OPTIMAL MONETARY POLICY IN ECONOMIES WITH "STICKY-INFORMATION" WAGES

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Optimal Monetary Policy in Economies with "Sticky-Information" Wages[1]

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Abstract:

In economies with sticky-information wage setting, policymakers legitimately give attention to output stabilization as well as price-level or inflation stabilization. Consistent with Kydland and Prescott (1990), trend deviations in prices are predicted to be negatively correlated with trend deviations in output. A variant of the Taylor rule is optimal if household consumption decisions are forward-looking. Interestingly, it is essential that policy *not* be made contingent on the most up-to-date estimates of potential output, potential-output growth, or the natural real interest rate. New results on the "persistence problem" and a new rationalization for McCallum's P-bar inflation equation are also presented.

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I. INTRODUCTION

Fischer contracts specify a time-varying price rather than a price that is constant over the life of the contract. Real-world collective bargaining agreements often have this feature, and it was in a labor-market setting that overlapping Fischer contracts first found their way into the macroeconomics literature (Fischer 1977). However, attention shifted fairly quickly toward models with overlapping fixed-price contracts (Taylor 1980; Calvo 1983), which seemed capable of generating a more persistent real response to nominal shocks.

The early contracts literature assumed that price is set equal to the expected market-clearing price (Fischer) or an average of market-clearing prices expected to prevail over the life of the contract (Taylor and Calvo). These assumptions were bound to come under scrutiny. In a Fischer-style model, Koenig (1996, 1999a, 2000) showed that optimizing, monopolistically competitive wage setters will, in fact, often choose a contract wage that lies *between* the prevailing average and expected market-clearing wages. Consequently, nominal shocks can have substantially more persistent real effects than the early Fischer-contracting literature had suggested.[2] Mankiw and Reis (2002) obtain similar results in an economy with price-setting, monopolistically competitive yeoman farmers. Both Koenig and Mankiw-Reis emphasize that the generalized Fischer contracts model is potentially applicable whenever price (or wage) setters face significant information-gathering or information-processing costs, even if formal contracts are absent.

The current paper makes contributions in several areas. It takes a careful look at optimal monetary policy in economies with maximizing agents, imperfectly observed productivity shocks and sticky-information wages–complimenting and extending Ball, Mankiw and Reis' (2003) and Mankiw and Reis' (2003) analyses of optimal policy in

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[2] In contrast, Chari, Kehoe, and McGrattan (2000) show that the contract price typically overshoots the expected market-clearing price in economies with overlapping, fixed-output-price contracts and mobile labor. Hence, nominal shocks have substantially *less* persistent real effects, in such economies, than the early contracts literature had suggested. The tendency for staggered price setting to speed adjustment to nominal shocks is called “the persistence problem.”

economies with sticky-information prices. It goes beyond target rules to consider how the short-term interest rate might be manipulated to implement optimal policy. It extends the existing literature on the persistence of the real effects of nominal shocks. It links two heretofore distinct theories of disequilibrium price adjustment–the textbook expectations-augmented Phillips curve and McCallum's P-bar model.[3] Finally, the analysis allows for a completely general time-distribution of wage adjustments.

On optimal policy, a key finding is that the monetary authority must attend to output as well as the price level or price inflation. Output and the price level are predicted to be negatively correlated at business-cycle frequencies, consistent with Kydland and Prescott (1990). In Ball, Mankiw and Reis' analysis of sticky-information pricing, in contrast, the price level never deviates from a pre-announced target path under optimal policy.[4]

A variety of instrument rules can be used to implement optimal policy–including a version of the Taylor rule. One Taylor-rule ingredient is an estimate of potential output, and it is sometimes argued that the Taylor rule's performance suffers badly when the central bank's potential-output estimates are inaccurate (Orphanides and Williams 2002). In a sticky-information-wage economy, on the contrary, it is essential that the potential-output estimates used by the monetary authority *not* incorporate the latest information. Similarly, policymakers should not use the latest estimates of the "natural" real interest rate. Monetary policy must define a predictable price–output trade-off. The required predictability is lacking if policy is contingent on information unavailable when current wages were chosen.

A Taylor-rule policy is only appropriate if wages fully adjust to new information in finite time. Notably, this condition is *not* satisfied when a constant fraction of agents re-evaluate their wage plans each period, as proposed by Calvo and assumed in much of the

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[3] McCallum (1994). See, also, Barro and Grossman (1976) and Mussa (1981).

[4] The corresponding result, here, is that optimal policy eliminates aggregate-wage surprises over the interval required for new information to be fully reflected in wage decisions. (See Section III, below.) With price and output fluctuations negatively correlated and wage surprises eliminated, the real wage varies pro-cyclically under optimal policy.

recent literature. Because it implies that wage/price adjustment is never complete, there is concern, too, that Calvo timing artificially prolongs the economy's response to nominal shocks. The current paper avoids these problems by putting absolutely no restrictions on the time-pattern of wage re-evaluations. The price for this generality is a "speed effect" in aggregate inflation dynamics that is more complicated than would otherwise be the case.[5]

The current analysis also avoids the restrictive treatment given aggregate demand in much of the existing literature.[6] All main results are derived assuming only that policy defines an aggregate demand schedule. In this setting, the paper examines conditions under which staggered wage setting prolongs the response of the real economy to nominal shocks. The key finding is that staggering will *always* enhance persistence if policymakers choose the aggregate demand schedule's slope sensibly.

On inflation dynamics, the paper shows that a traditional Phillips curve applies at short forecast horizons, while McCallum's P-bar model is a good approximation at longer horizons. Intuitively, because the information embodied in wages is "sticky," the only way for policy shocks to affect near-term inflation is through the output gap–a key feature of standard Phillips curve models. As time passes, however, a larger and larger fraction of labor agreements will reflect agents' updated policy expectations, providing a channel independent of the output gap through which policy can affect inflation.

Section II presents the sticky-information wage-setting model and derives its aggregate implications. The model is shown to impose a cross-equation constraint on the impulse response functions of output and the price level–a dynamic aggregate supply schedule. Alternatively, the model implies an expectations-augmented Phillips curve with a

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[5] The speed effect is the tendency for inflation to respond to the rate of change of economic activity or economic slack, controlling for the level of slack and other variables. Empirical evidence of a speed effect goes all the way back to Phillips (1958).

[6] For example, Koenig (1996; 1999a,b; 2000), Mankiw-Reis (2002), Ball-Mankiw-Reis (2003), and Chari-Kehoe-McGrattan (2000) assume that money demand is proportional to nominal spending, and that the money supply is the central bank's chosen policy instrument.

built in speed effect. Section III shows that the effects of sticky information are completely offset if the monetary authority imposes an appropriately sloped and predictable price–output trade-off. Obeying the Taylor rule's interest-rate prescriptions is one (but only one) way that this trade-off can be established. Section IV presents examples illustrating how results can change if the monetary policy instrument must be set before current output and its price are revealed. With the money supply as the chosen instrument, it is typically no longer possible for policy to fully offset the effects of wage stickiness. Under interest-rate targeting, a full offset is possible only if the natural real interest rate is constant. Sections IV and V move on to present results on the persistence of nominal shocks and McCallum's P-bar inflation model, respectively. Section V contains a summary and concluding remarks.

II. STICKY-INFORMATION WAGE ADJUSTMENT

Tastes and Technology. The model assumes a continuum of monopolistically competitive, optimizing households and firms, indexed by $i \in [0,1]$ and $f \in [0,1]$, respectively. Firm f produces output according to

$$Y_f \equiv \Theta\left(\int_0^1 N_{fi}^{(\epsilon - 1)/\epsilon} di\right)^{\epsilon/(\epsilon - 1)}, \tag{1}$$

where N_{fi} is the quantity of labor purchased from household i, Θ is a productivity-shift variable, and $\epsilon > 1$ is the elasticity of substitution between the labor of different workers. Household i purchases output and chooses a wage path to maximize an instantaneous utility function of the form

$$U_i = \left(\frac{\sigma}{\sigma - 1}\right) C_i^{(\sigma - 1)/\sigma} - \left(\frac{1}{\lambda}\right) N_i^{\lambda} \tag{2}$$

where $N_i \equiv \int N_{fi} df$ is the total amount of labor sold by household i,

$$C_i \equiv (\int_0^1 C_{fi}^{(\eta - 1)/\eta} df)^{\eta/(\eta - 1)} \tag{3}$$

is household i's aggregate consumption, and C_{fi} denotes the rate at which firm f's output is purchased by household i. By assumption, the parameters η and λ are both greater than 1, while the elasticity of intertemporal substitution, σ, is greater than 0. Most empirical studies suggest that $\sigma \leq 1$. Household labor income is subsidized at a rate sufficient to offset the distortions created by firms' and households' monopoly power.[7]

The profit-maximizing price of each firm is a constant markup over unit labor costs. Since each firm faces the same optimization problem as every other firm and because, by assumption, there are no output-price contracts, firms all charge the same price:

$$P = P_f = (\frac{\eta}{\eta - 1})(\frac{W}{\Theta}). \tag{4}$$

where P_f is the price charged by firm f, P is the aggregate price level, and

$$W \equiv (\int_0^1 W_i^{1 - \epsilon} di)^{1/(1 - \epsilon)} \tag{5}$$

is the aggregate wage rate. Aggregate output is related to aggregate labor by the equation

$$Y = \Theta N. \tag{6}$$

where the aggregate quantity of labor is defined by

$$N \equiv \int_0^1 \int_0^1 N_{fi}(W_i/W) di df. \tag{7}$$

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[7] The subsidy is financed by lump-sum taxes. This device is common in the modern policy literature. See, for example, Woodford (2003) and Erceg, Henderson and Levin (2000). With an appropriate subsidy in place, the monetary authority can focus on keeping the economy at its market-clearing equilibrium, without worrying about a divergence between this equilibrium and the Pareto optimum.

Optimal Wage Setting. By assumption, households must preset their wage time paths. Consider household i at time $t - s < t$, choosing a wage to charge at time t. The first-order condition for household i's maximization problem takes the form:

$$E_{t-s}[\epsilon N^{\lambda}(\frac{W_i}{W})^{-(1 + \epsilon\lambda)}W^{-1}] = (1 + \tau)E_{t-s}[(\epsilon - 1)N(\frac{W_i}{W})^{-\epsilon}C_i^{-1/\sigma}P^{-1}], \tag{8}$$

where τ is the rate at which labor income is subsidized and where N, W_i, W, C_i, and P are all measured at time t. Intuitively, an increase in its wage increases household i's leisure and reduces the wage earnings available for household i's consumption. Utility maximization requires that the expected marginal utility from increased leisure equal the expected marginal disutility from reduced consumption.

After using Equations 4 and 6 to eliminate P and N, respectively, and taking logarithms, Equation 8 can be solved for household i's optimal wage:[8]

$$w_i(t) = E_{t-s}[w(t) + \beta z(t)]. \tag{9}$$

Here lower-case letters denote the logarithms of their upper-case counterparts,

$$\beta \equiv \frac{1/\sigma + (\lambda - 1)}{1 + \epsilon(\lambda - 1)}, \tag{10}$$

and $z \equiv \ln(Y/Y^*)$, where Y^* denotes the level of output that would be observed if wages were not preset:

$$Y^* = [(1+\tau)(\frac{\varepsilon - 1}{\varepsilon})(\frac{\eta - 1}{\eta})\Theta^{\lambda}]^{1/(1/\sigma+\lambda-1)}. \tag{11}$$

[8] In going from Equation 8 to Equation 9, households are assumed to insure themselves against idiosyncratic fluctuations in consumption due solely to cross-household differences in the timing of contract negotiations. Also, I suppress a risk premium. See Appendix A.

Thus, household i sets its time-t log wage equal to its expectation of the aggregate log wage, plus a correction for expected excess demand. With $1 + \tau = [\epsilon/(\epsilon - 1)][\eta/(\eta - 1)]$, the market-clearing equilibrium output level coincides with the economy's Pareto optimum.

Aggregate Wage and Price Adjustment. Suppose that we keep track of households not by their position on the unit interval, but by when they last had an opportunity to reset their wage paths. Denote by $W_s(t)$ ($s = 0, 1, 2, ...$) the wage charged at time t by households which last reset their wage paths at time $t - s$. Then, from the definition of the aggregate wage:

$$W(t) \;=\; [\Sigma_{s=0}^{\infty}\omega(s)W_s(t)^{1\,-\,\epsilon}]^{1/(1\,-\,\epsilon)}, \tag{5'}$$

where $\omega(s)$ is the fraction of workers who last adjusted their wage paths s periods ago. Take a log-linear approximation to Equation 5' about any point at which all households charge the same wage:

$$w(t) \;\approx\; \Sigma_{s=0}^{\infty}\omega(s)w_s(t), \tag{12}$$

where $w_s(t) \equiv \ln(W_s(t))$. Treating Equation 12 as an equality and using Equation 9,

$$w(t) \;=\; \Sigma_{s=1}^{\infty}\omega'(s)E_{t-s}[w(t) \;+\; \beta z(t)] \;+\; \omega'(0)\beta z(t), \tag{13}$$

where $\omega'(s) \equiv \omega(s)/(1 - \omega(0))$. Note that $\Sigma_{s=1}^{\infty}\omega'(s) \;=\; 1$ and that $\omega'(0) = \omega(0) = 0$ if the wage rate is set one or more periods in advance. Thus, the aggregate wage is determined by excess demand (insofar as $\omega'(0) > 0$) and a weighted average of past expectations of the current aggregate wage and excess demand. It is the presence of these latter terms that justifies the "sticky information" sobriquet.

Price adjustment is determined by Equations 4 and 13:

$$p(t) = \sum_{s=1}^{\infty}\omega'(s)E_{t-s}[p(t) + \beta z(t)] - [\theta(t) - \sum_{s=1}^{\infty}\omega'(s)E_{t-s}\theta(t)] + \omega'(0)\beta z(t) \tag{14}$$

where $p(t) \equiv \ln(P(t))$ and $\theta(t) \equiv \ln(\Theta(t))$. The only difference between price adjustment and wage adjustment is that prices respond directly to any productivity information that may have arrived since current wages were negotiated.

In Fischer's original analysis (Fischer 1977), $\omega(0) = 0$ and $\omega(1) = \omega(2) = ½$. There are no productivity shocks. On the demand side of the economy, it's assumed that $p + y = m$, where m is the (logarithm) of the exogenously determined money supply. The flexible-wage equilibrium price level is naturally defined by $p^* + y^* = m$, and it follows that $y - y^* = p^* - p$. Finally, Fischer simply assumes that $\beta = 1$, so that Equation 14 reduces to:

$$p(t) = \Sigma_{s=1}^{2} 1/2 E_{t-s}[p(t) + 1 \times (p^*(t) - p(t))] = \Sigma_{s=1}^{2} 1/2 E_{t-s}\, p^*(t).$$

The price level is the average of past expectations of the current market-clearing price level.

Relationship to the Yeoman-Farmer Model. Absent productivity shocks, the Mankiw-Reis (2002) model of yeoman farmers and sticky-information output prices is just a special case of the sticky-information wage model outlined above (Koenig 2000; Edge 2002). In Mankiw-Reis, N_i measures both the output produced by household i and household i's labor effort. W_i is household i's output price. (There is no wage, because there is no labor market.) The sole function of "firms" is to costlessly combine the goods produced by different households into composite bundles using the technology specified in Equation 1. These bundles are sold competitively ($\eta = \infty$).[9] Finally, following Calvo (1983), Mankiw-Reis limit their analysis to the case where $\omega(s) = \delta/(1 + \delta)^{s+1}$ for $s = 0, 1, 2, \ldots$.

Substantive differences between the sticky-information price and wage models arise once one introduces productivity shocks. In the wage model, productivity shocks have an immediate and direct impact on output prices. In the price model, in contrast, productivity

[9] Alternatively, firms can be dispensed with, and each household's utility be made a direct function of purchases from other households.

shocks are reflected in output prices only as price paths are reset: output prices are governed by Equation 13, instead of Equation 14.

In fact, Ball-Mankiw-Reis productivity shocks are mathematically equivalent, in the wage model, to shifts in each household's taste for leisure relative to consumption. The Ball-Mankiw-Reis policy results are, thus, relevant in an economy with sticky-information wages only insofar as taste shocks are an important source of macroeconomic fluctuations.

Impulse Response Functions. The supply-side implications of Fischer-style sluggish wage adjustment are conveniently summarized as constraints on the impulse response functions of the output gap and the wage rate or of output gap and the price level. From Equation 13:

$$E_t w(t+s) - E_{t-1} w(t+s) = \beta[\frac{\Omega(s)}{1 - \Omega(s)}][E_t z(t+s) - E_{t-1} z(t+s)], \qquad (15)$$

where $s \geq 0$ and $\Omega(s) \equiv \sum_{i=0}^{s} \omega(i)$. Similarly, the relationship between the price and output-gap impulse response functions is:

$$E_t p(t+s) - E_{t-1} p(t+s) = \beta[\frac{\Omega(s)}{1 - \Omega(s)}][E_t z(t+s) - E_{t-1} z(t+s)] - [E_t \theta(t+s) - E_{t-1} \theta(t+s)]. \quad (16)$$

The term $\beta\Omega(s)/(1 - \Omega(s)) \equiv \beta_1(s)$ on the right-hand side of these equations is the slope of the aggregate supply schedule s periods after an economic shock. It equals the sensitivity of an individual worker's wage to excess demand, when allowed to adjust, times the odds that a randomly chosen worker's wage has already adjusted. Thus, consistent with standard textbook treatments of aggregate supply, the model says that the wage–price response to any economic shock becomes larger and larger, over time, relative to the output-gap response. In addition, prices respond one-for-one (but inversely) to productivity innovations.

Koenig's (1999a) analysis is limited to the special case where $\omega(0) = 0$ (wage paths must be set at least one period in advance) and $\omega(s) = \delta/(1 + \delta)^s$ for $s = 1, 2, ...$, so that $\beta_1(s)$

$= \beta[(1 + \delta)^s - 1]$. In the Mankiw-Reis (2002) yeoman-farmer version of the sticky-information model, price adjustment is governed by Equation 15 instead of Equation 16, and it's assumed that $\omega(s) = \delta/(1 + \delta)^{s+1}$ for $s = 0, 1, 2, ...$, so that $\beta_1(s) = \beta[(1 + \delta)^{s+1} - 1]$.

The time required for wage adjustment to be complete (the smallest value of s such that $\Omega(s) = 1$), will be denoted by S. Under the Koenig (1999a) and Mankiw-Reis (2002) weighting schemes, S is obviously infinite. A finite S is obtained if, for example, wage adjustment is distributed uniformly over a finite interval–i.e., if either $\omega(s) = 1/(S + 1)$ for $s = 0, 1, 2, ..., S$, or $\omega(0) = 0$ and $\omega(s) = 1/S$ for $s = 1, 2, ..., S$. In the first case, the slope of the aggregate supply schedule is $\beta_1(s) = \beta(s + 1)/(S - s)$. In the second, it is $\beta_1(s) = \beta s/(S - s)$.

Inflation Dynamics. Equation 16 implies that

$$\pi(t) - E_{t-1}\pi(t) = \beta_1(0)[z(t) - E_{t-1}z(t)] - [\Delta\theta(t) - E_{t-1}\Delta\theta(t)]. \qquad (17)$$

where $\Delta\theta(t) \equiv \theta(t) - \theta(t-1)$ is productivity growth. Consistent with standard textbook treatments, realized inflation exceeds expected inflation if either excess demand turns out to be greater than anticipated or productivity growth turns out to be weaker than anticipated.

Appendix B shows that Equation 17 generalizes to an expectations-augmented Phillips curve of the form

$$\pi(t) = \pi^e(t) - [\Delta\theta(t) - \Delta\theta^e(t)] + (1 - b_1)\Delta\beta_1(1)z(t-1) + \sum_{s=0}^{S} B_s E_{t-s}\Delta z(t), \qquad (18)$$

where $\Delta z(t)$ and $\Delta\beta_1(s)$ are the rates of change of excess demand and the slope of the aggregate supply schedule, respectively, where $\pi^e(t) \equiv \sum_{s=1}^{S+1} b_s E_{t-s}\pi(t)$ is a weighted average of past inflation expectations, $\Delta\theta^e(t) \equiv \sum_{s=1}^{S+1} b_s E_{t-s}\Delta\theta(t)$ is a weighted average of past productivity-growth expectations, and the term $\sum_{s=0}^{S} B_s E_{t-s}\Delta z(t)$ is a distributed lag of past expectations of current excess-demand growth. The dependence of inflation on excess demand growth is known as the "speed effect."

The weight b_1 in the formulas for $\pi^e(t)$ and $\Delta\theta^e(t)$ is completely arbitrary, but it's sensible to restrict it to the unit interval. Given b_1,

$$b_s = (1 - b_1)\Delta\beta_1(1)\left[\frac{1}{\Delta\beta_1(s-1)} - \frac{1}{\Delta\beta_1(s)}\right], \tag{19}$$

for s = 2, 3, ...S - 1. With $b_1 \leq 1$, each b_s is non-negative provided the aggregate supply schedule's slope rises at an increasing rate with the passage of time. Finally, if the b_s are all to sum to 1, then b_S and b_{S+1} must satisfy the constraint

$$b_S + b_{S+1} = (1 - b_1)\left[\Delta\beta_1(1) / \Delta\beta_1(S-1)\right]$$

when S is finite. If S is infinite, the corresponding requirement is that $\lim_{s\to\infty}\Delta\beta_1(s) = \infty$.

What of the coefficients on past expectations of excess-demand growth (the speed effect) in Equation 18? They must satisfy:

$$B_0 = \beta_1(0),$$

$$B_1 = (1 - b_1)\beta_1(1) - \beta_1(0),$$

and

$$B_s = (1 - \textstyle\sum_{i=1}^{s} b_i)\beta_1(s-1) - (1 - \sum_{i=1}^{s-1} b_i)\beta_1(s-2)$$

for s = 2, 3, ...S. However, a little algebra establishes that

$$B_s = (1 - b_1)\Delta\beta_1(1)\left[\frac{1}{\Delta\beta_1(s) / \beta_1(s-1)} - \frac{1}{\Delta\beta_1(s-1) / \beta_1(s-2)}\right], \tag{20}$$

for s = 2, 3, ..., S - 1, which is non-negative if, and only if, the *percentage* change in the slope of the aggregate supply schedule *de*creases over time. Finally, if S is finite, then

$$B_S = (1 - b_1)\Delta\beta_1(1) - b_S\beta_1(S-1).$$

Examples will help make Equation 18 more concrete. In the geometric weighting scheme considered by Koenig (1999a), we know that the slope of the aggregate supply schedule is $\beta_1(s) = \beta[(1 + \delta)^s - 1]$, for $s = 0, 1, \ldots$. Setting $b_1 = \delta/(1 + \delta)$, Equation 18 takes an especially simple form:

$$\pi(t) = \pi^e(t) - [\Delta\theta(t) - \Delta\theta^e(t)] + \beta\left(\tfrac{\delta}{1+\delta}\right)z(t-1) + \beta\Delta z^e(t), \tag{18'}$$

where

$$\pi^e(t) = \delta \textstyle\sum_{s=1}^{\infty}[\, E_{t-s}\pi(t)/(1 + \delta)^s],$$

$$\Delta\theta^e(t) = \delta \textstyle\sum_{s=1}^{\infty}[\, E_{t-s}\Delta\theta(t)/(1 + \delta)^s],$$

and

$$\Delta z^e(t) \equiv \delta \textstyle\sum_{s=1}^{\infty}[\, E_{t-s}\Delta z(t)/(1 + \delta)^s].$$

Note that the weights attached to lagged expectations of the change in the output gap are identical to those attached to lagged expectations of inflation and productivity growth.

If the distribution of wage adjustments is uniform [so that $\beta_1(s) = \beta s/(S - s)$], rather than geometric, the speed effect is more complicated. Equation 18 reduces to

$$\pi(t) = \pi^e(t) - [\Delta\theta(t) - \Delta\theta^e(t)] + \left(\tfrac{\beta}{S+1}\right)\left[z(t-1) + \textstyle\sum_{s=1}^{S}\left(1 - \tfrac{2(s-1)}{S}\right)E_{t-s}\Delta z(t)\right], \tag{18''}$$

where,

$$\pi^e(t) = \left[\tfrac{2}{S(S+1)}\right]\textstyle\sum_{s=1}^{S}(S + 1 - s)E_{t-s}\pi(t),$$

and

$$\Delta\theta^e(t) = \left[\tfrac{2}{S(S+1)}\right]\textstyle\sum_{s=1}^{S}(S + 1 - s)E_{t-s}\Delta\theta(t),$$

assuming $b_1 \equiv 2/(S + 1)$ and $b_S \equiv 2/[S(S + 1)]$. The speed-effect weights are no longer identical to those attached to lagged expectations of inflation and productivity growth. Indeed, the speed-effect weights are negative for $s > 1 + S/2$.

In the special case where there is some immediate wage response to excess demand ($\omega(0) \neq 0$), one can set $b_1 = 1 - \beta_1(0)/\Delta\beta_1(1)$ and then rearrange Equation 18 to obtain:

$$\pi(t) = \pi^e(t) - [\Delta\theta(t) - \Delta\theta^e(t)] + \beta_1(0)z(t) + \sum_{s=1}^{S} B_s E_{t-s}\Delta z(t). \tag{21}$$

Now it's the *current* level of excess demand that appears on the right-hand side of the Phillips-curve equation, while the current *change* in excess demand does not. With geometric weighting, one has $\beta_1(0) = \beta\delta$, $b_s = \delta/(1 + \delta)^s$, and $B_s = \beta b_s$ for $s = 1, 2, 3, \ldots$. On the other hand, if wage adjustments are uniformly distributed, then $\beta_1(0) = \beta/S$, $b_s = 2(S + 1 - s)/[S(S + 1)]$, and $B_s = (\beta/S)[1 - 2s/(S + 1)]$ for $s = 1, 2, \ldots S$.

III. OPTIMAL POLICY

Introduction and Key Results. With an appropriate labor subsidy in place (to offset the systematic, employment and output-depressing effects of monopoly power in the labor and product markets), the proper objective of monetary policy is to keep aggregate output at its market-clearing, flexible-wage level. A potential objection is that the distribution of labor effort across households–not just the aggregate level of effort–matters for welfare.[10] Cross-household differences in labor effort arise whenever there are cross-household wage differences. In an economy with overlapping fixed-wage contracts, cross-household wage differences develop whenever the aggregate rate of wage inflation is non-zero, but in an economy with Fischer-style wage setting, they only result from the arrival of information that makes earlier wage-path decisions, in retrospect, sub-optimal. The observable manifestation of these shocks is unexpected movement in the aggregate wage, such that $w(t)$

[10] Ball-Mankiw-Reis (2003) emphasize this point in the context of their yeoman-farmer model of sticky output prices.

$\neq E_{t-s}w(t)$. However, Equation 15 tells us that any monetary policy that keeps aggregate output at its market-clearing level completely eliminates wage surprises. Hence, perfect aggregate output stabilization is a sufficient condition for welfare maximization.[11]

In pursuing their stabilization objective, policymakers are hampered by an inability to directly observe productivity shocks. They can, however, observe and respond to current-period movements in output and prices. (This assumption is relaxed in Section V.) The monetary authority is assumed to adjust its policy instrument as necessary to restrict the economy to an aggregate demand schedule of the form $p(t) = \alpha_0(t) - \alpha_1 y(t)$.[12] The question is whether and how $\alpha_0(t)$ and α_1 can be chosen to keep the output gap, $z(t)$, equal to zero.

By restricting the economy to an aggregate demand schedule, the monetary authority imposes a new constraint on the impulse response functions of output and prices:

$$E_t p(t+s) - E_{t-1}p(t+s) = [E_t\alpha_0(t+s) - E_{t-1}\alpha_0(t+s)] - \alpha_1[E_t y(t+s) - E_{t-1}y(t+s)] \quad (22)$$

for $s = 0, 1, 2, \ldots$ Equation 11, on the other hand, implies

$$E_t y^*(t+s) - E_{t-1}y^*(t+s) = [\tfrac{\lambda}{1/\sigma+\lambda-1}][E_t\theta(t+s) - E_{t-1}\theta(t+s)]. \quad (23)$$

Together, Equations 16, 22, and 23 can be solved for the responses of output and the price

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[11] The formal argument goes as follows: Cross-household wage differences arise when $w_s(t) \neq w_{s+1}(t)$ for some $0 \leq s < S$, where $w_s(t)$ is the period-t wage set at time $t - s$ and $w_{s+1}(t)$ is the period-t wage set at time $t - s - 1$. From Equation 9, however, $w_s(t) \neq w_{s+1}(t)$ if, and only if, $E_{t-s}[w(t) + \beta z(t)] \neq E_{t-s-1}[w(t) + \beta z(t)]$. Using Equation 15 to eliminate the wage terms from this last expression, $w_s(t) \neq w_{s+1}(t)$ if, and only if, $E_{t-s}z(t) \neq E_{t-s-1}z(t)$. It follows that any policy keeps $z(t) = 0$ also eliminates cross-household wage differences. Similarly, Ball-Mankiw-Reis (2003) find that optimal policy in an economy with sticky-information price setting eliminates price surprises.

[12] Fischer (1977) and Ball-Mankiw-Reis (2003), for example, close their models by assuming that $p(t) + y(t) = m(t)$, where the money supply, $m(t)$, is the policy instrument. This specification is consistent with the analysis presented below provided that the monetary authority adjusts $m(t)$ according to the rule $m(t) = \alpha_0(t) + (1 - \alpha_1)y(t)$. If $\alpha_1 < 1$, monetary policy accommodates swings in output. If $\alpha_1 > 1$, monetary policy "leans against the wind." If $\alpha_1 = 1$, the money supply is exogenous.

level to aggregate-demand and aggregate-supply shocks:

$$E_t y(t+s) - E_{t-1} y(t+s) = \left[\frac{1}{\alpha_1 + \beta_1(s)}\right][E_t \alpha_0(t+s) - E_{t-1}\alpha_0(t+s)]$$
$$+ \left[\frac{\left(\frac{1/\sigma + \lambda - 1}{\lambda}\right) + \beta_1(s)}{\alpha_1 + \beta_1(s)}\right][E_t y^*(t+s) - E_{t-1} y^*(t+s)] \tag{24}$$

$$E_t p(t+s) - E_{t-1} p(t+s) = \left[\frac{\beta_1(s)}{\alpha_1 + \beta_1(s)}\right][E_t \alpha_0(t+s) - E_{t-1}\alpha_0(t+s)]$$
$$- \alpha_1\left[\frac{\left(\frac{1/\sigma + \lambda - 1}{\lambda}\right) + \beta_1(s)}{\alpha_1 + \beta_1(s)}\right][E_t y^*(t+s) - E_{t-1} y^*(t+s)]. \tag{25}$$

As more and more workers have a chance to reset their wage paths (steepening the aggregate supply curve), output's response to demand shocks fades to zero while the price level's response becomes 1-for-1.[13] Supply shocks move the economy along the aggregate-demand schedule, raising output and lowering the price level (assuming $\alpha_1 \geq 0$). Eventually, supply shocks are reflected, 1-for-1, in output.

Applying Equation 24 recursively yields a reduced-form equation for the output gap:

$$E_t z(t+s) = \sum_{i=0}^{S-1-s}\left[\frac{1}{\alpha_1 + \beta_1(s+i)}\right]\{(\alpha_1^* - \alpha_1)[E_{t-i} y^*(t+s) - E_{t-i-1} y^*(t+s)]$$
$$+ [E_{t-i}\alpha_0(t+s) - E_{t-i-1}\alpha_0(t+s)]\}, \tag{26}$$

where S (as always) is the time required for all wage paths to be reset, and where $\alpha_1^* \equiv (1/\sigma + \lambda - 1)/\lambda > 0$. To ensure that the output gap, $z(t)$, is zero, it's both necessary and sufficient that the monetary authority adjust its policy instrument to set $\alpha_1 = \alpha_1^*$ and eliminate aggregate demand surprises [keep $E_{t-i}\alpha_0(t) = E_{t-i-1}\alpha_0(t)$] for as long as $\beta_1(i)$ is finite.

[13] Thus, the model satisfies a strict version of the natural rate hypothesis: There is no rule for setting α_0 that can keep the economy from converging to its market-clearing equilibrium. It follows that sticky-information models behave like real-business-cycle models in the long run. Models with fixed-price or fixed-wage contracts do not have this property.

The latter condition will be satisfied provided α_0 is announced S periods (or more) in advance. Effectively, $\alpha_0(t)$ can be made contingent on any variable known to wage-setters at t - S and must not depend on information revealed after t - S. Since union labor contracts often specify a 3-year wage path, S cannot reasonably be expected to be less than 3 years.

The fact that $\alpha_1^* > 0$ means that it can never be optimal for the monetary authority to adopt a pure price-level or pure inflation target. Under optimal policy, $p(t) = p^*(t) = \alpha_0(t) - \alpha_1^* y^*(t) = E_{t-S}[\alpha_0(t)] - \alpha_1^* y^*(t)$. Thus, the realized price level always depends on aggregate-supply shocks revealed over the preceding S periods. Note, also, that $p(t) + \alpha_1 y(t) = \alpha_0(t) = E_{t-S}[p(t) + \alpha_1 y(t)]$, so that $p(t) - E_{t-S}p(t) = -\alpha_1[y(t) - E_{t-S}y(t)]$. Thus, deviations in the price level from its trend are negatively correlated with trend deviations in output–a result consistent with Kydland and Prescott (1990).[14] Intuitively, if the monetary authority minimizes surprise shifts in aggregate demand (as is optimal) then output and price surprises will trace out the pre-announced, downward-sloping aggregate demand schedule.

While the price level is subject to unexpected swings under optimal policy, the wage rate is not. Indeed, it is precisely because it eliminates the need for wages to respond to new information, neutralizing wage inflexibility, that the policy described above *is* optimal. Thus, Equations 4, 23, and 25 together imply

$$E_t w(t+s) - E_{t-1} w(t+s) = \left[\frac{\beta_1(s)}{\alpha_1 + \beta_1(s)}\right] \{ [E_t \alpha_0(t+s) - E_{t-1}\alpha_0(t+s)] + (\alpha_1^* - \alpha_1)[E_t y^*(t+s) - E_{t-1} y^*(t+s)] \}. \tag{27}$$

Equation 27 says that $E_t w(t+s) = E_{t-1} w(t+s)$ provided that the monetary authority eliminates aggregate-demand surprises during the period over which $\beta_1(s)$ is finite and sets $\alpha_1 = \alpha_1^*$.[15]

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[14] If p and y have unit roots, $E_{t-S}p(t)$ and $E_{t-S}y(t)$ will approximate extrapolations of these series' Beveridge-Nelson trends for large S (Beveridge and Nelson 1981).

[15] This analysis suggests that the skill and reliability of the monetary authority will influence the frequency of wage negotiations in models where this frequency is endogenous.

Example Optimal Target Rules. A wide range of target policy rules are potentially optimal, depending on the parameters of the household utility function and policymakers' choice of $\alpha_0(t)$. Suppose, for example, that the elasticity of intertemporal substitution, σ, equals 1. Then $\alpha_1^* = 1$, and the monetary authority should adjust its policy instrument to keep nominal spending, $p(t) + y(t)$, equal to an arbitrary pre-announced target level, $\alpha_0(t)$.

Other interesting cases arise when the maximum interval between wage-path adjustments, S, is finite. For example, it is consistent with optimal policy for the monetary authority to define

$$\alpha_0(t) \equiv p(t - S) + \pi^T \times S + \alpha_1^* [y(t - S) + \mathrm{E}_{t-S}(y^*(t) - y^*(t - S))], \tag{28}$$

where π^T is the authority's long-run inflation target, and then to adjust its policy instrument to keep $p(t) + \alpha_1^* y(t) = \alpha_0(t)$. Rearranging terms, policy will be optimal if the monetary authority succeeds in keeping

$$\mu\left[\Delta^S p(t) - \pi^T\right] + (1 - \mu)\left[\Delta^S y(t) - E_{t-S}\Delta^S y^*(t)\right] = 0, \tag{29}$$

where $\mu \equiv 1/(1 + \alpha_1^*)$ and $\Delta^S x(t) \equiv [x(t) - x(t - S)]/S$ for any variable "x." In words, the monetary authority should respond to a weighted average of deviations in inflation from target and deviations in output growth from expected potential growth, along the lines suggested by Orphanides and Williams (2002). Both inflation and output growth are retrospective, calculated over an interval that extends back (at least) S periods. Inflation is allowed to exceed its target only to the extent that realized output growth proves to be weaker than the economy's estimated growth potential.

Alternatively, the monetary authority can adjust its policy instrument to keep

$$p(t) + \alpha_1^* y(t) = p(t - S) + \pi^T \times S + \alpha_1^* E_{t-S} y^*(t) \equiv \alpha_0(t), \tag{30}$$

which is equivalent to keeping

$$\mu'\left[\Delta^S p(t) - \pi^T\right] + (1 - \mu')\left[y(t) - E_{t-S} y^*(t)\right] = 0, \tag{31}$$

where $\mu' \equiv S/(S + \alpha_1^*) \geq \mu$. Here, the monetary policy prescription is similar, in spirit, to the Taylor rule: Inflation is allowed to exceed target only to the extent that the realized output level falls short of an estimate of potential output.

Equations 29 and 31 have identical implications for the S-period inflation rate, $\Delta^S p(t)$:

$$\Delta^S p(t) = \pi^T - \alpha_1^*[y^*(t) - E_{t-S}y^*(t)]/S. \tag{32}$$

It follows that S-period-ahead expected inflation is completely tied down by policymakers' inflation target ($E_{t-S}\,\Delta^S p(t) = \pi^T$), with actual inflation exceeding target inflation if, and only if, realized potential output falls short of what was expected S periods ago.

Paradoxically, to keep output and employment at their market-clearing levels, it is essential that monetary policy *not* be made contingent on the most up-to-date estimates of potential output or its growth rate: Neither the estimate of potential output growth in Equation 29 nor the estimate of the level of potential output in Equation 31 is based on the latest-available information. The trick to optimal policy is for the monetary authority to announce the period-t position of the aggregate demand schedule ($\alpha_0(t)$) well in advance, so that all workers have a chance to factor this information into their period-t wage demands. Then, if the slope of the demand schedule (α_1) has been properly fixed by the monetary authority, realized output will respond appropriately to shifts in aggregate supply without conscious effort on the part of policymakers. To this extent, the analysis presented here buttresses the case against policy "fine tuning."

Example Optimal Instrument Rules. Target rules like Equations 29 and 31 (or the more general prescription that the monetary authority ought to eliminate wage surprises) provide a means for judging the after-the-fact success of policy, but provide little guidance on how success is actually to be achieved. For this one needs an "instrument rule," which tells the monetary authority how it ought to adjust the monetary base or a short-term interest rate in response to available information. The downside of optimal instrument rules is that they very much depend on the assumed links between the policy instrument and the economy. The following analysis provides some instrument-rule examples for an economy with sticky-information wage adjustment, but is by no means exhaustive.

One popular model of aggregate demand assumes that nominal spending is linked to the money supply by the equation $p(t) + y(t) = m(t)$, where $m(t)$ is the policy instrument.[16] In this case, optimality requires that

$$m(t) = \alpha_0(t) + [(\sigma - 1)/\sigma\lambda]y(t), \tag{33}$$

where the only restriction on $\alpha_0(t)$ is that it be announced at least S periods in advance.[17] Whether or not monetary policy accommodates output movements or "leans against the wind" is, thus, completely determined by whether the elasticity of intertemporal substitution, σ, is greater or less than 1.

If S is finite and $\alpha_0(t)$ is defined as in Equation 28, the optimal money supply rule takes the form:

$$\Delta^S m(t) = \pi^T + \alpha_1^* E_{t-S}\Delta^S y^*(t) + (1 - \alpha_1^*)\Delta^S y(t), \tag{33'}$$

Recalling that $\alpha_1^* \equiv (1/\sigma + \lambda - 1)/\lambda$, policy follows Milton Friedman's constant money-growth rule (with appropriate allowance for changes in long-run potential output growth) when $\sigma = 1$. In the more realistic case where $\sigma < 1$, optimal policy restrains money growth relative to the Friedman prescription whenever realized output growth exceeds a (lagged) estimate of growth in potential output.

Still assuming that S is finite, suppose that $\alpha_0(t)$ is defined as in Equation 30. Then Equation 33 says that money growth should be governed by

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[16] Tacking a money demand function on to our model in this way is obviously not entirely satisfactory. A simple trick for generating household demand for money without complicating the policy optimization problem is presented by Erceg, Henderson and Levin (2000).

[17] See Note #12, and recall that $\alpha_1^* = (1/\sigma + \lambda - 1)/\lambda$.

$$\Delta^S m(t) = \pi^T + \Delta^S y(t) - \alpha_1^* \left[\frac{y(t) - E_{t-S} y^*(t)}{S} \right]. \tag{33''}$$

Money growth fully accommodates output growth except insofar as realized output exceeds an estimate of potential output.

To obtain an *interest-rate* instrument rule, one must first model the linkages between interest rates and the real economy. A popular approach is to use the household intertemporal Euler equation for this purpose (Koenig 1993, McCallum and Nelson 1999). Accordingly, suppose that

$$y(t) = E_t y(t+1) - \sigma[r(t) - \rho], \tag{34}$$

where $r(t)$ is the real 1-period interest rate and ρ is the representative household's rate of time preference. Then one will have $p(t) + \alpha_1^* y(t) = \alpha_0(t)$, as is required for optimal policy, if and only if,

$$r(t) = \rho + (1/\sigma)[E_t y(t+1) + (p(t) - \alpha_0(t))/\alpha_1^*], \tag{35}$$

where the aggregate-demand intercept, $\alpha_0(t)$, can be any function of information available at $t - S$.[18] If $\alpha_0(t)$ is defined as in Equation 28, for example, policy will be optimal provided

$$r(t) = E_{t-S} r^*(t) + \left(\frac{S+1}{\sigma}\right)\left[E_t \Delta^{S+1} y(t+1) - E_{t-S} \Delta^{S+1} y^*(t+1)\right] + \left(\frac{S}{\sigma\alpha_1^*}\right)\left[\Delta^S p(t) - \pi^T\right], \tag{35'}$$

where $r^*(t) \equiv \rho + (1/\sigma)E_t \Delta y^*(t+1)$ is the real interest rate in market-clearing equilibrium–i.e., the "natural" rate. Equation 35' says that the monetary authority should drive the market interest rate up relative to an estimate of the natural rate whenever expected output growth exceeds an estimate of potential growth or realized inflation exceeds target inflation. If $\alpha_0(t)$ is defined as in Equation 30, in contrast, then policy will be optimal provided

[18] I ignore complications that can arise when the nominal interest rate is bounded below [Benhabib, Schmitt-Grohe and Uribe (2001); Woodford (2003), Chapter 2, Section 4].

$$r(t) = E_{t-S}r^*(t) + \left(\tfrac{1}{\sigma}\right)\left[E_t y(t+1) - E_{t-S}y^*(t+1)\right] + \left(\tfrac{S}{\sigma\alpha_1}\right)\left[\Delta^S p(t) - \pi^T\right]. \tag{35''}$$

Now, the monetary authority drives the market interest rate up relative to the estimated natural rate whenever there is expected to be a gap between actual output and trend potential output or realized inflation is above target.

Equation 35" is a form of Taylor rule. It relates the real short-term interest rate to output and inflation gaps. (In this case, the output gap is *pro*spective while the inflation gap is *retro*spective.) Equation 35' is very like a rule that Orphanides and Williams (2002) claim is likely to do better than the Taylor rule whenever the size of the output gap is highly uncertain. In the model of Fischerian wage setting developed here, however, the policy rules defined by Equations 35' and 35" do equally well. Moreover, their success hinges on the fact that they do *not* make the short-term interest rate contingent on best-available estimates of the current output gap, current potential growth, or the current natural rate of interest. Efforts to improve these estimates are, according to the analysis presented here, misplaced.

IV. IMPERFECT IMPLEMENTATION OF OPTIMAL POLICY

A Modified Output-Gap Equation. Suppose that the monetary authority cannot respond to new information with less than a one-period lag. The result is that the authority loses control of the current-period position and slope of the aggregate demand schedule. Formally, although the authority can ensure that $E_{t-1}p(t) = \alpha_0(t) - \alpha_1 E_{t-1}y(t)$ for pre-announced $\alpha_0(t)$ and α_1, the realized period-t aggregate demand schedule is $p(t) = a_0(t) - a_1 y(t)$, where $a_0(t)$ and a_1 depend on the instrument used to implement policy, but are *not* period-t choice variables. The reduced-form equation for the output gap (Equation 26) becomes:

$$z(t) = \left[\frac{1}{\alpha_1 + \beta_1(0)}\right]\{(\alpha_1^* - \alpha_1)[y^*(t) - E_{t-1}y^*(t)] + [\alpha_0(t) - E_{t-1}\alpha_0(t)]\}$$

$$+ \sum_{i=1}^{S-1}\left[\frac{1}{\alpha_1 + \beta_1(i)}\right]\{(\alpha_1^* - \alpha_1)[E_{t-i}y^*(t) - E_{t-i-1}y^*(t)] + [E_{t-i}\alpha_0(t) - E_{t-i-1}\alpha_0(t)], \quad (36)$$

where α_1^* is defined as in Equation 26. As examples, we consider money-supply targeting and nominal-interest-rate targeting, in turn. Under money-supply targeting, policymakers cannot completely offset money demand shocks nor guarantee that output will move 1-for-1 with changes in potential output. Properly conducted nominal-interest-rate targeting, in contrast, produces a 1-for-1 response of output to potential provided that expected future *growth* in potential is constant. However, time-preference shocks cannot be fully offset. The bottom line is that nominal-interest-rate targeting is fully successful at maintaining a zero output gap only in the absence of shocks to the natural real interest rate.

As noted above, the monetary authority must be concerned about cross-household variation in labor effort insofar as there are innovations to $z(t)$ (c.f. Note #11). For future reference, Equation 36 implies that

$$z(t) - E_{t-1}z(t) = \left[\frac{1}{\alpha_1 + \beta_1(0)}\right]\{(\alpha_1^* - \alpha_1)[y^*(t) - E_{t-1}y^*(t)] + [\alpha_0(t) - E_{t-1}\alpha_0(t)]\} \quad (36')$$

and

$$E_{t-i}z(t) - E_{t-i-1}z(t) = \left[\frac{1}{\alpha_1 + \beta_1(i)}\right]\{(\alpha_1^* - \alpha_1)[E_{t-i}y^*(t) - E_{t-i-1}y^*(t)] + [E_{t-i}\alpha_0(t) - E_{t-i-1}\alpha_0(t)]\}$$

(36")

for $i = 1, 2, ..., S - 1$.

Money as a Policy Instrument. Suppose that the demand for money equals $p(t) + y(t) + \nu(t)$, where $\nu(t)$ is an exogenous disturbance that is uncorrelated with shifts in labor productivity. The monetary authority must choose the money supply, $m(t)$, one period in advance, and sets $m(t) = \alpha_0(t) + E_{t-1}\nu(t) + (1 - \alpha_1)E_{t-1}y(t)$ for some pre-announced $\alpha_0(t)$ and

α_1. By equating money demand to money supply, and taking expectations, it's easily verified that $E_{t\text{-}1}p(t) = \alpha_0(t) - \alpha_1 E_{t\text{-}1}y(t)$. Thus, $\alpha_0(t)$ and α_1 determine a "long-run" aggregate demand schedule with vertical intercept $\alpha_0(t)$ and slope $-\alpha_1$. Once in period t, the money supply is pre-determined and, so, price and output are constrained by a short-run aggregate demand equation of the form $p(t) = [m(t) - \nu(t)] - y(t)$. In the notation introduced above, $a_0(t) = m(t) - \nu(t)$ and $a_1 = 1$. Substituting into Equations 36 and 36':

$$z(t) = \left[\frac{1}{1+\beta_1(0)}\right]\{(\alpha_1^* - 1)[y^*(t) - E_{t-1}y^*(t)] - [\nu(t) - E_{t-1}\nu(t)]\}$$
$$+ \sum_{i=1}^{S-1}\left[\frac{1}{\alpha_1+\beta_1(i)}\right]\{(\alpha_1^* - \alpha_1)[E_{t-i}y^*(t) - E_{t-i-1}y^*(t)] + [E_{t-i}\alpha_0(t) - E_{t-i-1}\alpha_0(t)]\}. \quad (37)$$

$$z(t) - E_{t-1}z(t) = \left[\frac{1}{1+\beta_1(0)}\right]\{(\alpha_1^* - 1)[y^*(t) - E_{t-1}y^*(t)] + [\nu(t) - E_{t-1}\nu(t)]\}. \quad (37')$$

Equation 36" remains valid without any modification. Clearly, the period-t innovation to $z(t)$ (Equation 37') is completely outside of the monetary authority's control. In contrast, earlier innovations to $z(t)$ (Equation 36") are eliminated provided that the monetary authority announces $\alpha_0(t)$ at least S periods in advance and sets $\alpha_1 = \alpha_1^*$. This same strategy minimizes the variance of $z(t)$: There is no conflict between the goal of minimizing the variance of $z(t)$ and the goal of minimizing cross-household labor variation.

Under optimal policy, Equation 37 collapses to

$$z(t) = \left[\frac{1}{1+\beta_1(0)}\right]\{(\alpha_1^* - 1)[y^*(t) - E_{t-1}y^*(t)] - [\nu(t) - E_{t-1}\nu(t)]\}. \quad (37^*)$$

According to 37*, whether or not output responds appropriately to current-period supply shocks depends entirely on the elasticity of intertemporal substitution, σ. If $\sigma > 1$, then $\alpha_1^* < 1$ and output under-reacts to potential-output shocks. If $\sigma < 1$–as suggested by empirical studies–then $\alpha_1^* > 1$ and output over-reacts. Only when $\sigma = 1$ does output move 1-for-1 with potential output. Money-demand shocks cause output to drop below potential. Neither type of shock has persistent output-gap effects if policy is optimal.

The Short-Term Interest Rate as a Policy Instrument. Analysis of imperfect policy implementation is considerably more complicated under interest-rate targeting than under money-supply targeting. We begin by generalizing Equation 34 to allow for variation in the representative household's rate of time preference (with any such variation assumed to be uncorrelated with potential-output shocks):

$$y(t) = E_t y(t+1) - \sigma\{R(t) - [E_t p(t+1) - p(t)] - \rho(t)\}. \tag{38}$$

Here $R(t)$ is the period-t short-term nominal interest rate. Given our assumption that the monetary authority must set $R(t)$ using information available in period t - 1, Equation 38 says that the current-period aggregate demand schedule has a slope with absolute value $a_1 = 1/\sigma$ and vertical intercept $a_0(t) = (1/\sigma)E_t y(t+1) + E_t p(t+1) + \rho(t) - R(t)$.

To impose a long-run aggregate demand schedule of the form $E_{t-1}p(t) = \alpha_0(t) - \alpha_1 E_{t-1}y(t)$, the monetary authority need only set

$$R(t) = E_{t-1}\rho(t) + E_{t-1}\pi(t+1) + (1/\sigma)\{E_{t-1}y(t+1) + [E_{t-1}p(t) - \alpha_0(t)]/\alpha_1\} \tag{39}$$

for some pre-announced $\alpha_0(t)$. It follows that Equations 24 and 25 remain valid when evaluated at s = 1, and can be used to eliminate $E_t y(t+1)$ and $E_t p(t+1)$ from the expression for $a_0(t)$ given above. In particular:

$$a_0(t) - E_{t-1}a_0(t) = [\rho(t) - E_{t-1}\rho(t)] + (\tfrac{1}{\sigma} - \alpha_1)\left[\frac{\alpha_1^* + \beta_1(1)}{\alpha_1 + \beta_1(1)}\right][E_t y^*(t+1) - E_{t-1}y^*(t+1)]$$
$$+ \left[\frac{\frac{1}{\sigma} + \beta_1(1)}{\alpha_1 + \beta_1(1)}\right][E_t\alpha_0(t+1) - E_{t-1}\alpha_0(t+1)]. \tag{40}$$

From Equation 36, it follows that:

$$z(t)=\left[\frac{\frac{1}{\sigma}-\alpha_1}{\frac{1}{\sigma}+\beta_1(0)}\right]\left[\frac{\alpha_1^*+\beta_1(1)}{\alpha_1+\beta_1(1)}\right][E_t\Delta y^*(t+1)-E_{t-1}\Delta y^*(t+1)]+\left[\frac{\alpha_1^*-\alpha_1}{\frac{1}{\sigma}+\beta_1(0)}\right][y^*(t)-E_{t-1}y^*(t)]$$

$$+\left[\frac{1}{\alpha_1+\beta_1(1)}\right]\left[\frac{\frac{1}{\sigma}+\beta_1(1)}{\frac{1}{\sigma}+\beta_1(0)}\right][\alpha_0(t+1)-E_{t-1}\alpha_0(t+1)]+\left[\frac{1}{\frac{1}{\sigma}+\beta_1(0)}\right][\rho(t)-E_{t-1}\rho(t)]$$

$$+\textstyle\sum_{i=1}^{S-1}\left[\frac{1}{\alpha_1+\beta_1(i)}\right]\{(\alpha_1^*-\alpha_1)[E_{t-i}y^*(t)-E_{t-i-1}y^*(t)]+[E_{t-i}\alpha_0(t)-E_{t-i-1}\alpha_0(t)]\}. \qquad (41)$$

According to this equation, policymakers' choice of α_1 is key for determining how the economy reacts to innovations in potential output and its future growth rate. If policymakers set $\alpha_1 = 1/\sigma$, then current output will be insulated from shocks to expected future potential growth, but will respond inappropriately to innovations in current potential output. If they set $\alpha_1 = \alpha_1^*$, on the other hand, then output responds 1-for-1 to current potential output, but also reacts to new information about future growth in potential output when it should not. There are two special cases in which this tension regarding α_1 disappears. The first is when $\alpha_1^* = 1/\sigma$, which (from the definition of α_1^*) occurs precisely when $\sigma = 1$. The other special case is when potential-growth forecasts aren't revised, so that the only source of unexpected variation in the natural real interest rate (c.f. Equation 35') is time-preference shocks. This case applies, for example, when potential output follows a random walk with drift.

The random-walk-with-drift assumption is a reasonable approximation. Suppose that it holds. Then, the reduced-form output-gap equation becomes:

$$z(t)=\left[\frac{\alpha_1^*-\alpha_1}{\frac{1}{\sigma}+\beta_1(0)}\right][y^*(t)-E_{t-1}y^*(t)]$$

$$+\left[\frac{1}{\alpha_1+\beta_1(1)}\right]\left[\frac{\frac{1}{\sigma}+\beta_1(1)}{\frac{1}{\sigma}+\beta_1(0)}\right][\alpha_0(t+1)-E_{t-1}\alpha_0(t+1)]+\left[\frac{1}{\frac{1}{\sigma}+\beta_1(0)}\right][\rho(t)-E_{t-1}\rho(t)]$$

$$+\textstyle\sum_{i=1}^{S-1}\left[\frac{1}{\alpha_1+\beta_1(i)}\right]\{(\alpha_1^*-\alpha_1)[E_{t-i}y^*(t)-E_{t-i-1}y^*(t)]+[E_{t-i}\alpha_0(t)-E_{t-i-1}\alpha_0(t)]\}. \quad (42)$$

The period-t output-gap innovation takes the form:

$$z(t) - E_{t-1}z(t) = \left[\frac{\alpha_1^* - \alpha_1}{\frac{1}{\sigma} + \beta_1(0)}\right][y^*(t) - E_{t-1}y^*(t)]$$
$$+ \left[\frac{1}{\alpha_1 + \beta_1(1)}\right]\left[\frac{\frac{1}{\sigma} + \beta_1(1)}{\frac{1}{\sigma} + \beta_1(0)}\right][\alpha_0(t+1) - E_{t-1}\alpha_0(t+1)] + \left[\frac{1}{\frac{1}{\sigma} + \beta_1(0)}\right][\rho(t) - E_{t-1}\rho(t)]. \tag{42'}$$

Earlier innovations remain governed by Equation 36".

Interestingly, Equations 42 and 42' imply that the monetary authority can reduce both the variability of output about potential and the cross-household variation in labor effort by promising to make *next* period's interest rate contingent on *this* period's time-preference shock.[19] The easiest case is when wage adjustment is complete after one period, so that the summation term drops off of Equation 42. Then, to ensure that $z(t) = 0$ at all times the monetary authority need only set $\alpha_1 = \alpha_1^*$ and $\alpha_0(t + 1) = E_{t-1}\alpha_0(t + 1) - [\rho(t) - E_{t-1}\rho(t)]$, where $E_{t-1}\alpha_0(t + 1)$ is an arbitrary function of information available in period $t - 1$. From Equation 39 (updated one period), policymakers should respond to an unexpected increase in the rate of time preference by announcing they will raise the period $t + 1$ interest-rate target.

If wage adjustment takes more than one period ($S > 1$), there is no interest-rate rule that keeps output equal to potential in the face of shocks to the rate of time preference. Keeping in mind both the variance of the output gap and the variances of innovations to the output gap, the best the monetary authority can do is to set $\alpha_1 = \alpha_1^*$ and

$$\alpha_0(t) = E_{t-S}\alpha_0(t) - \phi[\rho(t - 1) - E_{t-2}\rho(t - 1)], \tag{43}$$

for all t, where the coefficient ϕ is chosen by the monetary authority to minimize a weighted sum of the variance of $z(t)$ and the variance of $z(t) - E_{t-1}z(t)$. It follows that $0 < \phi < [\alpha_1^* + \beta_1(1)]/[1/\sigma + \beta_1(1)]$. As in the case where $S = 1$, Equations 39 and 43 say that policymakers should announce an increase in next period's interest rate whenever there is a shock to $\rho(t)$.

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[19] I'm grateful to Nathan Balke for calling my attention to this implication.

If the monetary authority can credibly commit to setting $\alpha_0(t)$ according to Equation 43, then the output gap will evolve according to the equation

$$z(t) = \psi\left[\frac{\rho(t) - E_{t-1}\rho(t)}{\frac{1}{\sigma} + \beta_1(0)}\right] - (1-\psi)\left[\frac{\rho(t-1) - E_{t-2}\rho(t-1)}{\frac{1}{\sigma} + \beta_1(1)}\right], \qquad (42^*)$$

where

$$0 < \psi \equiv \phi\frac{[\frac{1}{\sigma} + \beta_1(1)]}{[\alpha_1^* + \beta_1(1)]} < 1.$$

Thus, the output gap follows a moving average process with negative serial correlation. The only source of variation in the gap is unexpected variation in the rate of time preference (*IS* shocks). In contrast to results under money-supply targeting (Equation 37'), changes in the level of potential output are always fully reflected in actual output.

Suppose that, convinced by this analysis, the monetary authority sets $\alpha_1 = \alpha_1^*$ and adjusts $\alpha_0(t)$ in accordance with Equation 43. However, at some point private agents revise upward their expectations of the economy's future growth potential. Something similar may have occurred in the 1990s, when an acceleration in trend productivity growth was not immediately apparent to policymakers. According to Equation 41, whether current output responds to the expectations revision by contracting or by expanding depends on whether α_1^* is greater than or less than $1/\sigma$. Since α_1^* is always less than $1/\sigma$ in the empirically realistic case where $\sigma < 1$, an interest-rate policy that assumes stable growth in potential output will likely prove too easy in the event that the potential-growth outlook suddenly improves.

V. THE PERSISTENCE (NON)PROBLEM

Chari, Kehoe and McGrattan (2000) (hereafter, C-K-M) show that economies with staggered price setting do not necessarily adjust more gradually to demand shocks than do economies with synchronous price setting. Indeed, if labor is mobile and wages are flexible, staggered price setting is likely to reduce persistence, not increase it.[20] The intuition is that the employment expansion that accompanies a demand shock drives the market wage up sharply, so that firms with the chance to do so set their price at a level that overshoots the market-clearing price. When wage contracts are in place, upward wage pressure is muted, so that the C-K-M results are overturned.[21]

The first step in the formal argument is to note that Equation 25 says that the fraction $\beta_1(s)/[\alpha_1 + \beta_1(s)]$ of the eventual total price adjustment is completed s periods after a demand shock hits a sticky-wage economy.[22] If wage setting is synchronous, but randomly distributed over time in the same way as wage-adjustment opportunities in the staggered-wage-setting model, the corresponding fraction is $\Omega(s)$, on average, where $\Omega(\bullet)$ is the cumulative distribution function of wage-adjustment opportunities (c.f. Equation 15). Thus, staggered wage setting contributes to persistence if, and only if, $\beta_1(s)/[\alpha_1 + \beta_1(s)]$ is less than $\Omega(s)$. Using the definition of $\beta_1(s)$ (which immediately follows Equation 16) and a little algebra, the key inequality holds precisely when $\beta < \alpha_1$, where β measures the sensitivity of wages (and prices) to excess demand (c.f. Equation 10).

Clearly, the specification of aggregate demand–the monetary authority's choice of α_1, in particular–is important for determining whether staggered wage adjustment enhances persistence.[23] It is often assumed, without much discussion, that the money supply is

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[20] C-K-M assume that each firm chooses a fixed price rather than a price path. However, Koenig (1999b) demonstrates that the same results hold in a world of sticky-information price setting, provided labor is free to move between firms.

[21] This intuition is further developed in Koenig (2000). Of course, the same conclusion holds for sticky-price models with immobile labor, because they are isomorphic to sticky-wage models, absent productivity shocks. See Koenig (2000) and Edge (2002).

[22] This is also the fraction of the initial output response eliminated after s periods, where "the initial output response" is defined to be the horizontal shift in the aggregate demand schedule.

[23] In contrast, the distribution of wage-adjustment opportunities is irrelevant.

exogenous and rigidly linked to nominal spending, so that $\alpha_1 = 1$.[24] Under this assumption there is a persistence problem if, and only if, $(1/\sigma - 1) > (\lambda - 1)(\varepsilon - 1)$ –a condition that may or may not hold. However, it may make better sense to assume that even if the monetary authority fails to completely eliminate aggregate-demand shifts, it at least gets the slope of the aggregate-demand schedule right, and sets $\alpha_1 = \alpha_1^* = (1/\sigma + \lambda - 1)/\lambda$. A little algebra reveals that $\beta < \alpha_1^*$ if, and only if, ε and λ both exceed 1, which is *always* true. In fact,

$$\frac{\beta_1(s)}{\alpha_1^* + \beta_1(s)} = \Omega(s)\left[\frac{\lambda}{\lambda + (1-\Omega(s))(\varepsilon - 1)(\lambda - 1)}\right],$$

which is necessarily less than $\Omega(s)$, as required for staggering to add to persistence. More generally, persistence is a non-problem whenever the monetary authority's aggregate-demand-slope choice is even approximately optimal.

VI. INFLATION DYNAMICS: FROM PHILLIPS CURVE TO P-BAR

Introduction. Standard Phillips curves relate inflation to lagged inflation or lagged inflation expectations, productivity-related supply-side shocks (captured by relative price changes in empirical work), and the output gap. Monetary policy innovations affect inflation only through this last term. McCallum's P-bar model, in contrast, says that inflation equals the market-clearing inflation rate plus an output-gap adjustment that acts like an error-correction term (McCallum 1994). Monetary policy can affect inflation by changing the market-clearing inflation rate even if the output gap is zero. On the other hand, productivity shocks have no effect on inflation except through the market-clearing inflation rate and/or the output gap, according to McCallum. The sticky-information wage-setting model bridges the differences between the Phillips curve and P-bar models. It predicts that

[24] C-K-M always make the former assumption, and sometimes the latter. Koenig (1999b, 2000) and Devereux and Yetman (2003) are guilty on both counts.

near-term inflation movements are well-described by a traditional Phillips curve, but that longer-horizon inflation behavior is essentially as described by McCallum. In particular, policy innovations impact near-term inflation exclusively through the output gap, but they can affect longer-term inflation expectations independent of the gap.

Inflation Dynamics. The shortest-of-short-run Phillips curves is Equation 17, repeated here for convenience:

$$\pi(t) - E_{t-1}\pi(t) = \beta_1(0)[z(t) - E_{t-1}z(t)] - [\Delta\theta(t) - E_{t-1}\Delta\theta(t)]. \tag{17}$$

Consistent with traditional Phillips-curve models, Equation 17 says that analysts should revise their inflation forecasts upward only when new information suggests that the output gap is larger than previously believed or that productivity is lower than previously believed.[25] Monetary-policy innovations affect inflation only through the output gap.

Equation 17 makes no assumptions at all about the demand side of the economy. However, any inflation model that relates price changes to the market-clearing inflation rate–as does McCallum's–is necessarily a mix of demand-side and supply-side elements. Here, our only assumption is that monetary policy determines an aggregate demand schedule with vertical intercept $\alpha_0(t)$ and slope $-\alpha_1 < 0$.[26] The market-clearing price level is naturally defined by $p^*(t) \equiv \alpha_0(t) - \alpha_1 y^*(t)$.

Taking the first difference of the basic aggregate supply relationship, Equation 16, and using the fact that $z(t) = [p^*(t) - p(t)]/\alpha_1$, one obtains a formula relating the impulse response function of inflation to the impulse response functions of the market-clearing

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[25] A standard accelerationist wage Phillips curve has the form $\Delta w(t) = \Delta w(t - 1) + bz(t)$, which implies that $\pi(t) = \pi(t - 1) + bz(t) - [\Delta\theta(t) - \Delta\theta(t - 1)]$. Take expectations as of $t - 1$, subtract, and you get a relationship that looks exactly like Equation 17 (with $b = \beta_1(0)$).

[26] We've already discussed how such a policy can be implemented through control of the money supply or the short-term interest rate. See Section III.

inflation rate, the output gap, and productivity growth:

$$
\begin{aligned}
E_t\pi(t+s) - E_{t-1}\pi(t+s) = &\left[\frac{\beta_1(s-1)}{\alpha_1 + \beta_1(s-1)}\right]\left[E_t\pi^*(t+s) - E_{t-1}\pi^*(t+s)\right] \\
&+ \left[\frac{\alpha_1\Delta\beta_1(s)}{\alpha_1 + \beta_1(s-1)}\right]\left[E_t z(t+s) - E_{t-1}z(t+s)\right] \\
&- \left[\frac{\alpha_1}{\alpha_1 + \beta_1(s-1)}\right]\left[E_t\Delta\theta(t+s) - E_{t-1}\Delta\theta(t+s)\right], \qquad (44)
\end{aligned}
$$

for $s \geq 1$. The three coefficients in square brackets depend on the conduct of monetary policy (through α_1, the slope of the aggregate demand curve) as well as the slope of the aggregate supply schedule, $\beta_1(s)$.

The coefficient of $[E_t\pi^*(t + s) - E_{t-1}\pi^*(t + s)]$ in Equation 44 rises toward 1 as s increases toward S, while the coefficient of $[E_t\Delta\theta(t+s) - E_{t-1}\Delta\theta(t+s)]$ falls toward 0. For $s \approx S$, then, inflation behaves as in McCallum's P-bar model, responding 1-for-1 to innovations in the market-clearing inflation rate. Productivity shocks have no independent relevance.

Special Cases. Suppose that the economy was initially expected to be in market-clearing equilibrium (with $E_{t-1}\pi(t + s) = E_{t-1}\pi^*(t + s)$ and $E_{t-1}z(t + s) = 0$). Equation 44 becomes

$$
\begin{aligned}
E_t\pi(t+s) = &\left[\frac{\beta_1(s-1)}{\alpha_1 + \beta_1(s-1)}\right]E_t\pi^*(t+s) + \left[\frac{\alpha_1}{\alpha_1 + \beta_1(s-1)}\right]E_{t-1}\pi(t+s) \\
&+ \left[\frac{\alpha_1\Delta\beta_1(s)}{\alpha_1 + \beta_1(s-1)}\right]E_t z(t+s) \\
&- \left[\frac{\alpha_1}{\alpha_1 + \beta_1(s-1)}\right]\left[E_t\Delta\theta(t+s) - E_{t-1}\Delta\theta(t+s)\right]. \qquad (45)
\end{aligned}
$$

The coefficient on the output gap in Equation 45 is positive in every period during which wage plans are subject to revision. The coefficients of $E_{t-1}\pi(t + s)$ and $E_t\pi^*(t + s)$ are both non-negative and sum to 1. The longer the forecast horizon (the larger is s), the greater the weight placed on the market-clearing inflation rate, and the smaller the weight placed on lagged inflation expectations. The weight on productivity-growth innovations also declines the further into the future one peers. Inflation behaves less and less as predicted by a traditional Phillips curve, and more and more as predicted by McCallum.

Suppose, for example, that $\beta_1(0) = 0$ and $\beta_1(s) = \beta[(1 + \delta)^s - 1]$ for $s \geq 1$, consistent with a geometric distribution of wage adjustments. Then, Equation 45 becomes

$$\begin{aligned} E_t\pi(t+s) = {} & \left[\frac{\beta[(1+\delta)^{s-1}-1]}{\alpha_1 + \beta[(1+\delta)^{s-1}-1]}\right]E_t\pi^*(t+s) + \left[\frac{\alpha_1}{\alpha_1 + \beta[(1+\delta)^{s-1}-1]}\right]E_{t-1}\pi(t+s) \\ & + \left[\frac{\alpha_1\delta\beta(1+\delta)^{s-1}}{\alpha_1 + \beta[(1+\delta)^{s-1}-1]}\right]E_t z(t+s) \\ & - \left[\frac{\alpha_1}{\alpha_1 + \beta[(1+\delta)^{s-1}-1]}\right][E_t\Delta\theta(t+s) - E_{t-1}\Delta\theta(t+s)], \end{aligned} \tag{46}$$

which reduces to a traditional Phillips curve,

$$E_t\pi(t+1) = E_{t-1}\pi(t+1) + \delta\beta E_t z(t+1) - [E_t\Delta\theta(t+1) - E_{t-1}\Delta\theta(t+1)] \tag{46'}$$

at $s = 1$, and approaches a McCallum-style equation,

$$\begin{aligned} E_t\pi(t+s) &= E_t\pi^*(t+s) + \alpha_1\delta E_t z(t+s) \\ &= E_t\pi^*(t+s) - \delta E_t[p(t+s) - p^*(t+s)], \end{aligned} \tag{46''}$$

in the limit, as $s \to \infty$. Comparing Equations 46' and 46", whether inflation is more or less sensitive to the output gap at long horizons than at short horizons is determined by whether α_1 is greater than or less than β. From our discussion of the persistence problem, however, we know that $\alpha_1^* > \beta$. If the monetary authority sets $\alpha_1 = \alpha_1^*$, therefore, the inflation impact of the output gap will be larger at longer horizons.

VII. SUMMARY AND CONCLUSION

Sluggish nominal wage adjustment doesn't matter (is non-distorting) if wage setters never have a reason to regret their decisions. If wage adjustment is sluggish because of the costs of information gathering ("sticky information"), eliminating wage regrets means eliminating aggregate wage surprises over the horizon of the longest labor contract or, more generally, over the interval required for new information to be fully reflected in wages. The basic reasoning is the same as that used by Ball, Mankiw and Reis (2003) in their analysis of optimal policy in an economy with sticky-information price setting, but the implications for output and prices are quite different. Whereas the sticky-information-price model says that policymakers ought to focus solely on hitting a pre-announced price-level target, the sticky-information-wage model says that the monetary authority should give explicit attention to output stabilization. It predicts that trend deviations in prices will be negatively correlated with trend deviations in output (Kydland and Prescott 1990).

A variety of familiar interest-rate and money growth rules are potentially consistent with optimal policy. For example, Friedman's constant-money-growth rule is optimal under certain conditions, and a version of the Taylor rule is optimal if household consumption decisions are forward looking.

It's essential that the potential-output (or potential-output growth) estimates that enter the monetary authority's instrument rule *not* incorporate the latest information. Similarly, policymakers do not need–and should not use–the latest estimates of the "natural" real interest rate. Monetary policy must define a predictable price–output trade-off. The required predictability is lacking if policy is contingent on information unavailable when current wages were set.

If the monetary authority must choose a value for its policy instrument in advance, it may be difficult for the authority to generate the proper short-run trade-off between output and its price. A full offset of the effects of sluggish wage adjustment can be achieved under interest-rate targeting, for example, only if the natural real interest rate is constant.

Results are derived using a completely general specification of the timing of wage adjustments. The geometric timing distribution assumed in the existing literature is a special case. Although it remains true that inflation dynamics are captured by an expectations-augmented Phillips curve, the form of the so-called "speed effect" is sensitive to changes in wage-timing assumptions. More importantly, certain policy rules–including the Taylor rule–are optimal only if new information is completely reflected in wages in finite time.

The demand side of the economy is also treated with greater generality, here, than in much of the existing literature. Relaxing demand-side restrictions reveals that staggering wage adjustments in a sticky-information economy almost certainly prolongs the economy's response to nominal shocks, in sharp contrast to results obtained by Chari, Kehoe and McGrattan (2000) in an economy with staggered price setting.

Finally, this paper unifies different strands of the literature on the dynamics of inflation. At short horizons, inflation's response to shocks matches that predicted by a traditional expectations-augmented Phillips curve. At somewhat longer horizons, inflation is well-described by McCallum's (1994) P-bar model. Finally, at horizons long enough that shocks are fully reflected in wages, inflation behaves as in a real-business-cycle model.

In future research, it will be desirable to allow for staggered price adjustment as well as staggered wage adjustment.[27] In such an economy, there will be endogenous variation in the markup of price over unit labor cost. Moreover–as discussed by Erceg, Henderson, and Levin (2000) in the context of an economy with Calvo-style fixed-price and fixed-wage contracts–it will likely be impossible for monetary policy to obtain the Pareto optimum.

[27] Two efforts in this direction are Mankiw-Reis (2003) and Koenig (2000). Both papers use a sticky-information framework. Mankiw and Reis analyze optimal policy in a 1-period, multi-sector, yeoman-farmer economy in which one sector "produces" and sells a composite good called labor. Koenig adds sticky-information prices to a simplified version of the model developed here. His focus is on implications for persistence rather than optimal policy.

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APPENDICES

Appendix A: The Wage Decision

After using Equations 4 and 6 to eliminate P and N, Equation 8 in the main text can be solved for W_i:

$$W_i = [(\frac{1}{1+\tau})(\frac{\epsilon}{\epsilon-1})(\frac{\eta}{\eta-1})\frac{E_{t-s}((Y/\Theta)^{\lambda}W^{\epsilon\lambda})}{E_{t-s}(Y^{(\sigma-1)/\sigma}W^{\epsilon-1})}]^{\frac{1}{1+\epsilon(\lambda-1)}}. \qquad \text{(A.1)}$$

Here, it is assumed that households are able to insure themselves against idiosyncratic fluctuations in consumption due solely to cross-household differences in the timing of contract negotiations. (Hence, $C_i = Y$.) Rearranging:

$$W_i = [\frac{E_{t-s}[Y^{*(\sigma-1)/\sigma}Z^{\lambda}W^{\epsilon\lambda}]}{E_{t-s}[Y^{*(\sigma-1)/\sigma}Z^{(\sigma-1)/\sigma}W^{\epsilon-1}]}]^{\frac{1}{1+\epsilon(\lambda-1)}}, \qquad \text{(A.2)}$$

where $Z \equiv Y/Y^*$.

Finally, assume that $w \equiv ln(W)$, $y^* \equiv ln(Y^*)$, and $z \equiv ln(Y/Y^*)$ have a trivariate normal distribution conditional on information available at time $t - s$, and that the conditional variances and covariances of w, y^*, and z depend only on s. Then

$$w_i(t) = E_{t-s}[w(t) + \beta z(t)] + \Re(s), \qquad \text{(A.3)}$$

where $\Re(s)$ is a risk premium that depends only on conditional variances and covariances. Apart from this last term (which is suppressed in subsequent analysis), Equation A.3 matches Equation 9 in the main text. [Alternatively, following Woodford (2003), one can obtain Equation 9 by solving a log-linear approximation to A.2.]

Appendix B: The Expectations-Augmented Phillips Curve

Equation 23 in the main text generalizes to

$$E_{t-i}\pi(t) - E_{t-i-1}\pi(t) = \beta_1(i)[E_{t-i}z(t) - E_{t-i-1}z(t)] - \beta_1(i-1)[E_{t-i}z(t-1) - E_{t-i-1}z(t-1)]$$
$$-[E_{t-i}\Delta\theta(t) - E_{t-i-1}\Delta\theta(t)], \qquad \text{(A.1)}$$

for $i = 1, 2, 3, ...S - 1$. By applying this last equation recursively, one can extend Equation 23 back in time to obtain

$$\pi(t) - E_{t-s}\pi(t) = \sum_{i=0}^{s-1} \beta_1(i)[E_{t-i}z(t) - E_{t-i-1}z(t)] - \sum_{i=1}^{s-1} \beta_1(i-1)[E_{t-i}z(t-1) - E_{t-i-1}z(t-1)]$$

$$- [\Delta\theta(t) - E_{t-s}\Delta\theta(t)], \tag{A.2}$$

for $s = 2, 3, ...S$, and

$$\pi(t) - E_{t-s}\pi(t) = \sum_{i=0}^{S-1} \beta_1(i)[E_{t-i}z(t) - E_{t-i-1}z(t)] - \sum_{i=1}^{S} \beta_1(i-1)[E_{t-i}z(t-1) - E_{t-i-1}z(t-1)]$$

$$- [\Delta\theta(t) - E_{t-s}\Delta\theta(t)], \tag{A.2'}$$

for $s = S + 1, S + 2, S + 3, ...$. The s-period revision to inflation expectations is increasing in a distributed lag of past revisions to expectations of *current* excess demand, decreasing in a distributed lag of past revisions to expectations of one-period *lagged* excess demand, and decreasing in the s-period revision to expectations of productivity growth. A given revision to excess-demand expectations has a bigger inflation impact the earlier the revision occurs.

Equation 24 in the main text is obtained by taking a weighted average of Equations 23, A.2, and A.2'.