Inflation Expectations, Real Interest Rate and Risk Premiums—Evidence from Bond Market and Consumer Survey Data

Dong Fu

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Dong Fu†

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Abstract

This paper extracts information on inflation expectations, the real interest rate, and various risk premiums by exploring the underlying common factors among the actual inflation, University of Michigan consumer survey inflation forecast, yields on U.S. nominal Treasury bonds, and particularly, yields on Treasury Inflation Protected Securities (TIPS). Our findings suggest that a significant liquidity risk premium on TIPS exists, which leads to inflation expectations that are generally higher than the inflation compensation measure at the 10-year horizon. On the other hand, the estimated expected inflation is mostly lower than the consumer survey inflation forecast at the 12-month horizon. Survey participants slowly adjust their inflation forecasts in response to inflation changes. The nominal interest rate adjustment lags inflation movements too. Our model also edges out a parsimonious seasonal AR(2) time series model in the one-step-ahead forecast of inflation.

JEL classification: E43, G12, C32

Key words: Inflation Expectations, Treasury Inflation Protected Securities (TIPS), Survey Inflation Forecast, Kalman Filter

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†Research Department, Federal Reserve Bank of Dallas and Southern Methodist University. Correspondence: dong_fu@yahoo.com

1 Introduction

Timely and accurate estimates of inflation expectations are gaining importance in monetary policy as inflation targeting, formal or informal, has become a widely adopted practice (Leiderman and Svensson, 1995; Bernanke, Laubach, Mishkin and Posen, 1998). Specifically, they help monetary authorities in setting the short term real interest rate at the appropriate level and provide observers a tool to analyze whether a central bank's inflation targeting is credible or not.

Many economic indicators contain information on inflation and expectations of future inflation. In macroeconomics, the Phillips curve has played a central role in relating future inflation to current real activities (Stock and Watson, 1999; Gali and Gertler, 1999). However, many of these real macroeconomic variables are subject to routine revisions, sometimes rather significant ones. These revisions present serious problems for real time analysis and forecasting. Two other channels, which are less subject to data revisions, have also been used to gauge inflation expectations, namely, survey inflation forecasts and the term structure of interest rates. In the latter approach, inflation compensation, defined as the yield spread between nominal and indexed government bonds of the same maturity, is often taken as a proxy for the expected future inflation. However, surveys of inflation expectations tend to reflect partial and incomplete updating in response to macroeconomic news (Mankiw, Reis and Wolfers, 2003), while the simple inflation compensation measure ignores various risk premiums involved in bond pricing (Berger, Stortelder and Wurzburger, 2001). More importantly, using these measures individually may not efficiently capture the information on the underlying inflation process.

This paper seeks to use the information contained in different channels simultaneously in measuring inflation expectations. Ours is a dynamic factor model, which extracts information on inflation expectations, the real interest rate and risk premiums by exploring the underlying common factors among a group of observed variables, including the actual inflation, University of Michigan consumer survey inflation forecast, yields on nominal Treasury bonds, and particularly, yields on Treasury Inflation Protected Securities (TIPS). We use the Kalman filter in maximum likelihood estimation of the state variables that drive the inflation and real interest rate processes. These state variables include unobserved permanent and temporary components of the real interest rate; permanent, temporary and seasonal components of the inflation rate; idiosyncratic excess return factors on T-bonds and TIPS, which represent risk premiums; and an idiosyncratic factor representing the survey inflation forecast error. Our sample spans from 1997, when TIPS were first issued, to 2003.

Our estimate of U.S. inflation expectations, which incorporates information from both the survey inflation forecast and bond market, seems much more sensitive to negative shocks to the U.S. economy than both the survey inflation forecast and the inflation compensation suggest. At the 10-year horizon, this estimate is generally higher than the inflation compensation measure. On the other hand, the estimated inflation expectation is mostly lower than the University of Michigan consumer survey inflation forecast at the 12-month horizon. The finding that 10-year inflation expectations are higher than the inflation compensation suggests a significant liquidity risk premium of TIPS over nominal T-bonds. In our model, the difference between the excess returns of 10-year T-bonds and TIPS can be viewed as a proxy for the inflation risk premium embodied in the 10-year T-bond yield. We find the difference to be negative in part of the sample period, pointing to a possible liquidity risk premium of TIPS over T-bonds. Our finding, thus, is consistent with Shen and Corning (2001), Sack (2000, 2004), D'Amico, Kim and Wei (2007). Sack (2000, 2004) uses off-the-run T-bond yields to better match up with the TIPS yields on a liquidity basis. However, using off-the-run T-bonds in our model doesn't eliminate the negative values. The implication is that there is a significant liquidity risk premium of TIPS over T-bonds, no matter whether the latter are on-the-run or off-the-run. We also find that the expected real interest rate has trended downward since the end of 2000.

Innovations in the real interest rate and inflation are found to be strongly negatively correlated, implying that nominal interest rate adjustments lag the inflation changes, which is consistent with the findings of Barr and Campbell (1997), Pennacchi (1991) and Summers (1983). We also find negative correlation between innovations in inflation and the survey inflation forecast error factor, suggesting survey participants only partly adjust their forecasts in response to underlying inflation changes, in line with Keane and Runkle (1990), Mankiw, Reis and Wolfers (2003). In addition, we find innovations in permanent and temporary real interest rates to be positively correlated, as are innovations in permanent and temporary inflation. Our estimated nominal term premium follows the observed term spread closely.

The model offers us the opportunity to combine the information from the bond market, survey inflation forecast and actual realized inflation in conducting forecasts. First, the simple out-of-sample forecast suggests that the real interest rate is likely to stay low as of yearend 2003, while inflation remains stable. Then we conduct one-step ahead out-of-sample forecasts of the inflation rate by incorporating new data observations at each step, but without re-estimating the model parameters because of the considerable cost involved in such estimations. Our model seems to do a little better than a parsimonious seasonal AR(2) time series model of inflation, in terms of producing a lower RMSE for the out-of-sample forecast period from January 2004 to December 2005.

Our modelling of T-bond and TIPS yields follows a rich literature, dating back to Fisher (1896), of analyzing the relationship between inflation and interest rate. However, here we draw a distinction between our model and the popular Affine Term Structure Model (ATSM) (Cox, Ingersoll and Ross, 1985; Dai and Singleton, 2000)[1]. In ATSM, bond prices (yields) are linear functions of unobserved state variables (or latent factors), which reflect sources of uncertainty and are usually termed as "level", "slope" and "curvature" factors. D'Amico, Kim and Wei (2007) use such a three-factor term structure model to study the informational content of TIPS. Meanwhile, it remains a difficult task to link these latent factors to macroeconomic dynamics, even with recent progress made by Wu (2005), and Ang and Piazzesi (2003), among others. So instead of relying on ATSM, this paper derives the bond price (yield) from the standard asset pricing equation (Cochrane, 2001; Balke and Wohar, 2000 & 2002; Pennacchi 1991). Bond prices (yields) are functions of underlying factors with explicit economic meanings—the real interest rate, inflation, and idiosyncratic factors embodying risk premiums. These factors follow mutually dependent processes and we assume conditional homoskedasticity[2].

Methodology-wise, this paper is done in the same vein as Pennacchi (1991), which also expresses bond prices and inflation forecasts as linear functions of state variables such as the real interest rate and inflation, but includes only constants instead of time-varying idiosyncratic factors in the equations. Our other novelty here is the use of TIPS to supplement traditional T-bonds, together with the survey inflation forecast, in estimating the underlying factors. TIPS and other forms of inflation indexed bonds provide additional information on the real interest rate and inflation compared with the traditional T-bonds (Deacon and Derry, 1994; Campbell and Shiller, 1996; Evans, 1998 and Emmons, 2000). Since U.S. TIPS' appearance, inflation compensation has often been used as a measure of expected inflation (Kliensen and Schmid, 2004). This approach ignores various risk premiums involved in bond pricing. Barr and Campbell (1997) estimate the expected future real interest rate and inflation rate from observed prices of U.K. nominal and indexed government bonds. They too ignore both the real term premium and inflation risk premium, while Brown and Schaefer (1994) allow for the real term premium in estimating a real yield curve of the U.K. inflation indexed bond using ATSM. In this paper, we model the excess return on an n-period T-bond or TIPS as a time-varying maturity-specific idiosyncratic factor, which incorporates both the real term premium and the inflation risk premium. In addition, we explicitly consider the seasonal factor contained

[1] Estimation is either based on GMM, as in Gibbons and Ramaswamy (1986) and Heston (1988); or maximum likelihood, as in Pearson and Sun (1988), Gong and Remolona (1996).

[2] Campbell and Viceira (2001) has a similar setting, although more akin to the ATSM.

in the non-seasonally adjusted Consumer Price Index used for the indexation of TIPS. We also account for the TIPS indexation lag, as Barr and Campbell (1997) suggest, while early studies, such as Brown and Schaefer (1994), assume perfect indexation.

The remainder of the paper is organized as follows. Section 2 explores bond price (yield) equations for both indexed and nominal bonds from a standard asset pricing setting. Section 3 lays out the formula for the consumer survey inflation forecast. Section 4 details the processes of underlying dynamic factors that determine the behavior of observed variables. Section 5 describes the state space model and the Kalman filter technique used in estimation. Section 6 reports our major findings. Section 7 forecasts the inflation and real interest rate. Section 8 concludes.

2 Bond Price

2.1 General Bond Price

The standard asset pricing equation is

$$1 = E_t[M_{t+1}R_{t+1}] \tag{1}$$

where M_{t+1} is the stochastic discount factor and R_{t+1} is the real gross return from holding an asset from time t to time $t+1$. When the asset is an n-period zero-coupon bond, (1) becomes

$$1 = E_t\left[M_{t+1}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{Q_{n-1,t+1}}{Q_{n,t}}\right)\right]$$

$Q_{n,t}$ is the nominal price at t of this bond, which matures at $t+n$. P_t is the general price level at t. So the bond price can be rewritten iteratively as

$$Q_{n,t} = E_t\left[\left(\prod_{i=1}^{n} M_{t+i}\right)\left(\prod_{i=1}^{n}\frac{P_{t+i-1}}{P_{t+i}}\right)Q_{0,t+n}\right] \tag{2}$$

$Q_{0,t+n}$ is the nominal price of this bond at the maturity date $t+n$. For a nominal bond, we routinely standardize $Q_{0,t+n}$ to 1. When $Q_{0,t+n}$ is indexed to reflect general price level changes over time, the bond becomes an inflation indexed bond. In the following discussion, we assume all data are of monthly frequency.

2.2 Inflation Indexed Bond Price

2.2.1 Fully Indexed Bond Price

For a fully indexed bond, we have

$$Q_{0,t+n} = 1 \cdot \frac{P_{t+n}}{P_t}$$

From (2), the price at time t of a n-period fully indexed zero-coupon bond, denoted by $Q^R_{n,t}$, is

$$Q^R_{n,t} = E_t \left[\left(\prod_{i=1}^{n} M_{t+i} \right) \right] \tag{3}$$

So this bond becomes a real bond. Under log normality, we have

$$\log Q^R_{n,t} = E_t \left[\sum_{i=1}^{n} m_{t+i} \right] + \frac{1}{2} Var_t \left[\sum_{i=1}^{n} m_{t+i} \right]$$

where $m_t = \log(M_t)$. The yield to maturity of this real bond is the n-period real interest rate, denoted by $i^R_{n,t}$.

$$i^R_{n,t} = -\left(\frac{1}{n}\right) \left\{ E_t \left[\sum_{i=1}^{n} m_{t+i} \right] + \frac{1}{2} Var_t \left[\sum_{i=1}^{n} m_{t+i} \right] \right\}$$

When $n = 1$, $i^R_{1,t}$ is simply the expected one-period real interest rate

$$E_t [r_{t+1}] = i^R_{1,t} = -\left\{ E_t [m_{t+1}] + \frac{1}{2} Var_t [m_{t+1}] \right\}$$

When $n > 1$, $i^R_{n,t}$ can be rewritten as

$$i^R_{n,t} = \left(\frac{1}{n}\right) \sum_{i=1}^{n} E_t [r_{t+i}] - \left(\frac{1}{n}\right) \sum_{i=j+1}^{n} \sum_{j=1}^{n} Cov_t [m_{t+i}, m_{t+j}] \tag{4}$$

where $\left(\frac{1}{n}\right) \sum_{i=1}^{n} E_t [r_{t+i}]$ is the average expected one-period real interest rate over the bond's lifetime and $-\left(\frac{1}{n}\right) \sum_{i=j+1}^{n} \sum_{j=1}^{n} Cov_t [m_{t+i}, m_{t+j}]$ is the real term premium investors demand in order to invest in the n-period real bond instead of the one-period real bond.

2.2.2 TIPS Price

Lately, inflation indexed bonds have gained increasing popularity in a number of countries. In reality, these bonds all deviate one way or another from the fully indexed bond. The U.S. TIPS are a case in point. TIPS are coupon bonds, with both coupon payments within the term and principal repayment at the maturity date indexed to reflect the changes over time of the non-seasonally adjusted Consumer Price Index (CPI). For convenience in analysis, we assume TIPS are zero-coupon bonds. In fact, TIPS yields are routinely quoted on a zero-coupon basis in practice (for example, in McCulloch and Kochin, 2000; see data appendix A.2).

Assume an n-period zero-coupon TIPS is issued at t. Ideally, at its maturity date $t+n$, TIPS' nominal price should reflect the CPI level change between the issuing date t and the maturity date $t+n$. In reality, TIPS are indexed to the CPI with a short indexation lag of 3 months[3] (We assume $n > 3$ holds for TIPS in all following discussions). So,

$$Q_{0,t+n} = 1 \cdot \frac{P_{t+n-3}}{P_{t-3}}$$

From (2), the price at t of a n-period TIPS, denoted by $Q^L_{n,t}$, is

$$Q^L_{n,t} = E_t\left[\left(\prod_{i=1}^{n} M_{t+i}\right)\left(\frac{P_{t+n-3}}{P_{t+n}}\right)\left(\frac{P_t}{P_{t-3}}\right)\right] \tag{5}$$

Similar to Barr and Campbell (1997), we assume $E_t(P_t) = P_t$[4]. Thus (5) becomes

$$Q^L_{n,t} = \left(\frac{P_t}{P_{t-3}}\right) E_t\left[\left(\prod_{i=1}^{n} M_{t+i}\right) / \left(\prod_{i=1}^{3} \frac{P_{t+n+1-i}}{P_{t+n-i}}\right)\right]$$

Under log normality, with $m_t = \log(M_t)$ and $\pi_t = \log(P_t/P_{t-1})$, the yield to maturity of TIPS is

$$\begin{aligned} i^L_{n,t} \quad = \quad & \left(\frac{1}{n}\right)\sum_{i=1}^{n} E_t\left[r_{t+i}\right] - \left(\frac{1}{n}\right)\sum_{i=j+1}^{n}\sum_{j=1}^{n} Cov_t\left[m_{t+i}, m_{t+j}\right] - \\ & \left(\frac{1}{n}\right)\log\left(\frac{P_t}{P_{t-3}}\right) + \left(\frac{1}{n}\right)\sum_{i=1}^{3} E_t\left[\pi_{t+n+1-i}\right] - \\ & \left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{3}\pi_{t+n+1-i}\right] - Cov_t\left[\sum_{i=1}^{n} m_{t+i}, \sum_{i=1}^{3}\pi_{t+n+1-i}\right]\right\} \end{aligned} \tag{6}$$

Unlike the fully indexed bond, TIPS is exposed to a small amount of inflation risk due the indexation lag. Appendix B compares TIPS with the fully indexed bond in more details. Next, we define an idiosyncratic excess return factor $\lambda_{n,t}$ on TIPS, which contains all the variance-covariance terms in (6). We assume it is dependent on maturity n and time varying, similar to Balke and Wohar (2000, 2002). So the TIPS yield can be rewritten as

$$i^L_{n,t} = -\left(\frac{1}{n}\right)\log\left(\frac{P_t}{P_{t-3}}\right) + \left(\frac{1}{n}\right)\sum_{i=1}^{n} E_t\left[r_{t+i}\right] + \left(\frac{1}{n}\right)\sum_{i=1}^{3} E_t\left[\pi_{t+n+1-i}\right] + \lambda_{n,t} \tag{6'}$$

[3]Strictly speaking, the TIPS indexation lag is 2.5 months. The indexation lag for the U.K. inflation indexed bond is 8 months. See Barr and Campbell (1997), Deacon and Derry (1994), McCulloch and Kochin (2000) for details.

[4]In expectation models, it is customary to assume a random variable X_t's conditional expectation $E_t(X_t) = X_t$. In other words, X_t is observed at t. However, in reality, X_t is usually not observed until some time after t. For example, CPI is normally observed with a lag. In fact, it is precisely because of this lag that TIPS is indexed to CPI with a lag of 3 months in practice.

2.3 Nominal Treasury Bond Price

Because $Q_{0,t+n} = 1$ for a nominal bond, from (2), the price of an n-period nominal T-bond at t, still denoted by $Q_{n,t}$, is

$$Q_{n,t} = E_t \left[\left(\prod_{i=1}^{n} M_{t+i} \right) / \left(\prod_{i=1}^{n} \frac{P_{t+i}}{P_{t+i-1}} \right) \right] \tag{7}$$

Under log normality, the yield to maturity of this n-period nominal t-bond is

$$\begin{aligned} i_{n,t} \quad = \quad & \left(\frac{1}{n}\right) \sum_{i=1}^{n} E_t \left[r_{t+i} \right] - \left(\frac{1}{n}\right) \sum_{i=j+1}^{n} \sum_{j=1}^{n} Cov_t \left[m_{t+i}, m_{t+j} \right] + \left(\frac{1}{n}\right) \sum_{i=1}^{n} E_t \left[\pi_{t+i} \right] - \\ & \left(\frac{1}{n}\right) \left\{ \frac{1}{2} Var_t \left[\sum_{i=1}^{n} \pi_{t+i} \right] - Cov_t \left[\sum_{i=1}^{n} m_{t+i}, \sum_{i=1}^{n} \pi_{t+i} \right] \right\} \end{aligned} \tag{8}$$

Compared with TIPS yield, nominal T-bond yield covers more completely the expected inflation and inflation risk premium during the entire lifetime of n periods. Appendix B provides more details. We define a time-varying and maturity-specific idiosyncratic excess return factor $\xi_{n,t}$ on a nominal T-bond as containing all the variance-covariance terms in (8), similar to the case of TIPS. So a nominal T-bond yield can be rewritten as

$$i_{n,t} = \left(\frac{1}{n}\right) \sum_{i=1}^{n} E_t \left[r_{t+i} \right] + \left(\frac{1}{n}\right) \sum_{i=1}^{n} E_t \left[\pi_{t+i} \right] + \xi_{n,t} \tag{8'}$$

2.4 Inflation Compensation

From (6') and (8'), inflation compensation, defined as the difference between the yield of a nominal T-bond and the yield of a TIPS of the same maturity (Sack, 2000; Kliensen and Schmid, 2004), is

$$i_{n,t} - i_{n,t}^{L} = \left(\frac{1}{n}\right) \log \left(\frac{P_t}{P_{t-3}} \right) + \left(\frac{1}{n}\right) \sum_{i=1}^{n-3} E_t \left[\pi_{t+i} \right] + \left(\xi_{n,t} - \lambda_{n,t} \right) \tag{9}$$

We can see it contains three parts. First, existing price change between $t-3$ and t. Second, the expected inflation between t and $t+n-3$. The sum of the two is a proxy for the expected inflation during the bonds' entire lifetime from t to $t+n$. The third part $\left(\xi_{n,t} - \lambda_{n,t} \right)$ stands for the difference between the excess return on the nominal T-bond and the excess return on the TIPS.

$$
\begin{aligned}
&\xi_{n,t}-\lambda_{n,t} \\
= \quad & -\left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{n}\pi_{t+i}\right]-Cov_t\left[\sum_{i=1}^{n}m_{t+i},\sum_{i=1}^{n}\pi_{t+i}\right]\right\}- \\
& \left\{-\left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{3}\pi_{t+n+1-i}\right]-Cov_t\left[\sum_{i=1}^{n}m_{t+i},\sum_{i=1}^{3}\pi_{t+n+1-i}\right]\right\}\right\} \qquad (10)
\end{aligned}
$$

Here, $-\left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{n}\pi_{t+i}\right]-Cov_t\left[\sum_{i=1}^{n}m_{t+i},\sum_{i=1}^{n}\pi_{t+i}\right]\right\}$ is the risk premium due to inflation uncertainty in n periods. $-\left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{3}\pi_{t+n+1-i}\right]-Cov_t\left[\sum_{i=1}^{n}m_{t+i},\sum_{i=1}^{3}\pi_{t+n+1-i}\right]\right\}$ is the risk premium due to inflation uncertainty during the indexation lag period. As long as n is much larger than 3, the first term dominates the second term.

2.5 Summary

According to (6'), the TIPS yield is a function of the expected real short interest rate $E_t\left[r_{t+i}\right]$, expected inflation $E_t\left[\pi_{t+i}\right]$ and the excess return $\lambda_{n,t}$. And according to (8'), the nominal T-bond yield is a function of $E_t\left[r_{t+i}\right]$, $E_t\left[\pi_{t+i}\right]$ and the excess return $\xi_{n,t}$. The time varying excess returns $\lambda_{n,t}$ and $\xi_{n,t}$ incorporate both real term premium and inflation risk premium (see appendix B for details). The measure of inflation risk premium is particularly important. One of the major reasons for issuing the indexed bond is that it is supposedly shielded from the inflation risk. So the Treasury department doesn't have to pay an inflation risk premium, resulting in a lower cost of financing government debts. Equally important, once we have an estimate of the inflation risk premium, we can screen it out from the inflation compensation measure to obtain a more precise estimate of underlying expected inflation.

Barr and Campbell (1997) directly model the prices, instead of the yields, of U.K. nominal and indexed bonds as functions of the expected real interest rate and the expected inflation rate. However, they assume the expected real holding period returns on both nominal and indexed bonds at any point in time are the same as the short term real interest rate. This actually contains two assumptions. First, the real holding period return equals the short term real interest rate—the log pure expectation hypothesis of the real interest rate, so there is no real term premium. Second, expected real holding period returns on nominal bonds equal returns on indexed bonds, implying zero inflation risk premium. As no risk premium is considered, their model equates to assuming the variance and covariance terms in our bond price equations are all zeros.

In reality, taxation affects the perception of returns and risks involved in pricing TIPS and nominal T-bonds. However, we choose not to model the tax effects explicitly in this paper, because our focus is on the common factors behind inflation and the real interest rate, based on both TIPS and nominal T-bonds yields. Fitting of either type of bond yields is of secondary importance. In any event, tax effects will only show up in the discount factor (real interest rate), but will not affect the inflation process. If the tax treatment of investing in TIPS is the same as that of investing in nominal T-bonds, we can simply regard the discount factor m_t as the after-tax discount factor, taking into account that the tax rate is also time varying. Similarly, the real interest rate r_t can be simply viewed as the after-tax real interest rate. If tax treatments differ, we argue that the common underlying discount factor (real interest rate) movement should still be picked up by m_t (r_t) in our setup while the differential taxation effect is pushed into the idiosyncratic excess return factor of either TIPS or nominal T-bonds. Another practical difficulty in modeling the tax effect is that different investors face different marginal tax rates on investing in the bond market. Anyway, considering that TIPS only make up a small portion of the U.S. government bond market at the moment, differential taxation effects overall should be small.

3 Survey Inflation Forecast

Survey inflation forecasts have been frequently used as benchmarks for measuring inflation expectations. Examples include the inflation forecast from the Federal Reserve Bank of Philadelphia's survey of professional forecasters (Croushore, 1993; Zarnowitz and Braun, 1992; Giordani and Soderlind, 2002), the University of Michigan consumer survey inflation forecast (Curtin, 1996), and the Blue Chip forecast of inflation. Thomas (1999) and Mehra (2002) summarize recent developments in these survey inflation forecasts.

In this paper, we use the University of Michigan consumer survey inflation forecast, which is available monthly and has seen wide usage in practice (see data appendix A.3). Keane and Runkle (1990) suggest surveys based on polling non-professional forecasters, who lack the incentive to compile accurate forecasts compared with professional forecasters, may not rationally utilize all available information. As non-professional survey participants are usually not as financially liable as professionals, they can be tardy in updating their forecasts, thus leading to potentially persistent forecast errors. Mankiw, Reis and Wolfers (2003) also find surveys of inflation expectations tend to reflect partial and incomplete updating in response to macroeconomic news. Accordingly, we model the consumer survey inflation forecast at t for future n periods, denoted by $\pi_{n,t}^{survey}$, as

$$\pi_{n,t}^{survey} = \frac{1}{n} E_t \left[\sum_{i=1}^{n} \pi_{t+i} \right] + e_{n,t} \tag{11}$$

We assume the persistent forecast error is reflected in an idiosyncratic factor $e_{n,t}$, which is dependent on the forecasting horizon n and is time varying. Thus, the consumer survey inflation forecast is a function of expected inflation $E_t[\pi_{t+i}]$ and the forecast error factor $e_{n,t}$.

4 Underlying Factor Processes

So far, we have shown that the observed TIPS yield, T-bond yield and survey inflation forecast are all determined by unobserved factors. Among them, common factors $E_t[\pi_{t+i}]$ and $E_t[r_{t+i}]$ are shared by multiple observed variables. By exploring the cross-equation restriction imposed by these common factors on the observed variables, we can extract information on the underlying factors. To do so, we need to put more structure on the factors' generating processes.

We assume the short term (one-month) real interest rate consists of a permanent (trend) component r_t^p and a temporary (cyclical) component r_t^a

$$r_t = r_t^p + r_t^a \tag{12}$$

r_t^p follows a random walk and r_t^a follows AR(2)

$$\begin{aligned} r_t^p &= r_{t-1}^p + v_t^{rp} && (13) \\ r_t^a &= \rho_1^{ra} \cdot r_{t-1}^a + \rho_2^{ra} \cdot r_{t-2}^a + v_t^{ra} && (14) \end{aligned}$$

Because TIPS indexation is based on the nonseasonally-adjusted CPI[5], we use the nonseasonally-adjusted CPI month-over-month inflation in the model. Following Harvey (1989, 1990, 1993), Koopman, Harvey, Doornik and Shephard (2000), the CPI inflation consists of a permanent (trend) component π_t^p, a temporary (cyclical) component π_t^a and a seasonal component π_t^s

$$\pi_t = \pi_t^p + \pi_t^a + \pi_t^s \tag{15}$$

π_t^p follows a random walk and π_t^a follows AR(2)

$$\begin{aligned} \pi_t^p &= \pi_{t-1}^p + v_t^{\pi p} && (16) \\ \pi_t^a &= \rho_1^{\pi a} \cdot \pi_{t-1}^a + \rho_2^{\pi a} \cdot \pi_{t-2}^a + v_t^{\pi a} && (17) \end{aligned}$$

The seasonal component π_t^s can be represented by γ_t, which evolves according to

[5] "Summary of Marketable Treasury Inflation-Indexed Securities" (http://www.publicdebt.treas.gov/gsr/gsrlist.html)

$$
\begin{pmatrix} \gamma_t \\ \gamma_{t-1} \\ . \\ . \\ . \\ \gamma_{t-9} \\ \gamma_{t-10} \end{pmatrix} = \begin{pmatrix} -1 & -1 & . & . & . & -1 & -1 \\ 1 & & & & & & \\ & . & & & & & \\ & & . & & & & \\ & & & . & & & \\ & & & & 1 & & \\ & & & & & 1 & 0 \end{pmatrix} \begin{pmatrix} \gamma_{t-1} \\ \gamma_{t-2} \\ . \\ . \\ . \\ \gamma_{t-10} \\ \gamma_{t-11} \end{pmatrix} + \begin{pmatrix} v_t^{\gamma} \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{pmatrix} \tag{18}
$$

So the process for inflation can be rewritten as

$$
\pi_t = \pi_t^p + \pi_t^a + \gamma_t \tag{15'}
$$

We assume the excess return factors in the nominal T-bond and TIPS yields follow AR(2).

$$
\xi_{n,t} = \rho_{n,1}^{\xi} \cdot \xi_{n,t-1} + \rho_{n,2}^{\xi} \cdot \xi_{n,t-2} + v_{n,t}^{\xi} \tag{19}
$$

$$
\lambda_{n,t} = \rho_{n,1}^{\lambda} \cdot \lambda_{n,t-1} + \rho_{n,2}^{\lambda} \cdot \lambda_{n,t-2} + v_{n,t}^{\lambda} \tag{20}
$$

So does the survey inflation forecast error factor.

$$
e_{n,t} = \rho_{n,1}^{e} \cdot e_{n,t-1} + \rho_{n,2}^{e} \cdot e_{n,t-2} + v_{n,t}^{e} \tag{21}
$$

5 State Space Model

Now we build a state space model following Harvey (1993), Hamilton (1994) and Harvey, Koopman and Shephard (2004). Appendix C.1 provides an overview of the state space model. In our case, the observed variables Y_t include the 3-month T-bill yield, 10-year T-bond yield, 10-year TIPS yield, 12-month inflation forecast from the University of Michigan consumer survey, as well as actual monthly inflation (see data appendix A for more detailed descriptions). All data are monthly and quoted in annual rates. The sample period is from January 1997, when TIPS were first issued, to December 2003. We choose to work with demeaned data Y_t, different from Pennacchi (1991) and Barr and Campbell (1997)[6]. The resulting state space model is more parsimonious as it contains no intercept in either the observation equation or the state equation.

[6] Pennacchi (1991) models bond prices and survey inflation forecasts as functions of expected real interest rate and inflation in a similar state space setting. He also defines state variables in terms of deviations from their unconditional means, but retains observed variables without demeaning. In comparison, Barr and Campbell (1997) impose trend-

5.1 State Equation

Under the assumption of rational expectations, and using equations (12) through (21), we transform the initial unobserved underlying factors $E_t\left[r_{t+i}\right]$ contained in (6') and (8'), $E_t\left[\pi_{t+i}\right]$ in (6'), (8') and (11), $\lambda_{n,t}$ in (6'), $\xi_{n,t}$ in (8') and $e_{n,t}$ in (11) into an expanded state vector, which evolves according to the following state equation:

$$
\begin{pmatrix} r_t^p \\ \pi_t^p \\ \gamma_t \\ \gamma_{t-1} \\ ... \\ \gamma_{t-10} \\ r_t^a \\ \pi_t^a \\ \xi_{3m,t}^a \\ \xi_{10yr,t}^a \\ \lambda_{10yr,t}^a \\ e_{12m,t}^a \\ r_{t-1}^a \\ \pi_{t-1}^a \\ \xi_{3m,t-1}^a \\ \xi_{10yr,t-1}^a \\ \lambda_{10yr,t-1}^a \\ e_{12m,t-1}^a \end{pmatrix}
=
\begin{pmatrix}
1 & & & & & & & & & & & \\
 & 1 & & & & & & & & & & \\
 & & -1 & ... & -1 & -1 & & & & & & \\
 & & 1 & & & & & & & & & \\
 & & & ... & & & & & & & & \\
 & & & & 1 & 0 & & & & & & \\
 & & & & & & \rho_1^{ra} & & & \rho_2^{ra} & & \\
 & & & & & & & . & & & . & \\
 & & & & & & & . & & & . & \\
 & & & & & & & . & & & . & \\
 & & & & & & & . & & & . & \\
 & & & & & & & & \rho_{12m,1}^{ea} & & & \rho_{12m,2}^{ea} \\
 & & & & & & 1 & & & & & \\
 & & & & & & & . & & & & \\
 & & & & & & & . & & & & \\
 & & & & & & & . & & & & \\
 & & & & & & & . & & & & \\
 & & & & & & & & 1 & & &
\end{pmatrix}
\cdot
\begin{pmatrix} r_{t-1}^p \\ \pi_{t-1}^p \\ \gamma_{t-1} \\ \gamma_{t-2} \\ ... \\ \gamma_{t-11} \\ r_{t-1}^a \\ \pi_{t-1}^a \\ \xi_{3m,t-1}^a \\ \xi_{10yr,t-1}^a \\ \lambda_{10yr,t-1}^a \\ e_{12m,t-1}^a \\ r_{t-2}^a \\ \pi_{t-2}^a \\ \xi_{3m,t-2}^a \\ \xi_{10yr,t-2}^a \\ \lambda_{10yr,t-2}^a \\ e_{12m,t-2}^a \end{pmatrix}
+
\begin{pmatrix} v_t^{rp} \\ v_t^{\pi p} \\ v_t^{\gamma} \\ 0 \\ 0 \\ 0 \\ v_t^{ra} \\ v_t^{\pi a} \\ v_{3m,t}^{\xi a} \\ v_{10yr,t}^{\xi a} \\ v_{10yr,t}^{\lambda a} \\ v_{12m,t}^{ea} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}
$$

$$S_t = F \cdot S_{t-1} + V_t \tag{22}$$

with $V_t \sim N(0, Q)$. As we are particularly interested in the relationship among various factors, we want to impose a minimum number of restrictions on Q. The only restriction on Q is that there

stationary AR(1) processes directly on the log expected inflation and expected real interest rate, which are then estimated from cross section data of observed bond prices at a particular time t. Their data are not demeaned either. The intercepts of the asset pricing equations based on the underlying inflation and real short interest rate are identified as part of the AR(1) processes.

is no correlation between the innovation in inflation's seasonal factor v_t^{γ} and innovations in other factors.

5.2 Observation Equation

5.2.1 Yields of Nominal 3-Month T-bill and 10-Year T-bond

Under rational expectations, the yield of a n-period nominal T-bond in (8'), is written as a function of the state vector.

$$\begin{aligned} i_{n,t} &= \left(\frac{1}{n}\right) E_t\left[\sum_{i=1}^{n} r_{t+i}\right] + \left(\frac{1}{n}\right) E_t\left[\sum_{i=1}^{n} \pi_{t+i}\right] + \xi_{n,t} \\ &= \left(\frac{1}{n}\right)\left[H_r \sum_{i=1}^{n} F^i S_t + H_\pi \sum_{i=1}^{n} F^i S_t\right] + H_{\xi,n} S_t \end{aligned}$$

So the 3-month T-bill yield is

$$i_{3m,t} = \left(\frac{1}{3}\right)\left[H_r \sum_{i=1}^{3} F^i S_t + H_\pi \sum_{i=1}^{3} F^i S_t\right] + H_{\xi,3m} S_t \tag{23}$$

and the 10-year T-bond yield is

$$i_{10yr,t} = \left(\frac{1}{120}\right)\left[H_r \sum_{i=1}^{120} F^i S_t + H_\pi \sum_{i=1}^{120} F^i S_t\right] + H_{\xi,10yr} S_t \tag{24}$$

where

$$\begin{aligned}
H_r &= \begin{array}{c} \begin{array}{ccccccccccccccc} & & & 4th & & 13th & & & & & & & 20th & & 25th\text{ element} \end{array} \\ \left(\begin{array}{ccccccccccccccc} 1 & 0 & 0 & 0 & \ldots & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \end{array}\right) \end{array} \\
H_\pi &= \begin{array}{c} \begin{array}{ccccccccccccccc} & & & 4th & & 13th & & & & & & & 20th & & 25th\text{ element} \end{array} \\ \left(\begin{array}{ccccccccccccccc} 0 & 1 & 1 & 0 & \ldots & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 \end{array}\right) \end{array} \\
H_{\xi,3m} &= \begin{array}{c} \begin{array}{ccccccccccccccc} & & & 4th & & 13th & & & & & & & 20th & & 25th\text{ element} \end{array} \\ \left(\begin{array}{ccccccccccccccc} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 \end{array}\right) \end{array} \\
H_{\xi,10yr} &= \begin{array}{c} \begin{array}{ccccccccccccccc} & & & 4th & & 13th & & & & & & & 20th & & 25th\text{ element} \end{array} \\ \left(\begin{array}{ccccccccccccccc} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \ldots & 0 \end{array}\right) \end{array}
\end{aligned}$$

5.2.2 Yield of 10-Year TIPS

Similarly, under rational expectations, the yield of a n-period TIPS in (6') follows

$$\begin{aligned} i_{n,t}^{L} &= -\left(\frac{1}{n}\right)\log\left(\frac{P_t}{P_{t-3}}\right) + \left(\frac{1}{n}\right) E_t\left[\sum_{i=1}^{n} r_{t+i}\right] + \left(\frac{1}{n}\right) E_t\left[\sum_{i=1}^{3} \pi_{t+n+1-i}\right] + \lambda_{n,t} \\ &= -\left(\frac{1}{n}\right)\log\left(\frac{P_t}{P_{t-3}}\right) + \left(\frac{1}{n}\right)\left[H_r \sum_{i=1}^{n} F^i S_t + H_\pi \sum_{i=1}^{3} F^{n+1-i} S_t\right] + H_{\lambda,n} S_t \end{aligned}$$

So the 10-year TIPS yield is

$$i^L_{10yr,t} = -\left(\frac{1}{120}\right)\log\left(\frac{P_t}{P_{t-3}}\right) + \left(\frac{1}{120}\right)\left[H_r\sum_{i=1}^{120}F^iS_t + H_\pi\sum_{i=1}^{3}F^{120+1-i}S_t\right] + H_{\lambda,10yr}S_t \tag{25}$$

If we rename $\left[i^L_{10yr,t} + \left(\frac{1}{120}\right)\log\left(\frac{P_t}{P_{t-3}}\right)\right]$ as $i^{TIPS}_{10yr,t}$, we then have

$$i^{TIPS}_{10yr,t} = \left(\frac{1}{120}\right)\left[H_r\sum_{i=1}^{120}F^iS_t + H_\pi\sum_{i=1}^{3}F^{120+1-i}S_t\right] + H_{\lambda,10yr}S_t \tag{26}$$

where

$$H_{\lambda,10yr} = \begin{array}{c} \begin{array}{ccccccccccccccc} & & & 4th & & 13th & & & & & & & 20th & & 25th \text{ element} \end{array} \\ \left(\begin{array}{ccccccccccccccc} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \ldots & 0 \end{array}\right) \end{array}$$

5.2.3 Monthly Inflation

According to (15'), the monthly inflation is

$$\pi_t = H_\pi S_t \tag{27}$$

5.2.4 Survey Inflation Forecast

According to (11), together with (15') and (18), the 12-month consumer survey inflation forecast under rational expectations is written as

$$\begin{aligned} \pi^{survey}_{12m,t} &= \frac{1}{12}E_t\left[\sum_{i=1}^{12}\pi_{t+i}\right] + e_{12m,t} \\ &= \frac{1}{12}H_\pi\sum_{i=1}^{12}F^iS_t + H_{e,12m}S_t \end{aligned} \tag{28}$$

where

$$H_{e,12m} = \begin{array}{c} \begin{array}{ccccccccccccccc} & & & 4th & & 13th & & & & & & & 20th & & 25th \text{ element} \end{array} \\ \left(\begin{array}{ccccccccccccccc} 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \ldots & 0 \end{array}\right) \end{array}$$

5.2.5 Summary

All variables in Y_t are assumed to be subject to measurement errors, denoted by $w^i_{3m,t}$, $w^i_{10yr,t}$, $w^{TIPS}_{10yr,t}$, w^π_t and $w^{survey}_{12m,t}$. Measurement errors are important in modeling bond yields, particularly TIPS yields, because of the relatively small number of outstanding TIPS and the different methods used in estimating the yield curve in practice[7]. However, the measurement error in bond yields may be eclipsed by idiosyncratic excess return factors. Similarly, measurement errors' effect on the survey inflation forecast may be small as the idiosyncratic factor is likely to pick up most of the variation. On the other hand, measurement errors may affect observed inflation to a larger extent as no idiosyncratic factor is there to pick up the residual variation in data. Now the combined observation equation in the state space model is

$$\begin{pmatrix} i_{3m,t} \\ i_{10yr,t} \\ i^{TIPS}_{10yr,t} \\ \pi_t \\ \pi^{survey}_{12m,t} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{3}\right)(H_r + H_\pi)\sum_{i=1}^{3} F^i + H_{\xi,3m} \\ \left(\frac{1}{120}\right)(H_r + H_\pi)\sum_{i=1}^{120} F^i + H_{\xi,10yr} \\ \left(\frac{1}{120}\right)\left(H_r \sum_{i=1}^{120} F^i + H_\pi \sum_{i=1}^{3} F^{120+1-i}\right) + H_{\lambda,10yr} \\ H_\pi \\ \left(\frac{1}{12}\right) H_\pi \sum_{i=1}^{12} F^i + H_{e,12m} \end{pmatrix} \cdot S_t + \begin{pmatrix} w^i_{3m,t} \\ w^i_{10yr,t} \\ w^{TIPS}_{10yr,t} \\ w^\pi_t \\ w^{survey}_{12m,t} \end{pmatrix}$$

$$Y_t = H \cdot S_t + W_t \tag{29}$$

with $W_t \sim N(0, R)$. R is assumed to be diagonal, indicating no cross correlation among observation errors.

5.3 Estimation

The state space model usually has a large number of unknown parameters. However, in this model, the H matrix in the observation equation (29) involves only constant vectors such as H_r and H_π, except the unknown F matrix, which also belongs to the state equation (22). This H matrix in fact puts strong cross-equation restrictions on parameters in both the state and observation equations. These restrictions go a long way in helping to achieve identification. We estimate the state space model using a combination of the EM algorithm (Watson and Engle, 1983) and the traditional Maximum Likelihood estimation method (Hamilton, 1994). Appendix C.2 covers the estimation technique in detail. We are able to achieve convergence in the maximization process. Appendix C.3 provides goodness of fit statistics. In general, the model performs reasonably well. Appendix C.4 details the estimated AR(2) coefficients. The excess return factors of the 10-year T-bond yield and TIPS yield are highly persistent, more than the excess return factor in the 3-month T-bill yield. The forecast error factor in the survey inflation forecast is also fairly persistent. We find low persistence in the temporary real interest rate and temporary inflation.

[7] Our dynamic factor model in the state space form offers the flexibility to model measurement errors in bond yields, similar to Pennacchi (1991). In comparison, Affine Term Structure Model (ATSM) is more restrictive in that the number of bonds subject to measurement errors is assumed to be the same as the number of underlying factors.

6 Major Findings

6.1 Correlations of Innovations in the State Factors

By examining the variance-covariance/correlation matrix Q of the state equation in table 1, we can establish the relationship among inflation, the real interest rate and survey inflation forecast. The variance-covariance terms are on and below the diagonal, while the correlation coefficients are above the diagonal.

Table 1

Variance-covariance/Correlation Matrix Q of the State Equation

	v_t^{rp}	$v_t^{\pi p}$	v_t^{γ}	v_t^{ra}	$v_t^{\pi a}$	$v_{3m,t}^{\xi a}$	$v_{10yr,t}^{\xi a}$	$v_{10yr,t}^{\lambda a}$	$v_{12m,t}^{ea}$
v_t^{rp}	0.4602	-0.85	0	0.71	-0.58	-0.49	-0.49	-0.98	0.87
$v_t^{\pi p}$	-0.3403	0.3514	0	-0.72	0.69	-0.04	0.06	0.81	-0.98
v_t^{γ}	0	0	0.0000	0	0	0	0	0	0
v_t^{ra}	0.7299	-0.6491	0	2.3229	-0.98	0.01	-0.09	-0.65	0.83
$v_t^{\pi a}$	-0.6235	0.6509	0	-2.3643	2.5124	-0.21	-0.07	0.52	-0.79
$v_{3m,t}^{\xi a}$	-0.0843	-0.0056	0	0.0042	-0.0862	0.0656	0.83	0.51	0.01
$v_{10yr,t}^{\xi a}$	-0.1170	0.0121	0	-0.0479	-0.0380	0.0758	0.1256	0.60	-0.05
$v_{10yr,t}^{\lambda a}$	-0.4536	0.3288	0	-0.6713	0.5629	0.0897	0.1441	0.4647	-0.82
$v_{12m,t}^{ea}$	0.1891	-0.1865	0	0.4035	-0.4030	0.0004	-0.0057	-0.1790	0.1029

An innovation in the permanent real interest rate is highly negatively correlated with innovations in both permanent inflation (-0.85) and temporary inflation (-0.58). In addition, an innovation in the temporary real interest rate is highly negatively correlated with innovations in both permanent inflation (-0.72) and temporary inflation (-0.98). These suggest that the real interest rate and inflation tend to move in opposite directions. An increase in the real interest rate typically coincides with a decrease in inflation. Or alternatively, an increase in inflation coincides with a decrease in the real interest rate, reflected by the nominal interest rate not increasing as much as inflation. So in general, the nominal interest rate doesn't adjust one for one in response to, but lags behind, the inflation movement. This is consistent with the findings of Barr and Campbell (1997) based on U.K. bond data, Pennacchi (1991) based on U.S. data, and other earlier studies such as Summers (1983).

Of particular interest is that an innovation in either permanent inflation or temporary inflation is highly negatively correlated (-0.98 and -0.79 respectively) with an innovation in the survey inflation forecast error factor. So a shock that increases inflation is cancelled out by a shock to the survey inflation forecast error factor in the opposite direction. This suggests that inflation survey participants are slow to adjust to changes in underlying actual inflation, which is consistent with our initial assumption that persistent forecast errors exist among survey participants. As pointed out earlier, the real interest rate and inflation tend to move in opposite directions. So it is not surprising that an innovation in either the permanent or the temporary real interest rate is highly positively correlated (0.87 and 0.83 respectively) with an innovation in the survey inflation forecast error factor.

Innovations in excess returns in the 3-month T-bill and 10-year T-bond are highly positively correlated (0.83). They are also positively correlated (0.51 and 0.60 respectively) with an innovation in the excess return in 10-year TIPS, although to a lesser extent than the correlation between themselves. This is consistent with the fact that short and long nominal bonds are subject to the same set of real interest risk and inflation risk, while TIPS are largely shielded from inflation risk.

Innovations in the permanent and temporary real interest rate are highly positively correlated (0.71). So a shock that increases the permanent real interest rate coincides with an increase in the temporary real interest rate. The real interest rate is likely to show high volatility. The correlation between innovations in permanent inflation and temporary inflation is also highly positive (0.69). So a shock that increases permanent inflation coincides with an increase in temporary inflation. Inflation, as a sum of the permanent, temporary and seasonal components, is likely to show high volatility too. Considering the data are of monthly frequency, it is not surprising that the real interest rate and inflation tend to show high volatility.

Even though we have imposed only the minimum restriction on matrix Q of no correlation between innovations in the seasonal factor of inflation and other factors, our model establishes reasonable relationships among major factors. Appendix C.5 performs a robustness check on these relationships, with additional restrictions that an innovation in the survey inflation forecast error factor is uncorrelated with innovations in other factors. Our findings on the relationships among major factors, particularly between inflation and the real interest rate, are untouched. However, with the additional restrictions, we lose the interesting finding that survey participants adjust to inflation changes with considerable inertia. So we prefer the current model in general.

6.2 Contributions of Factors to Observed Variables

Equation 30 estimates $\overline{Y}_t$, given the filtered state vector $S_{t|t}$, which is based on the information up to time t.

$$\overline{Y}_t = H \cdot S_{t|t} \tag{30}$$

$$\begin{pmatrix} \bar{i}_{3m,t} \\ \bar{i}_{10yr,t} \\ \bar{i}^{TIPS}_{10yr,t} \\ \overline{\pi}_t \\ \overline{\pi}^{survey}_{12m,t} \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{pmatrix} \cdot S_{t|t}$$

Now we can analyze the contributions of the state vector to each observed variable. Because our model uses demeaned data, all contributions discussed here need to be interpreted as made toward the observed variables relative to their means.

First, we look at contributions of the state vector to observed inflation. From (30), we have $\overline{\pi}_t = H_4 \cdot S_{t|t} = H_\pi \cdot S_{t|t}$, which can be decomposed into contributions of permanent, temporary and seasonal inflation to observed inflation π_t. Figure 1 indicates that the seasonal component explains only a minor part of π_t, when compared with permanent and temporary inflation combined. Figure 2 suggests that permanent inflation and temporary inflation tend to move together, which is consistent with the positive correlation between innovations in permanent and temporary inflation. This leads to high volatility of inflation, particularly in the post-1999 period in our sample. Temporary inflation explains more of this volatility than permanent inflation.

Then, we analyze the contributions of the state vector to interest rates. From (30), we have $\bar{i}_{3m,t} = H_1 \cdot S_{t|t}$, which can be decomposed into contributions of the real interest rate, inflation and excess return to the observed $i_{3m,t}$. From figure 3, we can see the 3-month T-bill yield's movement is largely due to the real interest rate, consistent with the finding by Mishkin (1990a) that, at the shorter end, the term structure of nominal interest rates contains a great deal of information about the real term structure. Meanwhile, contributions of the real interest rate and inflation tend to move in opposite directions. For example, immediately after the 9/11 terrorist attack, the real interest rate shot up while inflation dropped sharply, keenly reflecting the market sentiment then. This is consistent with our earlier finding of negative correlation between innovations in the real interest rate and inflation. Furthermore, the permanent real interest rate explains more of the 3-month T-bill yield's movement than the temporary real interest rate (Figure 4), while neither the permanent nor the temporary inflation seems to explain much of the variation in the 3-month T-bill yield (Figure

5). The latter is again consistent with Mishkin (1990a)'s finding that the term structure of nominal interest rates for maturities of six months or less provides little information about the future path of inflation.

From (30), we have $\bar{i}_{10yr,t} = H_2 \cdot S_{t|t}$, which can be decomposed into contributions of the real interest rate, inflation and excess return to the observed $i_{10yr,t}$. Figure 6 shows contributions of the real interest rate and inflation still tend to move in opposite directions. However, inflation's contribution to the 10-year T-bond yield is more pronounced than its contribution to the 3-month T-bill yield. This is consistent with Mishkin (1990a, 1990b) and Fama (1990)'s findings that there is substantial information in the longer maturity term structure about future inflation. The short term nominal interest rate mostly reflects the real interest rate fluctuation, while the long term nominal interest rate contains more information on underlying inflation. Inflation's contribution to the 10-year T-bond yield is mostly due to permanent inflation (Figure 7) and the real interest rate's contribution is largely due to the permanent real interest rate (Figure 8). In addition, a significant portion of the movement in the 10-year T-bond yield is explained by the idiosyncratic excess return factor. Notably, since 2001, the idiosyncratic factor has trended up (Figure 6).

Similarly, $\bar{i}^{TIPS}_{10yr,t} = H_3 \cdot S_{t|t}$ can be decomposed into contributions of the real interest rate, inflation and excess return to the observed $i^{TIPS}_{10yr,t}$. As Figure 9 shows, inflation's contribution is negligible. This is only natural, considering that the TIPS is shielded for the most part from price changes. The TIPS yield's movement is accounted for by the real interest rate (almost entirely due to the permanent real interest rate in turn) and the idiosyncratic excess return factor. This idiosyncratic factor has trended up since 2001, just as the case with the idiosyncratic factor in the 10-year T-bond yield.

Finally, we look at contributions of the state vector to the survey inflation forecast. From (30), we have $\bar{\pi}^{survey}_{12m,t} = H_5 \cdot S_{t|t}$, which can be decomposed into contributions of inflation and the idiosyncratic forecast error factor to the observed $\pi^{survey}_{12m,t}$. Figure 10 indicates there is a significant forecast error. It tends to move in the opposite direction of inflation, which is consistent with the negative correlation found between innovations to inflation and the forecast error factor. Figure 11 further suggests that permanent inflation carries more weight than temporary inflation in the survey inflation forecast.

6.3 Expected Inflation Based on Filtering

From (22) and (29), we derive the expected future inflation at $t+k$ based on the information available at t as

$$
\begin{aligned}
E_t\left[\pi_{t+k}\right] &= E_t\left[H_\pi S_{t+k} + w^\pi_{t+k}\right] \\
&= H_\pi \cdot F^k S_{t|t}
\end{aligned}
$$

So the average expected inflation for future n months[8] is defined as

$$
\pi^e_n = \left(\frac{1}{n}\right) H_\pi \sum_{k=1}^{n} F^k S_{t|t}
$$

Figure 12 shows that over our sample there is little difference between the expected future inflation at the 12-month horizon and that at the 10-year horizon. Figure 13 compares our measure of expected 12-month inflation with the 12-month inflation forecast from the University of Michigan consumer survey. Our estimated series follows the survey inflation forecast's general trend, but remains lower than the survey inflation forecast most of the time. Particularly, when there is a sharp downturn in the consumer survey forecast of 12-month inflation, our measure, which incorporates additional bond market information, tends to predict an even steeper decline in future inflation. For example, it showed a greater drop in the aftermath of 9/11, and again in mid 2002, than the survey inflation forecast. These suggest survey participants may be slow to adjust their forecast, consistent with our finding of negative correlation between innovations to inflation and the survey forecast error. Figure 14 compares our measure of 10-year expected inflation based on the information at time t with the inflation compensation measure, i.e., the yield spread between 10-year nominal T-bond and 10-year TIPS. Contrary to the case at the 12-month horizon, our measure of 10-year expected inflation has been generally higher than inflation compensation since 1998. Again, it picked up a much sharper downturn than the inflation compensation measure did after 9/11 and in mid 2002. In general, our estimated expected inflation seems to be more sensitive to negative shocks to the economy than both the inflation survey forecast and inflation compensation suggest.

6.4 Inflation Risk Premium vs. Liquidity Risk Premium

From (9), inflation compensation at the 10-year horizon is

$$
i_{10yr,t} - i^L_{10yr,t} = \left(\frac{1}{120}\right) \log\left(\frac{P_t}{P_{t-3}}\right) + \left(\frac{1}{120}\right) \sum_{i=1}^{120-3} E_t\left[\pi_{t+i}\right] + \left(\xi_{10yr,t} - \lambda_{10yr,t}\right)
$$

[8] Because the model estimation is done with demeaned data on observed variables, we need to add back the sample mean of monthly inflation to get the correct measure of expected inflation for future n months.

From (10), $\left(\xi_{10yr,t} - \lambda_{10yr,t}\right)$ is dominated by the risk premium due to inflation uncertainty in the 10-year lifetime of the bonds. Thus it can be treated as a measure of inflation risk premium, denoted by $\varkappa_{10yr,\ t}$. This is given by

$$\varkappa_{10yr,\ t|t} = \left(i_{10yr,t} - i_{10yr,t}^{TIPS}\right) - \left(w_{10yr,t|t}^{i} - w_{10yr,t|t}^{TIPS}\right) - \left(\frac{1}{120}\right)\sum_{i=1}^{120-3}\left(H_{\pi} \cdot F^{i} S_{t|t}\right) \tag{31}$$

with the means of $i_{10yr,t}$, $i_{10yr,t}^{TIPS}$ and π_t added back correspondingly.

For part of the sample period, the estimated series is actually negative, as displayed in Figure 15. Normally, 10-year T-bonds trade at a premium over TIPS beyond expected inflation because nominal T-bonds need to compensate for inflation risks due to the uncertainty in future inflation, while TIPS are largely shielded from inflation risks. If $\varkappa_{10yr,\ t|t}$ is attributed solely to the inflation risk premium, it can not assume negative values. However, as McCulloch and Kochin (2000), Sack (2000, 2004), Shen and Corning (2001), D'Amico, Kim and Wei (2007) point out, TIPS tend to be less liquid than nominal T-bonds, particularly when investors are not familiar with TIPS. This suggests $\lambda_{10yr,t}$ contains a liquidity risk premium in the 10-year TIPS relative to the 10-year T-bond. Then $\varkappa_{10yr,\ t|t}$ is actually made up by two parts: the inflation risk premium of the 10-year t-bond over the TIPS and the liquidity risk premium of the 10-year TIPS over the T-bond. The former takes a positive value, while the latter a negative value. In fact, our earlier finding that the 10-year expected inflation has been generally higher than inflation compensation since 1998 mirrors the liquidity risk premium.

Unfortunately, without imposing further structural assumptions, we cannot pin down either the inflation risk premium or the liquidity risk premium individually. Nonetheless, we may still derive some insights based on the current result. From Figure 15, we can see that $\varkappa_{10yr,\ t|t}$ remained positive in the early sample period, roughly from 1997 to mid 1998. This suggests the inflation risk premium on the T-bond dominated the liquidity risk premium on the TIPS. Then $\varkappa_{10yr,\ t|t}$ turned mostly negative from mid 1998 to 2002, suggesting the liquidity risk premium on the TIPS exceeded the inflation risk premium on the T-bond. If we assume there has been no dramatic change in the liquidity risk premium on the TIPS since 1997, then it means the inflation risk has been significantly reduced in the same period. The only exception is that right after 9/11, $\varkappa_{10yr,\ t|t}$ saw a sudden increase before quickly returning to the negative territory. This could be explained by either a jump in the inflation risk premium, or by a drop in the liquidity risk premium. The former is consistent with the large variation we have estimated in underlying inflation in the 9/11 aftermath. The latter is consistent with investors shifting to TIPS from T-bonds or other financial instruments during a possible flight to safety. The fact that $\varkappa_{10yr,\ t|t}$ came down quickly afterwards suggests that people's perception of risks returned to normal in a short period of time. Only in 2003, did $\varkappa_{10yr,\ t|t}$ turn positive. This suggests that the inflation risk premium may have increased in recent years. Another possibility is that the liquidity risk premium may have declined (D'Amico, Kim and Wei, 2007).

One way to limit the influence of the TIPS' liquidity risk premium on asset pricing equations is to use the 10-year off-the-run T-bond yield instead of the normal 10-year T-bond yield. Off-the-run T-bonds are those which are no longer the most recently issued by the Treasury. They have lower liquidity than on-the-run nominal t-bonds. So the 10-year off-the-run T-bond yield is more comparable with the 10-year TIPS yield in terms of liquidity. McCulloch and Kochin (2000), Sack (2000, 2004) and Shen and Corning (2001) all explore this point. We re-estimate the model using the 10-year off-the-run T-bond yield. The results are largely unchanged. Figure 16 shows the re-estimated $\varkappa_{10yr,\ t|t}$. It is still negative in part of the sample period. So using the 10-year off-the-run T-bond yield doesn't seem to eliminate the liquidity risk premium. However, the re-estimated $\varkappa_{10yr,\ t|t}$ does tend to rise above zero more often in the late sample period, suggesting the liquidity risk premium's magnitude may be smaller. In conclusion, the 10-year TIPS yield contains a significant liquidity risk premium, whether compared with the normal on-the-run or the off-the-run 10-year T-bond yield.

6.5 Expected Real Interest Rate

We calculate the expected real interest rate within the sample period. From (12), (13), (14) and (22), the expected real interest rate at $t+k$ given the information available at t is

$$\begin{aligned} E_t\left[r_{t+k}\right] &= E_t\left[H_r S_{t+k}\right] \\ &= H_r F^k S_{t|t} \end{aligned}$$

This is based on demeaned data, so we can interpret $E_t\left[r_{t+k}\right]$ as relative to its mean. Figure 17 displays the 1-month real interest rate (ex ante) $E_t\left[r_{t+1}\right] = H_r F S_{t|t}$, which shows high volatility. Still it clearly has trended down since the end of 2000. It experienced a sudden but short-lived spike following 9/11, reflecting the prevailing pessimistic market sentiment at that time, before quickly resuming its downward movement. To smooth out the volatility, we also include in Figure 17 the average 12-month real interest rate, defined as

$$\begin{aligned} r_{12}^{e} &= \left(\frac{1}{12}\right)\sum_{k=1}^{12} E_t\left[r_{t+k}\right] \\ &= \left(\frac{1}{12}\right) H_r \sum_{k=1}^{12} F^k S_{t|t} \end{aligned}$$

We can see more clearly the downward movement after yearend 2000, till yearend 2003. Much of this period corresponds to the period in which the Federal Reserve cut the benchmark short term nominal interest rate to an unprecedented low level[9]. The lowering of the real interest rate may also be connected with the so-called global saving glut (Bernanke, 2005). Interestingly, our sample period coincides roughly with the sample period that Bernanke originally used to analyze the global saving glut and its economic and policy implications for the U.S.

6.6 Expected Nominal Term Premium Based on Filtering

The expected nominal term premium is defined as the excess return of investing in a 10-year T-bond (held to maturity) relative to rolling over a 3-month T-bill consecutively in the 10-year period, all based on the information available at time t. The former implies simply yield $i_{10yr,t}$. The latter is equivalent to 40 investments spread out at time t, $t+3$, ... $t+117$[10]. Using (22) and (29), the implied yield is calculated by

$$\begin{aligned} i_{3m,t}^{40} &= \left(\frac{1}{40}\right)\left\{E_t\left[i_{3m,t}\right] + E_t\left[i_{3m,t+3}\right] + ... + E_t\left[i_{3m,t+117}\right]\right\} \\ &= \left(\frac{1}{40}\right)\left[i_{3m,t} + H_1(F^3 + ...F^{117})S_{t|t}\right] \end{aligned}$$

Figure 18 compares our estimated expected nominal term premium with the simple observed (ex post) term spread between the 10-year T-bond and the 3-month T-bill. Our estimated nominal term premium follows the actual observed term spread's general trend closely.

[9] Between January 2001 and June 2003, the Federal Open Market Committee cut the Federal Funds target rate 13 times consecutively, from 6.5 percent to 1.0 percent. It was not until June 30, 2004, that the Federal Reserve raised the target for the first time in the current cycle, from 1.0 to 1.25 percent.

[10] We also need to add the sample mean of $i_{3m,t}$ back in order to produce the actual expected yield of rolling over a 3-month t-bill for 10 years.

7 Forecasting Inflation and Real Interest Rate

7.1 Simple Out-of-sample Forecast

We conduct out-of-sample forecast of the real interest rate following Koopman, Shephard and Doornik (1999, 2002). Starting from the end of the sample period, denoted by time T, the forecast of the state vector can be computed recursively by

$$E_T\left[S_{T+i+1}\right] = F \cdot E_T\left[S_{T+i}\right]$$

and the conditional variance based on information at time T by

$$Var_T\left[S_{T+i+1}\right] = F \cdot Var_T\left[S_{T+i}\right] \cdot F' + Q$$

for $i = 1, 2, ...$, with $Var_T\left[S_{T+1}\right] = P_{T+1|T}$ and $E_T\left[S_{T+1}\right] = F \cdot S_{T|T}$ obtained by the Kalman filter at T. Then the forecast of the real interest rate is simply

$$E_T\left[r_{T+i}\right] = H_r \cdot E_T\left[S_{T+i}\right]$$

and the variance conditional on information at time T is

$$Var_T\left[r_{T+i}\right] = H_r \cdot Var_T\left[S_{T+i}\right] \cdot H_r'$$

for $i = 1, 2, ...$ Figure 19 shows the out-of-sample forecast for 12 months, from January to December 2004, with the confidence band based on the conditional standard deviation. It remains significantly negative in the first 3 months. It remains negative for the rest of the forecast period, although the confidence band contains the zero line.

Similar to the forecast of the real interest rate, the forecast of inflation can be calculated from

$$E_T\left[\pi_{T+i}\right] = H_\pi \cdot E_T\left[S_{T+i}\right]$$

and the variance conditional on information at time T from

$$Var_T\left[\pi_{T+i}\right] = H_\pi \cdot Var_T\left[S_{T+i}\right] \cdot H_\pi' + Var(w_t^\pi)$$

Figure 20 indicates that the out-of-sample forecast of inflation for 12 months, from January to December 2004, has a stable outlook as of yearend 2003. Because of the relatively large observation error, the forecast is much less volatile than in-sample observations.

7.2 One-step-ahead Forecast on Inflation

There have been mixed evidences on different economic indicators' capability to forecast inflation out of sample. Ang, Bekaert and Wei (2007) provide an extensive summary of this topic before reaching their own conclusion that surveys outperform others. Our model in this paper offers the opportunity to produce a model-based inflation forecast, combining the information from the bond market, survey inflation forecast and actual realized inflation. Specifically, we perform one-step-ahead out-of-sample forecasts on inflation based on the estimated state space model, incorporating new information available at each step. The forecasting period starts in January 2004 and ends December 2005. The end of the in-sample period, T, is extended by one month at a time. However, the state space model is not re-estimated. It remains the same as being already done based on the sample from 1997 to 2003. Instead, the Kalman filter is applied to the newly expanded sample and a one-step ahead forecast of inflation is calculated by

$$\begin{aligned} E_T\left[\pi_{T+1}\right] &= H_\pi \cdot E_T\left[S_{T+1}\right] \\ &= H_\pi \cdot F \cdot S_{T|T} \end{aligned}$$

The alternative approaches are either a simple out-of-sample multi-step forecast, which we have already done in the previous subsection, or a one-step ahead forecast based on a repeatedly estimated model with each new data observation (recursive forecasts and rolling regressions). Considering the estimation cost involved in the latter, we think our current approach is sensible in evaluating the model's forecasting capability, while provides more sophisticated analysis compared with the former.

We collect the one-step ahead forecasts over the chosen forecasting period in a time series. We analyze the forecast validity of our model by comparing these one-step ahead forecasts of inflation with the actual out-of-sample realized inflation. The RMSE is 4.696. Next, we pit our model against the Box Jenkins model, which has proven quite successful in univariate time series forecasting in practice. We fit the inflation series with a parsimonious seasonal AR(2) and conduct a similar one-step-ahead forecast using continuously updated data, but without re-estimating the model. This seasonal AR(2) yields forecasts with RMSE of 4.742. Our model seems to be doing a little better than the seasonal AR(2) in one-step ahead forecast.

8 Concluding Remarks

In this paper, we use yields on both U.S. nominal T-bonds and TIPS, together with consumer survey inflation forecast and actual realized inflation, to extract information on the underlying inflation and real interest rate processes. The estimated inflation expectation is generally lower than the consumer survey inflation forecast at the 12-month horizon. However, it turns out to be higher than the inflation compensation measure at the 10-year horizon in part of the sample period. Corrected for observation errors, this finding implies that the risk premium, measured by the difference between the excess returns on the 10-year T-bond and the TIPS, is not entirely due to inflation risk. Instead, the risk premium is likely a combination of the inflation risk premium embodied in the T-bond yield and the liquidity risk premium embodied in the TIPS yield. When we fit the model using the off-then-run T-bond yield to better match the TIPS' liquidity, the result still suggests the existence of a liquidity risk premium, but maybe to a lesser extent.

One important feature of the model is that we allow interdependence not only between the real interest rate and inflation, but among other factors. The empirical results confirm that innovations in the real interest rate and inflation are strongly negatively correlated, in terms of both permanent and temporary components. The nominal interest rate adjustment lags inflation changes. These results are robust, whether or not we allow for correlations between the innovation in the survey inflation forecast error factor and innovations in other underlying factors. If we allow for such correlations, we find innovations in the survey inflation forecast error factor and inflation are negatively correlated, suggesting that survey participants adjust their forecast of future inflation only gradually in response to underlying inflation movement.

The in-sample estimated real interest rate has trended down since the end of 2000. The out-of-sample forecast of the real interest rate suggests it would remain below its mean as of yearend 2003, while the out-of-sample forecast of inflation shows stability. Our model outperforms a parsimonious seasonal AR(2) time series model in one-step ahead forecasts of inflation, albeit with a small margin.

We conclude by noting several possible avenues for future research. First, our model is hampered by the short sample period available for the TIPS yield. This problem will be gradually alleviated as the number of TIPS issued and outstanding increases, and longer TIPS yield series become available. A longer sample period may also reveal more low frequency movements in the inflation rate, which will help to improve the fitting of the observation equation on inflation. Secondly, we can explore the model's forecasting capability more vigorously, particularly in real time recursive forecasts. Thirdly, the method we develop in this paper may also be applied to studies of other economies, possibly with more established indexed bond markets. Finally, there has been a rapidly growing effort to connect the macroeconomic literature with the term structure literature in finance. It would be worthwhile to compare our model with, and maybe incorporate elements from, these new macro-term structure models, in studying the inflation dynamics.

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Figure 1: The Seasonal Component Explains

Only a Minor Part of Inflation

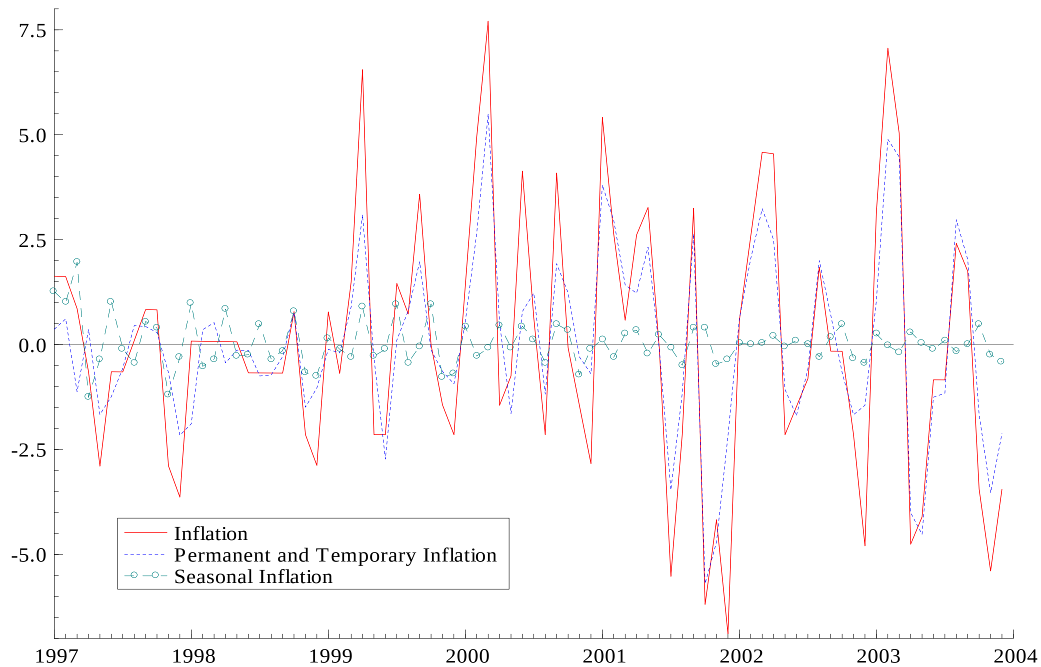


Figure 2: Permanent and Temporary Inflation

Tend to Move Together

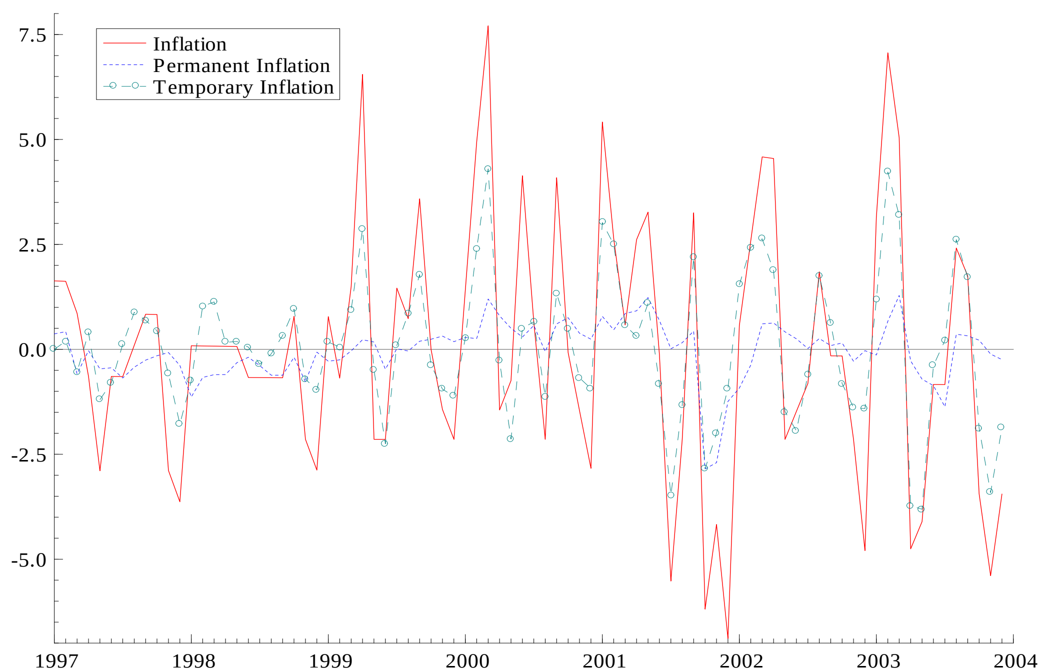


Figure 3: 3-Month T-bill's Movement Is Largely Due to Real Interest Rate

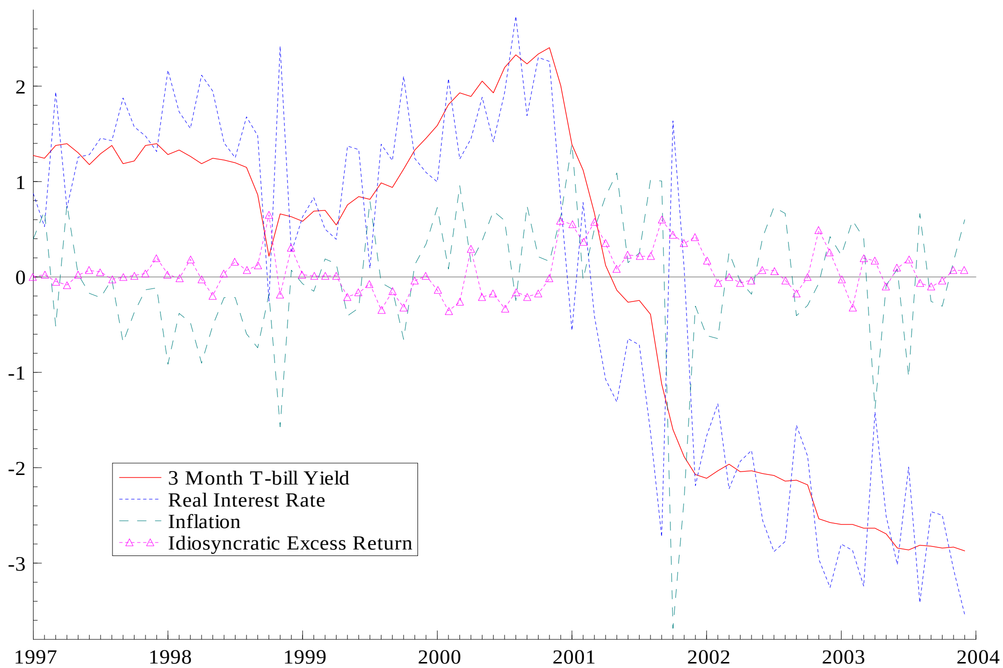


Figure 4: Permanent Real Interest Rate Accounts for More of 3-Month T-bill Yield than Temporary Real Interest Rate

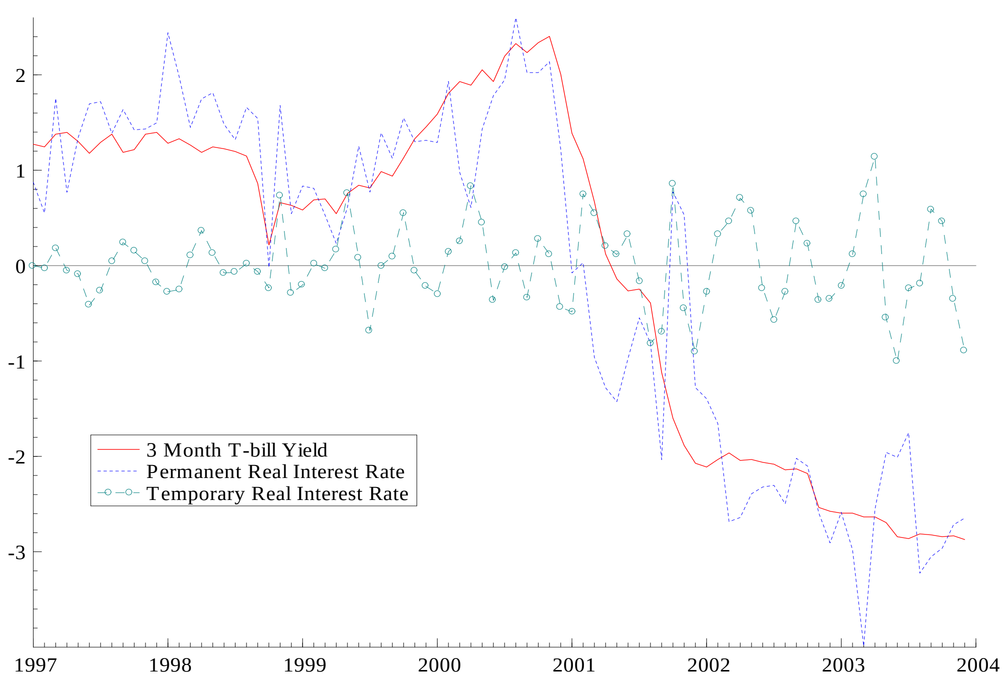


Figure 5: Neither Permanent nor Temporary Inflation

Explains Much of the 3-Month T-bill Yield

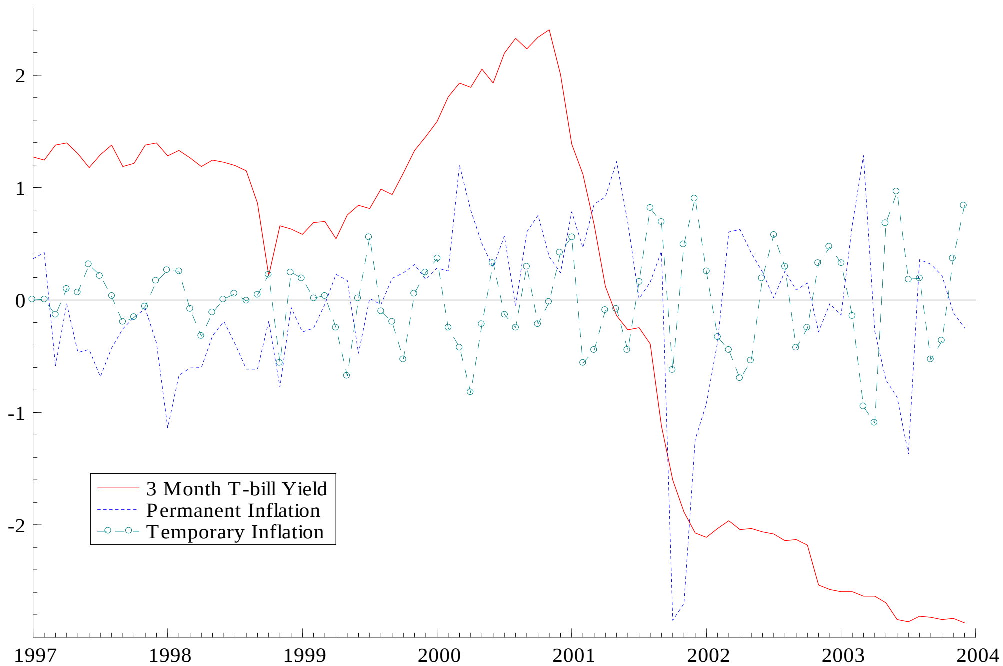


Figure 6: Inflation's Contribution to the 10-Year T-bond

Is More Pronounced

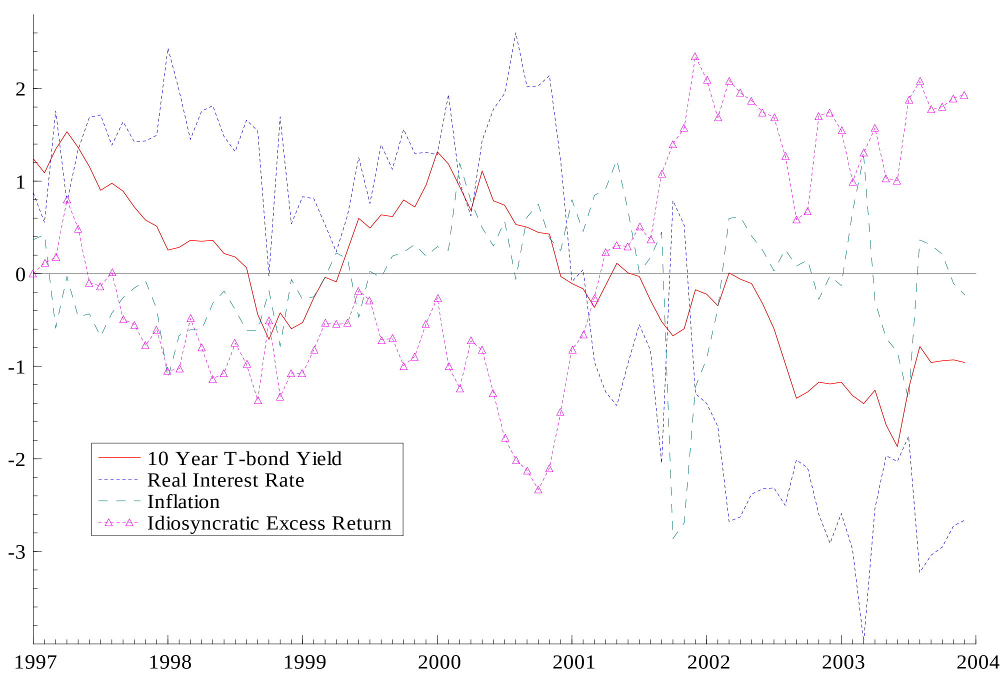


Figure 7: Permanent Inflation Dominates Temporary Inflation

in Explaining the 10-Year T-bond Yield

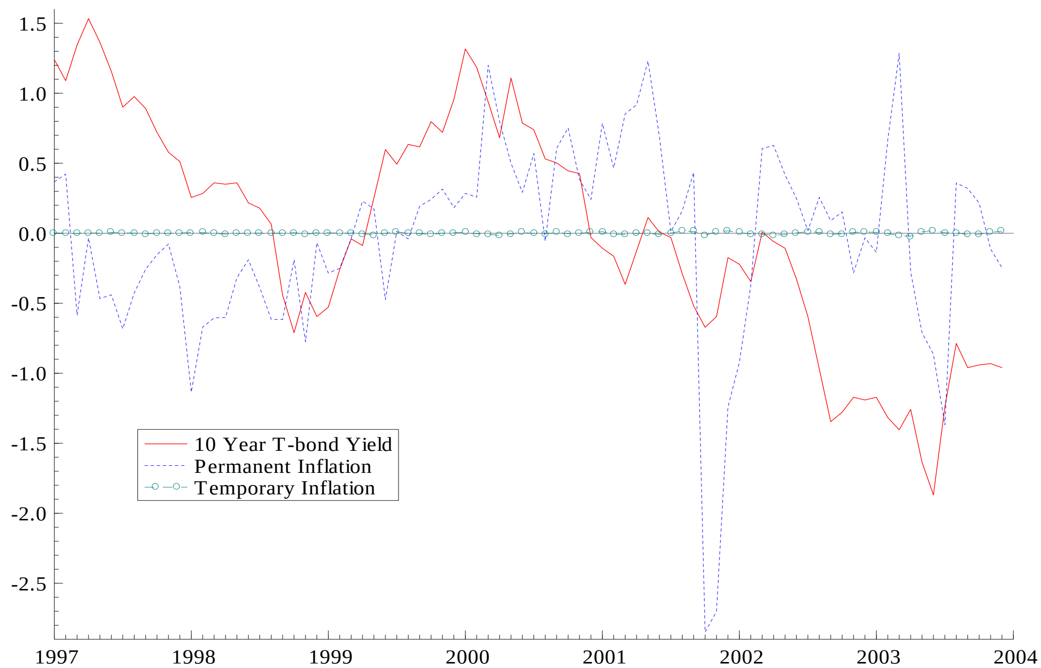


Figure 8: Permanent Real Interest Rate Dominates Temporary Real Interest Rate

in Explaining the 10-Year T-bond Yield

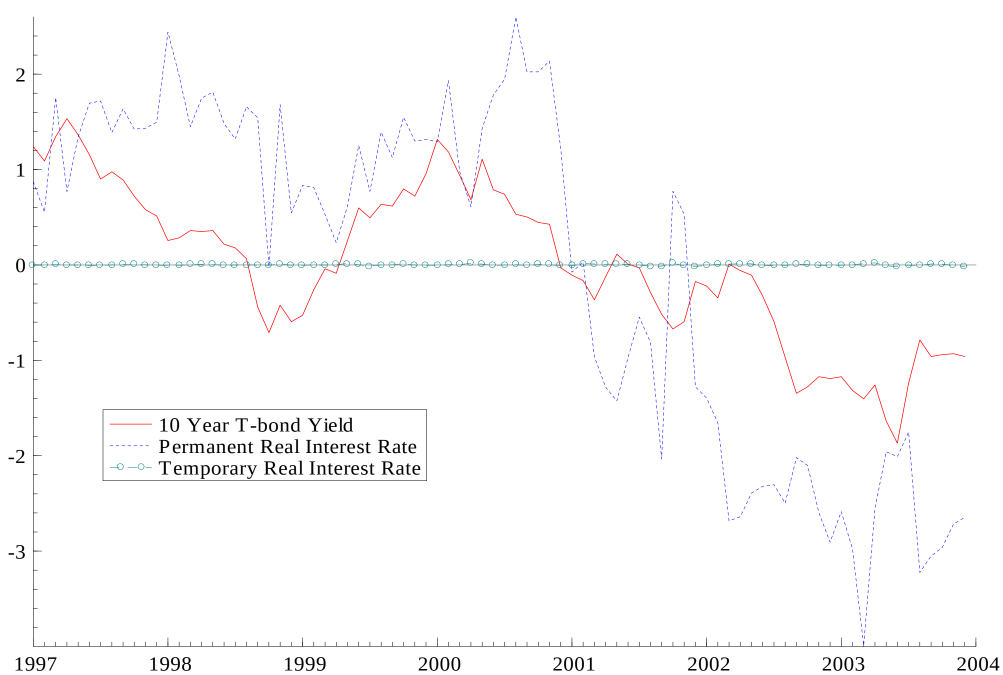


Figure 9: The 10-Year TIPS Yield is Almost Entirely Determined by the Real Interest Rate and Excess Return

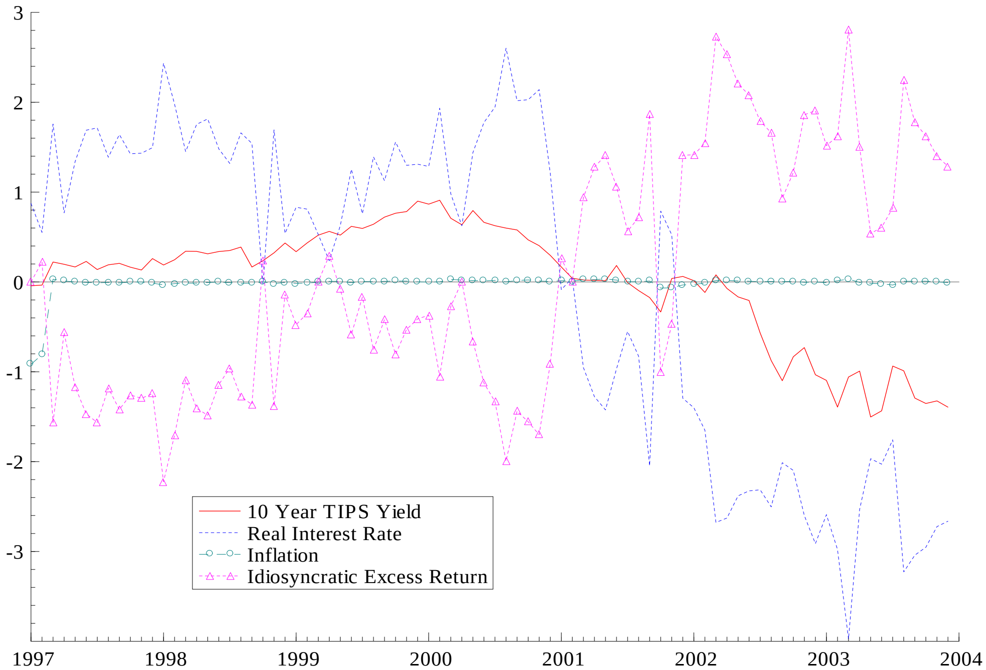


Figure 10: There Is a Significant Forecast Error Involved in the Consumer Survey Inflation Forecast

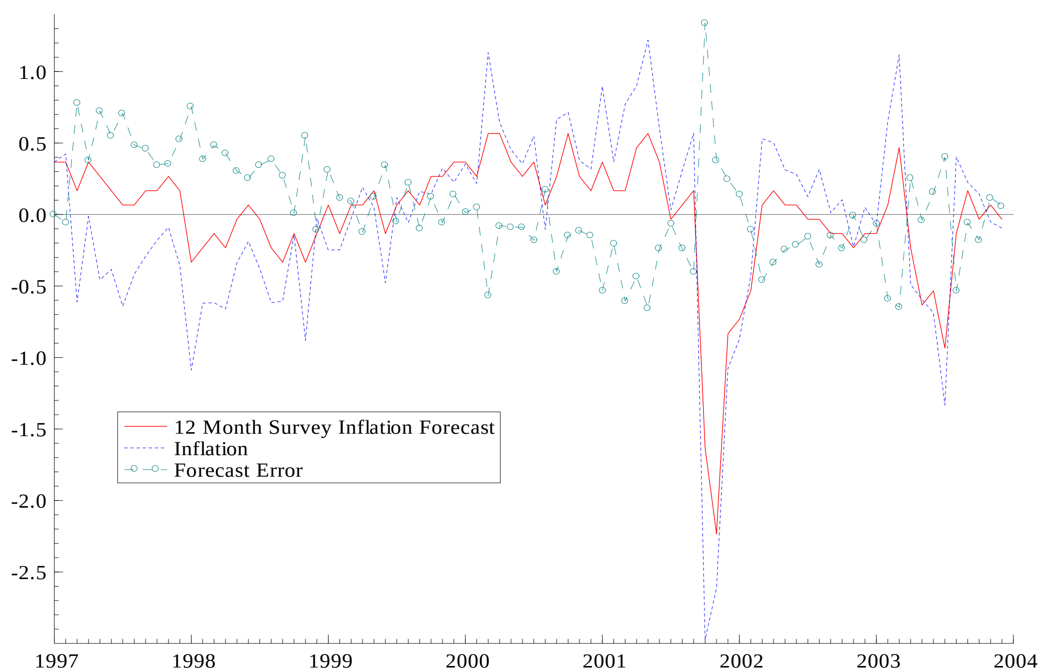


Figure 11: Permanent Inflation Carries More Weight than Temporary Inflation in the Survey Forecast

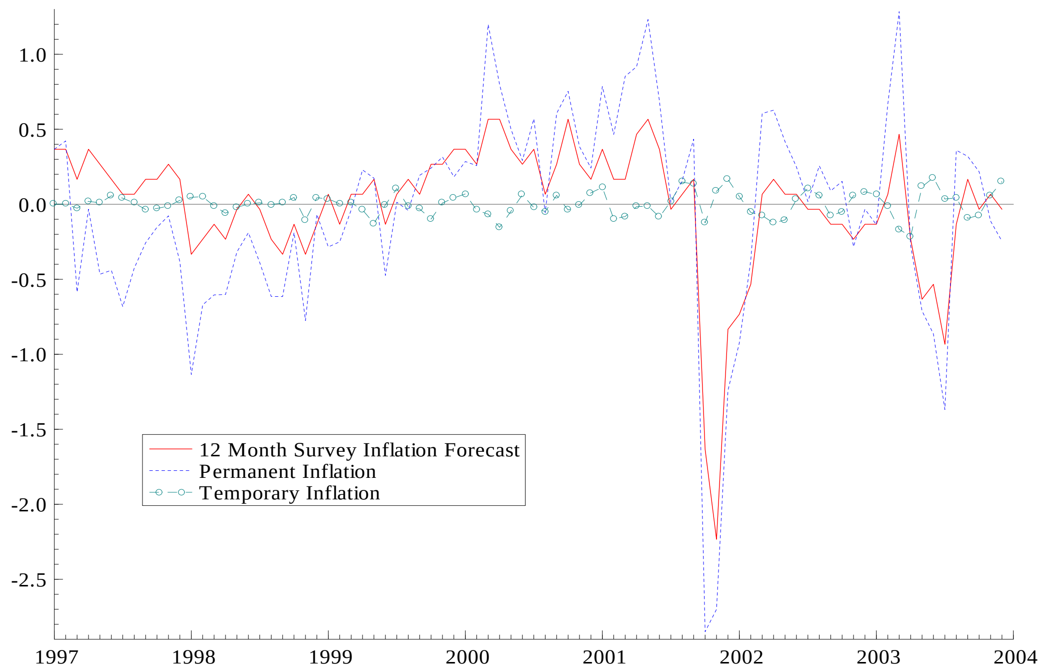


Figure 12: 12-Month and 10-Year Expected Inflation Appear Similar

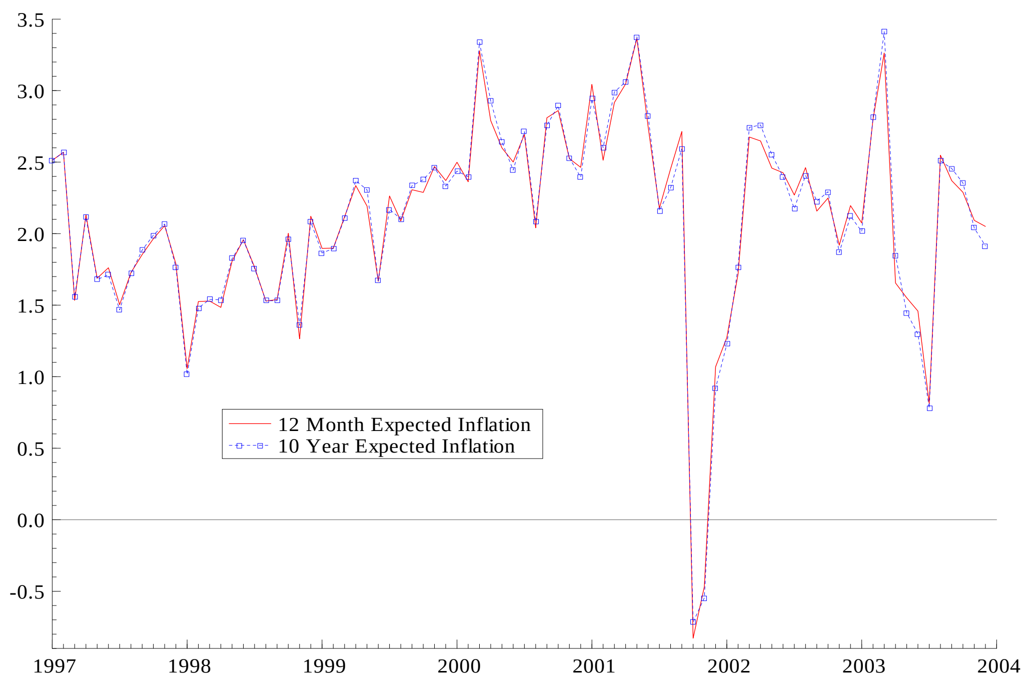


Figure 13: 12-Month Expected Inflation is Generally Lower than the Consumer Survey Inflation Forecast

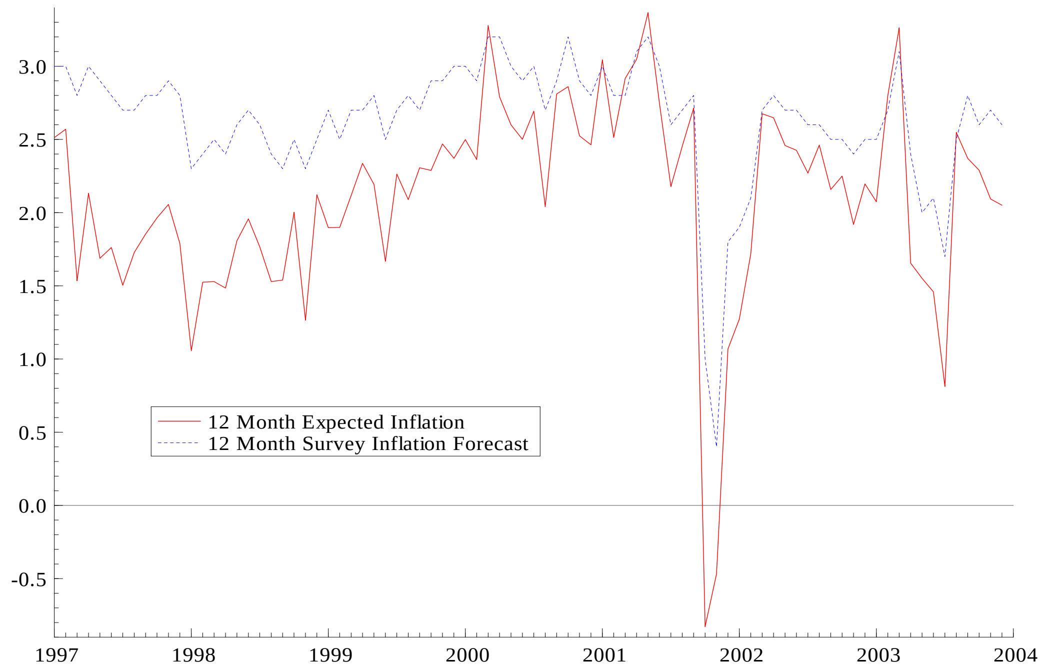


Figure 14: 10-Year Expected Inflation is Mostly Higher than the Inflation Compensation Measure

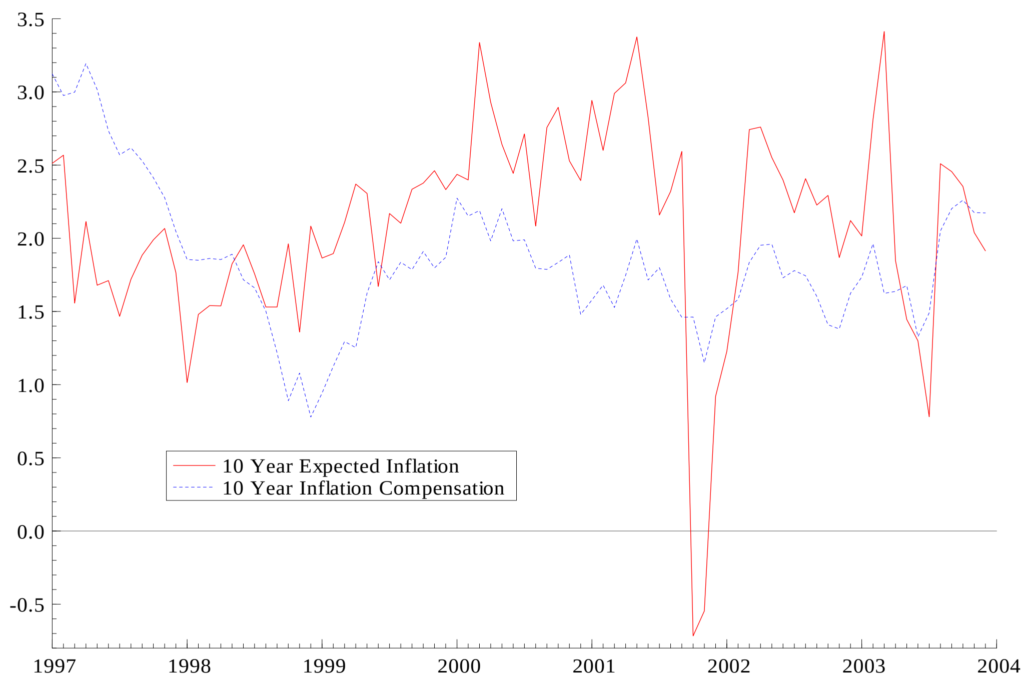


Figure 15: The Difference of Excess Returns in 10-Year T-bond and TIPS Suggests Liquidity Risk Premium

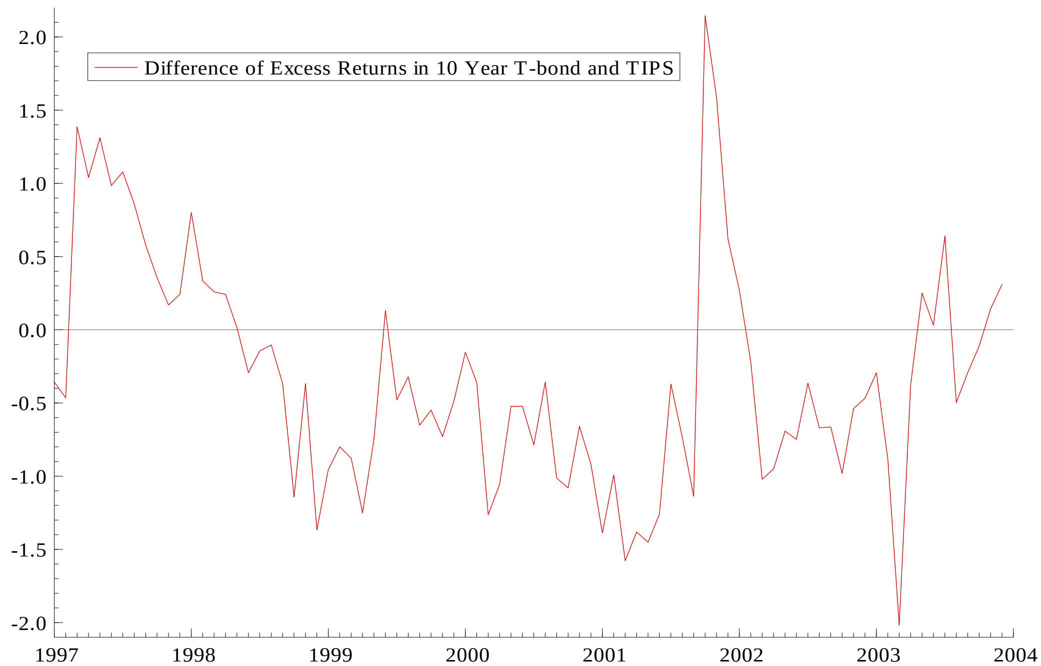


Figure 16: Using Off-the-run T-bond Yield Doesn't Eliminate the Liquidity Risk Premium

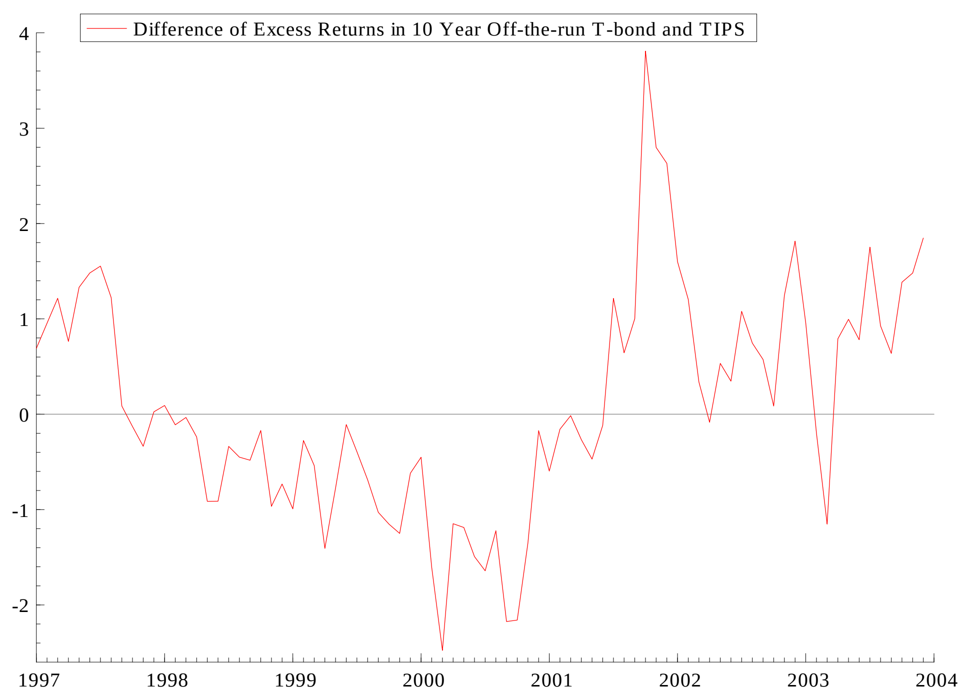


Figure 17: Expected Real Interest Rate Has

Trended Down since 2001

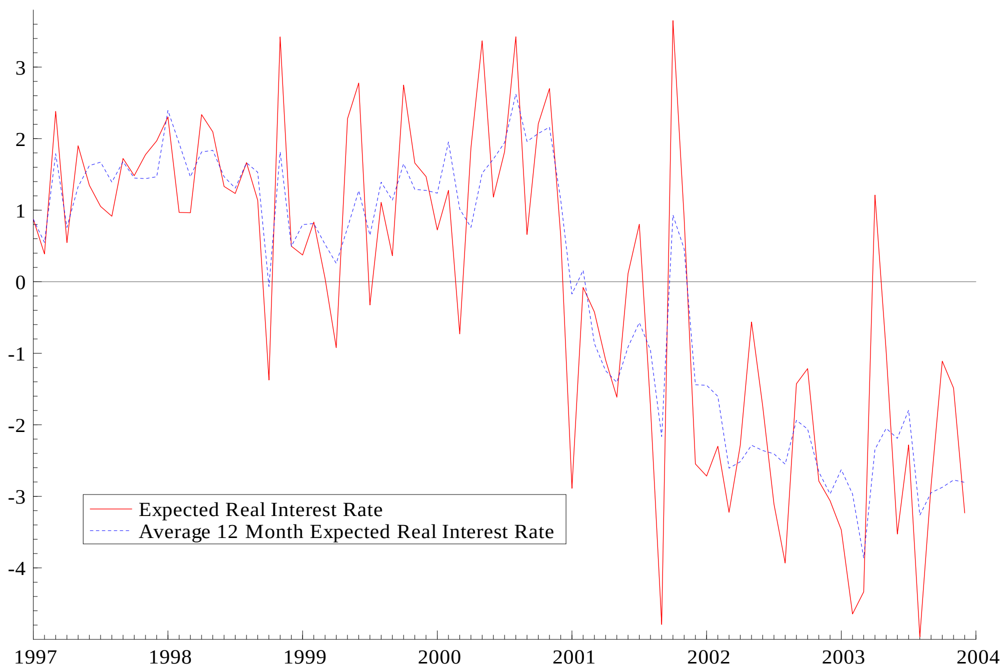


Figure 18: Expected Nominal Term Premium Resembles

the Realized 10-Year 3-Month Spread

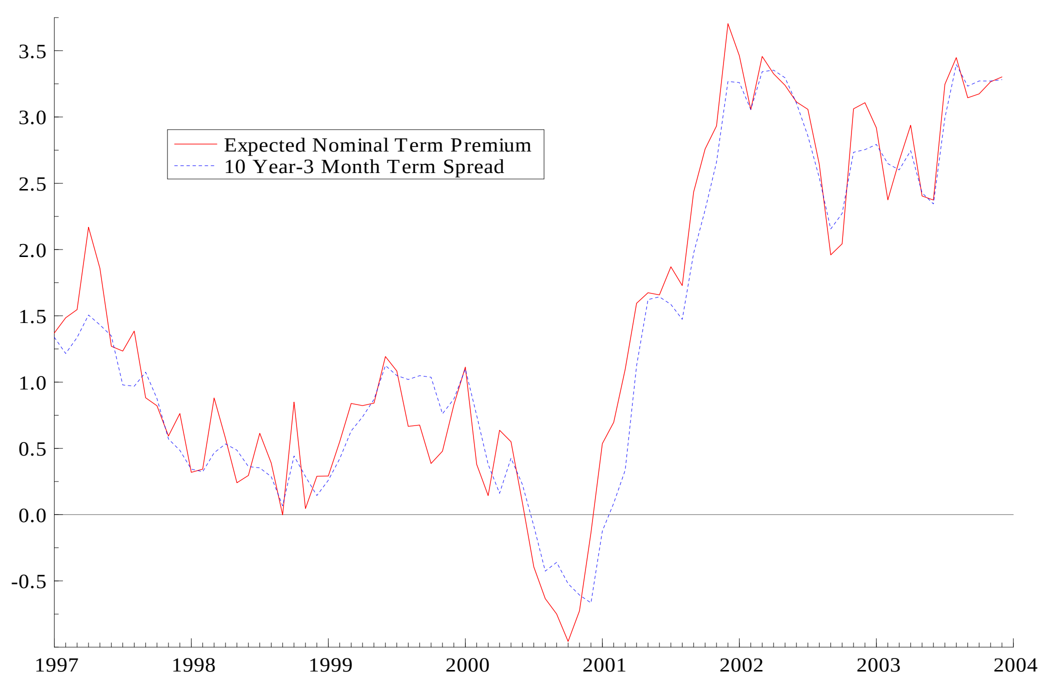


Figure 19: The Real Interest Rate is Predicted to

Remain Low at Yearend 2003

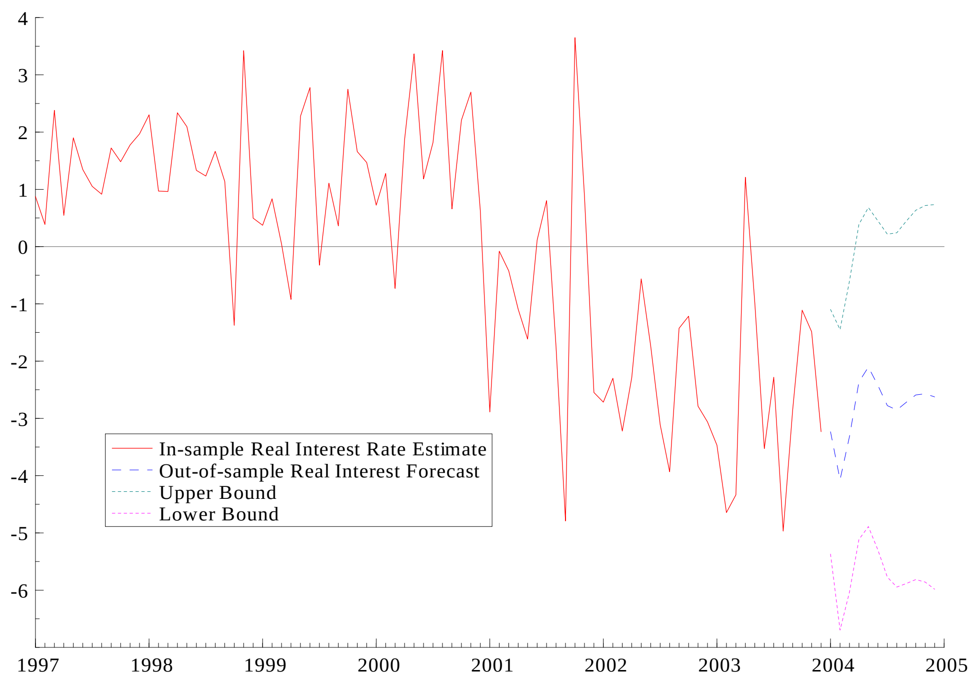
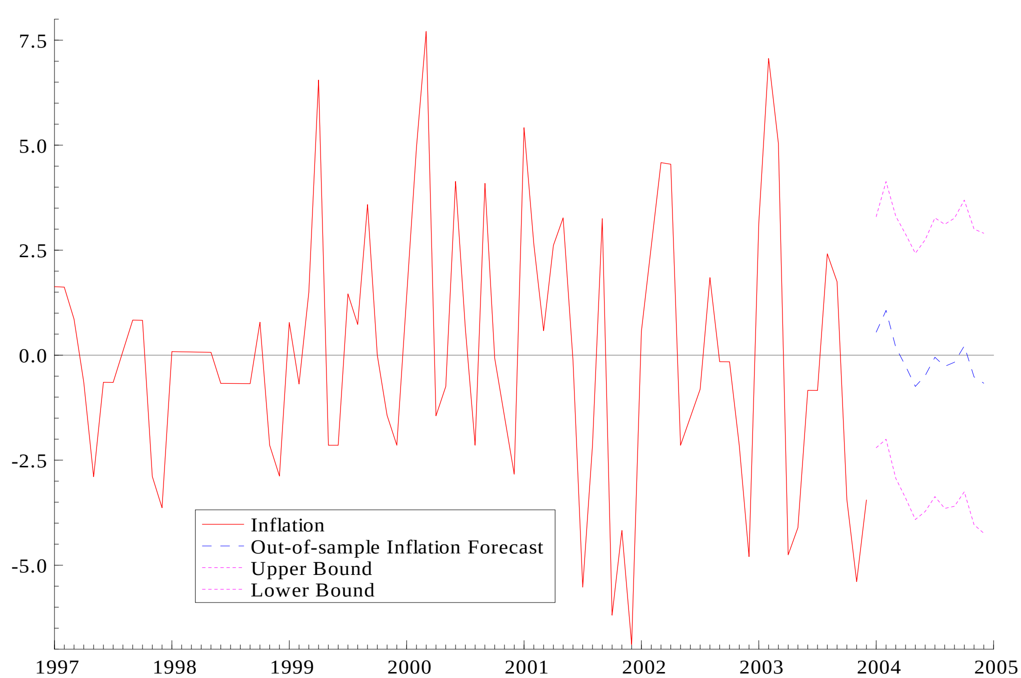


Figure 20: Looking Ahead, Inflation is Likely to

Remain Stable at Yearend 2003



A Data Appendix

A.1 Nominal T-bond Yield

Both the 3-month T-bill yield and the 10-year T-bond yield are from the Federal Reserve Board of Governors' Statistical Release H.15—Selected Interest Rate. The 10-year off-the-run T-bond yield is also from the Federal Reserve Board of Governors' estimate. Data are monthly, quoted in annual rates in percentage points.

A.2 TIPS Yield

The 10-year TIPS yield is based on the archive of the U.S. real term structure estimated by J. H. McCulloch[11]. McCulloch and Kochin (2000) explains the methodology in detail. Data are monthly, quoted in annual rates in percentage points.

Starting from January 2, 2004, the U.S. Treasury Department began publishing daily Treasury real yield curve rates. They are also reported in the Federal Reserve Board of Governors' Statistical Release H.15. However, monthly data only starts from January 2003.

A.3 Survey Inflation Forecast

The survey inflation forecast is from Surveys of Consumers conducted by the University of Michigan. The actual survey question is "By what percent do you expect prices to go up/down, on the average, during the next 12 months?" as described in Curtin (1996). So we interpret the answer to this question as a forecast of

$$\left(\frac{1}{12}\right) \cdot 100 \cdot \left[12 \cdot \log\left(\frac{P_{t+1}}{P_t}\right) + 12 \cdot \log\left(\frac{P_{t+1}}{P_t}\right) + ... + 12 \cdot \log\left(\frac{P_{t+12}}{P_{t+11}}\right)\right]$$

where P_t is the price level at t. We can see it is the same as

$$100 \cdot \log\left(\frac{P_{t+12}}{P_t}\right).$$

In other words, the average month-over-month inflation (in annual rate) for 12 months is the same as the year-over-year inflation. As in Pennacchi (1991), we use the median survey response to prevent extreme values from contaminating the survey result.

[11] "The US Real Term Structure Of Interest Rates", http://economics.sbs.ohio-state.edu/jhm/ts/ts.html

A.4 Inflation

We use non-seasonally adjusted Consumer Price Index (CPI) as the price measure all over the model because it is the basis for TIPS indexation. Data is from the Bureau of Labor Statistics. The month-over-month inflation rate (in annual rate) is calculated as

$$100 \cdot 12 \cdot \log\left(\frac{CPI_t}{CPI_{t-1}}\right)$$

B Bond Price Appendix

We keep the same equation number as in the text for convenience in the discussion below. The yield to maturity of an n-period fully indexed bond is

$$i^R_{n,t} = \left(\frac{1}{n}\right)\sum_{i=1}^{n} E_t\left[r_{t+i}\right] - \left(\frac{1}{n}\right)\sum_{i=j+1}^{n}\sum_{j=1}^{n} Cov_t\left[m_{t+i}, m_{t+j}\right] \tag{4}$$

where $\left(\frac{1}{n}\right)\sum_{i=1}^{n} E_t\left[r_{t+i}\right]$ is the average expected one-period real interest rate over the bond's lifetime and $-\left(\frac{1}{n}\right)\sum_{i=j+1}^{n}\sum_{j=1}^{n} Cov_t\left[m_{t+i}, m_{t+j}\right]$ the real term premium.

The yield to maturity of an n-period TIPS is

$$\begin{aligned} i^L_{n,t} = {} & \left(\frac{1}{n}\right)\sum_{i=1}^{n} E_t\left[r_{t+i}\right] - \left(\frac{1}{n}\right)\sum_{i=j+1}^{n}\sum_{j=1}^{n} Cov_t\left[m_{t+i}, m_{t+j}\right] - \\ & \left(\frac{1}{n}\right)\log\left(\frac{P_t}{P_{t-3}}\right) + \left(\frac{1}{n}\right)\sum_{i=1}^{3} E_t\left[\pi_{t+n+1-i}\right] - \\ & \left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{3}\pi_{t+n+1-i}\right] - Cov_t\left[\sum_{i=1}^{n} m_{t+i}, \sum_{i=1}^{3}\pi_{t+n+1-i}\right]\right\} \end{aligned} \tag{6}$$

Comparing the yield of an n-period TIPS in (6) with the yield of an n-period real bond in (4), we can see they both contain the average expected short real interest rate $\left(\frac{1}{n}\right)\sum_{i=1}^{n} E_t\left[r_{t+i}\right]$, and the real term premium $-\left(\frac{1}{n}\right)\sum_{i=j+1}^{n}\sum_{j=1}^{n} Cov_t\left[m_{t+i}, m_{t+j}\right]$.

The difference is also evident. Unlike a real bond, a TIPS with its explicit indexation lag is not fully shielded from the inflation risk. The TIPS yield contains $\left(\frac{1}{n}\right)E_t\left[\sum_{i=1}^{3}\pi_{t+n+1-i}\right]$, which covers expected inflation in the indexation lag period between $t+n-3$ and $t+n$. This term tends to put an upward bias on the TIPS yield. This bias is corrected by $-\left(\frac{1}{n}\right)\log\left(\frac{P_t}{P_{t-3}}\right)$, which represents the existing price change between $t-3$ and t. As long as n is much larger than 3, the net should be small in value[12].

[12] A special case is that if inflation turns out to be constant, $\left(\frac{P_{t+n-3}}{P_{t+n}}\right)$ and $\left(\frac{P_t}{P_{t-3}}\right)$ will cancel out each other. Then the TIPS is fully indexed and becomes a real bond.

The TIPS yield contains $-\left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{3}\pi_{t+n+1-i}\right]-Cov_t\left[\sum_{i=1}^{n}m_{t+i},\sum_{i=1}^{3}\pi_{t+n+1-i}\right]\right\}$, which reflects the additional excess return demanded by investors on TIPS compared with the real bond because of the indexation lag. It represents the risk premium due to inflation uncertainty during the indexation lag period. As long as n is much larger than 3, the exposure of the TIPS to the future inflation risk during the last 3 months before it matures should be small, as McCulloch and Kochin (2000) point out.

The yield to maturity of an n-period nominal T-bond is

$$
\begin{aligned}
i_{n,t} \quad = \quad & \left(\frac{1}{n}\right)\sum_{i=1}^{n}E_t\left[r_{t+i}\right]-\left(\frac{1}{n}\right)\sum_{i=j+1}^{n}\sum_{j=1}^{n}Cov_t\left[m_{t+i},m_{t+j}\right]+\left(\frac{1}{n}\right)\sum_{i=1}^{n}E_t\left[\pi_{t+i}\right]- \\
& \left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{n}\pi_{t+i}\right]-Cov_t\left[\sum_{i=1}^{n}m_{t+i},\sum_{i=1}^{n}\pi_{t+i}\right]\right\} \qquad (8)
\end{aligned}
$$

Compare the yield of an n-period nominal T-bond in (8) with the yield of an n-period TIPS in (6), we can see again they both contain $\left(\frac{1}{n}\right)\sum_{i=1}^{n}E_t\left[r_{t+i}\right]$, the average expected short real interest rate, and $-\left(\frac{1}{n}\right)\sum_{i=j+1}^{n}\sum_{j=1}^{n}Cov_t\left[m_{t+i},m_{t+j}\right]$, the real term premium.

The nominal T-bond yield includes expected inflation for the bond's lifetime, $\left(\frac{1}{n}\right)\sum_{i=1}^{n}E_t\left[\pi_{t+i}\right]$. In comparison, the TIPS yield contains only $\left(\frac{1}{n}\right)E_t\left[\sum_{i=1}^{3}\pi_{t+n+1-i}\right]$, the expected inflation in the indexation lag period, and even this part is corrected by $-\left(\frac{1}{n}\right)\log\left(\frac{P_t}{P_{t-3}}\right)$, as shown in (6). The term $-\left(\frac{1}{n}\right)\left\{\frac{1}{2}Var_t\left[\sum_{i=1}^{n}\pi_{t+i}\right]-Cov_t\left[\sum_{i=1}^{n}m_{t+i},\sum_{i=1}^{n}\pi_{t+i}\right]\right\}$ reflects the additional excess return investors demand on the nominal T-bond compared with the real bond. It represents the risk premium due to inflation uncertainty in n periods.

C State Space Model

C.1 Overview

The state space model expresses observed variables as functions of unobserved state variables. Harvey (1993), Harvey, Koopman and Shephard (2004), Hamilton (1994), Durbin and Koopman (2001), Kim and Nelson (1999) provide extensive coverages of state space models. Assume Y_t is a vector of observed variables. The state space representation of the dynamics of Y_t is given by the observation equation and the state equation. The observation equation describes Y_t as a function of the vector of underlying state variables S_t

$$Y_t = X_y + H \cdot S_t + W_t$$

with intercept X_y and white noise disturbance $W_t \sim N(0,R)$. The state equation describes the state variables S_t as following AR(1)

$$S_t = X_s + F \cdot S_{t-1} + V_t$$

with intercept X_s and white noise disturbance $V_t \sim N(0,Q)$. V_t and W_t are assumed to be uncorrelated at all lags. When using demeaned data Y_t, the resulting state space model is more parsimonious as it contains no intercept in either the observation equation or the state equation. Both the observed variables and the state variables can be interpreted as deviations from the means. So the state space model involves the following observation equation and state equation

$$\begin{aligned} Y_t &= H \cdot S_t + W_t \\ S_t &= F \cdot S_{t-1} + V_t \end{aligned}$$

C.2 Estimation Method

To estimate the state space model with the observation equation (29) and state equation (22), we first calculate the one-month-ahead prediction error

$$Y_t - Y_{t|t-1} = Y_t - H \cdot S_{t|t-1} = H \cdot \left(S_t - S_{t|t-1}\right) + W_t$$

Assume $P_{t|t-1} = Var\left(S_t - S_{t|t-1}\right)$, then the prediction error variance is

$$E\left[\left(Y_t - Y_{t|t-1}\right)\left(Y_t - Y_{t|t-1}\right)'\right] = H \cdot P_{t|t-1} \cdot H' + R$$

So Y_t conditional on the information available at $t-1$ follows normal distribution with mean $(H \cdot S_{t|t-1})$ and variance $\left(H \cdot P_{t|t-1} \cdot H' + R\right)$. Then the sample log likelihood is

$$L = -\left(\frac{nt \cdot nv}{2}\right)\log(2\pi) - \left(\frac{1}{2}\right)\sum_{t=1}^{nt}\left[\begin{array}{c} \log\left|H \cdot P_{t|t-1} \cdot H' + R\right| + \\ \left(Y_t - H \cdot S_{t|t-1}\right)'\left(H \cdot P_{t|t-1} \cdot H' + R\right)^{-1}\left(Y_t - H \cdot S_{t|t-1}\right) \end{array}\right]$$

nt is the number of observations in the sample and nv is the number of observed variables included in Y_t. This is also referred to as the prediction error decomposition (Harvey, 1989, Durbin and Koopman, 2001). The estimation of parameters is based on maximizing L. $S_{t|t-1}$ and $P_{t|t-1}$ are calculated by the Kalman filter. The initial condition is set as $S_{1|0} = 0$. For the stationary state variables r_t^a, π_t^a, $\xi_{3m,t}^a$, $\xi_{10yr,t}^a$, $\lambda_{10yr,t}^a$ and $e_{12m,t}^a$, the corresponding $P_{1|0}$ is solved using $vec(P_{1|0}) =$

$(I - F_a \otimes F_a)^{-1} \cdot vec(Q_a)$, as pointed out by Hamilton (1994). F_a and Q_a are the components of F and Q corresponding to the stationary state variables. For the nonstationary state variables r_t^p, π_t^p and γ_t, the corresponding $P_{1|0}$ is treated as diffuse, following Koopman, Shephard and Doornik (1999). We first use the EM algorithm developed by Watson and Engle (1983), which is more robust to the initial condition. Then we shift to the traditional maximum likelihood algorithm to obtain the final estimates.

C.3 Goodness of Fit

First, In table C3-1, we look at the R^2 for one-month-ahead prediction $Y_{t|t-1} = H \cdot S_{t|t-1}$ and Q-statistics for one-month-ahead prediction error $Y_t - Y_{t|t-1} = Y_t - H \cdot S_{t|t-1}$ in each observation equation. The R^2 is based on 54 free parameters (12 coefficients of the autoregressive terms, 5 in R and 37 in Q variance-covariance matrices). Except the π_t equation, generally speaking, the R^2 for our state space model is quite good. Given the lack of persistence demonstrated by the monthly inflation π_t in our sample period, it is only natural that we have a low R^2 for the π_t equation. In comparison, a parsimonious seasonal AR(2) fit to π_t would have produced an even lower R^2.

Except in the π_t equation, we cannot reject the null hypothesis of no serial correlation in the one-month-ahead prediction error, based on the Q-statistics calculated on $\min\left(nt/4, 3\sqrt{nt}\right) = 21$ lags. Looking more closely, however, for the one-month-ahead prediction error in π_t, it is the autocorrelation at lag 9 that seems to exert the most influence. In fact, the Q-statistic based on only the first 8 lags is 11.472 with a p-value of 0.176. Considering that we are using the non-seasonally adjusted CPI inflation data for π_t, the jump at an irregular lag length, such as 9 in this case, may not be that surprising. It is not clear to us whether adding more autoregressive terms to the underlying temporary inflation process would correct this problem. This is particularly the case when we already have a permanent component included in π_t. The key observation here is that, unfortunately, within our sample period when TIPS yields are available, there is just not that much persistence in the inflation series. Our emphasis in this paper is to extract the common factors among the group of observed variables, not only relying on the observed inflation rate. Following the principle of parsimony, we choose to stick to our current specification, and leave possibly less parsimonious specifications for future explorations.

Table C3-1

Goodness of Fit Statistics

	R^2	Q-statistic (p-value)
$i_{3m,t}$	0.9257	15.820 (0.780)
$i_{10yr,t}$	0.9016	16.708 (0.729)
$i^{TIPS}_{10yr,t}$	0.9466	24.149 (0.286)
π_t	0.0903	60.894 (0.000)
$\pi^{survey}_{12m,t}$	0.4706	20.874 (0.467)

Secondly, our state space model implies conditional homoskedasticity in the observation equation. Following Pennacchi (1991), we plot the one-month-ahead prediction error $Y_t - Y_{t|t-1} = Y_t - H \cdot S_{t|t-1}$ (not shown) to get a rough idea whether the assumption of homoskedasticity holds. Generally speaking, we think that homoskedasticity is a reasonable assumption.

We also examine the R matrix for the observation equation in table C3-2. The variance of π_t is fairly big, suggesting the monthly inflation rate is subject to more pronounced observation errors compared with other four variables. This is partly due to the fact that by our model design, inflation is the only variable whose variation is not picked up by any idiosyncratic factor.

Table C3-2

Variance-covariance Matrix R of the Observation Equation

	$w^i_{3m,t}$	$w^i_{10yr,t}$	$w^{TIPS}_{10yr,t}$	w^π_t	$w^{survey}_{12m,t}$
$w^i_{3m,t}$	$8.97{*}10^{-12}$				
$w^i_{10yr,t}$		$2.67{*}10^{-11}$			
$w^{TIPS}_{10yr,t}$			$2.50{*}10^{-3}$		
w^π_t				2.82	
$w^{survey}_{12m,t}$					$2.59{*}10^{-12}$

C.4 AR(2) Coefficients

Table C4-1 shows the autoregressive coefficients' estimates with standard errors. The excess return factors in the 10-year T-bond yield and the TIPS yield are highly persistent. The excess return factor in the 3-month T-bill yield is less so. The forecast error factor in the survey inflation forecast is also fairly persistent. For the idiosyncratic factors in the temporary real interest rate and temporary inflation, there is much less persistence.

Table C4-1

AR(2) Coefficients' Estimates

	ρ_1	ρ_2
r^a	0.6835 (0.0386)	-0.5207 (0.0624)
π^a	0.6482 (0.0486)	-0.4942 (0.0744)
ξ^a_{3m}	0.3627 (0.1350)	0.1211 (0.0616)
ξ^a_{10yr}	1.1978 (0.0488)	-0.2286 (0.0486)
λ^a_{10yr}	0.9315 (0.0196)	0.0303 (0.0193)
e^a_{12m}	0.4466 (0.0359)	0.5100 (0.0357)

C.5 Variance-covariance/Correlation Matrix Q with Additional Restrictions

We impose additional restrictions that an innovation in the survey inflation forecast error factor is uncorrelated with innovations in other factors, in addition to the restriction of no correlation between innovations in the seasonal factor of inflation and in other factors in our model. Table C5-1 shows the re-estimated Q matrix of the state equation with the correlation coefficients on the upper diagonal.

The real interest rate and inflation still tend to move in opposite directions. An innovation in the permanent real interest rate is negatively correlated with innovations in both permanent inflation (-0.69) and temporary inflation (-0.43). An innovation in the temporary real interest rate is also negatively correlated with both permanent inflation (-0.52) and temporary inflation (-0.91). So, our finding that the nominal interest rate lags behind the inflation movement still holds. In addition, innovations in the excess returns in the 3-month T-bill and the 10-year T-bond remain positively correlated (0.59). They still remain positively correlated (0.69 and 0.77 respectively) with an innovation in the excess return in TIPS. However, the magnitudes are higher than the correlation between themselves, contrary to what we have shown in table 1 in the main text. The positive correlations between the permanent and the temporary real interest rate (0.29), between permanent and temporary inflation (0.35) still hold. We also notice the variances of the temporary real interest rate and temporary inflation and the covariance between the two have all increased compared with those in table 1.

Table C5-1

Variance-covariance/Correlation Matrix Q of the State Equation, with Additional Restrictions

	v_t^{rp}	$v_t^{\pi p}$	v_t^{γ}	v_t^{ra}	$v_t^{\pi a}$	$v_{3m,t}^{\xi a}$	$v_{10yr,t}^{\xi a}$	$v_{10yr,t}^{\lambda a}$	$v_{12m,t}^{ea}$
v_t^{rp}	0.0913	-0.69	0	0.29	-0.43	-0.61	-0.57	-0.95	0
$v_t^{\pi p}$	-0.0537	0.0663	0	-0.52	0.35	0.19	0.07	0.60	0
v_t^{γ}	0	0	0.0000	0	0	0	0	0	0
v_t^{ra}	0.2176	-0.3277	0	6.0614	-0.91	-0.40	0.20	-0.15	0
$v_t^{\pi a}$	-0.3497	0.2432	0	-6.0884	7.3380	0.70	0.10	0.34	0
$v_{3m,t}^{\xi a}$	-0.0207	0.0054	0	-0.1113	0.2142	0.0127	0.59	0.69	0
$v_{10yr,t}^{\xi a}$	-0.0491	0.0049	0	0.1362	0.0757	0.0187	0.0799	0.77	0
$v_{10yr,t}^{\lambda a}$	-0.1020	0.0554	0	-0.1331	0.3274	0.0275	0.0775	0.1269	0
$v_{12m,t}^{ea}$	0	0	0	0	0	0	0	0	0.0145