Pricing-to-Market with State-Dependent Pricing

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Abstract

In an attempt to capture the incomplete pass-through of exchange rate movements, the open economy macroeconomic literature with nominal rigidities has recently concentrated on market segmentation for tradable goods or so-called pricing-to-market models. This paper studies the implications that such pricing structure has for the dynamics of real and nominal economic activity within a simple open economy macroeconomic model which embodies elements of state-dependent pricing and strategic complementarity. In contrast to its time-dependent variants, a domestic monetary shock spills over to foreign consumption as movements in the distributions of price-setters influence foreign aggregate prices.

Keywords: international business cycle transmission, pricing-to-market, state-dependent pricing, complementarity in price-setting

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1 Introduction

Traditional Keynesian models of international macroeconomics assume full pass-through of exchange rate changes to consumer prices. This assumption is central to the expenditure-switching effect of exchange rate, the key feature giving monetary policy its efficacy under flexible exchange rate regimes or allowing exchange rate changes to counter country-specific shocks. Yet, there is a growing body of evidence suggesting that exchange rate changes do not fully pass-through to consumer prices. For example, Engel (1993) and Engel and Rogers (1996) found that consumer prices are not much affected by nominal exchange rate changes in the short-run.

In an attempt to capture the incomplete pass-through of exchange rate movements, the open economy macroeconomic literature has recently concentrated on market segmentation for tradable goods in which prices are sticky in the currency of the consumer or so-called Pricing-to-Market (PTM) models.[1] Early contributions of PTM models have been made by Betts and Devereux (1996, 2000), Chari, Kehoe, and McGrattan (2002), and Kollmann (2001). Unlike the traditional framework, such models imply a radical rethinking of the expenditure-switching role of exchange rates as domestic monetary shocks do not spill over to foreign consumption. Following a domestic monetary shock, foreign consumers do not perceive any change in the relative price of imports (since import prices are temporarily fixed in consumer currency units) and consequently do not altered their expenditure decisions. The foreign expenditure dynamics implied by those models has lead some to argue that this behavior can largely insulate an economy from foreign monetary shocks (see Devereux and Engel (2003)).

A standard but much-criticized element of this literature is an exogenously imposed timing of the opportunity that firms have for nominal price adjustments. This paper studies the implications of PTM for the dynamics of real and nominal economic activity within a simple state-dependent pricing open economy macroeconomic model. We augment our model with variable demand elasticities to induce complementarity in price-setting. Together, state-dependent pricing and complementarity in price-setting generate aggregate price inertia that mimic those observed in the data (see Dotsey and King (2005)). Relative to previous PTM work, there are new implications for the dynamics of real and nominal economic activity: in contrast to its time-dependent variant, a domestic monetary shock spills over to foreign consumption as movements in the distributions of price-setters influence foreign aggregate prices.

Why does the state-dependent pricing structure leads to a different outcome than its time-dependent variant following a domestic monetary expansion? Time-dependent pricing models imply that the distributions of firms over prices remain constant through time while market segmentation breaks the link between domestic

[1] Also named Local-Currency Pricing (LCP).

and foreign demand. These factors imply that domestic firms have no incentives to adjust prices in the foreign market since foreign aggregate prices and demand remain constant. However, state-dependent pricing models imply that the distributions of firms over prices vary with economic conditions. A domestic monetary expansion generates a currency depreciation which increases domestic firms markup on exported goods and induces firms to compete in the foreign market: since firms prices are roughly set as a markup over marginal cost, firms that reoptimize prices choose a lower prices in the export market. This implies cheaper imported goods for foreign consumer and also implies that foreign consumer can reallocate their money balance to the consumption of domestic goods. This increase in foreign aggregate demand raises the value of the price adjusting firms and ultimately leads to higher foreign aggregate prices. With foreign money supply held constant, this generates a foreign consumption crash.

The remainder of the paper is organized as follows. Section 2 describes our open economy macroeconomic model. Section 3 discusses the model's implications using a two-country framework calibrated to match some US facts. In this section, we analyze the endogenous evolution of price distributions in response to an expansionary monetary policy shock, describe the way these distributions influence international economic activity, and contrast the implications of our model with its corresponding time-dependent variant which is used as a reference case because of its popularity in the current literature. Finally, section 4 concludes.

2 Structure of the Model

The open economy macroeconomic literature builds small scale dynamic general equilibrium models for open economy macroeconomics and is the departure point for our work.[2] The world economy consists of two countries each having (i) a representative infinitely lived household, (ii) a continuum of firms indexed on the unit interval, and (iii) a monetary authority. In what follows, each variable is represented by a country-specific subscript (i.e.: 1 and 2 for Country 1 and Country 2 respectively). When three subscripts are attached to a single variable, the first and second denote the country of production and the country of consumption respectively, and the third subscript denotes time.

2.1 The Households

Households are identical across countries except for the local bias introduced in consumption. They demand consumption goods produced in both countries and supply

[2] Examples include Betts and Devereux (2000), Chari et al. (2002), Kollmann (2001), Bergin and Feenstra (2001), and Obstfeld and Rogoff (1995, 2000).

factors of production on a competitive basis. Households in both countries maximize the following time separable objective function defined over consumption goods c and labor supply n:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u\left(c_t\right) - v\left(n_t\right)\right) \tag{1}$$

where β is the subjective discount factor and $u\left(c_t, n_t\right)$ is the momentary utility function with characteristics $u_c > 0$, $u_{cc} < 0$, $v_n > 0$, and $v_{nn} > 0$. These characteristics imply that $u(c)$ is increasing and concave, and that $v(n)$ is increasing and convex. Concavity of $u(c)$ indicates diminishing marginal utility of consumption, while convexity of $v(n)$ suggests increasing marginal disutility from labor supply. More specifically, our momentary utility function, separable in consumption and leisure, has the following form, where σ governs the intertemporal elasticity of substitution and η governs the elasticity of labor supply:

$$u\left(c_t\right) - v\left(n_t\right) = \frac{1}{1-\sigma} c_t^{1-\sigma} - \frac{\chi}{1+\eta} n_t^{1+\eta}. \tag{2}$$

We assume that households prefer to consume locally produced goods. This feature generates movements in relative prices and reinforces the terms of trade as an important channel through which country-specific output movements affect welfare: following a decline in imported good prices, households do not fully substitute domestic for imported goods in their consumption basket. Instead, households consume a relatively fixed basket with a fraction $(1-\theta)$ of domestic goods, and the remaining θ of foreign goods. This specification is consistent with the data since the ratios of imports to GDP are relatively stable in the long-run. We let the $(1-\theta)$ determines the degree of home bias in the steady-state, and γ the elasticity of substitution between domestic and imported goods. The consumption indices for both countries are defined as:

$$\begin{aligned} c_{1,t} &= \left((1-\theta_1)^{\frac{1}{\gamma}} c_{1,1,t}^{\frac{1-\gamma}{\gamma}} + \theta_1^{\frac{1}{\gamma}} c_{2,1,t}^{\frac{1-\gamma}{\gamma}} \right)^{\frac{\gamma}{1-\gamma}} \\ c_{2,t} &= \left((1-\theta_2)^{\frac{1}{\gamma}} c_{2,2,t}^{\frac{1-\gamma}{\gamma}} + \theta_2^{\frac{1}{\gamma}} c_{1,2,t}^{\frac{1-\gamma}{\gamma}} \right)^{\frac{\gamma}{1-\gamma}} \end{aligned} \tag{3}$$

In this context, the following equations define the optimal allocations between domestic and imported consumption

$$\begin{aligned} c_{1,1,t} &= (1-\theta_1) \left(\frac{P_{1,1,t}^P}{P_{1,t}^C} \right)^{-\gamma} c_{1,t} \qquad & c_{2,1,t} &= \theta_1 \left(\frac{P_{2,1,t}^P}{P_{1,t}^C} \right)^{-\gamma} c_{1,t} \\ c_{2,2,t} &= (1-\theta_2) \left(\frac{P_{2,2,t}^P}{P_{2,t}^C} \right)^{-\gamma} c_{2,t} \qquad & c_{1,2,t} &= \theta_2 \left(\frac{P_{1,2,t}^P}{P_{2,t}^C} \right)^{-\gamma} c_{2,t} \end{aligned} \tag{4}$$

which depend on overall consumption, domestic and imported producer price indices (hereafter PPIs) denoted by P^P, and the overall consumer price indices (hereafter CPIs) denoted by P^C.

Our benchmark economy evolves under complete domestic and international financial markets. This implies that households can freely reallocate risk through a complete set of state-contingent nominal bonds b and corresponding stochastic discount factor D, such that $E_t[D_{t+1}b_{t+1}] = \sum_{s_{t+1}} \rho(s_{t+1}|s_t)D(s_{t+1}|s_t)b(s_{t+1})$ where $\rho(s_{t+1}|s_t)$ denotes the probability of state s_{t+1} given s_t. The households also receive nominal wages W from labor supply, and a series of dividend payments Z from firms. The sequence of intertemporal budget constraints can be represented in terms of aggregates as:

$$\begin{aligned} P_{1,t}^C c_{1,t} + E_t[D_{t+1}b_{1,t+1}] &\leq b_{1,t} + W_{1,t}n_{1,t} + Z_{1,t} \\ P_{2,t}^C c_{2,t} + E_t[D_{t+1}b_{2,t+1}] &\leq b_{2,t} + W_{2,t}n_{2,t} + Z_{2,t}. \end{aligned} \tag{5}$$

We assume that prices are set in the currency of the consumer or so-called Pricing-to-Market/Local Currency Pricing. In this environment, households choose consumption, labor supply, and portfolio holdings to maximize their lifetime utility (1) subject to a sequence of intertemporal budget constraints (5) and allocation of time. The maximization problem implies the following risk sharing condition with the real exchange rate defined as $q_t = S_t \cdot \left(P_{2,t}^C/P_{1,t}^C\right)$ and a constant reflecting initial wealth differences κ:

$$q_t = \kappa \cdot \frac{\lambda_{2,t}}{\lambda_{1,t}} \tag{6}$$

where, S represents the nominal exchange rate defined as the price of one unit of foreign currency in terms of the domestic currency. Equation (6) implies that the ratios of marginal utilities of consumption λ are equalized across countries such that the levels of consumption defined in (3) differ only to the extent that the real exchange rate deviates from its steady-state value. Finally, the level of nominal aggregate demand is governed by a money demand relationship of the form $M_t/P_t^C = c_t$ along with country-specific monetary policies.

2.2 Modelling Complementarity in Price-Setting

Complementarity in price-setting is introduced by allowing for variable demand elasticities following the work of Kimball (1995). This approach is consistent with microeconomic evidence suggesting that competitors' actions play a central role in the behavior of price adjustments:[3] the Kimball demand function makes it more costly for firms to get their prices out of line with the average price set by other firms than

[3] Examples include Bills and Klenow (2004), and Blinder (1991, 1994) for the United States.

the standard Dixit-Stiglitz demand as illustrated by the relative demand and corresponding profit functions plotted in Figure 1.[4] This approach is also consistent with the evidence that firms tend to change their price more in response to a cost increase than decrease (Peltzman (2000)) as seen by the curvature of the profit function. This concept has been introduced by Stiglitz (1979), Ball and Romer (1990), and Dotsey and King (2005), and more recently within the open economy macroeconomic literature by Bergin and Feenstra (2001), Bouakez (2005), and Gust, Leduc, and Vigfusson (2006). However, as opposed to the literature where the timing of price adjustment is fixed, state-dependent pricing and variable demand elaticities increases the interaction between firms as they opt to keep their price in line with the general level of product prices: variable demand elasticities makes it desirable for firms to keep their prices similar to those of others while state-dependent pricing makes it feasible for them to do so.

2.2.1 Demand Aggregators and Firm's Relative Demand

We follow the approach outlined by Kimball (1995) and consider the following general expenditure minimization problem for each market:

$$\min_{d(z)} \int_0^1 P(z)d\left(z\right)dz \qquad \text{subject to} \qquad \int_0^1 \Gamma\left(d\left(z\right)/d\right)dz = 1 \tag{7}$$

where d represents a country-specific aggregate demand for each market which is implicitly defined by a demand aggregator Γ such that an aggregate producer price index P^P holds for each country's domestic and exported markets. In this environment, each firm produces a differentiated product such that $P\left(z\right)$ identifies the price of the good charged by an individual firm z with corresponding relative demand $d\left(z\right)/d$. Moreover, our specific aggregator Γ is an increasing and concave function reflecting diminishing demand elasticity, and is defined over the parameters φ and ϱ which govern the curvature of the demand function[5]

$$\Gamma\left(d\left(z\right)/d\right) = \frac{1}{\left(1+\varphi\right)\varrho}\left[\left(1+\varphi\right)\left(d\left(z\right)/d\right)-\varphi\right]^{\varrho} - \left[1+\frac{1}{\left(1+\varphi\right)\varrho}\right]. \tag{8}$$

This demand aggregator implicitly defines firm's relative demand as the ratio of firm z in a country-specific aggregate demand d, and is a function of individual and aggregate

[4]The open economy macroeconomic literature typically assume that firms face a constant elasticity of demand. This assumption implies that the optimal price-setting rule is a constant markup over marginal cost. Therefore, cost considerations become central to a firm's price setting decision leaving little room for interactions between competitors.

[5]The parameter ϱ determines the elasticity of demand at the average level of product prices while φ determines the curvature of the demand function. A nice property of this specification is that the Dixit and Stiglitz aggregator is a special case represented by $\varphi = 0$. Derivation of the above equations are provided in Appendix B.

prices

$$\frac{d_t\left(z\right)}{d_t}=f\left(\frac{P(z)}{P^P},\varphi,\varrho\right).$$

2.2.2 Price Indices

The PPIs are given as a weighted sum of prices over individual firm ratios

$$P_{1,1,t}^{P}=\int_0^1 P_t\left(z\right)\left(d_{1,1,t}\left(z\right)/d_{1,1,t}\right)dz \qquad P_{1,2,t}^{P}=\int_0^1 P_t\left(z\right)\left(d_{1,2,t}\left(z\right)/d_{1,2,t}\right)dz \tag{9}$$

$$P_{2,2,t}^{P}=\int_0^1 P_t\left(z\right)\left(d_{2,2,t}\left(z\right)/d_{2,2,t}\right)dz \qquad P_{2,1,t}^{P}=\int_0^1 P_t\left(z\right)\left(d_{2,1,t}\left(z\right)/d_{2,1,t}\right)dz$$

and the CPIs follow a weighted sum of domestic and imported good prices

$$\begin{aligned} P_{1,t}^{C} &= \left((1-\theta_1)\left(P_{1,1,t}^{P}\right)^{1-\gamma}+\theta_1\left(P_{2,1,t}^{P}\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \\ P_{2,t}^{C} &= \left((1-\theta_2)\left(P_{2,2,t}^{P}\right)^{1-\gamma}+\theta_2\left(P_{1,2,t}^{P}\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \end{aligned} \tag{10}$$

2.3 The Firms

There exists a continuum of monopolistically competitive firms located on the unit interval and indexed by z in each country. At any date t, a firm is identified by its current prices (i.e.: $P_{1,1,t}(z)$ and $P_{1,2,t}(z)$ in Country 1 and $P_{2,2,t}(z)$ and $P_{2,1,t}(z)$ in Country 2), and its current menu cost of price adjustment $\xi_t(z)\in\left[0,\overline{B}\right]$. The menu cost is denominated in labor hours and drawn from a time-invariant distribution $G(\xi)$ that is common across all country-specific firms. Since the indices z are uncorrelated over time, and there are no other state variables attached to individual firms, all country-specific price-adjusting firms simultaneously choose the same optimal price $\widehat{P}$ for each market. We restrict ourselves to environments with positive steady-state inflation rates so that the benefit of price adjustment becomes infinitely large as the number of periods for which the price has been fixed grows. Given that the support of the distribution $G(\xi)$ is finite, there exist finite fractions of firms sharing a common price in each country denoted by J_1 and J_2 and defined as vintages.

2.3.1 Production and Demand

In this environment, labor used for price adjustment is denoted $n^a(z)$ and labor used for production is denoted $n^y(z)$. The total amount of labor is thus $n^a(z)+n^y(z)=n(z)$. Technology is linear in labor, and firms are subject to a common country-specific stochastic total factor productivity a such that production by an individual firm is represented by $y_t(z)=a_t n_t^y(z)$.

Using (4), aggregate demand d is determined by domestic and exported consumption

$$\begin{aligned} d_{1,t} &= d_{1,1,t} + d_{1,2,t} = (1-\theta_1)\left(\frac{P^P_{1,1,t}}{P^C_{1,t}}\right)^{-\gamma} c_{1,t} + \theta_2\left(\frac{P^P_{1,2,t}}{P^C_{2,t}}\right)^{-\gamma} c_{2,t} \\ d_{2,t} &= d_{2,2,t} + d_{2,1,t} = (1-\theta_2)\left(\frac{P^P_{2,2,t}}{P^C_{2,t}}\right)^{-\gamma} c_{2,t} + \theta_1\left(\frac{P^P_{2,1,t}}{P^C_{1,t}}\right)^{-\gamma} c_{1,t} \end{aligned} \tag{11}$$

such that production by an individual firm corresponds to a fraction of its country aggregate demand

$$\begin{aligned} y_{1,1,t}(z) &= f\left(\tfrac{P_{1,1,t}(z)}{P^P_{1,1,t}}, \varphi, \varrho\right) \cdot d_{1,1,t} \qquad y_{1,2,t}(z) = f\left(\tfrac{P_{1,2,t}(z)}{P^P_{1,2,t}}, \varphi, \varrho\right) \cdot d_{1,2,t} \\ y_{2,2,t}(z) &= f\left(\tfrac{P_{2,2,t}(z)}{P^P_{2,2,t}}, \varphi, \varrho\right) \cdot d_{2,2,t} \qquad y_{2,1,t}(z) = f\left(\tfrac{P_{2,1,t}(z)}{P^P_{2,1,t}}, \varphi, \varrho\right) \cdot d_{2,1,t} \end{aligned} \tag{12}$$

Equation (12) illustrates that production by an individual firm depends on its price relative to other domestic firms (PPI), and on country-specific aggregate demand (11) which is determined by the degree of home bias, the elasticity of substitution between domestic and imported goods, the PPI to CPI ratios, and aggregate domestic and foreign consumption.

2.3.2 Pricing Policy

In both state- and time-dependent pricing frameworks, the firm's optimal decision can be represented using a dynamic programming approach: given the level of technology, demand (12), the current menu cost of price adjustment $\xi(z)$, the current real prices $p^C(z)$, and the prevailing real wage rate w, individual firms decide whether or not to adjust their prices with respect to a state vector s. Accordingly, firms in each country have a real value function of the form:

$$\begin{aligned} & v\left(p^C_{1,1,t}, p^C_{1,2,t}, \xi_{1,t} | s_t\right) \\ = \; & \max \left\{ \begin{array}{c} v_{1,j,t} = \pi\left(p^C_{1,1,t}, p^C_{1,2,t} | s_t\right) \\ +\beta E_t \Lambda_{1,t,t+1} v\left(p^C_{1,1,j+1,t+1}, p^C_{1,2,j+1,t+1}, \xi_{1,t+1} | s_{t+1}\right), \\ v_{1,0,t} = \max_{\widehat{p}^C_{1,1,t}, \widehat{p}^C_{1,2,t}} \pi\left(\widehat{p}^C_{1,1,j,t}, \widehat{p}^C_{1,2,j,t} | s_t\right) \\ +\beta E_t \Lambda_{1,t,t+1} v\left(p^C_{1,1,j+1,t+1}, p^C_{1,2,j+1,t+1}, \xi_{1,t+1} | s_{t+1}\right) - w_{1,t}\xi_{1,t} \end{array} \right\} \\ & v\left(p^C_{2,2,j,t}, p^C_{2,1,j,t}, \xi_{2,t} | s_t\right) \\ = \; & \max \left\{ \begin{array}{c} v_{2,j,t} = \pi\left(p^C_{2,2,j,t}, p^C_{2,1,j,t} | s_t\right) \\ +\beta E_t \Lambda_{1,t,t+1} v\left(p^C_{2,2,j+1,t+1}, p^C_{2,1,j+1,t+1}, \xi_{2,t+1} | s_{t+1}\right), \\ v_{2,0,t} = \max_{\widehat{p}^C_{2,2,t}, \widehat{p}^C_{2,1,t}} \pi\left(\widehat{p}^C_{2,2,j,t}, \widehat{p}^C_{2,1,j,t} | s_t\right) \\ +\beta E_t \Lambda_{2,t,t+1} v\left(p^C_{2,2,j+1,t+1}, p^C_{2,1,j+1,t+1}, \xi_{2,t+1} | s_{t+1}\right) - w_{2,t}\xi_{2,t} \end{array} \right\} \end{aligned} \tag{13}$$

with the value if the individual firm does (v_{0t}) or does not (v_{jt}) adjust, and real profits π defined as

$$\begin{aligned}
\pi\left(p^C_{1,1,j,t}, p^C_{1,2,j,t}|s_t\right) &= \pi\left(p^C_{1,1,j,t}|s_t\right) + \pi\left(p^C_{1,2,j,t}|s_t\right) \qquad (14)\\
&= \left(\frac{P_{1,1,t}(z)}{P^C_{1,t}} - \psi_{1,t}\right)\cdot y_{1,1,t}(z) + \left(\frac{S_t P_{1,2,t}(z)}{P^C_{1,t}} - \psi_{1,t}\right)\cdot y_{1,2,t}(z)\\
\pi\left(p^C_{2,2,j,t}, p^C_{2,1,j,t}|s_t\right) &= \pi\left(p^C_{2,2,j,t}|s_t\right) + \pi\left(p^C_{2,1,j,t}|s_t\right)\\
&= \left(\frac{P_{2,2,t}(z)}{P^C_{2,t}} - \psi_{2,t}\right)\cdot y_{2,2,t}(z) + \left(\frac{P_{2,1,t}(z)}{S_t P^C_{2,t}} - \psi_{2,t}\right)\cdot y_{2,1,t}(z)
\end{aligned}$$

In these functions, $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$ denotes the ratio of future to current marginal utility and is the appropriate discount factor for future real profits, ψ_t represents real marginal cost which is equal to $\psi_t = w_t/a_t$, p^C represents the domestic and foreign firms' relative prices, $\widehat{p}^C$ represents the domestic and foreign optimal price chosen by adjusting firms, and y represents domestic and foreign demand. Both, the optimal and current real price are relative to domestic CPI which are the appropriate prices in the firm's decision making.

Equation (13) shows that the firm must weight the current and future benefits of adjusting its prices versus the status-quo. Firms that decide to adjust set prices optimally and choose cost-minimizing levels of input. Firms that decide not to adjust prices take their output as given and simply choose input to minimize cost. In this environment, the country-specific endogenous adjustment fractions $\alpha_{j,t}$ are determined by the menu cost of the marginal firms being just equal to the value gained such that

$$\xi\left(\alpha_{j,t}\right)\cdot w\left(s_t\right) = v_{0,t}(s_t) - v_{j,t}(s_t). \qquad (15)$$

Notice that the firm's pricing decision considers simultaneous adjustment of domestic and exported prices: firms face a single cost of price adjustment for both the domestic and exported markets. This cost of price adjustment specification implies that individual firm consider the evolution of both markets in choosing the timing and magnitude of domestic and export price adjustment. This specification may represent managerial costs or common costs across markets for the price adjusting firms.[6]

Finally, the dynamic program (13) implies that the optimal price satisfies an Euler equation that involves balancing pricing effects on current and expected future profits.

[6] Their is few research on costs behavior associated with firm's international price adjustment decisions. It might be the case that these costs are distinct across markets. A natural extension could study costs behavior in more details and allows firms to have distinct domestic and exported pricing decisions.

That is, as part of an optimal plan, price-adjusting firms choose prices that satisfy balancing current and expected future profits

$$\begin{aligned}
0 &= \frac{\partial \pi\left(\widehat{p}^C_{1,1,t}|s_t\right)}{\partial \widehat{p}^C_{1,1,t}} + \beta E_t\left[\Lambda_{1,t,t+1}\cdot\frac{\partial v\left(\widehat{p}^C_{1,1,t},\xi_{1,t+1}|s_{t+1}\right)}{\partial \widehat{p}^C_{1,1,t}}\right] \\
0 &= \frac{\partial \pi\left(\widehat{p}^C_{1,2,t}|s_t\right)}{\partial \widehat{p}^C_{1,2,t}} + \beta E_t\left[\Lambda_{1,t,t+1}\cdot\frac{\partial v\left(\widehat{p}^C_{1,2,t},\xi_{1,t+1}|s_{t+1}\right)}{\partial \widehat{p}^C_{1,2,t}}\right]
\end{aligned} \tag{16}$$

$$\begin{aligned}
0 &= \frac{\partial \pi\left(\widehat{p}^C_{2,2,t}|s_t\right)}{\partial \widehat{p}^C_{2,2,t}} + \beta E_t\left[\Lambda_{2,t,t+1}\cdot\frac{\partial v\left(\widehat{p}^C_{2,2,t},\xi_{2,t+1}|s_{t+1}\right)}{\partial \widehat{p}^C_{2,2,t}}\right] \\
0 &= \frac{\partial \pi\left(\widehat{p}^C_{2,1,t}|s_t\right)}{\partial \widehat{p}^C_{2,1,t}} + \beta E_t\left[\Lambda_{2,t,t+1}\cdot\frac{\partial v\left(\widehat{p}^C_{2,1,t},\xi_{2,t+1}|s_{t+1}\right)}{\partial \widehat{p}^C_{2,1,t}}\right].
\end{aligned}$$

Iterating the Euler equation (16) forward, the country-specific firm nominal optimal prices $\widehat{P}$ can be expressed as an explicit function of current and expected future variables:

$$\begin{aligned}
\widehat{P}_{1,1,t} &= \frac{\sum_{j=0}^{J1-1}\beta^j E_t\left[\Omega_{1,j,t,t+j}\cdot\Lambda_{1,t,t+j}\cdot\epsilon_{1,j,t+j}\cdot\psi_{1,t+j}\cdot P_{1,1,t+j}\cdot d_{1,j,t+j}\right]}{\sum_{j=0}^{J1-1}\beta^j E_t\left[\Omega_{1,j,t,t+j}\cdot\Lambda_{1,t,t+j}\cdot(\epsilon_{1,j,t+j}-1)\cdot\left(P^P_{1,1,t+j}/P^C_{1,t+j}\right)\cdot d_{1,1,j,t+j}\right]} \\
\widehat{P}_{1,2,t} &= \frac{\sum_{j=0}^{J1-1}\beta^j E_t\left[\Omega_{1,j,t,t+j}\cdot\Lambda_{1,t,t+j}\cdot\epsilon_{1,j,t+j}\cdot\psi_{1,t+j}\cdot P_{1,2,t+j}\cdot d_{1,j,t+j}\right]}{\sum_{j=0}^{J1-1}\beta^j E_t\left[\Omega_{1,j,t,t+j}\cdot\Lambda_{1,t,t+j}\cdot(\epsilon_{1,j,t+j}-1)\cdot\left(S_{t+j}P^P_{1,2,t+j}/P^C_{1,t+j}\right)\cdot d_{1,2,j,t+j}\right]}
\end{aligned}$$

$$\begin{aligned}
\widehat{P}_{2,2,t} &= \frac{\sum_{j=0}^{J2-1}\beta^j E_t\left[\Omega_{2,j,t,t+j}\cdot\Lambda_{2,t,t+j}\cdot\epsilon_{2,j,t+j}\cdot\psi_{2,t+j}\cdot P_{2,2,t+j}\cdot d_{2,2,j,t+j}\right]}{\sum_{j=0}^{J2-1}\beta^j E_t\left[\Omega_{2,j,t,t+j}\cdot\Lambda_{2,t,t+j}\cdot(\epsilon_{2,j,t+j}-1)\cdot\left(P^P_{2,2,t+j}/P^C_{2,t+j}\right)\cdot d_{2,2,j,t+j}\right]} \\
\widehat{P}_{2,1,t} &= \frac{\sum_{j=0}^{J2-1}\beta^j E_t\left[\Omega_{2,j,t,t+j}\cdot\Lambda_{2,t,t+j}\cdot\epsilon_{2,j,t+j}\cdot\psi_{2,t+j}\cdot P_{2,1,t+j}\cdot d_{2,1,j,t+j}\right]}{\sum_{j=0}^{J2-1}\beta^j E_t\left[\Omega_{2,j,t,t+j}\cdot\Lambda_{2,t,t+j}\cdot(\epsilon_{2,j,t+j}-1)\cdot\left(P^P_{2,1,t+j}/S_{t+j}P^C_{2,t+j}\right)\cdot d_{2,1,j,t+j}\right]}
\end{aligned} \tag{17}$$

where $\Omega_{j,t,t+j}$ represents the probability of non-adjustment from t to $t+j$, and $\epsilon_{j,t+j}$ denotes the elasticity of demand facing the individual firm. Accordingly, the optimal price is a fixed markup over real marginal cost if the demand elasticities, the price levels, and real marginal cost are expected to be constant over time.

The optimal pricing rules (17) are generalizations of the types derived in open economy macroeconomic models with exogenous probabilities. They also represent an open economy version of the closed economy state-dependent pricing rules of Dotsey, King, and Wolman (1999), and Dotsey and King (2005). However, in contrast to their closed economy counterparts, foreign economic conditions and the nominal exchange rate enter the decision of the value maximizing firms and henceforth influence the endogenous adjustment probabilities. The pricing rules illustrate that optimal prices vary with adjustment probabilities, discount factors, demand elasticities, real marginal costs, domestic PPIs and CPIs, and current and expected future demand (which includes global consumption, domestic and foreign CPIs, domestic PPIs, and the nominal exchange rate).

2.4 Monetary Policies

The monetary policy rules are specified as exogenous money supply rules. More specifically, the nominal money supply growth follows an autoregressive process in both countries

$$\begin{aligned} \Delta M_{1,t} &= \rho_1 \Delta M_{1,t-1} + \vartheta_1 \Delta M_{2,t} + \nu_{1,t} \\ \Delta M_{2,t} &= \rho_2 \Delta M_{2,t-1} + \vartheta_2 \Delta M_{1,t} + \nu_{2,t} \end{aligned} \tag{18}$$

where ρ describes the coefficients of autocorrelation, ϑ admits for the possibility of monetary policy comovements, and ν_t are independently and identically distributed zero-mean disturbances.

2.5 General Equilibrium

In this environment, the aggregate state of the economy at time t is a vector $s_t = (M_{1,t}, M_{2,t}, \Theta_{1,t}, \Theta_{2,t})$ where M represents the exogenous state variables, and Θ represents the evolution of producer prices within each country (country's specific vector of prices and corresponding density distribution of firms across prices). Given the aggregate state, a general equilibrium for the economy is a collection of sequences satisfying a set of equilibrium conditions: a collection of allocations for consumers $c_{1,t}, n_{1,t}, b_{1,t+1}$ and $c_{2,t}, n_{2,t}, b_{2,t+1}$, a collection of allocations and price for firms $y_{1,t}(z), n_{1,t}(z), P_{1,1,t}(z), P_{1,2,t}(z)$ and $y_{2,t}(z), n_{2,t}(z), P_{2,2,t}(z), P_{2,1,t}(z)$, and a collection of prices $P^P_{1,1,t}, P^P_{1,2,t}, P^C_{1,t}, W_{1,t}, D_{1,t+1}$ and $P^P_{2,2,t}, P^P_{2,1,t}, P^C_{2,t}, W_{2,t}, D_{2,t+1}$ such that (i) consumers maximize their utilities, (ii) firms maximize their values, and (iii) aggregate consistency conditions hold. These aggregate consistency conditions include market clearing conditions in the goods and labor markets, and consistency for the time-varying distributions of firms in each country.

2.6 Solution and Benchmark Parameterization

2.6.1 Solution

We use numerical methods to solve the model and study its behavior. First, we compute the steady state equilibrium by imposing trade account balance to the long-run behavior of the model. The steady-state equilibrium for this economy involves the lowest values of vintages that generates unconditional adjustment by all firms in each country. Second, we take a linear approximation of the behavioral equations around the steady state equilibrium and compute the resulting linear rational expectations equilibrium using an algorithm developed by King and Watson (1998). The introduction of a Pricing-to-Market environment add an extra distortion associated with the prevailing rate of inflation experiences in both countries. Unlike a model of in which producers set prices in domestic currency units, a model like ours implies a long-run interdependence between countries: trading partner policies or economic structures has an effect on domestic real and nominal variables. For example, trading with countries experiencing higher levels of inflation decreases consumption as more resources are devoted to price adjustments and not to production of goods.

2.6.2 Benchmark Parameterization

We study the model's implication using a calibrated model that matches the relative size and degree of home bias shared by the US with an aggregate of developed economy composed of Japan, Germany, France, and the UK called G4 economy. We use parameter values generally accepted in the macroeconomic and open economy literatures. A time period of the model corresponds to a quarter of a year. The subjective discount factors β imply annual real rate of returns of 4.1 percent. We choose preference parameter values that produce a low elasticity of marginal cost with respect to real output by setting the parameters governing the degrees of risk aversion σ to 0.25 and the parameters governing the elasticities of labor supply η to 0.05. Those parameters generate elasticities of marginal cost of approximately 0.3.[7] Agents work 20 percent of their time endowment. Country 1 is characterized by a degree of home bias of 3.75 percent and represent half of the world's GDP. The former correspond to the average share of US exports to GDP over the sample while the later correspond to the ratio of US to G4 GDPs. We set the elasticities of substitution between domestic and imported consumption goods γ to unity. Bergin (2004) offers empirical evidence from macro-level data which supports this common practice in the literature. The two countries share similar levels of productivity a. Finally, we set steady-state money growth rates μ to 0.01 which correspond to growth rates of 4 percent on an annual basis, and the autocorrelations of the money growth processes ρ to 0.5.

[7] Given that the households efficiency condition is $w_t = c_t^{\sigma} n^{\eta}$, and that consumption and labor are approximately equal to output, the elasticities of marginal cost are approximately equal to $\sigma + \eta$.

2.6.3 Demand Structure and Price Distributions

The variable elasticity demand curves are parametrized by choosing values of ϱ so that demand curves have elasticities of 10 at $d(z)/d = 1$. Restricting φ to take values of 1.02 implies that a 1 percent increase in price decreases demand by 13 percent, which is somewhere between the response assumed Kimball (1995), and Bergin and Feenstra (2001).

The remaining parameters involve the distributions of adjustment costs which, alongside the demand functions, determine the timing and distributions of prices. Table 2 displays the steady-state fractions of price-adjusting firms as well as the population densities associated with the parametrized model for both countries.[8] The chosen adjustment costs structure leads to a steady-state hazard function that is roughly quadratic in the log relative price deviation as suggested by Caballero and Engle (1993). It implies an average age of prices of less then 2.72 quarters, and an expected price duration of 4 quarters under the steady-state inflation rate of 4 percent. Together, the demand and adjustment costs specifications provide a good approximation of the main features governing the pattern of price adjustments and pricing policies observed in empirical studies on pricing behavior in developed economies.

3 Understanding the Model and its Implications

We analyze the model's responses to a monetary policy shock and contrast these responses with those from a time-dependent variant more closely related to standard open economy macroeconomic works. We subject Country 1 to a monetary policy shock in which the money stock increases by 1 percent on impact with a long-run response approaching 2 percent above its initial level. Chari, Kehoe, and McGrattan (2002) and Christiano, Eichembaum, and Evans (2005) support this pattern of money growth as a good approximation to a US interest rate rule.

3.1 Open Economy Business Cycle Statistics

First, we compared the filtered moments generated by the model to the filtered moments generated by the data. The filter used is the approximate business cycle band-pass filter suggested by Baxter and King (1999). When comparing the moments, we implicitly assumed that monetary disturbances are the only exogenous shocks in the model. Table 3 displays the correlations that occur at business-cycle frequencies for the US and a aggregate of developed economy composed of Japan, Germany, France,

[8] Choice of the adjustment costs parameters are detailed in Appendix B.

and the UK called G4 economy and our calibrated model that matches the relative size and degree of home bias shared by those economies. The US data are from the BEA, and the Japanese, German, French, and British data are from OECD Economic Outlook. The quarterly data cover the period 1974Q1 to 2005Q4 and when necessary are expressed in per capita 2000 US dollars at purchasing-power-parity. Prior research summarized by Baxter (1995) shown that open economy fluctuations are similar among developed economies. The terms of trade statistics are calculated as the ratio of imports to exports prices.[9]

In the data, output, consumption, exports, imports, and inflation tend to be strongly procyclical. The trade balance is countercyclical with respect to output and consumption. As for international prices, the nominal exchange rate is acyclical relative to output but procyclical relative to the trade balance, the terms of trade, and inflation while the terms of trade is countercyclical relative to output but procyclical relative to the trade balance. In terms of cross-country correlation, we observed a higher cross-country output correlation relative to consumption correlation. Finally, CPI inflation across countries are highly correlated.

Altogether, our simple state-dependent pricing model does better than its time-dependent variant in replicating the moments of interest: the within- and cross-country correlations are fairly closed to the data in all but the ones related to the nominal exchange rate. The state-dependent model generates the procyclicality of imports and exports and the countercyclicality of the trade balance.[10] The state-dependent model also replicates the sign and the magnitude of the correlation between the trade balance and the terms of trade, as well as the sign and magnitude of the cross-correlation of output and consumption.

3.2 Open Economy Business Cycle Fluctuations

Figures 2 to 6 illustrate the impulse response of microeconomic and macroeconomic aggregates over horizons of 16 quarters. The solid lines represent the state-dependent version of the model, while the dashed lines represent its time-dependent counterpart. The time-dependent counterpart is calibrated so that the fractions of price-adjusting firms are held fixed at steady-state values. To get a better understanding of the mechanism through which money affects international economic activity, we start by exploring the reaction of individual firms to the monetary policy shock and then turn to the aggregate implications.

[9] The data sources and aggregation are described in Appendix A.

[10] Note that the state-depedent model replicates the relatively high correlation between output and imports versus output and exports.

3.2.1 Firms' Reactions to a Monetary Shock

The top row of Figure 2 displays firms' reactions while the bottom rows displays the domestic and exported optimal prices chosen by the price-adjusting firms. Figure 3 displays the associated producer and consumer price indices. A novel feature of the state-dependent pricing open economy model is the evolving distribution of price-adjusting firms across countries. In Country 1, rising product demand and rising aggregate prices generated by the domestic monetary expansion enforce the extent of adjustment of individual firms, and consequently result into increasing deviations in fractions of price-adjusting firms. Notice that the introduction of variable demand elasticities generates smooth movements in adjusting fractions as firms bunch their actions: (i) initially, the monetary shock translates very little to individual and aggregate prices because firms are not willing to price differently from one another, (ii) then rising domestic aggregate prices enforce the extent of adjustment of individual firms and consequently result into increasing deviations in fractions of price-adjusting firms, and finally (iii) the fractions of price adjusting firms returns to its long-run value as decreased demand offset higher aggregate prices.

Associated with movements in adjusting fraction are the optimal domestic and exported prices charged by price-adjusting firms. The behavior of the optimal price movements under the time-dependent pricing environment is in sharp contrast with the behavior of optimal prices under the state-dependent pricing environment. Under the time-dependent pricing environment, the domestic optimal price jumps before converging to its new long-run value while the exported optimal price remains at steady-state value. The former is the front-loading aspect common to time-dependent pricing models. The later is an outcome of time-dependent pricing and market segmentation: time-dependent pricing models imply that the distributions of firms over prices remain constant through time while market segmentation breaks the link between domestic and foreign demand. Under the state-dependent pricing environment, the domestic optimal price reacts very little on impact. On the one hand, the forward looking price-setting firm would prefer to raise its price in light of the monetary policy shock. On the other hand, the firm knows that it has the possibility to reset its prices at any time in the future, and would rather do so than lose market share by pricing high relative to its competitors. In contrast, the exported optimal price oscillates arround the steady-state. On impact, the nominal exchange rate depreciation generated by the domestic monetary expansion increases domestic firms markup on exported goods and induce firms to compete in the export market: since firms prices are roughly set as a markup over marginal cost, firms that reoptimize prices choose a lower prices in the export market. Thereafter, movements in the nominal exchange rate coupled with demand and cost movements generate markup fluctuations on exported goods and ultimately leads to fluctuation on exported optimal prices.

In Country 2, rising exported product demand and rising domestic competition induces firms to reset their domestic and exported optimal prices. This results into a positive deviation in the fraction of price-adjusting firms and an incease in domestic and exported optimal prices. Once again, the behavior of the optimal price movements under the time-dependent pricing environment is very different from the behavior of optimal prices under the state-dependent pricing environment. Under the time-dependent pricing environment, the domestic optimal price remains at steady-state value while the exported optimal price increases to follow Country's 1 CPI and to offset nominal exchange rate movements. Under the state-dependent pricing environment, the domestic optimal price decreases slightly to compete against lower imported good prices before surging in positive territory as demand for domestic good increases. At the same time, the exported optimal price increases to follow the Country's 1 CPI and to offset nominal exchange rate movements.

3.2.2 Aggregate Implications of a Monetary Shock

Price Indices We now turn to the aggregate implications of our model. As displayed in Figure 3, the two environments have very different implications for the dynamics of prices faced by consumers. Under the time-dependent model, consumers in Country 1 see raising movements in prices while consumers in Country 2 see no movements in prices as the optimal prices as well as the distributions of firms over prices remain constant. In contrast, the distributions of prices over firms evolve under the state-dependent pricing model and consequently altered the prices faced by consumers in both countries.

Output, Consumption, and Inflation Dynamics Output and consumption responses are displayed in the top rows of Figure 4. Under the time-dependent model, Country 1's output and consumption respond positively after the monetary shock. In Country 2, output respond positively has the economy supplies more goods to Country 1 while consumption doesn't respond has prices faced by consumer remain constant. As in time-dependent pricing model, Country 1's output and consumption respond positively after the monetary shock under the state-dependent pricing model. However, while Country 2's output respond positively, its consumption contracts as the consumer price index moves above its steady-state value. Although the cross-correlation of output and consumption are in line with data, the models fail to generate a consumption boom abroad following a monetary shock as suggested by the empirical work of Faust and Rogers (2003), Kim (2001), and Sims (1992).

The bottom row of Figure 4 displays CPI inflation dynamics. Under the time-dependent model, CPI inflation responds positively in Country 1 while its response is nil in Country 2. In contrast, an important strength of the state-dependent model is its ability to generate delayed responses in CPI inflations in both countries: CPI inflation peaks 5 quarters after the monetary shock in Country 1 and 8 quarters after the monetary shock in Country 2.

Nominal Exchange Rate, Terms of Trade, and Trade Dynamics Figure 5 displays nominal and real exchange rates, as well as the trade balance and the terms of trade for Country 1. The monetary shock induces a significant and persistent depreciation in the nominal exchange rate, and displays the delayed overshooting effect stressed by Eichenbaum and Evans (1995) empirical study on the effects of US monetary policy shocks on exchange rates. Most of the short-term responses of the nominal exchange rate are in the real exchange rate: these are relative price changes that affect demand composition. At longer horizons, the nominal exchange rate is mainly affected by the domestic price level in a close to neutral manner.[11]

A central idea in the Keynesian approach to international macroeconomics is the expenditure-switching effect of nominal exchange rate changes: a country with a depreciating currency will experience a fall in the relative price of its exports and a resulting redirection of world expenditure in favor of its products. The results are clearly at odd with this proposition. Under the time-dependent model, their is no movements in domestic exports since foreign agents do not perceived any price changes. Under the state-dependent pricing model, exports quickly moves in negative territories as domestic firms raise their foreign prices. On the imports side, the increase in income mixed with an overvalued exchange rate raises the demand for imports in the short-run. The behavior of exports and imports translate into a trade balance deficit under both time- and state-dependent models.

Alternatively, one can look at the terms of trade defined as the ratio of home-currency price paid for goods imported from abroad over the home-currency price received on home goods exported abroad. Under traditional Keynesian models, the terms of trade raises following a domestic currency depreciation as the home-currency price paid for good imported increases. We say that the terms of trade are favorable since domestic goods are now cheaper on the world market. Obstfeld and Rogoff (2000) offers empirical evidence of a positive comovements between the nominal exchange rate and the terms of trade. However, in the current PTM environment, the opposite arises: domestic and imported goods prices move in tandem while the domestic currency depreciates. Hence, a currency depreciation causes a worsening of the terms of trade. Therefore, although the model generates a positive correlation between the trade balance and the terms of trade, terms of trade movements contradict Obstfeld and Rogoff conjecture. Moreover, in contrast to its time-dependent

[11] Figure 5 aknowledges the model's failure to generate real and nominal exchange rate movements that are highly correlated as in the data. However, most of the correlation between real and nominal exchange rates often found in the open economy macroeconomic literature results from simultaneous front-loading aspects of prices and consumption common to time-dependent pricing models.

variant, the state-dependent model displays (i) greater variations in the terms of trade because of the greater variability in consumer prices, and (ii) an improvement in the terms of trade after 3 years as the price paid for imported goods is above the price received for exported goods.

From the perspective of Country 2, real economic activity and trade dynamics are best understood by undertaking decompositions of output and consumption into their domestic and foreign components as illustrated in Figure 6. In the time-dependent model, the expansion of output is solely generated by the increase in exports as domestic consumption remains at steady-state value. In the state-dependent model, output is influenced by a domestic and exported components: the exported component lunches the output boom while the domestic component dampens it after a few quarters as domestic consumption falls. Similarly, consumption is influenced by a domestic and imported components. An interesting implication is that the bulk of the deviation in consumption comes from falling domestic demand as the monetary shock influences the foreign price distribution. This implies that the foreign economy is not isolated from Country 1's actions and consequently suggests a role for the foreign monetary authority in a PTM environment.

4 Conclusion

This paper develops an open economy state-dependent pricing that embodies a PTM environment. Relative to its time-dependent variants, our state-dependent pricing model implies that domestic monetary shocks spill over to foreign consumption as movements in the distributions of price-setters influence foreign aggregate prices. This result contradicts the current wisdom and has important implications for the design of international monetary policy in a PTM environment.

Endogenizing the timing of firm's response is clearly a feature that bring the current open economy macroeconomic models closer to reality. An interesting avenue of research will look at the foreign monetary authority responses to change the distribution of prices and expected path of expenditure and inflation. For example, endogenizing monetary policy could potentially eradicate the puzzling nature of foreign expenditure by easing foreign monetary policy following a domestic monetary shock. This is clearly something to investigate in future research.

4.1 Appendix A: Data

Source: BEA

- Quarterly real gross domestic product
- Quarterly real personal consumption expenditure
- Quarterly personal consumption expenditure price deflator
- Quarterly exports price index
- Quarterly imports price index
- Quarterly working-age population
- Monthly effective Federal Funds rate
- Monthly M1 money stock
- Monthly Trade Weighted exchange rate index - Major Currencies
- Monthly exports/imports to/from Japan
- Monthly exports/imports to/from Germany
- Monthly exports/imports to/from United Kingdom
- Monthly exports/imports to/from France

Source: OECD Economic Outlook

- Quarterly nominal gross domestic product for Japan, Germany, France, and the UK.
- Quarterly nominal private final consumption expenditure for Japan, Germany, France, and the UK.
- Quarterly gross domestic product deflator for Japan, Germany, France, and the UK.
- Quarterly private final consumption expenditure deflator for Japan, Germany, France, and the UK.
- Quarterly Working-age population for Japan, Germany, France, and the UK.
- Annual Purchasing Power Parity in US dollars for Japan, Germany, France, and the UK.

G4 aggregates are built using working-age population weights and translated in 2000 US dollars using the year 2000 PPP values.

4.2 Appendix B: Demand Aggregators

We consider the following general expenditure minimization problem for each country:

$$\min_{d(z)} \int_0^1 P(z) d(z)\, dz \qquad \text{subject to} \qquad \int_0^1 \Gamma\left(d(z)/d\right) dz = 1 \tag{B.1}$$

The country-specific aggregate demands d for goods are implicitly defined by a demand aggregator Γ such that an aggregate producer price index P^P holds for each country. The first order condition of the expenditure minimization problem yields:

$$P(z) = Z \cdot \Gamma' \left(\frac{d(z)}{d}\right) \tag{B.2}$$

where Z is the Lagrange multiplier on the constraint. Consequently, the first order condition can be solved to yield demand curves of the form:

$$\left(\frac{d(z)}{d}\right) = \Gamma'^{-1} \left(\frac{P(z)}{Z}\right) \tag{B.3}$$

Given the demand curves and the multipliers, the aggregate producer price indices are determined by

$$\int_0^1 \left(\frac{P(z)}{P^P}\right) \left(\frac{d(z)}{d}\right) dz = 1 \tag{B.4}$$

4.3 Appendix C: Adjustment Costs Structure

We adopt the costs structure used in Dotsey and King (2005). The adjustment costs are stochastic and idiosyncratic across firms, and are governed by country-specific cumulative distribution functions (CDF) $G(x)$ on the interval $0 \leq x \leq \overline{B}$ and corresponding density functions $g(x)$. Under the adjustment rules, a country-specific firm's probability of adjustment is:

$$\alpha(\xi) = G(\xi) = \int_0^{\xi} g(x)\, dx \tag{C.1}$$

or more intuitively

$$\alpha = G(\xi) = G\left(\frac{v_0 - v}{w}\right) \tag{C.2}$$

Hence, the fraction of price-adjusting firms in each vintage is determined by a marginal firm being indifferent to price adjustment.

The functional form used to derived the adjustment costs functions is the arctangent. This functional form is a monotonically increasing function that maps the real line into the interval $(-\pi, \pi)$ in different shapes. In this paper, we use the an interval of the arctangent $[\underline{x}, \overline{x}]$ and assume that

$$x(\alpha) = \alpha \cdot (\overline{x} - \underline{x}) + \underline{x} \tag{C.3}$$

where α is restricted to the range $0 \leq \alpha \leq 1$. Finally, we assume that the inverse of the CDF takes the form

$$\xi(\alpha) = K_1 s(\underline{x}) + K_2 \tag{C.4}$$

The inverse CDF takes on a zero value at $\alpha = 0$ and a value of $\overline{B}$ at $\alpha = 1$:

$$\begin{aligned} 0 &= K_1 s(\underline{x}) + K_2 \\ \overline{B} &= K_1 s(\underline{x}) + K_2 \end{aligned} \tag{C.5}$$

so that the values of the parameters are given by

$$\begin{aligned} K_1 &= \frac{\overline{B}}{s(\overline{x}) - s(\underline{x})} \\ K_2 &= \frac{\overline{B} s(\underline{x})}{s(\overline{x}) - s(\underline{x})} \end{aligned}$$

The results reported in the paper use value of $B = 0.015$ and $x \in [0, 4]$. Since the steady-state fractions of households' times devoted to production are $n = 0.2$, setting $B = 0.015$ involves that the maximum adjustment costs are 7.5 percent of production times in the hypothetical economy. This also implies that the resources devoted to price adjustments correspond roughly to 0.8 percent of firm's revenues with a maximum adjustment cost of 8.2 percent of revenues.

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Parameter values governing:		**Country 1**	**Country 2**
Preferences			
β	Discount rate	0.99	0.99
σ	Intertemporal elasticity of substitution	0.25	0.25
η	Elasticity of labor supply	0.05	0.05
n	Fraction of time working	0.20	0.20
Demands			
ϱ	Demand curvature	1.02	1.02
ϵ	Elasticity of demand at 1	10	10
Countries			
s	Country's relative size	0.50	0.50
θ	Degree of home bias	0.0375	0.0.75
γ	Elasticity of substitution - Country	1	1
Productivity			
a	Total factor productivity	1	1
Monetary policies			
μ	Steady-state money growth rate	0.01	0.01
ρ	Money growth autocorrelation	0.50	0.50

Table 1: Benchmark parameters

		Quarter(s) since last adjustment						
		0	**1**	**2**	**3**	**4**	**5**	**6**
α_j	Probability of adjustment	—	0.04	0.11	0.22	0.39	0.67	1
ω_j	Population density	0.25	0.24	0.21	0.17	0.10	0.03	—

Table 2: Stationary distributions of firms across countries

Table 3: Domestic and International Cross-Correlations

	Data	TDP	SDP
Country 1 Cross-Correlations with Output			
Consumption	**0.86**	1.00	1.00
Exports	**0.28**	-0.39	0.76
Imports	**0.72**	1.00	0.92
CPI Inflation	**0.18**	0.96	0.31
Trade balance	**-0.26**	-0.99	-0.51
Terms of trade	**-0.17**	-0.76	0.47
Nominal exchange rate	**0.09**	0.24	-0.62
Country 1 Cross-Correlations between ...			
Consumption - trade-balance	**-0.36**	-0.98	-0.54
Trade-balance - terms of Trade	**0.40**	0.85	0.50
Nominal exch. - trade-balance	**0.20**	-0.39	-0.54
Nominal exch. - terms of trade	**0.43**	-0.74	-0.89
Nominal exch. - Inflation	**0.35**	0.40	0.35
International Cross-Correlations between ...			
Home output - foreign output	**0.43**	1.00	0.59
Home cons. - foreign cons.	**0.29**	-0.04	0.55
Home CPI - foreign CPI	**0.88**	1.00	0.34

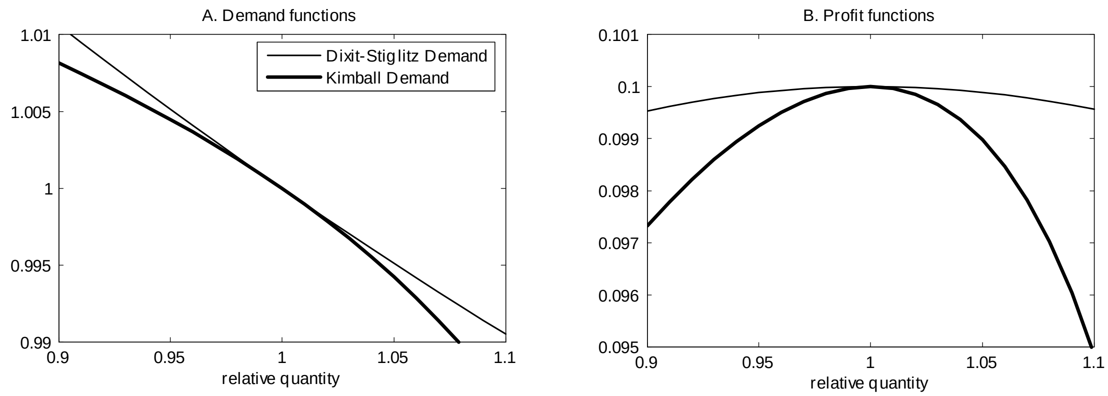

Figure 1: Dixit-Stiglitz and Kimball demand and profit functions

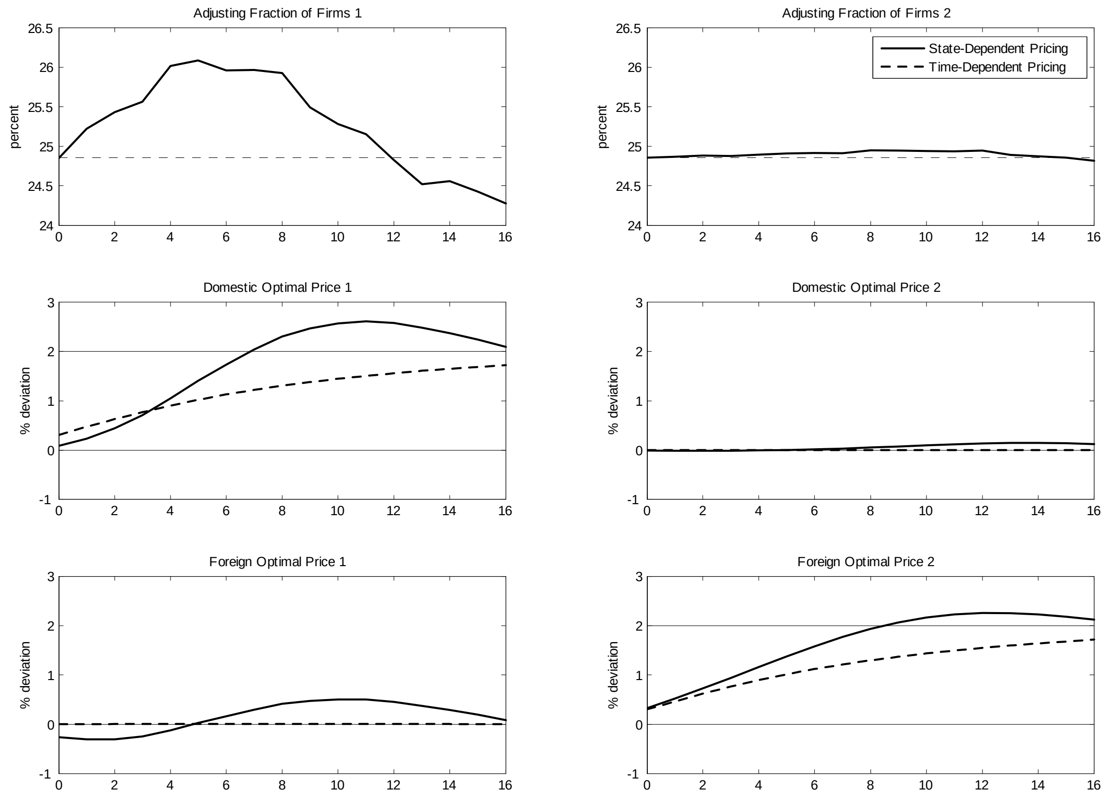

Figure 2: Firms' reaction

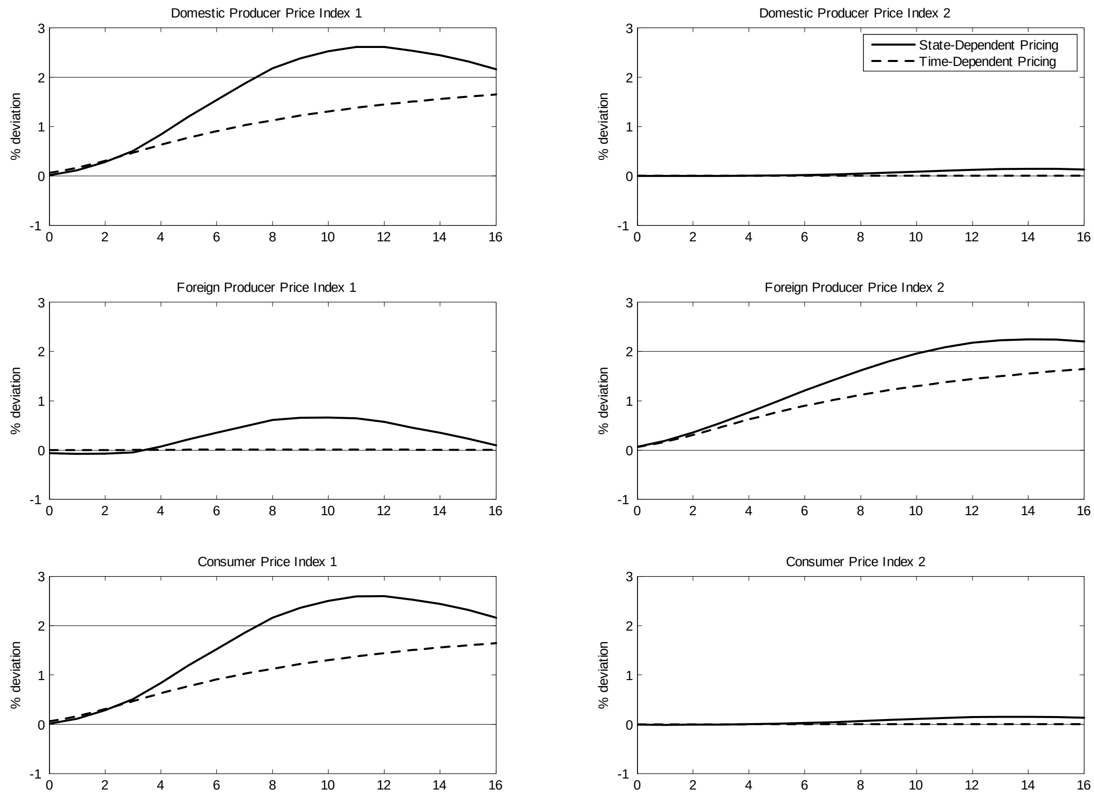

Figure 3: Price indices

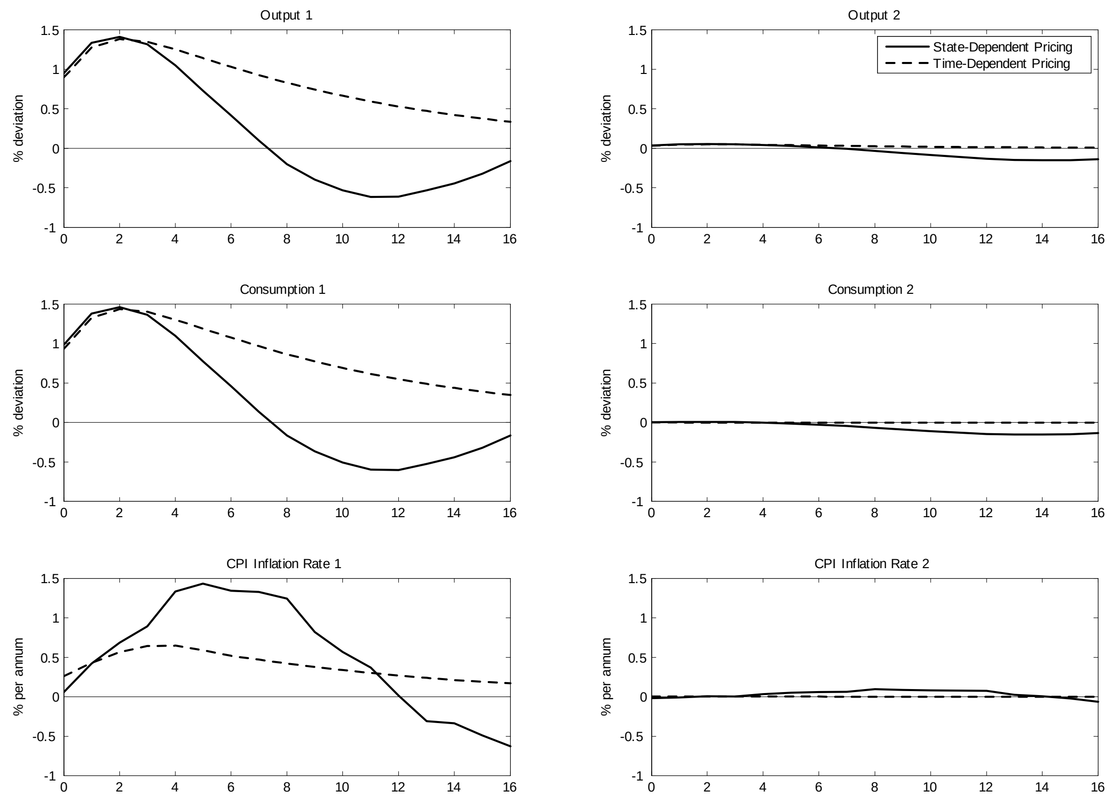

Figure 4: Output, consumption, and CPI inflation

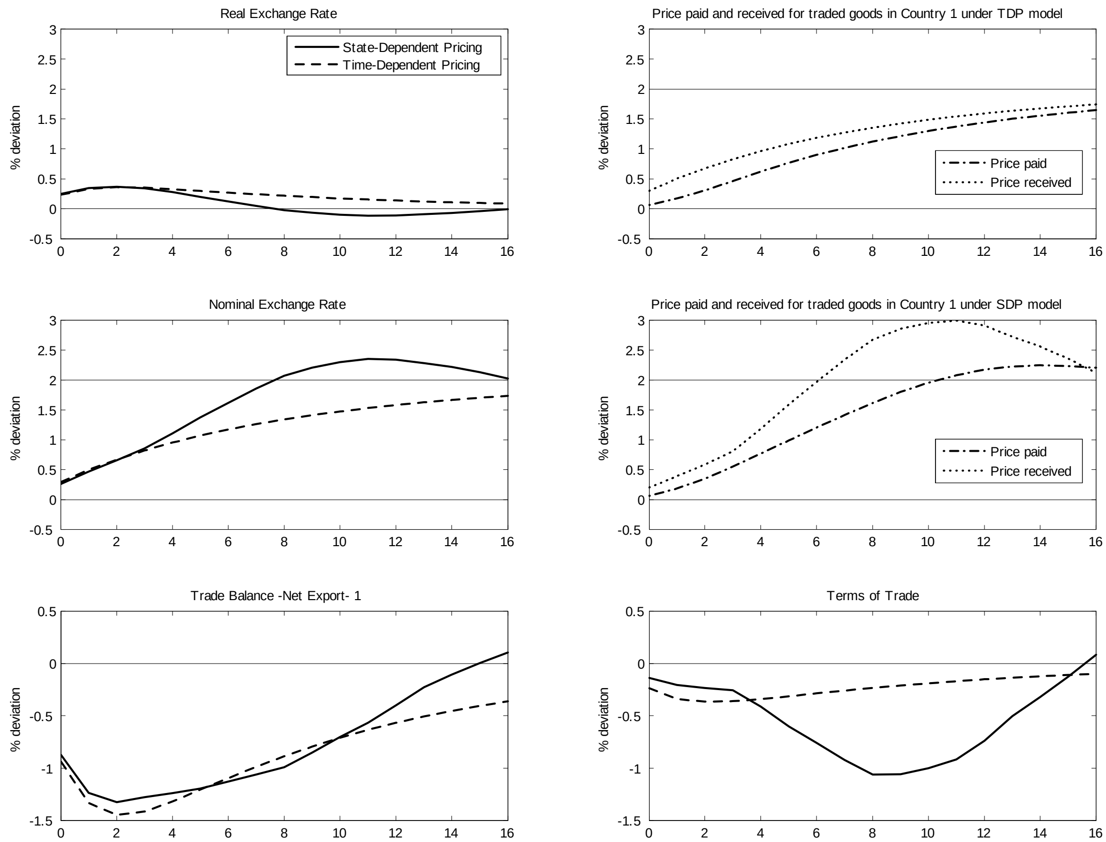

Figure 5: Exchange rate and trade

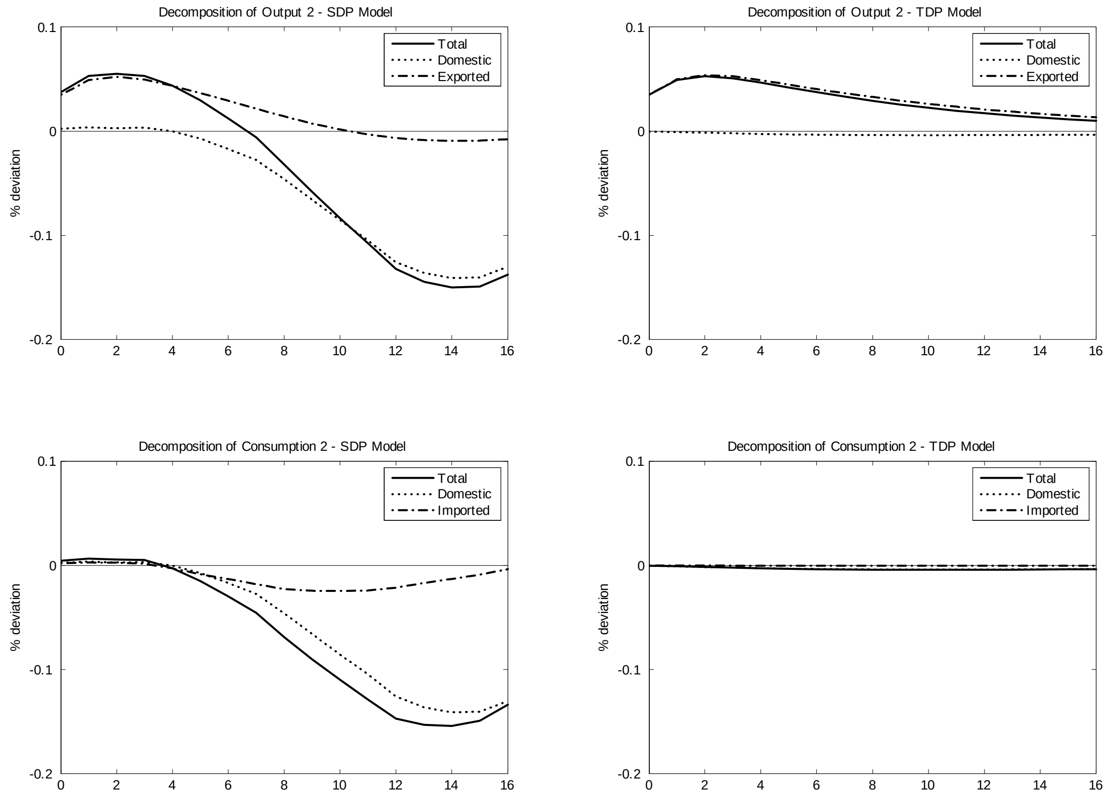

Figure 6: Output and consumption components in Country 2