A Role for Government Policy and Sunspots in Explaining Endogenous Fluctuations in Illegal Immigration[1]

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Abstract

In this paper we provide an alternative explanation for why illegal immigration can exhibit substantial fluctuations despite a constant wage gap. We develop a model economy in which migrants make decisions in the face of uncertain border enforcement and lump-sum transfers from the host country. The uncertainty is extrinsic in nature, a sunspot, and arises as a result of ambiguity regarding the commodity price of money. Migrants are restricted from participating in state-contingent insurance markets in the host country, whereas host country natives are not. We establish the existence of sunspot equilibria that are not mere randomizations over certainty equilibria. Volatility in migration flows stems from two distinct sources: the tension between transfers inducing migration and enforcement discouraging it and secondly the existence of a sunspot. Finally, we examine the impact of a change in tax/transfer policies by the government on migration.

1 Introduction

1.1 Overview

Understanding what motivates migration, especially the large swings in illegal migration, and providing a cogent theoretical explanation consistent with observed migration dynamics has proven to be a daunting task for economists. In particular, most neoclassical economic models have not been able to account for significant changes in migration patterns in the face of constant or diminishing wage gaps.[1]

Explicitly modeling network externalities has been one theoretical tack for explaining fluctuations given a constant wage differential.[2] Networks induce agents to migrate by reducing the transactions costs associated with moving. While the marginal benefits in the form of wage differentials are unchanged, the marginal costs associated with moving are reduced — resulting in greater migration. However, this literature focuses chiefly on explaining increases to migration, and does not easily lend itself to account for the volatility observed in actual migration patterns — as it cannot account for falling, or even stable, migration flows.

In this paper we offer an alternative explanation; fluctuations can arise as a result of both (i) a lack of a coordinating mechanism when multiple equilibria exist and (ii) sunspots (when either a unique equilibrium or multiple equilibria exist). As with the network externality literature, we emphasize the role of non-wage factors in accounting for fluctuations, but unlike this literature we emphasize economic variables that impact both the marginal costs *and* benefits of migration. Specifically, we focus on the role of government transfers, border enforcement, and human smugglers as the primary forces affecting migration fluctuations. Governments, like in the US, have used these policy instruments (border enforcement and transfers to immigrants) to stem rising illegal immigration in recent years.[3] Multiple

[1] This is not a new problem and was first noted by Todaro (1969) over thirty years ago.

[2] For recent literature in this vein, see Taylor (1986), Carrington *et al.* (1996), and Stark and Wang (2002).

[3] The Welfare Reform Act of 1996 is an example of changes in government transfer programs that may influence migration flows.

equilibria can arise as a result of the tensions existing between enforcement and transfers (as factors discouraging, encouraging migration respectively) and fluctuations can occur as there exists no mechanism for coordinating on an equilibria over time. In addition, we introduce extrinsic uncertainty, sunspots, regarding the real value of nominal taxes and transfers. As a result of the sunspot, not only do the values of government transfers and taxes vary, but also the resources devoted to border enforcement.

Sunspots have been used to study endogenous fluctuations in other literatures: most notably in the business cycle fluctuations literature.[4] As Figure 1 illustrates, the volatility in Border Patrol apprehensions of unauthorized immigrants (a proxy for illegal immigration) is much greater than volatility in output, as measured by GDP.[5] Indeed, the standard deviation in output is 0.84% between 1991 and 2002 while the standard deviation of apprehensions is 14.6%. Insofar as endogenous fluctuations can explain observed volatility in output, they may also represent a partial explanation for observed volatility in illegal immigration.

Our goal in this paper is two-fold. First, we seek to develop a theoretical sunspot-based immigration model, characterized by real world entities such as human smugglers, border patrols and government transfers, which can account for fluctuations in migration flows (both increases and decreases) over time. Second, we wish to explore the impact of changes in policy instruments on the levels of illegal immigration and smuggler usage

Our analysis builds on the methods developed in Bhattacharya *et al.* (1998), henceforth BGS.[6] In that paper, the authors derive conditions under which stationary sunspot equilibria are not mere randomizations over certainty equilibria, and thus, sunspots matter in a ma-

[4]See, for instance, Azariadas and Guesnerie (1986), Woodford (1987), Farmer and Guo (1994), Farmer and Woodford (1997), and Benhabib and Kazuo (1998) for examples of model economies in which sunspots are offered as a possible explanation for the volatility observed in real GDP at business cycle frequencies.

[5]Figure 1 plots the detrended (log) levels of apprehensions by the U. S. border patrol. The data are quarterly observations for the period 1991:4 through 2002:4. We detrend the data by applying the HP filter. For reference we have also include a detrended (log) levels (different scale) of real GDP. Finally, we use border patrol apprehensions of undocumented immigrants as a proxy for the flow of illegal immigrants into the United States since actual data regarding the number of illegal immigrants crossing the border are unavailable.

[6]Although our paper and results most closely emulated those of BGS, they also depend on the generalization of BGS found in Keister (1998).

terial sense. Our paper closely follows the methodology of BGS, except for which groups of individuals have access to insurance markets. In our model, there is a "natural" restriction in which individuals not born in the host country do not have access to the state-contingent insurance markets available to host-country workers, although these restricted individuals are still assumed to be born after the state of nature is revealed.

1.2 Model Description and Main Findings

The basic structure of the economy is as follows. Individuals are born in either of two countries, hereafter designated as the home country and the host country. For simplicity, we assume that only some individuals from the home country migrate. Host-country individuals are divided into skilled and unskilled workers. Both sets of individuals supply labor, earn wages, pay taxes, and consume. The nominal value of taxes is known, while the goods value is not — due to uncertainty regarding the price level. Individuals born in the host country have access to a market in which state-contingent securities are traded and use this market to partially insure against price level uncertainty, and hence consumption volatility. Tax revenues are distributed between two government activities: border enforcement and transfers to migrant workers. Since the only uncertainty in this economy stems from the commodity price of money being unknown, all uncertainty in our model is extrinsic in nature.[7]

Some individuals born in the home country may choose to emigrate, spending a fraction of their time working in the home country and the rest crossing the border (evading enforcement) and working in the host country. Home-country workers are unskilled. In addition to wage income, migrant workers receive benefits from the host-country transfer program (e.g., education and health services). In addition to potential migrants, a fraction of the home country population is engaged in human smuggling. Smugglers are endowed with some knowledge of border enforcement techniques and divide their labor time between

[7] For a more complete discussion of extrinsic uncertainty, see Cass and Shell (1983) and Balasko (1983). Also related, Balasko and Shell (1986) study an overlapping-generations model in which lump-sum taxes and transfers are present.

acquiring additional information regarding border patrols and arranging for border crossings. Migrants who use the services of smugglers spend less time crossing the border and more time working in the host country relative to migrants who do not use smugglers.

Finally, individuals born in the home country do not have access to state-contingent securities markets in the host country. They represent the restricted set of individuals who are assumed to be born after the realization of the sunspot. As has been shown in previous literature, participation can matter for equilibria allocations under extrinsic uncertainty.[8]

Our results are easily summarized as follows. First, we show that sunspot equilibria exist in this economy. Moreover, the sunspot equilibria are not mere randomizations over certainty equilibria. Second, we classify two distinct sources of potential volatility which help to explain fluctuations in migration that are neither wage driven nor dependent on networks. Specifically, we derive conditions under which there exist multiple (two), non-sunspot equilibria in this economy. Multiple equilibria derive from the interplay between transfers received from migrating (migrants benefits) and the resulting impact these have on border crossing frictions (migrant costs). These frictions, in turn, depend on the level of border enforcement and the smuggling industry. Thus by explicitly, and more realistically, modeling the economic variables affecting both marginal costs and benefits, we derive combinations of migrant flows, transfers and enforcement levels at which the utility maximizing migrant is indifferent. However, there is no mechanism for coordinating on a particular equilibria, and over time, nothing prevents the economy from switching between equilibria, for a given realization of the sunspot. Hence volatility in migration patterns may result from the interactions between enforcement and transfers. In addition, even when there exists a unique equilibria, the two states of nature associated with the sunspot offers another source of volatility to the migration pattern. Third, we consider the effects that a change in transfer payments (or conversely taxes) would have on equilibrium outcomes. As expected, the exact impact of policy changes is highly dependent on which equilibrium and which state of nature

[8]See Cass and Shell (1983), Woodford (1986), Cass (1992), Balasko and Shell (1993) for a thorough development of the role of limited participation on fluctuations in stationary sunspot equilibria.

prevails.

The remainder of the paper is arranged as follows. The basic model is outlined in Section 2. Section 3 defines a sunspot equilibria and shows existence of pure sunspot equilibria while Section 4 describes the equilibrium values for the level of migration and quantity of smugglers services. We obtain the comparative static results in Section 5. Section 6 offers a brief summary and conclusions.

2 The Model

We consider a world consisting of two countries: a home country, from which individuals may choose to emigrate, and a host country, to which individuals illegally immigrate and from which there is no emigration. The economies of both countries are characterized by a standard two-period lived, overlapping generations model with production.[9] Time is discrete and indexed by $t = 0, 1, 2, ...$ For every date $t \geq 1$, there are N individuals born in both the host and home countries, where N is assumed to be finite and greater than or equal to two.[10] All individuals, regardless of their country of origin, are identical with respect to their preferences and time endowments; they are endowed with one unit of labor time when young and nothing when old, and value only old age consumption. Individuals differ with respect to skill levels, as discussed below.

We also introduce extrinsic uncertainty (i.e., sunspots) into the model economy. It is assumed that all taxes and transfers are denominated in nominal units, which for simplicity we will refer to as dollars. The goods price of these dollars, ρ, is assumed to be sunspot dependent.[11] We assume that there are two possible states of nature: $s \in \{\alpha, \beta\}$. Thus the

[9] As will be detailed below, production in both economies will be labor only (i.e., no capital required) and wages are fixed. Alternatively, one can reinterpret our economy as an endowment economy where the amount of the endowment received depends on the quantity of time spent in a respective country.

[10] There is no loss in generality by assuming that the populations of the two countries are identical. In addition, we could also assume a continuum of individuals with unit mass in both countries. A finite number of individuals is assumed solely for expository convenience.

[11] We follow Shell (1977), Cass and Shell (1983), Bhattacharya *et al.* (1998), Keister (1998), etc. in having the sunspot dependent variable be the price of money.

value of a unit of taxes will either be $\rho(\alpha)$ or $\rho(\beta)$ depending on which state occurs. Finally, we assume that the likelihood of state α (β) occurring is given by the probability π $(1-\pi)$ respectively.

2.1 Host Country

Individuals born in the host country are distinguished by their skill level. For simplicity, we assume there are two types of individuals: skilled and unskilled. An individual's skill level is common knowledge and an indication of their marginal productivity. At each date $t \geq 1$, the host-country production technology transforms labor into a single, homogeneous consumption good and workers are paid their marginal product. In addition, it is assumed that host-country natives do not migrate.

All host-country workers, regardless of skill level, have access to state-contingent markets for trading securities prior to the realization of the state, s and spot markets for trading goods after the realization of state s. The former allows all host country workers the ability to (partially) insure themselves against the uncertainty regarding the future state of nature.

2.1.1 Skilled Worker's Problem

A fraction ε of individuals within any given host-country generation are highly skilled workers. These individuals live for two periods and inelastically supply their one unit of labor when young. They earn a fixed wage ω^H.[12] In addition, they also face a lump-sum tax, τ_t^H, which is denominated in dollars and which, in real terms, cannot exceed the income earned by the worker. Thus, the value of these taxes in state s, is given by $\rho_t(s)\,\tau_t^H$. Individuals also have the opportunity to buy and sell state-contingent claims.[13] Since only old-age consumption is valued, individuals save their entire wage income net of taxes and any state contingent

[12] Alternatively, one can interpret this as an endowment economy where some individuals (high-skilled) receive a large endowment while others (low-skilled) receive a smaller endowment.

[13] One can think of individuals as issuing IOU's which are contingent upon whether state α or β prevails. We restrict individuals to having an overall non-negative position with respect to the total value of IOUs issued across the two states of nature.

securities in the form of a simple storage technology, which yields λ units of consumption in period $t+1$ for every unit of savings at t. Thus we can write the skilled worker's problem as

$$\max_{c_{t+1}^{H}(\alpha),c_{t+1}^{H}(\beta)} \pi U\left(c_{t+1}^{H}(\alpha)\right)+(1-\pi)U\left(c_{t+1}^{H}(\beta)\right) \tag{SHP}$$

subject to

$$\lambda\left[p_t^*(\alpha)\,\omega^H-p_t^m(\alpha)\,\tau_t^H\right]+\lambda\left[p_t^*(\beta)\,\omega^H-p_t^m(\beta)\,\tau_t^H\right]=p_t^*(\alpha)\,c_{t+1}^H(\alpha)+p_t^*(\beta)\,c_{t+1}^H(\beta)$$

where p_t^* is the price of the consumption good and p_t^m is the price of money. The price of the host-country consumption good is strictly positive while the price of money is nonnegative. It is useful to represent the host-country individual's resources available for consumption *after taxes.* We define the tax-adjusted endowment, $\bar{\omega}^H$, by

$$p_t^*(s)\,\bar{\omega}^H=p_t^*(s)\,\omega^H-p_t^m(s)\,\tau_t^H=p_t^*(s)\left[\omega^H-\frac{p_t^m(s)}{p_t^*(s)}\tau_t^H\right]$$

where

$$\frac{p_t^m(s)}{p_t^*(s)}=\rho_t(s)$$

is the goods price of money. The skilled worker's problem can be rewritten as

$$\max_{c_{t+1}^{H}(\alpha),c_{t+1}^{H}(\beta)} \pi U\left(c_{t+1}^{H}(\alpha)\right)+(1-\pi)U\left(c_{t+1}^{H}(\beta)\right)$$

subject to

$$\left[p_t^*(\alpha)\,\bar{\omega}^H+p_t^*(\beta)\,\bar{\omega}^H\right]\lambda=p_t^*(\alpha)\,c_{t+1}^H(\alpha)+p_t^*(\beta)\,c_{t+1}^H(\beta)\,.$$

As written, the individual's budget constraint is properly interpreted as workers having unrestricted access to the state-contingent claims markets. Bhattacharya *et al.* (1998), in their appendix, show the equivalence between the above formulation of an individual's budget constraint and ones where these individuals trade on spot markets and other contingent commodity markets and/or contingent money markets.

2.1.2 Unskilled Worker's Problem

A fraction $1-\varepsilon$ of individuals within any given host-country native generation are unskilled (or low-skilled) workers. Individuals live for two periods and inelastically supply their one unit of labor when young. They earn a fixed wage ω^L. In addition, they also face a lump-sum tax, τ_t^L, which is denominated in dollars. Thus, the value of these taxes in state s, is given by $\rho_t(s)\tau_t^L$. As with the skilled workers, unskilled workers also have the opportunity to participate in state-contingent claims markets. Since only old-age consumption is valued, individuals save their entire wage income net of taxes and any state contingent securities in the form of a simple storage technology, which yields λ units of consumption in period $t+1$ for every unit of savings at t. Thus we can write the skilled worker's problem as

$$\max_{c_{t+1}^L(\alpha), c_{t+1}^L(\beta)} \pi U\left(c_{t+1}^L(\alpha)\right) + (1-\pi) U\left(c_{t+1}^L(\beta)\right) \tag{UHP}$$

subject to

$$\left[p_t^*(\alpha)\omega^L - p_t^m(\alpha)\tau_t^L\right]\lambda + \left[p_t^*(\beta)\omega^L - p_t^m(\beta)\tau_t^L\right]\lambda = p_t^*(\alpha)c_{t+1}^L(\alpha) + p_t^*(\beta)c_{t+1}^L(\beta).$$

It will again be useful to represent the host-country individual's resources available for consumption *after taxes.* We define the tax-adjusted endowment, $\bar{\omega}^L$, by

$$p_t^*(s)\bar{\omega}^L = p_t^*(s)\omega^L - \rho_t^m(s)\tau_t^L = p_t^*(s)\left[\omega^L - \frac{p_t^m(s)}{p_t^*(s)}\tau_t^L\right].$$

The unskilled worker's problem can thus be rewritten as

$$\max_{c^L_{t+1}(\alpha), c^L_{t+1}(\beta)} \pi U\left(c^L_{t+1}(\alpha)\right) + (1-\pi) U\left(c^L_{t+1}(\beta)\right)$$

subject to

$$\left[p^*_t(\alpha)\bar{\omega}^L + p^*_t(\beta)\bar{\omega}^L\right]\lambda = p^*_t(\alpha)\, c^L_{t+1}(\alpha) + p^*_t(\beta)\, c^L_{t+1}(\beta).$$

2.1.3 Government Problem

The host-country government engages in three related activities: collecting taxes, enforcing the border, and providing basic services to illegal immigrants. Taxes are collected from both skilled and unskilled workers from the host countries. These funds are then used to provide basic services for illegal immigrants and to fund the desired level of border enforcement, denoted by e_t. We assume that the level of service provided to illegal immigrants, a, is constant (on a per person basis) over time but that the quantity of services received in total depends on the fraction of time that immigrants spend getting to and working in the host country, $T(\mu)$.[14] In addition we assume that the government runs a balanced budget. Thus the government's budget constraint is given by

$$\rho_t(s)\left[\varepsilon\tau^H_t + (1-\varepsilon)\tau^L_t\right] = \gamma T(\mu_t)\rho_t(s)\, a + e_t \tag{1}$$

for $s = \alpha, \beta$, where γ represents the fraction of home-country individual's who are migrants. The properties of the transfer-proportion function are described in detail when we discuss the migrant's maximization problem.

[14] The idea is that all illegal immigrants consume some government-provided goods and services. Migrants who are caught crossing the border are provided basic services at detention centers and are returned home at government expense. Migrants who successfully cross the border receive services such as education and emergency health care – even though they are usually not eligible for welfare or most other assistance programs. For simplicity, we assume the government provided goods are perfect substitutes for the consumption good and that they are transformed at a one-for-one rate. Finally, although amount of time crossing the border and working is denoted by $1-\mu$, we have chosen to define T in terms of μ for expositional efficiency.

2.2 Home Country

The home country is characterized by two classes of individuals: migrants and smugglers. All individuals are assumed to be born after the state of nature has been realized, and thus, home-country individuals represent the class of restricted participants in our model.[15] Smugglers work only in the smuggling industry while migrants divide their time between home production, crossing the border, and host-country production. Migrant production in the home country is characterized by a labor only production process. It is assumed that migrants produce a single homogenous final good, which is produced and saved in the migrant's first period of life, and then consumed when old. Finally, all migrant workers are assumed to be unskilled.

2.2.1 Migrant's Problem

A fraction γ of individuals within any given home-country generation are potential migrants. Each generation of migrants is endowed with one unit of labor when young and nothing when old. There is no initial old generation of migrants. Since only old-age consumption is valued, this labor is supplied inelastically when young. The migrant must decide what fraction of her labor time, μ_t, to spend working in the home country and what fraction, $1-\mu_t$, to spend crossing the border and working in the foreign country. However, merely deciding to go and work in the host country does not guarantee that the migrant will be successful in her attempt(s) to cross the border. Thus, the fraction of time spent emigrating from the home country, $1-\mu$, is further divided into two activities; time spent actually working in the host country $M(\cdot)$ and time spent crossing the border, $1-M(\cdot)$.[16]

The amount of time used in crossing the border depends on the level of border enforce-

[15] Although we do not address it in this paper, one could also conceive of a model where home country individuals are born prior to the realization of the sunspot but are restricted from participating in contingent-claims markets due to geographical restrictions. In this case migrants and smugglers would most likely attempt to self-insure against the sunspot via their choice of time allocations (i.e., fraction of time spent in each country and fraction of time devoted to learning about enforcement respectively.)

[16] We define M to lie inside the unit interval.

ment implemented by the host country, e_t, and the amount of services, q_t, a migrant obtains from smugglers. Thus, the amount of time spent working in the host country is a fraction of the time allotment not spent working in the home country; that is, $M(q_t, e_t)(1 - \mu_t)$, where $0 \leq M(q_t, e_t) \leq 1$. Conversely, the time lost crossing the border is given by $[1 - M(q_t, e_t)](1 - \mu_t)$. The level of border enforcement, e_t, is taken as given by the migrant. It is assumed that if $e_t = 0$, then there is no border enforcement and $M(q_t, 0) = 1$ for all $q_t \geq 0$.[17] In addition we assume that $0 > M_e > -\infty$. Thus, an increase in the level of enforcement reduces the amount of time spent working in the host country.

Since crossing the border is time consuming, smugglers exist to reduce the crossing time. At date t, migrants can purchase a quantity q_t of smuggling services, taking the price, p_t, as given; where p_t is measured in units of the home-country production good. It is assumed that the greater the quantity of smuggling services obtained, the less time is used to cross the border, that is, $M_q > 0$, and that there are decreasing returns to additional units of smuggling services, $M_{qq} < 0$. In addition, it is assumed that $M_q < \infty$ and $0 < M(0, e_t,) \leq 1$.[18]

Migrants who work in the home country earn a fixed wage ω per unit of time spent in home production.[19] Any income not spent on smuggling services is saved via a simple storage technology in the home country. For every unit of output saved at time t, the migrant receives one unit of consumption good at date $t + 1$. Migrants who are successful in crossing the border earn a fixed wage ω^{*L} in the host country and save in the host country via the same storage method as in the home country.[20] In addition, the migrant receives transfer payments, a_t, denominated in dollars, from the host country government. The quantity of dollars received is assumed to be proportional to the quantity of time spent crossing the

[17] Open borders correspond to perfect labor mobility.

[18] The latter assumption implies that even without the aid of the smuggler, a migrant will eventually cross the border and spend some time working in the host country.

[19] Alternatively, one can think of ω as an endowment which the migrant receives continuously throughout his young period life. Thus, if the migrant choose to stay in the home country for μ fraction of his young life, then she will receive only $\omega\mu$ of the total endowment possible.

[20] It is assumed that migrants are low skilled and thus enter that segment of job market paying lower wages, ω^L. We assume that this wage is fixed and thus think of this as a minimum wage earned by all low skilled workers in the host country. Alternatively, one can also think of this as an endowment earned by migrants in the host country which will be dependent on the fraction of time actually spent in the host country.

border and working in the host country, captured by $T(\mu_t)$. We assume that T lies in the unit interval (i.e., $T : [0,1] \rightarrow [0,1]$) and possesses the following properties: $T(0) \leq 1$, $T(1) = 0$, $T'(\mu_t) < 0$, $T''(\mu_t) < 0$.[21] Thus the total, goods value of transfer payments is given by $T(\mu_t)\rho_t(s)a_t$.

Finally, unlike the workers in the host-country, migrants are born after the realization of the sunspot and are not able to trade on contingent claims market. We additionally assume that migrants spend their retirement in the host country.

We can formally write the migrant's problem as

$$\max_{\mu_t, q_t} U(c_{t+1}(s)) \tag{MWP}$$

subject to

$$c_{t+1} = \omega\mu_t - p_t q_t + \left\{\omega^L M(q_t, e_t)[1-\mu_t] + T(\mu_t)\rho_t(s)a_t\right\}\lambda$$

and

$$\begin{aligned} \omega\mu_t &\geq p_t q_t \\ 0 &\leq \mu_t \leq 1 \end{aligned}$$

for $s \in \{\alpha, \beta\}$. First order conditions yield the set of equations

$$\omega = \left\{\omega^L M(q_t, e_t) - T'(\mu_t)a_t\rho_t(s)\right\}\lambda \tag{2}$$

$$p_t = \omega^L(1-\mu_t)\lambda M_q(q_t, e_t). \tag{3}$$

[21] Although the time spent crossing the border and working in the host country is given by $1-\mu_t$, given that μ_t lies in the unit interval it is immaterial whether we define $T(\cdot)$ as a function of μ_t or $1-\mu_t$. In addition, despite the technical nature of these assumptions, the intuition behind them is quite reasonable. If $T(\mu)$ satisfies these conditions, then illegal immigrants obtain most of their transfers (total, goods value of social services) after first arriving in the host-country and then government provided services decline with duration of stay (assimilation).

Equation (2) indicates the trade-off associated with migrating; the income (wage) earned per unit of time in the home country must equal the income earned per unit of time in the host-country (the sum of the wages in the host country and transfer payments). Equation (3) indicates that the marginal cost of the smuggling service is equal to the marginal income gain from using smuggling services, where the marginal gain in time working in the host-country labor market is measured by the product $M_q(q_t, e_t)(1 - \mu_t)$. We assume that $U(c_{t+1})$ satisfies all the standard conditions necessary for an interior solution; namely $U(0) = 0$ and $U'(c_{t+1}) > 0$.

2.2.2 Smuggler's Problem

In each generation, a fraction, $1 - \gamma$, of the home-country population are smugglers. Like migrants, smugglers live for two periods. In contrast, smugglers are restricted to producing smuggling services and may not migrate or work in the home-country production sector. When young, smugglers are endowed with one unit of labor that they supply inelastically. As with migrants, smugglers value only old age consumption and are retired when old. Thus they consume the gross return from investing their savings in the same simple storage technology as migrants. Finally, there exists an initial old generation of smugglers who possess smuggling capital h_0.

A smuggler's unit of labor is divided between two activities when young: accumulating smuggling capital (research and development), $h_{t,}$ and selling border crossings. For a smuggler, these operations are ordered sequentially; that is, the young smuggler first accumulates smuggling capital by crossing people, then begins selling services. We think of smuggling capital as the knowledge of methods and means for circumventing host-country border enforcement. The smuggler uses the remaining time endowment to arrange border crossings. We let d_t represent the fraction of time which smugglers devote to accumulating smuggling capital and $(1 - d_t)$ be the fraction of time devoted to arranging crossings.[22]

[22] One can think of the smuggler's first period as divided into two distinct subperiods. The initial subperiod of his young life is spent as an apprentice to an old smuggler, who has institutional knowledge about crossing

When determining the amount of time to devote to accumulating smuggling capital in period t, we assume that the quantity of smuggling capital (knowledge) previously acquired by all past generations, h_{t-1}, is available to the current generations of smugglers; that is, there is no depreciation of smuggling capital. We let the function $g(d_t, h_{t-1})$ represent the process by which time devoted to capital accumulation is transformed into smuggler's capital. Thus we have

$$h_t = g(d_t, h_{t-1}) \tag{4}$$

where $0 \leq d_t \leq 1$. We assume that $g(d_t, h_{t-1})$ has the following properties: g_d and $g_h > 0$ and g_{dd} and $g_{hh} < 0$. Let $g(0, h_{t-1}) = 0$, that is, a smuggler must devote some time to actually smuggling people over the border in order to develop knowledge about effective crossing methods and techniques. Finally, we assume that for $d_t > 0$, $g_h(d_t, 0) > 1$. Thus, taking the time to accumulate smuggler capital pays bigger dividends when there exists little smuggler capital from previous generations.

The smuggler arranges migration services in a perfectly competitive environment. As such, the representative smuggler takes the price of smuggling services, p_t, as given. In addition, the smuggler also takes as given the level of enforcement, e_t, in period t. Finally, it is only the process of arranging for migrant crossings that generates income. To produce migration services, the smuggler must devote sufficient time to capital accumulation, so that he may overcome the anticipated level of enforcement. Formally, let the quantity of migration services supplied be given by

and enforcement. In this subperiod, the smuggler undertakes the actual process of crossing migrants over the border. While the apprenticeship provides no income, it provides the required knowledge to make income-generating arrangements for migrant crossings during the second subperiod.

This is not unlike arrangements smugglers currently make on the U.S.-Mexican border. In practice, apprentice smugglers "run" the migrants across until they have been caught so many times (usually ten) that they risk prosecution if caught again. They then become coordinators and recruiters charged with getting clients for the new generation of runners. For more information on how smugglers operate, see Spener (2002).

$$
\begin{aligned}
Q_t &= B\left[h_t - e_t\right]\left(1 - d_t\right) \text{ for } h_t \geq e_t \\
&= 0 \text{ otherwise and}
\end{aligned} \tag{5}
$$

where $B > 0$ is a constant scale factor, $h_t - e_t$ is the effectiveness of the smuggling methods relative to enforcement methods, and $1 - d_t$ is the fraction of time devoted to selling migration services.

We can therefore write the smuggler's maximization problem as

$$
\max_{d_t} U\left(c_{t+1}^c\right) \tag{CP}
$$

subject to the constraints

$$
\begin{aligned}
c_{t+1}^c &= p_t B\left[h_t - e_t\right]\left(1 - d_t\right), \text{ and} \\
h_t &= g\left(d_t, h_{t-1}\right) \\
h_t &\geq e_t,
\end{aligned}
$$

where c^c denotes consumption by the smuggler. Given the interior solution guaranteed by the properties of the utility function, the efficiency condition for the smuggler is

$$
p_t B\left\{g_d\left(d_t, h_{t-1}\right)\left(1 - d_t\right) - \left[g\left(d_t, h_{t-1}\right) - e_t\right]\right\} = 0 \tag{6}
$$

Equation (6) describes the smuggler's trade-off. The first term inside the brackets represents the marginal gain from capital accumulation while the second term inside the brackets represents the marginal cost of time allocated to capital accumulation — time not spent arranging migration services.

3 Existence of Sunspot Equilibria

Before focusing on existence and properties of the equilibrium values for migration, smuggler services, etc., it will be useful to establish the existence of sunspot equilibria. Establishing this result requires only examining equilibrium consumption allocations for host-country individuals. This is the result of the fact that skilled and unskilled host-country workers will insure themselves, via trade in contingent-claims markets, prior to the realization of the sunspot and that, migrants, who enter the host country after the state of nature has been revealed, cannot participate in these markets and thus will merely consume the proceeds from their work. Thus, the existence of sunspots (and in particular sunspot equilibria which are not mere randomizations over certainty equilibria) is strictly a matter of examining the equilibrium allocations of host-country workers.[23] Given this, and the fact that the setup of host-country workers is almost identical to the unrestricted agents in Bhattacharya *et al.* (1998), their results (from section 3.3) are directly applicable to our economy. Hence we now focus on defining equilibrium in the host-country.

A host-country competitive equilibrium for the sunspot economy must satisfy the following

Definition 1 *A sunspot equilibrium consists of: (i) a sequence of allocations by the host-country workers,* $\left\{c_t^{*H}(\alpha), c_t^{*H}(\beta), c_t^{*L}(\alpha), c_t^{*L}(\beta)\right\}_{t=0}^{\infty}$*,that satisfies problems (SHP) and (UHP); (ii) a price vector,* $\left\{p_t^m(\alpha), p_t^m(\beta), p_t^*(\alpha), p_t^*(\beta)\right\}_{t=0}^{\infty}$*, that clears the host-country goods market; and (iii) for some* t*, either* $c_t^{*H}(\alpha) \neq c_t^{*H}(\beta)$ *or* $c_t^{*L}(\alpha) \neq c_t^{*L}(\beta)$*.*

As in Bhattacharya *et al.* (1998) we assume a log utility function and at least one unrestricted individual to have no tax burden.[24] For ease of exposition, we also assume that taxes,

[23] We are thus following the structure and methods developed in Bhattacharya *et al.* (1998) and Keister (1998) to verify the existence of stationary sunspot equilibria. As Bhattacharya *et al.* (1998) point out, "...equilibrium in this economy reduces to the determination of equilibrium in a smaller economy with no restrictions on market participation but...in which uncertainty is intrinsic."

[24] Keister (1998) shows that in a one period setup, which is essentially what we have, that the results which follow hold for more general specifications of the utility function. The following proposition is based on this generalization.

for both high and low skilled workers, are time independent, i.e., $\tau_t^H = \tau^H$ and $\tau_t^L = \tau^L$. Given these assumptions, we can show the following.

Proposition 1 *There exist equilibria in which sunspots affect the consumption of host country workers if and only if* $\tau^H + \tau^L \neq 0$. *There will exist sunspot equilibria whenever* $\rho_t(\alpha) \neq \rho_t(\beta)$.

The results follow directly from Proposition 2 in Keister (1998). The basic intuition is that for $\rho_t(\alpha) \neq \rho_t(\beta)$ (an assumption we make), the tax adjusted Edgeworth box is not square. Because equilibrium consumption bundles will lie on the diagonal of the Edgeworth box, it follows that consumption will not be identical across the two states of nature.

Proposition 2 *There exist sunspot equilibria which are not mere randomizations over the certainty equilibria.*

This result is proved in Proposition 3.1 in Bhattacharya *et al.* (1998). Their result is proved by setting $\tau^L = 0$. Thus, the unskilled host-country workers in the certainty economy face no taxes and would merely eat their endowment (savings in our model). Any randomization over the certainty economy would require that unskilled native workers consume only their endowment (savings). However, in the sunspot economy they would choose to insure against the sunspot — thus not consuming their endowment (savings) and the resulting equilibrium would not be a randomization over the certainty economy.

Remark: Taken together, these propositions have interesting implications regarding the impact of immigrants (or any new, younger job entrant) on unskilled (older), native workers. Specifically, they highlight the fact that any detrimental impact to unskilled workers is not the result of immigrants but rather is the result of choices made by unskilled workers in the face of uncertain future tax burdens. Consider the unskilled worker's consumption. With $\tau^L = 0$, it follows immediately that in the absence of a market in which state-contingent securities are traded, this unskilled worker would simply consume her savings. Proposition 2 implies that access to a market trading state-contingent securities results in consumption of the unskilled worker varying with the state of nature.[25]

Most importantly, this variance of consumption (well-being) does not depend on the usual story in which low-skilled workers suffer wage reductions because of the inflow of migrant workers. Here the wages for low-skilled workers are constant. Consequently, the variance stems from the fiat redistribution scheme (taxes and transfers) and the *ex ante* incentive to participate in the market trading state-contingent securities. In short, unskilled workers "pay" for the (preferred) redistribution scheme even though their explicit tax burden is zero. In our setting, the trade-off between *ex ante* efficient participation and *ex post* variance is a by-product of the sunspot, restricted participation, and the monetary redistribution scheme and *not* the presence of migrant (new) workers.

4 Migration and Smuggling Equilibria

The uncertainty over the state of nature, and the resulting existence of sunspot equilibria, also affects the equilibrium levels of migration, smuggling services, and smugglers' allocation of time. Although individuals in the home-country are restricted from participating in contingent claims markets, their decisions regarding allocation of time and the level of smuggler services to acquire, will differ depending on which state of nature, s, prevails. We begin by first discussing some properties of enforcement and smuggling in equilibrium. We then describe the equilibrium laws of motion governing the system and state conditions under which multiple equilibria will exists. Finally we end this section by examining the impact of sunspots on the equilibrium values of migration, smuggler services and smugglers' time allocation decision.

[25] Bhattacharya *et al.* (1998) use the term volatility to refer to the range of equilibrium allocations and prices we are referring to as variation in consumption. Formally, $\left[c_t^{*L}(\alpha), c_t^{*L}(\beta)\right]$ is a proper subset of $\Re^{++}$ and has positive measure because $c_t^{*L}(\alpha) \neq c_t^{*L}(\beta)$. In contrast, $\left[c_t^{*L}(s_t)\right]$ is a singleton and therefore is a measure zero set when the low-skilled worker has access to the market for state-contingent securities.

4.1 Enforcement and Smugglers

It will be useful to first describe the smuggler's decision on how to allocate her time, d_t, between capital accumulation and arranging border crossings. The choice of d_t depends on the level of enforcement, which is given by equation (1), and can be written as

$$e_t = \rho_t(s)\left[\varepsilon\tau^H + (1-\varepsilon)\tau^L - \gamma T(\mu_t)a\right] \tag{7}$$

for $s = \alpha, \beta$. As with the taxes paid by host-country workers, we assume that the marginal dollar value of transfers received by migrants is time invariant — $a_t = a$. Thus, we can rewrite this expression as $e_t^* = e^*\left(\mu_t, \tau^H, \tau^L, a, \rho_t(s), \varepsilon, \gamma\right)$. The following lemma states selected properties about the level of enforcement.

Lemma 1 *a)* $e_\mu^* > 0$, *b)* $e_\rho^* > 0$, *c)* $e_\tau^* > 0$, *and d)* $e_a^* < 0$.

These results follow directly from simply differentiating equation (7) and hence a proof is omitted. The intuition behind these results is straightforward. Parts (a) through (c) state that if transfers decrease (because migrants spend more time in the home-country), the goods value of dollars increase, or taxes increase, ceteris paribus, then there will be greater funds available for enforcement. Part (d) says that an increase in the dollar value of transfers will decrease the funds available for enforcement for a given level of tax revenue.

From equation (6), one obtains

$$g_d(d_t(s), h_{t-1})(1-d_t(s)) - g(d_t(s), h_{t-1}) = -e_t. \tag{8}$$

Using equation (8) and the implicit function theorem we can solve for the fraction of time smugglers spend acquiring smuggling capital, $d_t^* = d^*\left(\mu_t, \tau^H, \tau^L, a, h_{t-1}, \rho_t(s)\right)$. We derive the effect of changes in several variables of interest on the time allotted for research activity in the following lemma.

Lemma 2 *a)* $d_\mu^* > 0$, *b)* $d_\rho^* > 0$, *c)* $d_\tau^* > 0$, *d)* $d_a^* < 0$ *and e) for* $g_h > g_{dh}$, *then* $d_{h_{t-1}}^* < 0$

The results of Lemma 2 follow directly from differentiating equation (8).[26] The first three results (Parts (a)-(c)) are tantamount to increasing enforcement and part (d) is equivalent to decreasing enforcement and thus is the converse of the first three parts. An increase in border enforcement results in the smuggler allocating greater time to research and development in order to overcome the greater level of enforcement. Finally, an increase in the amount of prior smuggler capital (Part (e)) will lead to less research and development since the marginal payoff to additional effort is lower. In effect, with an increase in accumulated knowledge the young smugglers are reaping the rewards from research efforts undertaken by previous generations.

We use the results of Lemma 2 to characterize the effect that changes in enforcement on the equilibrium quantity of smuggling services, q^*. Recall that the quantity of smuggler service supplied was given by

$$\begin{aligned} Q_t &= B\left[h_t - e_t\right](1-d_t) \\ &= B\left[h_t - e^*\left(\mu_t, \tau^H, \tau^L, a, \rho_t(s), \varepsilon, \gamma\right)\right]\left(1 - d^*\left(\mu_t, \tau^H, \tau^L, a, h_{t-1}, \rho_t(s)\right)\right). \end{aligned}$$

Thus, in equilibrium market clearing implies that

$$\gamma q_t = (1-\gamma) B\left[h_t - e_t^*\right](1-d_t^*), \tag{9}$$

or rewriting this equation letting q_t^* be the equilibrium quantity of services and using equation (4) we obtain

$$q_t^* = \frac{(1-\gamma)B}{\gamma}\left\{\left[g(d_t) - e_t^*\right]\left[1-d_t^*\right]\right\}. \tag{10}$$

Applying the implicit function theorem yields $q_t^* = q^*\left(\mu_t, h_{t-1}, \tau^H, \tau^L, a, \rho_t(s), \varepsilon, \gamma\right)$. The properties of q^* with respect to key variables are described in the following lemma.

Lemma 3 *a)* $q_\mu^* < 0$, *b)* $q_\rho^* < 0$ *c)* $q_\tau^* < 0$, *d)* $q_a^* > 0$, *and e)* $q_{h_{t-1}}^* > 0$.

[26] This result is analogous to Lemma 2 in Guzman *et al.* (2001).

The results of Lemma 3 follow directly from differentiating equation (10) and applying the results of Lemma (2).[27] The intuition is as follows. For parts (a)-(c), changes in μ, ρ, and τ lead to greater enforcement, which leads to less smuggling activity, as smugglers devote greater time to learning about these new enforcement levels and thus less time actually arranging for crossings. Thus, these 3 parts of Lemma 3 are capturing the effect that a change in border patrol has on the quantity of smuggling services through the effect on the smuggler's time allocation. Part (d) is merely the reverse as greater transfers lead to less enforcement and thus more smuggling activity. For a given level of enforcement, smugglers with a higher level of accumulated knowledge (smuggler's capital) – part (e) – will choose to arrange for a greater number of illegal border crossings.

4.2 Equilibrium Laws of Motion

We can now condense the equilibria of this economy down to two equations (laws of motion). We can rewrite equation (10) as

$$q_t^* = \frac{(1-\gamma)\,B}{\gamma}\left\{g\left(d_t^*, h_{t-1}\right) - e^*\right\}\left[1 - d_t^*\right], \tag{11}$$

where the long list of arguments in d_t^* and e_t^* are omitted for ease of exposition. The second equilibrium condition comes from the migrants maximization problem and is given by

$$\frac{\omega^*}{\lambda} = \omega^{*L} M\left[q_t^*, e_t^*\right] - T'\left(\mu_t^*\right)\rho_t\left(s\right)a \tag{12}$$

To prove existence of and ascertain the number of equilibria, it will be useful to know the properties of the above two equations. First, from Lemma (3), we know that differentiating equation (11) with respect to the level of migration, μ, yields

$$\frac{dq^*}{d\mu} = e_\mu^* \frac{(1-\gamma)}{\gamma} B\left\{-\left(1 - d_t^*\right)\right\} < 0.$$

[27] See Guzman *et al.* (2001), Lemma 3.

Thus, equation (11) is downward sloping in (μ, q) space. It will be important when analyzing the impact of sunspots on the volatility of migration to know whether this equation is concave, convex, or some combination of the two. To simplify our analysis we henceforth make the following assumption.

Assumption 1 (A.1) *Let the function $T(\mu)$ be such that for $0 \leq \mu \leq 1$ and $0 \leq q$, then*

$$\frac{T'(\mu_t)}{T''(\mu_t)} > \frac{1 - d_t^*}{d_\mu}.$$

This assumption on the curvature of the function $T(\mu)$ guarantees that equation (11) is convex, as represented in Figures 2 and 3.[28]

Next, differentiating equation (12), we obtain

$$\frac{dq^*}{d\mu} = \frac{a\rho_t(s)}{M_q \omega^{*L}} \left(T'' + \omega^{*L} M_e \gamma T' \right).$$

The sign of $dq^*/d\mu$ depends on the sign of $T'' + \omega^{*L} M_e \gamma T'$. Recall that we assumed that $T'(\mu)$ and $T''(\mu)$ were both negative and $M_e < 0$. We further make the following assumptions.

Assumption 2 (A.2) *: The functions $M(q, e)$ and $T(\mu)$ are such that they satisfy the following conditions for $0 \leq \bar{\mu}$ and $0 \leq \bar{q}$:*

i) $M_{eq} = M_{qe} > 0$ and $M_{ee} \leq 0$,

ii) $T'''(\mu) \in (-\varepsilon, \infty)$

Finally, let $(\bar{\mu}, \bar{q})$ represent the locus of points which satisfy the following

iii) $T''(\bar{\mu}) / \omega^{*L} = -M_e(\bar{q}, e(\bar{\mu})) \gamma T'(\bar{\mu})$

The first two parts of this assumption guarantee that part (iii) defines a upward sloping locus of points $(\bar{\mu}, \bar{q})$. For combinations of μ and q which lie below this locus we have $dq^*/d\mu < 0$

[28] The propositions which follow in the next few sections and the analysis of comparative statics is, of course, dependent on whether equation (11) is concave, convex, or contains (multiple) points of inflection. However, the propositions (and analysis) which follow can be straight-forwardly modified based on the curvature properties of equation (11). Hence we focus only on the convex case for equation (11).

and for (μ, q) combinations above this locus close we have $dq^* / d\mu > 0$. Thus equation (12) has the general hill-shape depicted in Figures 2 and 3.

4.2.1 Existence of Equilibria

Existence of equilibria requires that a) equations (11) and (12) intersect and that b) any equilibrium pair (μ^*, q^*) satisfy the conditions that $0 \leq \mu^* \leq 1$ and $0 \leq q^*$. Although there are a myriad of different sufficient conditions one could state that would guarantee the existence of a unique equilibria (or none at all), we restrict ourselves to studying the cases under which two equilibria exist. Prior to stating necessary and sufficient conditions for multiple equilibria, it will be useful to make the following definitions, some depicted in Figure 4.

Definition 2 *Let μ^- and μ^+ be the values of μ such that both $(\mu^-, 0)$ and $(\mu^+, 0)$ satisfy equation (12), where $\mu^- < \mu^+$.*

Definition 3 *Let $\mu_{\max} = \min[\mu^+, 1]$, and let $q_{\max}$ be defined such that equation (12) holds at $(\mu_{\max}, q_{\max})$, i.e.*

$$\frac{\omega^*}{\lambda} = \omega^{*L} M [q_{\max}, e^* (\mu_{\max})] - T' (\mu_{\max}) \rho_t (s) a.$$

Definition 4 *Let $\mu_{\min} = \max[\mu^-, 0]$, and let $q_{\min}$ be defined such that equation (12) holds at $(\mu_{\min}, q_{\min})$, i.e.*

$$\frac{\omega^*}{\lambda} = \omega^{*L} M [q_{\min}, e^* (\mu_{\min})] - T' (\mu_{\min}) \rho_t (s) a.$$

Definition 5 *Let (μ^{*2}, q^{*2}) represent any values of μ and q that satisfy equation (12), that is*

$$\frac{\omega^*}{\lambda} = \omega^{*L} M \left[q^{*2}, e^* \left(\mu^{*2}\right)\right] - T' \left(\mu^{*2}\right) \rho_t (s) a.$$

Given these definitions, we now state necessary and sufficient conditions under which two equilibria exist.

Proposition 3 *If there exists a pair* (μ^{*2}, q^{*2}) *such that*

i) $0 \leq \mu^{*2} \leq 1$ *and* $0 \leq q^{*2}$ *and*

ii)

$$q^{*2} > \frac{(1-\gamma)\,B}{\gamma}\left[g\left(d_t^*\left(\mu^{*2}\right), h_{t-1}\right) - e^*\left(\mu^{*2}\right)\right]\left[1 - d_t^*\left(\mu^{*2}\right)\right]$$

iii) and if both

$$q_{\min} \leq \frac{(1-\gamma)\,B}{\gamma}\left[g\left(d_t^*\left(\mu_{\min}\right), h_{t-1}\right) - e^*\left(\mu_{\min}\right)\right]\left[1 - d_t^*\left(\mu_{\min}\right)\right]$$

and

$$q_{\max} \leq \frac{(1-\gamma)\,B}{\gamma}\left[g\left(d_t^*\left(\mu_{\max}\right), h_{t-1}\right) - e^*\left(\mu_{\max}\right)\right]\left[1 - d_t^*\left(\mu_{\max}\right)\right]$$

then there exists two equilibria: a high-migration, high-smuggler use equilibrium and a low-migration, low-smuggler use equilibrium.

Figures 2 and 3 depict the two possible situations in which there exist two equilibria. For the remainder of the paper we focus on these two generic cases with two equilibria. However, the results which follow below also encompass those situations where a unique equilibria exists. Finally, we will use the following short-hand notation when referring to the two possible equilibria: the high level of migration and smuggler service utilization will be referred to as the high-migration equilibria and the low level of migration and quantity of smuggler services will be referred to as the low-migration equilibria.

One important point of note is that there exists volatility in migration independent of the sunspot in this model, as evidenced by Figures 2 and 3 — which depict multiple equilibria within a given state of nature. Because there is no selection mechanism for choosing an equilibrium and because the model is static from the migrants perspective, at any given date

t, either equilibria is equally likely to prevail and it is possible to switch between the two equilibria regularly – thus generating volatility.

The source of this volatility, as evidenced by Assumption A.2, is the fact that service obtained by migrants from the government are non-linear (in fact concave) in the quantity of time spent working in the home country. The net result of this is that at low levels of migration, migrants are willing to acquire additional smuggler services because the marginal return to migrating is sufficiently high. However, as migration increases, this results in a decrease in the level of enforcement, as (made clear in equation (7)) enforcement is merely the residual of what is left-over from taxes after paying for migrant services. Thus increased migration lessens the funds available for, and hence level of, enforcement. At some point, this reduced enforcement will lead to a drop in smuggler services as they are not needed to overcome the waning enforcement level. Thus the marginal return to migrating is equal when there is low migration, high enforcement, as when there exists high-migration resulting in lower levels of enforcement. This existence of multiple equilibria combined with a lack of a coordinating mechanism implies that it is entirely possible to observe switching (in no particularly structured way) between equilibria, thus generating a volatile path with respect to migration flows.

4.2.2 Sunspots and Equilibrium Migration

In addition to the volatility possible as a result of multiple equilibria and no coordinating mechanism, the sunspot nature of the economy adds another layer of potential volatility to the level of illegal immigration. The impact of sunspots on equilibrium values often depends on induced changes in migration flows relative to changes in enforcement. As such, we begin this section by detailing when changes in equilibrium values are definitive or ambiguous and then proceed to explain the sources of any ambiguities. Finally, we examine the extent to which sunspots lead to greater volatility in migration and smuggler services.

Since both equations (11) and (12) depend on $\rho(s)$, the equilibrium levels of migration,

μ^*, and quantity of smuggler services, q^*, will depend on which state of nature, $s = \alpha, \beta$, prevails. Without loss of generality, we henceforth assume that $\rho(\alpha) > \rho(\beta)$. To understand the impact that a change in the real value of money will have, we examine the relative positions of equations (11) and (12) as depicted in Figures 2 and 3 for the two states of nature. We begin by differentiating equations (11) and (12) with respect to $\rho(s)$.

Differentiating equation (11) with respect to goods price of money, ρ, yields

$$\frac{dq^*}{d\rho} < 0.$$

Thus, equation (11) has the configuration denoted in Figures 5-7 for the respective states of nature $s = \alpha, \beta$. Similarly, differentiating equation (12) yields

$$\frac{dq^*}{d\rho} = \frac{\frac{T'a}{\omega^{*L}} - M_e e_\rho}{M_q}.$$

The impact of an increase in the goods price of money depends on when

$$T'a - \omega^{*L} M_e e_\rho \gtreqless 0. \tag{13}$$

We restrict ourselves to examining three generic sets of cases regarding equation (13) and let $(\mu(\alpha), q(\alpha))$ and $(\mu(\beta), q(\beta))$ denote values of μ and q associated with state α and β respectively.[29]

Case 1 *Suppose that for every pair* (μ, q) *satisfying* $0 \leq \mu \leq 1$ *and* $0 \leq q$ *and equation (12) that either*

$$T'a - \omega^{*L} M_e e_\rho > 0 \textit{ for all } \mu$$

or

$$T'a - \omega^{*L} M_e e_\rho < 0 \textit{ for all } \mu.$$

[29] Although these three sets of cases are not exhaustive of all possibilities, they do provide the general methodology and explanations needed to look at more specific cases the reader could imagine.

Figure 5 depicts the two possible situations when either $\frac{dq^*}{d\rho} > 0$ or $\frac{dq^*}{d\rho} < 0$. When $\frac{dq^*}{d\rho} > 0$ prevails, then at the high-migration equilibria, an increase in $\rho(s)$ increases the quantity of migration (lowers μ) and the impact on the quantity of smuggler services, q, is ambiguous (depending on whether equation (11) or (12) shifts more). At the low-migration equilibria, an increase in ρ decreases both the quantity of migration and smuggler services. When $\frac{dq^*}{d\rho} < 0$, then an increase in $\rho(s)$ has an ambiguous impact on migration in both equilibria. The quantity of smuggler services decreases in the high-migration equilibria and is ambiguous in the low-migration equilibria.

Case 2 *Suppose that for every pair* (μ, q) *satisfying* $0 \leq \mu \leq 1$ *and* $0 \leq q$*, one of the following conditions prevails:*

$$\begin{aligned} T'a - \omega^{*L} M_e e_\rho &> 0 \text{ for all } \mu < \mu_{\min}(\beta) \text{ and} \\ T'a - \omega^{*L} M_e e_\rho &< 0 \text{ for all } \mu > \mu_{\max}(\alpha) \end{aligned}$$

or

$$\begin{aligned} T'a - \omega^{*L} M_e e_\rho &< 0 \text{ for all } \mu < \mu_{\min}(\alpha) \text{ and} \\ T'a - \omega^{*L} M_e e_\rho &> 0 \text{ for all } \mu > \mu_{\max}(\beta). \end{aligned}$$

Figure 6 depicts the two possible situations, which amount to shifting equation (12) to the left or right respectively. As Figure 6 shows, at the high-migration equilibria an increase in ρ increases the quantity of migration (lowers μ) and the impact on the quantity of smuggler services, q, is ambiguous. At the low-migration equilibria, an increase in ρ has an ambiguous effect on both the level of migration and the quantity of smuggling services. When equation (12) "shifts right," then at the high-migration equilibrium, the quantity of smuggler services falls while the impact on migration is ambiguous. At the low-migration equilibrium, both the quantity of smuggler services and level of migration fall.

Case 3 *Suppose that for every pair* (μ, q) *satisfying* $0 \leq \mu \leq 1$ *and* $0 \leq q$, *one of the following conditions prevails:*

$$
\begin{aligned}
T'a - \omega^{*L} M_e e_\rho &> 0 \textit{ for all } \mu < \mu_{\min}(\beta) \textit{ and for all } \mu > \mu_{\max}(\beta) \textit{ and} \\
T'a - \omega^{*L} M_e e_\rho &< 0 \textit{ for at least one } \mu_{\min}(\beta) < \mu < \mu_{\max}(\beta)
\end{aligned}
$$

or

$$
\begin{aligned}
T'a - \omega^{*L} M_e e_\rho &< 0 \textit{ for all } \mu < \mu_{\min}(\alpha) \textit{ and for all } \mu > \mu_{\max}(\alpha) \textit{ and} \\
T'a - \omega^{*L} M_e e_\rho &> 0 \textit{ for at least one } \mu_{\min}(\alpha) < \mu < \mu_{\max}(\alpha).
\end{aligned}
$$

Figure 7 depicts the two possible situations. In the first sub-case (where equation (12) flattens), an increase in $\rho(s)$ decreases the quantity of smuggler services while the impact on migration levels is ambiguous in both equilibria. In the second subcase, the impact on migration levels is ambiguous for a given increase in $\rho(s)$. However, at the high-migration equilibrium, the quantity of smuggler services will decrease while at the low-migration equilibrium the impact of smuggler services is ambiguous.

Remark: Although the impact of changes to the goods price of money are not necessarily clear or consistent across the different cases, the intuition as to why the results are sometimes ambiguous and asymmetric with respect to the high- and low-migration equilibria is straightforward. The two key factors driving our results are a) the non-linear (and opposing) nature of both $T(\mu)$ and $M(q, e)$ and b) the fact that enforcement funding equals the residual tax income obtained after paying for migrant services. As equation (7) makes clear, the level of enforcement varies with the level of migration. Thus a change in the goods price of taxes and transfers will have different impacts on enforcement depending on the initial level of migration. In addition, given the non-linear nature of $T(\mu)$, the marginal impact of changes to $\rho(s)$ are likely to be even more pronounced for different initial levels of migration.

Thus, the marginal benefit of an increase in transfer payments will depend crucially on the current level of migration. Obviously, this marginal benefit can vary significantly depending on whether the high or low-migration equilibria prevail.

This benefit must be weighed against the costs of an increased goods value of tax revenue – namely greater funds available for enforcement. Greater enforcement increases the time spent crossing the border; thus effectively decreasing the wage income from migrating. Given the non-linear nature of $M(q, e)$, the marginal impact of a change in enforcement will differ (potentially significantly) depending on which equilibrium prevailed prior to the change in the goods price of money. Thus, the impact of a change in $\rho(s)$ on the high/low-migration equilibria depends entirely on the relative curvatures of the function $T(\mu)$ and $M(q, e)$.

Most importantly however, when examining Figures 5 - 7 it becomes obvious that only in vary rare cases will either the level of migration or the quantity of smuggler services (but neither both) be unchanged when comparing high (low) migration equilibria across the two states of nature. Thus the introduction of sunspots into the model economy will affect the volatility of both migration flows and quantity of smuggler services.[30] The source of this volatility is described above: namely the marginal trade-off which occurs between additional enforcement and migrant services associate with different goods prices of taxes and transfers. Finally, it is also worth noting that in the case where there exists a unique equilibrium for a given state of nature (i.e., when equations (11) and (12) intersect only once for $0 \leq \mu \leq 1$ and $0 \leq q$ and $s = \alpha$ or β) then volatility arises only as a result of the sunspot and not from a lack of a coordinating mechanism.

4.3 Smuggler Capital Evolution

Finally, although the previous section details the properties of the equilibria with respect to smuggler services and migration, even when the equilibrium values of q and μ remain constant over time the evolution of smuggler capital is not static. This follows from two

[30] In the analytical framework we have, it is not possible to determine the size of this impact on volatility without specifying simple functional forms for many variables and simulating results on a computer.

facts. First, smuggler capital depends not only on the state variables but also on previously accumulated smuggler capital. Second the amount of time devoted to acquiring smuggler capital, d_t depends on the level of migration – which can take on one of four values depending on whether the high or low-migration equilibria prevails and whether state α or β is realized. We start be describing a simple example before generalizing to our more complicated model.

4.3.1 Unchanging Equilibrium and State

Consider the case when the same state of nature, for example α, and when the same equilibrium, for example high-migration, always prevails. Denote the equilibrium values by $(\mu_H(\alpha), q_H(\alpha))$. For simplicity we will use the notation α_H to denote this set. The evolution of smuggler capital is given by

$$h_t = g(d_t, h_{t-1}).$$

In state α, the high-migration equilibrium yields the following smuggler capital accumulation equation,

$$\begin{aligned} h_t &= g\left(d\left(\mu_H(\alpha), \tau^H, \tau^L, a, h_{t-1}, \rho(\alpha)\right), h_{t-1}\right) \\ &= g(d(h_{t-1}), h_{t-1}; \alpha_H) \\ &\equiv \tilde{g}(h_{t-1}; \alpha_H), \end{aligned}$$

which is a standard first order difference equation. It is straightforward to show that if $g(d(h_{t-1}), h_{t-1}; \alpha_H)$ is a standard Cobb-Douglas production function, for example with d and h having equal shares, then $\tilde{g}(h_{t-1}; \alpha_H)$ is concave and has the following two properties: $\tilde{g}' > 0$ and $\tilde{g}'' < 0$. Recall from section 2.2.2 that $g_h(d_{t,}0) > 1$ for any $d_t > 0$. In this case, the difference equation, $h_t = \tilde{g}(h_{t-1}; \alpha_H)$ has the shape depicted in Figure 8 and consequently has a unique steady state level of capital accumulation $\tilde{h}(\alpha_H)$ to which the system converges regardless of the initial condition h_0. Thus, even though the quantity of migration and smuggler services provided are unchanging, smugglers will continually adjust the time spent accumulating smuggler capital until the quantity of smuggler capital accumulated, h, and the effort placed into acquiring more capital d, approach the steady state value $\tilde{h}(\alpha_H)$ and $d\left(\tilde{h}(\alpha_H)\right)$. We now turn to the case described in our model: two equilibria and states of nature.

4.3.2 Two Equilibria and Two States of Nature

The above example holds only for the case when the state of nature, s, and the particular equilibrium remain constant over time. However, there is nothing to rule out the switching between states of nature over time as well as between high and low-migration equilibria. In this case, the level of smuggler capital will not converge to a unique steady state but rather jump around between the four possible steady states $\left(\tilde{h}(\alpha_H), \tilde{h}(\beta_H), \tilde{h}(\alpha_L), \tilde{h}(\beta_L)\right)$ (one for each high/low-migration equilibrium and state of nature α or β). Although convergence of h_t to a unique steady state will not occur, it is possible to discuss the bounds between which all value for h_t will eventually lie.

With out loss of generality, let α and β be such that $\rho(\alpha) > \rho(\beta)$. In addition, and also without loss of generality, we assume that $\mu_H(\alpha) < \mu_H(\beta) < \mu_L(\alpha) < \mu_L(\beta)$,where H denotes the high-migration equilibrium and L the low-migration equilibrium.[31] As before we let α_H denote that equilibrium values associated with the high-migration equilibrium when state α occurs. Thus we have $\alpha_H < \beta_H < \alpha_L < \beta_L$. The laws of motion for the four equations

$$h_t(s_{H,L}) = \tilde{g}(h_{t-1}(s_{H,L}))$$

for $s = \alpha, \beta$ and the two possible equilibria are depicted in Figure 9.

Let $\bar{h}$ denote the value of smuggler capital such that for a given state and equilibrium,

[31] This particular configuration corresponds to Figure 2. The lemmas which follow would need to be appropriately modified to match other configurations. However, regardless of the relative positions of the laws of motions in the different states of nature, it will always be the case evolution of smuggler capital will "almost surely" be bounded above and below for sufficiently large t.

$\bar{h} = \tilde{g}\left(\bar{h}\right)$. We know the following three facts.

Lemma 4 *If $\bar{h}(\alpha_H) \leq h_0 \leq \bar{h}(\beta_L)$, then for all $t > 0$, $\bar{h}(\alpha_H) \leq h_t \leq \bar{h}(\beta_L)$. Thus h_t will cycle between the four $\tilde{g}(h)$'s but has and upper and lower bound.*

Lemma 5 *Suppose $h_0 < \bar{h}(\alpha_H)$, then at date $t > 0$, with probability $\delta \in \left[1 - [p(\alpha)]^t, 1\right]$, $\bar{h}(\alpha_H) \leq h_t \leq \bar{h}(\beta_L)$. Thus for sufficiently large t, the level of smuggler capital "almost surely" will be bounded by the upper and lower "steady states."*

Lemma 6 *Suppose $h_0 > \bar{h}(\beta_L)$, then at date $t > 0$, with probability $\delta \in \left[1 - [p(\beta)]^t, 1\right]$, $\bar{h}(\alpha_H) \leq h_t \leq \bar{h}(\beta_L)$. Thus for sufficiently large t, the level of smuggler capital "almost surely" will be bounded by the upper and lower "steady states."*

Thus, although the level of smuggler capital may not converge, it will be bounded above and below by the steady state $\bar{h}(\alpha_H)$ and $\bar{h}(\beta_L)$ respectively. Although our discussion of volatility has centered on levels of migration, this sections highlights yet another level of volatility – namely in smuggler's capital accumulation. Our model would suggest that even though the levels of migration may vary (between at most four possible levels), the path of smuggler's capital accumulation could be much more erratic and following no particular pattern. Finally, the next section explores the impact of changes in fiscal policy of the equilibrium levels of migration and smuggler services.

5 Comparative Statics: Taxes

Although the government in this model simply collects taxes to make transfers and enforce the borders, changes in either the taxes collected or the transfers made has an impact on the equilibria because both indirectly impact the funds available to enforce the border. However, as one would expect the impact of changes in taxes or transfers will largely depend on which equilibrium prevails.

Consider the effect of a change in taxes (on either the skilled or unskilled). Differentiating equation (11) yields the following expression:

$$\frac{dq^*}{d\tau} = e_\tau \frac{(1-\gamma)}{\gamma} B \left\{ -\left(1-d\right) \right\} < 0.$$

Note that an increase in taxes has qualitatively the same effect as an increase in the commodity price of money. Differentiate equation (12) one obtains

$$\frac{dq}{d\tau} = \frac{-M_e e_\tau}{M_q} > 0.$$

Figure 10 depicts the impact of an increase in taxes on the equilibria, for a given state of nature. There are qualitative differences depending on which equilibria we study. At low-migration equilibrium, an increase in τ, for example, results in a reduction in smuggling services and a decrease in migration. Conversely, when evaluated at the high-migration equilibrium, an increase in taxes results in an increase in migration while the effect on the quantity of smuggling services is ambiguous.

It is not so unusual for the comparative static responses to differ. We offer some intuition to account for why these differences arise. The common thread between the two equilibria is simple; an increase in taxes, for example, increases the level of border enforcement. Other things being equal, the increase in border patrol induces less migration. We see this mechanism operating in the comparative statics evaluated at the low-migration equilibrium; workers respond to the reduced incentive to migrate, spend more time in the home country, and purchase fewer smuggling services.

The general equilibrium effects are more pronounced in the high-migration equilibrium. In particular, the migrant worker's endogenous response can reduce the level of border enforcement. By increasing the amount of time spent in the host country, migrant workers receive a larger transfer payment. From Part (a) in Lemma (1), border enforcement is inversely related to worker's time spent in host country. Thus, migrant workers have an incentive to increase the level of migration. Indeed, this incentive to increase migration is the dominant force operating in our analysis of the tax effect evaluated at the high-migration equilibrium. Along with the increased incentive to migrate, there is additional incentive to purchase smuggling services. Our results indicate that we cannot infer which of the two countervailing effects dominate. Hence, the total effect on equilibrium quantity of smuggling services is ambiguous when evaluated at the high-migration equilibrium.[32]

6 Conclusion

In this paper, we examine a two-country model with one-way migration. The destination country uses an interdiction policy — a border patrol — to inhibit this movement. It is natural to interpret such cross-country migration as illegal immigration. Further, we introduce smuggling services into the model economy to provide a market solution that assists migrant workers in their efforts to circumvent the border patrol.

The key contribution of this paper is to account for fluctuations in the flow of illegal immigrants in the face of constant wage differentials. In contrast to earlier papers that have stressed network externalities, we offer two sources for the endogenous fluctuations: sunspots and a lack of a coordination mechanism when multiple equilibria exist. Under our setup, we derive conditions under which the presence of sunspots results in equilibria that are not mere randomizations over the certainty equilibria. The importance of these findings is that endogenous fluctuations can account for volatility in illegal immigration flows in the face of constant wage differentials. As such, our results offer an alternative view to the network externality hypothesis.

Another advantage to our approach is that we introduce the sunspot as impacting the value of transfer payments (or services) offered to illegal immigrants and to the intensity of

[32] Essentially, Part (c) versus Part (a) in Lemma (3).

Note that the same intuition applies if we consider an increase in marginal rate of nominal transfer payments, a. For the sake of saving space, we omit the analysis in the paper. We make this result available upon request.

border enforcement. Thus, unlike the network externality approach, our approach develops a direct link between policy variables and the volatility in illegal immigration. Both transfer payments and border patrol are frequently discussed when policymakers debate efforts aimed at changing the flow of illegal immigration. To our knowledge, this is the first paper in which these policy variables play a central role in affecting the fluctuations.

Finally, we consider a case in which there are two equilibria. Because we are essentially dealing with a static decision problem, the two equilibria add another layer of endogenous fluctuations to the layer already associated with the sunspot. In other words, the model economy shows that one source of endogenous fluctuation owes to the existence of the sunspot while another contributing factor is the equilibria — the high-migration or the low-migration — on which migrant workers coordinate. We do not address the coordination issue in this paper as it pertains to the equilibrium selection mechanism, but simply point out that an added degree of endogenous volatility could be contributing to volatility observed in the illegal immigration data.

There is much room for further research in this model. One issue that deserves attention is to formulate this model with physical capital so that dynamics could be formally developed and the implications studied. This may provide one avenue to resolving the lack of a coordinating mechanism which is generating some of the volatility in our model. Additionally, allowing the migrants and smugglers to be born prior to the realization of the sunspot, but restricting them from participating in state-contingent markets due to geographical limitations, is likely to have interesting implications. In particular, one could conjecture that both migrants and smugglers would attempt to self-unsure by means of their choice of time spent working at home and time spent accumulation capital respectively. This would likely eliminate volatility in migrant flows and smuggler services and result in volatility resulting only from coordination problems.

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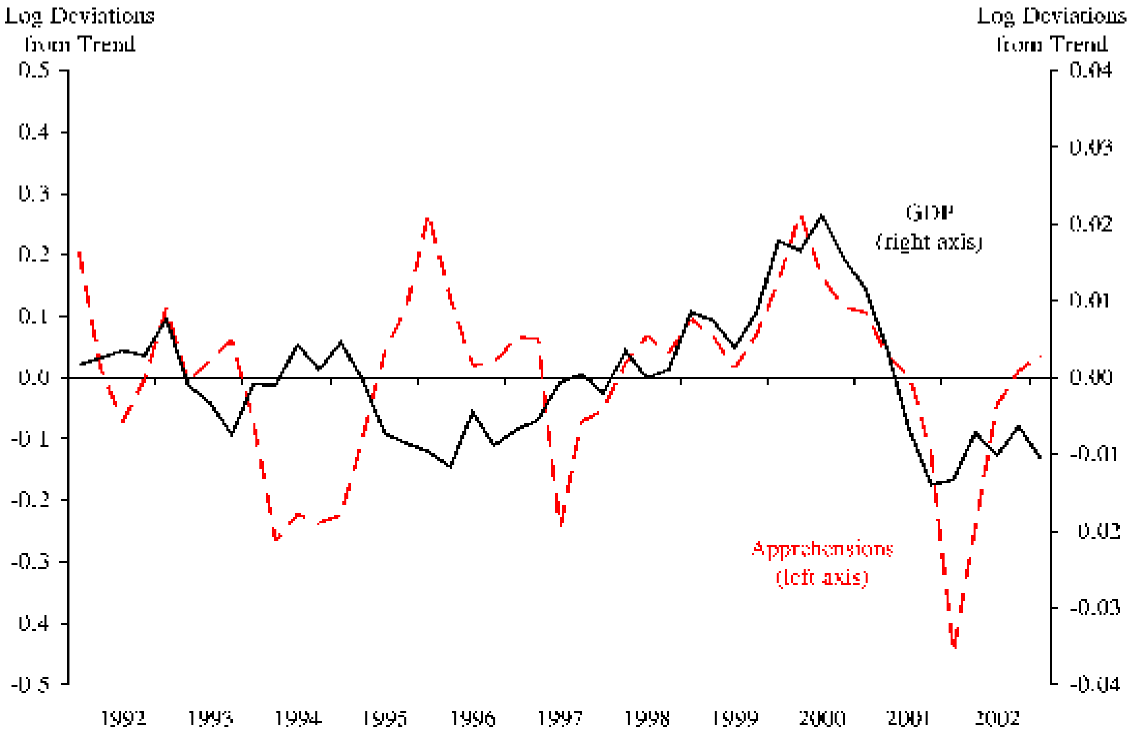

Figure 1: Volatility: Detrended Apprehensions and GDP

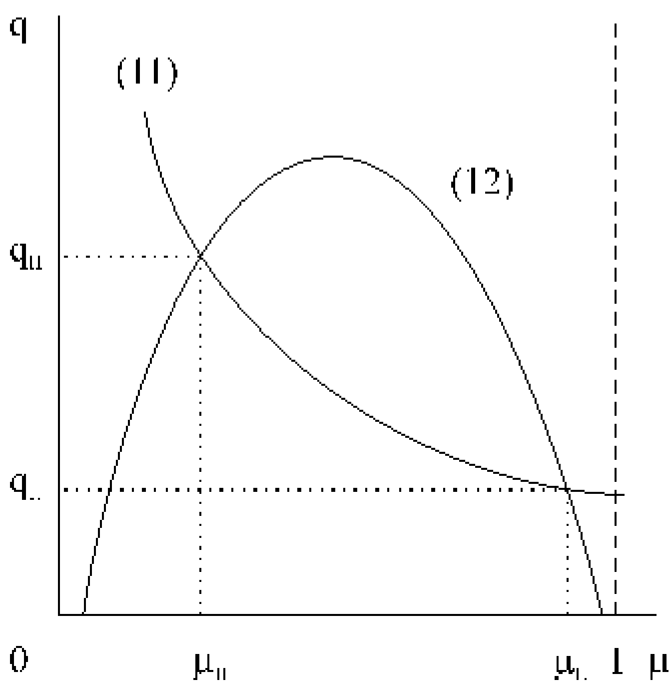

Figure 2: Multiple Equilibria with High and Low Migration Levels

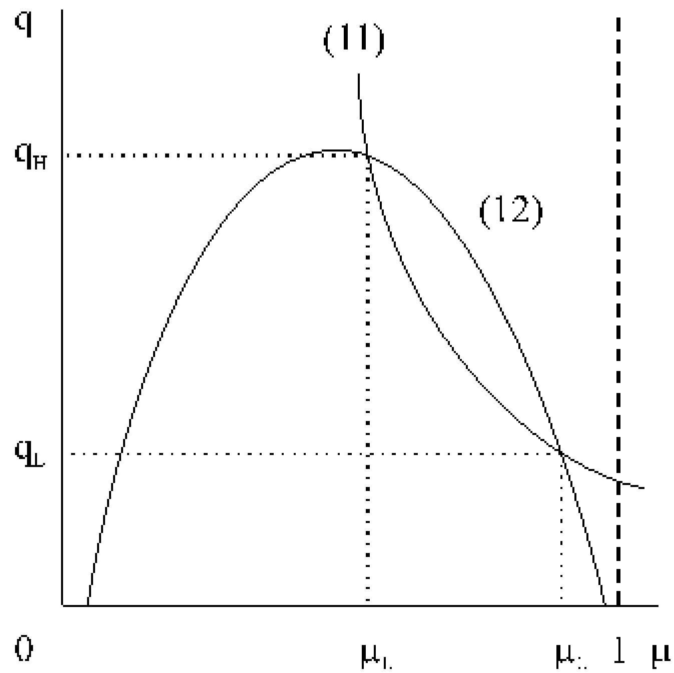

Figure 3: Multiple Equilibria with Low and Lower Migration Levels

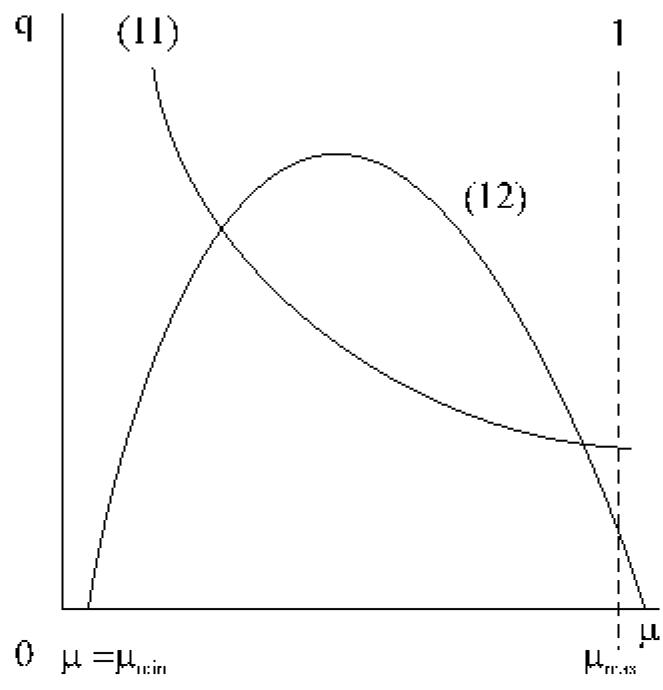

Figure 4: One Possible Depiction of Definitions 2, 3, and 4

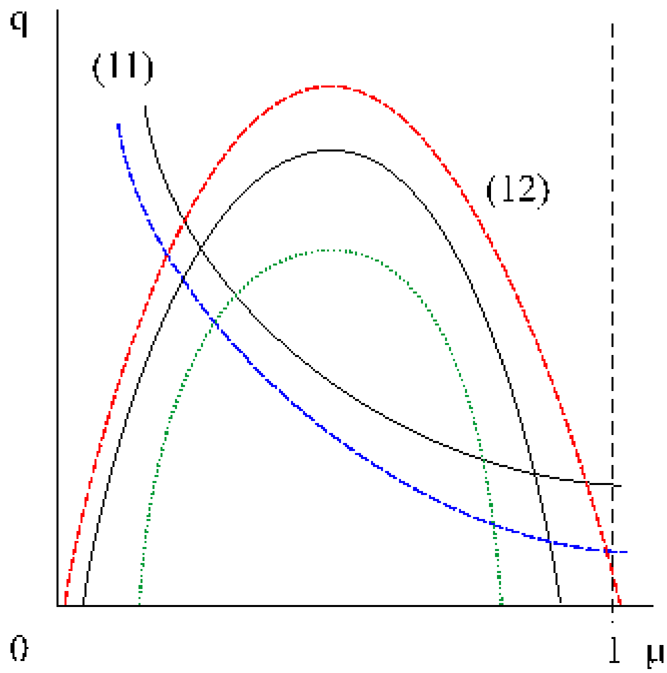

Figure 5: Equation (12) shifts up (down) as a result of an increase in the goods price of money

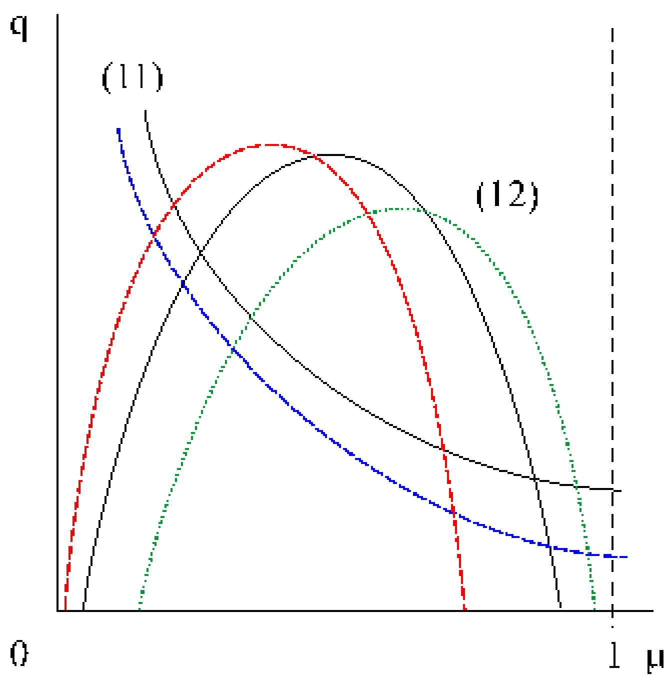

Figure 6: Equation (12) shifts left (right) as a result of an increase in the goods price of money

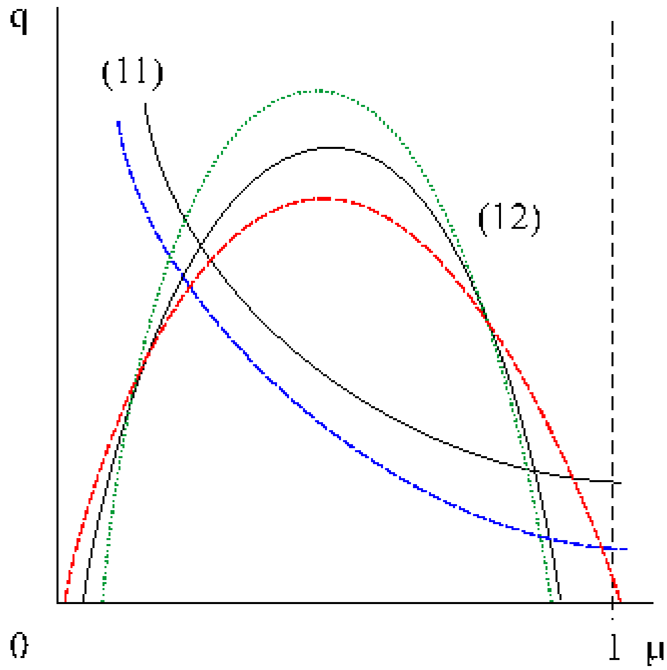

Figure 7: Equation (12) flattens (elongates) as a result of an increase in the goods price of money

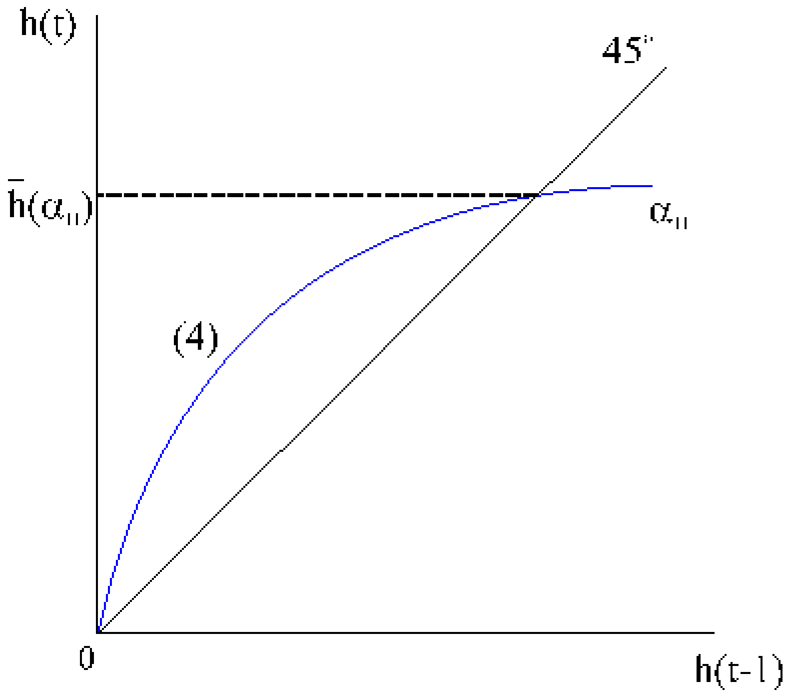

Figure 8: Smuggler Capital Evolution with one state and one equilibrium

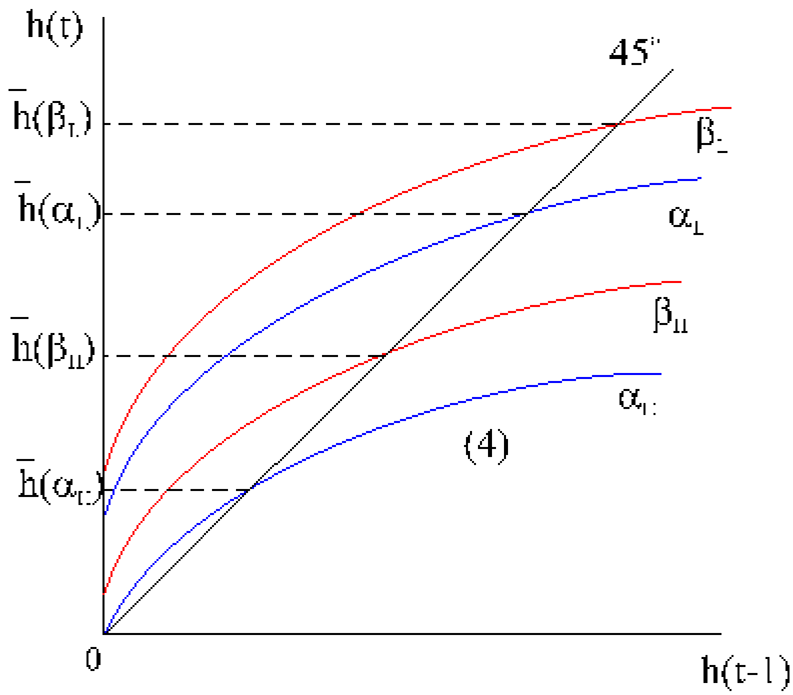

Figure 9: Smuggler Capital Evolution with multiple states and equilibria

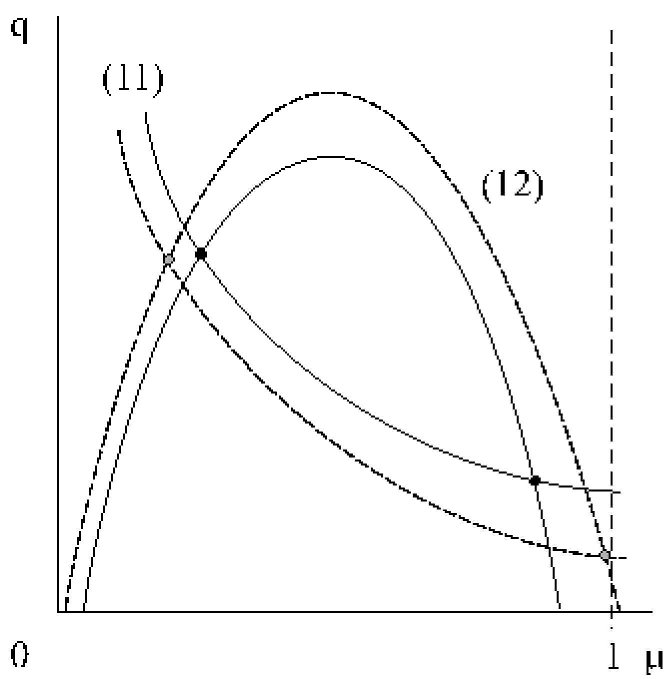

Figure 10: Increase in Taxes Paid