Implications of Asset Market Data for Equilibrium Models of Exchange Rates*

Zhengyang Jiang
Northwestern Kellogg, NBER

Arvind Krishnamurthy
Stanford GSB, NBER

Hanno Lustig
Stanford GSB, NBER, and CEPR

September 30, 2023

Abstract

We characterize the relation between exchange rates and their macroeconomic fundamentals without committing to a specific model of preferences, endowment or menu of traded assets. When investors can trade home and foreign currency risk-free bonds, the exchange rate (conditionally) appreciates in states of the world that are worse for home investors than foreign investors. This prediction is at odds with the empirical evidence and can only be overturned (unconditionally) if the deviations from U.I.P. are large and exchange rates are highly predictable. Without bond Euler equation wedges, it is impossible to match the empirical exchange rate cyclicality (the Backus-Smith puzzle) and the deviations from U.I.P. (the Fama puzzle) as well as the lack of predictability (the Meese-Rogoff puzzle). To relax this trade-off, we need Euler equation wedges consistent with a home currency bias, home bond convenience yields or financial repression.

Key Words: Exchange Rate Cyclicality, Exchange Rate Predictability, Incomplete Markets, International Risk-Sharing

*First draft: Feb. 2023. We are especially grateful to Andy Atkeson for his help on the first draft. For comments and feedback, we thank Mike Chernov, Sebastian Di Tella, Ben Hebert, Matteo Maggiori, Chris Tonetti, Monika Piazzesi, Martin Schneider, and Chenzi Xu. We thank Xuning Ding for excellent research assistance.

1 Introduction

We start by listing four key stylized facts about exchange rates. First, real exchange rates are only weakly positively correlated with relative aggregate consumption growth [Kollmann, 1991, Backus and Smith, 1993]. The home currency tends to depreciate when the home investors experience adverse macro-economic shocks and thus have high marginal utility growth. Exchange rates are weakly pro-cyclical. Second, more generally, exchange rates seem disconnected from the other macro variables that should determine them [Obstfeld and Rogoff, 2000]. Third, as documented by Tryon [1979], Hansen and Hodrick [1980] and Fama [1984], interest rate differences do not predict changes in exchange rates with the right sign to enforce the uncovered interest rate parity (U.I.P.). Instead, currency returns are predictable, but exchange rates themselves are not. In order to explain the negative slope coefficients, risk premia have to be extremely volatile [Fama, 1984]. Fourth, other macro variables also fail to predict exchange rates, as shown by Meese and Rogoff [1983]. It is hard to beat a random walk when predicting exchange rates.

We show that these four stylized facts are not separate phenomena in a large class of international real business cycle models, as long as investors can trade home and foreign risk-free bonds. We cannot match all of these facts without imputing a home currency bias to bond market investors, effectively deviating from the standard Euler equations and segmenting bond markets by their currency denomination.

In a complete-market setting, real exchange rates have to appreciate—conditionally and unconditionally—when the domestic marginal utility growth is higher than the foreign marginal utility growth to enforce no arbitrage. When domestic investors have a higher marginal willingness than foreign investors to pay for consumption in some state tomorrow, i.e. to save into that state, then the state-contingent interest rate is correspondingly lower at home, and the real exchange rate has to appreciate in that state to keep arbitrageurs from borrowing domestically and investing abroad in that state of the world. The exchange rate has to appreciate to keep the state prices at home and abroad aligned state-by-state. When investors have power utility, this state-contingent version of interest rate parity induces a perfectly negative correlation with aggregate consumption growth. That's why international economists refer to the first stylized fact as the Backus-Smith puzzle.

Shutting down trade in other securities markets does not alleviate the puzzle. An average version of this state-contingent interest rate parity prediction survives even when we restrict the menu of assets traded. The exchange rate has to appreciate in bad states for the home investor to keep the state prices at home and abroad aligned and enforce the bond Euler equations. As long as we also want to match the third and fourth stylized facts, it is not possible to jointly match the first and second stylized facts in a large class of models with cross-currency bond investments.

Standard two-country international real business cycle (IRBC) models feature four Euler equations that must hold in equilibrium. These equations implicitly describe the risk-adjusted returns home and foreign investors require for holding home and foreign risk-free bonds. More concretely, let $m_{t,t+1}$ and $m^*_{t,t+1}$ denote the home and foreign SDF in log, let r_t and r^*_t denote the home and foreign risk-free rates in log, and let s_t denote the log spot exchange rate in units of foreign currency per dollar. When s_t increases, the home currency appreciates. Then, the four bond Euler equations are given by:

$$
\begin{aligned}
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1} + r_t)\right], \\
1 &= \mathbb{E}_t \left[\exp(m_{t,t+1} - \Delta s_{t+1} + r^*_t)\right], \\
1 &= \mathbb{E}_t \left[\exp(m^*_{t,t+1} + r^*_t)\right], \\
1 &= \mathbb{E}_t \left[\exp(m^*_{t,t+1} + \Delta s_{t+1} + r_t)\right].
\end{aligned}
$$

The first two equations are the Euler equations for the home investor investing in domestic and foreign currency risk-free bonds. The second set of two Euler equations pertain to the foreign investor. In this paper, we start from these Euler equations and show how they impose strong restrictions on the exchange rate's cyclicality, which is defined as the covariance between the exchange rate movement Δs_{t+1} and the SDF differential $m_{t,t+1} - m^*_{t,t+1}$. A positive covariance means that the home currency's exchange rate tends to appreciate when the home investors' marginal utility growth rate is higher than the foreign investors'.

First, we characterize the conditional exchange rate cyclicality. We obtain a stark result, building on the work by Lustig and Verdelhan [2019]. The conditional covariance is still positive, which means that the exchange rate has to be conditionally counter-cyclical to enforce these bond Euler equations. A more restrictive version of this conditional result was derived by Lustig and Verdelhan [2019]. Second, we characterize the unconditional exchange rate cyclicality in order to link our result directly to the Kollmann [1991] and Backus and Smith [1993] evidence.[1] To replicate unconditionally pro-cyclical exchange rates, the first stylized fact, we need very volatile forward premia or highly predictable exchange rates. The first condition is at odds with the Tryon [1979], Hansen and Hodrick [1980], and Fama [1984] evidence on the violation of U.I.P, the second stylized fact. The

[1] The unconditional cyclicality is equal to the mean of the conditional exchange rate cyclicality plus a term that captures the extent to which exchange rates can be predicted by the expected SDF differential.

second condition is at odds with the Meese and Rogoff [1983] evidence, the third stylized fact.

We end up with an impossibility result: when investors can trade domestic and foreign currency bonds, we cannot generate procyclical exchange rates (stylized fact 1) while matching the observed U.I.P. deviations (stylized fact 3) and the lack of exchange rate predictability (stylized fact 4). Our results allow for arbitrary currency risk premia. While U.I.P. deviations may help to generate an exchange rate disconnect, we show that they are not sufficient. Our analytical results show that we also need a counterfactual amount of exchange rate predictability to deliver pro-cyclical exchange rates.

Our results have the following implications for equilibrium models of exchange rates. First, the key ingredient needed to make progress on the disconnect and Backus-Smith puzzles is not the market incompleteness itself. It is not sufficient to shut down asset markets other than the bond market. Second, our work distinguishes between currency risk premium shocks and cross-currency bond Euler equation wedges as sources of U.I.P. deviations. In the literature, these are sometimes referred to collectively as U.I.P. or financial shocks [see, e.g., Farhi and Werning, 2014, Itskhoki and Mukhin, 2021]. As long as all of the cross-currency bond Euler equations hold, we find that the Backus-Smith puzzle reappears. Financial shocks cannot overturn this result unless the Euler equations are violated. Third, we show analytically that cross-currency bond Euler equation wedges are needed which impute a home currency bias to bond investors. They act as if they face large (perceived or real) transaction costs or inconvenience yields associated with buying bonds denominated in a currency that is different from their home currency. Conversely, domestic investors need to act as if they derive large convenience yields from bonds denominated in domestic currency. We need to effectively shut domestic investors out of the foreign currency bond market. Large convenience yields on domestic bonds may instead be a symptom of governments engaging in financial repression, a catch-all term for measures aimed at lowering the government's cost of borrowing below the market rate.

There is a wealth of evidence to support the notion that investors act as if they face large transaction costs in buying foreign securities [Lewis, 1995]. Moreover, there is recent empirical evidence to specifically support a home currency bias in bonds. Maggiori, Neiman, and Schreger [2020] report strong evidence of a home currency bias in international mutual fund holdings of corporate bonds. The only exception is the dollar, the international reserve currency. Investors will buy dollar-denominated bonds. Maggiori et al. [2020] attribute this home currency bias to the costs of currency hedges and behavioral bias.

Surprisingly, we find that only the mean of the cross-currency bond Euler equation wedges matters. The covariance of these wedges with the SDFs does not matter for disconnect. The level of the wedges matters, because these cross-currency wedges are substitutes for covariance of the pricing kernel with the exchange rate. The average wedge replaces the currency risk premium in generating U.I.P. deviations.

Chari, Kehoe, and McGrattan [2002] analyze a complete-market model of exchange rates with sticky prices and identify the Backus-Smith puzzle as the key failure of their model. Corsetti, Dedola, and Leduc [2008], Pavlova and Rigobon [2012] consider incomplete market models of exchange rates. In their model, domestic and foreign investors only invest in a risk-free bond that pays off in a global numéraire, implicitly dropping all 4 Euler equations. This implicitly introduces wedges in one of the four bond Euler equations. They report progress on the Backus-Smith puzzle.

A different strand of the literature segments the markets and thus introduces wedges into the bond Euler equations. Alvarez, Atkeson, and Kehoe [2002a, 2009a] consider a Baumol-Tobin style model in which investors pay a cost to transact in currency and bond markets. Relatedly, Jiang, Krishnamurthy, and Lustig [2018], Jiang, Krishnamurthy, Lustig, and Sun [2021] explore the dollar exchange rate implications of convenience yields earned on dollar safe assets, another type of the Euler equation wedges. To the extent that investors derive larger convenience yields from foreign bonds, these will exacerbate the Backus-Smith puzzle.[2]

Finally, a third strand of the literature imputes a central role to financial intermediaries, drawing on insights from the literature on intermediary asset pricing. Gabaix and Maggiori [2015], Itskhoki and Mukhin [2021], Fukui, Nakamura, and Steinsson [2023] all consider models in which most investors cannot directly access currency markets. In Itskhoki and Mukhin [2021], domestic investors only invest in the domestic bond market. Only the intermediaries can trade foreign currencies. This approach implies bond Euler equation wedges for domestic and foreign investors. Similarly, Gourinchas, Ray, and Vayanos [2020], Greenwood, Hanson, Stein, and Sunderam [2020] study models with preferred-habitat investors and global arbitrageurs. This class of models remove all Euler equations we consider and only keep one Euler equation that captures the global arbitrageurs' long-short portfolio decision.

Other recent work by Hassan [2013], Dou and Verdelhan [2015], Chien, Lustig, and Naknoi [2020], Jiang and Richmond [2023] takes a different tack by introducing heterogeneity in household trading technologies. Active households can freely trade bonds and other state contingent claims, whereas the inactive households have no access to the asset

[2] There is a growing literature on convenience yields in bond markets, starting with Krishnamurthy and Vissing-Jorgensen [2012]

market. In this case, while the four Euler equations we consider in this paper hold for the active households without any wedges, their marginal utilities have different cyclicality than the country-level aggregate marginal utilities. As a result, the model disconnects aggregate consumption from the SDF of the Euler equation to which the model applies.

Lustig and Verdelhan [2019] ask whether market incompleteness helps to resolve outstanding currency puzzles. They focus only on the conditional Backus-Smith puzzle, not the unconditional version, the focus of our paper. In closely related work, Chernov, Haddad, and Itskhoki [2023] conclude that the moments of asset returns in financial markets are not informative about exchange rates, once they rule out the asset market structure with 4 bond Euler equations as implausible. We reach a different conclusion, because we keep the asset structure, but allow for wedges. Instead, we derive tight restrictions on the bond Euler equation wedges from the same asset return moments to make progress on these puzzles.

The paper is organized as follows. We start by discussing the benchmark complete-market case in section 2. Next, section 3 discuss the conditional Backus-Smith puzzle in the incomplete-market case. Section 4 analyzes the unconditional Backus-Smith puzzle in the incomplete-market case. Finally, section 5 inserts bond Euler equation wedges. In this section, we characterize the restrictions these wedges need to satisfy in order to make progress on the unconditional Backus-Smith puzzle. Lastly, Appendix contains the proof of the propositions.

2 Complete Markets and Exchange Rate Puzzles

In the case of complete markets, exchange rates act as shock absorbers for the shocks to the pricing kernels: $\Delta s_{t+1} = m_{t+1} - m^*_{t+1}$. This expression for the log change in the real exchange rate has puzzling implications.

1. **Volatility puzzle**: As was noted by Brandt, Cochrane, and Santa-Clara [2006b], the implied volatility of the exchange rate will be too high if the market price of risk clears the Hansen-Jagannathan bounds, unless the pricing kernels are highly correlated across countries.

$$\begin{aligned} var_t(\Delta s_{t+1}) &= var_t(m^*_{t+1}) + var(m_{t+1}) \\ &- 2\rho_t(m_{t+1}, m^*_{t+1}) std_t(m_{t+1}) std_t(m^*_{t+1}). \end{aligned}$$

We would need a correlation of the pricing kernels $\rho_t(m_{t+1}, m^*_{t+1})$ close to one. In the case of the standard Breeden-Lucas SDF $m_{t+1} = \log \delta - \gamma \Delta c_{t+1}$, this would imply close to perfectly correlated consumption growth across countries: $\rho_t(\Delta c_{t+1}, \Delta c^*_{t+1})$. This prediction is counterfactual [see Backus, Kehoe, and Kydland, 1992].

2. **Counter-cyclicality/Backus-Smith puzzle**:

$$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0.$$

When markets are complete, the unconditional exchange rate cyclicality satisfies

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var(\Delta s_{t+1}) > 0.$$

We obtain a very general result: in any complete-market models, the unconditional exchange rate cyclicality is always positive: a higher marginal utility growth in the home country is associated with a home currency appreciation. The model can only generate exchange rate disconnect by shrinking the variance of the exchange rate to zero. In the Breeden-Lucas case, the implied changes in the log exchange rates are perfectly negatively correlated with consumption growth differences $\rho_t(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta s_{t+1}) = -1$, which is strongly counterfactual [Kollmann, 1991, Backus and Smith, 1993].

These puzzles are partially governed by the specific nature of the pricing kernel. The Breeden-Lucas SDF assumes time-additive utility. Colacito and Croce [2011] impute a preference for early resolution to uncertainty to the stand-in investor in an endowment economy with long run risks [Bansal and Yaron, 2004]. In this LRR economy, it is feasible to make progress on the volatility puzzle by choosing highly correlated persistent components of consumption growth, while still matching the low correlation of consumption growth observed in the data. In this LRR economy, we can push the correlation of the pricing kernels $\rho_t(m_{t+1}, m^*_{t+1})$ to one by choosing perfectly correlated long-run consumption growth $\rho_t(x_{t+1}, x^*_{t+1}) = 1$. However, this mechanism does not resolve the Backus-Smith puzzle. In their benchmark calibration, $\rho_t(\Delta c_{t+1} - \Delta c^*_{t+1}, \Delta s_{t+1}) = -0.8$. Next, we examine this cyclicality of the exchange rate when we shot down some asset markets. In closely related work, Verdelhan [2010] explores the habit model's exchange rate implications, and concludes that this model cannot entirely resolve the Backus-Smith puzzle.

3 Conditional Exchange Rate Cyclicality and Incomplete Markets

We start by assuming that investors can invest in risk-free bonds denominated in domestic and foreign currency. Our analysis is silent on the rest of the market structure.

3.1 Gaussian case

We assume that the return, exchange rate and pricing kernel innovations are jointly normally distributed. Then, the four risk-free bond Euler equations imply:

$$\begin{aligned}
0 &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) + r_t, \\
0 &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m_{t,t+1}, -\Delta s_{t+1}) + r_t^*, \\
0 &= \mathbb{E}_t[m^*_{t,t+1}] + \frac{1}{2}var_t(m^*_{t,t+1}) + r_t^*, \\
0 &= \mathbb{E}_t[m^*_{t,t+1}] + \frac{1}{2}var_t(m^*_{t,t+1}) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m^*_{t,t+1}, \Delta s_{t+1}) + r_t.
\end{aligned}$$

Reorganizing the terms, we can obtain two expressions that relate the expected excess return of a strategy that goes long in foreign currency and borrows at the domestic risk-free rate to the riskiness of the exchange rate

$$\begin{aligned}
(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}, -\Delta s_{t+1}), \\
(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m^*_{t,t+1}, \Delta s_{t+1}).
\end{aligned}$$

The first expression takes the home investors' perspective. If the foreign currency tends to appreciate (i.e., higher $-\Delta s_{t+1}$) in the home investors' high marginal utility states (i.e., higher $m_{t,t+1}$), then, the foreign currency is a good hedge and the home investors demand lower expected returns to hold it. As a result, the foreign currency has a lower expected excess return leading to a lower $(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1})$ on the left-hand side. Similarly, the second expression takes the foreign investors' perspective, and relates the currency excess return to the covariance between the foreign investors' SDF and the exchange rate movement.

Given the exchange rate variance $var_t(\Delta s_{t+1})$ is positive, these expressions imply

$$\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^* > cov_t(m_{t,t+1}, -\Delta s_{t+1}),$$

$$-(\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^*) > cov_t(m^*_{t,t+1}, \Delta s_{t+1}),$$

which leads to the following proposition.

Proposition 1. *In the log-normal case, the conditional exchange rate cyclicality satisfies*

$$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0. \tag{1}$$

Lustig and Verdelhan [2019] derive a version of this result assuming incomplete market wedges that are jointly log-normal with the SDF and the exchange. Our derivation does not use incomplete market wedges.

In this paper, we define the exchange rate cyclicality as the covariance between the exchange rate movement Δs_{t+1} and the SDF differential $m_{t,t+1} - m^*_{t,t+1}$. This proposition shows that exchange rate innovations are counter-cyclical in a Gaussian model. The home currency's exchange rate tends to unexpectedly appreciate when the home investors' marginal utility growth is unexpectedly higher than the foreign investors'.

While Lustig and Verdelhan [2019] find that market incompleteness helps with the Brandt, Cochrane, and Santa-Clara [2006a] puzzle in reducing volatility, it cannot change the sign of the conditional covariance. We have not assumed that markets are complete to derive this result. In the case of complete markets, state-contingent interest rate parity obtains $m_{t,t+1} - m^*_{t,t+1} = \Delta s_{t+1}$ and this covariance result is directly obtained. Our proposition shows that this result is much more general, as long as investors can freely trade home and foreign risk-free bonds. In other words, the risk-free bond Euler equations discipline the joint dynamics of the exchange rates and marginal utility growth to imply counter-cyclical exchange rates.

3.2 Non-Gaussian Case

Some dynamic asset pricing models are not conditionally Gaussian [Rietz, 1988, Longstaff and Piazzesi, 2004, Barro, 2006, Farhi and Gabaix, 2016]. This result can be extended to non-normal settings. We can define conditional entropy as follows:

$$L_t(X_{t+1}) = (\log \mathbb{E}_t[X_{t+1}] - \mathbb{E}_t[x_{t+1}]) \, .$$

We can use μ_{it} to denote the i-th central conditional moment of $\log X$. Then we can state:

$$\log \mathbb{E}_t \exp(s x_{t+1}) = \sum_{j=1}^{\infty} s^j \kappa_{j,t}/j! = k_t(x_{t+1}; s)$$

where $\kappa_{1t} = \mu_{1t}, \kappa_{2t} = \mu_{2t}, \kappa_{3t} = \mu_{3t}, \kappa_{4t} = \mu_{4t} - 3\mu_{2t}^2$. This implies that the conditional entropy can be stated as the sum of the higher order *cumulants*:

$$L_t(X_{t+1}) = \sum_{j=2}^{\infty} \kappa_{j,t}/j! = k_t(x_{t+1};1) - \kappa_1.$$

The log of the currency risk premium (in levels) earned by domestic investors can be stated as:

$$(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + L_t\left[\frac{S_t}{S_{t+1}}\right] = -C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right),$$

where co-entropy is defined as $C_t(x_{t+1}, y_{t+1}) = L_t(x_{t+1}y_{t+1}) - L_t(x_{t+1}) - L_t(y_{t+1})$ [Backus, Boyarchenko, and Chernov, 2018]. If x_{t+1} and y_{t+1} are independent, then $C_t(x_{t+1}, y_{t+1}) = 0$. If we define the cumulant generating function,

$$\log \mathbb{E}_t \exp(s_1 x_{t+1} + s_2 y_{t+1}) = k_t(s_1, s_2)$$

then $C_t(x_{t+1}, y_{t+1}) = k_t(1,1) - k_t(1,0) - k_t(0,1)$. A long position in foreign currency is risky for the domestic investor when the foreign currency tends to depreciate (and the home currency appreciates) in worse states for the domestic investor, i.e. when

$$C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right) < 0$$

is more negative. Similarly, the log of the currency risk premium (in levels) earned by domestic investors can be stated as the co-entropy of the domestic SDF with the domestic currency's rate of appreciation:

$$(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + L_t\left[\frac{S_{t+1}}{S_t}\right] = -C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t}).$$

Proposition 2. *In the non-normal case, the conditional exchange rate cyclicality satisfies*

$$-C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t}) - C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right) = L_t\left[\frac{S_t}{S_{t+1}}\right] + L_t\left[\frac{S_{t+1}}{S_t}\right] > 0. \tag{2}$$

In the normal case, we recover the covariance result in Proposition 1. When the domestic currency appreciates in worse states for the domestic investor, we have $C_t\left(M_{t+1}, \frac{S_t}{S_{t+1}}\right) < 0$. Similarly, when the foreign currency appreciates in worse states for the foreign investor, we have $C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t}) < 0$.

Because the right-hand side is positive, as entropy is non-negative, we know that exchange rates will have to be counter-cyclical for at least one of the countries, and possibly both.

4 Unconditional Exchange Rate Cyclicality and Incomplete Markets

In IRBC models that link the SDFs to aggregate consumption shocks, we are interested in understanding how the exchange rate moves in response to relative consumption growth. Backus and Smith [1993] summarize this relationship by regressing the exchange rate movement on relative consumption growth, and find procyclical exchange rates. To relate our result to this Backus-Smith coefficient, we need to characterize the unconditional exchange rate cyclicality. To do so, we use the law of total covariance:

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = \mathbb{E}[cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})] + cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]).$$

Our previous result shows the conditional exchange rate cyclicality $cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})$ is always positive. To generate a negative unconditional cyclicality $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})$, we need a negative $cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}])$ that is greater in magnitude than the average conditional cyclicality $\mathbb{E}[cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1})]$. In other words, the exchange rate movement needs to be strongly predictable by the expected SDF differential $\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}]$. This connects the unconditional exchange rate cyclicality to exchange rate predictability.

Recall the case of complete markets, $\Delta s_{t+1} = m_{t,t+1} - m^*_{t,t+1}$. The exchange rate has to absorb all of the shocks to the pricing kernels in each state of the world. When markets are complete, the unconditional exchange rate cyclicality satisfies

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var(\Delta s_{t+1}) > 0.$$

In any complete-market models (which by definition allows home and foreign agents to trade risk-free bonds), the unconditional exchange rate cyclicality is always positive: a higher marginal utility growth in the home country is associated with a home currency appreciation. The model can only generate exchange rate disconnect by shrinking the variance of the exchange rate to zero.

Next, we consider a more general case of less than complete markets in which we shut down some trade in non-bond asset markets. We begin by introducing some concepts.

The Fama Regression Coefficient b We use f_t to denote the log of the one-period forward exchange rate in units of foreign currency per dollar. The log excess return on buying foreign currency forward is

$$rx_{t+1} = f_t - s_{t+1} = -\Delta s_{t+1} + f_t - s_t,$$

where $f_t - s_t$ denotes the forward discount and Δs_{t+1} denotes the appreciation of the home currency. When the Covered Interest Rate Parity holds, we further obtain $f_t - s_t = r_t^* - r_t$ and, as a result, we can restate the log excess return on a long position in foreign currency as $rx_{t+1} = -\Delta s_{t+1} + r_t^* - r_t$.

Now, consider the standard Tryon [1979], Hansen and Hodrick [1980], Fama [1984] time-series regression:

$$\Delta s_{t+1} = a + b(f_t - s_t) + \varepsilon_{t+1}.$$

In the data, the slope coefficient b tends to be negative: a higher-than-usual foreign interest rate predicts further appreciation of the foreign currency. Following Fama [1984], we use $p_t = \mathbb{E}_t[rx_{t+1}] = f_t - \mathbb{E}_t[s_{t+1}]$ to denote the currency risk premium and $q_t = \mathbb{E}_t[\Delta s_{t+1}]$ to denote the expected exchange rate movement. The forward discount can be decomposed as $f_t - s_t = p_t + q_t$. When the Covered Interest Rate Parity holds, $p_t = r_t^* - r_t - \mathbb{E}_t[\Delta s_{t+1}]$, and $p_t + q_t = r_t^* - r_t$.

As shown by Fama, the slope coefficient in this regression can be restated as:

$$\frac{cov(f_t - s_t, \mathbb{E}_t[\Delta s_{t+1}])}{var(f_t - s_t)} = \frac{cov(p_t + q_t, q_t)}{var(p_t + q_t)} = \frac{cov(p_t, q_t) + var(q_t)}{var(p_t + q_t)}$$

To get negative slope coefficient b, we need $\frac{cov(p_t, q_t)}{var(q_t)} < -1$. Two necessary conditions have to be satisfied in order to obtain negative slope coefficients: $corr(p_t, q_t) < 0$ and $std(p_t) > std(q_t)$. Risk premia have to be more volatile than the expected change in the spot rate. Backus, Foresi, and Telmer [2001] analyze sufficient conditions for these U.I.P. violations in a large class of affine asset pricing models.

The Meese-Rogoff R^2 The Meese and Rogoff [1983] puzzle states that exchange rates are hard to forecast. Put differently, the $R^2 = var(\mathbb{E}_t[\Delta s_{t+1}])/var(\Delta s_{t+1})$ in a forecasting regression is low. In a recent survey of exchange rate predictability, Rossi [2013] concludes that the Meese-Rogoff findings have not been conclusively overturned.[3] There is some

[3] At higher frequencies ranging from one day to one month, order flow seems to predict changes in the spot exchange rate [see Evans and Lyons, 2002, 2005]. This data is proprietary and may not be available

limited evidence of exchange rate predictability but the evidence usually is specific to certain countries, horizons and the predictability is not stable.

When the linear projection yields the best forecast, we can obtain this R^2 from a projection of the exchange rate changes on its predictors. We will assume the linear predictor yields the best forecast. Let R^2 denote the fraction of the predictable variation in the exchange rate:

$$R^2 = \frac{var(\mathbb{E}_t[\Delta s_{t+1}])}{var(\Delta s_{t+1})},$$

and let R^2_{Fama} denote the R^2 of the Fama regression:

$$R^2_{Fama} = \frac{var(b(f_t - s_t))}{var(\Delta s_{t+1})}.$$

Our main result characterizes the unconditional exchange rate cyclicality. Without loss of generality, we assume that the covariance between the home SDF's conditional variance and the exchange rate movement from the home perspective is higher than the covariance between the foreign SDF's conditional variance and the exchange rate movement from the foreign perspective:

$$cov\left(var_t(m_{t,t+1}), \mathbb{E}_t[\Delta s_{t+1}]\right) \geq cov\left(var_t(m^*_{t,t+1}), -\mathbb{E}_t[\Delta s_{t+1}]\right).$$

If this condition is not satisfied, we simply need to swap the labeling of the home and foreign countries.

Proposition 3. *Each of the following is a necessary condition for a negative unconditional exchange rate cyclicality, i.e.,* $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$*:*

(a)

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}} + \sqrt{R^2}\left(\frac{1}{b}\frac{R^2_{Fama}}{R^2} - 1\right). \tag{3}$$

If the Fama regression yields the best predictor of the exchange rate movement, then, we can simplify this formula to

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}} + \sqrt{R^2}\left(\frac{1}{b} - 1\right) = \frac{1}{\sqrt{R^2}} - \sqrt{R^2} + sign(b)\frac{std(f_t - s_t)}{std(\Delta s_{t+1})}. \tag{4}$$

in real-time to all investors. Gourinchas and Rey [2007] report evidence that the net foreign asset position predicts changes in the exchange rate out-of-sample.

(b)

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t+1}])}{std(\mathbb{E}_t[\Delta s_{t+1}])} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{std(\Delta s_{t+1})}{std(\mathbb{E}_t[\Delta s_{t+1}])} = \frac{1}{\sqrt{R^2}}.$$

Between these conditions,

- *(a) $\Rightarrow$ (b).*
- *If Fama regression yields the best predictor and $b \notin (0,1)$, (b) $\Rightarrow$ (a); otherwise (b) is a weaker condition.*

Conditions (a) and (b) are necessary, but not sufficient conditions for a negative unconditional exchange rate cyclicality. The bounds tighten as the R^2 decreases: as exchange rates become less predictable, we need more variations in the conditional risk premia and the conditional price of risk to generate a negative unconditional exchange rate cyclicality. In the limit, as we approach the Meese and Rogoff [1983]'s benchmark random walk case in which exchange rate movements are not predictable, $1/\sqrt{R^2}$ on the right-hand side approaches infinity. The model simply cannot deliver pro-cyclical exchange rates. As such, these bounds deliver an impossibility result: if we take Meese and Rogoff [1983] random walk result regarding *exchange rate predictability* at face value, then we cannot make progress on the Backus and Smith [1993] puzzle regarding *exchange rate cyclicality*.

In the data, the exchange rate movements are only moderately predictable. Let us consider a simple calculation. Suppose $R^2 = 10\%$ at the one-year horizon, and the Fama regression coefficient is $b = -1$. Then, Eq. (4) becomes $std\left(var_t(m_{t,t+1})\right)/std(\Delta s_{t+1}) \geq 2.53$. With an annualized exchange rate volatility of $std(\Delta s_{t+1}) = 10\%$, then, the SDF needs to have a very high variability in its variance $std\left(var_t(m_{t,t+1})\right) \geq 25.3$. As the variance is non-negative and a two-standard-deviation band is $25.3 \times 2 = 50.6$, in certain states, the log SDF needs to have a variance of 50.6 per annum, and a volatility of $\sqrt{50.6} = 7.1 = 711\%$ per annum. This implies the existence of an asset with a very high Sharpe ratio according to the Hansen-Jagannathan bound. In other words, when the investors can freely trade risk-free bonds, a negative unconditional exchange rate cyclicality requires a very volatile SDF.

On the other hand, while it is enticing to conclude that, holding R^2 constant, a small but negative Fama coefficient b can lower the right-hand side of Eq. (3) and make this condition more likely to hold, we note that $\sqrt{R^2}$ and b are closely related. When the b shrinks towards zero, the R^2 shrinks towards zero as well. In fact, Eq. (4) shows that, holding R^2 constant, a small but negative b means a high forward premium volatility

$std(f_t - s_t)$ relative to the exchange rate volatility $std(\Delta s_{t+1})$, which is also rejected by the data.

We make two more observations. First, in the case of U.I.P. ($b = 1$), Eq. (4) becomes:

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}}.$$

A natural case to consider under the U.I.P. is the case of constant market prices of risk. Then, we get an impossibility result: $0 \geq 1/\sqrt{R^2}$, so the unconditional exchange rate cyclicality cannot be negative.

Second, in the case of a fully predictable exchange rate movement, the R^2 tends to one and Eq. (4) becomes:

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{b}.$$

As long as the slope coefficient b is negative, then the bound is trivially satisfied, even when the R^2 is very high but not equal to 1.

4.1 Role of the Horizon

We consider a long-horizon Fama regression:

$$\Delta s_{t,t+k} = a_k + b_k(f_t^k - s_t) + \varepsilon_{t+k}.$$

Similarly, we use $R^2_{Fama,k}$ to denote the R^2 of this regression, and we define

$$R_k^2 = \frac{var(\mathbb{E}_t[\Delta s_{t,t+k}])}{var(\Delta s_{t,t+k})}.$$

There are opposing forces on the left-hand side of the bounds we derived. As we increase the horizon, $std\left(var_t(m_{t,t+k})\right)$ increases faster than the $\sqrt{k}$, while $std(\mathbb{E}_t[\Delta s_{t,t+k}])$ converges to zero if a long-run version of PPP holds and the real exchange rate is stationary.[4] On the other hands, as $k \to \infty$, a long-run version of U.I.P kicks in and $b_k \to 1$. In fact, long-run U.I.P. is implied by no arbitrage when real exchange rates are stationary [Lustig, Stathopoulos, and Verdelhan, 2019].

Proposition 4. *Each of the following is a necessary condition for a negative unconditional exchange rate cyclicality, i.e., $cov(m_{t,t+k} - m^*_{t,t+k}, \Delta s_{t,t+k}) < 0$:*

[4] As noted by Rogoff [1996], the real exchange rate's rate of convergence to its long-run mean is slow.

(a)

$$\frac{std\left(var_t(m_{t,t+k})\right)}{std(\Delta s_{t,t+k})} \geq \frac{1}{\sqrt{R_k^2}} + \sqrt{R_k^2}\left(\frac{1}{b_k}\frac{R^2_{Fama,k}}{R_k^2} - 1\right).$$

If the Fama regression yields the best predictor of the exchange rate movement, then, we can simplify this formula to

$$\frac{std\left(var_t(m_{t,t+k})\right)}{std(\Delta s_{t,t+k})} \geq \frac{1}{\sqrt{R_k^2}} + \sqrt{R_k^2}\left(\frac{1}{b_k} - 1\right) = \frac{1}{\sqrt{R_k^2}} - \sqrt{R_k^2} + sign(b_k)\frac{std(f_t^k - s_t)}{std(\Delta s_{t,t+k})}$$

(b)

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t,t+k}])}{std(\mathbb{E}_t[\Delta s_{t,t+k}])} + \frac{std\left(var_t(m_{t,t+k})\right)}{std(\mathbb{E}_t[\Delta s_{t,t+k}])}} \geq \frac{std(\Delta s_{t,t+k})}{std(\mathbb{E}_t[\Delta s_{t,t+k}])} = \frac{1}{\sqrt{R_k^2}}.$$

Among these conditions,

- *(a) $\Rightarrow$ (b).*
- *If Fama regression yields the best predictor and $b \notin (0,1)$, (b) $\Rightarrow$ (a); otherwise (b) is a weaker condition.*

5 Bond Euler Equation Wedges

Our result shows that IRBC models with the four bond Euler equations cannot simultaneously generate a negative Backus-Smith coefficient and replicate the Fama regression coefficient and the Meese-Rogoff puzzle.

We need to entertain models that break these four Euler equations in one way or another. First, Corsetti et al. [2008], Pavlova and Rigobon [2012] consider incomplete-market settings in which only one type of bond is traded. When the bond is denominated in a country's numéraire, this set-up drops two of our four Euler equations. When the bond is denominated in a basket of country-level numéraires, this set-up drops all of our four Euler equations and supplement with two new ones.

Second, Alvarez, Atkeson, and Kehoe [2002b, 2009b] consider models in which agents need to pay a cost to access the financial market, which is equivalent to adding additional wedges in the four Euler equations we consider in this paper. Relatedly, Jiang et al.

[2018], Jiang, Krishnamurthy, and Lustig [2020], Jiang et al. [2021] introduce bond convenience yields to the dollar safe assets, which introduce another type of the Euler equation wedges. However, we show that one version of the convenience yield dynamics which is supported by the data – convenience yields on dollar bonds – does not resolve the exchange rate cyclicality puzzle.

Third, Gabaix and Maggiori [2015], Itskhoki and Mukhin [2021], Fukui et al. [2023] consider models in which domestic investors can only hold local bonds. They introduce a financial intermediary who can trade currencies. This set-up effectively removes the two cross-country Euler equations out of the four Euler equations we consider in this paper, and replaces them with an additional Euler equation that captures the trade-off of the international intermediary. Similarly, Gourinchas et al. [2020], Greenwood et al. [2020] study models with preferred-habitat investors and global arbitrageurs, which remove all Euler equations we consider and only keep one Euler equation that captures the global arbitrageurs' long-short portfolio decision.

We explain how these models implicitly insert cross-currency Euler equation wedges that are consistent with a home currency bias in bonds to improve the model's fit with the data. There is a wealth of empirical evidence that investors act as if they face large transaction costs, capital controls or other frictions when buying foreign securities [Lewis, 1995]. This is as typically referred to as the home bias puzzle. Alternatively, they may be inserting domestic bond Euler equation wedges as well.

We start by analyzing the case in which we only allow cross-currency Euler equation wedges in section 5.1. Next, in section 5.2, we allow for wedges in the domestic bond Euler equations as well.

5.1 Cross-Currency Wedges and the Home Currency Bias

We start by allowing for wedges only in the Euler equations of investors buying foreign currency risk-free bonds. These wedges for the domestic and foreign investor respectively are denoted (ξ_t, ξ_t^*). Then, the four Euler equations can be expressed as follows:

$$\begin{aligned}
1 &= \mathbb{E}_t\left[\exp(m_{t,t+1} + r_t)\right], \\
\exp(\xi_t) &= \mathbb{E}_t\left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right], \\
1 &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + r_t^*)\right], \\
\exp(\xi_t^*) &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + \Delta s_{t+1} + r_t)\right].
\end{aligned}$$

These wedges are security-specific. They only apply to the bonds denominated in a currency different from the home currency. Investors apply a lower SDF to the bond payoffs that accrue in foreign currency. Positive wedges are akin to inconvenience yields derived from holding bonds denominated in foreign currency. They effectively segment the home currency from the foreign currency bond markets.

This approach nests the models by Gabaix and Maggiori [2015], Itskhoki and Mukhin [2021] because they remove the two cross-country Euler equations. This approach nests the model by Alvarez et al. [2002b, 2009b] who consider models in which agents need to pay a cost to access securities and currency markets. This approach nests convenience yields earned by foreign investors Jiang et al. [2018, 2021] which would correspond to negative Euler equation wedges. This also nests Corsetti et al. [2008], Pavlova and Rigobon [2012] who consider incomplete-market settings in which only one type of bond denominated in a global numéraire is traded.

Reorganizing the terms, we obtain two expressions that relate the expected excess return of a strategy that goes long the foreign bond to the perceived risks from the home and foreign perspectives as well as the wedges:

$$
\begin{aligned}
(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}, -\Delta s_{t+1}) + \xi_t, \\
(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2} var_t(\Delta s_{t+1}) &= -cov_t(m^*_{t,t+1}, \Delta s_{t+1}) + \xi_t^*.
\end{aligned}
$$

Combining these expressions, we directly obtain the following result.

Proposition 5. *In the presence of Euler equation wedges, the conditional exchange rate cyclicality is given by:*

$$
cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) - (\xi_t^* + \xi_t).
$$

The conditional correlation is given by:

$$
corr_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = \frac{std_t(\Delta s_{t,t+1})}{std_t(m_{t,t+1} - m^*_{t,t+1})} \left(1 - \frac{(\xi_t^* + \xi_t)}{var_t(\Delta s_{t,t+1})}\right).
$$

In order to obtain conditionally pro-cyclical exchange rates $cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$, we need positive wedges that exceed the exchange rate variance, i.e., $var_t(\Delta s_{t+1}) < (\xi_t^* + \xi_t)$. There is no need to shrink the conditional variance to zero in order to generate an exchange rate disconnect between innovations to the SDF and innovations to the exchange rate. We need positive wedges to mitigate the conditional version of the Backus-

Smith puzzle. These wedges reduce the need for exchange rates to respond to shocks to the pricing kernel in order to enforce the bond Euler equation wedges.

A natural example would be a model in which foreign investors cannot invest in the domestic risk-free bond, but the domestic investors can invest in the foreign risk-free bond. In this case, $\xi_t = 0$. In order to obtain conditionally pro-cyclical exchange rates, we need a home currency bias. Domestic investors are willing to forgo a return equal to at least the variance of exchange rates when considering foreign currency bonds: $\xi_t^* > var_t(\Delta s_{t+1})$.

Proposition 6. *In the presence of only cross-border Euler equation wedges, each of the following is a necessary condition for a negative unconditional exchange rate cyclicality, i.e.,* $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$*:*

(a)

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}}\left(1 - \frac{\mathbb{E}(\xi_t^* + \xi_t)}{var(\Delta s_{t+1})}\right) + \sqrt{R^2}\left(\frac{1}{b}\frac{R^2_{Fama}}{R^2} - 1\right).$$

If the Fama regression yields the best predictor of the exchange rate movement, then, we can simplify the formula to

$$\begin{aligned}\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} &\geq \frac{1}{\sqrt{R^2}}\left(1 - \frac{\mathbb{E}(\xi_t^* + \xi_t)}{var(\Delta s_{t+1})}\right) + \sqrt{R^2}\left(\frac{1}{b} - 1\right) \\ &= \frac{1}{\sqrt{R^2}}\left(1 - \frac{\mathbb{E}(\xi_t^* + \xi_t)}{var(\Delta s_{t+1})}\right) - \sqrt{R^2} + sign(b)\frac{std(f_t - s_t)}{std(\Delta s_{t+1})}.\end{aligned}$$

(b)

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t+1}])}{std(\mathbb{E}_t[\Delta s_{t+1}])} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{1}{R^2}\left(1 - \frac{\mathbb{E}(\xi_t^* + \xi_t)}{var(\Delta s_{t+1})}\right).$$

Among these conditions,

- *(a) $\Rightarrow$ (b).*
- *If Fama regression yields the best predictor and $b \notin (0, 1)$, (b) $\Rightarrow$ (a); otherwise (b) is a weaker condition.*

Armed with a home currency bias, these conditions can now be satisfied even if the exchange rate is close to a random walk and $R^2 = 0$, as long as the sum of the wedges exceeds the variance of the spot exchange rate changes. If these wedges are large enough to flip the sign on the right-hand side, then we do not need to rely on a highly volatile market price of risk and/or large U.I.P. deviations. This being the case, condition (a) is always satisfied if $b \leq 0$, even if the market price of risk is constant, while condition (b) is satisfied even in the case of U.I.P with constant market prices of risk when $b = 1$.

Importantly, only the first moment of the cross-currency wedges matter. These wedges are substitutes for covariance between the pricing kernel and the change in the spot exchange rate. The variance of the wedge is irrelevant. The wedge can even be fixed.

5.2 Home Bond Wedges, Convenience Yields and Financial Repression

Next, we allow for wedges in the Euler equations of investors buying risk-free domestic bonds. These home wedges for the domestic and foreign investor respectively are denoted (ϕ_t, ϕ_t^*). Then, the four Euler equations can be expressed as follows:

$$
\begin{aligned}
\exp(\phi_t) &= \mathbb{E}_t\left[\exp(m_{t,t+1} + r_t)\right], \\
\exp(\xi_t) &= \mathbb{E}_t\left[\exp(m_{t,t+1} - \Delta s_{t+1} + r_t^*)\right], \\
\exp(\phi_t^*) &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + r_t^*)\right], \\
\exp(\xi_t^*) &= \mathbb{E}_t\left[\exp(m_{t,t+1}^* + \Delta s_{t+1} + r_t)\right].
\end{aligned}
$$

Assuming log-normality, we obtain

$$
\begin{aligned}
\phi_t &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) + r_t, \\
\xi_t &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m_{t,t+1}, -\Delta s_{t+1}) + r_t^*, \\
\phi_t^* &= \mathbb{E}_t[m_{t,t+1}^*] + \frac{1}{2}var_t(m_{t,t+1}^*) + r_t^*, \\
\xi_t^* &= \mathbb{E}_t[m_{t,t+1}^*] + \frac{1}{2}var_t(m_{t,t+1}^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m_{t,t+1}^*, \Delta s_{t+1}) + r_t.
\end{aligned}
$$

Reorganizing the terms, we obtain two expressions that relate the expected excess return of a strategy that goes long the foreign bond to the perceived risks from the home and foreign perspectives as well as the wedges:

$$
\begin{aligned}
(r_t^* - r_t) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}, -\Delta s_{t+1}) + \xi_t - \phi_t, \\
(r_t - r_t^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) &= -cov_t(m_{t,t+1}^*, \Delta s_{t+1}) + \xi_t^* - \phi_t^*.
\end{aligned}
$$

By combining these equations, we directly obtain

$$cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) - (\xi^*_t + \xi_t) + (\phi_t + \phi^*_t).$$

In order to obtain pro-cyclical exchange rates $cov_t(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$, we need $var_t(\Delta s_{t+1}) < (\xi^*_t + \xi_t) - (\phi_t + \phi^*_t)$. Clearly, in addition to positive cross-currency Euler equation wedges, negative domestic bond equation wedges will help to satisfy this condition.

Next, we derive restrictions on the wedges that are needed to change the sign of the unconditional exchange rate cyclicality.

Proposition 7. *Let* $\omega = \mathbb{E}[-(\xi^*_t + \xi_t) + (\phi_t + \phi^*_t)] - cov\,(\phi^*_t, q_t) + cov\,(\phi_t, q_t)$ *denote the new adjustment term that arises from the wedges. In the presence of Euler equation wedges, each of the following is a necessary condition for a negative unconditional exchange rate cyclicality, i.e.,* $cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) < 0$*:*

(a)

$$\frac{std\,(var_t(m_{t,t+1}))}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) + \sqrt{R^2}\left(\frac{1}{b}\frac{R^2_{Fama}}{R^2} - 1\right)$$

If the Fama regression yields the best predictor of the exchange rate movement, then, we can simplify the formula to

$$\begin{aligned}\frac{std\,(var_t(m_{t,t+1}))}{std(\Delta s_{t+1})} &\geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) + \sqrt{R^2}\left(\frac{1}{b} - 1\right) \\ &= \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) - \sqrt{R^2} + sign(b)\frac{std(f_t - s_t)}{std(\Delta s_{t+1})}.\end{aligned}$$

(b)

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t+1}])}{std(\mathbb{E}_t[\Delta s_{t+1}])} + \frac{std\,(var_t(m_{t,t+1}))}{std(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{1}{R^2}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right).$$

Among these conditions,

- *(a)* $\Rightarrow$ *(b).*
- *If Fama regression yields the best predictor and* $b \notin (0, 1)$*, (b)* $\Rightarrow$ *(a); otherwise (b) is a weaker condition.*

As was the case of conditional covariance, this proposition shows that a sufficiently negative ω, which can be obtained by assuming positive transaction costs, i.e., $\xi_t + \xi^*_t > 0$, is able to lower the right-hand side of the inequalities (a)–(b), and make it easier to attain a negative unconditional exchange rate cyclicality. Alternatively, we need a sufficiently negative Euler equation wedge for domestic bonds $\phi_t + \phi_t^* < 0$, which can be interpreted either as a convenience yield from holdings of domestic bonds, or, alternatively, a symptom of financial repression. Governments routinely adopt measures to allow themselves to borrow at below-market rates. This is usually referred to as financial repression [see Reinhart, Kirkegaard, and Sbrancia, 2011, Chari, Dovis, and Kehoe, 2020].

During the Great Financial Crisis, banks were induced by their national governments to buy the sovereign debt of their countries [Acharya and Steffen, 2015, De Marco and Macchiavelli, 2016, Ongena, Popov, and Van Horen, 2019]. Since the 2008 GFC, central banks in advanced economies have increased the size of their balance sheets to purchase government bonds, a new wave of financial repression [see Hall and Sargent, 2022, for a comparison of the pandemic and two World Wars]. Financial repression come in other forms, including macro-prudential regulation that favors government bonds, direct lending to the government by domestic pension funds and banks, moral suasion used to increase domestic bank holdings of government bonds [see Acharya and Steffen, 2015, De Marco and Macchiavelli, 2016, Ongena et al., 2019, for examples from Europe during the GFC].[5] Japan is a textbook example of financial repression. The Bank of Japan has implemented a yield curve control policy.

6 Conclusion

In order to break the relation between exchange rates and macro fundamentals, models need to impute a home currency bias to bond market investors. Alternatively, these investors need to derive large convenience yields from their holdings of domestic bonds.

[5] Chari et al. [2020] derive conditions under which forcing banks to hold government debt may be optimal, because it acts as a commitment device.

References

Viral V. Acharya and Sascha Steffen. The "greatest" carry trade ever? understanding eurozone bank risks. *Journal of Financial Economics*, 115(2):215–236, 2015. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2014.11.004. URL https://www.sciencedirect.com/science/article/pii/S0304405X14002505.

Fernando Alvarez, Andrew Atkeson, and Patrick J. Kehoe. Money, interest rates and exchange rates with endogenously segmented markets. *Journal of Political Economy*, 110(1):73–112, 2002a.

Fernando Alvarez, Andrew Atkeson, and Patrick J Kehoe. Money, interest rates, and exchange rates with endogenously segmented markets. *Journal of political Economy*, 110 (1):73–112, 2002b.

Fernando Alvarez, Andrew Atkeson, and Patrick J. Kehoe. Time-varying risk, interest rates and exchange rates in general equilibrium. *Review of Economic Studies*, 76:851 – 878, 2009a.

Fernando Alvarez, Andrew Atkeson, and Patrick J Kehoe. Time-varying risk, interest rates, and exchange rates in general equilibrium. *The Review of Economic Studies*, 76(3): 851–878, 2009b.

David Backus, Silverio Foresi, and Chris Telmer. Affine term structure models and the forward premium anomaly. *Journal of Finance*, 56(1):279–304, 2001.

David Backus, Nina Boyarchenko, and Mikhail Chernov. Term structures of asset prices and returns. *Journal of Financial Economics*, 129(1):1–23, 2018.

David K Backus and Gregor W Smith. Consumption and real exchange rates in dynamic economies with non-traded goods. *Journal of International Economics*, 35(3-4):297–316, 1993.

David K Backus, Patrick J Kehoe, and Finn E Kydland. International real business cycles. *Journal of political Economy*, 100(4):745–775, 1992.

Ravi Bansal and Amir Yaron. Risks for the long run: A potential resolution of asset pricing puzzles. *Journal of Finance*, 59:1481–1509, August 2004.

Robert Barro. Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, 121:823–866, 2006.

Michael W. Brandt, John Cochrane, and Pedro Santa-Clara. International risk-sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics*, 53(4):671–698, May 2006a.

Michael W Brandt, John H Cochrane, and Pedro Santa-Clara. International risk sharing is better than you think, or exchange rates are too smooth. *Journal of Monetary Economics*, 53(4):671–698, 2006b.

V. V. Chari, Alessandro Dovis, and Patrick J. Kehoe. On the optimality of financial repression. *Journal of Political Economy*, 128(2):710–739, 2020. doi: 10.1086/704575. URL https://doi.org/10.1086/704575.

Varadarajan V. Chari, Patrick J. Kehoe, and Ellen R. McGrattan. Can sticky price models generate volatile and persistent real exchange rates? *The Review of Economic Studies*, 69 (240):533–564, July 2002.

Mikhail Chernov, Valentin Haddad, and Oleg Itskhoki. What do financial markets say about the exchange rate? https://www.wu.ac.at/fileadmin/wu/d/ri/isk/VSFX_2023/Paper/23_HADDAD_FMER_230223_MC.pdf, 2023. Accessed: 2023-6-23.

YiLi Chien, Hanno Lustig, and Kanda Naknoi. Why are exchange rates so smooth? a household finance explanation. *Journal of Monetary Economics*, 112:129–144, 2020.

Riccardo Colacito and Mariano M Croce. Risks for the long run and the real exchange rate. *Journal of Political economy*, 119(1):153–181, 2011.

Giancarlo Corsetti, Luca Dedola, and Sylvain Leduc. International risk sharing and the transmission of productivity shocks. *The Review of Economic Studies*, 75(2):443–473, 2008.

Filippo De Marco and Marco Macchiavelli. The political origin of home bias: The case of europe. March 2016.

Winston Wei Dou and Adrien Verdelhan. The volatility of international capital flows and foreign assets. *Unpublished working paper, MIT*, 2015.

Martin D. D. Evans and Richard K. Lyons. Order flow and exchange rate dynamics. *Journal of Political Economy*, 110:170–180, 2002.

Martin D.D. Evans and Richard K. Lyons. Meese-rogoff redux: Micro-based exchange rate forecasting. *American Economic Review*, 95(2):405–414, 2005.

Eugene F Fama. Forward and spot exchange rates. *Journal of monetary economics*, 14(3): 319–338, 1984.

Emmanuel Farhi and Xavier Gabaix. Rare disasters and exchange rates. *The Quarterly Journal of Economics*, 131(1):1–52, 2016.

Emmanuel Farhi and Iván Werning. Dilemma not trilemma? capital controls and exchange rates with volatile capital flows. *IMF Economic Review*, 62(4):569–605, November 2014.

Masao Fukui, Emi Nakamura, and Jón Steinsson. The macroeconomic consequences of exchange rate depreciations. Working Paper 31279, National Bureau of Economic Research, May 2023. URL http://www.nber.org/papers/w31279.

Xavier Gabaix and Matteo Maggiori. International liquidity and exchange rate dynamics. *The Quarterly Journal of Economics*, 130(3):1369–1420, 2015.

Pierre-Olivier Gourinchas, Walker D Ray, and Dimitri Vayanos. A preferred-habitat model of term premia, exchange rates, and monetary policy spillovers. Technical report, National Bureau of Economic Research, 2020.

Pierre-Olivier Gourinchas and Hélène Rey. International financial adjustment. *Journal of Political Economy*, 115(4):665–703, 2007. ISSN 00223808, 1537534X. URL http://www.jstor.org/stable/10.1086/521966.

Robin Greenwood, Samuel G Hanson, Jeremy C Stein, and Adi Sunderam. A quantity-driven theory of term premia and exchange rates. Technical report, National Bureau of Economic Research, 2020.

George J. Hall and Thomas J. Sargent. Three world wars: Fiscal–monetary consequences. *Proceedings of the National Academy of Sciences*, 119(18):e2200349119, 2022. doi: 10.1073/pnas.2200349119. URL https://www.pnas.org/doi/abs/10.1073/pnas.2200349119.

Lars Peter Hansen and Robert J. Hodrick. Forward exchange rates as optimal predictors of future spot rates: An econometric analysis. *Journal of Political Economy*, 88(5):829–853, October 1980.

Tarek A Hassan. Country size, currency unions, and international asset returns. *The Journal of Finance*, 68(6):2269–2308, 2013.

Oleg Itskhoki and Dmitry Mukhin. Exchange rate disconnect in general equilibrium. *Journal of Political Economy*, 129(8):2183–2232, 2021.

Zhengyang Jiang and Robert J Richmond. Origins of international factor structures. *Journal of Financial Economics*, 147(1):1–26, 2023.

Zhengyang Jiang, Arvind Krishnamurthy, and Hanno Lustig. Foreign safe asset demand and the dollar exchange rate. Technical report, National Bureau of Economic Research, 2018.

Zhengyang Jiang, Arvind Krishnamurthy, and Hanno Lustig. Dollar safety and the global financial cycle. Technical report, National Bureau of Economic Research, 2020.

Zhengyang Jiang, Arvind Krishnamurthy, Hanno Lustig, and Jialu Sun. Beyond incomplete spanning: Convenience yields and exchange rate disconnect. Technical report, 2021.

Robert M.W.K. Kollmann. *Essays on International Business Cycles*. PhD thesis, University of Chicago, Department of Economics, 1991.

Arvind Krishnamurthy and Annette Vissing-Jorgensen. The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267, 2012.

Karen K. Lewis. Puzzles in international financial markets. In G. Grossman and K. Rogoff, editors, *Handbook of International Economics*, pages 1913–1971. Elsevier Science B.V., Amsterdam, 1995.

Francis A Longstaff and Monika Piazzesi. Corporate earnings and the equity premium. *Journal of financial Economics*, 74(3):401–421, 2004.

Hanno Lustig and Adrien Verdelhan. Does incomplete spanning in international financial markets help to explain exchange rates? *American Economic Review*, 109(6):2208–44, 2019.

Hanno Lustig, Andreas Stathopoulos, and Adrien Verdelhan. The term structure of currency carry trade risk premia. *American Economic Review*, 109(12):4142–77, December 2019. doi: 10.1257/aer.20180098. URL https://www.aeaweb.org/articles?id=10.1257/aer.20180098.

Matteo Maggiori, Brent Neiman, and Jesse Schreger. International currencies and capital allocation. *Journal of Political Economy*, 128(6):2019–2066, 2020. doi: 10.1086/705688. URL https://doi.org/10.1086/705688.

Richard A Meese and Kenneth Rogoff. Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of international economics*, 14(1-2):3–24, 1983.

Maurice Obstfeld and Kenneth Rogoff. The six major puzzles in international macroeconomics: Is there a common cause? *NBER Macroeconomics Annual*, 15:339–390, January 2000.

Steven Ongena, Alexander Popov, and Neeltje Van Horen. The invisible hand of the government: Moral suasion during the european sovereign debt crisis. *American Economic Journal: Macroeconomics*, 11(4):346–79, October 2019. doi: 10.1257/mac.20160377. URL https://www.aeaweb.org/articles?id=10.1257/mac.20160377.

Anna Pavlova and Roberto Rigobon. Equilibrium portfolios and external adjustment under incomplete markets. In *AFA 2009 San Francisco Meetings Paper*, 2012.

Carmen M. Reinhart, Jacob F. Kirkegaard, and M. Belen Sbrancia. Finance and development. https://www.imf.org/external/pubs/ft/fandd/2011/06/reinhart.htm, June 2011. Accessed: 2022-11-25.

Thomas A. Rietz. The equity risk premium: A solution. *Journal of Monetary Economics*, 22: 117–131, 1988.

Kenneth Rogoff. The purchasing power parity puzzle. *Journal of Economic Literature*, XXXIV:647–668, 1996.

Barbara Rossi. Exchange rate predictability. *Journal of Economic Literature*, 51(4):1063–1119, December 2013. doi: 10.1257/jel.51.4.1063. URL https://www.aeaweb.org/articles?id=10.1257/jel.51.4.1063.

Ralph Tryon. Testing for rational expectations in foreign exchange markets. International Finance Discussion Papers, 1979.

Adrien Verdelhan. A habit-based explanation of the exchange rate risk premium. *Journal of Finance*, 65(1):123–145, 2010.

Appendix

A Proof

A.1 Proposition 1

Proof. Combining

$$\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^* = cov_t(m_{t,t+1}, -\Delta s_{t+1}) + \frac{1}{2}var_t(\Delta s_{t+1}),$$
$$-(\mathbb{E}_t[\Delta s_{t+1}] + r_t - r_t^*) = cov_t(m_{t,t+1}^*, \Delta s_{t+1}) + \frac{1}{2}var_t(\Delta s_{t+1}),$$

we directly obtain

$$cov_t(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) = var_t(\Delta s_{t+1}) > 0.$$

□

A.2 Proposition 2

Proof. The US Euler equations are given by

$$\mathbb{E}_t[\exp(m_{t+1} + r_t)] = 1$$
$$\mathbb{E}_t[\exp(m_{t+1} + r_t^* - \Delta s_{t+1})] = 1$$

where $m_{t+1} = \log M_{t+1}$. By the definition of entropy and co-entropy, we recast the equations as follows

$$\begin{aligned}
0 &= \log \mathbb{E}_t[\exp(m_{t+1} + r_t)] \\
&= \log \mathbb{E}_t[\exp(m_{t+1})] + r_t \\
&= \log \mathbb{E}_t[m_{t+1}] + L_t(M_{t+1}) + r_t \\
0 &= \log \mathbb{E}_t[\exp(m_{t+1} + r_t^* - \Delta s_{t+1})] \\
&= \log \mathbb{E}_t[\exp(m_{t+1} - \Delta s_{t+1})] + r_t^* \\
&= \mathbb{E}_t[m_{t+1} - \Delta s_{t+1}] + L_t(M_{t+1}\frac{S_t}{S_{t+1}}) + r_t^* \\
&= \mathbb{E}_t[m_{t+1} - \Delta s_{t+1}] + r_t^* + C_t(M_{t+1}, \frac{S_t}{S_{t+1}}) + L_t(M_{t+1}) + L_t(\frac{S_t}{S_{t+1}})
\end{aligned}$$

Subtract the first equation from the second to get

$$r_t^* - r_t + \mathbb{E}_t[\Delta s_{t+1}] + L_t(\frac{S_t}{S_{t+1}}) = -C_t(M_{t+1}, \frac{S_t}{S_{t+1}}).$$

Similarly, from the foreign Euler equations we obtain

$$r_t - r_t^* - \mathbb{E}_t[\Delta s_{t+1}] + L_t(\frac{S_{t+1}}{S_t}) = -C_t(M_{t+1}^*, \frac{S_{t+1}}{S_t})$$

Add up the two equations to get the proposition. Note that entropy is always greater than zero, which ensures the inequality in the proposition.

□

A.3 Proposition 3

Proof. Using the definition of p_t and q_t, we can restate the covariance as follows:

$$cov(\mathbb{E}_t[m_{t,t+1} - m_{t,t+1}^*], \mathbb{E}_t[\Delta s_{t+1}]) = cov(p_t + q_t, q_t) + \frac{1}{2}cov\left(var_t(m_{t,t+1}^*), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right).$$

Note $cov(p_t + q_t, q_t) = b \times var(p_t + q_t)$ by the construction of the Fama regression. Then,

$$cov(\mathbb{E}_t[m_{t,t+1} - m_{t,t+1}^*], \mathbb{E}_t[\Delta s_{t+1}]) = b \times var(p_t + q_t) + \frac{1}{2}cov\left(var_t(m_{t,t+1}^*), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right).$$

A negative unconditional exchange rate cyclicality then implies

$$\begin{aligned} cov(m_{t,t+1} - m_{t,t+1}^*, \Delta s_{t+1}) &= \mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(p_t + q_t) \\ &\quad + \frac{1}{2}cov\left(var_t(m_{t,t+1}^*), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) \leq 0 \end{aligned}$$

Rearranging terms,

$$\frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) + \frac{1}{2}cov\left(var_t(m_{t,t+1}^*), -q_t\right) \geq \mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(p_t + q_t)$$

By assumption,

$$\begin{aligned} cov\left(var_t(m_{t,t+1}), q_t\right) &\geq \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) + \frac{1}{2}cov\left(var_t(m_{t,t+1}^*), -q_t\right) \\ &\geq \mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(p_t + q_t) \end{aligned}$$

Hence, a necessary (but not sufficient) condition is given by:

$$
\begin{aligned}
std\left(var_t(m_{t,t+1})\right) &\geq \frac{\mathbb{E}[var_t(\Delta s_{t+1})] + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\
std\left(var_t(m_{t,t+1})\right) &\geq \frac{var(\Delta s_{t+1}) - var(\mathbb{E}_t[\Delta s_{t+1}]) + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\
std\left(var_t(m_{t,t+1})\right) + std(\mathbb{E}_t[\Delta s_{t+1}]) &\geq \frac{var(\Delta s_{t+1}) + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 &\geq \frac{var(\Delta s_{t+1}) + b \times var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])}
\end{aligned}
$$

which implies

$$
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])} \geq \frac{var(\Delta s_{t+1})}{var(\mathbb{E}_t[\Delta s_{t+1}])} = \frac{1}{R^2} \tag{5}
$$

Now, notice that

$$
b = \frac{cov(\Delta s_{t+1}, f_t - s_t)}{var(f_t - s_t)} = \frac{std(\Delta s_{t+1})}{std(f_t - s_t)} corr(\Delta s_{t+1}, f_t - s_t).
$$

We obtain

$$
\begin{aligned}
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})\sqrt{R^2}} + 1 - b \times \frac{var(f_t - s_t)}{var(\Delta s_{t+1})R^2} &\geq \frac{1}{R^2} \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} + \sqrt{R^2} - b \times \frac{corr(\Delta s_{t+1}, f_t - s_t)^2}{b^2\sqrt{R^2}} &\geq \frac{1}{\sqrt{R^2}} \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} &\geq \frac{1}{\sqrt{R^2}} - \sqrt{R^2} + \frac{R^2_{Fama}}{b\sqrt{R^2}}
\end{aligned}
$$

When Fama Regression yields the best predictor, $R^2_{Fama} = R^2$, the formula is simplified to

$$
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}} - \sqrt{R^2} + \frac{1}{b}\sqrt{R^2},
$$

where

$$
\frac{1}{b}\sqrt{R^2} = \frac{1}{b}\frac{|b| std(f_t - s_t)}{std(\Delta s_{t+1})} = sign(b)\frac{std(f_t - s_t)}{\Delta s_{t+1}}.
$$

Hence, we arrive at condition (a).

Next, we show condition (b). By using the definition of covariance and imposing symmetry:

$$cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) = cov(p_t + q_t, q_t) - cov\left(var_t(m_{t,t+1}), q_t\right)$$
$$= var(q_t) + cov(p_t, q_t) - cov\left(var_t(m_{t,t+1}), q_t\right).$$

Using the definition of the covariance, this covariance expression on the left-hand side can be bounded below as follows:

$$cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \geq var(q_t) - std(p_t)std(q_t) - std(q_t)std\left(var_t(m_{t,t+1})\right).$$

This lower bound can be restated as:

$$cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \geq var(q_t)\left(1 - \frac{std(p_t)}{std(q_t)} - \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)}\right).$$

To get a negative unconditional Backus Smith coefficient, we need the following condition:

$$cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) = \mathbb{E}[var_t(\Delta s_{t+1})] + cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \leq 0.$$

This can be restated as follows:

$$-\mathbb{E}[var_t(\Delta s_{t+1})] \geq cov(\mathbb{E}_t[m_{t,t+1} - m^*_{t,t+1}], \mathbb{E}_t[\Delta s_{t+1}]) \geq var(q_t)\left(1 - \frac{std(p_t)}{std(q_t)} - \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)}\right).$$

Rearranging terms, we obtain the following result:

$$\frac{std(p_t)}{std(q_t)} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)} \geq 1 + \frac{\mathbb{E}[var_t(\Delta s_{t+1})]}{var(q_t)}.$$

Note that

$$1 + \frac{\mathbb{E}[var_t(\Delta s_{t+1})]}{var(q_t)} = \frac{var(\mathbb{E}_t[\Delta s_{t+1}]) + \mathbb{E}[var_t(\Delta s_{t+1})]}{var(\mathbb{E}_t[\Delta s_{t+1}])} = \frac{var(\Delta s_{t+1})}{var(\mathbb{E}_t[\Delta s_{t+1}])}.$$

Hence, we obtain the necessary condition (b) by using the definition of the unconditional variance:

$$\sqrt{\frac{std(p_t)}{std(q_t)} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(q_t)}} \geq \frac{std(\Delta s_{t+1})}{std(q_t)} = \frac{1}{\sqrt{R^2}}.$$

To compare conditions (a) and (b), note that condition (a) can be written as Eq. (5), reproduced below,

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])} \geq \frac{1}{R^2},$$

it suffices to compare the term $std(\mathbb{E}_t[rx_{t+1}])/std(\mathbb{E}_t[\Delta s_{t+1}]) = std(p_t)/std(q_t)$ in (b) with $1 - bvar(f_t - s_t)/var(\mathbb{E}_t[\Delta s_{t+1}]) = 1 - bvar(p_t + q_t)/var(q_t)$ in (a). Consider the general case, when Fama regression does not necessarily yield the best predictor. Take conditional expectation on both sides of the regression yields

$$q_t = a + b(p_t + q_t) + x_t$$

where $x_t = \mathbb{E}_t[\varepsilon_{t+1}]$ satisfies that

$$\begin{aligned} cov(x_t, p_t + q_t) &= cov(\varepsilon_{t+1}, p_t + q_t) - cov(\varepsilon_{t+1} - x_t, p_t + q_t) = 0 \\ cov(x_t, q_t) &= cov(x_t, b(p_t + q_t)) + var(x_t) = var(x_t) \end{aligned}$$

Hence, $var(q_t) = b^2 var(p_t + q_t) + var(x_t)$, and

$$\begin{aligned} 1 - b\frac{var(p_t + q_t)}{var(q_t)} &= 1 - \frac{1}{b}\left(1 - \frac{var(x_t)}{var(q_t)}\right) \\ &= \left(1 - \frac{1}{b}\right) + \frac{1}{b}\frac{var(x_t)}{var(q_t)}. \end{aligned}$$

On the other hand, $p_t = -a/b + (1/b - 1)q_t - x_t/b$, which implies

$$\begin{aligned} \frac{std(p_t)}{std(q_t)} &= \frac{1}{std(q_t)}\sqrt{\left(\frac{1}{b} - 1\right)^2 var(q_t) + \frac{1}{b^2}var(x_t) - \frac{2}{b}\left(\frac{1}{b} - 1\right)var(x_t)} \\ &= \sqrt{\left(\frac{1}{b} - 1\right)^2 + \left(\frac{2}{b} - \frac{1}{b^2}\right)\frac{var(x_t)}{var(q_t)}}. \end{aligned}$$

Note that

$$\begin{aligned} \left(1 - b\frac{var(p_t + q_t)}{var(q_t)}\right)^2 &= \left(\frac{1}{b} - 1\right)^2 + \left(\frac{2}{b} - \frac{2}{b^2}\right)\frac{var(x_t)}{var(q_t)} + \frac{1}{b^2}\left(\frac{var(x_t)}{var(q_t)}\right)^2 \\ &= \left(\frac{std(p_t)}{std(q_t)}\right)^2 - \frac{1}{b^2}\left(1 - \frac{var(x_t)}{var(q_t)}\right)\frac{var(x_t)}{var(q_t)} \end{aligned}$$

which implies that, when $var(x_t) > 0$, i.e., the Fama regression does not yield the best predictor, condition (a) is always tighter.

When $var(x_t) = 0$, i.e., Fama regression yields the best predictor, we obtain

$$\left(1 - b\frac{var(p_t + q_t)}{var(q_t)}\right)^2 = \left(\frac{std(p_t)}{std(q_t)}\right)^2,$$

and

$$1 - b\frac{var(p_t + q_t)}{var(q_t)} = \left(1 - \frac{1}{b}\right).$$

When $1 - 1/b > 0$, i.e., $b < 0$ or $b > 1$, condition (a) and (b) are equivalent. Otherwise, $1 - b\frac{var(p_t+q_t)}{var(q_t)} < 0 < \frac{std(p_t)}{std(q_t)}$, and condition (a) is tighter.

□

A.4 Proposition 4

The proof is identical to the proof of Proposition 3. Just replace the one-period objects (e.g., $m_{t,t+1}$) with the multi-period objects (e.g., $m_{t,t+k}$).

A.5 Propositions 6 and 7

Proof. From

$$\begin{aligned}
\phi_t &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) + r_t, \\
\xi_t &= \mathbb{E}_t[m_{t,t+1}] + \frac{1}{2}var_t(m_{t,t+1}) - \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m_{t,t+1}, -\Delta s_{t+1}) + r_t^*, \\
\phi_t^* &= \mathbb{E}_t[m_{t,t+1}^*] + \frac{1}{2}var_t(m_{t,t+1}^*) + r_t^*, \\
\xi_t^* &= \mathbb{E}_t[m_{t,t+1}^*] + \frac{1}{2}var_t(m_{t,t+1}^*) + \mathbb{E}_t[\Delta s_{t+1}] + \frac{1}{2}var_t(\Delta s_{t+1}) + cov_t(m_{t,t+1}^*, \Delta s_{t+1}) + r_t,
\end{aligned}$$

we obtain

$$\mathbb{E}_t[m_{t,t+1} - m_{t,t+1}^*] = \frac{1}{2}var_t(m_{t,t+1}^*) + r_t^* - \phi_t^* - \frac{1}{2}var_t(m_{t,t+1}) - r_t + \phi_t$$

Then

$$\begin{aligned}
cov(\mathbb{E}_t[m_{t,t+1} - m_{t,t+1}^*], \mathbb{E}_t[\Delta s_{t+1}]) &= cov(p_t + q_t, q_t) + \frac{1}{2}cov\left(var_t(m_{t,t+1}^*), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) \\
&\quad - cov\left(\phi_t^*, q_t\right) + cov\left(\phi_t, q_t\right) \\
&= b \times var(p_t + q_t) + \frac{1}{2}cov\left(var_t(m_{t,t+1}^*), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right)
\end{aligned}$$

$$- cov\left(\phi_t^*, q_t\right) + cov\left(\phi_t, q_t\right)$$

A negative unconditional exchange rate cyclicality then implies

$$\begin{aligned}
&cov(m_{t,t+1} - m^*_{t,t+1}, \Delta s_{t+1}) \\
=&\mathbb{E}[var_t(\Delta s_{t+1}) - (\xi_t^* + \xi_t) + (\phi_t + \phi_t^*)] + b \times var(p_t + q_t) \\
+&\frac{1}{2}cov\left(var_t(m^*_{t,t+1}), q_t\right) - \frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) - cov\left(\phi_t^*, q_t\right) + cov\left(\phi_t, q_t\right) \leq 0
\end{aligned}$$

Rearranging terms,

$$\begin{aligned}
&\frac{1}{2}cov\left(var_t(m_{t,t+1}), q_t\right) + \frac{1}{2}cov\left(var_t(m^*_{t,t+1}), -q_t\right) - cov\left(\phi_t, q_t\right) + cov\left(\phi_t^*, q_t\right) \\
\geq&\mathbb{E}[var_t(\Delta s_{t+1}) - (\xi_t^* + \xi_t) + (\phi_t + \phi_t^*)] + b \times var(p_t + q_t)
\end{aligned}$$

By assumption,

$$\begin{aligned}
&cov\left(var_t(m_{t,t+1}), q_t\right) - cov\left(\phi_t, q_t\right) + cov\left(\phi_t^*, q_t\right) \\
\geq&\mathbb{E}[var_t(\Delta s_{t+1}) - (\xi_t^* + \xi_t) + (\phi_t + \phi_t^*)] + b \times var(p_t + q_t)
\end{aligned}$$

Let $\omega = \mathbb{E}[-(\xi_t^* + \xi_t) + (\phi_t + \phi_t^*)] - cov\left(\phi_t^*, q_t\right) + cov\left(\phi_t, q_t\right)$ denote the new adjustment term that arises from the wedges. Then, a necessary (but not sufficient) condition is given by:

$$\begin{aligned}
std\left(var_t(m_{t,t+1})\right) &\geq \frac{\mathbb{E}[var_t(\Delta s_{t+1})] + \omega + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\
std\left(var_t(m_{t,t+1})\right) &\geq \frac{var(\Delta s_{t+1}) - var(\mathbb{E}_t[\Delta s_{t+1}]) + \omega + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\
std\left(var_t(m_{t,t+1})\right) + std(\mathbb{E}_t[\Delta s_{t+1}]) &\geq \frac{var(\Delta s_{t+1}) + \omega + b \times var(f_t - s_t)}{std(\mathbb{E}_t[\Delta s_{t+1}])} \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 &\geq \frac{var(\Delta s_{t+1}) + \omega + b \times var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])}
\end{aligned}$$

which implies

$$\begin{aligned}
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])} + 1 - b \times \frac{var(f_t - s_t)}{var(\mathbb{E}_t[\Delta s_{t+1}])} &\geq \frac{var(\Delta s_{t+1}) + \omega}{var(\mathbb{E}_t[\Delta s_{t+1}])} \\
&= \frac{1}{R^2}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right)
\end{aligned}$$

Recall that $R^2_{Fama} = b^2 var(f_t - s_t)/var(\Delta s_{t+1})$. Hence,

$$\begin{aligned}
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})\sqrt{R^2}} + 1 - b \times \frac{var(f_t - s_t)}{var(\Delta s_{t+1})\sqrt{R^2}} &\geq \frac{1}{R^2}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} + \sqrt{R^2} - \frac{R^2_{Fama}}{b} &\geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) \\
\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} &\geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) - \sqrt{R^2} + \frac{R^2_{Fama}}{b\sqrt{R^2}}
\end{aligned}$$

When Fama Regression yields the best predictor, $R^2_{Fama} = R^2$, the formula is simplified to

$$\frac{std\left(var_t(m_{t,t+1})\right)}{std(\Delta s_{t+1})} \geq \frac{1}{\sqrt{R^2}}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right) - \sqrt{R^2} + \frac{\sqrt{R^2}}{b}$$

where

$$\frac{1}{b}\sqrt{R^2} = \frac{1}{b}\frac{|b| std(f_t - s_t)}{std(\Delta s_{t+1})} = sign(b)\frac{std(f_t - s_t)}{\Delta s_{t+1}}.$$

Hence, we arrive at condition (a).

Similarly, condition (b) can be derived as

$$\sqrt{\frac{std(\mathbb{E}_t[rx_{t+1}])}{std(\mathbb{E}_t[\Delta s_{t+1}])} + \frac{std\left(var_t(m_{t,t+1})\right)}{std(\mathbb{E}_t[\Delta s_{t+1}])}} \geq \frac{1}{R^2}\left(1 + \frac{\omega}{var(\Delta s_{t+1})}\right).$$

the relation between condition (a) and (b) depends only on the Fama regression but not the Euler equations, thus are identical to that in Proposition 3. □