A Portfolio Theory of International Capital Flows

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Abstract

This paper constructs a model in which the currency composition of national portfolios is an essential element in facilitating capital flows between countries. In a two country environment, each country chooses optimal nominal bond portfolios in face of real and nominal risk. Current account deficits are financed by increases in domestic currency debt, but balanced by increases in foreign currency credit. This is combined with an evolution of risk-premiums such that the rate of return on the debtor country's gross liabilities is lower than the return on its gross assets. This ensures stability of the world wealth distribution.

1 Introduction

Recent global imbalances have revived interest in models of current account dynamics and sustainability[1]. One of the key new messages in this literature is that current account adjustment may depend critically on the structure of international financial markets. In particular, Gourinchas and Rey (2004) and Tille (2004) note that the U.S. net international portfolio involves substantial gross liabilities held in U.S. dollar denominations, but also substantial gross assets in non-U.S. currencies. As shown by Tille (2004), this significantly alters the link between the exchange rate and the current account. Much of the real adjustment to a large current account deficit could take place automatically through re-valuation effects on the U.S. net international portfolio.

In light of the importance of the structure of external country portfolios in understanding current account imbalances, Lane and Milesi-Ferretti (2004) and Obstfeld (2004) have called for a renewed effort in integrating portfolio structure into theoretical dynamic open economy models. Traditional open economy portfolio balance models (see Kouri 1976, Dooley and Isard 1982) have been limited both by their empirical failure as well as their lack of clear micro-foundations. On the other hand, technical difficulties limit the application of the recent generation of dynamic open economy models (e.g. Obstfeld and Rogoff 2002) to questions where portfolio structure is of first order importance.

This paper takes a first step at bridging this gap. We develop a stochastic, continuous time model in which trade in nominal bonds represents an essential component of the external adjustment process. Specifically, our model has two countries in which independent monetary authorities pursue inflation targeting objectives, and private agents can issue internationally traded nominal bonds in the currency of either country. All international asset trade is mediated through the use of these nominal bonds.

We find that there is a unique optimal portfolio structure for each country. The form of national portfolios depends critically on the stance of monetary policies. When the price level is counter-cyclical, countries hold short positions in their own currency, and long positions in the Foreign currency. Moreover, the structure of portfolios is an essential component of current account adjustment. Capital flows will take the form of 'cross-hauling', whereby a country in current account deficit will borrow by issuing debt in its own country, but simultaneously accumulate assets in Foreign currencies. Moreover, debtor countries will pay a lower rate of interest on their gross liabilities than they receive on their gross assets.

In the model with two nominal bonds, we find that the share of world wealth held by

[1] See, for instance, Gourinchas and Rey (2004), Obstfeld (2004), Obstfeld and Rogoff (2004), Tille (2004), and Lane and Milesi Ferretti (2004).

any country follows a symmetric, stationary distribution. But the presence of diversified nominal bond portfolios is an essential element of this stationarity result. If returns on nominal bonds were independent (not correlated with technological shocks at all), then the wealth distribution would be no longer stationary.

Finally, our model implies a critical role for monetary policy. As in the model of Neumeyer (1997), eliminating exchange rate movements by a fully pegged exchange rate, or a single currency area would reduce welfare, so would the elimination of nominal assets which operate as a risk-sharing mechanism. By contrast, the model implies a unique welfare maximizing monetary rule for each country to follow (which requires flexible exchange rates).

There are limitations of our analysis. To focus exclusively on a portfolio approach to the current account, we have only a single world commodity, and full purchasing power parity (PPP). This means that the real exchange rate plays no role at all in current account adjustment. In this sense, our analysis is strictly a marriage of the inter-temporal approach to the current account (Obstfeld and Rogoff 1996) with a Merton (1971) type consumption-portfolio model. In addition we restrict preferences to have an inter-temporal elasticity of substitution of unity. This is the only tractable approach to portfolio dynamics in an economy with time-varying rates of return (see for instance, Devereux and Saito 1997).

Section 2 develops the basic model. Section 3 explores the equilibrium portfolio holdings in the model, and analyses the interaction between portfolio structure, capital flows, and the world wealth distribution. Section 4 briefly discusses the models implications for the current account and optimal monetary policy. Section 5 concludes.

2 The Model

We take a one-good two country model of a world economy. In each country there is a risky linear technology which uses capital and generates expected instantaneous return α_i with standard deviation σ_i, where $i = h$ or f, signifying the 'Home' and 'Foreign' country. Capital can be turned into consumption without cost. The return on technology i (in terms of the homogeneous good) is given by:

$$\frac{dQ_i}{Q_i} = \alpha_i dt + \sigma_i dB_i,$$

for $i = h$ or f, where dB_i is the increment to a standard Weiner process. For simplicity, we assume that the returns on the two technologies are independent, so that

$$\lim_{\Delta t \to 0} \frac{Cov_t\left(\Delta B_h(t+\Delta t), \Delta B_f(t+\Delta t)\right)}{\Delta t} = 0.$$

We will assume that residents of one country cannot directly own the technology of the other country. Hence, shares in the technology are not traded across countries. Nominal bonds can be traded between the countries, however. Bonds may be denominated in Home or Foreign currency. Although nominal bonds are risk-free in currency terms, their real returns are subject to inflation risk. We assume that inflation in country i may be represented as[2]:

$$\frac{dP_i}{P_i} = \Pi_i dt + v_i dM_i.$$

Thus, inflation has mean Π_i and standard deviation v_i, $i = h$ and f. dM_i represents the increment to a standard Weiner process. The monetary policy followed by country i is represented by the parameters Π_i and v_i, and the covariance of dM_i with dB_i. We let

$$\lim_{\Delta t \to 0} \frac{Cov_t\left(\Delta M_i(t+\Delta t), \Delta B_i(t+\Delta t)\right)}{\Delta t} = \lambda,$$

and

$$\lim_{\Delta t \to 0} \frac{Cov_t\left(\Delta M_i(t+\Delta t), \Delta M_j(t+\Delta t)\right)}{\Delta t} = 0.$$

for $i \neq j$.

The covariance term λ is the most critical parameter for the analysis. It describes the cyclical characteristics of the price level, and hence the real return on nominal bonds. In general we allow for any value of λ, such that $-1 < \lambda < 1$. Most of our discussion however will focus on the case where $\lambda < 0$, so that the price level is countercyclical[3]. The second condition here says that inflation shocks are independent across countries. This is not critical, but simplifies the algebra.

Let the instantaneous nominal return on currency i bonds be $\widehat{R}_i$. Then the real return on bond i is

$$(R_i - \Pi_i)dt - v_i dM_i,$$

where $R_i = \widehat{R}_i + v_i^2$ is an adjusted nominal interest rate[4]. This will be determined endoge-

[2] We do not explicitly model a source of demand for money. As in Woodford (2003), we can think of the model as representing a 'cashless economy'. What matters is that there is an asset whose payoff depends on the price level, and monetary policy can generate a particular distribution for the price level.

[3] Evidence for this comes from various sources. See, for instance, Kydland and Prescott (1990). Note that an alternative approach to interest rate determination would be to allow the monetary authorities to set short term interest rates (e.g. money market rates) directly, and allowing bond market rates to equal the policy determined interest rate, plus an endogenous risk premium term, plus a disturbance (diffusion term). When the innovation to interest rates was positively correlated with domestic GDP, this would represent the equivalent to the $\lambda < 0$ case in the present model.

[4] The adjustment factor comes from a Jensen's inequality term in evaluating the real return on nominal

nously as part of the world bond market equilibrium.

The budget constraint for the Home country may then be written as:

$$\begin{aligned} dW_h &= W_h(\omega_T^h(\alpha_h - r_h) + \omega_h^h(R_h - \Pi_h - r_h) + \omega_f^h(R_f - \Pi_f - r_h) + r_h)dt \\ &\quad -Cdt + W_h\left(\omega_T^h\sigma_h dB_h - \omega_h^h v_h dM_h - \omega_f^h v_f dM_f\right), \end{aligned} \tag{1}$$

where ω_T^h, ω_h^h, and ω_f^h are the portfolio shares, respectively, of the domestic technology, Home currency nominal bonds, and Foreign currency nominal bonds. It facilitates the presentation of the model to allow for a non-traded domestic risk-free real bond, with return r_h. The equilibrium value of r_h can be used as a measure the risk-free real interest rate in the Home economy. Since all Home agents are alike, this bond is in zero supply in equilibrium. C denotes consumption of the representative Home household[5].

Assume that each country is populated by a continuum of identical agents, that preferences are identical across countries, and are given by logarithmic preference, or

$$E_0 \int_0^\infty \exp(-\rho t) \ln C(t) dt,$$

where ρ is the rate of time preference.

2.1 Optimal Consumption and Portfolio Rules

We derive optimal consumption and portfolio rules in our context. At present, we simplify the analysis by assuming identical drift and diffusion parameters across countries[6]. Given logarithmic utility, expected utility maximizing agents follow the myopic consumption rule

$$C = \rho W.$$

The optimal portfolio rules may obtained as the solution to:

bonds.

[5]Note that despite the fact that all capital flows are facilitated with nominal bonds, the exchange rate plays no independent role in our analysis. Since we have one world good and PPP holds, then the rate of the change in the exchange rate S $(=\frac{P_h}{P_f})$ is just determined residually by:

$$d\ln S = \left(\Pi_h - \frac{1}{2}v_h^2 - \Pi_f + \frac{1}{2}v_f^2\right)dt + v_h dM_h - v_f dM_f.$$

The model could be extended to a multi-good environment with the real exchange rate playing a critical role. At present, however, we focus on the single-good case so as to emphasize the role of portfolio structure in current account adjustment.

[6]This assumption is not at all important for the results of the paper.

$$\begin{bmatrix} \omega_T^h \\ \omega_h^h \\ \omega_f^h \end{bmatrix} = \begin{bmatrix} \sigma^2 & -\lambda\sigma v & 0 \\ -\lambda\sigma v & v^2 & 0 \\ 0 & 0 & v^2 \end{bmatrix}^{-1} \begin{bmatrix} \alpha - r_h \\ R_h - \Pi - r_h \\ R_f - \Pi - r_h \end{bmatrix}$$

A similar set of conditions hold for the Foreign economy. In what follows we make the following parameter assumptions governing the behavior of optimal portfolios: $\sigma^2 + \lambda v\sigma > 0$ and $v^2 + \lambda v\sigma > 0$. These conditions ensure that the behavior of portfolio demands satisfy regularity properties. In particular, the first condition ensures that a rise in the risk-free rate reduces the demand for domestic currency nominal bonds, while the second ensures that a rise in the risk free rate reduces the demand for shares in the domestic technology. These conditions are not necessary for the key stability results of the paper developed below, but they make the exposition of the results substantially easier.

2.2 Autarky vs. Complete Markets

In order to provide a reference point, we describe the characteristics of the model first when there is no asset trade of any kind, and secondly, when there are complete markets (full trade in shares of each country's technology).

Without any asset trade, each country is in autarky. The risk-free rate in each country is given by $r = \alpha - \sigma^2$. In this case, equilibrium nominal bond holdings (in either currency) are zero, so that $\omega_T^i = 1$. The equilibrium nominal interest rate on Home currency bonds is $R_h = \Pi + \alpha - \sigma^2 - \lambda v\sigma$. This includes a risk premium term $\lambda v\sigma$. When $\lambda < 0$, the nominal bond is a bad hedge against technology risk, and must have a return higher than the risk-free rate, adjusted for inflation. When $\lambda > 0$, the opposite logic applies. The zero-trade equilibrium interest rate on Foreign currency bonds is $R_f = \Pi + \alpha - \sigma^2$. Since the Foreign price level is independent of Home output, it is a better hedge against consumption risk when $\lambda < 0$, and therefore carries a lower autarky return than the Home bond.

From a welfare perspective, with preferences given by (2), the relevant measure of expected consumption (or wealth) growth in any equilibrium is the *risk-adjusted growth rate*, given by:

$$\lim_{\Delta t \to 0} \frac{\Delta \ln C(t+\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{E_t\left(\frac{\Delta C(t+\Delta t)}{C(t)}\right) - \frac{1}{2}Var_t\left(\frac{\Delta C(t+\Delta t)}{C(t)}\right)}{\Delta t}.$$

In an autarky equilibrium, the risk-adjusted growth rate is given by $\alpha - \rho - \frac{1}{2}\sigma^2$.

With trade in shares of technology, there are effective complete markets. In this case, trade in nominal assets is redundant, as full diversification can be attained through trading in shares. The equilibrium share of each technology will be $\omega_{Ti} = \frac{1}{2}$, and the equilibrium risk free rate will be $r = \alpha - \frac{1}{2}\sigma^2$. Hence, the risk-pooling effect of complete markets implies a higher risk-free interest rate than under autarky. Finally, the risk-adjusted growth rate with complete markets is $\alpha - \rho - \frac{1}{4}\sigma^2$. Risk pooling between the countries raises welfare by raising the risk-adjusted consumption growth rate.

2.3 Equilibrium

Since autarky nominal returns on Home and Foreign currency bonds differ, we anticipate that there will be trade between countries in the two bonds. At any moment in time, an equilibrium in the market for Home and Foreign currency bonds determines the nominal rates of return R_h and R_f. Nominal bonds in each currency are in zero net supply, so that the sum of Foreign and domestic holdings is zero:

$$\omega_h^h W_h + \omega_h^f W_f = 0, \tag{2}$$

and

$$\omega_f^h W_h + \omega_f^f W_f = 0. \tag{3}$$

In addition, since equilibrium holdings of the non-traded risk free bond must be zero, in each country, the portfolio shares of the domestic technology plus the two nominal bonds must add to one, in each country:

$$\omega_T^h + \omega_h^h + \omega_f^h = 1, \tag{4}$$

and

$$\omega_T^h + \omega_h^h + \omega_f^h = 1. \tag{5}$$

These four conditions may be solved for R_h, R_f, r_h, and r_f. Define $\theta = \frac{W_f}{W_h + W_f}$ as the ratio of Foreign wealth to world wealth. From equations (2) through (5), it is clear that we may write the solution nominal interest rates and domestic risk-free rates as $R_h(\theta)$, $R_f(\theta)$, $r_h(\theta)$, and $r_f(\theta)$. The solutions are quite tedious to derive, but can be written as:

$$R_h(\theta) = \left[\alpha + \Pi - \sigma^2 + \frac{1}{2}\left(\sigma^2\lambda^2 - \lambda\sigma v\right)\right] + \Phi_h(\theta), \tag{6}$$

$$R_f(\theta) = \left[\alpha + \Pi - \sigma^2 + \frac{1}{2}\left(\sigma^2\lambda^2 - \lambda\sigma v\right)\right] + \Phi_f(\theta), \tag{7}$$

$$r_h(\theta) = \left[\alpha - \sigma^2 + \frac{1}{2}\sigma^2\lambda^2\right] + \Gamma_h(\theta), \tag{8}$$

and

$$r_f(\theta) = \left[\alpha - \sigma^2 + \frac{1}{2}\sigma^2\lambda^2\right] + \Gamma_f(\theta), \tag{9}$$

where we define the expressions Φ_i and Γ_i as:

$$\Phi_h(\theta) = \frac{\lambda\sigma(2\theta-1)(v+\lambda\sigma(1-2\theta))\left(v+2\lambda v\sigma-\sigma^2\left(\lambda^2-2\right)\right)}{2\left(v^2+2\lambda v\sigma+\sigma^2\left(2-(1-2\theta)^2\lambda^2\right)\right)},$$

$$\Phi_f(\theta) = \frac{v-\lambda\sigma(1-2\theta)}{v+\lambda\sigma(1-2\theta)}\Phi_h(\theta),$$

$$\Gamma_h(\theta) = \frac{\lambda^2\sigma^2(2\theta-1)\left(v^2+2\lambda v\sigma\theta+\sigma^2(2\theta-1)\left(\lambda^2-2\right)\right)}{2\left(v^2+2\lambda v\sigma+\sigma^2\left(2-(1-2\theta)^2\lambda^2\right)\right)},$$

and

$$\Gamma_f(\theta) = \frac{\lambda^2\sigma^2(2\theta-1)\left(v^2+2\lambda v\sigma(1-\theta)+\sigma^2(2\theta-1)\left(\lambda^2-2\right)\right)}{2\left(v^2+2\lambda v\sigma+\sigma^2\left(2-(1-2\theta)^2\lambda^2\right)\right)}.$$

Using these solutions with the optimal rules from the portfolio problem allows us to write the equilibrium Home country portfolio shares as:

$$\omega_h^h = \frac{\theta\lambda\sigma(v^2+2\sigma^2+3\lambda\sigma v-2\lambda\sigma v\theta)}{v\left(v^2+2\sigma^2+2\lambda\sigma v-\sigma^2\lambda^2+4\sigma^2\lambda^2\theta(1-\theta)\right)}, \tag{10}$$

and

$$\omega_f^h = -\frac{\theta\lambda\sigma(v^2+2\sigma^2+\lambda\sigma v+2\lambda\sigma v\theta)}{v\left(v^2+2\sigma^2+2\lambda\sigma v-\sigma^2\lambda^2+4\sigma^2\lambda^2\theta(1-\theta)\right)}. \tag{11}$$

The model has an appealing recursive structure. Given the myopic consumption rule, portfolio equilibrium has the property that returns depend only on the current world distribution of wealth, captured by the term θ. Even with this, however, portfolio holdings and returns are complicated functions of θ. It is helpful therefore to focus first on a special case of an equal wealth levels in each country.

3 Portfolios, Returns, and Capital Flows

3.1 A Special Case: $\theta = 0.5$

Take first the case where $\theta = 0.5$, so that wealth levels are equal in the Home and Foreign countries. From inspection of the above solutions for interest rates, we see that $\Phi_i(0.5) = 0$ and $\Gamma_i(0.5) = 0$. Hence,

$$R_i = \alpha + \Pi - \sigma^2 + \frac{1}{2}\left(\sigma^2\lambda^2 - \lambda\sigma v\right),$$

and

$$r_i = \alpha - \sigma^2 + \frac{1}{2}\sigma^2\lambda^2.$$

The real risk-free interest rate is between that of autarky and complete markets. The critical parameter determining the equilibrium real risk-free rate is λ. Since $-1 < \lambda < 1$, generically, the real risk-free rate is higher than under autarky, but lower than under complete markets. Recall that λ captures the correlation between the return on the Home (Foreign) nominal bond and the return on the Home (Foreign) technology. When $\lambda = 0$, the nominal bond can play no role at all as a hedge against technology (and therefore consumption) risk. Hence the presence of nominal bonds does not affect the equilibrium real risk-free rate, which is equal to the autarky rate. But as λ rises toward unity in absolute terms, bonds can act as a real hedge against technology risk, and the fall in consumption risk raises the real risk-free rate in each country. In the limit, as λ goes to unity in absolute value, we approach the risk-free rate under complete markets.

In the symmetric economy with nominal bond trading, nominal interest rates on Home and Foreign currency bonds are equalized across countries. The movement of the nominal interest rate, relative to autarky, depends on the sign of λ. When $\lambda < 0$, the Home currency bond is a poor hedge against consumption risk for the Home economy, and will have a high autarky return. By the same token, however, this bond is a relatively good hedge against Foreign consumption risk, and would have a low autarky return in the Foreign economy. Hence, upon opening international nominal bond markets, the Home country will sell Home currency bonds to Foreign residents, and the new equilibrium nominal return will be in between the (high) Home country autarky return and the (low) Foreign country autarky return. Of course, the same mechanism works for the Foreign currency bond. With $\lambda < 0$, its equilibrium return is less than the autarky return in the Foreign economy, but greater than the autarky return in the Home economy[7].

[7] Our simplifying assumption $\sigma^2 + \lambda v\sigma > 0$ ensures this intuitive result.

When $\theta = 0.5$, the risk-adjusted growth rate may be written as:

$$\alpha - \rho - \frac{1}{2}\sigma^2\left(1 - \frac{\lambda^2}{2}\right).$$

Again, we find that this falls between that of autarky and complete Markets. When $\lambda = 0$, the risk-adjusted growth rate is identical to that under autarky. When $|\lambda| = 1$, the bond trading regime attains the risk-adjusted growth rate under complete markets.

How does the presence of nominal bonds support risk-pooling across countries? This happens because, in face of country specific risk on the real technology, each country can hold a portfolio of domestic and Foreign currency bonds to hedge this risk. In the symmetric case with $\theta = 0.5$, net Foreign assets are zero in each country, so that, for the Home country, for instance, we have $\omega_h^h + \omega_f^h = 0$. But in order to hedge technology risk, countries find it advantageous to hold different gross positions in each currency. The optimal portfolio in this case is:

$$\omega_h^h = \frac{1}{2}\frac{\lambda\sigma}{v}, \qquad \omega_f^h = -\frac{1}{2}\frac{\lambda\sigma}{v}.$$

When $\lambda < 0$, the Home country takes a short position in domestic currency bonds and a long position in Foreign currency bonds. Since in this case, the return on domestic nominal bonds is pro-cyclical, it can use a negative gross holdings of domestic currency bond as an effective hedge against consumption risk. Similarly, a positive gross holding of Foreign currency bonds allows it to share in the Foreign technology process. This portfolio structure exploits the different returns processes on Home and Foreign currency bonds to allow the risk from technology shocks to be partly pooled across countries.

Of course when $\lambda > 0$, the process works in reverse. In this case, the Home currency bond is a good hedge against Home consumption risk. Then, in the Home country, it will have a lower autarky return than the Foreign currency bond. In a symmetric bond trading equilibrium with equal wealth, Home residents will therefore hold positive quantities of Home currency bonds, and negative amounts of Foreign currency bonds.

3.2 Effects of variation in θ

The results can be easily extended to general values of the relative wealth ratio $\frac{W_f}{W_h + W_f}$ or θ. First we focus on the expressions for the Home country portfolio, equations (10) and (11). Under the assumptions made so far and $\lambda < 0$, the first expression is negative, while the second is positive. Hence, for all levels of θ, the Home country issues its own currency bonds short, and holds positive quantities of the Foreign currency bonds. These solutions also confirm what we saw in the previous section; when $\theta = 0.5$, the net Foreign asset position is zero, since $\omega_h^h + \omega_f^h = 0$.

We may describe in more detail the behavior of both gross and net foreign assets as θ changes. Differentiating equations (10) and (11) at $\theta = 0.5$, we see that a rise in θ has the following effect on the Home country's portfolio:

$$\left.\frac{d\omega_h^h}{d\theta}\right|_{\theta=0.5} = \frac{\lambda\sigma(2\sigma^2 + \lambda\sigma v + v^2)}{v(2\sigma^2 + 2\lambda\sigma v + v^2)},$$

and

$$\left.\frac{d\omega_f^h}{d\theta}\right|_{\theta=0.5} = -\frac{\lambda\sigma(2\sigma^2 + 3\lambda\sigma v + v^2)}{v(2\sigma^2 + 2\lambda\sigma v + v^2)}.$$

When $\lambda < 0$, the first expression is negative, and the second is positive. Hence, beginning at $\theta = 0.5$, a rise in Foreign relative wealth will be followed by a rise in Home gross borrowing in domestic currency bonds, and a rise in gross lending in Foreign currency bonds.

These two effects do not cancel out, however. It is easy to see that:

$$\left[\frac{d\omega_h^h}{d\theta} + \frac{d\omega_f^h}{d\theta}\right]_{\theta=0.5} = -\frac{2\lambda^2\sigma^2}{2\sigma^2 + 2\lambda\sigma v + v^2},$$

which is negative. Hence, a rise in θ above $\theta = 0.5$ will lead to a rise in net Foreign borrowing in the Home country. This Foreign borrowing will be used to invest in the domestic technology, since from (4), it must be the case that

$$\left.\frac{d\omega_T^h}{d\theta}\right|_{\theta=0.5} = -\left[\frac{d\omega_h^h}{d\theta} + \frac{d\omega_f^h}{d\theta}\right]_{\theta=0.5} > 0.$$

Conversely, the Foreign economy will increase its net Foreign assets, as

$$\left[\frac{d\omega_h^f}{d\theta} + \frac{d\omega_f^f}{d\theta}\right]_{\theta=0.5} > 0,$$

and therefore it reduces its investment in its own domestic technology; that is, $\left.\frac{d\omega_T^f}{d\theta}\right|_{\theta=0.5} < 0$.

Thus, in the regions of a symmetric equilibrium, capital will flow to the less wealthy country. But this capital flow will take place through a 'cross-hauling' effect. When $\lambda < 0$, there will be a gross outflow to purchase Foreign currency assets, but a gross inflow for the sale of Home currency assets.

The net capital flow is reflected in the behavior of risk-free interest rates at $\theta = 0.5$.

Differentiating the impact of θ on the risk free rate, we obtain:

$$\left.\frac{dr_h}{d\theta}\right|_{\theta=0.5} = \frac{\lambda^2\sigma^2 v(\lambda\sigma + v)}{2\sigma^2 + 2\lambda\sigma v + v^2},$$

and

$$\left.\frac{dr_f}{d\theta}\right|_{\theta=0.5} = -\frac{\lambda^2\sigma^2 v(\lambda\sigma + v)}{2\sigma^2 + 2\lambda\sigma v + v^2}.$$

A rise in θ, evaluated at $\theta = 0.5$, leads to a rise in the Home risk-free rate, and a fall in the Foreign risk-free rate with our simplifying assumption $v^2 + \lambda v\sigma > 0$. This is associated with capital flows to the Home country.

Note however that capital flows can only be effected through movements in nominal bonds. The key feature of the model is the interaction between the portfolio position of each country and the share of aggregate world wealth held by the country, which in turn determines the net savings rate of the country. At $\theta = 0.5$, the two countries have exactly equal net wealth, and given the symmetry in the model, the current account of each country is zero. A rise in W_f, driven for instance by a positive technology shock in the Foreign country, will raise θ, and increase the Foreign country's demand for assets. Given that the Foreign optimal portfolio involves positive holdings of Home currency bonds and negative holdings of Foreign currency bonds, the rise in its asset demand raises world demand for Home bonds, and world supply of Foreign bonds. This leads to a fall in the return on Home bonds, and a rise in the return on Foreign bonds. From the solutions for R_h and R_f, we find that:

$$\left.\frac{dR_h}{d\theta}\right|_{\theta=0.5} = \frac{\lambda\sigma v(v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2\lambda^2)}{2\sigma^2 + 2\lambda\sigma v + v^2},$$

and

$$\left.\frac{dR_f}{d\theta}\right|_{\theta=0.5} = -\frac{\lambda\sigma v(v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2\lambda^2)}{2\sigma^2 + 2\lambda\sigma v + v^2}.$$

The first expression is negative while the second is positive, for $\lambda < 0$[8]. Thus there is a fall in the risk premium on Home currency bonds, thereby reducing the interest burden on the liabilities of the debtor country. On the other hand, there is a rise in the risk premium on Foreign country bonds, thereby increasing the return on liabilities of the creditor country. This fall in R_h and rise in R_f leads to a portfolio gain for the Home economy, given that it is

[8] Note that $v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2\lambda^2 > (v - \sigma)^2 + (1 - \lambda^2)\sigma^2 > 0$ as long as $\lambda > -1$. At this point, our simplifying assumption $\sigma^2 + \lambda v\sigma > 0$ or $v^2 + \lambda v\sigma > 0$ plays no role.

short in Home bonds and long in Foreign bonds. This reduces the effective cost of borrowing, leading it to a higher net foreign debt, and to increase investment in the domestic technology. In this manner, the original positive technology shock in the Foreign economy is shared by the Home economy.

3.3 Stability Properties

An implication of the model is that the country with a higher level of wealth is a net creditor. Using the solutions in (10) and (11) above, we can write the net foreign asset share in the Home country portfolio ($\omega_h^h + \omega_f^h$) as:

$$\omega_h^h + \omega_f^h = -\frac{2(2\theta - 1)\lambda^2\theta\sigma^2}{v^2 + 2\sigma^2 + 2\lambda\sigma v - \sigma^2\lambda^2 + 4\sigma^2\lambda^2\theta(1-\theta)}.$$

The denominator is always positive (whatever the sign of λ). Hence, as was inferred in the discussion above, Home country net Foreign assets are negative (positive) whenever $\theta > 0.5$ (< 0.5).

Is the wealth distribution stable? For this to be the case, it must be that Home wealth grows faster than Foreign wealth, when $\theta > 0.5$. To answer this question, we must explicitly derive the dynamics of θ. Using Ito's lemma and equation (1), we may write the diffusion process governing θ as:

$$d\theta = \theta\,(1-\theta)\,F(\theta)dt + \theta(1-\theta)G(\theta)dB, \tag{12}$$

where the functional forms of $F(\theta)$, $G(\theta)$, and dB are described in the Appendix. The asymptotic distribution of θ must satisfy either; (a) $\theta \to 1$, (b) $\theta \to 0$, or (c) θ follows a stable distribution in $(0, 1)$. Given the form of (12), clearly (a) and (b) are absorbing states. But the following proposition establishes the conditions under which (c) will apply.

Proposition 1 *For $\lambda \neq 0$, θ has a symmetric ergodic distribution in $(0, 1)$ centered at $\theta = \frac{1}{2}$.*
Proof. *See Appendix.* ■

The content of this proposition can be developed by showing the dependence of risk-adjusted growth rates of wealth on the relative wealth variable θ. As before, we define the risk-adjusted growth rate for country i as:

$$g_i(\theta) = \lim_{\Delta t \to 0} \frac{\Delta \ln W_i(t+\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{E_t\left(\frac{\Delta W_i(t+\Delta t)}{W_i(t)}\right) - \frac{1}{2}Var_t\left(\frac{\Delta W_i(t+\Delta t)}{W_i(t)}\right)}{\Delta t}.$$

Then, θ has an ergodic distribution if it cannot access the boundaries 0 or 1. Defining the difference between the Foreign and Home risk-adjusted growth rate as $\delta(\theta) = g_f(\theta) - g_h(\theta)$, this property holds if the probability of reaching either is zero. For the lower bound, this is the case if $\delta(0) > 0$. Likewise, the probability of reaching the upper bound is zero if $\delta(1) < 0$. This just says that as the Home country gets arbitrarily wealthy, relative to the Foreign country, the Foreign country's risk-adjusted growth rate exceeds that of the Home country. Likewise, if the Foreign country's wealth increases arbitrarily relative to that of the Home country, then the Home risk-adjusted growth rate will exceed that of the Foreign country. The Proposition establishes that, for $\lambda \neq 0$, this property always holds. We may show this directly by computing $\delta(\theta)$. The Appendix shows that $\delta(\theta)$ may be written as:

$$\delta(\theta) = \frac{\lambda^2\sigma^2(1-2\theta)(v^2+2\lambda\sigma v+2\sigma^2-\sigma^2\lambda^2)(v^2+2\lambda\sigma v+2\sigma^2)}{(4\sigma^2\theta\lambda^2(\theta-1)+\sigma^2\lambda^2-2\lambda\sigma v-2\sigma^2-v^2)^2}.$$

The denominator is always positive, and the numerator is positive (negative) for $\theta < 0.5$ (> 0.5), as long as $\lambda \neq 0$. Moreover, this satisfies the conditions

$$\delta(0) = \frac{\lambda^2\sigma^2(v^2+2\lambda\sigma v+2\sigma^2)}{v^2+2\lambda\sigma v+2\sigma^2-\sigma^2\lambda^2} > 0,$$

$$\delta(1) = -\frac{\lambda^2\sigma^2(v^2+2\lambda\sigma v+2\sigma^2)}{v^2+2\lambda\sigma v+2\sigma^2-\sigma^2\lambda^2} < 0,$$

$$\delta(0.5) = 0.$$

Hence, for $\theta > 0.5$, and the Foreign country is relatively wealthy, the Home risk-adjusted growth rate exceeds that of the Foreign country, and θ falls. The same dynamics occur in reverse when $\theta < 0.5$. These expressions also make clear that the distribution of θ is symmetric.

A key feature of the proof is the reliance on the parameter λ. This represents the correlation between prices and the real technology in each country. Note that stationarity is ensured whether λ is positive or negative. In either case, agents can make use of nominal bonds to hedge, internationally, against consumption risk, holding short the domestic (Foreign) currency bonds if $\lambda < 0$, and conversely if $\lambda > 0$. But if $\lambda = 0$, then nominal bond returns are independent of consumption risk in either country, and they will not be held in equilibrium (i.e. $\omega_j^i = 0$, for all i and j). In this case, the stationarity result fails.

To see the relationship between the portfolio structure of international assets markets and the stationarity of the world wealth distribution, take the case $\lambda < 0$. Then when $\theta > 0.5$, the Foreign country is wealthier than the Home country. The Foreign country is also a net creditor. But the positive holdings of net international assets are based on positive holdings of Home currency bonds, and negative holdings on Foreign currency bonds. As we have seen, Home currency bonds pay a relatively low return, while Foreign currency bonds pay a relatively high return. This endogenous fall in the risk-premium on the Home portfolio reduces the effective cost of borrowing for the Home economy, encouraging it to invest more in its domestic technology. Since the expected return on the domestic technology exceeds that on its nominal asset portfolio, this increases the risk-adjusted expected growth rate for the Home country, relative to the Foreign country. As a result, θ is driven back towards 0.5 again. In effect, it is the portfolio composition and its interaction with the evolution of the global wealth distribution that represents an essential element in the stability of the wealth distribution itself. Thus, current account imbalances are naturally self-correcting when agents hold an optimal currency portfolio of international debt.

Another perspective can be given from a comparison of the movement of returns relative to their autarky values. As one country begins to dominate world wealth, it will push bond returns towards their autarky values for that country. If this is the Foreign country, for instance, this will lead Home currency bond returns to fall, and Foreign country bond returns to rise, since Foreign consumption risk is relatively well hedged by Home currency bonds, and badly hedged by Foreign currency bonds. Thus, Home bond returns will approach $\alpha + \Pi - \sigma^2$, and Foreign bond returns approach $\alpha + \Pi - \sigma^2 - v\lambda\sigma$. This process reduces the net return that the Home country must pay on its international debt; that is, a key mechanism in generating a stable wealth distribution.

While this interpretation is based on a negative value of λ, this is not necessary for the stability result. If $\lambda > 0$, then the equivalent stabilizing force takes place, but now with the Foreign country holding positive (negative) amounts of Foreign (Home) currency bonds.

But for this to occur, it is essential that $\lambda \neq 0$. If $\lambda = 0$, the portfolio composition is indeterminate, since bonds can then play no role as a hedge against technology risk. In fact, agents will hold no bonds at all. Since technologies are identical, there can be no gains from trade in international bonds at all. Any innovations to wealth are permanent. Clearly then the wealth distribution will not be stable. In fact, θ will be characterized by hysteresis in technology shocks will give rise to an expected permanent increase in wealth without international asset trade at all.

As demonstrated in the Appendix, the model allows for an explicit solution for the distribution of wealth. Figure 1 illustrates the distribution of $\ln(\frac{W_f}{W_h}) = \ln(\frac{\theta}{1-\theta})$ for different values of λ with $\sigma = v = 0.02$.[9] In each case, the distribution has mean zero. But the λ

[9] A major reason for adopting $\ln \frac{W_f}{W_h}$ instead of $\frac{W_f}{W_h + W_f}$ is that the former definition can illuminate the behavior at tails of distribution with a support of $(-\infty, +\infty)$ rather than $(0, 1)$.

parameter matters substantially for the shape of the distribution. For high absolute values of λ, the distribution is tightly centered around zero. But as λ falls in absolute value, the distribution becomes substantially spread out. This means that the speed of convergence in the wealth distribution depends critically on the size of λ. For high absolute values of λ, convergence is much faster.

3.4 Characteristics of Equilibrium

Figures 2 through 5 describe some features of the equilibrium. The Figures illustrate the relationship between the relative wealth ratio θ, country portfolio weights, interest rates, and risk-adjusted growth rates. We choose the following set of parameter values for the Figures:

Table 1: Parameter Values

α	σ	λ	v	π
0.03	0.02	-0.50	0.02	0.02

Figure 2 shows how the Home country's portfolio weights depend on θ. As θ falls towards zero, the Home country dominates the world capital market, and its bond portfolio shares in both currencies are very low. But as θ rises, it increases its holdings of Foreign currency bonds, and balances this by issuing Home currency bonds. As we saw above, for $\theta < 0.5$, ($\theta > 0.5$) the former (latter) effect dominates, and it is a net creditor (debtor).

Figure 3 illustrates the pattern of real risk free rates as a function of θ. From the analysis at $\theta = 0.5$, we know that the Home country risk-free rate is increasing in θ at $\theta = 0.5$, and the Foreign risk-free rate is decreasing in θ at that point. But the behavior of risk-free rates away from $\theta = 0.5$ may be quite different. In the Figure, we see that risk-free rates may be non-monotonic in θ. As θ rises above 0.5, the Home country risk-free rate first rises, as income growth rises when capital flows go into Home technology. But, for relatively low v, this also involves increasing the riskiness of the Home portfolio, which tends to push down the risk-free rate. As θ continues to increase, this second effect can dominate, and r_h may fall in θ. But if monetary uncertainty is high enough (i.e. v high enough), then the Home risk-free rate will be monotonically increasing in θ. The logic is as follows. As θ rises towards unity, R_h approaches $\alpha + \Pi - \sigma^2$, while R_f approaches $\alpha + \Pi - \sigma^2 - \lambda\sigma v$. Since the Home country holds positive amounts of the Foreign currency bond, this pushes up its growth rate, and hence the Home risk free rate.

Figure 4 shows a similar pattern but in terms of nominal returns R_h and R_f. As the Home (Foreign) country increases its share of world wealth, it pushes down the return on the

Foreign (Home) currency bond, since that bond is a better hedge against Home (Foreign) consumption risk, when $\lambda < 0$. Similarly, because Home (Foreign) issues its own currency bond in these circumstances, the return on this bond must increase when it dominates world wealth.

Finally, Figure 5 illustrates the relationship between θ and risk-adjusted growth rates. For the reasons discussed above, risk-adjusted growth rates are monotonic in θ. As θ approaches either boundary, the risk-adjusted growth rate of the smaller country will always exceed that of the larger country.

4 Applications of the basic model

4.1 Current Account Adjustment

In our symmetric model, the mean of the distribution of θ is 0.5. This implies that the current account will have a zero long run mean with zero holdings of net Foreign assets. How does the current account adjust when we are away from the long run mean? We may define the current account as the change in net Foreign assets. Net Foreign assets of the Home country is written as:

$$(\omega_h^h + \omega_f^h)W_h = \frac{2\theta(2\theta-1)\sigma^2\lambda^2}{4\theta(\theta-1)\sigma^2\lambda^2 + \sigma^2\lambda^2 - 2\lambda\sigma v - 2\sigma^2 - v^2}W_h.$$

Since W_h is non-stationary, the level of net Foreign assets will not converge. But we may rewrite this in terms of net Foreign assets, relative to world wealth $W_h + W_f$. Hence we have:

$$NFA(\theta) = (\omega_h^h + \omega_f^h)(1-\theta) = \frac{2\theta(2\theta-1)(1-\theta)\sigma^2\lambda^2}{4\theta(\theta-1)\sigma^2\lambda^2 + \sigma^2\lambda^2 - 2\lambda\sigma v - 2\sigma^2 - v^2}.$$

From what we've seen already, it is clear that the sign of normalized net Foreign assets satisfies the conditions $NFA(\theta) < 0$ (> 0) for $\theta > 0.5$ (< 0.5). The current account is defined as the change in the value of net foreign assets. Therefore, we may define the process of the normalized current account of the Home country as:

$$dNFA(\theta) = NFA'(\theta)d\theta + \frac{1}{2}NFA''(\theta)d\theta^2.$$

Then, using the solution for the Ito process for θ defined above, we have:

$$dNFA(\theta) = \left[NFA'(\theta)\theta(1-\theta)F(\theta) + \frac{1}{2}NFA''(\theta)\theta^2(1-\theta)^2G(\theta)^2\right]dt + NFA'(\theta)\theta(1-\theta)G(\theta)\,dB, \tag{13}$$

where again, the expressions $F(\theta)$, $G(\theta)$, and dB are defined in the Appendix.

Equation (13) allows us to describe the behavior of the drift and diffusion of the normalized current account, as a function of θ. Using the same parameters as in Table 1 ($\sigma = v = 0.02$ and $\lambda = -0.5$), we may illustrate the drift term in Figure 6, or the term $NFA'(\theta)\theta(1-\theta)F(\theta) + \frac{1}{2}NFA''(\theta)\theta^2(1-\theta)^2G(\theta)^2$.

When $\theta > 0.5$, the expected current account surplus is positive for Home country, as its relative wealth growth is higher than that of the Foreign country. But for very high values of θ, the expected current account surplus becomes negative. This is because, when θ is close to unity, the Home country actively exploits a short position in Home currency bonds, which become more and more appealing with low borrowing costs. Of course, the case when $\theta < 0.5$ is just the mirror image of the $\theta > 0.5$ case.

Figure 7 illustrates the relationship between θ and the diffusion term (or volatility) in equation (13). The volatility of the (normalized) current account is highest at $\theta = 0.5$, but volatility also increases close to the boundaries, as the smaller countries increase their borrowing more in that neighborhood.

4.2 Monetary Policy Rules

An immediate implication of our model is that, in an economy where risk-sharing must be achieved by trade in nominal assets, monetary policy rules have direct implications for welfare. In the symmetric economy we saw that the complete markets allocation was attained in the limit as $\lambda \to \pm 1$. Absent problems of international monetary policy coordination, it then follows that a welfare optimal monetary rule would be to have prices perfectly countercyclical (or perfectly pro-cyclical) in each country. This guarantees that full risk sharing can be achieved through trade in nominal bonds. Moreover, the higher is λ in absolute terms, the faster will θ converge. As λ approaches one, the θ distribution collapses to its mean point of 0.5.[10]

[10] Once the absolute value of λ is one, the complete markets allocation is immediately achieved with extremely large sizes of nominal bond trading, and the wealth distribution stays forever at any point of the initial distribution, not necessarily at $\theta = 0.5$. A reason for this is that if $\lambda = -1$ for example, then the real return on the Home (Foreign) technology is exactly equal to that of the Home (Foreign) currency bond, and any size of short positions in its own currency bond is perfectly offset by the same amount of investment in its domestic technology.

An additional implication of the analysis is that it is important to have separate monetary policies. As in the model of Neumeyer (1997), eliminating exchange rate movements by a fully pegged exchange rate or a single currency area would reduce welfare, so would the elimination of nominal assets which operate as a risk-sharing mechanism.[11]

5 Conclusions

This paper develops a tractable model of international capital flows in which the existence of nominal bonds and the portfolio composition of net Foreign assets is an essential element in facilitating capital flows between countries. National monetary policies make domestic and Foreign currency denominated bonds differ in the degree to which they can hedge country specific consumption risk. This leads countries to have distinct composition of currency-denominated bonds in their national portfolios. By adjusting their gross positions in each currency's bonds, countries can achieve an optimally hedged change in their net Foreign assets (or their current account), thus facilitating international capital flows. Moreover, the risk characteristics of optimal portfolios ensures that current account movements are sustainable - net debtor countries pay lower rates of return on their gross liabilities than they receive on their gross assets. This ensures that the distribution of wealth across countries is stationary.

The modeling approach can be extended in a number of dimensions. First, we could do a more explicit welfare evaluation, comparing welfare across different bond trading regimes, as well as computing the welfare implications of alternative monetary policy rules. Secondly, we could also allow for differences in growth and volatilities of technologies across countries, as well as differences in monetary policy rules. This would allow us to calibrate the model in the direction of developing an understanding of the empirical structure of national portfolios as described by Tille (2004) and Lane and Milesi Ferretti (2004).

[11] It is tempting to conclude from this that a pegged exchange rate would allow no capital flows at all, because such a regime would have only a single nominal bond. But in fact this is not true. In a pegged exchange rate version of our simple model, there would be one world nominal price level (because we have continuous PPP). But if the covariance of the world price level and the home technology differs from the covariance with the foreign technology, then agents may find it advantageous to take a position in nominal international bonds, thus allowing some partial lending and borrowing. But the risk-sharing possibilities would still be inferior to those analyzed in this paper.

Appendix

Process of wealth distribution θ

To obtain the process of wealth distribution θ $(=\frac{W_f}{W_h+W_f})$, we define

$$m_h(\theta) = \lim_{\Delta t \to 0} \frac{E_t\left[\frac{\Delta W_h(t+\Delta t)}{W_h(t)}\right]}{\Delta t}, \quad m_f(\theta) = \lim_{\Delta t \to 0} \frac{E_t\left[\frac{\Delta W_f(t+\Delta t)}{W_f(t)}\right]}{\Delta t},$$

$$n_h(\theta) = \lim_{\Delta t \to 0} \frac{Var_t\left[\frac{\Delta W_h(t+\Delta t)}{W_h(t)}\right]}{\Delta t}, \quad n_f(\theta) = \lim_{\Delta t \to 0} \frac{Var_t\left[\frac{\Delta W_f(t+\Delta t)}{W_f(t)}\right]}{\Delta t},$$

and

$$n_{hf}(\theta) = \lim_{\Delta t \to 0} \frac{Cov_t\left[\frac{\Delta W_h(t+\Delta t)}{W_h(t)}, \frac{\Delta W_f(t+\Delta t)}{W_f(t)}\right]}{\Delta t}.$$

Then, using Ito's lemma, we can derive the process of wealth distribution θ $(=\frac{W_f}{W_h+W_f})$ as

$$d\theta = \theta(1-\theta)\left[F(\theta)dt + G(\theta)dB\right], \tag{14}$$

where

$$F(\theta) = m_f(\theta) - m_h(\theta) - \theta n_f(\theta) + (1-\theta)n_h(\theta) + (2\theta - 1)n_{hf}(\theta),$$
$$G(\theta) = \sqrt{n_h(\theta) + n_f(\theta) - 2n_{hf}(\theta)},$$

and

$$dB = \frac{1}{G(\theta)}\left[\left[\omega_T^f(\theta)\sigma dB_f - \omega_f^f(\theta)v dM_f - \omega_h^f(\theta)v dM_h\right] - \left[\omega_T^h(\theta)\sigma dB_h - \omega_h^h(\theta)v dM_h - \omega_f^h(\theta)v dM_f\right]\right].$$

$dB(t)$ is newly defined as the increment to a standard Brownian motion. Note that

$$\lim_{\Delta t \to 0} \frac{E_t\left[\Delta B(t+\Delta t)\right]}{\Delta t} = 0, \quad \lim_{\Delta t \to 0} \frac{Var_t\left[\Delta B(t+\Delta t)\right]}{\Delta t} = 1.$$

Stationarity of wealth distribution θ

To make theorems 16 and 18 of Skorohod (1989) applicable, we consider the process of κ or $\ln\frac{\theta}{1-\theta}$ $(= \ln\frac{W_f}{W_h})$ instead of θ. The process of κ is derived as

$$d\kappa = \delta(\theta)dt + G(\theta)dB, \tag{15}$$

where $\theta = \frac{\exp(\kappa)}{1+\exp(\kappa)}$, and $\delta(\theta) = g_f(\theta) - g_h(\theta)$.As defined in the main text, $\delta(\theta)$ represents a difference in risk-adjusted wealth growth between the two countries. Given equilibrium asset pricing characterized by equations (6) through (9), $\delta(\theta)$ is computed as

$$\delta(\theta) = \frac{\lambda^2\sigma^2(1-2\theta)(v^2+2\lambda\sigma v+2\sigma^2-\sigma^2\lambda^2)(v^2+2\lambda\sigma v+2\sigma^2)}{(4\sigma^2\theta^2\lambda^2-4\theta\lambda^2\sigma^2+\sigma^2\lambda^2-2\lambda\sigma v-2\sigma^2-v^2)^2}. \tag{16}$$

We then introduce the following integrals:

$$\begin{aligned} I_1 &= \int_{-\infty}^{0} \exp\left[-\int_0^w c(u(v))dv\right] dw, \\ I_2 &= \int_{0}^{\infty} \exp\left[-\int_0^w c(u(v))dv\right] dw, \end{aligned}$$

and

$$M = \int_0^\infty \left[\frac{2}{G(u(w))^2} \exp\left[\int_0^w c(u(v))dv\right]\right] dw,$$

where

$$c(u(v)) = \frac{2\delta(u(v))}{G(u(v))^2}, \tag{17}$$

and

$$u(v) = \frac{\exp(v)}{1+\exp(v)}.$$

According to the above theorems of Skorohod (1989), if $I_1 = \infty$, $I_2 = \infty$, and $M < \infty$, then κ has a unique ergodic distribution in $(-\infty, +\infty)$; accordingly, θ has a unique ergodic distribution in $(0, 1)$.

A function $c(\quad)$ characterized by equation (17) plays a key role in determining stationarity of κ. Saito (1997) demonstrates that if $c(0) > 0$ and $c(1) < 0$, then κ (θ) has a unique ergodic distribution under some regulatory conditions. The process of κ or equation (15) always satisfies $c(0) > 0$ and $c(1) < 0$, because from equation (16),

$$\begin{aligned} \delta(0) &= \frac{\lambda^2\sigma^2(v^2+2\lambda\sigma v+2\sigma^2)}{v^2+2\lambda\sigma v+2\sigma^2-\sigma^2\lambda^2} > 0, \\ \delta(1) &= -\frac{\lambda^2\sigma^2(v^2+2\lambda\sigma v+2\sigma^2)}{v^2+2\lambda\sigma v+2\sigma^2-\sigma^2\lambda^2} < 0, \end{aligned}$$

given finite $G(0)$ and $G(1)$. Note that $v^2+2\lambda\sigma v+2\sigma^2 > (v-\sigma)^2 > 0$ and $v^2+2\lambda\sigma v+2\sigma^2 - \sigma^2\lambda^2 > (v-\sigma)^2 + (1-\lambda^2)\sigma^2$ as long as $\lambda > -1$.

Density function of wealth distribution $\ln \frac{W_f}{W_h}$

According to Gihman and Skorohod (1972), given the process of κ $(= \ln \frac{W_f}{W_h})$ or equation (15), a density function of κ is derived as

$$\frac{2\mu}{G(u(\kappa))^2} \exp\left[\int_0^{\kappa} c(u(v))dv\right],$$

where μ is chosen such that $\mu \int_0^{\infty} \left[\frac{2}{G(u(w))^2} \exp\left[\int_0^{w} c(u(v))dv\right]\right] dw = 1$. Figure 1 depicts density functions of $\ln \frac{W_f}{W_h}$ for $\lambda = -0.9$, -0.8, -0.5, and -0.3 when $\sigma = v = 0.02$.

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Figure 1: Density Functions of Wealth Distribution $\ln\frac{W_f}{W_h}$ for Various Correlation Coefficients λ ($\sigma = v = 0.02$)

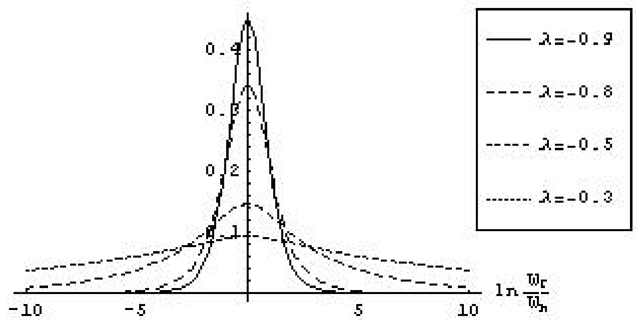

Figure 2: Portfolio Weights of Nominal Bonds for Home Country (ω_h^h and ω_f^h)

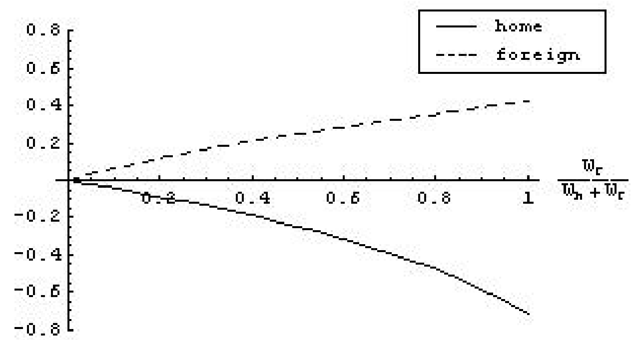

Figure 3: Real Risk-free Rates ($r_h(\theta)$ and $r_f(\theta)$)

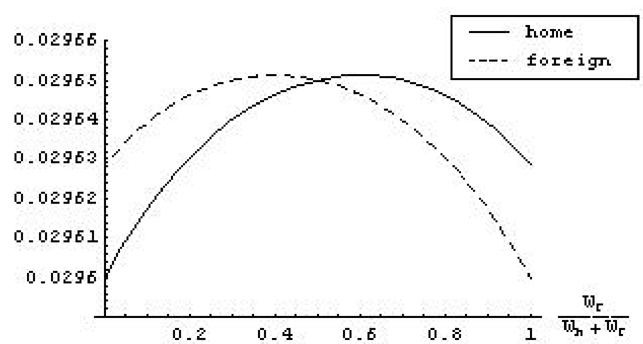

Figure 4: Nominal Interest Rates ($R_h(\theta)$ and $R_f(\theta)$)

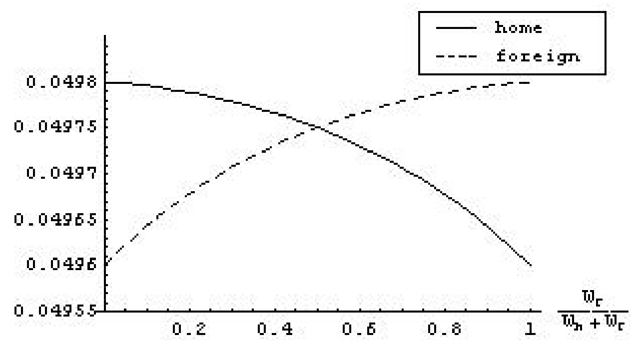

Figure 5: Risk-adjusted Growth Rates ($g_h(\theta)$ and $g_f(\theta)$)

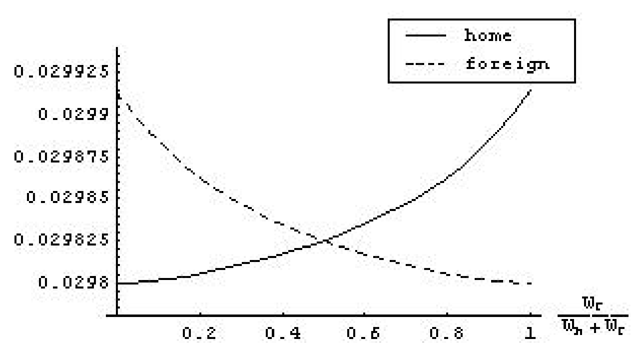

Figure 6: Drift Term of Relative Net Foreign Assets ($\sigma = v = 0.02$, $\lambda = -0.5$)

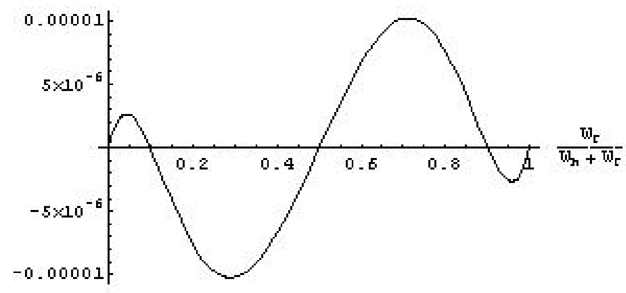

Figure 7: Diffusion Term of Relative Net Foreign Assets ($\sigma = v = 0.02$, $\lambda = -0.5$)