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**Can Interest Rate Factors Explain Exchange Rate Fluctuations?\***

Julieta Yung  
Federal Reserve Bank of Dallas

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**Abstract**

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This paper explores whether interest rate factors, derived from the yield curve, can explain exchange rate fluctuations at different horizons. Using a dynamic term structure model under no-arbitrage, exchange rates are modeled as the ratio of two countries' stochastic discount factors. Key to this framework is that factors are observable, which allows the model to be estimated by Maximum Likelihood. Results show that interest rate factors can explain half of the variation in one-year exchange rates and up to ninety percent of five-year movements, for free-floating currencies from 1999 to 2014. These findings suggest that yield curves contain important information for modeling exchange rate dynamics, particularly at longer horizons.

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\* Julieta Yung, Federal Reserve Bank of Dallas, Research Department, 2200 N. Pearl Street, Dallas, TX 75201. 214 922-5443. [julieta.yung@dal.frb.org](mailto:julieta.yung@dal.frb.org). I am grateful to Tom Cosimano, Jun Ma, Mark Wohar, and many others for helpful comments and suggestions. This paper also benefited from comments made by the participants at the Macroeconomics Seminar and the Mathematical Finance Research Group at the University of Notre Dame. The views in this paper are those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

# 1 Introduction

The motivation for this paper arises from the idea that a few common factors drive most of the movement in asset prices. Investors deciding how to allocate their wealth consider portfolios with different risk profiles, assets with various time horizons, and investment opportunities in different countries. We can therefore think of a unique pricing mechanism that simultaneously determines prices of assets, such as bonds and exchange rates, given a limited set of information available at any point in time. In this paper, I explore whether interest rate factors extracted from the yield curve can explain exchange rate fluctuations across a number of different horizons.

Exchange rates are modeled as the ratio of two countries' stochastic discount factors, derived from a dynamic term structure model with observable interest rate factors. In this context, exchange rate movements are driven by three components: (i) the interest rate differential, which is linear in the factors, (ii) a nonlinear risk premium that varies throughout time, and (iii) the difference in shocks to the prices of risk. The interest rate factors driving exchange rate movements, the level, slope, and curvature, are principal components from the yield curve that capture over 99% of cross-sectional variation within a country. This paper contributes to the international and macro-finance literature by modeling movements in the exchange rate from one month to five years, through a dynamic no-arbitrage term structure model, using only factors from the yield curve.

The results suggest that yield curves contain important information for modeling exchange rate dynamics during the 1999-2014 period, particularly at longer horizons. For example, three observable interest rate factors from each country can explain about half of the variation in one-year exchange rates for countries with free-floating currencies relative to the U.S. dollar: Australia (57%), Canada (45%), Japan (61%), Norway (49%), Sweden (70%), Switzerland (42%), and the U.K. (61%). As the horizon increases, between 55% (Swiss franc) and 92% (British pound) of the five-year exchange rate fluctuations are accounted for by these observable factors alone. Results are robust to correcting for small-sample bias with different simulation-based bootstrapping methods and adjusting for potential heteroskedasticity in the error term, suggesting that interest rate factors can better explain fluctuations in exchange rates at longer terms. These findings are in line with work by Mark (1995), who finds that exchange rate fluctuations become more predictable at longer horizons. Key to my paper is that factors are observable, which allows the no-arbitrage term structure model to be estimated by Maximum Likelihood, as in Joslin, Singleton, and Zhu (2011).

The main driver of exchange rate fluctuations in this model is a time-varying risk premium, nonlinear in the factors, derived from the countries' stochastic discount factors. This component is consistent with the Fama (1984) conditions: it is negatively correlated with the interest rate differential and exhibits a significantly higher volatility. I therefore study the model-implied risk premium in the context of the uncovered

interest parity condition, which links expected exchange rate movements to the interest rate differential between two countries with a coefficient of one. The puzzle in this literature arises when the coefficient is not only different from one, but also negative. I project log exchange rate changes in the data onto the interest rate differential and the risk premium components implied by the model. After including the risk premium, the coefficient of the interest rate differential becomes consistently positive (except for Sweden at the one- and three-month horizons), approaches one, and is statistically significant at longer horizons. The coefficient of the risk premium is always positive, approaches one, and is statistically significant for all currencies at horizons greater than one month. This suggests that the risk premium implied by the model is not only consistent with the theory, but also important in explaining exchange rate dynamics.

The ability of interest rate factors to explain cross-sectional variation in the yield curve decreases with the number of factors. The yield curve's third factor, or curvature, can explain less than 1% of interest rate movements for most countries. Interestingly, the model finds that the curvature factor significantly contributes to explaining exchange rate fluctuations despite its low explanatory power for the cross-section of interest rates. For example, a model with two factors (level and slope) can only account for 28% of variation in the one-year Japanese yen, compared to 61% with a three-factor model, and this is true for most currencies studied. One exception is the Canadian dollar, since one-year fluctuations are well captured by two factors alone, and incorporating a curvature factor only improves the fit of the model from 42% to 45%.

The findings in this paper are not driven by the U.S. dollar or any one country in particular. I explore exchange rates in terms of the British pound, instead of the U.S. dollar, which implies U.K. interest rate factors are extracted from the yield curve. In general, results are very similar regardless of the chosen base, and the fit of the model improves at longer horizons, consistent with the baseline results. For some countries, for example Canada, modeling one-year exchange rate fluctuations with respect to the British pound significantly improves the model's fit from 45% to 64%. This suggests that the U.S. term structure does not contain more information on exchange rate movements than the U.K. Regardless of the country base, exchange rate fluctuations are well captured in the long run by the same factors that explain cross-sectional interest rate variation.

My model also allows one to explore a variety of questions; for example, which countries under the euro can better explain exchange rate movements? I extract interest rate factors from countries under the euro regime: Austria, Belgium, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal, and Spain. In the baseline framework, each country's interest rate factors capture the dynamics of the exchange rate at different horizons, relative to the U.S. dollar. In the case of the euro, however, individual countries exhibit very different yield curves, and it is unclear which country's term structure contains more information on the path of the euro. The model yields evidence that different European countries have roughly the same

ability to explain one-year euro-dollar fluctuations, from a stable country such as Germany (46%) to a much more volatile one like Italy (58%). Moreover, at the five-year horizon, every country can explain more than 83% of exchange rate movements. This suggests that expectations on euro fluctuations are well captured by the cross-sectional movement of the yield curves in all European countries.

The literature has mixed evidence on whether the information contained in both the domestic and foreign yield curves alone is useful in accounting for exchange rate movements. For example, Inci and Lu (2004) extend a quadratic term structure model in which unobservable interest rate factors feed into the stochastic discount factor of different countries. They provide evidence that interest rates factors alone may be insufficient to determine exchange rate movements for the U.S.-U.K. and U.S.-Germany country pairs. Li and Yin (2014) reach similar conclusions in a no-arbitrage macro-finance framework, in which unobservable yield curve factors alone can only explain 13% of the one-month euro-U.S. dollar exchange rate. They find that incorporating macro variables helps explain exchange rate movements, although 76% of the variation they find is driven by innovations to the factors.

Diez de Los Rios (2009) develops a two-country term structure model in continuous time, where the exchange rate is determined by each country's risk-free rate. This framework is able to provide statistically better forecasts than those produced by a random walk for the British pound and the Canadian dollar, although the results are negative for the German mark/euro and the Swiss franc, which are explained by a rejection of the restrictions imposed by the term structure model. Clarida and Taylor (1997) estimate a vector-error-correction model for the term structure of the forward premium (interest rate differentials) and conclude that interest rates from two countries contain useful information for predicting exchange rates. More recently, work by Ang and Chen (2010) finds that changes in the interest rates and slope of the yield curve are significant predictors of foreign exchange returns.<sup>1</sup> Although close to my work, these studies only focus on very short-term exchange rate fluctuations.

My work also builds upon those who have studied the model-implied risk premium in the context of the uncovered interest parity puzzle, such as Frachot (1996), Backus, Foresi, and Telmer (2001), Dewachter and Maes (2001), Ahn (2004), and Alexius and Sellin (2012), among many others. I also explore the euro dynamics as in Egorov, Li, and Ng (2011), although I pose a very different question: Which European countries are able to better explain changes in the euro by only using their interest rate factors?

This paper contributes to this rapidly growing literature that studies the joint dynamics of exchange

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<sup>1</sup>These papers are not alone in making significant contributions to the literature, but the number of studies devoted to this area is too large to summarize in this introduction alone. Some recent studies on the field are Dong (2006), who develops a no-arbitrage two-country term structure model to explain exchange rate fluctuations between the U.S. and Germany; Benati (2006), who studies U.S.-U.K. exchange rate fluctuations with unobservable factors; Chabi-Yo and Yang (2007), who introduce a New Keynesian model to study the Canadian dollar with macroeconomic variables; Pericola and Taboga (2011), who use observable and unobservable variables to explore exchange rate dynamics with German data; and more recently, Bauer and Diez de Los Rios (2012), who introduce a multicountry framework with interest rate, macroeconomic, and exchange rate variables.

rates and yield curves through term structure models in the following dimensions. First, it only introduces observable interest rate factors extracted from the countries' yield curves. There are no other sources of risk (such as macro risk or exchange rate risk), other than the ones spanned by the yield curve, which allows the model to estimate all parameters by Maximum Likelihood. That is because the goal of this paper is to explore whether the cross-sectional variation of the term structure alone can help explain movements in other assets, such as the exchange rate. Second, this paper focuses on a more recent period, 1999 to 2014, which is particularly interesting given the difficulty of modeling exchange rates during different financial crises, severe changes to the term structure of interest rates, the creation of the euro, and the more recent financial integration of asset markets across the world. Third, I consider not only one country pair but eight different currencies relative to the U.S. dollar or the British pound. The model successfully explains exchange rate fluctuations for every currency using only their interest rate factors. Fourth, I model exchange rate changes at different horizons, from one month to five years, which has been largely unexplored in the context of no-arbitrage term structure models. My results are in line with work by Mark (1995) that finds short-term fluctuations are very difficult to explain but at longer horizons, there are significantly predictable components driving exchange rate fluctuations.

The remainder of the paper is organized as follows. Section 2 describes the two-country term structure framework in which exchange rates are modeled as the ratio of the countries' stochastic discount factors. The model is then estimated by Maximum Likelihood. Section 3 introduces the data and the interest rate factors used to model both interest rates and exchange rates simultaneously. Section 4 presents the results of the model as well as measures of its ability to explain exchange rate movements at different horizons, along with a series of robustness checks. I also evaluate the model in the context of the uncovered interest parity puzzle and study the model's implications for the euro with a new set of European countries. Section 5 concludes.

## 2 Term Structure Model and Estimation

I derive a dynamic term structure model in discrete time in the spirit of Joslin, Singleton, and Zhu (2011). This type of model was originally developed to explain the cross-sectional variation in yields.<sup>2</sup> I expand its purpose to not only account for the term structure of two countries simultaneously, but to also account for movements in their exchange rate at different horizons.

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<sup>2</sup>Term structure models originated with Vasicek (1977) in the finance literature, who presumed that the short-term interest rate was a linear function of a vector that followed a Gaussian diffusion (joint normal distribution). This has been the building block of a large family of term structure models, formalized by Duffie and Kan (1996), characterized by Dai and Singleton (2000, 2003), and expanded by many others. Comprehensive surveys on the literature can be found in Diebold, Piazzesi, and Rudebusch (2005), Piazzesi and Schneider (2007), Piazzesi (2010), and Gürkaynak and Wright (2012).

The key assumption is that there are no arbitrage opportunities in the bond markets. This assumption implies that investors receive the same risk-adjusted compensation for bonds of different maturities. No-arbitrage is implemented in the form of cross-equation restrictions that make the yield curve consistent at every point in time. Hence, the yields are modeled as linear functions of the factors, in which the coefficients are restricted so that no-arbitrage holds, as in Duffie and Kan (1996). Although in reality this assumption may not hold at every point in time, given the liquidity of well-developed bond markets, violations to the arbitrage-free assumption are usually short lived, since bank holding companies and other large investors are able to immediately take advantage of those opportunities, acting as arbitrageurs in the economy.

## 2.1 General Framework

I extend the term structure framework to allow for a domestic and a foreign country, where  $i = \{d, f\}$ . Let  $\mathcal{P}_t^i$  be a set of  $N$  observable factors for each country. These factors are constructed as  $N$  linear combinations of zero coupon yields,  $y_t^i$ , such that

$$\mathcal{P}_t^i = W^i \cdot y_t^i, \quad (1)$$

where  $W$  is an  $(N \times J)$  matrix containing the weights that factors have on the  $J$  maturity yields. The model assumes that  $\mathcal{P}_t$  and  $y_t$  are priced perfectly, but the observed yields,  $y_t^{obs}$ , are measured with error; such that  $e_t^y = y_t^{obs} - y_t$ . The errors are assumed to be conditionally independent of their lagged values and satisfy consistency conditions as specified in Joslin, Singleton, and Zhu (2011). Each country is affected by their own set of observable interest rate factors,  $\mathcal{P}_t^d$  and  $\mathcal{P}_t^f$ . Factors are extracted by principal component analysis, as it is standard in the literature, for which they are orthogonal linear combinations of the country yields.<sup>3</sup>

Models based on a reduced set of factors have proven to be a good representation of interest rate movements. Hence the assumption that a small number of factors drive the entire cross-section of interest rates is not only parsimonious but also effective. Typically, three factors alone are enough to explain almost the entire cross-sectional variation of interest rates, with the first -and most persistent- factor having the largest explanatory power over interest rate movements. In this context these same factors will also drive exchange rate movements at different horizons between the countries.

As in Vasicek (1977), interest rates are linear in the factors: Each country has its own one-period annualized risk-free rate or short rate,

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<sup>3</sup>More details on what the factors are and how they are extracted from the interest rate data can be found in section 3.

$$r_t^i = \rho_{0\mathcal{P}}^i + \rho_{1\mathcal{P}}^{i'} \cdot \mathcal{P}_t^i, \quad (2)$$

and its own set of yields at different  $m$  maturities, which are linear in the factors as well,

$$y_t^{m,i} = A_{\mathcal{P}}^{m,i} + B_{\mathcal{P}}^{m,i} \cdot \mathcal{P}_t^i, \quad (3)$$

where coefficients  $A_{\mathcal{P}}$  and  $B_{\mathcal{P}}$  are restricted to satisfy internal consistency conditions, so that  $A_{\mathcal{P}}^{m,d} \cdot W^d = A_{\mathcal{P}}^{m,f} \cdot W^f = 0$  and  $B_{\mathcal{P}}^{m,d} \cdot W^d = B_{\mathcal{P}}^{m,f} \cdot W^f = I$ . When dealing with observable factors constructed from the cross-section of yields, Dai and Singleton (2003) argue that internal consistency conditions need to be satisfied in order for the model to correctly price the state variables at each point in time.

## 2.2 Bond Pricing and the Stochastic Discount Factor

The stochastic discount factor (SDF) is the pricing mechanism that investors in each country develop in order to price assets in the local currency. Let  $P_t^m$  be the price at time  $t$  of an  $m$  maturity zero coupon bond (no intermediate cashflows). Asset prices are risk-adjusted expected values of their future payoffs, so that

$$P_t^m = E_t^{\mathbb{P}} [M_{t+1} P_{t+1}^{m-1}], \quad (4)$$

where  $M_{t+1}$  is the SDF that accounts for the adjustment of risk. The investors demand this risk premium to compensate for unaccounted fluctuations or uncertainty in the economy. The SDF is also called *pricing kernel* in the finance literature and can be specifically linked to preferences through the intertemporal rate of substitution. SDFs are also related to the beta in conditional versions of the classical Capital Asset Pricing Model (CAPM) and its multifactor extensions.<sup>4</sup> The basic intuition of the CAPM still holds true: the factors that determine asset prices also determine the risk premium, which compensates investors for weathering losses during bad times. However, the main goal of CAPM is to express expected excess returns in terms of a security's beta with a benchmark portfolio without taking a stand on the characteristics of the pricing relationship. Term structure models parametrize the SDF directly as a function of the state vector implied by the market prices of risk, relating the risk premium to bonds across the yield curve without relying on assumptions about preferences and technology. Let  $\mathbb{P}$  represent the *physical* or *actual* process, such that  $E_t^{\mathbb{P}}$  represents realized expectations of a *historical* process.

Let  $\mathbb{Q}$  represent the *risk-neutral* process, such that asset prices are expected values of their future payoffs discounted at the risk-free rate:

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<sup>4</sup>A detailed discussion on the discount factor can be found in Singleton (2006).

$$P_t^m = e^{-r_t} E_t^{\mathbb{Q}} [P_{t+1}^{m-1}]. \quad (5)$$

Equations (4) and (5) are equivalent representations of the asset pricing mechanism. However, under risk-neutrality, the prices have already been adjusted for risk and there is no need for a pricing kernel. The existence of a risk-neutral distribution is guaranteed by the absence of arbitrage. Moreover, if  $\mathbb{Q}$  is unique then bond markets are complete.

The evolution of each country's factors is expressed under the two equivalent probability measures  $\mathbb{P}$  and  $\mathbb{Q}$ :

$$\Delta \mathcal{P}_t^i = K_{0\mathcal{P}}^{\mathbb{P},i} + K_{1\mathcal{P}}^{\mathbb{P},i} \cdot \mathcal{P}_{t-1}^i + \Sigma_{\mathcal{P}}^i \epsilon_t^{\mathbb{P},i}, \quad (6)$$

$$\Delta \mathcal{P}_t^i = K_{0\mathcal{P}}^{\mathbb{Q},i} + K_{1\mathcal{P}}^{\mathbb{Q},i} \cdot \mathcal{P}_{t-1}^i + \Sigma_{\mathcal{P}}^i \epsilon_t^{\mathbb{Q},i}. \quad (7)$$

Under  $\mathbb{P}$  the factors move by an unrestricted vector auto-regressive (VAR) process that describes the evolution of the yields. Under  $\mathbb{Q}$ , its equivalent Martingale measure, the process has already been adjusted for risk, allowing for a risk-neutral representation of the movement of factors over time.  $(K_{0\mathcal{P}}^{\mathbb{P}}, K_{0\mathcal{P}}^{\mathbb{Q}})$  are  $(N \times 1)$  constant vectors that can be interpreted as the long run average of the factors,  $(K_{1\mathcal{P}}^{\mathbb{P}}, K_{1\mathcal{P}}^{\mathbb{Q}})$  are  $(N \times N)$  matrices that account for the speed of mean reversion in the process towards its long run unconditional mean, and the shocks that disturb the state vector from moving back to its mean are i.i.d Gaussian,  $\epsilon_t^{\mathbb{P}}, \epsilon_t^{\mathbb{Q}} \sim N(0, I_N)$ .  $\Sigma_{\mathcal{P}}$  is a lower triangular matrix such that  $\Sigma_{\mathcal{P}} \Sigma_{\mathcal{P}}'$  is the  $(N \times N)$  variance-covariance matrix of innovations to  $\mathcal{P}_t$ . Changing distributions from  $\mathbb{P}$  to  $\mathbb{Q}$  removes the drift from the stochastic process, which is essentially a shift in the mean and not the variance of  $\mathcal{P}$ , for which  $\Sigma_{\mathcal{P}}^{\mathbb{Q}} = \Sigma_{\mathcal{P}}^{\mathbb{P}} = \Sigma_{\mathcal{P}}$ .<sup>5</sup>

The risk-free rate equation, together with the equation of motion for the factors under  $\mathbb{P}$ , help recover the historical moments of all bond returns. The equation for the short rate, along with the equation of motion for the factors under  $\mathbb{Q}$ , price all the assets in the economy. The reason it is necessary to specify the distribution of the state variables under both measures, is because what lays between the  $\mathbb{P}$  and  $\mathbb{Q}$  distributions of the state vector are adjustments for the *market prices of risk*: terms that capture agents' attitudes toward risk. Term structure models evaluate the evolution of the factors under both distributions to determine more information on the pricing kernel as a function of a known state vector. As in Duffee (2002), the price of risk is determined by the difference in the coefficients of the equations of motion,

$$\Lambda_0^i = K_{0\mathcal{P}}^{\mathbb{P},i} - K_{0\mathcal{P}}^{\mathbb{Q},i}, \quad (8)$$

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<sup>5</sup>This is an application of the diffusion invariance principle, which describes the dynamics of stochastic processes when the physical measure is changed to an equivalent martingale measure but only the drift changes (Girsanov, 1958).

$$\Lambda_1^i = K_{1\mathcal{P}}^{\mathbb{P},i} - K_{1\mathcal{P}}^{\mathbb{Q},i}. \quad (9)$$

The price of risk coefficients measure how much the investor has to be compensated for fluctuations in the state vector.  $\Lambda_t$ , the market price of risk, takes a linear functional form with respect to the factors,

$$\Lambda_t^i = \Sigma_{\mathcal{P}}^{i-1} (\Lambda_0^i + \Lambda_1^i \mathcal{P}_t^i), \quad (10)$$

which captures the prices of shocks to the risk factors per unit of volatility. Since it is independent of cash-flow patterns of the security being priced, it is common to all securities with payoffs that are functions of the factors, and is implied by the fact that factors are Gaussian under both distributions. Duffee (2002) and Dai and Singleton (2002) show that this equation is able to match many features of the historical distribution of yields really well.  $\Lambda_0$  represents the average prices of the factors assigned by investors, whereas  $\Lambda_1$  measures how the market prices vary with respect to the risk levels summarized in  $\mathcal{P}_t$ . Finally,  $\Sigma_{\mathcal{P}}^{-1}$  captures the interaction of the shocks to the factors.

The stochastic discount factor is then specified under the absence of arbitrage as an exponential quadratic function that depends on each country's set of factors through their risk-free rate and price of risk equations,

$$M_{t+1}^i = \exp\left(-r_t^i - \frac{1}{2} \left(\Lambda_t^{i'} \Lambda_t^i\right) - \Lambda_t^{i'} \epsilon_{t+1}^i\right), \quad (11)$$

where  $\epsilon_{t+1}$  is the vector of shocks to the factors. Notice that each country's stochastic discount factor depends on its own interest rate factors alone.

### 2.3 Exchange Rates

By no arbitrage, the exchange rate between two countries is governed by the ratio of their stochastic discount factors (Bekaert, 1996). Let  $S_t$  be the nominal spot exchange rate at time  $t$ ; i.e., the domestic price of one unit of the foreign currency. We thus have:

$$\frac{S_{t+1}}{S_t} = \frac{M_{t+1}^f}{M_{t+1}^d}, \quad (12)$$

or, if extended to  $k$  horizons (in months),

$$\frac{S_{t+k}}{S_t} = \frac{M_{t+k}^f}{M_{t+k}^d}, \quad (13)$$

where

$$M_{t+k}^i = \prod_{j=1}^k M_{t+j}^i. \quad (14)$$

By taking logarithms, so that  $\Delta s_{t+1} = \log(S_{t+1}) - \log(S_t)$  and  $\log(M_{t+1}^i) = -r_t^i - \frac{1}{2}(\Lambda_t^{i'} \Lambda_t^i) - \Lambda_t^{i'} \epsilon_{t+1}^i$ , the log exchange rate change can be expressed as

$$\Delta s_{t+1} = (r_t^d - r_t^f) + \frac{1}{2}(\Lambda_t^{d'} \Lambda_t^d - \Lambda_t^{f'} \Lambda_t^f) + (\Lambda_t^{d'} \epsilon_{t+1}^d - \Lambda_t^{f'} \epsilon_{t+1}^f). \quad (15)$$

This relationship formally describes the log exchange rate change as the sum of three different components: (i) the interest rate differential between the countries, (ii) the exchange rate risk premium, and (iii) the difference in the shocks to the price of risk assigned by investors from each country. Recall that the risk-free rate and the prices of risk are linear functions of the factors  $\{\mathcal{P}_t^d, \mathcal{P}_t^f\}$ , from equations (2) and (10). In this context, the same factors that drive variation in the cross-section of interest rates also drive the exchange rate changes between them, but they do so in a nonlinear way through the risk premium component. Also, this specification for exchange rates yields a time-varying risk premium, which is consistent with earlier findings from Campbell and Shiller (1991) and Backus et. al (2001), among others, that provide evidence from yield and forward rate regressions for a time-varying risk premium. Time-variance is also important to match the dynamics of yields in the data and deviations from the Expectations Hypothesis, as shown in Dai and Singleton (2002).

Finally, the model assumes that  $\mathcal{P}_t$  and  $\{y_t^d, y_t^f, \Delta s_t\}$  are priced perfectly, but the observed variables,  $\{y_t^{d,obs}, y_t^{f,obs}, \Delta s_t^{obs}\}$ , are measured with error; such that the vector of errors is

$$e_t = \begin{bmatrix} e_t^{d,y} \\ e_t^{f,y} \\ e_t^{\Delta s} \end{bmatrix} = \begin{bmatrix} y_t^{d,obs} - y_t^d \\ y_t^{f,obs} - y_t^f \\ \Delta s_t^{obs} - \Delta s_t \end{bmatrix}. \quad (16)$$

## 2.4 Estimation: Maximum Likelihood

If the factors are latent, estimates governing the physical distribution necessarily depend on those of the risk-neutral distribution. In this case, the factors are observable variables captured from the yield curve of each country. I estimate the model following Joslin, Singleton, and Zhu (2011), who show that when factors are observable, a *separation property* holds: the parameters that govern the physical and risk-neutral distributions are different, except for the variance-covariance matrix of the innovations to the factors. The separation argument holds since the Maximum Likelihood estimates of the parameters are independent of the variance-covariance matrix, as shown in Zellner (1962). Their work has simplified the computational

burden and provided state of the art estimation techniques that allow for more sophisticated specifications of term structure models.

The first step in the estimation strategy is to optimize over  $\{\Sigma_{\mathcal{P}}, \lambda^{\mathbb{Q}}\}$ , where  $\Sigma_{\mathcal{P}} = \{\Sigma_{\mathcal{P}}^d, \Sigma_{\mathcal{P}}^f\}$  and  $\lambda^{\mathbb{Q}} = [\lambda^{d,\mathbb{Q}} \quad \lambda^{f,\mathbb{Q}}]'$ .  $\Sigma_{\mathcal{P}}$  is parametrized by the lower triangular Cholesky factorization of the variance-covariance matrices from the vector auto-regressions under  $\mathbb{P}$  for both countries. I do not impose any restrictions between factors of different countries since their dynamics are modeled separately; Joslin, Le, and Singleton (2013) argue that these initial estimations of  $\Sigma_{\mathcal{P}}$  will be very close to the optimal values.  $\lambda^{\mathbb{Q}}$  is a vector of ordered eigenvalues in the  $K_{1\mathcal{P}}^{\mathbb{Q}}$  matrix. It can be interpreted as the speed of mean reversion of the factors under risk-neutrality and is assumed to be real, parametrized by the difference in ordered eigenvalues. Therefore, the model is  $\mathbb{Q}$  stationary because the difference in eigenvalues is negative by construction.  $K_{0\mathcal{P}}^{\mathbb{Q}}$  can be expressed in terms of  $k_{\infty}^{\mathbb{Q},i} = -r_{\infty}^{\mathbb{Q},i} \lambda_1^{\mathbb{Q},i}$ , where  $r_{\infty}^{\mathbb{Q},i}$  is the mean of the short rate for country  $i$ .

The optimal estimates of  $\{\Sigma_{\mathcal{P}}, \lambda^{\mathbb{Q}}\}$  are obtained by nonlinear optimization, in which the eigenvalues are bounded so that  $\lambda^{\mathbb{Q}} < 0$ .  $\Sigma_{\mathcal{P}}$  is unconstrained except for the diagonal elements in each country, which are bounded from below by 0 to preserve singularity. Forcing a positive diagonal for the state transition matrix implies the stability criteria for arbitrage free models and does not conflict with the trends of dynamic models.<sup>6</sup> To summarize, the parameters to be estimated are  $\Theta = \{K_{0\mathcal{P}}^{\mathbb{P},i}, K_{1\mathcal{P}}^{\mathbb{P},i}, \Sigma_{\mathcal{P}}, \lambda^{\mathbb{Q}}, k_{\infty}^{\mathbb{Q},i}\}$  and the observable (interest rate) factors are  $\mathcal{P}_t = \begin{bmatrix} \mathcal{P}_t^d & \mathcal{P}_t^f \end{bmatrix}'$ .

The conditional log likelihood function can be factorized into the risk-neutral and the physical conditional densities of the observed data:

$$f(z_t^{obs} | z_{t-1}^{obs}; \Theta) = f^{\mathbb{Q}}(z_t^{obs} | \mathcal{P}_t) + f^{\mathbb{P}}(\mathcal{P}_t | \mathcal{P}_{t-1}), \quad (17)$$

where  $z_t^{obs}$  is the vector of observed yields for the domestic and foreign countries and the observed log exchange rate changes at different horizons between the countries:  $z_t^{obs} = \begin{bmatrix} y_t^{d,obs} & y_t^{f,obs} & \Delta s_t^{obs} \end{bmatrix}'$ . The  $\mathbb{P}$  conditional density of the observed vector captures the time series properties of the factors, whereas the  $\mathbb{Q}$  conditional density captures the cross-sectional properties of yields.

Using the assumption that  $\mathcal{P}_t$  is conditionally Gaussian, the log likelihood under  $\mathbb{P}$  can be expressed as

$$f(\mathcal{P}_t | \mathcal{P}_{t-1}) = \frac{N^T}{2} \cdot \log(2\pi) + \frac{1}{2} \cdot \log |\Sigma'_{\mathcal{P}} \Sigma_{\mathcal{P}}| + \frac{1}{2} \sum_{t=1}^T \left( (\mathcal{P}_t - E_{t-1}[\mathcal{P}_t]) \cdot (\Sigma'_{\mathcal{P}} \Sigma_{\mathcal{P}})^{-1} \cdot (\mathcal{P}_t - E_{t-1}[\mathcal{P}_t]) \right), \quad (18)$$

where  $T$  is the total number of time observations,  $N^T = N^d + N^f$  is the total number of factors in the

<sup>6</sup>Details of the model can be found in the appendix.

model,

$$E_{t-1} [\mathcal{P}_t^i] = K_0^{\mathbb{P},i} + (I + K_1^{\mathbb{P},i}) \mathcal{P}_{t-1}^i, \quad (19)$$

$$\Sigma_{\mathcal{P}}' \Sigma_{\mathcal{P}} = \begin{bmatrix} \Sigma_{\mathcal{P}}^{d'} \Sigma_{\mathcal{P}}^d & (\Sigma_{\mathcal{P}}^{f'} \Sigma_{\mathcal{P}}^f) (\Sigma_{\mathcal{P}}^{d'} \Sigma_{\mathcal{P}}^d)' \\ (\Sigma_{\mathcal{P}}^{d'} \Sigma_{\mathcal{P}}^d) (\Sigma_{\mathcal{P}}^{f'} \Sigma_{\mathcal{P}}^f)' & \Sigma_{\mathcal{P}}^{f'} \Sigma_{\mathcal{P}}^f \end{bmatrix}. \quad (20)$$

Imposing zeros on the off-diagonal elements of the joint variance-covariance matrix of the factors instead does not lead to different results; these numbers are actually very small. The parameters  $(K_0^{\mathbb{P},i}, K_1^{\mathbb{P},i})$  that maximize the log likelihood function  $f$  conditional on  $t = 0$  information are given by the OLS estimates of the conditional mean of  $\mathcal{P}_t$ .

The log likelihood under  $\mathbb{Q}$  can be expressed as

$$f(z_t^{obs} | \mathcal{P}_t) = \frac{1}{2} \sum_{t=1}^T e_t^2 / \Sigma_e^2 + \frac{1}{2} (J^T - N^T) \cdot \log(2\pi) + \frac{1}{2} (J^T - N^T) \cdot \log(\Sigma_e^2), \quad (21)$$

where  $e_t^2$  is the vector with square errors between the actual and the model-implied yields and exchange rates as in (16) and  $\Sigma_e = \sqrt{\frac{\sum_{t=1}^T e_t^2}{T(J^T - N^T)}}$ , where  $J^T$  is the number of observed variables.

### 3 Data and Interest Rate Factors

#### 3.1 Interest Rate and Exchange Rate Data

Monthly interest rate data from January 1999 to January 2014 are obtained from Bloomberg, L.P. 2014, for a total of 181 observations. This time period is particularly interesting since it experiences an increase in the relevance of asset prices given the integration of financial markets, as well as drastic changes in interest rate behavior, the creation of the Euro, and different financial crises around the globe. The dataset contains interest rates of 12 different maturities: 3 months, 6 months, and 1 to 10 years for countries with free-floating currencies for which interest rate data are available: Australia, Canada, Japan, Norway, Sweden, Switzerland, the United Kingdom, and the United States. Yields are in percentage, end-of-month, annualized, and expressed in the local currency. Bloomberg uses coupon bonds, bills, swaps, or a combination of these instruments to derive the discount factors for each country using standard bootstrapping. The zero-coupon yields are then calculated step by step using the estimated discount factors with a minimum of four instruments at different tenors for each observation.

Figure 1 shows the cross-sectional variation of interest rates throughout time for each country. In the U.S., short-term interest rates increased steadily starting April 2004 to reach around 5% by the beginning

of 2007. In September 2008 the short rate dropped below 1% and has remained close to the zero lower bound ever since. This pattern of low short-term rates is observed in most other countries as well during the past few years. Australia's rates tend to be slightly higher on average and Norway is the only country to experience yield curve inversion in 1999. However, Japan is the country that exhibits the most dissimilar term structure, since interest rates have been close to the zero lower bound for the entire sample.

Table 1 shows summary statistics for one-year yields. On average, Australia has the highest one-year yields (4.82%) and Japan has the lowest (0.20%). The well established stylized facts on U.S. yield curves also apply for the rest of the countries in the sample.<sup>7</sup> A few features of interest rates during this period that are worth highlighting are:

1. *The yield curves are typically upward sloping.* For the most part of the sample, low-maturity yields are lower than high-maturity yields, except for times of crisis in which the yield curve flattens and even tends to revert - the magnitudes depending on each country's vulnerability to financial contagion, among other factors. During 2008, for example, Switzerland's and Norway's short-term rates not only increased significantly, but the yield curves even inverted, being short-term rates higher than long-term yields for subsequent months.
2. *Standard deviation tends to be lower for long-term yields than short-term yields.* This is particularly true during the earlier years in the sample, where short-term yields exhibit high volatility. During the last few years, however, yields have stayed close to the zero lower bound, exhibiting less variability at the short end of the yield curve. Also, some countries exhibit more volatility than others. Japan's one-year yield, for example, has a standard deviation of 0.21 compared to 2.05 for the U.S. during the same period.
3. *Yields are very persistent.* The first order autocorrelation is higher than 90% for all countries, with the most persistent countries being Canada, Norway, the U.K., and the U.S., with an autocorrelation higher than 65% after one year, as seen in table 1. The least persistent country is Australia, with a one-year autocorrelation of 35%.
4. *Most yields are close to a normal distribution over the sample period.* This is consistent with contemporary work that suggests U.S. yields have become more Gaussian in recent years - see for example, Piazzesi (2010). For most countries, skewness is around zero, except for Japan (1.34) and kurtosis ranges from 1.5 (U.K.) to 2.5 (Australia). This suggests that modeling yields under the assumption of a Gaussian distribution is quite reasonable, especially during the time frame considered in the sample.

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<sup>7</sup>For analysis on stylized facts on U.S. yield curves refer to Piazzesi and Schneider (2007).

5. *Yields are highly correlated across countries.* Table 2 shows the correlation between one-year interest rates of different countries. Interest rates of the same maturity tend to be very correlated across countries. Although the Japanese yield curve is quite different from other countries' term structure, particularly during the earlier years, it is most correlated with Australia's (50%) and Switzerland's (61%). The U.K., the U.S., and Canada share a correlation above 89%.
6. *Yields of near maturity are highly correlated.* Table 3 shows the correlation of the 3-month interest rates of each country with respect to different maturities from 6 months to 10 years. For all countries, the closest the maturity range, the highest the correlation between interest rates of the same country. For example, 3-month and 6-month rates are highly correlated, moving almost one-to-one, but 3-month and 10-year rates tend to differ by country, ranging from 37% (Japan) to 84% (the U.K. and Norway).

The exchange rate data are also from Bloomberg L.P, 2014. There are a total of 181 monthly observations per currency from January 1999 to January 2014. All the end-of-month exchange rates are expressed in terms of U.S. dollars (USD) for the following countries: Australia (AUD), Canada (CAD), Japan (JPY), Norway (NOK), Sweden (SEK), Switzerland (CHF), and the United Kingdom (GBP).

Table 4 summarizes the main characteristics of exchange rates during this period. Exchange rates, like interest rates, are very persistent, with an autocorrelation between 57% (GBP) and 79% (CAD) after one year. They are also highly related among themselves, sharing a correlation of 88% or higher, except for the Japanese yen and the British pound, which are the most dissimilar currencies in the sample. The most volatile currencies during this period are the Australian dollar, the Swiss franc, and the British pound with a standard deviation above 0.17, and the least volatile currency is the Japanese yen (0.00). All the currencies exhibit a skewness around zero, from -0.39 (SEK) to 0.83 (JPY), and kurtosis ranges from 1.5 (CAD) to 2.6 (JPY). For the estimation, I calculate annualized log exchange rate changes at different horizons: 1, 3, and 6 months, and 1 to 5 years, in 6-month increments.

### 3.2 Interest Rate Factors

Given the high correlation of yields within each country, the literature has taken advantage of dimensionality-reduction techniques to find the underlying drivers of interest rates. I extract the three main principal components or factors of each country by the eigenvalue-eigenvector decomposition of the variance-covariance matrix of the yields. Typically, the first two or three factors are enough to explain most of the cross-sectional variability in all of the yields. In general, the factors for each country tend to have similar shapes, with Japan's being the most different. This corresponds to the fact that Japan's interest rates have been very low for the past decade relative to the rest of the sample.

The first factor is downward sloping, capturing the drop in interest rates that most countries have experienced in the last few years. Table 5 presents summary statistics on the country-specific components. In every country, the first factor extracted from the data is responsible for more than 81% of the cross-sectional variation and the first three factors combined account for more than 99% of total variability. Since the first factor captures most of the variation in all yields within a country, it fluctuates more and has higher standard deviation (around 3.4) than the second (0.8) and third (0.2) factors. Interest rate factors are also very persistent, with the first factor being more persistent than the second and third ones. In term structure models, yields are usually modeled as linear functions of the factors. In this type of setting, that persistence of yields comes from persistent factors, as observed in the data.

Figure 2 shows the loadings or weights that each factor places on the different maturities for every country. The factor loadings exhibit the same pattern: horizontal for the first factor (around 0.3), downward sloping for the second factor, and U-shaped for the third factor. Litterman and Scheinkman (1991) attributed the labels *level*, *slope*, and *curvature* to these three factors given the effect they have on the yield curve. For example, the first factor affects all yields the same, hence shocks to the first factor generate parallel shifts on the curve, changing the *level* or average of yields. The second factor loads positively for short-term maturities and negatively for long-term maturities, thus shocks to the second factor move long and short yields in opposite directions, changing the *slope* of the yield curve. Finally, the third factor loads positively on short- and long-term maturities, but negatively on mid-range maturities, thus shocks to the third factor affect the *curvature* of the yield curve. The Litterman and Scheinkman (1991) interpretation of the factors is very common in the literature, although it remains agnostic on links to the macroeconomy.

Recent work provides evidence that the level factor is partly correlated with inflation (Diebold et. al, 2006), aggregate supply shocks from the private sector (Wu, 2006), and shocks to preferences for current consumption and technology (Evans and Marshall, 2007). The slope factor seems to be related to monetary policy shocks (Evans and Marshall, 1998, 2007; Wu 2006) and real activity (Diebold et. al, 2006). However, the curvature factor has not yet been linked to any specific variables. Through my model, the curvature factor helps explain exchange rate fluctuations despite its low contribution to explaining cross-sectional variability in yields. I find some evidence that higher order variables of the yield curve, such as the curvature factor, might be useful in explaining exchange rate movements.

## 4 Results

The baseline model is estimated by Maximum Likelihood using three observable interest rate factors from each country: level, slope, and curvature. The domestic country is the U.S., therefore exchange rates are

expressed in terms of USD for the 1999-2014 period. First, I present the results for the base line model parameters in annual terms, as well as the term structure fit for each country pair. Next, I evaluate the ability of the model to capture exchange rate movements at different horizons by only using interest rate factors. I explore different model specifications, such as a model with less factors (only the countries' level and slope) and a model with a different base country instead of the U.S. (the U.K.), as well as several robustness checks. I also explore the implications the model has on the uncovered interest parity puzzle, one of the most relevant puzzles in the exchange rate literature. Finally, I study the implications of my model for ten other countries under a common currency, the euro, and their ability to predict euro-USD fluctuations.

Table 6 shows the parameter estimates with asymptotic standard errors in parentheses that correspond to the evolution of the factors under the physical distribution as an unrestricted VAR(1) in equation (6). All parameters are annualized. The first column shows  $K_{0\mathcal{P}}^{\mathbb{P}} \times 100$ . The coefficient of the first factor is always positive, although only sometimes significant, with magnitudes ranging from 0.14 (Canada) to 0.94 (U.S.). The coefficient of the second factor is negative for Australia, Japan, the U.K., and the U.S. and positive for the rest, although not statistically significant. The coefficient of the third factor is statistically significant for all the countries and always positive, except for Switzerland (-0.08). Table 6 also reports estimates of  $K_{1\mathcal{P}}^{\mathbb{P}} + I$ . As expected, the diagonal elements are close to one and statistically significant at the 99% level, capturing the fact that interest rate factors are very persistent. The off-diagonal elements of the first and second columns in the VAR are small and insignificant, although the estimates in the third column that correspond to the first factor are larger and negative (around -0.40), ranging from -0.65 (Japan) to -0.14 (Canada), except for Switzerland (0.41), which is positive. Finally, the table reports the lower triangular Cholesky factorization of the variance-covariance matrix,  $\Sigma_{\mathcal{P}} \times 100$ , which contains large, positive, and significant diagonal elements.

Table 7 reports the parameters of the risk-free rate equation (2),  $\rho_{0\mathcal{P}}$  and  $\rho_{1\mathcal{P}}$ , along with the largest eigenvalues of the  $K_{1\mathcal{P}}^{\mathbb{Q}} + I$  matrix, and a measure of goodness of fit of model-implied yields.  $\rho_{0\mathcal{P}}$  is always negative between -0.017 and -0.002,  $\rho_{1\mathcal{P}}$  is always positive, and each factor's estimate of  $\rho_{1\mathcal{P}}$  increases, being the third factor estimates close to one for all countries, except for Switzerland (-1.054). The eigenvalues of the transition matrix under risk-neutrality are large and close to one, hence shocks to these factors have permanent effects in their evolution over time. As a measure of goodness of fit, the last column reports root mean square errors (RMSE) in basis points for the cross-section of yields. The RMSE is around 10 basis points, with the best fit for the Japanese yen (4.7) and Australian dollar (4.9) and the worst fit for the Swiss franc (11.8). Overall, the goodness of fit for interest rates is very good, as it is standard in term structure models of interest rates, which suggests that the incorporation of exchange rates into the model does not compromise the fit of the yield curve.

## 4.1 Exchange Rate Fit

In order to obtain a general measure of goodness of fit for exchange rates, I project the exchange rate data on the model-implied exchange rates. Hence, given the model equation describing annualized log exchange rate changes at different  $k$  horizons,

$$\Delta s_{t+k} = r_{t+k-1}^d - r_{t+k-1}^f + \frac{1}{2} \left[ \Lambda_{t+k-1}^{d'} \Lambda_{t+k-1}^d - \Lambda_{t+k-1}^{f'} \Lambda_{t+k-1}^f \right] + \Lambda_{t+k-1}^{d'} \epsilon_{t+i}^d - \Lambda_{t+k-1}^{f'} \epsilon_{t+i}^f, \quad (22)$$

I perform an ordinary least squares (OLS) regression of the data on the model-implied exchange rate equations for each currency  $i$ ,

$$\Delta s_{t+k}^{obs,i} = \alpha^i + \beta^i \Delta s_{t+k}^i + \mu_{t+k}^i. \quad (23)$$

Table 8 displays the results from regression (23), which account for the goodness of fit of the annualized model-implied exchange rate changes at different  $k$  horizons, from 1 month to 5 years. This model determines the exchange rate changes as a function of six total factors (the level, slope, and curvature of each country). The domestic country is chosen to be the U.S. so that the exchange rates are in terms of USD prices. Therefore, an increase in  $\Delta s_t$  represents a depreciation of the U.S. dollar with respect to the foreign currency,  $i$ . The results for  $\beta^i$ , along with the corresponding standard errors and the adjusted  $R^2$ , are reported for selected horizons: 1 , 3 , and 6 months, and 1 , 1.5 , 2 , 3 , 4 , and 5 years.

At the one-month horizon, the fit of the model, as measured by the  $R^2$ , ranges from 1% and insignificant (Switzerland) to 7% and statistically significant at the 99% confidence level (Japan). Very short exchange rate fluctuations are quite difficult to predict. Meese and Rogoff (1983) claimed that the best forecasting model is obtained under the assumption that the exchange rate follows a random walk. However, at longer horizons the predictability of exchange rates improves significantly. This type of behavior is observed in other assets as well. Although short-term movements are not well understood, it seems that at longer horizons the factors are better able to capture the signal that drives movements in asset returns. Figure 3 shows the annualized one-month log changes in exchange rates for all the currencies relative to the USD as well as the model-implied expected exchange rate fluctuations. Overall, the expected exchange rate changes implied by the model are not as volatile; however, they capture the data movements on average quite well.

In general, the regression coefficients increase in magnitude and significance as the horizons increase. At the one-year horizon and thereafter, the coefficients of all currencies are close to one. Approximately half of the variation in one-year exchange rate movements can be explained by the model using interest rate factors

alone, for all currencies relative to the USD. Figure 4 shows the data and model-implied one-year expected fluctuations in the exchange rate. The fit of the model at this horizon ranges from 41.8% for the Swiss franc to 70.3% for the Swedish krona. The model is even more successful at capturing five-year exchange rate movements, depicted in figure 5, ranging from 55% (Swiss franc) to 91.6% (British pound).

One of the potential concerns within this framework is small-sample bias. Because the closer to a unit root, the larger the small-sample bias in Maximum Likelihood estimates, Joslin, Singleton, and Zhu (2011) enforce a high degree of persistence under the physical distribution of the factors to ensure a stationary process, by which they constrain all the eigenvalues that govern the speed of mean reversion to be strictly positive. Small-sample bias arises due to the high persistence of the factors, and becomes particularly problematic when the sample period is short. To this end, I estimate robust (White) standard errors and Newey-West standard errors to account for small-sample bias and for potential correlation and heteroskedasticity in the innovation vector. This generates larger standard errors that specify the precision of the estimated coefficients. Similar to Lustig, Roussanov, and Verdelhan (2011), I follow Andrews (1991) to choose the number of lags for the Newey-West estimation to be the horizon  $k$  (in months) plus one. After the three-month horizon, all of the estimates are statistically significant at the 99% confidence level regardless of the method used to calculate standard errors, except for the Norwegian krone under Newey-West standard errors that is significant at the 95% level.

Bauer, Rudebusch, and Wu (2012, 2014) argue that this method still leads to large biased estimates that may yield an inconsistent risk premium. Given the separation argument, only the parameters that characterize the physical density of yields are affected. They propose to replace the initial estimates of the factors' vector auto-regressive process under  $\mathbb{P}$  with simulation-based bias-corrected estimates and then proceed to the Maximum Likelihood estimation. I therefore implement their suggested inverse bootstrap bias-corrected method: For each of the 5,000 bootstrap iterations (after discarding the first 1,000), I simulate a set of 50 bootstrap samples using some trial parameters and calculate the mean or median of the OLS estimator to be equal to the original OLS estimate. The bias-corrected estimates then reflect more persistent factor dynamics, as measured by the largest eigenvalues of the transition matrix. More details on the estimation of bias-corrected factor dynamics and the consequences of OLS bias in term structure models can be found in Bauer, Rudebusch, and Wu (2012, 2014).

Table 9 shows the exchange rate fit for the model after adjusting for mean and median bias-corrected estimates for selected maturities with the coefficients from regression (23), Newey-West standard errors in parenthesis, and  $R^2$  adjusted for degrees of freedom. As expected, the VAR physical dynamics are more persistent after correcting for bias, either by mean or median correction, although estimates for  $\Sigma_{\mathcal{P}}$  remain close to the estimates of the unrestricted VAR. After estimates are corrected for bias (either by the mean

or the median) interest rate factors are still able to explain as much variation in exchange rates. The same patterns as the base model can be observed: at the one-month horizon, the best fit is the Japanese yen (6%) and the worst is the Swiss franc (0%), and as the horizon increases, the fit of the model improves for all currencies. Exchange rate changes at the one-year horizon and thereafter have coefficients around one that are statistically significant at the 99% confidence level, and about 50% of one-year exchange rate movements can be explained by the model, from 43% (Swiss franc and Canadian dollar) to 60% (Japanese yen and Swedish krona). For some currencies, correcting for bias reduces the fit of the exchange rate model, but for some currencies (Norwegian krone and Swiss franc), it improves it. Most importantly, the results are still quite similar, and at the five-year horizon, the model accounts for 57% to 91% of exchange rate movements using interest rate factors alone.

I next evaluate the performance of the model with a different number of factors. Table 10 reports the estimates for selected maturities of a linear projection of the data on a model with only one factor, the level, or two factors, the level and the slope, for each country. With fewer interest rate factors, the root mean square errors that measure the term structure fit of each country increase relative to the three-factor model. The two-factor model has RMSEs that are from 1.4 basis points (Switzerland) to 17.1 basis points (Canada) higher than the three-factor model. In contrast to the base model, the two-factor model exhibits coefficients of  $\rho_{0P}$  that are positive for all countries, except for Sweden, and the coefficients of  $\rho_{1P}$  are positive, except for Japan's first factor, with the second coefficient being larger than the first. The physical distribution parameters do not exhibit significant differences in signs or magnitudes. The one-factor model exhibits larger RMSEs than the other models, as expected, ranging from 43.4 basis points for Norway to 103.9 for Canada.

In general, the same pattern emerges for all models: the exchange rate fit improves at longer horizons. In every case, the two-factor model outperforms the one-factor model, although they are both well below the performing levels of the three-factor model. That is, adding a third factor to the model, the curvature factor, significantly improves the model's ability to explain exchange rates, despite having low-content information to explain cross-sectional interest rate variation.

A few highlights of these lower-factor models are worth mentioning. For Norway, Sweden, and the U.K., the one-factor model can explain less than 5% of exchange rate movements at horizons one-year or lower, so for these countries most of the valuable information in explaining short-term exchange rate fluctuations actually comes from the other components, the slope and curvature. However, the one-factor model does remarkably well for Australia and Japan, over 20% of one-year changes are captured by the countries' level alone, which suggests that Japan's and Australia's level factor may contain a lot of information about their exchange rate movements. Finally, the two-factor model explains exchange rate changes extremely well for

Canada relative to the three-factor model, which suggests that incorporating the curvature factor does not seem to significantly contribute to modeling the CAD-USD exchange rate path. Given the relevance of the curvature factor in modeling most currency movements in the long run, the third yield curve component could be capturing long run expectations on exchange rate fluctuations or other fundamentals relative to international markets that influence the exchange rate.

I also re-estimate the model considering a different base country, the U.K., so that exchange rates are in terms of the British pound, instead of the U.S. dollar. Table 11 reports the coefficients, robust and Newey-West standard errors, and the adjusted  $R^2$  for all countries in which the log exchange rate changes are modeled in terms of the British pound. The model parameters are very similar, for example,  $\rho_{0P}$  and  $\rho_{1P}$  have the same sign and similar magnitude regardless of the base country, including  $\rho_{13P}$  for Switzerland, which is negative one as well. Notice that modeling the USD-GBP exchange rate is the same as modeling the GBP-USD exchange rate, since results are symmetric. This is because it does not matter what country is selected as domestic and what country is selected as foreign; the information content in the factors is still the same. Hence the regression results for the U.S. dollar in terms of the British pound from table 11 are the same as the results for the British pound in terms of the U.S. dollar from table 8.

For the other countries results are similar, although not the same, since instead of the U.S. term structure we are now modeling exchange rates with U.K. interest rate factors, which exhibit different shapes. In general, results are similar whether one is using the U.S. or the U.K. as a base: The coefficients are significant at the 99% confidence level after 3 months (except for Norway under N.W. standard errors, which is significant at a 95% confidence level, just as in the base model).

For exchange rate fluctuations in the Australian dollar, Canadian dollar, Japanese yen, and Swiss franc, results are either slightly better or significantly better when using the U.K. as a country base. For example, the Swiss franc, which has the worst fit at most maturities when modeled with respect to the U.S. dollar, significantly improves the fit with a 47%  $R^2$  at the one-year horizon and 92% at the five-year horizon, in contrast with 41% and 55% under the USD, respectively. In the base model, 45% of one-year CAD-USD exchange rate fluctuations are explained by the factors, whereas the model with U.K. factors accounts for more than 64% of the CAD-GBP one-year exchange rate. Moreover, except for Sweden, all other currencies experience a better fit at the five-year horizon when modeled in terms of the British pound, with  $R^2$  from 80% (NOK) to 92% (JPY and CHF). Estimates for the Nordic countries explain less of the exchange rate movements when the GBP is used as a base. For Norway, results are slightly worse, whereas for Sweden, a lot worse. This may suggest that some countries have more information than others about expected exchange rate movements, although we can say with certainty that the U.S. is not particularly more informative than other countries, such as the U.K.

## 4.2 Implications in the context of the UIP Puzzle

In this section, I evaluate my model in the context of the Uncovered Interest Parity (UIP) puzzle. UIP claims that any excess return on foreign-denominated deposits is offset by the expected depreciation against the domestic currency. That is, if we run the following regression

$$\Delta s_{t+k}^{obs} = \alpha + \beta_{UIP} \left( r_{t+k-1}^d - r_{t+k-1}^f \right) + \mu_{t+k} \quad (24)$$

we would expect that  $\alpha = 0$ ,  $\beta_{UIP} = 1$ .<sup>8</sup> However, the data do not hold against this proposition, since the coefficient of the interest rate differential is not only different from one, but also negative, and the explanatory power of the interest rate differential is very low. Several papers have focused on studying this phenomenon.<sup>9</sup> In my model, the UIP condition does not hold, since the expected log exchange rate change is not only a function of the interest rate differential but also a function of a risk premium term implied by my framework:

$$\Delta s_{t+k}^{obs} = \alpha + \beta_{IRD} \left( r_{t+k-1}^d - r_{t+k-1}^f \right) + \beta_{RP} \left( \frac{1}{2} \left[ \Lambda_{t+k-1}^{d'} \Lambda_{t+k-1}^d - \Lambda_{t+k-1}^{f'} \Lambda_{t+k-1}^f \right] \right) + \mu_{t+k}. \quad (25)$$

This is consistent with the idea proposed by Fama (1984) and Hansen and Hodrick (1983), that attribute the anomalous findings to a time-varying risk premium. I evaluate the ability of my model to reproduce the UIP puzzle when equation (24) is estimated and I examine to what extent, the augmented UIP regression implied by my model, equation (25), can help solve the puzzle.

Table 12 shows the results for the UIP and augmented-UIP regressions. The coefficient of the UIP regression,  $\beta_{UIP}$ , is sometimes positive, sometimes negative, but never statistically significant in the short run. At longer horizons (after two years), some coefficients are significant but always large and negative. Moreover, the adjusted  $R^2$  is below 5% for one-year or lower frequency changes for every country. This is consistent with the UIP puzzle observed in the data.

When incorporating the model-implied risk premium, the adjusted  $R^2$  always increases, the coefficient of the interest rate differential,  $\beta_{IRD}$  is always positive (except for Switzerland at the one-month and three-month horizon) although only statistically significant at longer horizons. For the three-year horizon, the interest rate differential is close to one for every currency and the coefficients are statistically significant at

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<sup>8</sup>Notice that this regression is a slight variation from the standard UIP regression, since the interest rate differential is lagged one period with respect to the log exchange rate changes. This is because the interest rate differential component comes from the model and it is constructed as a linear function of the factors. Standard UIP regressions use the actual interest rate differential from the data with yields of the corresponding maturity  $k$ , such as, for example, Flood and Taylor (1996) or Dong (2006). The idea in this case is to evaluate the interest rate differential implied by the model.

<sup>9</sup>Surveys on the UIP literature can be found in Hodrick (1987), Froot and Thaler (1990), Lewis (1995), and Engel (1996).

the 99% confidence level, with the lowest coefficient being 0.60 for the Swiss franc. The coefficient of the risk premium,  $\beta_{RP}$ , is always positive, approaches one, and becomes statistically significant at the 99% confidence level for every country after the three-month horizon (except for Norway, which is statistically significant at the 95% level). These results suggest that the risk premium term is clearly important in modeling exchange rates, even when only interest rate factors are considered, and contributes to closing the gap with respect to the UIP puzzle.

Besides attributing the failure of UIP to the presence of a time-varying risk premium, Fama (1984) identified two conditions necessary for the risk premium to solve the UIP puzzle. First, the risk premium has to be more volatile than the interest rate differential, and second, the model-implied risk premium and the interest rate differential need to be negatively correlated. Table 13 shows that the Fama conditions hold for all currencies and for all horizons considered. The variance of the risk premium is higher than the variance of the interest rate differential by a magnitude of two or higher. Also, the correlation between the interest rate differential and the risk premium is always negative and large as it increases in horizon, as expected. This suggests that the risk premium implied by the model with interest rate factors is a key driver of exchange rates, which is consistent with Fama (1984) and Hansen and Hodrick (1983) explanation of the UIP puzzle, and does not rely solely on large shocks to drive movements in the exchange rate.

### 4.3 Implications for the Euro Dynamics

In the baseline model, the factors extracted from two countries help determine their exchange rate fluctuations through each country's stochastic discount factor. However, when multiple countries share the same currency, what are the factors that determine movements in, for example, the euro-U.S. dollar exchange rate?

I explore the determination of the path of the euro by estimating the stochastic discount factor of ten countries under the euro regime: Austria, Belgium, Finland, France, Germany, Ireland, Italy, the Netherlands, Portugal, and Spain. That is, I estimate pricing kernels for those countries individually to explore whether their interest rate factors alone contribute to explaining changes in the euro (EUR). My model allows one to observe whether any particular country in the Euro area has more useful information on the EUR-USD exchange rate fluctuations than others.

Figure 6 shows the evolution of yield curves for each country. Although some term structures exhibit similarities, such as Germany's, Finland's, and the Netherlands', other countries behave quite differently, such as Ireland, Portugal, Italy, or Spain; these countries exhibit different yield curve movements, particularly during the 2011-2012 European crisis. Although the short rate increased for every European country during

that time, some were more susceptible to the Greek crisis than others, with their short rates reaching significantly higher levels. For example, Portugal's interest rates rose to 17% in January 2012. Because these countries exhibit different yield curves, interest rate factors extracted from each term structure will also be quite different.

Which country can explain euro fluctuations the most? As expected, the yield curve factors for a country such as Germany are important in determining changes in the euro. However, risk factors from countries with more volatile interest rates, such as Italy and Spain, also have high explanatory power on the exchange rate dynamics. For example, Italy has higher explanatory power over euro-USD fluctuations than Germany at every horizon. The results suggest that the risk factors that govern fluctuations in the euro are priced to a similar extent in the yield curves of different European countries.

Table 14 reports the results of the linear projection of euro movements on the model-implied exchange rates for ten European countries in terms of the U.S. dollar. All countries seem to have similar degrees of explanatory power on the euro. In general, the same conclusion holds for this set of countries: exchange rate fit improves at longer horizons and the addition of a curvature factor generally improves the fit of the model. At the one-year exchange rate horizon, all the countries are able to explain around 50% of exchange rate movements. Ireland and Finland exhibit the worst fit (39% and 40%), while Italy, Spain, Portugal, and Belgium show the best fit (58%, 51%, 50%, and 50%). However, at the five-year horizon, every country is able to explain 83% or more of the total variation in the EUR-USD exchange rate, with the lowest being Austria (83.8%) and Spain (88.6%) the highest. After the three-month horizon, every country is statistically significant at the 99% confidence level, except for Finland and Portugal, which are significant at the 95% level with N.W. standard errors. Results are quite similar regardless of what country is the base, the GBP or the USD. These results suggest that currency risk in the euro is priced in every European country, and term structure cross-sectional variation captures those expectations really well.

## 5 Conclusion

This paper examines whether interest rate factors alone can help determine fluctuations in the exchange rate using a no-arbitrage term structure model. I propose a framework that models exchange rate changes at different horizons as the ratio of two countries' stochastic discount factors in which most of the variation is driven by a nonlinear risk premium. The results suggest that interest rate factors are indeed helpful in modeling exchange rates, particularly at longer horizons. These results are in line with findings by Mark (1995), who identifies a predictable component in exchange rate fluctuations as the horizon increases. Key to my results is the fact that interest rate factors extracted from the yield curve are observable, which allows

for a straight forward estimation of the model by Maximum Likelihood as in Joslin, Singleton, and Zhu (2011).

Interest rate factors, level, slope, and curvature, can explain half of the variation in one-year exchange rates for countries with free-floating currencies from 1999 to 2014: Australia, Canada, Japan, Norway, Sweden, Switzerland, and the U.K., with respect to the U.S. dollar. Moreover, the fit of the model improves as the horizon increases, suggesting that long-term expected fluctuations in the exchange rate are captured by yield curve factors. Results are robust to accounting for small-sample bias, potential heteroskedasticity in the error term, and across multiple currencies relative to a different base. A drawback in the literature is that there is no clear link between interest rate factors and the macroeconomy, particularly the curvature factor. In this paper, I present evidence that despite of the curvature factor explaining a small fraction of yield curve movements, it provides a significant contribution to explaining exchange rates. When using fewer factors, either the level or the level and slope, the explanatory power of the model decreases, suggesting that the curvature factor is important in explaining exchange rate fluctuations for most currencies in the sample.

A key implication of the model is that a risk premium, nonlinear in the interest rate factors, drives most of the variation in exchange rates. Further analysis in the context of the uncovered interest parity puzzle suggests that the model-implied risk premium satisfies the Fama (1984) conditions: is negatively correlated with the interest rate differential and exhibits significantly larger variance. This two-country term structure model with observable interest rate factors is quite successful at deriving stochastic discount factors that imply a theory-consistent risk premium. Moreover, this paper explores the ability of ten European countries to account for fluctuations in the euro by only using their own interest rate factors. Although European countries exhibit different yield curves, they all capture exchange rate movements to a similar extent, suggesting that exchange rate risk is implicit in yield curves of every European country in the sample. Further work on this area includes exploring whether these results are time-independent, or if during crises, certain European countries capture exchange rate information better than others.

There remain multiple avenues to be explored within this framework. Given that this work presents evidence of interest rate factors' explanatory power over exchange rate movements at different horizons, a possible extension is to investigate the factors' ability to forecast exchange rates out of sample. Despite the large body of literature devoted to modeling exchange rates, there is a high degree of uncertainty about how exchange rates react to changes in expectations, particularly out of sample.

Motivated by the idea that a unique stochastic discount factor should be able to account for prices of different assets, another extension of this paper is to incorporate equities. Given that the same stochastic discount factor prices securities in the bond and foreign exchange markets, it should price securities in the equity market as well. Similarly to exchange rates, the equity returns literature also finds a long-run

predictable component in stock returns: the price-dividend ratio, e.g., Fama and French (1988), which is a characteristic observed in other assets as well.

One of the present challenges the literature faces is that the stochastic discount factor is not derived within a general equilibrium framework in terms of consumption preferences and technology parameters. Integrating term structure models into a preference-based framework is of interest not only to academics seeking to understand the theoretical connections of asset pricing dynamics, but also to monetary policy makers, bond investors, and other financial market participants concerned with fluctuations in asset prices from a general equilibrium perspective.

## References

- [1] Alexius A. and P. Sellin (2012), “Exchange Rates and Long-Term Bonds,” *Scandinavian Journal of Economics* 114(3), 974-990.
- [2] Ahn, D. (2004), “Common Factors and Local Factors: Implications for Term Structures and Exchange Rates,” *Journal of Financial and Quantitative Analysis* 39(1), 69-102.
- [3] Andrews, D. (1991), “Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation,” *Econometrica* 59(3), 817-58.
- [4] Ang, A. and J. Chen (2010), “Yield Curve Predictors of Foreign Exchange Returns,” Working Paper, March 2010 version.
- [5] Backus, D. K., S. Foresi, A. Mozumdar, and L. Wu (2001), “Predictable Changes in Yields and Forward Rates,” *Journal of Financial Economics* 59(3), 281-311.
- [6] Backus, D. K., S. Foresi, and C. I. Telmer (2001), “Affine Term Structure Models and the Forward Premium Anomaly,” *Journal of Finance* 56(1), 279-304.
- [7] Bauer, G. H. and A. Diez de Los Rios (2012), “An International Dynamic Term Structure Model with Economic Restrictions and Unspanned Risks,” Bank of Canada Working Paper 12-5, January 2012 version.
- [8] Bauer, M. D., G. D. Rudebusch, and J. C. Wu (2012), “Correcting Estimation Bias in Dynamic Term Structure Models,” *Journal of Business and Economic Statistics* 30(3), 454-467.
- [9] Bauer, M. D., G. D. Rudebusch, and J. C. Wu (2014), “Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset: Comment,” *American Economic Review* 104(1), 323-337.
- [10] Bekaert, G. (1996), “The Time Variation of Risk and Return in Foreign Exchange Markets: A General Equilibrium Perspective,” *Review of Financial Studies* 9(2), 427-70.
- [11] Benati, L. (2006), “Affine term structure models for the foreign exchange risk premium,” Bank of England Working Paper 291.
- [12] Campbell, J. Y. and R. Shiller (1991), “Yield Spreads and Interest Rate Movements: A Bird’s Eye View,” *Review of Economic Studies* 58, 495–514.

- [13] Chabi-Yo, F. and J. Yang (2007), “A No-Arbitrage Analysis of Macroeconomic Determinants of Term Structures and the Exchange Rate,” Working Paper 2007-21.
- [14] Clarida, R. H. and M. P. Taylor (1997), “The Term Structure of Forward Exchange Premiums and the Forecastability of Spot Exchange Rates: Correcting the Errors,” *The Review of Economics and Statistics* 79(3), 353-361.
- [15] Dai, Q. and K. Singleton (2000), “Specification Analysis of Affine Term Structure Models,” *Journal of Finance* 55, 1943-78.
- [16] Dai, Q. and K. Singleton (2002), “Expectation Puzzles, Time-varying Risk Premia, and Affine Models of the Term Structure,” *Journal of Financial Economics* 63, 415-441.
- [17] Dai, Q. and K. Singleton (2003), “Term Structure Dynamics in Theory and Reality,” *Review of Financial Studies* 16, 631-678.
- [18] Dewachter H. and K. Maes (2001), “An Affine Model for International Bond Markets,” Working Paper, February 2001 version.
- [19] Diebold, F. X., M. Piazzesi, and G. D. Rudebusch (2005), “Modeling Bond Yields in Finance and Macroeconomics,” *American Economic Review P&P* 95(2), 415-520.
- [20] Diebold, F.X., G. D. Rudebusch, B. and Aruoba (2006), “The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach,” *Journal of Econometrics* 131, 309-338.
- [21] Diez de Los Rios, A. (2009), “Can Affine Term Structure Models Help us Predict Exchange Rates?,” *Journal of Money, Credit and Banking* 41 (4), 755-766.
- [22] Dong, S. (2006), “Macro Variables Do Drive Exchange Rate Movements: Evidence from a No-Arbitrage Model,” Working Paper, July 2006 version.
- [23] Duffee, G. (2002), “Term Premia and Interest Rate Forecasts in Affine Models,” *Journal of Finance* 57, 405-443.
- [24] Duffie, D. and R. Kan (1996), “A Yield-Factor Model of Interest Rates,” *Mathematical Finance* 6, 379-406.
- [25] Egorov, A. V., H. Li, and D. Ng (2011), “A tale of two yield curves: Modeling the joint term structure of dollar and euro interest rates,” *Journal of Econometrics* 162(1), 55-70.

- [26] Engel, C. (1996), “The Forward Discount Anomaly and the Risk Premium: A Survey of Recent Evidence,” *Journal of Empirical Finance* 3(2), 123-192.
- [27] Evans, C. L. and D. A. Marshall (1998), “Monetary Policy and the Term Structure of Nominal Interest Rates: Evidence and Theory,” *Carnegie-Rochester Conference Series on Public Policy* 49(1), 53-111.
- [28] Evans, C. L. and D. A. Marshall (2007), “Economic Determinants of the Nominal Treasury Yield Curve,” *Journal of Monetary Economics* 54(7), 1986-2003.
- [29] Fama, E. (1984), “Forward and Spot Exchange Rates,” *Journal of Monetary Economics* 14, 319-338.
- [30] Fama, E. F. and K. R. French (1988), “Dividend Yields and Expected Stock Returns,” *Journal of Financial Economics* 33, 3-25.
- [31] Flood, R. P. and M. P. Taylor (1996), “Exchange Rate Economics: What’s Wrong with the Conventional Macro Approach?,” NBER Chapters, in: *The Microstructure of Foreign Exchange Markets*, 261-302.
- [32] Frachot, A. (1996), “A reexamination of the uncovered interest parity hypothesis,” *Journal of International Money and Finance* 15(3), 419-437.
- [33] Froot, K. and R. Thaler (1990), “Anomalies: Foreign Exchange,” *Journal of Economic Perspectives* 4(3), 179-192.
- [34] Girsanov, I. (1958), “On Transforming a Certain Class of Stochastic Processes by Absolutely Continuous Substitution of Measures,” *Theory Probability and Its Applications* 5, 285-301.
- [35] Gürkaynak, R. and J. Wright (2012), “Macroeconomics and the Term Structure,” *Journal of Economic Literature* 50(2), 331-67.
- [36] Hansen, L. and R. Hodrick (1983), “Risk Averse Speculation in the Forward Exchange Markets: An Econometric Analysis of Linear Models,” In J. A. Frenkel (Ed.), *Exchange Rates and International Economics*. Chicago: University of Chicago Press.
- [37] Hodrick, R. (1987), “The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets,” Chur, Switzerland: Harwood Academic Publishers.
- [38] Inci, A. and B. Lu (2004), “Exchange Rates and Interest Rates: Can Term Structure Models Explain Currency Movements?,” *Journal of Economic Dynamics & Control* 28, 1595-1624.
- [39] Joslin, S., A. Le, and K. Singleton (2013), “Why Gaussian Macro-Finance Term Structure Models Are (Nearly) Unconstrained Factor-VARs,” *Journal of Financial Economics* 109, 604-622.

- [40] Joslin, S., K. Singleton, and H. Zhu (2011), “A New Perspective on Gaussian Dynamic Term Structure Models,” *Review of Financial Studies* 24, 926-970.
- [41] Lewis, K. (1995), “Puzzles in International Financial Markets,” in G. Grossman and K. Rogoff, eds. *Handbook of International Economics* (North Holland: Amsterdam).
- [42] Li, J. and W. Yin (2014), “Macroeconomic Fundamentals and the Exchange Rate Dynamics: A No-Arbitrage Macro-Finance Approach,” *Journal of International Money and Finance* 41, 46-64.
- [43] Litterman, R. and J. Scheinkman (1991), “Common factors affecting bond returns,” *Journal of Fixed Income* 1, 54-61.
- [44] Lustig, H., N. Roussanov, and A. Verdelhan (2011), “Common Risk Factors in Currency Markets,” *Review of Financial Studies* 24 (11), 3731-3777.
- [45] Mark, N. C. (1995), “Exchange Rates and Fundamentals: Evidence on Long-Horizon Predictability,” *American Economic Review* 85(1), 201-218.
- [46] Meese, R. A. and K. Rogoff (1983), “Empirical Exchange Rate Models of the Seventies: Do they Fit Out of Sample?,” *Journal of International Economics* 14, 3-24.
- [47] Pericoli, M. and M. Taboga (2011), “Bond risk premia, macroeconomic fundamentals and the exchange rate,” *International Review of Economics & Finance* 22(1), 42-65.
- [48] Piazzesi, M. (2010), “Affine Term Structure Models,” *Handbook of Financial Econometrics* 12, 691-766, Elsevier.
- [49] Piazzesi, M. and M. Schneider (2007), “Equilibrium Yield Curves,” in Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, *NBER Macroeconomics Annual 2006*, Cambridge MA: MIT Press p. 389-442.
- [50] Singleton, K. J. (2006), “Empirical Dynamic Asset Pricing: Model Specification and Econometric Assessment,” Princeton University Press 2006.
- [51] Vasicek, O. (1977), “An Equilibrium Characterization of the Term Structure,” *Journal of Financial Economics* 5, 177-188.
- [52] Wu, T. (2006), “Macro Factors and the Affine Term Structure of Interest Rates,” *Journal of Money, Credit, and Banking* 38(7), 1847-1875.
- [53] Zellner, A. (1962), “An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation,” *Journal of the American Statistical Association* 57(298), 348-368.

## Tables and Figures

Table 1: Summary Statistics of One-Year Yields

Country	Mean	Median	Min.	Max.	Std. dev.	Skewness	Kurtosis	$\hat{\rho}(12)$
<i>Australia</i>	4.82	4.90	2.28	7.07	1.21	-0.42	2.48	0.35
<i>Canada</i>	2.85	2.82	0.49	6.34	1.64	0.30	1.99	0.66
<i>Japan</i>	0.20	0.12	0.01	0.81	0.21	1.34	3.54	0.56
<i>Norway</i>	3.80	2.88	1.26	7.59	2.04	0.43	1.64	0.66
<i>Sweden</i>	2.61	2.61	0.32	4.87	1.41	-0.05	1.61	0.58
<i>Switzerland</i>	1.43	0.96	0.01	3.97	1.13	0.63	2.08	0.56
<i>United Kingdom</i>	3.30	4.25	0.12	6.52	2.12	-0.37	1.51	0.74
<i>United States</i>	2.38	1.88	0.10	6.30	2.05	0.44	1.76	0.67

Note: Monthly interest rate data for one-year yields (in %) from January 1999 to January 2014, data source: Bloomberg, L.P. 2014.  $\hat{\rho}(12)$  refers to the autocorrelation function at the 12-month lag.

Table 2: Correlation of One-Year Yields

Country	AUL	CAN	JAP	NOR	SWE	SWI	UK	US
<i>Australia</i>	1							
<i>Canada</i>	0.71	1						
<i>Japan</i>	0.50	0.29	1					
<i>Norway</i>	0.53	0.76	0.26	1				
<i>Sweden</i>	0.70	0.82	0.30	0.92	1			
<i>Switzerland</i>	0.71	0.77	0.61	0.80	0.81	1		
<i>United Kingdom</i>	0.81	0.92	0.30	0.72	0.85	0.77	1	
<i>United States</i>	0.70	0.95	0.36	0.64	0.69	0.77	0.89	1

Note: Correlation of monthly interest rate data for one-year yields from January 1999 to January 2014, data source: Bloomberg, L.P. 2014.

Table 3: Correlation of 3-Month Yields With Other Maturities

Country	6 months	1 year	2 years	3 years	4 years	5 years	6 years	7 years	8 years	9 years	10 years
<i>AUL 3m</i>	1.00	0.98	0.93	0.88	0.85	0.82	0.80	0.77	0.75	0.73	0.72
<i>CAN 3m</i>	1.00	0.99	0.96	0.93	0.90	0.88	0.85	0.83	0.80	0.78	0.77
<i>JAP 3m</i>	0.99	0.97	0.89	0.81	0.73	0.64	0.56	0.46	0.40	0.39	0.37
<i>NOR 3m</i>	1.00	0.98	0.96	0.93	0.91	0.90	0.88	0.87	0.86	0.84	0.84
<i>SWE 3m</i>	1.00	0.99	0.95	0.91	0.88	0.86	0.84	0.83	0.81	0.80	0.79
<i>SWI 3m</i>	1.00	0.99	0.94	0.92	0.89	0.87	0.84	0.82	0.80	0.79	0.78
<i>UK 3m</i>	1.00	0.99	0.98	0.96	0.95	0.93	0.92	0.91	0.89	0.86	0.84
<i>USA 3m</i>	1.00	0.99	0.98	0.96	0.94	0.91	0.90	0.88	0.86	0.84	0.81

Note: Correlation of 3-month yields with interest rates at different maturities from January 1999 to January 2014, data source: Bloomberg, L.P. 2014.

Table 4: Summary Statistics of Exchange Rates Expressed in USD

Currency	Mean	Median	Min.	Max.	Std. dev.	Skewness	Kurtosis	$\hat{\rho}(12)$
<i>AUD</i>	0.78	0.76	0.49	1.10	0.17	0.11	1.92	0.75
<i>CAD</i>	0.84	0.86	0.62	1.06	0.14	-0.15	1.53	0.79
<i>JPY</i>	0.01	0.01	0.01	0.01	0.00	0.83	2.57	0.76
<i>NOK</i>	0.15	0.16	0.11	0.20	0.02	-0.31	2.16	0.68
<i>SEK</i>	0.13	0.14	0.09	0.17	0.02	-0.39	2.25	0.62
<i>CHF</i>	0.84	0.82	0.56	1.27	0.18	0.20	2.07	0.78
<i>GBP</i>	1.66	1.61	1.41	2.08	0.17	0.65	2.37	0.57

Note: Exchange rate data from January 1999 to January 2014, source: Bloomberg, L.P. 2014. All currencies for Australia, Canada, Japan, Norway, Sweden, Switzerland, and the United Kingdom are expressed in terms of the U.S. dollar.  $\hat{\rho}(12)$  refers to the autocorrelation function at lag 12.

Table 5: Summary Statistics of Interest Rate Factors

Factors:	Median			Minimum			Maximum			Std. dev.			$\hat{\rho}(12)$			% Variance Explained			
	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	Total
<i>Australia</i>	0.6	0.1	-0.0	-8.3	-2.4	-0.7	5.7	1.7	0.6	3.4	0.9	0.2	0.4	0.1	-0.1	92.9	6.7	0.3	99.9
<i>Canada</i>	0.7	0.1	0.0	-5.4	-2.1	-0.5	6.9	1.4	0.4	3.4	0.9	0.2	0.8	0.4	-0.1	93.8	6.0	0.2	100
<i>Japan</i>	-0.8	0.2	0.1	-4.9	-2.9	-1.4	7.9	3.1	0.8	3.1	1.4	0.4	0.5	0.4	-0.1	81.7	16.9	1.1	99.7
<i>Norway</i>	-0.5	0.1	0.0	-5.5	-1.8	-0.5	6.2	1.5	0.7	3.4	0.7	0.2	0.7	0.1	0.1	95.5	4.0	0.4	99.9
<i>Sweden</i>	0.6	0.0	0.0	-5.9	-1.9	-0.5	5.8	1.6	0.5	3.4	0.8	0.2	0.7	0.1	0.1	94.3	5.3	0.4	99.9
<i>Switzerland</i>	-0.3	0.0	-0.0	-5.5	-1.8	-0.5	7.0	1.9	1.3	3.4	0.8	0.3	0.7	0.3	0.2	93.4	5.8	0.6	99.8
<i>United Kingdom</i>	1.4	0.1	0.0	-6.8	-1.7	-0.6	5.4	1.4	0.4	3.4	0.7	0.2	0.7	0.3	-0.3	96.2	3.5	0.2	99.9
<i>United States</i>	-0.1	-0.0	-0.0	-5.4	-1.6	-0.4	7.1	1.3	0.4	3.4	0.7	0.2	0.7	0.3	-0.2	95.3	4.5	0.2	100

Note: Interest rate factors are the principal components extracted from each individual country. Principal component analysis is computed on the covariance matrix of standardized monthly interest rate data from January 1999 to January 2014. There are 12 yields per country (maturities 3 months, 6 months, and 1 to 10 years) and 181 observations per yield. 12 components are calculated but only the first 3 are reported. Mean of each factor is 0 by construction.  $\hat{\rho}(12)$  refers to the autocorrelation function at 12 months. % variance explained refers to the percentage of variance explained by each factor, whereas *Total* accounts for the cumulative variation of the first three factors combined.

Table 6: Parameters of Term Structure Model

Parameters:	$K_{0\mathcal{P}}^{\mathbb{P}}$ (S.E.)		$[K_{1\mathcal{P}}^{\mathbb{P}} + I]$ (S.E.)						$\Sigma_{\mathcal{P}}$ (S.E.)				
<i>Australia</i>	0.69 (0.61)		0.980 (0.02)	-0.12 (0.06)	-0.37 (0.31)				0.86 (0.25)				
	-0.07 (0.23)		0.02 (0.01)	0.94 (0.03)	-0.22 (0.12)			-0.11 (0.04)		0.31 (0.07)			
	0.35 (0.09)		0.00 (0.00)	0.01 (0.01)	0.78 (0.05)			-0.02 (0.04)		-0.02 (0.02)	0.13 (0.01)		
<i>Canada</i>	0.14 (0.22)		0.992 (0.01)	0.01 (0.04)	-0.14 (0.20)			0.69 (0.07)					
	0.14 (0.10)		0.00 (0.00)	0.97 (0.02)	-0.35 (0.09)			-0.11 (0.03)		0.28 (0.01)			
	0.14 (0.05)		0.00 (0.00)	0.01 (0.01)	0.78 (0.05)			-0.07 (0.02)		0.01 (0.01)	0.14 (0.01)		
<i>Japan</i>	0.45 (0.14)		0.943 (0.02)	0.01 (0.06)	-0.65 (0.23)			0.32 (0.04)					
	-0.09 (0.05)		0.01 (0.01)	0.95 (0.02)	0.05 (0.08)			-0.04 (0.05)		0.10 (0.01)			
	0.04 (0.02)		0.01 (0.00)	0.00 (0.01)	0.87 (0.04)			0.00 (0.03)		-0.02 (0.01)	0.05 (0.00)		
<i>Norway</i>	0.36 (0.23)		0.995 (0.01)	-0.02 (0.04)	-0.43 (0.13)			0.69 (0.06)					
	0.01 (0.15)		0.00 (0.01)	0.91 (0.03)	-0.32 (0.08)			-0.12 (0.03)		0.42 (0.02)			
	0.18 (0.07)		0.00 (0.00)	0.01 (0.01)	0.83 (0.04)			-0.05 (0.01)		0.01 (0.02)	0.20 (0.01)		
<i>Sweden</i>	0.15 (0.21)		0.993 (0.01)	-0.09 (0.05)	-0.42 (0.17)			0.69 (0.06)					
	0.02 (0.10)		0.01 (0.01)	0.94 (0.02)	-0.33 (0.08)			-0.11 (0.04)		0.32 (0.01)			
	0.10 (0.05)		0.00 (0.00)	0.01 (0.01)	0.86 (0.04)			-0.03 (0.03)		0.02 (0.01)	0.15 (0.00)		
<i>Switzerland</i>	0.22 (0.13)		0.996 (0.01)	-0.02 (0.04)	0.41 (0.13)			0.48 (0.03)					
	0.01 (0.07)		0.00 (0.01)	0.96 (0.02)	0.14 (0.07)			-0.08 (0.02)		0.24 (0.03)			
	-0.08 (0.04)		-0.00 (0.00)	0.00 (0.01)	0.84 (0.04)			0.04 (0.01)		-0.04 (0.01)	0.14 (0.00)		
<i>United Kingdom</i>	0.89 (0.37)		0.997 (0.01)	0.06 (0.05)	-0.43 (0.18)			0.66 (0.14)					
	-0.31 (0.21)		0.00 (0.00)	0.92 (0.03)	-0.02 (0.10)			-0.18 (0.07)		0.33 (0.02)			
	0.42 (0.10)		0.00 (0.00)	-0.00 (0.01)	0.74 (0.05)			0.02 (0.05)		0.01 (0.01)	0.18 (0.00)		
<i>United States</i>	0.94 (0.35)		0.992 (0.01)	0.05 (0.05)	-0.62 (0.22)			0.78 (0.07)					
	-0.15 (0.21)		0.00 (0.01)	0.92 (0.03)	-0.19 (0.13)			-0.31 (0.03)		0.35 (0.05)			
	0.18 (0.06)		0.00 (0.00)	0.00 (0.01)	0.84 (0.04)			-0.03 (0.01)		-0.00 (0.03)	0.14 (0.02)		

Note: Estimates and asymptotic standard errors in parentheses (S.E.) for country  $i$  from  $\Delta\mathcal{P}_t^i = K_{0\mathcal{P}}^{\mathbb{P},i} + K_{1\mathcal{P}}^{\mathbb{P},i} \cdot \mathcal{P}_{t-1}^i + \Sigma_{\mathcal{P}}^i \epsilon_t^{\mathbb{P},i}$ , where  $\mathcal{P}_t$  are the interest rate observable factors and  $\mathbb{P}$  represents the physical distribution. All parameters are annualized,  $K_{0\mathcal{P}}^{\mathbb{P}} \times 100$  and  $\Sigma_{\mathcal{P}} \times 100$  are reported, as well as  $K_{1\mathcal{P}}^{\mathbb{P}} + I$ .

Table 7: Parameters of Term Structure Model and Fit of Yields

Parameters:	$\rho_{0\mathcal{P}}$	$\rho_{1\mathcal{P}}$			$eig(K_{1\mathcal{P}}^{\mathbb{Q}} + I)$			RMSE (yields)
<i>Australia</i>	-0.002	0.265	0.579	0.825	0.998	0.928	0.927	0.049
<i>Canada</i>	-0.013	0.346	0.344	1.126	0.966	0.963	0.870	0.109
<i>Japan</i>	-0.002	0.053	0.402	0.821	0.993	0.990	0.933	0.047
<i>Norway</i>	-0.008	0.383	0.475	0.831	0.959	0.952	0.946	0.104
<i>Sweden</i>	-0.010	0.323	0.458	1.002	0.969	0.964	0.899	0.110
<i>Switzerland</i>	-0.009	0.343	0.443	-1.054	0.956	0.951	0.946	0.118
<i>United Kingdom</i>	-0.017	0.346	0.338	1.183	0.975	0.921	0.907	0.094

Note: Estimates for country  $i$  from equations  $r_t^i = \rho_{0\mathcal{P}}^i + \rho_{1\mathcal{P}}^{i'} \cdot \mathcal{P}_t^i$  and  $\Delta\mathcal{P}_t^i = K_{0\mathcal{P}}^{\mathbb{Q},i} + K_{1\mathcal{P}}^{\mathbb{Q},i} \cdot \mathcal{P}_{t-1}^i + \Sigma_{\mathcal{P}}^i \epsilon_t^{\mathbb{Q},i}$ , where  $r_t$  is the risk-free rate,  $\mathcal{P}_t$  are the interest rate observable factors, and  $\mathbb{Q}$  represents the risk-neutral distribution. All parameters are annualized. Root mean square errors (RMSE) are in basis points.

Table 8: Model Fit for Exchange Rate Changes (in USD)

Horizon:		1-month	3-month	6-month	1-year	1.5-year	2-year	3-year	4-year	5-year
<i>Australia</i>	$\beta$	0.492	0.668	0.827	0.977	1.018	1.023	1.024	1.033	1.099
	Robust	(0.197)**	(0.150)***	(0.115)***	(0.070)***	(0.041)***	(0.040)***	(0.047)***	(0.074)***	(0.072)***
	N.W.	(0.228)**	(0.204)***	(0.199)***	(0.131)***	(0.066)***	(0.046)***	(0.062)***	(0.072)***	(0.056)***
	$R^2$	0.023	0.113	0.259	0.567	0.788	0.770	0.774	0.593	0.718
<i>Canada</i>	$\beta$	0.466	0.528	0.646	0.961	1.080	1.055	1.029	0.990	1.038
	Robust	(0.208)**	(0.136)***	(0.108)***	(0.100)***	(0.062)***	(0.057)***	(0.039)***	(0.042)***	(0.064)***
	N.W.	(0.197)**	(0.181)***	(0.186)***	(0.216)***	(0.125)***	(0.091)***	(0.066)***	(0.065)***	(0.062)***
	$R^2$	0.019	0.074	0.158	0.451	0.667	0.698	0.771	0.725	0.767
<i>Japan</i>	$\beta$	0.890	0.859	0.875	1.004	1.033	1.040	1.005	0.958	1.026
	Robust	(0.211)***	(0.142)***	(0.109)***	(0.066)***	(0.052)***	(0.045)***	(0.030)***	(0.035)***	(0.040)***
	N.W.	(0.224)***	(0.213)***	(0.183)***	(0.147)***	(0.114)***	(0.097)***	(0.060)***	(0.051)***	(0.076)***
	$R^2$	0.071	0.159	0.275	0.614	0.728	0.805	0.878	0.854	0.822
<i>Norway</i>	$\beta$	0.515	0.644	0.851	1.147	1.098	1.090	1.029	1.054	1.007
	Robust	(0.208)**	(0.163)***	(0.157)***	(0.089)***	(0.050)***	(0.051)***	(0.046)***	(0.058)***	(0.046)***
	N.W.	(0.201)**	(0.248)**	(0.267)***	(0.161)***	(0.090)***	(0.047)***	(0.038)***	(0.051)***	(0.027)***
	$R^2$	0.026	0.094	0.204	0.494	0.748	0.726	0.729	0.646	0.786
<i>Sweden</i>	$\beta$	0.523	0.874	0.960	1.003	1.025	1.096	1.123	0.985	0.994
	Robust	(0.212)**	(0.129)***	(0.096)***	(0.052)***	(0.039)***	(0.039)***	(0.032)***	(0.042)***	(0.039)***
	N.W.	(0.212)**	(0.190)***	(0.181)***	(0.106)***	(0.090)***	(0.081)***	(0.047)***	(0.036)***	(0.062)***
	$R^2$	0.045	0.295	0.490	0.703	0.825	0.827	0.871	0.793	0.832
<i>Switzerland</i>	$\beta$	0.413	0.682	0.793	0.923	0.994	1.089	1.008	1.087	0.921
	Robust	(0.262)	(0.179)***	(0.117)***	(0.077)***	(0.056)***	(0.062)***	(0.061)***	(0.070)***	(0.076)***
	N.W.	(0.268)	(0.233)***	(0.202)***	(0.137)***	(0.092)***	(0.090)***	(0.084)***	(0.062)***	(0.066)***
	$R^2$	0.006	0.078	0.195	0.418	0.689	0.737	0.682	0.622	0.550
<i>United Kingdom</i>	$\beta$	0.577	0.821	0.862	0.925	0.997	0.985	1.008	1.007	1.039
	Robust	(0.190)***	(0.134)***	(0.122)***	(0.074)***	(0.042)***	(0.039)***	(0.024)***	(0.029)***	(0.022)***
	N.W.	(0.174)***	(0.217)***	(0.226)***	(0.144)***	(0.090)***	(0.058)***	(0.033)***	(0.033)***	(0.019)***
	$R^2$	0.045	0.194	0.300	0.606	0.820	0.797	0.910	0.865	0.916

Note: Estimates for  $\Delta s_{t+k}^{obs} = \alpha + \beta \Delta s_{t+k} + \mu_{t+k}$ , where  $\Delta s_{t+k}^{obs}$  is the observed annualized log exchange rate change in USD between  $t$  and  $t+k$  and  $\Delta s_{t+k}$  is the model-implied exchange rate change, modeled as the ratio of two countries' (log) pricing kernels, where the domestic country is the U.S. *Robust* and *N.W.* refer to robust (White) standard errors and Newey-West standard errors, where the lag is equal to the horizon  $k+1$  (in months). The significance level of the coefficient is determined by (\*) given the corresponding method to estimate standard errors, where \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .  $R^2$  is adjusted for degrees of freedom.

Table 9: Model Fit for Exchange Rate Changes (Bias-Corrected Estimates)

Horizon:		1-month		6-month		1-year		3-year		5-year	
Bias-Correction:		mean	median								
<i>Australia</i>	$\beta$	0.569**	0.566**	0.860***	0.858***	0.983***	0.983***	1.060***	1.058***	1.116***	1.115***
	N.W.	(0.244)	(0.243)	(0.212)	(0.212)	(0.147)	(0.146)	(0.072)	(0.072)	(0.091)	(0.089)
	$R^2$	0.022	0.022	0.229	0.230	0.510	0.513	0.773	0.773	0.670	0.672
<i>Canada</i>	$\beta$	0.476**	0.461**	0.678***	0.628***	0.978***	0.956***	1.057***	1.038***	1.054***	1.042***
	N.W.	(0.217)	(0.199)	(0.205)	(0.188)	(0.213)	(0.218)	(0.071)	(0.073)	(0.081)	(0.079)
	$R^2$	0.014	0.017	0.153	0.147	0.438	0.438	0.752	0.756	0.763	0.759
<i>Japan</i>	$\beta$	0.764***	0.813***	0.802***	0.826***	0.955***	0.964***	1.013***	1.014***	1.037***	1.038***
	N.W.	(0.203)	(0.213)	(0.174)	(0.178)	(0.130)	(0.133)	(0.063)	(0.063)	(0.069)	(0.069)
	$R^2$	0.062	0.065	0.261	0.266	0.608	0.608	0.871	0.874	0.824	0.825
<i>Norway</i>	$\beta$	0.649**	0.652**	1.021***	1.021***	1.109***	1.109***	1.031***	1.032***	0.972***	0.972***
	N.W.	(0.307)	(0.308)	(0.259)	(0.259)	(0.157)	(0.156)	(0.030)	(0.029)	(0.037)	(0.037)
	$R^2$	0.048	0.048	0.356	0.357	0.571	0.572	0.824	0.824	0.809	0.810
<i>Sweden</i>	$\beta$	0.490***	0.497***	0.763***	0.785***	0.951***	0.958***	1.185***	1.178***	1.060***	1.052***
	N.W.	(0.163)	(0.166)	(0.216)	(0.214)	(0.142)	(0.139)	(0.105)	(0.093)	(0.108)	(0.101)
	$R^2$	0.043	0.044	0.315	0.333	0.591	0.606	0.803	0.814	0.753	0.762
<i>Switzerland</i>	$\beta$	0.225	0.228	0.663***	0.660***	0.967***	0.966***	1.029***	1.028***	0.920***	0.920***
	N.W.	(0.250)	(0.248)	(0.216)	(0.215)	(0.116)	(0.114)	(0.090)	(0.089)	(0.092)	(0.094)
	$R^2$	0.000	0.000	0.162	0.162	0.436	0.438	0.719	0.719	0.574	0.575
<i>United Kingdom</i>	$\beta$	0.574***	0.576***	0.836***	0.836***	0.916***	0.917***	1.015***	1.015***	1.043***	1.042***
	N.W.	(0.171)	(0.171)	(0.227)	(0.226)	(0.147)	(0.146)	(0.033)	(0.033)	(0.018)	(0.018)
	$R^2$	0.042	0.043	0.279	0.279	0.584	0.586	0.910	0.909	0.912	0.912

Note: Estimates for  $\Delta s_{t+k}^{obs} = \alpha + \beta \Delta s_{t+k} + \mu_{t+k}$ , where  $\Delta s_{t+k}^{obs}$  is the observed annualized log exchange rate change in USD between  $t$  and  $t+k$  and  $\Delta s_{t+k}$  is the model-implied exchange rate change, modeled as the ratio of two countries' (log) pricing kernels, where the domestic country is the U.S. *N.W.* refers to Newey-West standard errors, where the lag is equal to the horizon  $k+1$  (in months). The significance level of the coefficient is determined by (\*) where \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .  $R^2$  is adjusted for degrees of freedom. Model is either mean or median-corrected to account for small-sample bias, as suggested by Bauer, Rudebusch, and Wu (2012, 2014).

Table 10: Model Fit for Exchange Rate Changes (One- and Two-Factor Model)

Horizon:		1-month		6-month		1-year		3-year		5-year	
Number of Factors:		1-factor	2-factor								
<i>Australia</i>	$\beta$	0.543	0.679	0.799	0.893	0.947	1.017	1.082	1.258	1.024	1.145
	Robust	(0.394)	(0.291)**	(0.139)***	(0.125)***	(0.099)***	(0.084)***	(0.087)***	(0.061)***	(0.127)***	(0.098)***
	N.W.	(0.398)	(0.271)**	(0.257)***	(0.218)***	(0.245)***	(0.166)***	(0.270)***	(0.133)***	(0.183)***	(0.193)***
	$R^2$	0.005	0.015	0.094	0.142	0.254	0.339	0.537	0.713	0.316	0.474
<i>Canada</i>	$\beta$	0.173	0.539	0.486	0.846	0.663	1.181	1.032	1.120	1.249	1.155
	Robust	(0.462)	(0.240)**	(0.177)***	(0.135)***	(0.118)***	(0.116)***	(0.060)***	(0.053)***	(0.098)***	(0.094)***
	N.W.	(0.477)	(0.214)**	(0.366)	(0.237)***	(0.274)**	(0.262)***	(0.121)***	(0.128)***	(0.126)***	(0.143)***
	$R^2$	0.000	0.013	0.021	0.159	0.099	0.417	0.473	0.684	0.640	0.706
<i>Japan</i>	$\beta$	0.740	0.560	0.700	0.558	0.771	0.690	0.987	1.037	1.057	1.149
	Robust	(0.277)***	(0.207)***	(0.159)***	(0.121)***	(0.128)***	(0.080)***	(0.068)***	(0.042)***	(0.067)***	(0.070)***
	N.W.	(0.303)**	(0.220)**	(0.308)**	(0.227)**	(0.360)**	(0.207)***	(0.238)***	(0.057)***	(0.180)***	(0.196)***
	$R^2$	0.028	0.027	0.110	0.115	0.218	0.283	0.471	0.624	0.668	0.658
<i>Norway</i>	$\beta$	0.130	0.965	0.118	1.125	0.344	1.269	0.619	0.990	1.606	0.985
	Robust	(1.448)	(0.374)**	(0.551)	(0.236)***	(0.397)	(0.118)***	(0.195)***	(0.070)***	(0.258)***	(0.049)***
	N.W.	(1.336)	(0.433)**	(1.144)	(0.418)***	(0.978)	(0.196)***	(0.360)*	(0.123)***	(0.231)***	(0.067)***
	$R^2$	0.000	0.044	0.000	0.202	0.000	0.388	0.033	0.568	0.245	0.732
<i>Sweden</i>	$\beta$	0.023	0.815	0.519	0.882	0.695	1.033	1.166	1.186	1.473	1.073
	Robust	(0.595)	(0.299)***	(0.295)*	(0.151)***	(0.216)***	(0.098)***	(0.178)***	(0.074)***	(0.355)***	(0.053)***
	N.W.	(0.669)	(0.314)**	(0.608)	(0.287)***	(0.569)	(0.175)***	(0.693)*	(0.227)***	(0.811)*	(0.098)***
	$R^2$	0.000	0.042	0.012	0.194	0.050	0.397	0.244	0.627	0.211	0.703
<i>Switzerland</i>	$\beta$	1.197	0.528	1.296	0.596	1.332	0.930	1.038	1.025	0.437	1.163
	Robust	(1.006)	(0.377)	(0.308)***	(0.185)***	(0.239)***	(0.128)***	(0.142)***	(0.110)***	(0.177)**	(0.111)***
	N.W.	(0.751)	(0.401)	(0.613)**	(0.327)*	(0.571)**	(0.264)***	(0.406)**	(0.348)***	(0.325)	(0.116)***
	$R^2$	0.007	0.006	0.091	0.063	0.182	0.245	0.296	0.444	0.033	0.451
<i>United Kingdom</i>	$\beta$	-0.726	0.644	-0.258	0.711	0.165	0.950	0.785	1.093	1.700	1.114
	Robust	(0.718)	(0.252)**	(0.356)	(0.150)***	(0.238)	(0.105)***	(0.163)***	(0.044)***	(0.255)***	(0.034)***
	N.W.	(0.810)	(0.222)***	(0.774)	(0.254)***	(0.681)	(0.228)***	(0.648)	(0.103)***	(0.528)***	(0.053)***
	$R^2$	0.000	0.027	0.000	0.118	0.000	0.347	0.105	0.780	0.451	0.841

Note: Estimates for  $\Delta s_{t+k}^{obs} = \alpha + \beta \Delta s_{t+k} + \mu_{t+k}$ , where  $\Delta s_{t+k}^{obs}$  is the observed annualized log exchange rate change in USD between  $t$  and  $t+k$  and  $\Delta s_{t+k}$  is the model-implied exchange rate change, modeled as the ratio of two countries' (log) pricing kernels, where the domestic country is the U.S. *Robust* and *N.W.* refer to robust (White) standard errors and Newey-West standard errors, where the lag is equal to the horizon  $k+1$  (in months). The significance level of the coefficient is determined by (\*) given the corresponding method to estimate standard errors, where \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .  $R^2$  is adjusted for degrees of freedom. The 1-factor model is estimated with the first factor extracted from interest rate data in each country (level), and the 2-factor model, with the first two factors (level and slope).

Table 11: Model Fit for Exchange Rate Changes (in GBP)

Horizon:		1-month	3-month	6-month	1-year	1.5-year	2-year	3-year	4-year	5-year
<i>Australia</i>	$\beta$	0.720	0.823	0.882	0.989	0.998	1.003	1.003	0.993	1.014
	Robust	(0.391)*	(0.154)***	(0.114)***	(0.060)***	(0.039)***	(0.047)***	(0.035)***	(0.031)***	(0.031)***
	N.W.	(0.291)**	(0.183)***	(0.169)***	(0.070)***	(0.042)***	(0.056)***	(0.043)***	(0.034)***	(0.022)***
	$R^2$	0.038	0.159	0.332	0.619	0.761	0.791	0.854	0.883	0.892
<i>Canada</i>	$\beta$	1.130	1.110	1.092	1.096	1.033	0.971	1.021	1.001	1.028
	Robust	(0.282)***	(0.169)***	(0.114)***	(0.061)***	(0.052)***	(0.050)***	(0.039)***	(0.036)***	(0.035)***
	N.W.	(0.239)***	(0.212)***	(0.172)***	(0.099)***	(0.078)***	(0.051)***	(0.031)***	(0.034)***	(0.022)***
	$R^2$	0.083	0.219	0.327	0.644	0.722	0.752	0.853	0.877	0.867
<i>Japan</i>	$\beta$	0.361	0.631	0.728	0.920	1.038	1.037	1.020	1.014	1.016
	Robust	(0.158)**	(0.120)***	(0.108)***	(0.069)***	(0.054)***	(0.040)***	(0.022)***	(0.020)***	(0.019)***
	N.W.	(0.147)**	(0.153)***	(0.169)***	(0.130)***	(0.105)***	(0.061)***	(0.029)***	(0.024)***	(0.027)***
	$R^2$	0.027	0.173	0.304	0.623	0.794	0.852	0.899	0.928	0.926
<i>Norway</i>	$\beta$	0.558	0.490	0.625	1.090	1.093	1.051	0.965	1.056	0.998
	Robust	(0.226)**	(0.142)***	(0.123)***	(0.083)***	(0.058)***	(0.050)***	(0.053)***	(0.049)***	(0.044)***
	N.W.	(0.238)**	(0.215)**	(0.241)**	(0.145)***	(0.081)***	(0.060)***	(0.061)***	(0.063)***	(0.047)***
	$R^2$	0.034	0.072	0.159	0.497	0.694	0.707	0.704	0.770	0.807
<i>Sweden</i>	$\beta$	0.438	0.563	0.582	0.912	1.078	1.082	1.052	0.991	1.056
	Robust	(0.268)	(0.137)***	(0.091)***	(0.094)***	(0.095)***	(0.099)***	(0.064)***	(0.059)***	(0.044)***
	N.W.	(0.239)*	(0.151)***	(0.142)***	(0.175)***	(0.212)***	(0.234)***	(0.143)***	(0.067)***	(0.049)***
	$R^2$	0.009	0.071	0.129	0.315	0.435	0.465	0.616	0.676	0.827
<i>Switzerland</i>	$\beta$	0.468	0.713	0.778	0.865	0.967	0.999	0.965	1.015	1.016
	Robust	(0.269)*	(0.157)***	(0.112)***	(0.066)***	(0.065)***	(0.039)***	(0.033)***	(0.031)***	(0.022)***
	N.W.	(0.206)**	(0.228)***	(0.206)***	(0.151)***	(0.122)***	(0.063)***	(0.053)***	(0.034)***	(0.026)***
	$R^2$	0.019	0.142	0.287	0.471	0.694	0.814	0.844	0.912	0.929
<i>United States</i>	$\beta$	0.576	0.821	0.862	0.925	0.997	0.986	1.009	1.007	1.039
	Robust	(0.188)***	(0.134)***	(0.122)***	(0.074)***	(0.042)***	(0.039)***	(0.024)***	(0.029)***	(0.022)***
	N.W.	(0.173)***	(0.218)***	(0.227)***	(0.144)***	(0.090)***	(0.058)***	(0.033)***	(0.033)***	(0.019)***
	$R^2$	0.045	0.194	0.300	0.606	0.819	0.796	0.910	0.865	0.916

Note: Estimates for  $\Delta s_{t+k}^{obs} = \alpha + \beta \Delta s_{t+k} + \mu_{t+k}$ , where  $\Delta s_{t+k}^{obs}$  is the observed annualized log exchange rate change in GBP between  $t$  and  $t+k$  and  $\Delta s_{t+k}$  is the model-implied exchange rate change, modeled as the ratio of two countries' (log) pricing kernels, where the domestic country is the U.K. *Robust* and *N.W.* refer to robust (White) standard errors and Newey-West standard errors, where the lag is equal to the horizon  $k+1$  (in months). The significance level of the coefficient is determined by (\*) given the corresponding method to estimate standard errors, where \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .  $R^2$  is adjusted for degrees of freedom.

Table 12: Uncovered Interest Parity Regressions

Horizon:		1-month			3-month			6-month		
		$\beta_{UIP}$	$\beta_{IRD}$	$\beta_{RP}$	$\beta_{UIP}$	$\beta_{IRD}$	$\beta_{RP}$	$\beta_{UIP}$	$\beta_{IRD}$	$\beta_{RP}$
<i>Australia</i>	$\beta$	-0.770	0.727	0.502**	-0.942	1.218	0.686***	-1.351	1.530	0.853***
	N.W.	(2.289)	(2.469)	(0.249)	(2.147)	(2.181)	(0.233)	(2.091)	(1.812)	(0.226)
	$R^2$	0.000	0.018		0.000	0.111		0.004	0.260	
<i>Canada</i>	$\beta$	0.733	1.444	0.475**	1.150	1.932	0.538***	0.842	1.751	0.654***
	N.W.	(2.514)	(2.479)	(0.201)	(2.635)	(2.486)	(0.184)	(2.750)	(2.268)	(0.191)
	$R^2$	0.000	0.014		0.000	0.075		0.000	0.160	
<i>Japan</i>	$\beta$	-0.526	1.128	0.876***	-0.492	1.136	0.860***	-0.588	1.048	0.877***
	N.W.	(1.279)	(1.264)	(0.235)	(1.215)	(1.164)	(0.224)	(1.180)	(1.000)	(0.194)
	$R^2$	0.000	0.062		0.000	0.152		0.001	0.268	
<i>Norway</i>	$\beta$	0.177	0.640	0.514**	0.309	1.047	0.643**	0.088	1.265	0.855***
	N.W.	(1.717)	(1.588)	(0.203)	(1.737)	(1.573)	(0.253)	(1.636)	(1.345)	(0.276)
	$R^2$	0.000	0.020		0.000	0.089		0.000	0.201	
<i>Sweden</i>	$\beta$	-0.162	0.201	0.524**	-0.309	0.508	0.870***	-0.515	0.575	0.954***
	N.W.	(1.987)	(1.687)	(0.212)	(1.974)	(1.292)	(0.193)	(1.989)	(1.082)	(0.186)
	$R^2$	0.000	0.039		0.000	0.289		0.000	0.485	
<i>Switzerland</i>	$\beta$	-1.216	-0.597	0.388	-1.452	-0.288	0.651***	-1.360	0.145	0.770***
	N.W.	(1.802)	(1.694)	(0.274)	(1.628)	(1.361)	(0.235)	(1.553)	(1.190)	(0.210)
	$R^2$	0.000	0.001		0.006	0.075		0.015	0.194	
<i>United Kingdom</i>	$\beta$	1.827	2.109	0.566***	1.836	2.322	0.810***	1.341	2.042	0.855***
	N.W.	(2.414)	(2.168)	(0.169)	(2.509)	(2.078)	(0.208)	(2.552)	(1.854)	(0.218)
	$R^2$	0.000	0.042		0.007	0.197		0.005	0.303	

Horizon:		1-year			2-year			3-year		
		$\beta_{UIP}$	$\beta_{IRD}$	$\beta_{RP}$	$\beta_{UIP}$	$\beta_{IRD}$	$\beta_{RP}$	$\beta_{UIP}$	$\beta_{IRD}$	$\beta_{RP}$
<i>Australia</i>	$\beta$	-2.264	1.289	0.990***	-3.206**	0.698*	1.000***	-2.921***	0.768***	0.996***
	N.W.	(1.936)	(1.138)	(0.157)	(1.511)	(0.382)	(0.041)	(1.112)	(0.269)	(0.048)
	$R^2$	0.049	0.566		0.184	0.772		0.251	0.774	
<i>Canada</i>	$\beta$	0.134	1.858	0.969***	-1.095	1.290**	1.058***	-1.523	1.193***	1.032***
	N.W.	(2.477)	(1.378)	(0.215)	(2.205)	(0.495)	(0.093)	(1.693)	(0.358)	(0.073)
	$R^2$	0.000	0.455		0.014	0.699		0.042	0.771	
<i>Japan</i>	$\beta$	-0.917	0.982	1.002***	-1.424	0.834***	1.021***	-1.492*	0.811***	0.984***
	N.W.	(1.167)	(0.640)	(0.154)	(1.005)	(0.282)	(0.096)	(0.835)	(0.192)	(0.059)
	$R^2$	0.026	0.611		0.112	0.804		0.174	0.878	
<i>Norway</i>	$\beta$	-0.337	1.393*	1.153***	-0.527	1.056***	1.084***	-0.356	0.963***	1.017***
	N.W.	(1.403)	(0.728)	(0.171)	(1.093)	(0.210)	(0.053)	(0.801)	(0.087)	(0.038)
	$R^2$	0.000	0.491		0.014	0.724		0.011	0.726	
<i>Sweden</i>	$\beta$	-1.091	0.381	0.992***	-1.657	0.518	1.069***	-1.752	0.870***	1.095***
	N.W.	(1.948)	(0.687)	(0.112)	(1.940)	(0.393)	(0.069)	(1.481)	(0.243)	(0.049)
	$R^2$	0.014	0.705		0.070	0.832		0.141	0.869	
<i>Switzerland</i>	$\beta$	-1.545	0.367	0.896***	-1.442	0.792***	1.065***	-1.437**	0.597***	0.954***
	N.W.	(1.322)	(0.677)	(0.142)	(1.017)	(0.245)	(0.081)	(0.600)	(0.150)	(0.078)
	$R^2$	0.045	0.420		0.083	0.737		0.169	0.688	
<i>United Kingdom</i>	$\beta$	-0.061	1.179	0.923***	-1.768	1.037*	0.984***	-3.613**	1.029***	1.007***
	N.W.	(2.538)	(1.202)	(0.144)	(2.125)	(0.592)	(0.068)	(1.425)	(0.246)	(0.036)
	$R^2$	0.000	0.604		0.043	0.795		0.232	0.910	

Note: The first column reports estimates for:  $\Delta s_{t+k}^{obs} = \alpha + \beta_{UIP}(r_{t+k-1}^d - r_{t+k-1}^f) + \mu_{t+k}$ . The second and third columns report the estimates for:  $\Delta s_{t+k}^{obs} = \alpha + \beta_{IRD}(r_{t+k-1}^d - r_{t+k-1}^f) + \beta_{RP}\left(\frac{1}{2}\left[\Lambda_{t+k-1}^{d'}\Lambda_{t+k-1}^d - \Lambda_{t+k-1}^{f'}\Lambda_{t+k-1}^f\right]\right) + \mu_{t+k}$ . *N.W.* refers to Newey-West standard errors where the lag is equal to the horizon  $k + 1$  (in months). Significance level: \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .  $R^2$  adjusted for degrees of freedom.

Table 13: Fama Conditions for the Risk Premium

Country		Condition I		Condition II	
		$var(IRD)$	$<$	$var(RP)$	$corr(IRD, RP) < 0$
<i>Australia</i>	1-month	0.02		2.62	-0.29
	6-month	0.83		61.54	-0.38
	1-year	3.03		176.33	-0.45
	2-year	9.36		396.59	-0.57
	3-year	15.90		448.62	-0.67
<i>Canada</i>	1-month	0.01		1.12	-0.12
	6-month	0.23		24.28	-0.13
	1-year	0.80		65.28	-0.19
	2-year	2.29		152.03	-0.27
	3-year	3.29		247.74	-0.30
<i>Japan</i>	1-month	0.04		1.22	-0.35
	6-month	1.47		31.86	-0.39
	1-year	5.60		100.85	-0.41
	2-year	18.27		270.38	-0.41
	3-year	30.94		386.22	-0.30
<i>Norway</i>	1-month	0.04		1.90	-0.13
	6-month	1.45		32.92	-0.29
	1-year	5.34		81.76	-0.40
	2-year	16.61		188.84	-0.48
	3-year	27.13		240.73	-0.53
<i>Sweden</i>	1-month	0.03		3.11	-0.07
	6-month	1.00		64.99	-0.14
	1-year	3.53		167.28	-0.21
	2-year	9.52		286.97	-0.37
	3-year	12.45		279.28	-0.47
<i>Switzerland</i>	1-month	0.02		1.02	-0.24
	6-month	0.79		25.06	-0.33
	1-year	2.79		76.19	-0.37
	2-year	7.83		162.28	-0.35
	3-year	11.86		202.23	-0.27
<i>United Kingdom</i>	1-month	0.01		1.37	-0.05
	6-month	0.43		30.19	-0.09
	1-year	1.53		92.62	-0.16
	2-year	4.46		215.58	-0.36
	3-year	7.49		288.39	-0.56

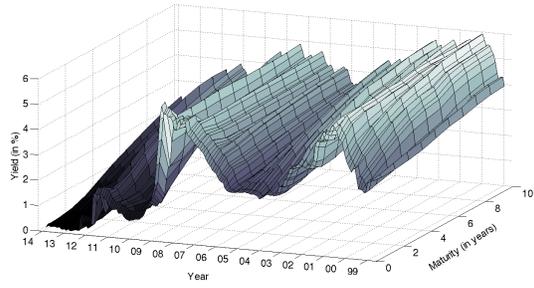
Note: Fama Conditions for the risk premium: (I) variance of  $IRD <$  variance of  $RP$ , and (II) correlation between  $IRD$  and  $RP < 0$ , where  $IRD_k = r_{t+k-1}^d - r_{t+k-1}^f$  is the interest rate differential and  $RP_k = \frac{1}{2} \left[ \Lambda_{t+k-1}^{d'} \Lambda_{t+k-1}^d - \Lambda_{t+k-1}^{f'} \Lambda_{t+k-1}^f \right]$  is the risk premium at horizon  $k$  in months. Variance reported  $\times 100$ .

Table 14: Model Fit for Exchange Rate Changes (Countries Under the Euro)

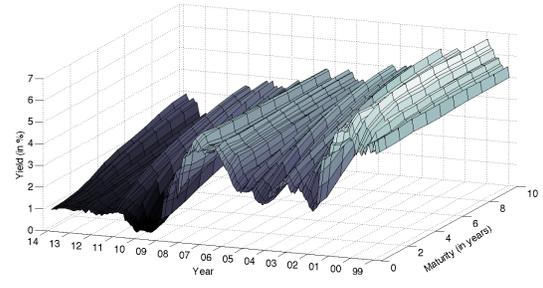
Horizon:		1-month	3-month	6-month	1-year	1.5-year	2-year	3-year	4-year	5-year
<i>Austria</i>	$\beta$	0.495**	0.687***	0.753***	0.907***	0.982***	1.089***	1.083***	1.023***	0.974***
	N.W.	(0.242)	(0.198)	(0.176)	(0.138)	(0.112)	(0.083)	(0.060)	(0.047)	(0.037)
	$R^2$	0.014	0.094	0.197	0.444	0.701	0.741	0.807	0.816	0.838
<i>Belgium</i>	$\beta$	0.433*	0.689***	0.846***	1.012***	1.076***	1.117***	1.049***	0.986***	0.953***
	N.W.	(0.225)	(0.194)	(0.190)	(0.136)	(0.093)	(0.083)	(0.067)	(0.044)	(0.023)
	$R^2$	0.010	0.091	0.230	0.504	0.782	0.779	0.810	0.818	0.870
<i>Finland</i>	$\beta$	0.531**	0.467**	0.531***	0.845***	1.047***	1.024***	1.108***	1.016***	1.014***
	N.W.	(0.223)	(0.211)	(0.176)	(0.124)	(0.096)	(0.115)	(0.075)	(0.069)	(0.030)
	$R^2$	0.022	0.048	0.112	0.403	0.724	0.648	0.777	0.750	0.853
<i>France</i>	$\beta$	0.523**	0.691***	0.777***	0.983***	1.041***	1.060***	1.071***	1.009***	0.960***
	N.W.	(0.257)	(0.239)	(0.215)	(0.148)	(0.102)	(0.106)	(0.091)	(0.061)	(0.025)
	$R^2$	0.015	0.085	0.189	0.474	0.746	0.727	0.798	0.796	0.863
<i>Germany</i>	$\beta$	0.524**	0.739***	0.818***	0.994***	1.044***	1.085***	1.071***	1.001***	0.960***
	N.W.	(0.265)	(0.236)	(0.208)	(0.143)	(0.108)	(0.105)	(0.090)	(0.061)	(0.025)
	$R^2$	0.014	0.091	0.194	0.458	0.724	0.727	0.796	0.793	0.861
<i>Ireland</i>	$\beta$	0.484*	0.673***	0.737***	0.826***	0.952***	1.084***	1.053***	1.062***	0.972***
	N.W.	(0.246)	(0.206)	(0.195)	(0.155)	(0.115)	(0.089)	(0.110)	(0.063)	(0.030)
	$R^2$	0.019	0.104	0.206	0.392	0.687	0.792	0.758	0.811	0.856
<i>Italy</i>	$\beta$	0.505***	0.662***	0.830***	0.969***	1.004***	1.046***	1.078***	1.015***	0.979***
	N.W.	(0.158)	(0.187)	(0.162)	(0.109)	(0.080)	(0.058)	(0.055)	(0.028)	(0.028)
	$R^2$	0.028	0.123	0.299	0.583	0.817	0.808	0.857	0.833	0.883
<i>Netherlands</i>	$\beta$	0.492*	0.725***	0.810***	0.988***	1.029***	1.061***	1.061***	0.999***	0.948***
	N.W.	(0.254)	(0.227)	(0.208)	(0.147)	(0.108)	(0.104)	(0.085)	(0.060)	(0.026)
	$R^2$	0.013	0.095	0.203	0.475	0.738	0.735	0.802	0.799	0.858
<i>Portugal</i>	$\beta$	0.223	0.488**	0.655***	0.874***	0.970***	0.981***	1.091***	1.026***	1.007***
	N.W.	(0.209)	(0.210)	(0.168)	(0.112)	(0.065)	(0.084)	(0.043)	(0.032)	(0.034)
	$R^2$	0.002	0.069	0.195	0.505	0.827	0.759	0.873	0.794	0.871
<i>Spain</i>	$\beta$	0.371	0.675***	0.770***	0.996***	1.050***	1.080***	1.082***	0.997***	0.998***
	N.W.	(0.250)	(0.217)	(0.191)	(0.113)	(0.068)	(0.049)	(0.050)	(0.030)	(0.017)
	$R^2$	0.009	0.101	0.210	0.512	0.817	0.810	0.865	0.796	0.886

Note: Estimates for  $\Delta s_{t+k}^{obs} = \alpha + \beta \Delta s_{t+k} + \mu_{t+k}$ , where  $\Delta s_{t+k}^{obs}$  is the observed annualized log exchange rate change in USD between  $t$  and  $t+k$  and  $\Delta s_{t+k}$  is the model-implied exchange rate change, modeled as the ratio of two countries' (log) pricing kernels, where the domestic country is the U.S. *N.W.* refers to Newey-West standard errors, where the lag is equal to the horizon  $k+1$  (in months). The significance level of the coefficient is determined by (\*), where \*\*\*  $p < 0.01$ , \*\*  $p < 0.05$ , \*  $p < 0.1$ .  $R^2$  is adjusted for degrees of freedom.

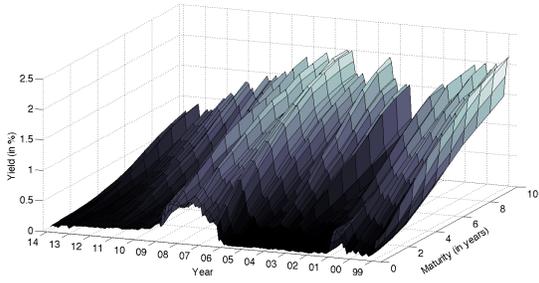
Figure 1: Evolution of Yield Curves (1999-2014)



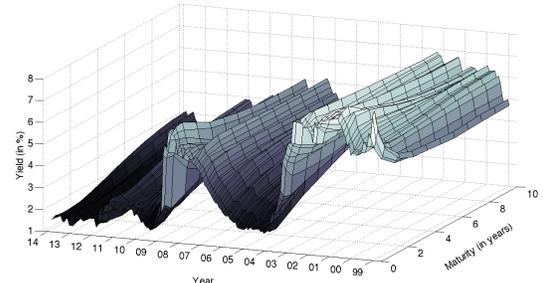
(a) Australia



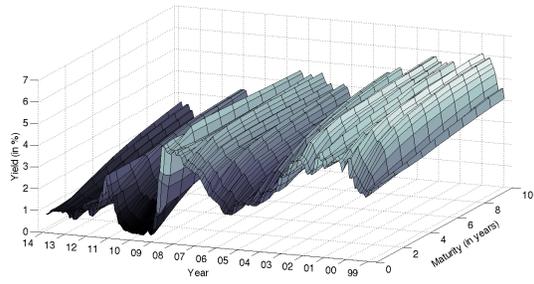
(b) Canada



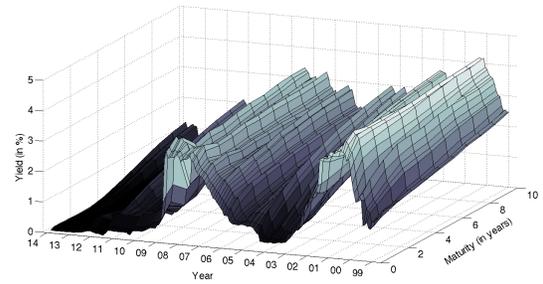
(c) Japan



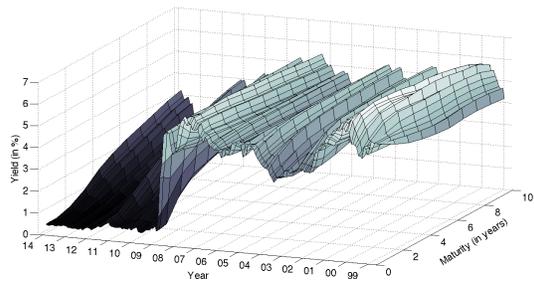
(d) Norway



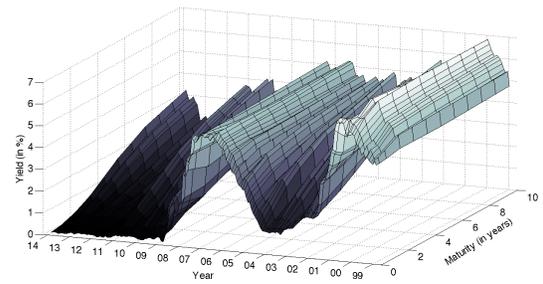
(e) Sweden



(f) Switzerland



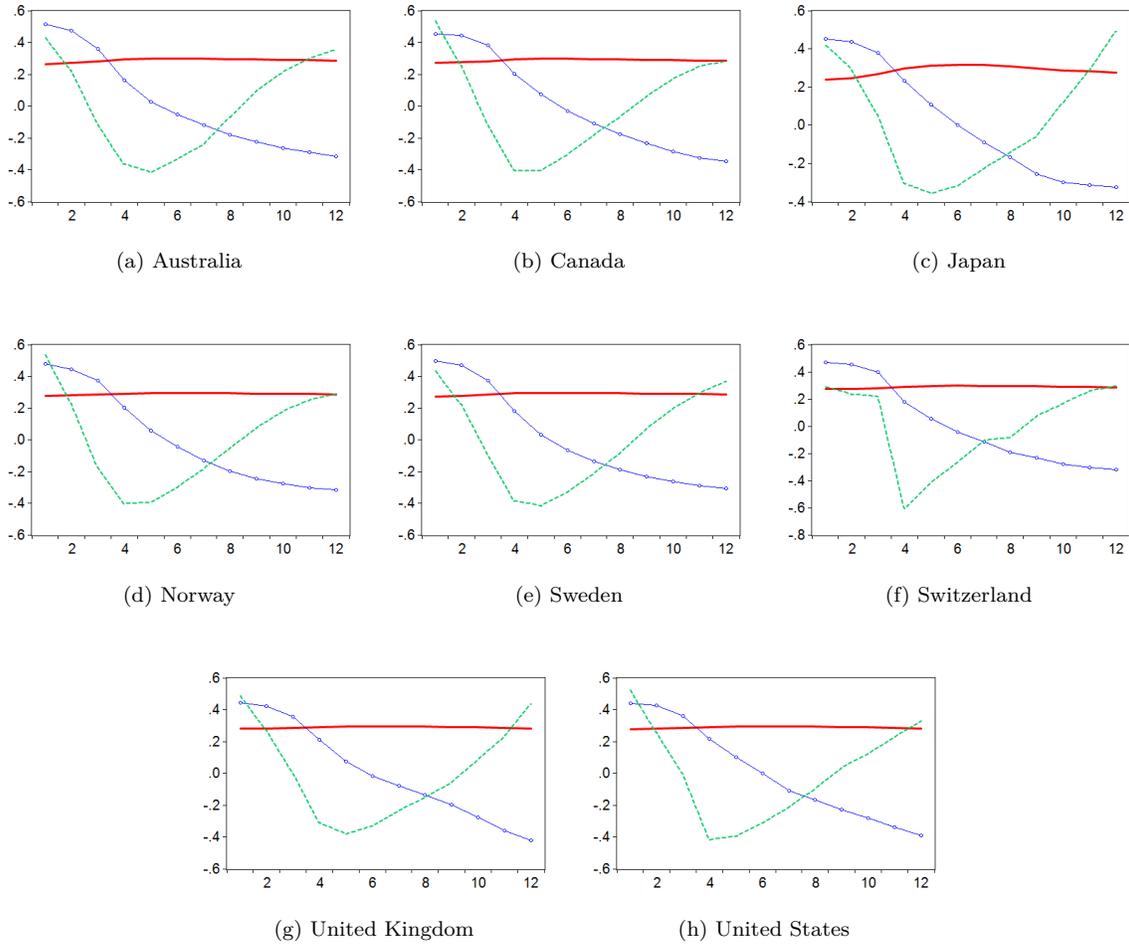
(g) United Kingdom



(h) United States

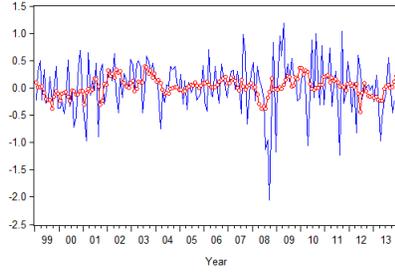
Note: Monthly zero coupon yields from January 1999 to January 2014, data source: Bloomberg, L.P. 2014. Yields are in percentage for maturities of 3 months, 6 months, and 1 to 10 years for each country.

Figure 2: Interest Rate Factor Loadings

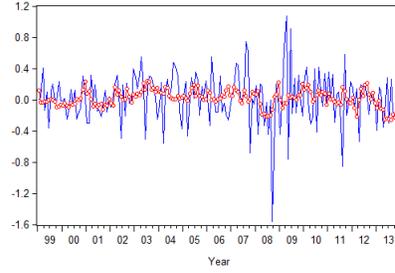


Note: Interest Rate Factor Loadings are the weights that principal components extracted from each individual country have on the corresponding maturity. On the vertical axis, the loading scores; on the horizontal axis, each maturity. The red solid line represents the first factor loading, the blue circles represent the second factor loading, and the green dashes represent the third factor loading. Principal component analysis is computed on the covariance matrix of standardized monthly interest rate data from January 1999 to January 2014. There are 12 yields per country (maturities 3 months, 6 months, and 1 to 10 years) and 181 observations per yield. 12 components are calculated but only the first 3 are reported.

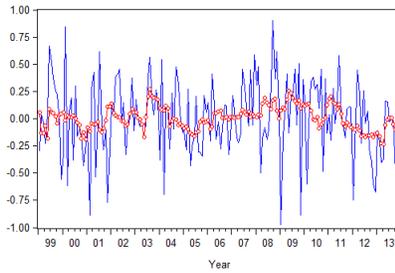
Figure 3: One-Month Exchange Rate Changes (in USD)



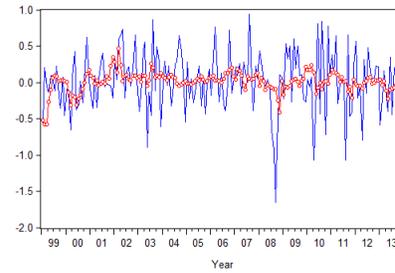
(a) Australian Dollar



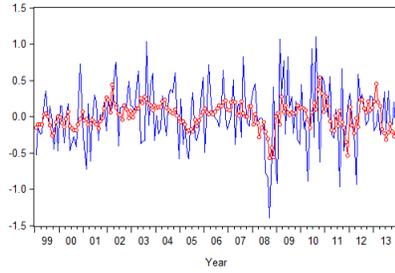
(b) Canadian Dollar



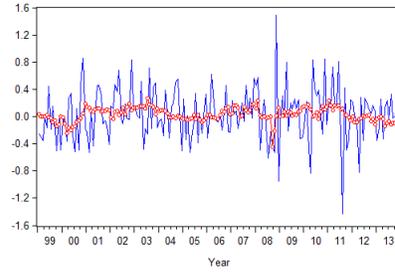
(c) Japanese Yen



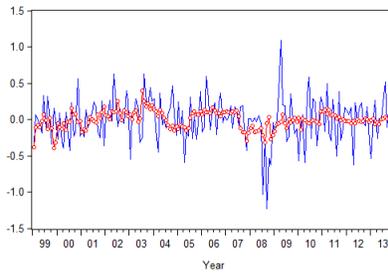
(d) Norwegian Krone



(e) Swedish Krona



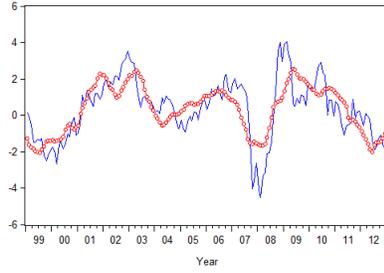
(f) Swiss Franc



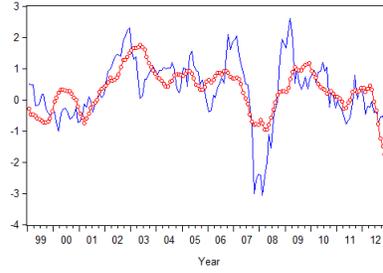
(g) British Pound

Note: The blue line represents the annualized changes in the log exchange rate in terms of the USD, the red circles represent the model-implied expected exchange rate changes, where  $E_t \Delta s_{t+k} = r_{t+k-1}^d - r_{t+k-1}^f + \frac{1}{2} [\Lambda_{t+k-1}^{d'} \Lambda_{t+k-1}^d - \Lambda_{t+k-1}^{f'} \Lambda_{t+k-1}^f]$ .

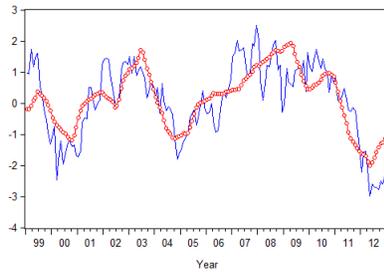
Figure 4: One-Year Exchange Rate Changes (in USD)



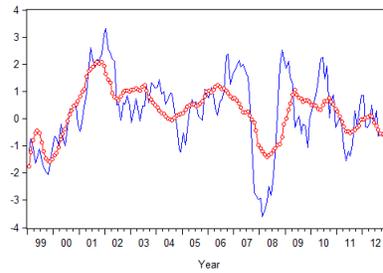
(a) Australian Dollar



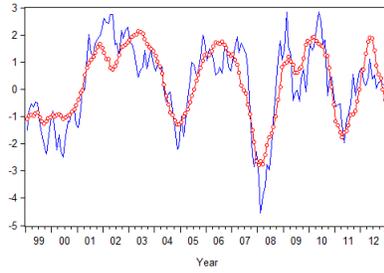
(b) Canadian Dollar



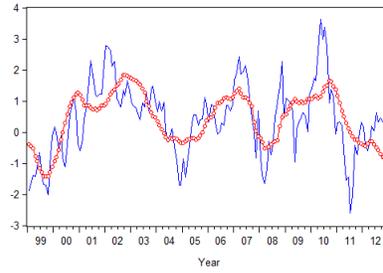
(c) Japanese Yen



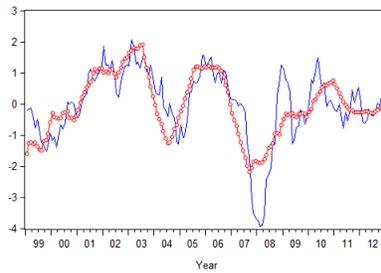
(d) Norwegian Krone



(e) Swedish Krona



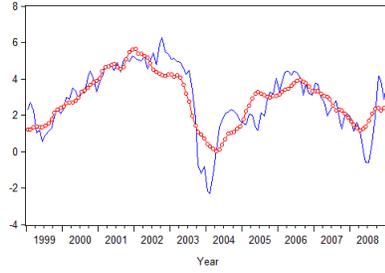
(f) Swiss Franc



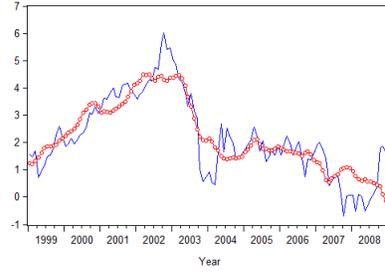
(g) British Pound

Note: The blue line represents the annualized changes in the log exchange rate in terms of the USD, the red circles represent the model-implied expected exchange rate changes, where  $E_t \Delta s_{t+k} = r_{t+k-1}^d - r_{t+k-1}^f + \frac{1}{2} [\Lambda_{t+k-1}^{d'} \Lambda_{t+k-1}^d - \Lambda_{t+k-1}^{f'} \Lambda_{t+k-1}^f]$ .

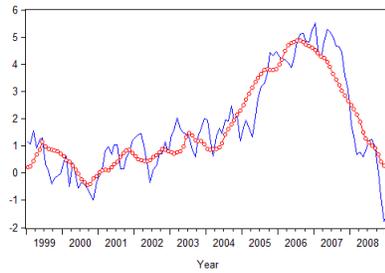
Figure 5: Five-Year Exchange Rate Changes (in USD)



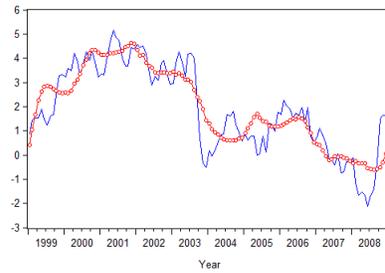
(a) Australian Dollar



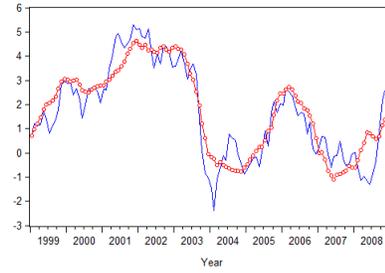
(b) Canadian Dollar



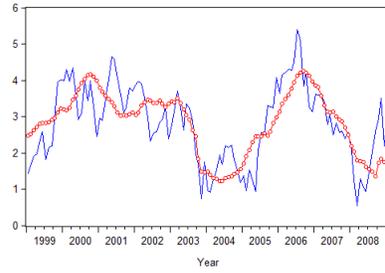
(c) Japanese Yen



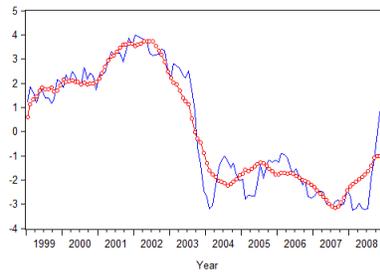
(d) Norwegian Krone



(e) Swedish Krona



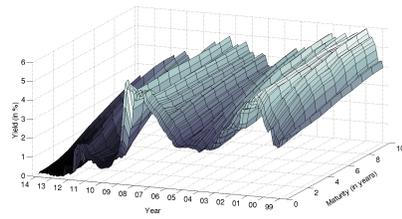
(f) Swiss Franc



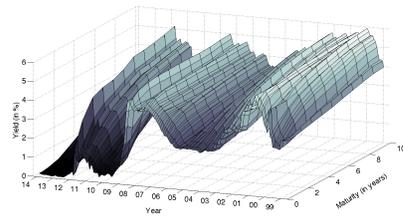
(g) British Pound

Note: The blue line represents the annualized changes in the log exchange rate in terms of the USD, the red circles represent the model-implied expected exchange rate changes, where  $E_t \Delta s_{t+k} = r_{t+k-1}^d - r_{t+k-1}^f + \frac{1}{2} [\Lambda_{t+k-1}^{d'} \Lambda_{t+k-1}^d - \Lambda_{t+k-1}^{f'} \Lambda_{t+k-1}^f]$ .

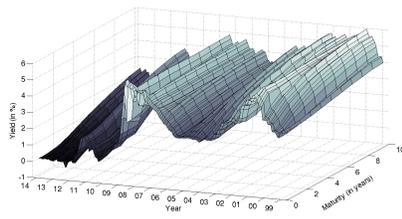
Figure 6: Evolution of Yield Curves for Countries Under the Euro (1999-2014)



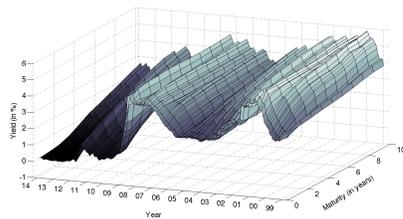
(a) Austria



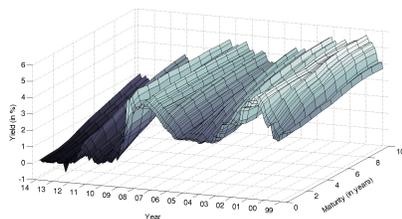
(b) Belgium



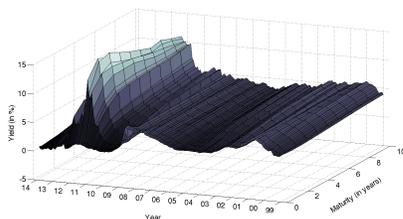
(c) Finland



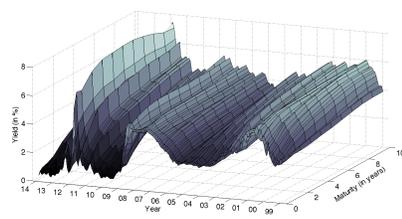
(d) France



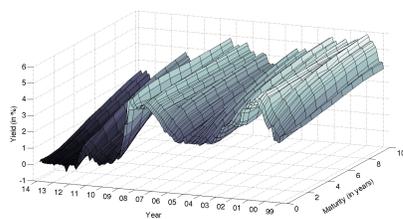
(e) Germany



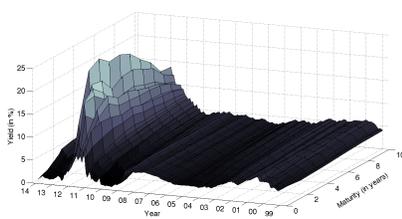
(f) Ireland



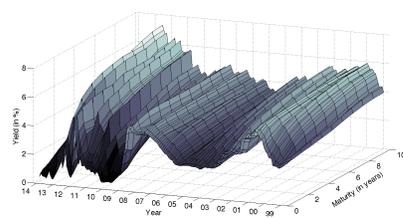
(g) Italy



(h) Netherlands



(i) Portugal



(j) Spain

Note: Monthly zero coupon yields from January 1999 to January 2014, data source: Bloomberg, L.P. 2014. Yields are in percentage for maturities of 3 months, 6 months, and 1 to 10 years for each country.

## Appendix: Details of the Term Structure Model

Observable factors can be linked to an  $N$ -dimensional latent state vector  $X_t$  through invariant transformations as in Joslin, Singleton, and Zhu (2011), where the risk-free rate is given by

$$r_t = \rho_{0X} + \rho'_{1X} \cdot X_t, \quad (26)$$

where  $\rho_{0X}$  is a constant and  $\rho_{1X}$  is an  $(N \times 1)$  vector. Given equation (26), the entire term structure of zero coupon bonds with maturity  $m$  (in years) can also be expressed in terms of  $X_t$ . The linearity in the factors allows for closed-form solutions to the prices of bonds, in order to avoid solution methods for partial differential equations or Monte Carlo methods to estimate yields, which are undesirable given their computational cost and limitations. This context assumes that no-arbitrage holds, which makes the time evolution of yields and the cross-sectional shape of the yield curve consistent at any point in time. By the fundamental theorem of asset pricing, the condition of no-arbitrage is equivalent to the existence of a risk-neutral measure,  $\mathbb{Q}$ . Therefore, the price of an  $m$ -maturity zero coupon bond at time  $t$ ,  $P_t^m$ , can be expressed as the expected value of the future payoffs (in this case = 1) discounted at the risk-free rate, under risk-neutrality ( $\mathbb{E}_t^{\mathbb{Q}}$ ):

$$P_t^m = \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \sum_{i=0}^{m-1} r_{t+i} \right) \right] = \exp [a_X^m + b_X^m \cdot X_t], \quad (27)$$

where  $a_X^m$  and  $b_X^m$  solve first-order Riccati difference equations with initial conditions  $a_X^0 = 0$  and  $b_X^0 = 0$  as in Duffie and Kan (1996). This allows one to model the yields at different horizons as a linear function of the state variable  $X_t$ , where the yield loadings are given by  $A_X^m = -a_X^m/m$  and  $B_X^m = -b_X^m/m$ ,

$$y_t^m = A_X^m + B_X^m \cdot X_t. \quad (28)$$

The evolution of the unobserved factors is expressed under two equivalent probability measures:  $\mathbb{P}$  and  $\mathbb{Q}$ .  $\mathbb{P}$  is the probability measure of the random process  $X$ , referred to as the actual or physical distribution of the state vector. This process can be represented, for example, by an unrestricted vector auto-regressive process that describes the evolution of the yields.  $\mathbb{Q}$  is an equivalent martingale measure,<sup>10</sup> referred to as the *risk-neutral* distribution of  $X$ . Under this distribution, the process has already been adjusted for risk, allowing for a risk-neutral representation of the movement of factors over time. The state unobserved vector then evolves as an auto-regressive process under the two distributions:

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<sup>10</sup>Defined as a finite stochastic process in which each realization is independent from the previous one.

$$\Delta X_t = K_{0X}^{\mathbb{P}} + K_{1X}^{\mathbb{P}} \cdot X_{t-1} + \Sigma_X \epsilon_t^{\mathbb{P}}, \quad (29)$$

$$\Delta X_t = K_{0X}^{\mathbb{Q}} + K_{1X}^{\mathbb{Q}} \cdot X_{t-1} + \Sigma_X \epsilon_t^{\mathbb{Q}}, \quad (30)$$

where  $(K_{0X}^{\mathbb{P}}, K_{0X}^{\mathbb{Q}})$  are  $(N \times 1)$  constant vectors,  $(K_{1X}^{\mathbb{P}}, K_{1X}^{\mathbb{Q}})$  are  $(N \times N)$  matrices, and errors are i.i.d Gaussian,  $\epsilon_t^{\mathbb{P}}, \epsilon_t^{\mathbb{Q}} \sim N(0, I_N)$ .  $\Sigma_X$  is a lower triangular matrix such that  $\Sigma_X \Sigma_X'$  is the  $(N \times N)$  variance-covariance matrix of  $X_t$ .

The estimations can be simplified by imposing identification conditions that yield an observationally equivalent Gaussian dynamic term structure model. Let  $\rho_{0X} = 0$  and  $\rho_{1X} = [1 \ 1 \ \dots \ 1]'$  so that the risk-free rate is the sum of the factors:  $r_t = \iota \cdot X_t$ , where  $\iota$  is a vector of ones, and let  $\Sigma_X$  be lower triangular with positive diagonal.  $K_{1X}^{\mathbb{Q}}$  is then parameterized as a diagonal matrix with ordered  $\mathbb{Q}$  eigenvalues. Finally,  $K_{0X}^{\mathbb{Q}}$  can be normalized by  $K_{0X}^{\mathbb{Q}} = [k_{\infty}^{\mathbb{Q}} \ 0 \ \dots \ 0]'$ , where  $k_{\infty}^{\mathbb{Q}}$  is proportional to the long run mean of the short rate whenever the model is stationary under  $\mathbb{Q}$ .<sup>11</sup> Given  $\lambda^{\mathbb{Q}}$  and  $k_{\infty}^{\mathbb{Q}}$ , there exists a unique set of parameters consistent with no-arbitrage. This invariant transformation of the model proposed by Joslin, Singleton, and Zhu (2011) simplifies the estimation process and significantly improves computational performance. As defined by Dai and Singleton (2000), this invariant property is an arbitrary combination of transformations that are possible because the factors are unobserved, the model is linear in the factors, and the admissibility condition is preserved (i.e., the model leads to well defined solutions to the bond pricing equation). Hence the identification conditions preserve the short rate and the bonds unchanged.

Let the observable factors be defined as  $\mathcal{P}_t = W \cdot y_t$ . From equation (28),  $\mathcal{P}_t$  can be expressed as a function of the latent vector  $X_t$

$$\mathcal{P}_t = W \cdot [A_X^m + B_X^m \cdot X_t] = W \cdot A_X^m + W \cdot B_X^m \cdot X_t. \quad (31)$$

Equivalently,

$$X_t = (W \cdot B_X^m)^{-1} \cdot (\mathcal{P}_t - W \cdot A_X^m). \quad (32)$$

Now, the yields can be expressed in terms of observable factors by substituting (32) into (28):

$$y_t = A_X^m + B_X^m \cdot \left[ (W \cdot B_X^m)^{-1} \cdot (\mathcal{P}_t - W \cdot A_X^m) \right], \quad (33)$$

---

<sup>11</sup>Note that this identification assumes that the factors are stationary under the risk-neutral distribution, which may not be true. It is not uncommon for the estimated roots to be close to 0, given the possibility of a unit root in the pricing factors. JSZ show that an equivalent re-normalization of  $K_0^{\mathbb{Q}}$  and  $\rho_0$  allows for the preservation of the canonical form in the presence of zero or negative  $\mathbb{Q}$  roots as well. Refer to Joslin, Singleton, and Zhu (2011) and their online appendix for more details.

where  $A_{\mathcal{P}}^m = A_X^m \cdot \left[ I - B_X^m \cdot (W \cdot B_X^m)^{-1} \cdot W \right]$  and  $B_{\mathcal{P}}^m = B_X^m \cdot (W \cdot B_X^m)^{-1}$ , such that  $y_t = A_{\mathcal{P}}^m + B_{\mathcal{P}}^m \cdot \mathcal{P}_t$ .

The risk-free rate can also be expressed in terms of observable factors:

$$r_t = \iota \cdot X_t \quad (34)$$

$$r_t = \iota \cdot \left[ (W \cdot B_X^m)^{-1} \cdot (\mathcal{P}_t - W \cdot A_X^m) \right], \quad (35)$$

where  $\rho_{0\mathcal{P}} = -\iota \cdot \left[ (W \cdot B_X^m)^{-1} \cdot W \cdot A_X^m \right]$  and  $\rho_{1\mathcal{P}} = \iota \cdot (W \cdot B_X^m)^{-1}$ , such that  $r_t = \rho_{0\mathcal{P}} + \rho_{1\mathcal{P}}' \cdot \mathcal{P}_t$ .

Finally, equation (29) can be expressed as  $\Delta \mathcal{P}_t = K_{0\mathcal{P}}^{\mathbb{P}} + K_{1\mathcal{P}}^{\mathbb{P}} \cdot \mathcal{P}_{t-1} + \Sigma_{\mathcal{P}} \epsilon_t^{\mathbb{P}}$ , where

$$K_{0\mathcal{P}}^{\mathbb{P}} = K_{0X}^{\mathbb{P}} \cdot (W \cdot B_X^m) - K_{1\mathcal{P}}^{\mathbb{P}} \cdot (W \cdot A_X^m), \quad (36)$$

$$K_{1\mathcal{P}}^{\mathbb{P}} = (W \cdot B_X^m) \cdot K_{1X}^{\mathbb{P}} \cdot (W \cdot B_X^m)^{-1}, \quad (37)$$

$$\Sigma_{\mathcal{P}} \Sigma_{\mathcal{P}}' = (W \cdot B_X^m)' \cdot \Sigma_X \Sigma_X' \cdot (W \cdot B_X^m). \quad (38)$$

Similarly for the evolution of factors under the risk-neutral distribution, equation (30).

Given the introduction of observable factors, the model can be now summarized by the following equations:

$$r_t = \rho_{0\mathcal{P}} + \rho_{1\mathcal{P}}' \cdot \mathcal{P}_t, \quad (39)$$

$$y_t^m = A_{\mathcal{P}}^m + B_{\mathcal{P}}^m \cdot \mathcal{P}_t, \quad (40)$$

$$\Delta \mathcal{P}_t = K_{0\mathcal{P}}^{\mathbb{P}} + K_{1\mathcal{P}}^{\mathbb{P}} \cdot \mathcal{P}_{t-1} + \Sigma_{\mathcal{P}} \epsilon_t^{\mathbb{P}}, \quad (41)$$

$$\Delta \mathcal{P}_t = K_{0\mathcal{P}}^{\mathbb{Q}} + K_{1\mathcal{P}}^{\mathbb{Q}} \cdot \mathcal{P}_{t-1} + \Sigma_{\mathcal{P}} \epsilon_t^{\mathbb{Q}}, \quad (42)$$

where internal consistency conditions are satisfied, i.e.,  $W \cdot A_{\mathcal{P}}^m = 0$  and  $W \cdot B_{\mathcal{P}}^m = I_N$ . The yield coefficients are rotated so that

$$A_{\mathcal{P}}^m = A_X^m \cdot \left[ I - B_X^m \cdot (W \cdot B_X^m)^{-1} \cdot W \right], \quad (43)$$

$$B_{\mathcal{P}}^m = B_X^m \cdot (W \cdot B_X^m)^{-1}. \quad (44)$$

The parameters under the risk-neutral distribution,  $\Theta^{\mathbb{Q}} = \left\{ K_{0\mathcal{P}}^{\mathbb{Q}}, K_{1\mathcal{P}}^{\mathbb{Q}}, \Sigma_{\mathcal{P}}, \rho_{0\mathcal{P}}, \rho_{1\mathcal{P}} \right\}$ , can be expressed in terms of  $\lambda^{\mathbb{Q}}$  and  $k_{\infty}^{\mathbb{Q}}$  given the mapping between  $X_t$  and  $\mathcal{P}_t$ , and the yield coefficients determined by

recursive rules  $\{A_X^m, B_X^m\}$  along with the weighting matrix  $W$ :

$$K_{1\mathcal{P}}^{\mathbb{Q}} = (W \cdot B_X^m) \cdot K_{1X}^{\mathbb{P}} \cdot (W \cdot B_X^m)^{-1}, \quad (45)$$

$$K_{0\mathcal{P}}^{\mathbb{Q}} = (W \cdot B_X^m) \cdot \begin{bmatrix} k_{\infty}^{\mathbb{Q}} & 0 & \dots & 0 \end{bmatrix}' - K_{1\mathcal{P}}^{\mathbb{Q}} \cdot (W \cdot A_X^m), \quad (46)$$

$$\rho_{1\mathcal{P}} = \iota \cdot (W \cdot B_X^m)^{-1}, \quad (47)$$

$$\rho_{0\mathcal{P}} = -\rho_{1\mathcal{P}} \cdot (W \cdot A_X^m). \quad (48)$$

The Joslin, Singleton, and Zhu (2011) transformation allows for a significant reduction in the number of parameters to be estimated by Maximum Likelihood, since the risk-neutral distribution can then be fully specified by  $\Theta^{\mathbb{Q}} = \{\lambda^{\mathbb{Q}}, k_{\infty}^{\mathbb{Q}}, \Sigma_{\mathcal{P}}\}$ . This *separation* in the parameters estimated under the different probability measures is a key property of the model, which I preserve when extending to a two-country framework and integrating exchange rates.