist class of models applying these foundations to monetary economies, also surveyed in Lagos et al. (2017); the literature on sequential search, whereby outside options are typically a key model ingredient; and the literature on models of rent-seeking.

The latter includes Farboodi, Jarosch and Menzio (2018), who study a search and bargaining model of financial intermediation where all agents can invest in bargaining power before entering the market. Relative to that paper and to CR24, our formalization assumes that acquiring the power to extract rents takes time. Producers must exert a flow effort until they acquire this ability. This formalization is similar in spirit to other papers in the New Monetarist literature who study the transitional dynamics of a state variable driven by a time-consuming investment process. For example, Choi and Rocheteau (2021b, 2022) model the process through which a money is mined. Agents decide how much to invest in this process. The mining of new coins follows a Poisson rate positively related to effort. Wang (2024) investigates how the acceptability of an asset as means of payment evolves over time in an environment where producers must invest in a technology to gain the ability to recognize the asset. Again, like in our model, this process takes both time and effort, and generates an endogenous distribution of producers.

The study of dynamic equilibria in New Monetarist models include Lagos and Wright (2003) and Rocheteau and Wright (2013). The continuous-time formalization allows to eliminate some exotic dynamics, such as cycles or chaotic dynamics, as shown in Choi and Rocheteau (2021).

A small literature investigates how real interest rates impact market power, e.g., Morlacco and Zeke (2021) and Liu et al. (2022). In both cases, low real interest rates induce larger firms (or, market leaders) to engage in relatively more strategic investment activities relative to smaller (or, follower) firms. In Morlacco and Zeke (2021), firms gain market power through investment in advertisement. In Liu et al., rent-seeking occurs through investment in productivity-enhancing technology. Relative to these papers, we focus on the impact of *nominal* interest rates, or inflation, on rent-seeking. Hence, although the resulting correlations are aligned (a low rate environment encourages market power formation), we provide an alternative, complementary explanation for the underlying mechanisms. We note that the mechanisms we describe operate through a direct impact of policy on consumers' demand, which only indirectly affects rent-seeking effort by producers. In the two papers above, low rates impact firms' strategic decisions directly.

Lastly, a few papers also investigate the relation between monetary policy and market power, however in the opposite direction than we do in this paper. Specifically, they study how market power impacts the transmission of monetary policy. Duval et al. (2024) find that in the US, the output of firms with higher market power does not respond as strongly to monetary policy shocks. Aquilante et al. (2019) add market power to an otherwise standard New Keynesian model and show that that the Phillips curve becomes steeper when firms market power increases.

2 Environment

Time is continuous and indexed by $t \in \mathbb{R}_+$. The economy is composed of two types of infinitely-lived agents: a unit measure of consumers and a unit measure of producers. There are two types of perishable goods: a good $c \in \mathbb{R}$ that is traded in an on-going competitive market and that is taken as the numéraire, and a good $y \in \mathbb{R}_+$, exclusively produced and consumed in pairwise meetings. The labels *consumer* and *producer* refer to agents' role in pairwise meetings.

Consumers can transition between two states: *idle* and *active*. We denote $n_a \in [0,1]$ the measure of active consumers. When idle, a consumer has no immediate desire to consume the good y. However, the urge to consume arises as per a Poisson process with a rate $\lambda > 0$, which shifts the consumer into the active state. In this state, the desire is either satisfied by consuming any amount $y > 0$, or fades on its own at a Poisson rate $\gamma > 0$. Once either outcome occurs, the consumer reverts to being idle. This feature of the model, introduced by CR24, is essential to generate meaningful outside options for consumers when they match with a producer, as it disconnects a consumer's desire to consume from her opportunities to consume.

The flow of pairwise meetings between active consumers and producers is given by $\bar{\alpha} n_a$, where $\bar{\alpha} > 0$. Consequently, the matching rate for an active consumer is $\alpha^b \equiv (\bar{\alpha} n_a)/n_a = \bar{\alpha}$. For producers, the matching rate is $\alpha^s \equiv \bar{\alpha} n_a$, as there is a unit measure of producers in the market. Figure 2 depicts consumers' transitions across states.

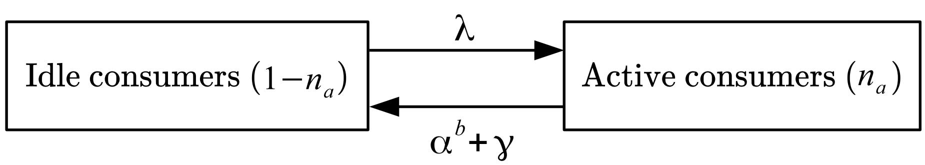

Figure 2: Consumers' transitions across two states, *idle* and *active*.

The lifetime expected discounted utility of a consumer is

$$\mathcal{U}^b = \mathbb{E}\left\{\sum_{n=1}^{+\infty} e^{-\rho T_n} u\left[y(T_n)\right] + \int_0^{\infty} e^{-\rho t} dC(t)\right\}, \tag{1}$$

where $C(t)$ is a measure of the cumulative net consumption of the numéraire good.[4] Negative net consumption is interpreted as production. If consumption (or production) of the numéraire happens in flows, then $C(t)$ admits a density, $dC(t) = c(t)dt$. If the consumer consumes or produces a discrete quantity of the numéraire good at some instant t, then $C(t^+) - C(t^-) \neq 0$. The first term between brackets on the right side of (1) accounts for the utility of consumption in pairwise meetings, while the second term accounts for the utility of consuming the numéraire good, or producing if $dC(t) < 0$. The process $\{T_n\}$ indicates the times at which the consumer is active and matched bilaterally with a producer. The utility from consuming y units of goods in pairwise meetings is $u(y)$, where u is strictly concave, $u(0) = 0$, $u'(0) = +\infty$, and $u'(\infty) = 0$. The rate of time preference is denoted by ρ.

The lifetime expected utility of a producer is

$$\mathcal{U}^s = \mathbb{E}\left\{-\sum_{n=1}^{+\infty} e^{-\rho T_n} y(T_n) + \int_0^{+\infty} e^{-\rho t} dC(t)\right\}. \tag{2}$$

[4] A similar cumulative consumption process is assumed in the continuous-time models of OTC trades of Duffie et al. (2005).

The first term corresponds to the disutility of producing y in pairwise meetings. The second term is the discounted linear utility from the consumption of the numéraire good.

In pairwise meetings, the terms of trade are determined as follows. A share n_1 of producers are able to appropriate the entire trade surplus (or, rents). We call these producers *perfect monopolists*.[5] The remaining share, akin to *perfect competitors*, extract no surplus. This formalization is equivalent to assuming that the terms of trade are determined by either Nash or Kalai proportional bargaining with the bargaining power of perfect monopolists being 1 and that of perfect competitors being 0. Alternatively, perfect monopolists make take-it-or-leave-it offers while perfect competitors receive take-it-or-leave-it offers. In aggregate, n_1 is equal to the share of aggregate rents appropriated by producers, which we interpret as an indicator of producer market power.

We assume that becoming a perfect monopolist requires both time and effort. Producers continuously choose how much effort, $e \in \mathbb{R}_+$, to exert, in order to develop their ability to appropriate rents. They incur a flow cost $v(e)$, with $v' > 0, v'' \geq 0, v(0) = 0$, and $v'(0) \geq 0$. A competitive producer who exerts an amount of effort e becomes a perfect monopolist at Poisson rate e. We assume that perfect monopolists exogenously and idiosyncratically transition back to being perfect competitors at Poisson rate δ. Figure 3 depicts the producer's transitions across these two states.

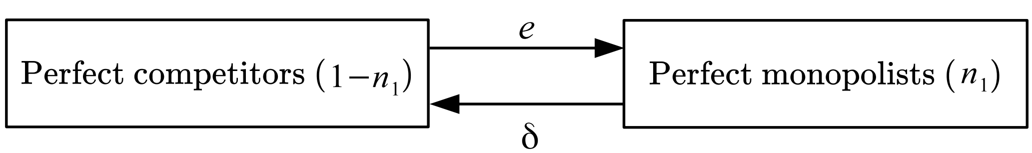

Figure 3: Producers' transitions across two states, *perfect competitors* and *perfect monopolists*.

The assumption that developing the ability to extract rents is a process that takes place over time is a key departure from CR24, who assume that producers can choose the share of rents they are able to extract, subject to some cost, at any point in time prior to a pairwise meeting. In our environment, producers can influence the speed at which they acquire bargaining power through effort, but this is not immediate. Hence, producers' share of rents becomes a state variable.[6]

Lastly, pairwise meetings are divided into two payment types. With probability $\chi^d \in [0, 1)$, the consumer can access the technology to produce the numéraire good while in the pairwise meeting. Agents can then trade y directly against the numéraire. Equivalently, the consumer cannot produce the numéraire in the meeting but can promise to deliver it in the future with perfect enforcement. Thus, one can think of χ^d as a measure of the access to credit. With complement probability $\chi^m = 1-\chi^d$, the consumer cannot produce the

[5]One could interpret this assumption as perfect monopolists having perfect information to extract the entirety of rents, i.e., they can perfectly price discriminate. See Choi & Rocheteau (2024b) for an alternative formalization of rent-seeking activities that operates through costly information acquisition.

[6]As we will see, an implication is that in our environment, there will exist a non-degenerate distribution of producer types, even though they are ex-ante homogeneous.

numéraire in the meeting and is not trusted to repay her debt in the future. In such meetings the consumer needs an alternative means of payment. There is a supply M_t of money. It has no intrinsic value, is perfectly recognizable, storable and durable, and thus may serve as a medium of exchange. The money growth rate is $\pi_t \equiv \dot{M}_t/M_t$, and the new money is injected in the economy through lump-sum transfers to consumers (or taxes if $\pi_t < 0$).[7] The price of money in terms of the numéraire is denoted ϕ_t. Hence, the return on holding money, r, is given by $\dot{\phi}/\phi = r$.

3 Equilibrium

Let $V^b(a)$ denote the value function of an active consumer with $a \in \mathbb{R}^+$ real balances (expressed in terms of the numéraire) and $W^b(a)$ the value function of an idle consumer. Due to the linearity of preferences with respect to the numéraire good, the value functions are linear in real balances, i.e. $V^b(a) = a + V^b$ and $W^b(a) = a + W^b$ (see Choi and Rocheteau, 2021) where V^b and W^b will be determined later. It is analogous to the linearity of the value functions in the discrete-time Lagos and Wright (2005) model. The difference, $Z^b \equiv V^b - W^b$, is the opportunity cost for an active consumer to accept a trade. It represents the value of the consumer's outside options, which consists in looking for an alternative trading partner. For producers, we denote $V_1^s(a)$ the value function of a perfect monopolist and $V_0^s(a)$ that of a perfect competitor. By the same logic as above, $V_i^s(a) = a + V_i^s$ for $i \in \{0, 1\}$, and V_i^s will be determined later.

3.1 Bilateral terms of trade

Consider a pairwise meeting between a consumer holding a units of real balances and a producer. Since producers have no motive to hold real balances, we assume, with no loss in generality, that they do not hold any money. A negotiation outcome is a pair, (y, p), that specifies a production of goods, y, by the producer in exchange for a payment, p by the consumer, where p is expressed in terms of the numéraire. The consumer's surplus is $u(y) + W^b(a-p) - V^b(a)$. From the linearity of V^b and W^b, it reduces to $u(y) - p - Z^b$. Similarly, the producer's surplus is $-y + V_i^s(p) - V_i^s(0)$, and therefore equal to his profits, $p - y$. The total surplus from a pairwise trade, or rents, is thus $u(y) - y - Z^b$. Denote y^* the quantity of goods traded such that unconstrained rents are maximized, $u'(y^*) = 1$. At the other extreme, let $\underline{y}$ denote the minimum trade quantity for rents to exist, $\{\underline{y} \equiv y < y^* : u(\underline{y}) - \underline{y} = Z^b\}$.

We first consider the fraction χ^d of meetings where payments can be made through credit or barter. A representation of the determination of rent sizes in these meetings is provided in the left panel of Figure 4. The value of the consumer's outside options is represented by the area with diagonal shading, while the rents available to share are represented by the dotted area. The latter are positive as long as $y^* \geq \underline{y}$. As the value of the consumer's outside option, Z^b, increases, $\underline{y}$ increases and the size of rents reduces.

[7] In the remaining of the paper, $\dot{x}_t$ denotes dx/dt.

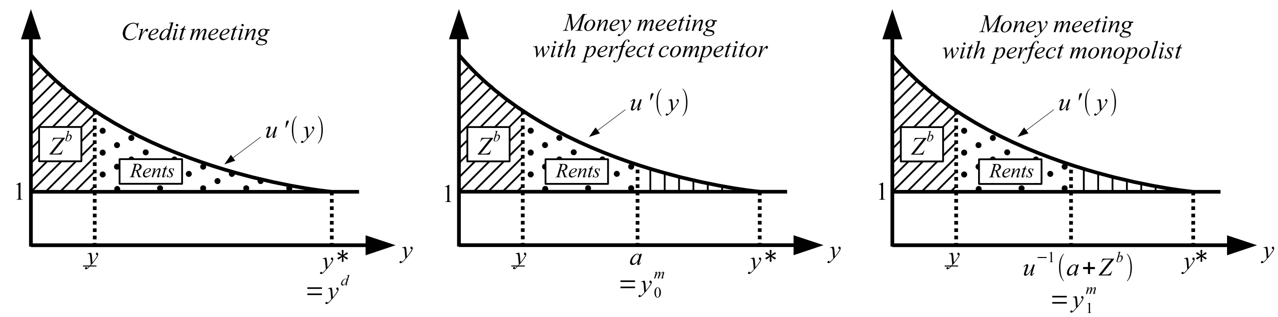

Figure 4: Rent size as a function of the type of meeting, the consumer's outside options Z^b, and her money balances a.

If the consumer trades with a perfect monopolist, the latter is able to extract the entirety of rents. Formally, the problem can be written as

$$\max_{y,p} \{p - y\} \quad \text{s.t.} \quad u(y) - p - Z^b = 0. \tag{3}$$

The solution is $y = y^* \mathbb{1}\{y^* \geq \underline{y}\} \equiv y^d$ and $p = [u(y^d) - Z^b]\mathbb{1}\{y^* \geq \underline{y}\}$. As long as the unconstrained rents are positive, they are fully extracted—the solution is Pareto efficient. The payment is set such that the producer perfectly price discriminates, and the consumer earns none of the rents. If there are no positive rents to extract due to the consumer's outside option being too high, then no trade takes place. Next, if the consumer instead trades with a perfect competitor, she is now able to extract the entirety of rents. Formally, the problem can be written as

$$\max_{y,p} \left\{u(y) - p - Z^b\right\} \quad \text{s.t.} \quad p - y = 0. \tag{4}$$

The solution is $y = p = y^d$. Again, unconstrained rents are fully extracted, but the payment is now set so that the producer receives none: the payment he receives exactly offset production costs.

We now consider meetings where credit or barter are not feasible, so that money must be used. The middle and right panels of Figure 4 highlight the determination of rents in those meetings. Liquidity constraints can now impact the size of rents that can be extracted. Formally, the terms of trade in meetings with a perfect monopolist and a perfect competitor still respectively satisfy (3) and (4), with the addition of the constraint $p \leq a$. It ensures that the payment does not exceed the consumer's real balances at the time she enters the meeting, a.

In a match with a perfect monopolist, the solution is $y = \min\{y^*, u^{-1}(a+Z^b)\}\mathbb{1}\{\min\{y^*, u^{-1}(a+Z^b)\} \geq \underline{y}\} \equiv y_1^m$, and $p = \{u(y_1^m) - Z^b\}\mathbb{1}\{\min\{y^*, u^{-1}(a+Z^b)\} \geq \underline{y}\}$. In both expressions, the indicator function takes the value one when there exists positive rents available to share. This occurs if the consumer carries at least enough money to purchase $\underline{y}$. If so, then there are two cases. If the consumer's money holdings are large enough, then y^* is traded. Otherwise, the consumer spends all her money and $y_1^m < y^*$. In either case, the perfect monopolist appropriates all the available rents. In a match with a perfect competitor, the solution is $y = \min\{y^*, a\}\mathbb{1}\{\min\{y^*, a\} \geq \underline{y}\} \equiv y_0^m$ and $p = y_0^m$. Again, the indicator function is equal to one when there exists positive rents to bargain over, and y^* is traded if the consumer carries enough money. Otherwise, she spends all her money and $y_0^m < y^*$. In either case, the consumer extracts the entirety of the rents: the payment exactly compensates the producer for the production costs. Just like in a money match with a perfect monopolist, the size of rents goes down when the consumer's real balances decrease. Graphically, the vertically-shaded area in the middle panel of Figure 4 grows larger as a diminishes. Different from the perfect monopolist case, however, an increase in the value of the consumer's outside options, Z^b, does not worsen the payment constraints. This is because the consumer's outside options do not impact the payment in a money match with a perfect competitor—it is entirely determined by the producer's revenue.

It will be helpful to study how bilateral rents behave as a function of the consumer's real balances and outside options, a and Z^b. We respectively denote $S^d \equiv [u(y^d) - y^d - Z^b]^+$, $S_0^m \equiv [u(y_0^m) - y_0^m - Z^b]^+$, and $S_1^m \equiv [u(y_1^m) - y_1^m - Z^b]^+$ the size of rents in credit meetings, money meetings with a perfect competitor, and money meetings with a perfect monopolist.[8] We focus on the case when $y^* > \underline{y}$, so that rents are strictly positive in the absence of liquidity constraints.

Intuitively, rents in credit meetings are independent of the consumer's real balances, $\partial S^d/\partial a = 0$. Rents in money meetings, however, increase with the consumer's holdings, $\partial S_1^m/\partial a \geq 0$ and $\partial S_0^m/\partial a \geq 0$. The inequalities are strict as long as a the trade quantity is lesser than y^*. Graphically, an increase in payment capacity a decreases the size of the area with vertical shading in the middle and right panels of Figure 4.

Next, outside options can impact the size of rents through two channels. First, given rents are equal to $u(y) - y - Z^b$, a change in the value of the consumer's outside options has a direct, one-to-one impact on rents conditional on the quantity traded, y. Through this channel, an increase in the value of outside options reduces the size of rents, and vice-versa. Graphically, an increase in Z^b increases the area with diagonal shading in all types of meetings. The second channel is indirect, and only operates in money meetings with a perfect monopolist. Recall that in these meetings, the payment is set so as to make the consumer indifferent between accepting or rejecting the deal. Consider a deal (y_1^m, a), whereby the consumer's liquidity constraint is binding, so $y_1^m < y^*$. Following an increase in the value of her outside options, the consumer will request a higher quantity of goods traded for the same payment a. Through this channel, an increase in the value of the consumer's outside option pushes rents upwards. Graphically, the area with vertical shading shrinks. This second effect is second order and therefore not large enough to offset the direct effect. Overall, $\partial S^d/\partial Z^b = \partial S_0^m/\partial Z^b = -1 < \partial S_1^m/\partial Z^b < 0$.

3.2 Consumers' problem

We showed earlier that due to the linearity of the preferences, $V_t^b(a) = a + V_t^b$. We now study the determination of V^b. It solves:

$$\rho V^b = \max_{a \geq 0} \left\{ -ia + \tau + \alpha^b \chi^m n_0 S_0^m(a) + \alpha^b \chi^d n_0 S^d - \gamma Z^b \right\} + \dot{V}_b, \tag{5}$$

[8] Note that bilateral rents are also equal to the producer's surplus when he is a perfect monopolist, and to the consumer's surplus when the producer is a perfect competitor.

where we used i to denote the flow spread between the discount rate and the rate of return on money, $\rho - r$. If $\rho > r$, then there is an opportunity cost (or user cost) to holding money. In addition, since money does not bear interest, its rate of return is the opposite of the inflation rate, $r = -\pi$. Hence, $i = \rho + \pi$ is also the nominal interest rate of an asset with real rate of return equal to ρ, i.e., an illiquid asset that cannot serve as means of payment in pairwise meetings.

Given that the consumer can adjust her real balances instantly at a unit cost at any point in time while searching for producers, a is treated as a control variable. The second term, τ, is the lump-sum transfer associated with money creation. According to the third and fourth terms, the consumer receives an opportunity to consume at Poisson arrival rate α^b. The meeting can be a monetary match (third term) or a credit/barter match (fourth term). The consumer extracts all the surplus from the match if she meets a producer who is a perfect competitor, which occurs with probability n_0.[9] The consumer becomes idle again after trading, or if her desire to consume disappears prior to getting the chance to trade, as represented by the last term within the brackets.

We now characterize the consumer's policy function. From the right side of (5), the consumer's optimal choice of real balances is given by

$$a^* \in \arg\max_{a \geq 0} \left\{ -ia + \alpha^b \chi^m n_0 S_0^m(a) \right\}. \tag{6}$$

The consumer chooses her real balances to maximize the rents in pairwise meetings with perfect competitors taking into account that holding money is costly as long as $i > 0$. When $\underline{y} \geq y^*$, $S_0^m = 0$ and the objective function strictly decreases in a. Intuitively, in this region, there are no rents for the consumer to extract in her meetings with perfect competitors due to her outside options being too high, so no trade takes place. Then, there is no benefit to carrying money and $a^* = 0$ as long as $i > 0$. When $\underline{y} < y^*$, the objective function admits a kink. It is linear with slope $-i$ for all $a \in [0, \underline{y}]$ and it is concave with slope $-i + \alpha^b \chi_m n_0 \{u'[y_0^m(a)] - 1\}$ for all $a > \underline{y}$. There are two candidate solutions for the consumer's optimal real balances: $a^* = 0$, or $a^* = \tilde{a}$, where $\tilde{a}$ is the solution to the first-order condition,

$$i = \alpha^b \chi^m n_0 \left[u'(\tilde{a}) - 1 \right]. \tag{7}$$

Note that $\tilde{a} = y^*$ when $i = 0$ and $\tilde{a} < y^*$ when $i > 0$. The right side of (7) is the marginal expected value of real money balances for the consumer. It is the product of three terms: the frequency of trading opportunities in which money can be used, $\alpha^b \chi^m$, the share of meetings in which the consumer can extract the (full) rent, n_0, and the marginal match surplus, $[u'(\tilde{a}) - 1]$. The left side of (7) is the marginal cost of holding real balances—the foregone opportunity cost of holding an illiquid asset with a nominal rate of return i.

[9] If she meets with a perfect monopolist, she receives zero surplus, thus this event does not appear in (5).

Lemma 1 ***(Optimal real money balances.)*** *The consumer's choice of real balances is given by*

$$a^* = \begin{cases} \tilde{a} \text{ if } Z^b \leq \{-i\tilde{a} + \alpha^b \chi^m n_0[u(\tilde{a}) - \tilde{a}]\}/(\alpha^b \chi^m n_0), \\ 0 \text{ otherwise.} \end{cases} \tag{8}$$

The solution is interior as long as the value of outside options is small enough.

Next, the HJB equation for the value function of an idle consumer, W^b, satisfies:

$$\rho W^b = \tau + \lambda \left(V^b - W^b\right) + \dot{W}^b. \tag{9}$$

The idle consumer receives the transfer from money creation, τ, and a preference shock with Poisson arrival rate λ, in which case she becomes active.

3.3 Consumers' outside options

The value of a consumer's outside options is obtained by taking the difference between (5) and (9). It solves:

$$(\rho + \lambda + \gamma) Z^b = -ia^* + \alpha^b n_0 \left[\chi^m S_0^m(a^*) + \chi^d S^d\right] + \dot{Z}^b. \tag{10}$$

The flow value of the outside options is equal to the consumer's expected trade surplus in (future) pairwise meetings, net of the continued cost of holding real balances. The former is the product of the arrival rate of trading opportunities, the proportion of these opportunities that involve a perfect competitor, and the expected rents from such trades, taking into account the type of payment.

We can see from (10) that in a static setting, and conditional on (n_0, a^*), the value of the consumer's outside options behaves intuitively, as described in CR24. It is positively correlated with the rate at which consumers get opportunities to trade, α^b and the share of credit matches, χ^d. We can also get preliminary insights into the relation between outside options and monetary policy. When the nominal interest rate, i, increases, the value of a consumer's alternatives goes down since the process of seeking new trading partners becomes more expensive. Indeed, in a monetary setting, a consumer who decides against trading with their current match must hold onto real balances until a new opportunity arises. Since holding such assets comes at a cost when $i > 0$, this creates a "hot potato" effect, whereby inflation drives up search costs and motivates the consumer to spend cash more quickly. As a result, the outside option of the consumer reduces, increasing the rent that can be extracted by a perfect monopolist producer. We will study these mechanisms in a dynamic, general equilibrium in Section 4.

3.4 Producers' investment in monopoly power

The value function of a perfect competitor producer solves

$$\rho V_0^s = \max_{e \geq 0} \left\{-\upsilon(e) + e\left(V_1^s - V_0^s\right) + \dot{V}_0^s\right\}. \tag{11}$$

Even though perfect competitors are matched with consumers and trade at the rate α^s, they get no surplus from those trades, explaining their absence in (11). Producers choose how much effort, e, to exert towards acquiring monopoly power, incurring the flow cost $\upsilon(e)$. Becoming a perfect monopolist, which occurs at rate e, is associated with the capital gain $V_1^s - V_0^s$. The value function of a producer with monopoly power solves

$$\rho V_1^s = \alpha^s(\chi^m S_1^m + \chi^d S^d) - \delta(V_1^s - V_0^s) + \dot{V}_1^s. \tag{12}$$

The producer enters a match at Poisson rate α^s. Whether it is a monetary match or a credit match, he extracts all of the rents. Perfect monopolists do no exert effort, as there is no benefit from doing so—they have already developed the ability to capture pairwise rents.

The optimal effort for perfect competitors satisfies

$$\upsilon'(e) = V_1^s - V_0^s. \tag{13}$$

Perfect competitors choose their rent-seeking effort so that the marginal cost of effort, on the left-hand side, equates the marginal gain, on the right-hand side. The marginal derivative of the rate at which producers can acquire monopoly power with respect to effort is 1, so the marginal benefit of effort is simply equal to the capital gain from becoming a perfect monopolist.

Making use of (13) and subtracting (11) from (12), we obtain the differential equation governing the dynamics of the amount of effort exerted by perfect competitors to obtain monopoly power,

$$(\rho + \delta)\,\upsilon'(e) = \alpha^s\left(\chi^m S_1^m + \chi^d S^d\right) + \upsilon(e) - e\upsilon'(e) + \upsilon''(e)\dot{e}. \tag{14}$$

3.5 Laws of motion of active consumers and perfect monopolists

The law of motion of the measure of active consumers is given by

$$\dot{n}_a = \lambda(1 - n_a) - (\alpha^b + \gamma)n_a, \tag{15}$$

where we assumed, without loss of generality, that all pairwise matches result in trade. Idle consumers become active when they get a consumption shock at the rate λ. Active consumers satiate their consumption desire by either trading with a producer (at rate α^b) or when the desire to fades away on its own (at rate γ). In steady state, the measure of active consumers is $n_a = \lambda/(\lambda + \gamma + \alpha^b)$.

The law of motion of the measure of perfect monopolists, also equal to the share of aggregate rents extracted by producers, is given by

$$\dot{n}_1 = e(1 - n_1) - \delta n_1. \tag{16}$$

The first term on the right side corresponds to the measure of perfect competitors who become perfect monopolists at rate e, and the second term corresponds to the measure of perfect monopolists who lose their status at rate δ. Along the $\dot{n}_1 = 0$ locus, the measure of perfect monopolists is given by $n_1 = e/(\delta + e)$ and increases with e.

We now have all the elements to define an equilibrium.

Definition 1 *An equilibrium is a list of time-paths,* $(a_t, Z_t^b, e_t, n_{1,t})$*, which jointly solve* (8)*,* (10)*,* (14)*, and* (16)*, taking* $n_{0,1}$ *as given and* $n_a = \lambda/(\lambda + \gamma + \bar{\alpha})$*.*

4 Dynamics of market power

We now leverage our model—an environment where producers can engage in costly activity to increase their ability to extract rents over time—to study the dynamics of market power. Specifically, we are interested in its relation with monetary policy. We first study how our equilibrium variables, and specifically the share of aggregate rents appropriated by producers, n_1, evolve following changes in monetary policy in an environment with no outside options ($Z^b = 0$). We then investigate the additional role played by outside options in equilibrium dynamics. In a third part, we introduce firm entry to endogenize market concentration, a key determinant of the meeting rate between consumers and producers. Lastly, we study the responses of two alternative measures of market power, markups and HHI.

4.1 Dynamics when outside options are muted

To shut down the outside option channel, we set $\lambda = +\infty$, meaning that consumers experience the desire to consumer at all times. Therefore, all consumers are active consumers, $n_a = 1$, and $Z^b = 0$, as can be checked from (10). Because consumers always have the desire to consume, a consumer who trades in a pairwise meeting with a producer no longer incurs the cost of losing future opportunities to satiate that desire with another producer. This environment is similar to the standard case in the New Monetarist literature, however with the addition of the ability for producers to acquire monopoly power in bilateral meetings over time. From (14), the law of motion for the amount of effort exerted by producers without monopoly power, e, obeys

$$(\rho + \delta + e)\, \upsilon'(e) = \alpha^s \{\chi^m \left[u\left(y_1^m\right) - y_1^m\right] + \chi^d \left[u\left(y^d\right) - y^d\right]\} + \upsilon(e) + \upsilon''(e)\dot{e}. \tag{17}$$

Along the $\dot{e} = 0$ locus, (17) gives a positive relationship between the effort level e and the bilateral trade quantities y_1^m and y^d. If producers anticipate they will be able to sell larger quantities in pairwise meetings, increasing the size of rents, they have higher incentives to invest effort into becoming perfect monopolists faster. In addition, recall that y_1^m is an increasing function of the consumer's real balances, since an increase in real balances relaxes the payment constraint in money meetings when $y < y^*$. As a result, there is a positive relationship between the consumers' choice of real balances and producers' effort to gain monopoly power along the $\dot{e} = 0$ locus.

Next, from (8), the consumer's optimal choice of real balances when $Z^b = 0$ simply solves

$$i = \alpha^b \chi^m (1 - n_1) \left[u'\left(a^*\right) - 1\right]. \tag{18}$$

We denote A the measure of aggregate real balances. Since the measure of consumers is equal to 1 and producers do not carry money, $A = a^*$. Using that $r = \dot{\phi}/\phi$, and imposing market clearing, according to which $A = \phi M$, we rewrite (18) as

$$\frac{\dot{A}}{A} = i - \alpha^b \chi^m (1 - n_1) \left[u'(A) - 1\right]. \tag{19}$$

An equilibrium reduces to a list of time-paths, $(e_t, n_{1,t}, A_t)$, that solves (16), (17), and (19). From (18), we express aggregate real balances as a function of i and n_1,

$$A = A(i, n_1) \equiv u'^{-1}\left(1 + \frac{i}{\alpha^b \chi^m (1 - n_1)}\right). \tag{20}$$

Note that $A(i, n_1)$ is decreasing in its two arguments. When either the cost of holding real balances or the measure of perfect monopolists increase, the equilibrium level of real balances held by consumers reduces—i.e., there is less liquidity.

Then, the equilibrium can further be reduced to a pair $(e_t, n_{1,t})$ which satisfies

$$(\rho + \delta + e)\, \upsilon'(e) = \alpha^s \{\chi^m \left[u \circ y_1^m(A(i, n_1)) - y_1^m(A(i, n_1))\right] + \chi^d \left[u(y^*) - y^*\right]\} + \upsilon(e) + \upsilon''(e)\dot{e}, \tag{21}$$

and

$$\dot{n}_1 = e(1 - n_1) - \delta n_1. \tag{22}$$

Proposition 1 ***(Steady-state equilibrium with muted outside options.)*** *Suppose $\lambda = +\infty$. If $\chi^m > 0$ and $\upsilon''(e) > 0$, then there always exists a unique positive steady-state monetary equilibrium, and it is saddle-point stable.*

We represent the phase diagram corresponding to the system of differential equations (21) and (22) in Figure 5. The e-isocline ($\dot{e} = 0$) gives a negative relationship between e and n_1. As the number of perfect monopolists rises, consumers hold fewer real balances, leading to lower rents in pairwise transactions. This, in turn, diminishes the producer's motivation to exert effort in gaining monopoly power. On the other hand, the n_1-isocline ($\dot{n}_1 = 0$) gives a positive relationship between e and n_1. As more competitive producers exert effort to become perfect monopolists, the number of perfect monopolists increases. Because the unique steady-state is a saddle, the equilibrium path is unique for any initial $n_{1,0}$. For example, assume that the share of perfect monopolists is lower than its steady-state value. Then, it slowly converges to the steady-state on the left branch of the blue saddle path. Note that the level of effort, while sufficient to keep the measure of perfect monopolists growing, reduces along the transition path. Indeed, the capital gains that stem from acquiring monopoly power decrease as the share of perfect monopolists goes up.

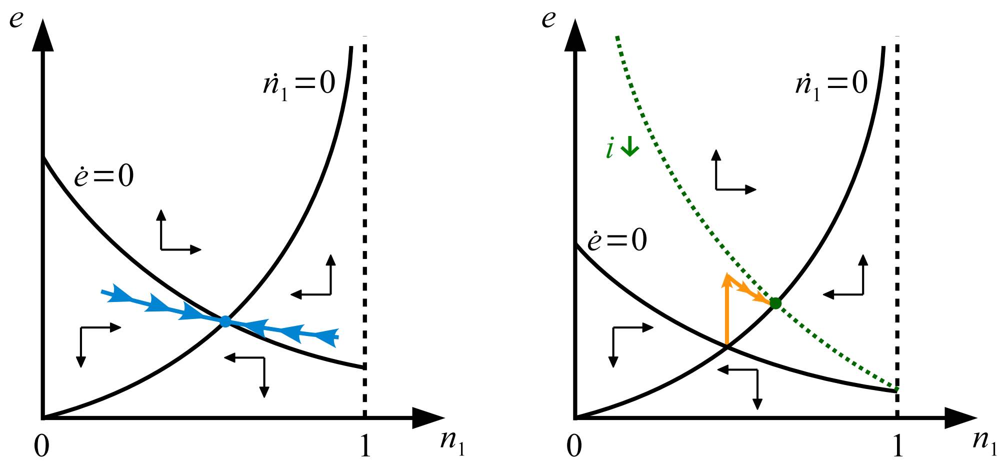

Figure 5: Left panel — Equilibrium determination when $\lambda = \infty$. The steady state is at the intersection of the two isoclines, and the blue path represents the saddle path. Right panel — Transition dynamics after a decrease in the nominal interest rate, i. Rent-seeking effort jumps on impact, and the measure of perfect monopolists gradually increases.

We now study the impact of monetary policy. We first consider a policymaker who targets the nominal interest rate, $i \equiv \rho - r$. Operationally, to increase i, monetary policy reduces the rate of return of money, r, by increasing its rate of growth, π. This takes place via increased lump-sum transfers to the consumers. Conversely, to reduce i, the rate of growth of money supply is slowed down.

Proposition 2 ***(Monetary policy and market power under a nominal interest rate target.)*** *Suppose the economy starts at the unique steady state. An unanticipated and permanent decrease (increase) in the nominal interest rate, i, generates an immediate increase (decrease) in rent-seeking effort. Effort then gradually reduces (increases) until it reaches its new, higher (lower) steady state level. The aggregate share of rents appropriated by producers, n_1, gradually increases (decreases).*

Consider a decrease in the nominal interest rate, i. As pictured in the right panel of Figure 5, the e-isocline shifts upward, as represented by the green-dotted curve. Indeed, the reduction in i decreases the consumer's cost of holding real balances. This encourages consumers to carry more, which raises producers' gain from being a perfect monopolist. Hence, producers who have not yet acquired monopoly power immediately ramp up their rent-seeking efforts. Graphically, e jumps to the new saddle path, as represented in orange. The measure of perfect monopolists then increases over time along the saddle path, until it reaches its new, higher steady state. During this transition, the level of effort exerted by producers to acquire monopoly power decreases. While it eventually settles at a higher level than it was before the reduction in i, this dynamic is due to the returns to effort decreasing as the level of real balances held by consumers goes down after jumping on impact.

We now examine the impact of a different type of policy, which targets the aggregate level of real balances in the economy, $A = A^p$. Operationally, the policymaker adjusts the nominal interest rate i at any point

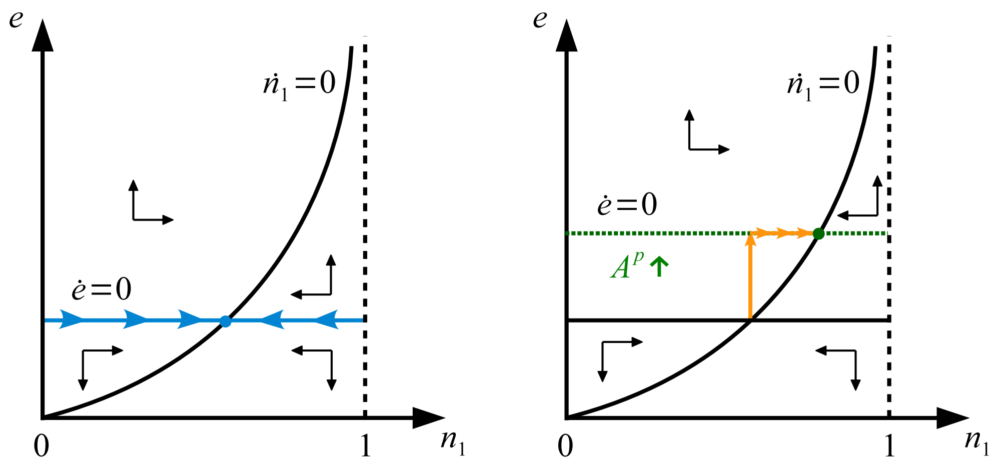

Figure 6: Left panel — Equilibrium determination when $\lambda = \infty$ and $A_t = A^p$. The steady state is at the intersection of the two isoclines, where the e-isocline coincides with the saddle path, in blue. Right panel — Transition dynamics after an increase in the aggregate real balances target, A^p. Rent-seeking effort jumps to its new steady-state value on impact, and the measure of perfect monopolists gradually increases.

in time so that aggregate real balances equate the target according to (20). If the measure of perfect monopolists goes up, the policymaker reduces the nominal interest rate to ensure A remains constant, and vice versa. Under this policy scheme, the level of effort is uniquely determined by (21) as a function of aggregate real balances, $e \equiv e(A^p)$, where $e'(A^p) > 0$. Graphically, the e-isocline becomes a horizontal line, as represented in Figure 6. The steady state remains unique and saddle-point stable, where the saddle path, in blue, coincides with the e-isocline.

Proposition 3 ***(Monetary policy and market power under a real balances target.)*** *Let $A^p \in (0, y^*)$ and i_t satisfy $A^p = A(i_t, n_{1,t})$.*

(i) For any $n_{1,0} \in [0,1]$, there exists a unique equilibrium path, given by

$$e_t = e(A^p) \text{ and } n_{1,t} = \frac{e(A^p)}{e(A^p)+\delta} - \left[\frac{e(A^p)}{e(A^p)+\delta} - n_{1,0}\right] \exp\left(-[e(A^p)+\delta]t\right).$$

(ii) Suppose the economy starts at the unique steady state. An unanticipated and permanent increase (decrease) in the real balances target, A^p, generates an immediate and permanent increase in rent-seeking activity. There is a gradual increase in the measure of perfect monopolists until the new steady state is reached. The nominal interest rate jumps down on impact, then gradually keeps decreasing until reaching a permanently lower steady state.

The first part of Proposition 3 is illustrated in the left panel of Figure (6). Starting from any $n_{1,0}$, the equilibrium path simply follows the blue horizontal saddle path towards the unique steady state. The second part of Proposition 3 is illustrated in the right panel. The increase in A^p shifts the e-isocline up, now represented by the green dotted line. Rent-seeking effort jumps on impact to the new saddle path, and the measure of perfect monopolists gradually increases towards its new steady state. The transition path is represented in orange. Targeting real balances requires reducing the nominal interest continuously until the new steady state is reached so as to counter the dampening effect of the rise in the measure of perfect monopolists on the consumers' holding of real balances.

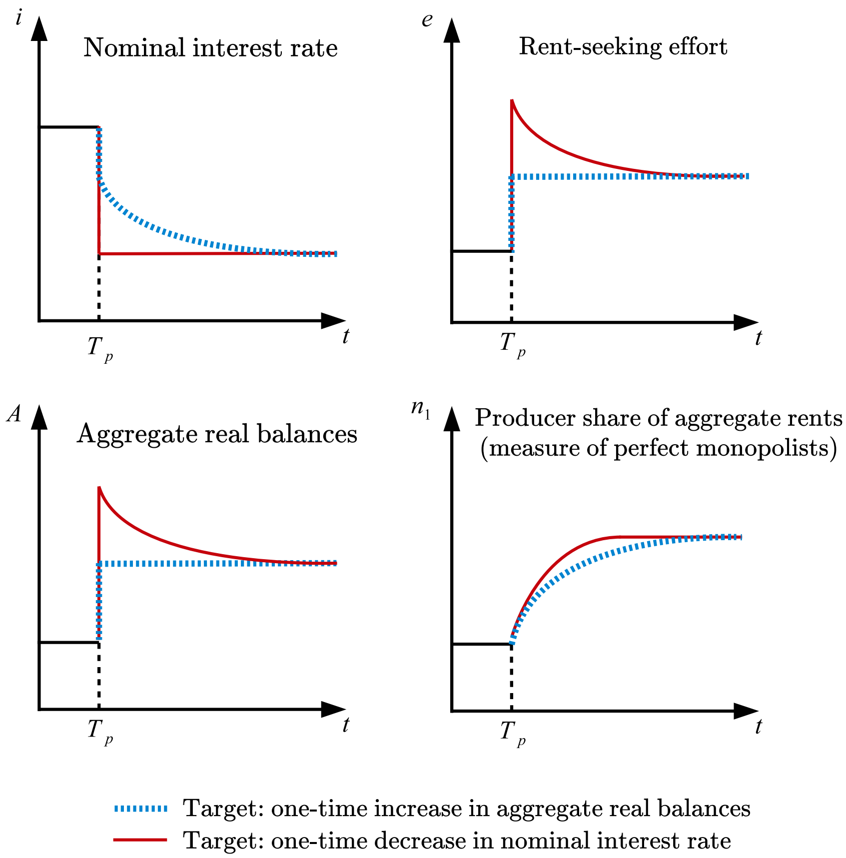

Figure 7: Timepaths after an expansionary policy shock at time T^p. The red paths follow a one-time reduction in the nominal interest rate. The blue paths follow a gradual reduction in the nominal interest rate engineered to generate a one-time permanent increase in aggregate real balances.

Overall, the two policy schemes have qualitatively similar effects in terms of the relation between monetary policy and firm market power as proxied by the aggregate share of rents appropriated by producers. A contractionary monetary policy reduces market power over time, while an expansionary policy does the opposite. Figure 7 summarizes the timepaths for (i, A, e, n_1) after an expansionary policy shock. The continuous red paths represent dynamics when the policymaker unexpectedly reduces the targeted nominal interest rate at time T_p. The blue dashed paths display dynamics when the policymaker engineers a permanent increase in the targeted quantity of real balances starting at time T_p. This is done through the immediate then gradual reduction in the nominal interest rate, as shown in the top left panel. Note that in this illustration, both policies are assumed to generate the same steady state in the long run.

Dynamic effects of alternative one-time shocks. While our focus lies primarily on the relation between monetary policy and the dynamics of market power, our framework allows us to equilibrium dynamics after a variety of alternative shocks. Assume that the nominal interest rate is constant, i.e., the policymaker is inactive.

Consider first an increase in the flow of matches between consumers and producers, $\bar{\alpha}$. It generates the same qualitative dynamics as a decrease in the nominal interest rate. As the measure of matches goes up, opportunities for perfect monopolists to extract rents are more frequent. Consumers also encounter trading opportunities more often and so, carry more real balances. Hence, producers exert more effort to gain monopoly power, thereby shifting the e-isocline upward and increasing the measure of perfect monopolists over time. An increase in the matching technology could arise from technology improvements, e.g., the development of online shopping.

Alternatively, if the cost of exerting effort, $v(e)$, rises in such a way that $v'(e)$ is now higher for all e, producers reduce their effort towards gaining monopoly power, as the process becomes more expensive. Again, an increase in $v(e)$ shifts the e-isocline downwards, decreasing the measure of perfect monopolists over time. This increase in costs could stem from factors like licensing and compliance requirements, where regulatory obligations—such as acquiring permits, or meeting environmental guidelines, add to the financial burden of attempting to monopolize a market.

Lastly, consider an increase in the rate at which existing perfect monopolists lose their ability to perfectly price discriminate, δ. The increase in δ reduces producers' incentive to invest effort into acquiring monopoly power conditional on the measure of perfect monopolists, shifting the e-isocline downwards. The n_1-isocline also shifts leftwards simultaneously. For a given amount of effort, the measure of perfect monopolists reduces since they exit the market faster. This encourages consumers to hold more real balances, which increases producers' incentives to exert effort everything else equal. Thus, the impact on the equilibrium level of effort depends on the relative strengths of the two counteracting forces. Either way, the measure of perfect monopolists gradually decreases towards the new steady-state. Such a rise in δ can result from various factors, including regulatory, legal, or technological influences. For instance, initiatives by the Federal Trade Commission to promote fair competition and ensure equitable trade practices may weaken perfect monopolistic control. Similarly, legal factors like patent expiration enable perfect competitors to enter markets previously dominated by firms relying on intellectual property protections. Technological advancements also play a crucial role; innovations can lower barriers to entry, allowing new players to compete, while the adoption of open-source platforms or standards can further erode perfect monopolistic advantages.[10]

4.2 Dynamics of outside options

Having understood the dynamics of rent-seeking effort and their relation to monetary policy, we now turn to studying the dynamics of an additional channel—consumers' outside options, which we reintroduce by setting

[10] We model firm entry with an entry cost in Section 4.3.

$\lambda < +\infty$. We assume that the policy maker targets $A_t = A^p$, keeping aggregate real balances constant at all times by correspondingly adjusting the nominal interest rate. This allows us to reduce the dimensionality of our dynamic system as well as investigate monetary policy shocks that target real activity.

From (10), the dynamics of the value of consumers' outside options are given by

$$\left[\rho + \lambda + \gamma + \alpha^b(1 - n_1)\right] Z^b = \alpha^b(1 - n_1)\left[u\left(A^p\right) - u'(A^p)A^p\right] + \dot{Z}^b, \tag{23}$$

The value of the outside options depends on the surpluses from trade with competitive producers, the value of which increases with A. The producer's value functions are still given by (11) and (12). From (14), the effort to gain monopoly power solves

$$(\rho + \delta + e)\, \upsilon'(e) = \alpha^s S_1^m(A^p, Z^b) + \upsilon(e) + \upsilon''(e)\dot{e}. \tag{24}$$

An equilibrium is now defined as a triple of time paths, $(Z_t^b, e_t, n_{1,t})$, that solves (16), (23), and (24), with an initial condition for the measure of perfect monopolists, $n_{1,0}$.

We now study the equilibrium. Consider the e-isocline, given by setting $\dot{e} = 0$ in (24). For a given A, this locus displays a negative relationship between the amount of effort exerted by producers to become perfect monopolists and the value of consumers' outside options. The intuition is straightforward: as consumers enjoy larger outside options, the size of rents that can be extracted by producers reduces, and so do the incentives to exert effort to acquire monopoly power. The n_1 isocline is still given by $n_1 = e/(e + \delta)$. Combining it with the e-isocline just described, we obtain a negative relation between outside options and the measure of perfect monopolists. As Z^b goes up and less effort is made by producers to acquire monopoly power, the equilibrium measure of perfect monopolists goes down. Next, consider the Z^b-isocline, which we obtain by setting $\dot{Z}^b = 0$ in (23). This locus also represents a negative relationship between Z^b and n_1. As the measure of perfect monopolists increases, the consumers' ex-ante chance of being able to extract rents in a random pairwise meeting goes down. Hence, for a consumer already in a pairwise meeting with a producer, the value of leaving the meeting to trade with another producer is reduced when n_1 is higher. Overall, the equilibrium is characterized by two downwards-sloping equations in the (n_1, Z^b) space, potentially leading to a multiplicity of equilibria.

We now present a simple example to illustrate this possibility analytically. Assume that the effort cost is linear, $\upsilon(e) = \bar{\upsilon}e$, and that effort is bounded, $e \in [0, \bar{e}]$. The e-isocline now becomes

$$e \begin{array}{c} = \bar{e} \\ \in [0, \bar{e}] \\ = 0 \end{array} \quad \text{if} \quad Z^b \begin{array}{c} < \\ = \\ > \end{array} u\left(A^p - \frac{(\rho + \delta)\,\bar{\upsilon}}{\alpha^s}\right) - A^p \equiv \tilde{Z}^b. \tag{25}$$

Due to the linearity of the effort cost, the optimal choice of effort by producers in steady-state is kinked. When $Z^b > \tilde{Z}^b$, the rents that a competitive producer can expect to extract if he becomes a perfect monopolist are too small to warrant the effort to transition, and thus $e = 0$. On the other hand, when $Z^b < \tilde{Z}^b$, the expected rents are large enough to encourage competitive producers to try and acquire monopoly power, and

e is at its maximum. When $Z^b = \tilde{Z}^b$, competitive producers are indifferent, and e is indeterminate along the isocline.

Plugging the correspondence (25) into the n_1-isocline, the latter becomes a piecewise function, as represented by the blue curves in Figure 8. The vertical segment when $n_1 = 0$ is such that $e = 0$, while the vertical segment when $n_1 > 0$ is such that $e = \bar{e}$. The red curve represents the Z^b-isocline. The next proposition characterizes the steady-state equilibria in this simple example.

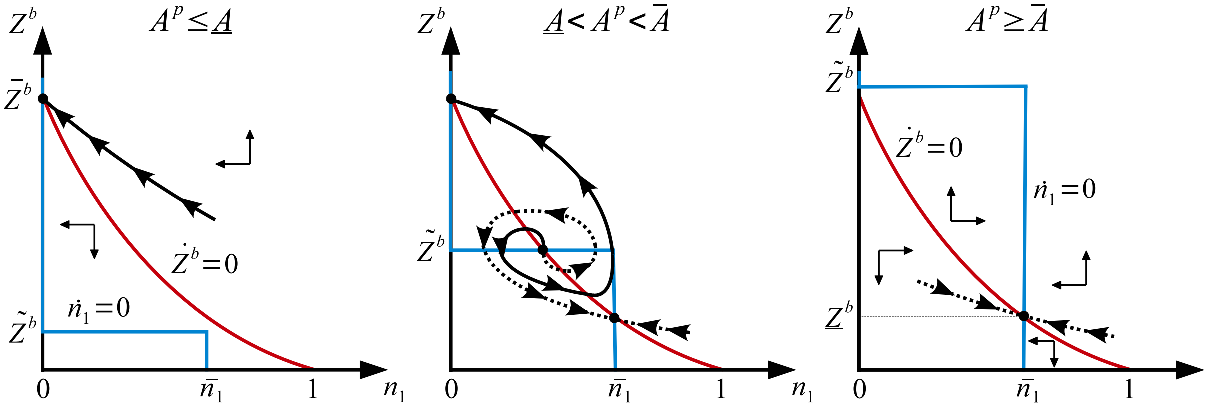

Figure 8: Equilibria with endogenous outside options and linear effort costs.

Proposition 4 ***(Steady-state equilibria with endogenous outside options)*** *Suppose $v(e) = \bar{v}e$ and $e \in [0, \bar{e}]$. There exists $\bar{v}_0 > 0$ such that for all $\bar{v} < \bar{v}_0$, there are $0 < \underline{A} < \bar{A}$ such that the following is true:*

1. *For all $A^p \leq \underline{A}$, there is a unique steady-state equilibrium. It is such that*

$$e = 0, n_1 = 0, \text{ and } Z^b = \frac{\alpha^b}{\alpha^b + \rho + \lambda + \gamma}[u(A^p) - u'(A^p)A^p] \equiv \bar{Z}^b.$$

2. *For all $A^p \geq \bar{A}$ there is a unique steady-state equilibrium. It is such that*

$$e = \bar{e}, n_1 = \bar{e}/(\delta + \bar{e}) = \bar{n}_1, \text{ and } Z^b = \frac{\alpha^b(1 - \bar{n}_1)}{\alpha^b(1 - \bar{n}_1) + \rho + \lambda + \gamma}[u(A^p) - u'(A^p)A^p] \equiv \underline{Z}^b.$$

3. *For all $A^p \in (\underline{A}, \bar{A})$, there exist three steady-state equilibria. The first two correspond to the equilibria described in cases 1 and 2. The third is such that*

$$Z^b = \tilde{Z}^b \in (\underline{Z}^b, \bar{Z}^b), n_1 = n_1(\tilde{Z}^b) \in (0, \bar{n}_1), \text{ and } e = \frac{\delta n_1(\tilde{Z}^b)}{1 - n_1(\tilde{Z}^b)} \in (0, \bar{e}).$$

When the target quantity of real balances is in the intermediate region, $A^p \in (\underline{A}, \bar{A})$, there is a multiplicity of steady states. The multiplicity is due to the complementarities between rent-seeking effort and rent sizes. As rent-seeking effort increases, leading to a higher share of perfect monopolists, consumers' outside options reduce, increasing rent sizes. As rent sizes increase, the expected gain from rent-seeking effort also increases. These complementarities lead to multiple steady states. In the steady state with $n_1 = 0$ and $Z^b = \bar{Z}^b$, there is no effort exerted by producers to acquire bargaining power. Thus, all producers are perfect competitors, which generates a high outside option value for consumers, as they can expect to extract the entirety of rents in any pairwise meeting. In turn, this discourages any producer from deviating and exerting effort to acquire monopoly power. Indeed, any consumer matched with a deviating perfect monopolist would simply exercise her outside option to match with a perfect competitor, preventing the perfect monopolist from trading. In the steady state with $n_1 = \bar{n}_1$ and $Z^b = \underline{Z}^b$, the opposite is true. Producers exert the maximum effort, so that there is a large share of perfect monopolists. The outside option value for consumers is low, which implies high rents available to extract, now consistent with competitive producers having incentives to invest in acquiring monopoly power. There is a third steady state in between, where rent-seeking effort, the measure of perfect monopolists, and the value of consumers' outside options are all intermediate.

When the target quantity of real balances is under some threshold $\underline{A}$, both the intermediate and the high effort-low outside option steady states disappear. Indeed, the low quantity of real balances limits the size of rents, and producers do not find it worthy to invest in monopoly power even when outside options are low. On the other hand, when the quantity of real balance is above some threshold $\bar{A}$, only the high effort-low outside option steady state remains. In this case, the payment capacity is so high that the size of rents remains high enough even if outside options were very strong. As a result, producers always find it worthwhile to invest in monopoly power, and the equilibrium without effort cannot be sustained.

Equilibrium paths determinacy and hysteresis. The two extreme steady states are saddle-point stable, while the intermediate steady state is an unstable spiral, as visible in Figure 8. When the steady state is unique, the equilibrium path is also unique starting from any $n_{1,0}$. Note that Z^b and n_1 are inversely correlated along the saddle paths, as we would expect. Effort is constant. When there are multiple equilibria, (1) the equilibrium path is not always determinate, and (2) the system exhibits hysteresis, that is, path dependence. As visible in the middle panel of Figure 8, if $n_{1,0}$ is in the region spanned by the spiral, there are multiple equilibrium paths leading to different steady states. This implies that due to coordination effects, two economies with the same fundamentals and the same starting point may experience different transition dynamics and long-run outcomes—one with producers appropriating a large share of aggregate rents, the other with producers appropriating none. The middle panel of Figure 8 also illustrates the possibility of path dependence. An economy with a low enough initial measure of perfect monopolists necessarily converges to the steady state with no perfect monopolists, while an economy with an intermediate measure of perfect monopolists can also converge to the steady state with the high measure of perfect monopolists.

These insights regarding the multiplicity of steady-state and dynamic equilibria do not hinge on the simplifying assumption that the effort cost is linear. Figure 9 presents the results from a numerical example where the effort cost is strictly convex, and parameter values were picked to generate multiple equilibria.[11]

[11] The model is parameterized as follows: $u(y) = y^{1/3}$, $v(e) = 0.5e^{3/2}$, $(\alpha, \gamma, \lambda, \delta, \rho) = (1, 0.5, 1, 0.1, 0.04)$, $\left(A^p_{\text{left}}, A^p_{\text{middle}}, A^p_{\text{right}}\right) = (3.8, 4.3, 4.5)$.

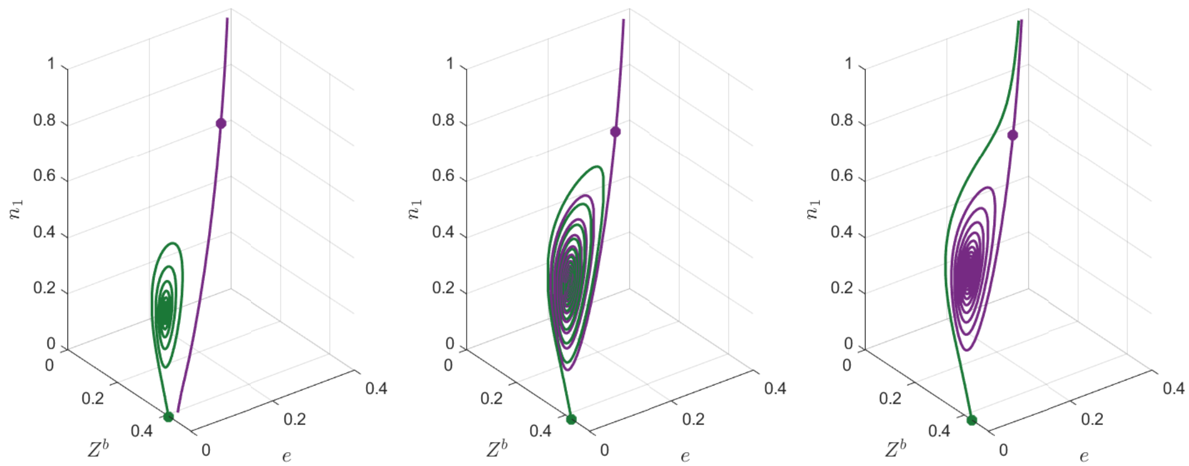

Figure 9: Numerical example — Multiplicity of equilibria with endogenous outside options and convex effort costs. From left to right, the aggregate real balances target, A^p, goes up.

The phase diagrams must now be displayed in three dimensions, as the dynamics of effort are no longer binary. The two stable steady states are marked with the green and purple dots. The two curves display the equilibrium paths starting from any $n_{1,0}$. Each panel is characterized by a different level of aggregate real balances. A projection of the phase diagrams onto the (Z^b, n_1) plane shows that, just like in the example with linear effort cost, the share of perfect monopolists is negatively correlated with the value of consumers' outside options along the equilibrium paths. Also like in the example with linear costs, the initial measure of perfect monopolists, $n_{1,0}$ impacts whether the equilibrium path is determinate and the set of steady states towards which the economy may converge.

Monetary policy and market power with endogenous outside options. In Section 4.1, we studied the impact of two types of monetary policies (nominal rate target and real balances target) on rent-seeking investment and market power, both over time and in the long run, in a context where outside options were shut down. We now proceed with a similar exercise with outside options being active. There are two main questions: (1) how do the dynamics of outside options impact the reaction of rent-seeking effort and the measure of perfect monopolists when the equilibrium is determinate? and (2) how does equilibrium indeterminacy and path dependence interact with monetary policy?

The simple example where $V(e) = \bar{v}e$ provides limited insights. When starting from one of the two stable steady-state equilibria, marginal changes in the target quantity of aggregate real balances have little impact. The value of consumers' outside options adjusts immediately to the new steady-state, and the measure of perfect monopolists remains unchanged as the rent-seeking effort stays at 0 or $\bar{e}$.

Our numerical example for the general case, where effort is strictly convex, is more interesting. Figure

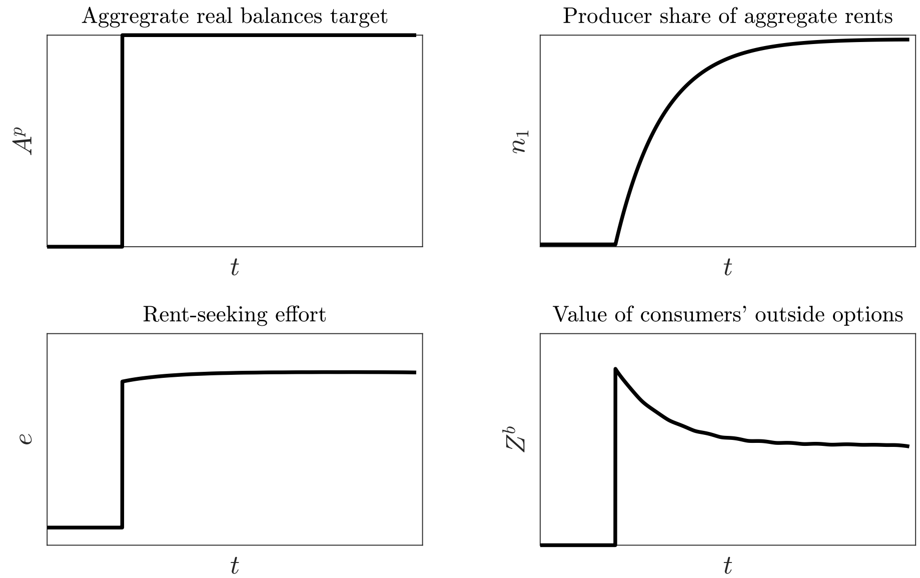

Figure 10: Numerical example under determinacy — Timepaths for the measure of perfect monopolists, n_1, rent-seeking effort, e, and consumers' outside options, Z^b, after an increase in the target aggregate real balances, A^p, with endogenous outside options and a determinate equilibrium.

10 presents the time-paths for (e, n_1, Z^b) after an increase in A^p.[12] The parametrization is such that the equilibrium path is determinate. This experiment provides some answers to question (1). First, note that n_1 behaves qualitatively in the same fashion with and without outside options, gradually increasing to a new, higher steady state after the increase in A^p. The rent-seeking effort directly jumps up, as before, but it also keeps increasing afterwards. Outside options jump on impact then go down, however settling to a higher steady state as well. The gradual reduction in the value of outside options along the transition path explains the gradual increase in rent-seeking effort, as rent sizes go up. Overall, the presence of outside options does not impact the qualitative response of market power to monetary policy much when considering a single equilibrium path. It certainly matters quantitatively, however, through its impact on the size of rents along the transition.

To answer question (2), we now exclusively consider regimes with multiple steady states. Consider an economy in the regime presented in the middle panel of Figure 9. Further assume the economy is in the low steady state, with $n_1 = e = 0$, represented by the green marker. Assume that the policy maker contracts A^p sufficiently for the system to look like the left panel. Effort and outside options immediately jump back onto either of the two paths depending on coordination. On the green path, the economy converges back to the low steady state. On the purple path, however, the economy now converges to the high steady state, represented by the purple marker. In this case, the contractionary monetary policy leads to an increase in

[12]The parametrization is the same as the previous numerical example, except $v(e) = e^2$. At time $t = 5$, A^p increases from 0.3 to 0.5.

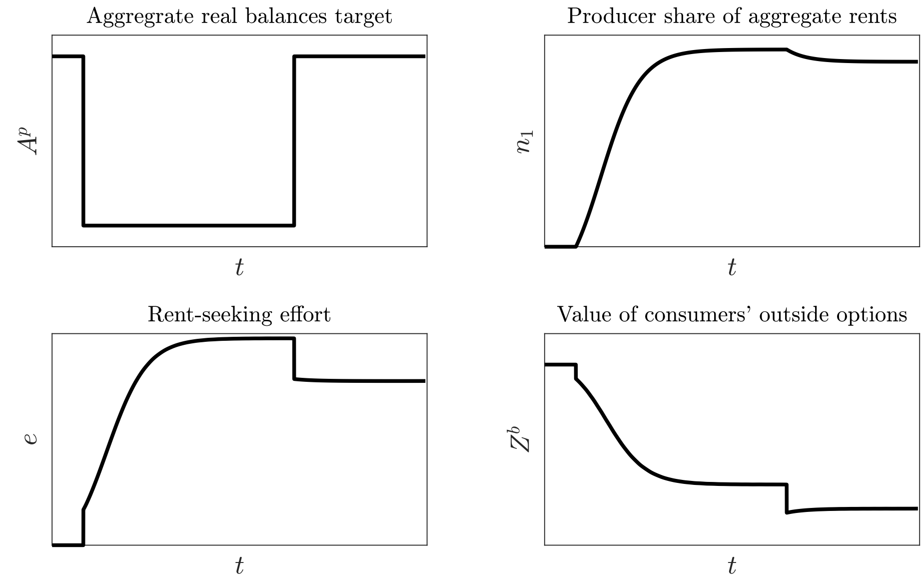

Figure 11: Numerical example with hysteresis — Timepaths for the measure of perfect monopolists, n_1 (top right), rent-seeking effort, e (bottom left), and consumers' outside options, Z^b (bottom right), following a decrease in the aggregate real balances target, A^p and its eventual reversion back to the original target (top left).

market power, as proxied through n_1, both along the transition and in the long run. Hence, in the presence of outside options, the impact of monetary policy on market power can go either way, and depends on the way agents coordinate. Intuitively, once off the steady-state, equilibrium indeterminacy implies that the economy could converge to a different steady state than the one it originated.

Lastly, the hysteresis described earlier also impacts the relation between monetary policy and market power. For example, consider the example described in the previous paragraph whereby the economy starts in the low steady state (green marker) in the middle panel of Figure 9, and the policy maker decreases A^p, shifting the system to the left panel. Assume that the economy converges towards the high steady state (purple marker), on the purple path. Now assume that the policymaker reverts the policy, shifting A^p back up to its original level, going back to the middle panel. If enough time has passed since the original decrease in A^p, reversing the policy will not enable the policymaker to revert back to the low steady state. Indeed, far enough along the purple path towards the high steady state, the equilibrium becomes determinate, and uniquely converges to the high steady state. These dynamics are visible in the numerical example provided in Figure 11. In this example, the original policy effectively "locks" the economy into the high-market-power steady state.

4.3 Dynamics of market concentration

Until now, the total measure of producers was fixed. We now endogenize it by allowing for firm entry. The goal is twofold. First, the previous section highlighted that the value of consumers' outside options plays a crucial role to explain market power dynamics. The rate at which consumers meet producers, α^b, is one of the main drivers of the value of outside options. Allowing for firm entry allows us to endogenize market tightness and matching rates. Second, we have established throughout the paper that the composition of producer types is a key determinant of measures of market power. We can expect composition dynamics to differ when the measure of producers is determined endogenously. For example, the average share of rents extracted by producers is n_1/n, where $n \equiv n_1 + n_0$ is the total measure of producers. Until now, we only considered changes in the numerator since we assumed that $n = 1$.

Assume that producers need to incur a one-time fixed cost, $k > 0$, to enter the market. They enter as perfectly-competitive producers, unable to extract rents in pairwise meetings. The flow of pairwise matches between active consumers and producers is now given by the CRS matching technology $\mathcal{M} = \mathcal{M}(n_a, n)$. Let $\theta \equiv n/n_a$ denote the market tightness. The rate at which an active consumer matches with a producer is given by $\alpha^b = \mathcal{M}(n_a, n)/n_a \equiv \alpha(\theta)$. It satisfies $\alpha(0) = 0, \alpha^{'}(0) = \infty, \alpha'(\theta) > 0$, and $\alpha''(\theta) < 0$. As market tightness increases, there are more producers per consumer, and consumers are matched at a higher rate. The matching rate of an individual producer is given by $\alpha^s = \mathcal{M}(n_a, n)/n \equiv \alpha(\theta)/\theta$. It is decreasing in θ—as market tightness increases, it takes longer for producers to be matched with a consumer.

For tractability, we revert to the environment where outside options are muted, $\lambda = \infty$, so $n_a = 1$ and $Z^b = 0$. As a result, $\theta = n$, thus market tightness is simply equal to the measure of producers.

The value function of a perfect competitor still satisfies (11). In equilibrium, new firms enter until profits from doing so are exhausted, i.e., $V_0 = k$. This condition implies that the value of being a perfect competitor is constant along the equilibrium path, pinned down by the entry cost. The capital gain of becoming a perfect monopolist is still given by (13), i.e., it is equal to the marginal cost of rent-seeking efforts. Combining these two equations, the rent-seeking effort of a producer solves

$$\rho k = -\upsilon(e) + e\upsilon'(e). \tag{26}$$

The right side, which is positive and increasing in e, pins down $e > 0$ uniquely, where e increases with k. The larger the entry cost, the more effort firms who enter then exert to gain monopoly power.

Note that the value of being a perfect monopolist is now pinned down by (13), and it is constant, $V_1 = \upsilon'(e) + k$. Next, recall that the value of a perfect monopolist also solves (12), however with $\alpha^s = \alpha(n)/n$. Making use of the previous results, we obtain the following equation,

$$(\rho + \delta)\, \upsilon' \circ e(k) = \frac{\alpha(n)}{n} \left[\chi^m S_1^m(a^*) + \chi^d S^d\right], \tag{FE}$$

which we denote the free entry (FE) condition. This condition pins down the measure of producers, n, as a function of the consumers' real balances, a^*. First note that $n(0) > 0$ if $\chi^d > 0$. In addition, the right-hand side is decreasing in n, so $\partial n/\partial a^* \geq 0$. If consumers hold more real balances, the incentives for producers to enter the market increase since they can extract more rents in monetary trades as potential perfect monopolists. We assume that fundamentals are such that $n > n_1$ at all dates, i.e., $n_0 > 0$, there are always competitive firms willing to participate in the market.

The demand for real balances solves (18), substituting $\alpha(n)(1-n_1)/n$ for $\alpha^b(1-n_1)$,

$$i = \alpha(n)\frac{1-n_1}{n}\chi^m\left[u'\left(a^*\right)-1\right]. \tag{27}$$

Again, consumers equate the expected marginal return of real balances to the marginal cost of holding them. Benefits from carrying money accrue when the consumer meets a producer, at rate $\alpha(n)$, the producer is competitive, with probability $(n-n_1)/n$, and money is necessary to pay. In such event, the marginal value of real balances is $u'\left(a^*\right)-1$.

The law of motion for the measure of perfect monopolists is

$$\dot{n}_1 = e(n-n_1) - \delta n_1. \tag{28}$$

The new flow of perfect monopolists is equal to the measure of producers with no bargaining power, $n-n_1$, multiplied by their rent-seeking effort, e. A flow δn_1 of perfect monopolists loses their ability to extract rents.

We first study steady-state equilibria. From (28), the share of perfect monopolists in the population of producers is

$$\frac{n_1}{n} = \frac{e}{\delta+e}. \tag{29}$$

The steady-state share of perfect monopolists, also equal to the share of aggregate rents extracted by producers, is independent of monetary policy. It only depends on barriers to entry through the entry cost k, which determines rent-seeking effort e. As barriers to entry increase, the rent-seeking effort of producers who have entered increases, and so does the share of aggregate rent that producers obtain. In other words, endogenizing the entry of firms makes the steady-state composition of firms independent of monetary policy.

Combining (29) with the equation that determines the optimal choice of real balances, we obtain

$$i = \alpha(n)\left(\frac{\delta}{\delta+e}\right)\chi^m\left[u'\left(a^*\right)-1\right], \tag{MD}$$

which uniquely determines a^* as a function of n alongside the n_1-isocline. We denote this equation the money demand (MD) condition. That condition implies that the consumer's real balances increase with the measure of firms. As n approaches 0, a^* tends to 0, and as n goes to $+\infty$, a^* tends to y^*. When the number of producers in the market grows, the marginal return on real balances improves because consumers encounter trading opportunities more frequently. As the proportion of perfect monopolists remains unchanged in the steady state, the likelihood of matching with a competitive producer, given a match occurs, stays constant. Consequently, the overall probability of consumers being able to match with producers where they can capture rents increases.

We define a steady-state equilibrium as the pair (A, n) that satisfies the money-demand (MD) and free-entry (FE) conditions with $a^* = A$. We denote $\underline{n}$ the solution to (FE) when $A = 0$, and $\bar{n}$ its solution when $A = y^*$. We now prove the existence and uniqueness of the steady-state equilibrium.

Proposition 5 ***(Steady-state equilibria with firm entry and muted outside options.)***
(i) If $\chi^d > 0$, so that $\underline{n} > 0$, then there exists at least one steady state.
(ii) Consider the steady state with the highest (A, n). Following a decrease (an increase) in the nominal interest rate, i, A and n both increase (decrease).

We cannot guarantee the uniqueness of a steady state as both the (MD) and (FE) curves are upward sloping. In the general case when there are multiple steady-states, we focus on the highest, which features the highest values for n and A. We illustrate the determination of that steady-state equilibrium in the left panel of Figure 12.

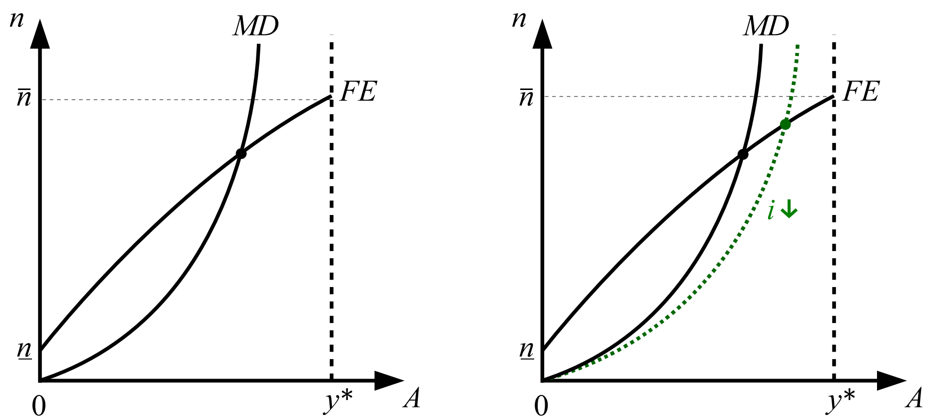

Figure 12: Equilibria with free entry of producers and muted outside options. Left panel — Steady state determination. Right panel — Steady-state impact of a decrease in the nominal interest rate, i.

The right panel illustrates the steady-state impact of a one-time, unanticipated, permanent decrease in the nominal interest rate, A. The money demand curve to shift downwards. Indeed, as the nominal interest rate decreases, the opportunity cost of holding real balances reduces. In response, consumers carry more real balances. This subsequently encourages producer entry, as the benefits of acting as a perfect monopolist in monetary transactions are magnified. The fact that i boosts producer entry implies that there are more perfect monopolists, but the ratio of perfect monopolists to competitive producers, n_1/n, remains constant.

We now turn to transitional dynamics in response to monetary policy. Again, for tractability, we assume that monetary policy targets aggregate real balances, $A = A^p$. Note that the target pins down the total number of producers, n. from the (FE) condition. Then, operationally, the policymaker adjusts i through the money growth rate so that (27) is satisfied at any given point given n_1.

Proposition 6 ***(Monetary policy and market power with firm entry under a real balances target.)*** *Let $A^p \in (0, y^*)$.*

(i) For any $n_{1,0} \in [0,1]$, there exists a unique steady-state with a unique equilibrium path, whereby the rent-seeking effort, e, and the measure of producers, n, are constant. The measure of perfect monopolists is given by

$$n_{1,t} = \frac{e}{e+\delta} n(A^P) - \left(\frac{e(A^p)}{e(A^p)+\delta} n - n_{1,0} \right) \exp\left[-(e+\delta)t\right]. \tag{30}$$

(ii) Suppose the economy starts at the unique steady state. An unanticipated an permanent increase (decrease) in the real balances target, A^p, generates an immediate increase (decrease) in total measure of producers. The share of perfect monopolists, n_1/n, also equivalent to the share of aggregate rents extracted by producers, falls (increases) on impact. Over time, the measure of perfect monopolists increases (decreases), and the share of aggregate rents extracted by producers returns gradually to its initial steady-state, $n_1/n = e/(e+\delta)$. The nominal interest rate jumps down (up) on impact then gradually diminishes (increases) to a permanently lower (higher) steady state.

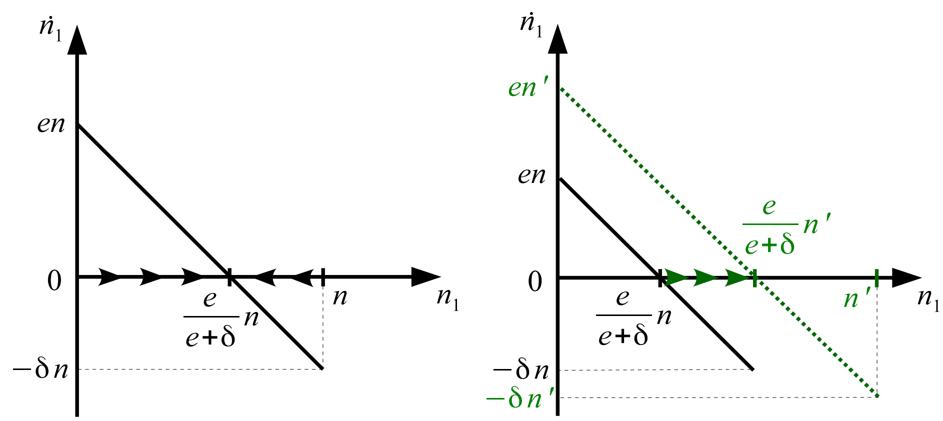

Figure 13: Left panel — Equilibrium path with entry of producers and targeted aggregate real balances. Right panel — Transition dynamic after increase in the real balances target, A^p.

The left panel of Figure 13 illustrates part (i) of the Proposition. Starting from any $n_1 \in [0, n]$, there is a unique equilibrium path along the $\dot{n}_1 = 0$ locus. It converges to the unique steady state, $n_1 = en/(e+\delta)$. Along this transition path, the rent-seeking effort and the total measure of producers is constant.

The right panel illustrates part (ii). We consider an increase in the target aggregate real balances, A^p, which occurs through an immediate then gradual decrease of the nominal interest rate. This expansionary policy induces more firm entry. As consumers increase their real balances, more producers are encouraged to participate in the market, as this allows them to secure higher gains in monetary exchanges via leveraging their position as potential perfect monopolists. Initially, the entry of new firms dilutes the market power of existing firms, i.e., n_1/n falls. However, the rise in firm entry also increases the steady-state measure of perfect monopolists. Over time, the number of perfect monopolists increases and the share of perfect monopolists returns to its original steady-state level. The corresponding time-paths are plotted in Figure 14.

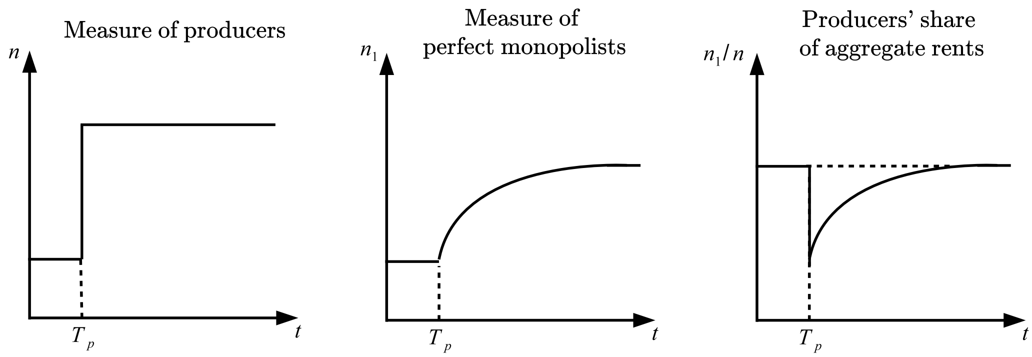

Figure 14: Timepaths after an increase in the target aggregate real balances, A^p, at time T_p., with firm entry.

We highlight a few implications regarding the impact of firm entry on the relation between monetary policy and market power. When focusing on the high equilibrium, an expansionary monetary policy dilutes market power, n_1/n, on impact, as new firms immediately enter. Hence, on impact, there is a negative relation between an expansionary policy and market power, unlike the results obtained in the regimes with determinate equilibrium paths in Sections 4.1 and 4.2. Second, while we do observe a negative relation between the aggregate market power and the nominal interest rate along the transition, like in previous sections, this correlation only occurs on the path back towards the original steady-state level of market power. Overall, allowing for producer entry can (1) reverse the effect of monetary policy in the short run, and (2) cancel out its effect in the long run.

4.4 Alternative measures of market power

So far, we have used the measure of perfect monopolists, n_1, as our indicator of market power. However, our framework also allows us to study alternative measures of market power, like aggregate markups. Because we have an endogenous distribution of producer types, it is also interesting to study concentration indices such as the Herfindahl–Hirschman Index (HHI), where composition effects matter. These alternative measures are not only empirically observable (unlike n_1), but may also exhibit distinct responses to monetary policy shocks, offering further insights. In this section, we examine the impact of monetary policy on the dynamics of aggregate markups and HHI, comparing these findings to its effect on n_1. For simplicity, throughout this section, we assume that monetary policy targets real aggregate balances, so A is constant unless mentioned.

Markups are defined as the percentage difference between the price of a good and its marginal cost of production. In pairwise meetings, the average price of a unit of output is defined as p/y and the marginal cost of production is 1. Hence, the markup in pairwise meetings is given by $(p/y) - 1$. Since competitive producers price good y at its cost, they do not earn any markups. In contrast, perfect monopolists achieve positive markups, the size of which depends on whether credit is available. In credit matches, the markup of a perfect monopolist producer is $MKUP^d = S^d/y^*$. In monetary matches, the markup of a perfect monopolist producer is $MKUP^m = S_1^m/y_1^m$. In both cases, the markup is proportional to the total surplus, or rents. In monetary transactions, the markup also depends on consumer' liquidity, a through y_1^m. We define the aggregate markup as the weighted average of markups in all types of pairwise meetings, $MKUP = (n_1/n)\left(\chi^m MKUP^m + \chi^d MKUP^d\right)$.

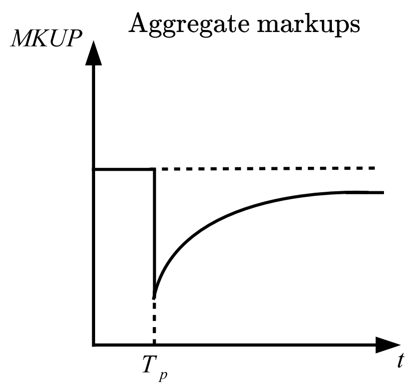

Figure 15: Impact of a one-time increase in the target aggregate real balances, A^p, at time T_p, on aggregate markups.

Figure 15 illustrates how a permanent increase in targeted aggregate real balances, A^p can affect aggregate markups. With endogenous firm entry, the share of perfect monopolists remains constant at the steady state, making aggregate markups solely dependent on A^p. When the level of real balances held by agents rises, more output is negotiated in a monetary match. This has two opposing effects on aggregate markups: on one hand, it increases the price charged by producers, leading to higher markups whereas on the other hand, it raises the aggregate cost of production, reducing markups. In example displayed here, the second effect dominates. As a result, aggregate markups decrease at the steady-state. Next, we look at the dynamics of aggregate markups. On impact, an increase in both A^p and subsequently $n(A^p)$ causes aggregate markups to drop down. The decline in markups due to higher firm entry stems from the reduced share of perfect monopolist producers. Over time, as the number of perfect monopolist producers grows, aggregate markups gradually increase during the transition, eventually stabilizing at a lower steady-state value. Thus, unlike the measure of perfect monopolists, aggregate markups drop immediately after the shock and settle at a lower steady-state level. This happens because aggregate markups are influenced not only by the number of rent-extracting perfect monopolists but also by the liquidity constraints faced by the consumers in monetary economies.

Regarding market concentration, we first define our model's counterpart to the Herfindahl–Hirschman Index (HHI). This index reflects the relative size of firms within an industry and serves as an indicator of the level of competition among them. It is calculated by summing the squares of the market shares of all competing firms. The market share of a producer is equal to the ratio of the producer's sales to aggregate sales,

$$s_j \equiv \frac{\alpha^s \bar{p}_j}{n_0 \alpha^s \bar{p}_0 + n_1 \alpha^s \bar{p}_1} = \frac{\bar{p}_j}{n_0 \bar{p}_0 + n_1 \bar{p}_1},$$

where $j = 1$ for a perfect monopolist and $j = 0$ for a competitive producer. Also, $\bar{p}_j$ denotes the expected payment in a match, i.e., $\bar{p}_j \equiv \chi^m p_j^m + \chi^d p_j^d$. It is equal to the expected revenue of the producer, $\alpha^s \bar{p}_j$, divided by the total revenue of all the producers, $n_0 \alpha^s \bar{p}_0 + n_1 \alpha^s \bar{p}_1$, where $\bar{p}_j$ is producer $j's$ expected revenue in a match with a consumer. Using that $\bar{p}_1 > \bar{p}_0$, perfect monopolists have a higher market share than competitive producers. The HHI is then defined as,

$$HHI = \int_0^n (s_i)^2 di = n_0 \left(\frac{\bar{p}_0}{n_0 \bar{p}_0 + n_1 \bar{p}_1} \right)^2 + n_1 \left(\frac{\bar{p}_1}{n_0 \bar{p}_0 + n_1 \bar{p}_1} \right)^2. \tag{31}$$

Note that $p_0^m = p_1^m = A^p$. Both the competitive producer and the perfect monopolist producer earn the same revenue in monetary meetings; the perfect monopolist only produces less. For simplicity, let us consider the case where the total number of firms is fixed.[13] As the target for real balances rises, we have to consider two mechanisms through which the HHI can react—first, the index depends on sales, which themselves depend on aggregate real balances in the case of money meetings. Second, the index depends on the composition of the pool of producers since it is a weighted average. Starting with the first mechanism, we know that as the target for real balances goes up, consumers spend more in any monetary meeting, which pushes up the sales of both types of producers. On one hand, higher revenue, everything else equal, increases a producer's market share, while on the other, the rise in the total size of the market dilutes it. For perfect monopolists, who have a larger market share than competitive producers, the latter effect prevails, leading to a reduction in their market share. Conversely, the first effect prevails for perfect competitors, and so their market share grows. As perfect monopolists' share declines and per competitors' share grows, the market becomes more competitive and the HHI decreases given n_1. Thus, on impact, when A^p increases, HHI drops down.

The second mechanism, whereby a change in policy impacts the HHI by changing the composition of the market, is more nuanced. There are two distinct channels through which n_1 affects HHI. First, as the measure of perfect monopolists goes up, the aggregate revenue keep increasing. This has a negative impact on the market share of all producers, and relatively more so for perfect monopolists, reinforcing the direct negative impact of the increase in A on the HHI. The second channel, a pure composition effect, acts in the opposite direction. As the measure of perfect monopolists goes up, it mechanically pushes HHI up since perfect monopolists have higher market shares.

[13] When firm entry is endogenous, an increase in A^p initially causes the HHI to decrease, followed by a gradual rise, ultimately settling at a lower steady-state. This dynamics mirrors the one depicted in Figure 15 and stands in contrast to that of n_1 in Figure 14.

Overall, the effect of n_1 on HHI is ambiguous. Numerically, we find that the the dilution channel dominates when n_1 is high, while the pure composition channel dominates when n_1 is low. The two cases are illustrated in Figure 16. In the former case, represented in the left panel, an increase in the target leads to HHI jumping down on impact, then decreasing gradually (due to increase in n_1) along the transition path to ultimately reach a lower steady-state. For low enough values of n_1 in equilibrium, the composition channel dominates, and can offset the negative direct effect of A on HHI. In that case, represented in the right panel, HHI gradually increases during the transition to ultimately reach a higher steady-state.

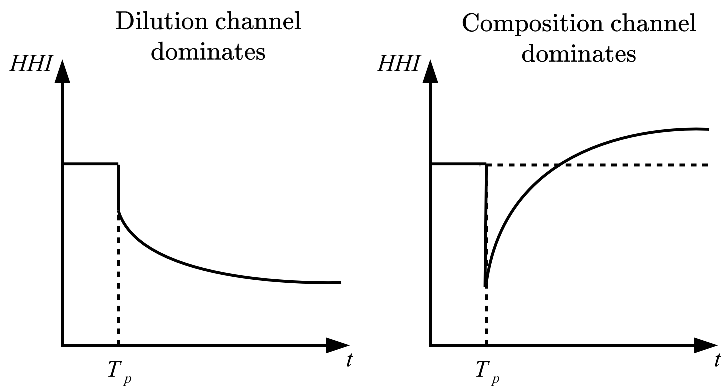

Figure 16: Impact of a one-time increase in the target aggregate real balances, A^p, at time T_p, on the HHI.

As illustrated in Figures 15 and 16, our model shows that the two commonly-used indicators of market power—markups and HHI—can respond inversely to a monetary policy shock that shifts aggregate real balances, both at the steady-state and along the transition. An increase in the targeted real balances encourages the entry of perfect monopolists, thereby raising aggregate markups in transition. However, it can also lead to a redistribution of market share between perfect monopolists and perfect competitors, reducing the dominance of the larger firms (i.e., the monopolists) and lowering the HHI. Since perfect competitors do not earn markups, for a given A, aggregate markups solely reflect the impact of perfect monopolist entry. In contrast, the HHI accounts for the sales of all types of producers and their distribution, thereby incorporating the composition effect as well.

5 Conclusion

In this paper, we investigated the dynamic relationship between monetary policy and market power in a decentralized monetary economy modeled in the spirit of the New Monetarist tradition. Building on CR24, market power is micro-founded through the existence of liquidity constraints, consumers' outside options, and rent-seeking effort by producers. Our key innovation is to model the ability to extract rents as a gradual process, making the transition dynamics of market power non-trivial. This enables us to highlight how monetary policy, implemented through changes in the nominal interest rate, can influence market structure over time.

A first key result is that in a simple model without outside options, reductions in the nominal interest rate stimulate rent seeking, and lead to an increase in producer market power in a deterministic fashion. Hence our model provides another rationale for the well-documented rise in market powers that has accompanied the decline in nominal interest rates between the 1980s and 2020.

A second key result is that the introduction of outside options for consumers, which can generate multiple equilibria, makes the dynamics of market power much more intricate. Indeterminacy and path-dependence can arise, implying that the impact of the same policy may differ in a given economy depending on the initial condition, or differ across two identical economies based on coordination.

Our findings underscore an unintended impact of monetary policy: while it targets inflation or real liquidity, it also affects the competitive structure of markets. An expansionary policy, while increasing liquidity, may inadvertently consolidate market power. This adds a new dimension to the welfare cost of inflation. By adjusting the inflation rate, the policy maker also impacts who appropriates rents in the economy, and how much wasteful rent-seeking effort is exerted by producers.

While we mostly focused on monetary policy in this study, our model could be adapted to study the dynamic impact of a wide range of alternative shocks on market power. The model could also be expanded to make the distribution of producer monopoly power richer instead of simply considering perfect monopolists and perfect competitors. We could also leverage the model to study the relation between monetary policy and market power from the opposite perspective, asking how market power impacts the transmission of monetary policy. These considerations open promising avenues for future research to deepen our understanding of the interplay between monetary policy and market power dynamics.

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Proofs of Propositions

Proof of Lemma 1. The objective function, given by the bracketed term in (6), admits a kink[14]. It is linear with slope $-i$ for all $a \in [0, \underline{y}]$ and it is concave with slope -$i + \alpha^b \chi^m n_0 (u'[y_0^m(a)] - 1)$ for all $a > \underline{y}$. It takes the value 0 at $a = 0$.

A positive solution of $a^* (= \tilde{a})$ exists provided that the objective function is positive. This condition can be expressed as

$$-i\tilde{a} + \alpha^b \chi^m n_0 (u(y_0^m) - y_0^m - Z^b) \geq 0.$$

From bargaining, $y_0^m = \tilde{a}$. We can rewrite the inequality as:

$$-i\tilde{a} + \alpha^b \chi^m n_0 [u(\tilde{a}) - \tilde{a}] \geq \alpha^b \chi^m n_0 Z^b$$

which implies

$$Z^b \leq \frac{-i\tilde{a} + \alpha^b \chi^m n_0 [u(\tilde{a}) - \tilde{a}]}{\alpha^b \chi^m n_0}.$$

■

Proofs of Proposition 1 and Proposition 2. A steady-state equilibrium (e, n_1), can be reduced to the following:

$$(\rho + \delta + e)\, \upsilon'(e) = \alpha^s \chi^m S_1^m \left[A(i, n_1)\right] + \alpha^s \chi^d S^d + \upsilon(e) \tag{32}$$

$$n_1 = \frac{e}{e + \delta}, \tag{33}$$

where the surplus of a credit match is $S^d = u(y^*) - y^*$ and the surplus of a monetary match is $S_1^m(A) = u(y_1^m) - y_1^m$ where $y_1^m(A) = \min\left\{u^{-1}(A), y^*\right\}$. From (33),

$$\frac{\partial n_1}{\partial e} = \frac{\delta}{(e + \delta)^2} > 0.$$

At $n_1 = 0, e = 0$ and at $n_1 = 1, e = \infty$. Thus, (33) is an upward sloping line starting form $e = 0$ to $e = \infty$. From (32), if $y_1^m = y^*$, then (32) is independent of n_1 and solves for a unique $e > 0$. Otherwise, if $y_1^m = u^{-1}(A)$,

$$\frac{\partial e}{\partial n_1} = \frac{\alpha^s \chi^m}{(\rho + \delta + e)\upsilon''(e)} \frac{[u'(A) - 1]}{(1 - n_1) u''(A)} \frac{[u'(u^{-1}(A)) - 1]}{u'(u^{-1}(A))} < 0$$

iff $\chi^m > 0$ and $\upsilon''(e) > 0$. This is because $y_1^m = u^{-1}(A) \implies u^{-1}(A) < y^*$, so $u'(u^{-1}(A)) > 1$. Similarly, $y_m^0 = A \implies A < y^*$, so $u'(A) > 1$. We assume utility function to be strictly concave, i.e., $u''(A) < 0$. Moreover, at $n_1 = 0$, we have $A > 0$ which implies $e > 0$ from (32). Thus, (32) is a downward sloping line

[14] See Figure 12 in Choi & Rocheteau (2024) for a graphical representation.

starting from a positive value of e. Coupled with an upward sloping (33) starting form origin, this yields a positive and unique steady-state monetary equilibrium, (e^*, n_1^*).

An equilibrium path can be reduced to a solution to the following system of ODEs:

$$(\rho+\delta+e)\,\upsilon'(e) = \alpha^s \chi^m S_1^m\left[A(i,n_1)\right] + \alpha^s \chi^d S^d + \upsilon(e) + \upsilon''(e)\dot{e}, \tag{34}$$

$$\dot{n}_1 = e(1-n_1) - \delta n_1. \tag{35}$$

We proved that there is a unique solution to $\dot{e} = \dot{n}_1 = 0$. Linearizing in the neighborhood of the steady state, the Jacobian matrix is:

$$J = \begin{pmatrix} \rho+\delta+e & -\alpha^m S'(A)\frac{\partial A}{\partial n_1} \\ 1-n_1 & -(e+\delta) \end{pmatrix}.$$

It can be checked that $\det J < 0$ (using that $\partial A/\partial n_1 \leq 0$). Hence, the eigenvalues are of opposite sign and the unique steady state is a saddle point. For a given $n_{1,0}$, the unique equilibrium is given by the saddle path leading to the steady state. As shown in the phase diagram in Figure 5, the saddle path is downward sloping. If i decreases, $A(i, n_1)$ increases for given n_1 and the e-isocline shifts upward. Hence, the new saddle path is located above the initial one. It means that e_t is larger than its initial steady-state value for all t. As a result, $n_{1,t}$ increases toward its new, higher steady-state value. ∎

Proof of Proposition 3.

Part 1. An equilibrium path is a solution to the following system of ODEs:

$$(\rho+\delta+e)\,\upsilon'(e) = \alpha^s \chi^m S_1^m(A^p) + \alpha^s \chi^d S^d + \upsilon(e) + \upsilon''(e)\dot{e}, \tag{36}$$

$$\dot{n}_1 = e(1-n_1) - \delta n_1. \tag{37}$$

With a real balance target, A^p, (36) now becomes independent of n_1. At steady-state, $\dot{e} = 0$ and (36) solves for a unique $e(A^p)$. The e-isocline is given by $e = e(A^p)$ with $\partial \dot{e}/\partial e > 0$. This gives a unique equilibrium path for $e : e_t = e(A^p)$. Given e_t, (37) is a first-order linear differential equation with solution

$$n_{1,t} = n_1^* + [n_1^* - n_{1,0}] \exp\left(-[e(A^p)+\delta]t\right)$$

where $n_1^* = e(A^p)/[e(A^p)+\delta]$.

Part 2. There is a unique solution to $\dot{e} = \dot{n}_1 = 0$ given by $e(A^p)$ and n_1^*. Linearizing in the neighborhood of the steady state, the Jacobian matrix is:

$$J = \begin{pmatrix} \rho+\delta+e & 0 \\ 1-n_1 & -(e+\delta) \end{pmatrix}.$$

It is easy to see that $\det J < 0$. Hence, unique steady state is a saddle point. For a given $n_{1,0}$, the unique equilibrium is given by the saddle path leading to the steady state. As shown in the phase diagram in Figure 6, the saddle path is the e-isocline. If A^P increases, the e-isocline shifts upwards as $e'(A^P) > 0$. Hence, the new saddle path is located above the initial one. It means that e_t is larger than its initial steady-state value for all t. As a result, $n_{1,t}$ increases toward its new, higher steady-state value. The nominal interest rate is set as per (20). When A^p increases, i reduces on impact as $\partial i/\partial A^p < 0$. During transition, n_1 gradually increases which further reduces i as $\partial i/\partial n_1 < 0$. ■

Proof of Proposition 4.

Part 1. From (25), the threshold for $\tilde{Z}^b$ above which $e = 0$ is

$$\tilde{Z}^b = u\left(A - \frac{(\rho+\delta)\,\bar{\upsilon}}{\bar{\alpha}n_a}\right) - A.$$

Provided $A > (\rho+\delta)\,\bar{\upsilon}/(\bar{\alpha}n_a)$, the right side is first increasing in A and then decreasing. If $A = (\rho+\delta)\,\bar{\upsilon}/(\bar{\alpha}n_a)$ then $\tilde{Z}^b < 0$. Hence, if

$$\max_{A \geq \frac{(\rho+\delta)\bar{\upsilon}}{\bar{\alpha}n_a}} \left\{u\left(A - \frac{(\rho+\delta)\,\bar{\upsilon}}{\bar{\alpha}n_a}\right) - A\right\} > 0, \tag{38}$$

then there exists a $\underline{A} > (\rho+\delta)\,\bar{\upsilon}/(\bar{\alpha}n_a)$ such that for all $A < \underline{A}$, $\tilde{Z}^b < 0$. If (38) does not hold, then $\tilde{Z}^b < 0$ for all $A > (\rho+\delta)\,\bar{\upsilon}/(\bar{\alpha}n_a)$, i.e., $\underline{A} = +\infty$. In both cases, if $\tilde{Z}^b < 0$, then any steady-state equilibrium is such that $e = 0$.

Part 2. If $n_1 = 0$ the stationary solution to (23) is

$$\bar{Z}^b = \frac{\bar{\alpha}}{\rho+\gamma+\lambda+\bar{\alpha}}\left[u(A) - u'(A)A\right].$$

There exists $\bar{\upsilon}_0 > 0$ such that $\bar{Z}^b = \tilde{Z}^b$ when $y = y^*$, i.e.,

$$u\left(y^* - \frac{(\rho+\delta)\,\bar{\upsilon}_0}{\bar{\alpha}n_a}\right) - y^* = \frac{\bar{\alpha}}{\rho+\gamma+\lambda+\bar{\alpha}}\left[u(y^*) - y^*\right].$$

For all $\bar{\upsilon} < \bar{\upsilon}_0$, $\bar{Z}^b < \tilde{Z}^b$ if $A = y^*$ and $\bar{Z}^b > \tilde{Z}^b$ if $A = \underline{A}$. Graphically, if $A = y^*$ then the intercept with the vertical axis of the curve representing $\dot{n}_1 = 0$ is larger than the intercept of $\dot{Z}^b = 0$. This case corresponds to the right panel of Figure 8. Hence, there exists $\bar{A} > \underline{A}$ such that for all $A > \bar{A}$, $\bar{Z}^b < \tilde{Z}^b$ and the unique steady state is such that $n_1 = 1$. By continuity, there are intermediate values for A such that there are three steady-state equilibria as illustrated in the middle panel of Figure 8. ■

Proof of Proposition 5.

Part 1. A steady-state equilibrium is the pair (A, n) that satisfies the following money-demand (MD) and free-entry (FE) conditions:

$$i = \alpha(n)\left(\frac{\delta}{\delta+e}\right)\chi^m\left[u'(A) - 1\right] \tag{MD}$$

$$(\rho+\delta)\,\upsilon'(e(k)) = \frac{\alpha(n)}{n}\left\{\chi^m\left[A - u^{-1}(A)\right] + \chi^d\left[u(y^*) - y^*\right]\right\}. \tag{FE}$$

From (MD),

$$\frac{\partial n}{\partial A} = \frac{-\alpha(n)u''(A)}{\alpha'(n)[u'(A)-1]} > 0$$

since we assume $u''(y) < 0, \alpha'(n) > 0$ and $y_0^m = A \implies A < y^*$, so $u'(A) > 1$. At $A = 0, n = 0$ and at $A = y^*, n = \infty$. This gives us the (MD) curve as an upward sloping curve starting from the origin to $n = \infty$ (as A goes to y^*).

From (FE),

$$\frac{\partial A}{\partial n} = \frac{-\alpha^{s\prime}(n)\{\chi^m\left[A - u^{-1}(A)\right] + \chi^d\left[u\left(y^*\right) - y^*\right]\}u'(u^{-1}(A))}{\alpha^s \chi^m[u'(u^{-1}(A)) - 1]} > 0$$

since we assume $\alpha^{s\prime}(n) < 0$ and following the same reasoning as in the proof of Proposition 1, $u'(u^{-1}(A)) > 1$. Moreover, if $\chi^d > 0$, then $A = 0$ yields a strictly positive $\underline{n}$; $A = y^*$ yields a finite $\overline{n}$. This gives us the (FE) curve as an upward sloping curve starting from $\underline{n}$ and increasing till $\overline{n}$.

Thus, the (MD) and (FE) curves intersect at least once to yield a positive steady-state equilibrium, (A^*, n^*).

Part 2. Now consider the highest steady state (A, n). A decrease in i does not affect the (FE) curve. However, it shifts the MD curve rightwards, as A must increase at the same level of n to satisfy (MD). This leads to an increase in both A and n at the new steady-state. ■

Proof of Proposition 6.

Part 1. From (MD), $A^p \in (0, y^*)$ for $i > 0$. With a real balance target, A^p, (FE) now becomes independent of A and uniquely pins down $n(A^p)$ with $n'(A^p) > 0$. e is uniquely pinned down by the entry cost, k, using (26). The equilibrium path for n_1 follows (28). Given e and $n(A^p)$, (28) is a first-order linear differential equation with solution

$$n_{1,t} = n_1^* + [n_1^* - n_{1,0}] \exp\left(-[e+\delta]t\right)$$

where $n_1^* = en(A^p)/[e+\delta]$.

Part 2. Suppose A^p increases. From (FE), $n(A^p)$ increases as $n'(A^p) > 0$. The share of perfect monopolist, n_1/n, drops down on impact as n increases. From (28), $\partial \dot{n}_1/\partial n_1 < 0$. When $n(A^p)$ increases, n_1^* increases, which shifts the n_1 isocline rightwards as shown in Figure 13. Since $\partial \dot{n}_1/\partial n_1 < 0$, n_1 gradually increases to reach the new steady-state. This slowly increases the share of perfect monopolists, till it reaches its original level, $n_1/n = e/(e+\delta)$ at the new steady-state. When A^p increases (and n increases), i reduces on impact to satisfy (27) as $\partial i/\partial A^p < 0$ and $\partial i/\partial n < 0$. During transition, n_1 gradually increases which further reduces i as $\partial i/\partial n_1 < 0$. ■