Central Bank Responsibility, Seigniorage, and Welfare

Joseph H. Haslag
Joydeep Bhattacharya

Research Department
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Joydeep Bhattacharya
Department of Economics
Iowa State University
Ames, IA 50011

Joseph H. Haslag
Research Department
Federal Reserve Bank of Dallas
Dallas, TX 75201

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Abstract

Historically, countries have relied on seigniorage. In this paper, we explore a set of features in which a benevolent government will rely on seigniorage. We use a simple overlapping generations model with return- dominated money. Money is valued because of a reserve requirement. The government *has* to raise a fixed amount of revenue solely for the purposes of making transfers to the old. It has two revenue generating options: lump-sum taxes (money creation) under the control of the treasury (central bank). We restrict the amount of seigniorage collected to be non-negative, and also require that the government's budget constraint be satisfied on a per-period basis. Our question is, Can we find stationary monetary competitive equilibria which are welfare maxima, given that the money stock cannot contract. Computational experiments reveal, somewhat surprisingly, that the answer is yes. Indeed, in our setup, benevolent governments may require that at least part, if not all of the revenue, be raised via money creation.

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1 Introduction

Why do countries raise *some* revenue from money creation? Figure 1 plots the average reliance on seigniorage revenue for 69 countries for the period 1965-94. Reliance is measured as the ratio of seigniorage revenue to federal expenditures (the change in high-powered money divided by the (nominal) level of government purchases). As Figure 1 shows, reliance varies from a maximum of nearly 31% to a minimum of 1%.[1] What strikes us most about Figure 1 is that the money stock does *not contract* in any one of these 69 countries; in fact, all of these 69 countries rely on positive seigniorage. In this paper, we explore the possibility that a simple welfare-based rationale could account for this observation.

Could it be that Figure 1 is the outcome of countries following Pareto efficient monetary policies? In models with infinitely-lived agents in which money enters the agent's utility function directly or indirectly, the Friedman rule identifies the Pareto efficient monetary policy as one in which money growth should be set equal to the agent's subjective time rate of preference (β). Then, with $\beta < 1$, the optimal monetary policy is to *contract* the money supply over time.[2] Wallace (1980) studies the same question for finitely-lived overlapping generations of agents, and concludes that a constant money stock (no-seigniorage) policy is Pareto efficient.[3] In an economy in which a storage technology exists alongside money, Wallace demonstrates that the optimal policy may require the money stock to contract. The bottom line is that Pareto efficient policies cannot account for the observations presented in Figure 1.[4]

[1] Click (1998) documents that between 1971-90, in a wide cross-section of countries, currency seignorage as percent of GDP ranged from 0.3% to 14% and seignorage as percent of government spending ranged from 1% to 148%.

[2] See Walsh (1998) and Correia and Teles (1999) for an extended discussion.

[3] With infinitely-lived agents, the Friedman rule stipulates that the money supply should contract, while Wallace finds that a constant money stock policy is optimal in an economy with finitely-lived agents. Freeman (1993) reconciles this apparent discrepancy.

[4] Non-welfare based explanations exist. For instance, Friedman's (1959) simple k% rule for the money supply could potentially account for the positive levels of seigniorage. This policy perscription is intended to counter "discretionary" policy and thus is presented in the context of monetary policy at business cycle frequencies. It is not a presecription dervied from some well specified objective. In addition, Haslag and Young (1998) compute the revenue-maximizing level of seigniorage. In that paper, the government would

In this paper, we search outside the set of Pareto efficient policies for an explanation for Figure 1. To that end, we focus on a second-best world inhabited by overlapping generations of two-period lived agents. There are two vehicles for income transfer over time: a linear storage technology and fiat money. The latter is rate of return dominated by the former. Fiat money is valued because there is a legal restriction of the type discussed in Wallace (1984). [5]The government *has* to make a fixed lump-sum transfer to each old agent every period. The government is comprised of a treasury that can raise lump-sum taxes from the young, and a central bank that may raise revenue from the printing of money.[6] Following Aiyagari and Gertler (1985), we assume that the government may raise the requisite revenue by using either taxes or seigniorage or both.[7] In contrast to Aiyagari and Gertler however, we restrict the government to a balanced budget every period, thus avoiding any intergenerational complications.[8]

Private agents take the policies of the government as given, and compute their own decision rules regarding how much to consume in each period. The government, in turn, takes these "policy reaction functions" as given, and chooses the mix of revenue-raising responsibilities for the treasury and the central bank, so as to maximize the welfare of a representative agent in a stationary setting. In other words, based on a simple welfare criterion, the government decides how much of the revenue for the old-age transfer ought

rely on seigniorage, explicitly taking into account the dynamic Lffer curve constraint.

[5]As such, there are two separate distortions built into the model economy, one arising from the assumption of overlapping-generations, and the other resulting from the legal restriction.

[6]Indeed, to make the inflation tax as odious as possible, we allow the government to have access to lump-sum taxes.

[7]Our work is closest in spirit to earlier work by Aiyagari and Gertler (1985). Indeed, like them, we work within a pure-exchange overlapping generations structure where money is return dominated. There are three differences though. First, fiat money is valued in our setup because there is a reserve requirement, whereas Aiyagari and Gertler apply the more general money-in-the-utility function approach. Second, we leave out government debt. Third, we focus on welfare consequences whereas Aiyagari and Gertler focus on the macroeconomic effects of different backing schemes.

[8]Aiyagari and Gertler (1985) rule out Ponzi schemes by the government. If the government can borrow at a positive nominal interest rate, policies initiated at date t might affect generations born at date t differently from generations born at date $t+1$. The government's financing decision affects the welfare comparisons for different generations along the transition path.

to come from lump-sum taxes (henceforth, the tax responsibility parameter). Because the government's program solves for the *fraction* of revenue coming from taxes (and money creation), the money growth rate is endogenous; it is the rate that satisfies the government budget constraint. Whatever is not backed by taxes is backed by seigniorage.[9] In keeping with the spirit of Figure 1, we focus only on equilibria where the tax-responsibility parameter cannot exceed unity, implying that we limit our focus only on equilibria in which the money stock cannot contract. Our question then is: will such a government ever wish to use the inflation tax as a revenue-raising tool?[10]

Our results are easily summarized. Because of the general-equilibrium effects, it is difficult to obtain definitive analytic results. We report the quantitative properties for a simple model economy in which agents have log preferences and have access to a linear storage technology. It is somewhat surprising that the highest welfare level is achieved for a case in which seigniorage is partially responsible for "backing" the old-age transfer. Indeed, partial backing is the most preferred financing scheme for a large part of the parameter space.

The intuition is straightforward. Given a fixed old-age transfer, suppose the government increases the tax responsibility parameter. Then, the central bank's revenue responsibility is satisfied at a lower money growth rate. With a reduction in the money growth rate, agents realize an increase in the return to savings. On the one hand, an increase to the gross real return can induce greater savings, while on the other hand, the increase in the first period lump-sum tax can reduce first-period disposable income, inducing less savings. As such, it is not clear if equilibrium savings goes up or down. Hence, second period income (from

[9] Note that this question does not bear on the issue of central bank independence. Independence would correspond to a situation in which the decision to raise any revenue from seigniorage is left to the central bank. Here, the central bank is required to back *whatever* lump-sum taxes do not. That is, the government outlines the revenue-generation responsibilities of both the treasury and the central bank based on welfare calculations, and these institutions perform their task *without* question.

[10] In other words, can we find welfare-maximizing monetary competitive equilibria in a regime where the money supply *expands*? Notice though that the Pareto efficient policy here is to keep the money stock constant and *not* use the inflation tax. Put sharply, our question and our analysis has nothing to add to the vast literature on the *optimal* inflation tax since we are *not* solving the social planner's problem.

savings) may fall or rise. We turn to numerical analyses, showing that for a large part of the parameter space, equilibrium savings fall. Thus, it is possible that agents would prefer to give up second period utility to get higher first period utility; that is, reduce lump-sum taxes and accepting the cost of a higher price for second-period consumption. Despite the availability of lump-sum taxes, a benevolent government would choose to use some money creation.

How robust is this result to changes in the model environment? We show that, ceteris paribus, if we abandoned the legal restriction on money holdings, the government's monetary policy choice would be the same as the Pareto efficient policy described in Wallace (1980). What if we eliminated the market incompleteness (inherent in the overlapping-generations environment) by allowing agents to leave bequests for their offspring, and retained the legal restriction? We show that even in such a setting, the government may still choose to make some use of the inflation tax. In addition, our results survive even if government spending has no purpose.

It is then the combination of the reserve requirement and the restriction on the money supply process, that is crucial to our results. Together, these two conditions ensure that the government cannot eliminate the distortion inherent in return-dominated money via monetary policy. Consequently, lump-sum taxes are not a "pure" policy option as they are in an undistorted environment. Here, changes in the usage of lump-sum taxes directly affects the money growth rate that the central bank can set, which indirectly changes the relative price of first and second period consumption. Changes in the reliance on seigniorage has similar distorting effects. As such, it is not obvious, a priori, which is the preferred poison.

Our experiments are motivated, in part, by the frequency with which countries employ reserve requirements. In this regard, two points deserve mention. First, our paper does not propose a theory of the coexistence of seigniorage and reserve requirements. Rather, it takes the presence of reserve requirements as exogenously given, and assesses whether seigniorage can be part of a benevolent government's financing scheme. Second, in making this latter assessment, we have also departed from the set of Pareto efficient policies. Put differently, the solution to the government's problem described here typically will not coincide with the solution to the social planner's problem (or the golden rule).

The rest of the paper is organized as follows. Section 2 lays out the description of the economic environment while Section 3 characterizes the equilibria. Section 4 reports the results of the various numerical computations, while Section 5 discusses the robustness of our results to alternative modeling assumptions. We conclude in Section 6. Appendix A contains a discussion of the dynamical equilibria, and Appendix B outlines a model similar to our baseline model where agents may leave bequests to their offspring.

2 The Model Economy

The economy is a modified version of Cass and Yaari's (1966) overlapping generations economy. There is an infinite sequence of periods indexed by $t = 1, 2, 3, ...$ Agents live for two periods. At each date $t \geq 1$, N people are born (hereafter the young) and N people are beginning the second period of their life (hereafter, the old). At date $t = 1$, there are N people who live only one period (hereafter, the initial old). Hereafter N is normalized to 1. In addition, there is a government that is infinitely lived.

Each agent is endowed with y units of a consumption good when young. For all dates $t \geq 1$, members of the generation born at date t do not receive any endowment of the consumption good at date $t + 1$. Date-t units of the consumption good spoil, rendering them useless (in the sense that units of the date-t good cannot contribute to the agent's utility) at date $t + 1$ or greater. Each agent born at date $t \geq 1$ has the same preferences over their young-age and old-age consumption. These preferences are summarized by a time-separable utility function,

$$U(c_{1t}, c_{2t+1}) = u(c_{1t}) + v(c_{2t+1}) \tag{1}$$

where c_{1t} denotes the consumption by a young agent born at date t and c_{2t+1} denotes consumption by that agent when old. We assume that u and v are twice continuously differentiable, and strictly concave; formally, $u', v' \geq 0$, $u'', v'' < 0$, $\lim_{c_1 \to 0} [u'(c_1)] = \infty$ $\lim_{c_2 \to 0} [v'(c_2)] = \infty$.

Agents receive a lump-sum transfer from the government when old. The government exogenously fixes the size of this transfer. The revenue needed to fund the old-age transfer payment comes from the revenues raised by the two wings of the government, the treasury and the central bank. The former collects lump-sum taxes from the young. The latter controls the nominal money stock, M, contributing to the government's revenue needs by creating money. Let ϕ denote the fraction of the transfer payment that lump-sum taxes will cover (henceforth, the tax-responsibility parameter). Throughout our analyses, we focus on cases in which the government picks ϕ to maximize the welfare of future generations in a stationary setting.

Agents have access to two vehicles for income transfer over time: a storage technology and money. The former yields a gross real return of x in that each unit of the consumption good placed into storage at date t yields $x > 1$ units of the consumption good at date $t+1$. Let p_t denote the time t price level.[11] Because fiat money does not pay any explicit interest, its gross real return between t and $t+1$ is $\frac{p_t}{p_{t+1}}$. Throughout this analysis, we restrict our attention to equilibria where money is dominated in rate of return, or $x > \frac{p_t}{p_{t+1}}$.

Agents cannot store their own goods. We assume that all storage activity is intermediated. Specifically, there is a composite asset, called "deposits", that are sold by banks. Banks operate in a perfectly competitive environment, taking the price of deposits and the gross real return on storage goods as given. There is no cost to creating these deposits. Let the gross real return on deposits between t and $t+1$ be represented by r_t.

We assume that banks are subject to a standard cash reserve requirement which restricts the bank to hold at least γ of each unit of the good deposited, in the form of fiat money.[12] In equilibrium, with $x > \frac{p_t}{p_{t+1}}$, banks will hold exactly a fraction γ in fiat money and no more. Let m denote nominal money balances per young person. Then, $m_t = \gamma p_t d_t$ holds. Consequently, the gross real return to deposits will be a weighted average of the returns to storage and money, the weights being pinned down by the reserve requirement ratio. Formally,

$$r_t = (1-\gamma)\, x + \gamma \frac{p_t}{p_{t+1}} \tag{2}$$

[11] More precisely, the price level is the number of units of fiat money traded for one unit of the consumption good.

[12] Our formulation of the reserve requirement is standard and follows Freeman (1987). The government picks γ at the beginning of time and does not change it ever.

must hold.

The agent solves a program to maximize lifetime welfare (1). Let a denote the quantity of goods transferred to each old agent, and τ be the quantity of goods that each young person pays in the form of a lump-sum tax. Agents take a and τ as given when computing their decision rules. The representative agent born at date $t \geq 1$ finds non-negative combinations of c_1 and c_2 such that (1) is maximized subject to the following per-period budget constraints:

$$y \geq c_{1t} + d_t + \tau_t$$

$$r_t d_t + a_{t+1} \geq c_{2t+1}$$

The necessary and sufficient conditions for the program yields a solution for the quantity of deposits, $d(.)$, that is defined as

$$d(.) = \arg\max \; \left[u(y - \tau - d) + v(rd + a) \right]. \tag{3}$$

The government budget constraint is represented (in per-young person terms) as

$$a_t = \tau_t + \frac{m_t - m_{t-1}}{p_t}. \tag{4}$$

As stated earlier, lump-sum taxes are the responsibility of the treasury, while seigniorage from money creation is the responsibility of the central bank. From (4), it is possible to assign revenue-generation responsibilities to the different wings of the government. Formally, let $\tau_t = \phi a_t$ and $\left(\frac{m_t - m_{t-1}}{p_t}\right) = (1 - \phi)\, a_t$, where $\phi \in [0, 1]$.[13] The special case, $\phi = 1$, is the "Ricardian" case (see Sargent [1982]) where taxes fully back the level of government spending. In contrast, with $\phi = 0$, the transfer is funded entirely through money creation.[14] The government chooses that ϕ which maximizes a representative agent's lifetime welfare.

[13] We restrict the value of ϕ to the $[0, 1]$ interval because transfer payments and taxes are paid in a lump-sum manner. $\phi < 0$ is equivalent to a case in which lump-sum transfers are defined as $a - \tau = a(1 - \phi)$.

[14] We distinguish between responsibility and independence. The central bank is *not* more independent as ϕ approaches zero. Our notion of responsibility is close to Aiyagari and Gertler's (1985) notion of backing. Ours differs in the sense that we restrict the government to a balanced budget at each date.

>From (4), it is clear that money growth plays a role in government financing whenever $\phi > 0$. Throughout the analysis, we assume that money growth is dictated by the rule, $m_t = \theta m_{t-1}$, where θ is the gross rate of money growth. The restrictions on ϕ imply that we will focus only on equilibria where $\theta \geq 1$ holds. The government budget constraint (4) may therefore be rewritten as:[15]

$$a_t = \tau_t + \frac{m_t}{p_t}\left(1 - \frac{1}{\theta}\right). \tag{5}$$

Here, θ is endogenous in the sense that changes in ϕ will prompt the central bank to adjust θ in order to satisfy (5) for all $t \geq 1$. Note that *both* the treasury and central bank are subservient to the government in this setting. The central bank is *not* independent in the sense that it can set the money growth rate without any regard to its revenue-generating responsibility. At the same time, the treasury cannot impose its will on the central bank. Both are equally subservient to the government which is coordinating their activities in order to satisfy its budget constraint.[16] In short, once ϕ has been set by the government, the revenue-generating responsibilities of both the treasury and the central bank are unquestionably pinned down.

3 Equilibrium

A valid perfect-foresight, competitive equilibrium for this economy is a set of allocations, $\{c_{1t}\}, \{c_{2t}\}$, and prices, $\{p_t\}, \{r_t\}$ for $t = 1, 2, 3, ...$such that

1. taking y, τ, a, x, γ, θ, p, and r as given, the agent's optimal savings behavior is defined by (3),

2. banks maximize the gross real return to deposits, taking x, γ, and $\frac{p_{t-1}}{p_t}$ as given;

3. markets clear; that is, $y = c_{1t} + c_{2t}$, $\frac{m_t}{p_t} = \gamma d_t$, and (5) is satisfied.

[15] Because we assume that population is constant, the rule stated in terms of money holdings per young person is identical to the rule stated in terms of aggregate quantities.

[16] Our setup differs from Sargent and Wallace (1981) where a dominant treasury ("fiscal leadership") existed alongside a subservient central bank.

In addition, d_t, r_t, and p_t must be positive at all dates, and $x > \frac{p_{t-1}}{p_t}$ must hold.

The necessary and sufficient condition for the agent's problem is:

$$u'(c_{1t}) = r_t v'(c_{2t+1}). \tag{6}$$

Equation (6) is a standard Euler equation; the agent chooses c_1 such that the marginal utility lost from foregoing a little bit of consumption when young is exactly equal to the marginal utility gained from adding to consumption when old. Banks maximize the gross real return to deposits when $r_t = (1-\gamma)\,x + \gamma\frac{p_{t-1}}{p_t}$. The equilibrium decision rule for deposits is implicitly defined by (6), the agent's budget constraint, and the responsibility constraint, $\tau = \phi a$, as follows:

$$d = d(r; y, \phi, a). \tag{7}$$

Throughout our analysis, we focus only on stationary equilibria.[17] In steady states, the money market clearing condition implies that $\frac{p_t}{p_{t-1}} = \frac{1}{\theta}$. Thus, $r = (1-\gamma)\,x + \frac{\gamma}{\theta}$. Since the central bank's responsibility is defined by $\gamma d\left(1 - \frac{1}{\theta}\right) = (1-\phi)\,a$, it is easy to see that

$$r \equiv r(\phi) = (1-\gamma)\,x + \gamma\left[1 - \frac{(1-\phi)\,a}{\gamma d}\right]. \tag{8}$$

The functions, $r(.)$ and $d\,(.)$, are the starting points for an analysis of the welfare effects associated with changes in the finance-responsibility parameter. Note that (8) and (7) jointly determine the gross real return to deposits and the equilibrium level of deposits.

3.1 Seigniorage

We undertake a brief detour and study seigniorage. Recall that the central bank's responsibility for revenue generation is entirely defined by

$$\frac{m_t - m_{t-1}}{p_t} = (1-\phi)\,a. \tag{9}$$

Noting that (from [2]),

$$\left(\frac{p_{t-1}}{p_t}\right) = \frac{r_{t-1} - (1-\gamma)x}{\gamma}$$

[17] We relegate the investigation of dynamical equilibria to the appendix.

and that $m_t = \gamma p_t d(r_t)$, it is possible to write the steady state version of (9) as

$$(1-\phi)\, a = \gamma d(r) - d(r)[r - (1-\gamma)x] \equiv S(r) \tag{10}$$

This is the exact analog of equation (7.54) in Sargent (1987). The left hand side of (10) captures the central bank's revenue responsibility while the right hand side is the amount of revenue that the central bank is potentially able to generate from money creation. Equilibrium considerations dictate that the central bank pick the money growth rate, for a given reserve requirement, that satisfies (10).

Define $\xi_r \equiv \frac{r}{d} d'(r)$ as the elasticity of deposits with respect to the return on deposits. For future reference, note that from (10),

$$S'(r) = [\gamma + (1-\gamma)x]d'(r) - d(r)\left[\xi_r + 1\right]$$

The following lemma outlines the behavior of the seigniorage Laffer curve.

Lemma 1 *a) Suppose either* $d'(r) > 0$ *and* $\xi_r < -1$*, or* $d'(r) < 0$ *and* $\xi_r > -1$*, then* $S'(r)$ *is strictly positive.*
b) Suppose $d'(r) < 0$ *and* $\xi_r < -1$*. Also, suppose* $[\gamma + (1-\gamma)x]d'(r^*) - d(r^*)\left[\xi_{r^*} + 1\right] = 0$*, and* $S''(r) < 0$*, and* $S(r^*) > (1-\phi)\, a$*. Then,* S *is hill-shaped.*

Part (b) of Lemma 1 says that on the upward (downward) sloping portion of the Laffer curve (drawn in the (S, θ) space), seigniorage increases (decreases) with an increase in the money growth rate. For this case, the Laffer curve is depicted as a continuous, hump-shaped curve; here, it is well known that the central bank can meet its fiscal responsibility with either a high money growth rate (low r) or a low money growth rate (high r). Also note that there is a maximum value of a that may be financed by seigniorage.[18]

As we shall see, with two candidate equilibria that satisfy the government budget constraint, changes in ϕ on variables of interest could be dramatically different depending on the equilibria.

[18] Similarly, the maximum a that may be financed by lump-sum taxes is y.

3.2 Welfare

The steady state level of welfare for all future generations is obtained by substituting the equilibrium decision rules into the agent's utility function. Formally,

$$W(\phi) = u\{y - d(y, \phi, r(.)) - \phi a\} + v\{r(.)d(y, \phi, r(.)) + a\} \tag{11}$$

From (11), the reader can see the different channels through which changes in the tax responsibility parameter affect lifetime welfare. In addition to the direct impact, there are two channels reflecting the general equilibrium effects that changes in ϕ have on welfare. We begin with brief overview of each.[19]

The direct effect is captured by the last term inside $u(.)$. Here, ceteris paribus, an increase in the tax-responsibility parameter, for example, results in a decline in the agent's first-period disposable income. If things stopped here, lifetime welfare would be decreasing in the tax-finance responsibility. As such, agents would prefer that all the revenue responsibility be borne by the central bank, that is, seigniorage would be the preferred way to finance the lump-sum transfer.

General-equilibrium effects, however, complicate any assessment of the impacts on lifetime welfare. Indeed, both the equilibrium level of deposits and the equilibrium gross real return to deposits are affected by changes in the tax-responsibility parameter. Suppose for now that deposits are invariant to changes in ϕ. Equation (8) indicates that an increase in tax-finance responsibility results in a higher gross real return to deposits, holding the level of deposits constant. The intuition behind this is straightforward. With lump-sum taxes bearing a larger share of the financing, money creation supports a smaller portion. With constant deposits, the economy is on the "good" side of the seigniorage Laffer curve; hence, the government budget constraint is satisfied with a lower money growth rate. With a decline in the money growth rate, the gross real return to deposits increases. It follows then from the first term in $v(.)$ that agents' second-period consumption would increase, resulting in an increase in lifetime utility.

[19] We use the description to develop some intuition. Recall that $d(.)$ and $r(.)$ are functions derived from optimizing behavior. To talk about these as if they were exogenous is not correct, but merely illustrative of the kinds of factors affecting lifetime utility.

In general, the equilibrium level of deposits will vary with ϕ. Indeed, the effect on deposits further muddles our efforts to assign a direction of change to lifetime welfare. Suppose, for instance, that ceteris paribus, deposits are an increasing function in ϕ. More deposits means that fewer goods are consumed in the first-period, while more second-period consumption is realized. To obtain further insight into this general-equilibrium effect, note that a formal expression of the total derivative of lifetime utility with respect to the tax-responsibility parameter is

$$W'(\phi) = u'(c_1)[-d_\phi - d_r r_\phi - a] + v'(c_2)[r_\phi d + d_\phi r].$$

Using the Euler equation (6), and the definition of the elasticity of deposits with respect to the gross real return, we can further reduce this expression to

$$W'(\phi) = v'(c_2)[d(.)r_\phi\{1 - \xi_r\} - ar(.)] \tag{12}$$

Obviously, the elasticity of deposits to changes in the gross real return play a crucial role in determining the impact of the tax-responsibility parameter on lifetime welfare. It is also important to note that there is a correspondence between the sign of $W'(\phi)$ and the sign of $S'(r)$. This is because the behavior of the return to deposits to changes in the financing responsibility parameter differs according to the side of the Laffer curve the economy finds itself. Of equal importance is the size of the transfer.

Proposition 2 *a) Suppose $\xi_r < 1$ and $r_\phi > 0$, then $W'(\phi)$ may be positive or negative depending on the size of a.*
b) Suppose $\xi_r > 1$ and $r_\phi > 0$, then $W'(\phi) < 0$ (people prefer seigniorage over taxes)
c) Suppose $\xi_r < 1$ and $r_\phi < 0$, then again $W'(\phi) < 0$ (people prefer seigniorage over taxes)
d) suppose $\xi_r < 1$ and $r_\phi < 0$, then $W'(\phi)$ may be positive or negative depending on the size of a.

From Wallace (1980), we know that the most desired policy is the one in which makes the rate of return on fiat money equal to the rate of return on storage. This would require the money stock to decline, implying that lump-sum taxes would have to finance *more* than 100% of the lump-sum transfer. Put another way, the contracting money supply would amount to another form of subsidy for agents. For us, the question is, given that the money stock *cannot* contract, what fraction of the lump-sum transfer would agents prefer to see coming from lump-sum taxes. Given the generality of the setup, a specific answer to this question has eluded us thus far. The next section proceeds with a numerical analysis of the model economy.

4 Computational experiments

In this section, the objective is to quantify the effects that different values of the tax-responsibility parameter have on the agent's decisions. In particular, our goal here will be to study the following question: if a benevolent government wants to choose a value for ϕ that maximizes the lifetime utility of a representative agent, what value would it choose? Our numerical analyses will permit us to assess whether an "interior" value for ϕ may be chosen, i.e., whether the treasury and the central bank will be asked to *share* the responsibility of raising the revenue. In order to proceed further, we first specify preferences are an additive log form and compute the decision rules.

4.1 Example with log utility

The utility function is represented as follows:

$$U(c_1, c_2) = \ln(c_1) + \ln(c_2) \tag{13}$$

For this specification, the decision rule for deposits is:

$$d = \frac{y - \tau}{2} - \frac{a}{2r}$$

With $\tau = \phi a$, this reduces to

$$d = \frac{y - \phi a}{2} - \frac{a}{2r}. \tag{14}$$

Using (14) and (8), it is possible to solve for the equilibrium return to deposits, which is (after some algebra):

$$r^2 (y - \phi a) + r \{2 (1 - \phi) a - a - (y - \phi a) [\gamma + (1 - \gamma) x]\} + a [\gamma + (1 - \gamma) x] = 0 \tag{15}$$

Using (15), it is straightforward to solve for the equilibrium level of deposits, consumption, etc. For our purposes, what is important is that the equilibrium decision rule for deposits is a function of the responsibility parameter, ϕ. Because the expression for the gross real return to deposits is a quadratic, there are two candidate equilibria, even for the case with log utility.

These two solutions correspond to the two interest rates at which the central bank can raise the requisite revenue. Formally, define $r = r_1$ and $r = r_2$ to be the two solutions to (15). Then,

$$r_1 = \frac{-\{2(1-\phi)a-a-(y-\phi a)[\gamma+(1-\gamma)x]\}+\sqrt[2]{\{2(1-\phi)a-a-(y-\phi a)[\gamma+(1-\gamma)x]\}^2-4(y-\phi a)a[\gamma+(1-\gamma)x]}}{2(y-\phi a)}$$

and

$$r_2 = \frac{-\{2(1-\phi)a-a-(y-\phi a)[\gamma+(1-\gamma)x]\}-\sqrt[2]{\{2(1-\phi)a-a-(y-\phi a)[\gamma+(1-\gamma)x]\}^2-4(y-\phi a)a[\gamma+(1-\gamma)x]}}{2(y-\phi a)}.$$

Clearly, $r_1 > r_2$. From (10), we have

$$(1-\phi)\,a = \gamma d(r) - d(r)[r-(1-\gamma)x] \equiv S(r).$$

Using (14) in (10), we get

$$S(r) = \gamma\left[\frac{y-\phi a}{2} - \frac{a}{2r}\right] - [r-(1-\gamma)x]\left[\frac{y-\phi a}{2} - \frac{a}{2r}\right]. \qquad (16)$$

By definition, r_1 and r_2 satisfy the government budget constraint; that is, $S(r) = (1-\phi)\,a$ holds in equilibrium.

For illustrative purposes, consider an economy with the following parameters: $y = 1$, $a = 0.06$, $\gamma = 0.173$, $x = 1.07$, and $\phi = 0.3$. Then $r_1 = 0.96$ and $r_2 = 0.06$. The seigniorage Laffer curve, $S(r)$ (see (16)) is illustrated in Figure 2. It is plotted along with the horizontal line $(1-\phi)\,a$. Two things are clear from the picture. First, the central bank's financing responsibility is met either at the low interest rate, $r_2 = 0.06$, or at the high interest rate, $r_1 = 0.96$. Second, an increase in the parameter ϕ would lower the central bank's financing responsibility. The horizontal line above would shift downwards. This would increase the rate of interest at r_1 and lower it at r_2. In other words, the central bank would have to lower the money growth rate at r_1 and raise it at r_2. It is therefore not surprising that the welfare results at the two interest rates will be very different, an issue we turn to below.

4.2 Numerical analysis with log utility

For the computational experiments, we work with the utility function as in (13) and set the baseline parameters as follows: $y = 1$, $a = 0.06$, $\gamma = 0.173$, and $x = 1.07$. We let ϕ vary between 0 and 1. Recall that an increase in ϕ lowers the central bank's responsibility, shifting greater burden on to the treasury. For these parameter settings, it is straightforward to compute the gross real return to deposits, and subsequently, the equilibrium level of deposits, the gross real return on fiat money (which is also the inverse of the money growth rate that satisfies the government budget constraint), and the steady-state level of welfare. It turns out that for the baseline parameters, there is a unique *valid* equilibrium corresponding to the high interest rate equilibrium alluded to in Section 4.1.[20]

We report on four endogenous variables: welfare, the gross real return to deposits, the quantity of deposits, and the growth rate of money as they relate to the tax-responsibility parameter. Figure 3 depicts these relationships in four separate panels. Panel a (upper left) shows that steady-state welfare reaches a maximum at around $\phi = 0.2$ indicating that agents in the model economy would prefer that the central bank and the treasury *share* the revenue raising responsibilities such that only about 20% of the revenues "should" come from lump-sum taxes. The remaining panels help to show why this result is obtained. Equation (12) indicates that the effect of changes in the tax-responsibility parameter on the gross real return to deposits *and* the effect on the elasticity of deposits to changes in the gross real return are important factors in determining the direction of change in steady-state welfare. Panel b (upper right) shows that the gross real return to deposits is increasing in the tax-responsibility parameter. In large part, Panel d (lower right) can account for the direction of change in r_ϕ. Indeed, the money growth rate that satisfies equation (10) decreases as the tax-responsibility parameter increases. The effect on agent's savings behavior is a bit muddled. On the one hand, an increase in ϕ translates into a higher tax burden on the

[20]See Appendix A for a complete discussion of the existence and uniqueness of valid steady states using the law of motion for real money balances. There it is shown that the only *valid* stationary competitive equilibrium is one with a "high" interest rate. The equilibrium with a "low" interest rate is not legitimate as it is associated with negative money growth rates, and hence negative price levels. Quite unsurprisingly, the high-return steady state is not stable (see Sargent, 1987, p. 282).

agent's first period (which serves to reduce savings); on the other hand, the rise in the interest rate makes saving more attractive. As Panel c (lower left) shows, the net effect is that equilibrium savings fall.

Perhaps it is more straightforward to concentrate on consumption. Figure 4 plots the decision rule for c_1 and c_2 for different levels of ϕ. The top panel shows that an increase in the tax-responsibility parameter reduces the agent's first-period consumption. Thus, the increase in lump-sum taxes more than offsets the decline in saving. As the bottom panel shows, despite a decline in saving, the increase in the return to deposits is enough to raise second period income, and consumption. In other words, an increased use of the lump-sum tax option (given a level of old-age transfer) increases old-age utility at the expense of young-age utility. Consequently, the overall effect on lifetime welfare is non-monotonic. As Figure 3 (panel a) illustrates, overall lifetime welfare goes up as ϕ rises up to a critical level, but beyond this, welfare will fall. As Figure 4 shows, the fall in first period consumption is *far* too severe to be made up by increased second-period consumption. Thus, the parameterized model economy yields results that match up with case (a) in Proposition 2.

This welfare result itself is somewhat counterintuitive. After all, seigniorage is acquired by using the distortionary inflation tax, whereas lump-sum taxes are not distortionary. So why is it that a benevolent government, faced with a option of choosing between a distortionary and a non-distortionary instrument, elects to use some of the latter anyway? Note that there is an explicit link between the tax-responsibility parameter and the money growth rate. Because there is a fixed level of government spending, changes in the usage of lump-sum taxes directly affects the money growth rate that the central bank can set, which indirectly changes the relative price of first and second period consumption. Changes in the reliance on seigniorage has similar distorting effects. As such, it becomes conceivable that a benevolent government may choose a combination financing scheme. Next, we conduct further computational experiments to assess whether our welfare results is sensitive to our parameter settings.

4.3 Assessing the importance of key parameters

One can easily quibble with the selection of the parameter values in the baseline computational experiments.[21] Here, we consider the effects of different parameter settings; specifically, we consider different levels of lump-sum transfer payments, reserve requirements and gross real return to storage.

We begin asking, what happens to lifetime welfare under different values of the old-age transfer?[22] Figure 5 plots the agent's lifetime welfare against the tax-responsibility parameter for two different values of a; $a = \{0.01, 0.15\}$. In either case, there is a single valid equilibrium corresponding to a high-return on deposits. The two panels of Figure 5 indicate that changes in the size of the lump-sum transfer precipitate changes in the welfare-maximizing choice of the tax-responsibility parameter. With $a = 0.01$, the top panel shows that agents would prefer seigniorage to account for 100% of the government's revenue. The most preferred financing scheme, however, shifts as the size of lump-sum transfers increases. Indeed, the bottom panel indicates that agents prefer lump-sum taxes to account for slightly over 80% of government revenues when $a = 0.15$. The computational experiments indicate that as the size of old-age transfers increase, agents are more willing to pay for them with lump-sum taxes. As the old-age transfer increases, for example, it becomes easier for agents to enjoy welfare-improving consumption allocations and hence agents want a smaller inflation tax on their savings. In other words, an increase in the old-age transfer payment effectively insures the agent, reducing the "cost" of young-age taxation. Consequently, the agent wants the smallest distortion to the return on their savings. Hence, they prefer lump-sum taxes.

Figure 6 reports the combinations of lifetime welfare and the tax-responsibility parameter with the reserve requirement set equal to 1%. We consider this case because all storage is intermediated in this model economy. Compared with those reported in the baseline setting, the results with $\gamma = 0.01$ are quantitatively different. As Figure 6 shows, welfare declines

[21] In some cases, the we use sample averages from data while in other cases, the parameters are selected. The selection is perhaps especially nettlesome with an overlapping gnerations model because the appropriate length of the period is 25-30 years if taken literally.

[22] In order to save space, we report only the combinations of steady-state welfare and the tax-responsibility parameter. Multi-panel figures (see Figures 3 and 4) are available from the authors upon request.

as the tax-responsibility parameter increases. Agents accordingly prefer seigniorage to pay for 100% of the revenues. With a lower reserve requirement, the inflation tax base declines as the reserve requirement declines. So does the distortion inherent to holding fiat money. For lifetime welfare, it is the trade-off between the gross real return to deposits and the burden of tax payments when young that matters. A quick glance at the expression for the equilibrium gross real return to deposits shows that r_ϕ is smaller (in absolute value) with a smaller reserve requirement. Consequently, agents prefer the smaller change in the price of old-age consumption to the change in young-age disposable income that accompanies a change in the tax-responsibility parameter.

In addition, we consider two alternative settings for the gross real return to capital. We set $x = 1.7$ to address concerns that a two-period overlapping generations economy if properly calibrated ought to correspond to gross real returns over periods of 20 to 30 years. We also consider a case in which $x = 1.007$ to indirectly address the importance of the rate of return dominance of money. In effect, the latter case corresponds to a situation where the return spread between the two vehicles for savings is tiny.

Figure 7 plots the steady state welfare level for both settings. The top panel plots combinations of W and ϕ with $x = 1.7$. This experiment shows that agents prefer seigniorage to account for 100% of the government's revenue. In contrast, with a low return to storage of $x = 1.007$ (the bottom panel), welfare increases as the tax-responsibility parameter increases. Here, agents prefer lump-sum taxes to account for 100% of the government's revenue. The results from these two experiments suggest that the size of rate-of-return dominance plays an important role in determining which financing scheme is preferred. When the yield spread is small between storage and money, agents are willing to take a decline in disposable income when young because the rate of return on deposits is relatively greatly affected when the government reduces its reliance on seigniorage. In contrast, with a higher gross real return to storage, the spread between capital and fiat money is greater. Greater reliance on lump-sum taxes means that agents suffer a greater decline in welfare for a given increase in the money growth rate. Hence, agents are more willing to substitute a loss of disposable income when young to minimize the spread between returns to capital and money.

In general, the results of the computational experiments indicate that welfare is highest when seigniorage accounts for some fraction of total government spending. As such, the model economy can account for the scatter-plot in Figure 1. In other words, we have articulated a theory as to why governments around the world use seigniorage to fund some portion of their spending. A more modest claim is that the model economy also offers an account for why the reliance on seigniorage (alluded to in the introduction) varies so greatly across countries. A tentative interpretation is that the sensitivity analyses indicates wide variety in the value of ϕ according to changes in some key parameters.

5 Assessing the crucial features

In this section, we delve deeper and attempt to identify the factors that are necessary to obtaining our findings. To that end, we ask: how robust are our findings to alternative modeling specifications? How important are the assumptions of an overlapping generations structure and a binding legal restriction, as far as delivering our main results? Do our results depend on whether the government uses the revenue to fund the old-age transfer or uses it to provide "useless" government goods? In other words, what is the minimal set of restrictions that are necessary to obtaining our results?

5.1 With bequests

What if we eliminated the market incompleteness (inherent in the overlapping-generations environment) by allowing agents to leave bequests for their offspring, and retained the legal restriction? Could we then justify the use of seigniorage on welfare grounds? The answer is yes.

We begin with a general version of the model economy that effectively mimics an infinite horizon economy. In particular, suppose that agents care about the utility of their offspring by choosing to leave non-negative bequests, b, to their children. Formally, they solve the following program:

$$\max_{c_{1t},c_{2t+1},b} U_t\left(c_{1t}, c_{2t+1}, U_{t+1}\right) = u\left(c_{1t}\right) + v\left(c_{2t+1}\right) + \beta U_{t+1}$$

subject to

$$y + b_t = c_{1t} + d_t + \tau_t,$$

$$r_{t+1} d_t + a_{t+1} = c_{2t+1} + b_{t+1},$$

and

$$b_{t+1} \geq 0.$$

It is straightforward to see that with $1 > \beta > 0$, this problem is identical to the program faced by an agent who lives forever. Following Freeman (1993), Appendix B sketches a version of our original model where agents are altruistic in that they care about the welfare of their children via the bequests they leave for them. We show that steady state welfare is maximum at some $\phi \in [0, 1)$ if: (i) bequests are positive; (ii) $\beta > 0.5$; and (iii) the return to storage is not "too big". With these three conditions satisfied, the best thing for the government is to ask the treasury and the central bank to *share* the financing responsibility, even though lump-sum taxes are available. Put differently, the model economy with bequests contains equilibria where a) the government's choice of monetary policy coincides with the "golden rule" policy in Wallace (1980), and b) where it does not, just as our original model.

With bequests, market incompleteness is eliminated. Bequests essentially allow the young to "borrow" from the old! If the old care about their children, they will leave enough bequests to try and "nullify" the present cost of the tax faced by the young who are getting hit with the tax bill. That is, they may leave enough bequests to make up for the loss in first-period disposable income of the young. Why may the old want to do such a thing? First, they care about their children, and second, compensating the young with bequests will leave the return on savings (deposits) relatively unaffected. Since the only vehicle of intertemporal consumption-smoothing is deposits, the old would want to take any action (leave bequests) that would reduce the distortion on this income transfer channel. All this comes with a cost: reduced second period income (because of the old-age bequest). As such, welfare may be maximized at some interior level for ϕ.

With money and capital in this model, the reserve requirement produces a wedge between the return to capital and the return to deposits. The Pareto optimal monetary policy is to set $\theta = \frac{1}{x}$. [23] Indeed, the way to remove the distortion of money is to make its real return equal to the real return on the alternative asset. Such a policy yields the Pareto efficient allocation whether the bequest motive is operational or not. The upshot is that with $x > 1$, the optimal monetary policy requires the money supply to contract over time. Alternatively, ϕ would have to set *higher than unity* in order to pay for the lump-sum transfer payment given the loss in revenue from the contracting stock of money. Because the money supply cannot contract, the government's choice of monetary policy will generically not coincide with the "golden rule" policy in Wallace (1980). In short, central bank responsibility may be positive if a binding reserve requirement coexists with the restriction that the money stock cannot contract.

5.2 Purposeless government expenditures

So far, we have modeled government spending as being purposeful, i.e., government expenditures can be consumed by agents. Would our central result about shared financing responsibility survive if we abandoned that assumption? To study this, we modify the model economy as follows. We posit that the government has to finance a fixed level of government expenditures, g, every period. Specifically, every period, the government appropriates g units of the consumption good from the agents (possibly) via a combination of lump-sum

[23]The planner solves:

$$\max u(c_1) + v(c_2)$$

s.t

$$c_1 + \frac{c_2}{x} = y$$

The first order conditions for the planners problem reveals:

$$\frac{u'(c_1)}{v'(c_2)} = x$$

>From the agent's problem (the competitive solution), we know:

$$\frac{u'(c_1)}{v'(c_2)} = r$$

Therefore, the planner must choose θ to ensure that $r = x$ which implies $\theta = \frac{1}{x}$. But, since $x > 1$, this is exactly the Friedman rule.

taxes and money creation.[24] These goods are then costlessly transformed into a "government good" which does not enter into the agents' utility function. It is as if the appropriated consumption goods are thrown in the ocean.

In this modified environment, the agent's problem then is to find non-negative combinations of first and second period consumption that maximize (1) subject to

$$c_1 = y - d - \tau$$

and

$$c_2 = rd.$$

If the utility function is specified as (13), it is easy to check that $d = \frac{y-\tau}{2}$. An exact analog of (8) for this economy may be written as

$$r \equiv r(\phi) = (1-\gamma)\,x + \gamma\left[1 - \frac{(1-\phi)\,g}{\gamma d}\right] = (1-\gamma)\,x + \gamma\left[1 - \frac{(1-\phi)\,g}{\gamma\left\{\frac{y-\phi g}{2}\right\}}\right].$$

In every other aspect, this economy is identical to the economy described in Section 2. We adopt the same baseline parameters as before: $y = 1$, $a = 0.06$, $\gamma = 0.173$, and $x = 1.07$. Then, it is straightforward to verify that as ϕ increases from 0 to 1, the money growth rate falls which raises the return on deposits. It is also the case that the volume of deposits falls, first period consumption falls and second period consumption rises. It turns out that agents in this economy would prefer that the treasury raise about 20% of the revenue needed to back this "purposeless" government spending. If g increases to 0.08, the agents would prefer that lump-sum taxes back about 30% of the government's spending. As such, our central result is immune to whether government spending affects agents' budget sets or not.

5.3 Remarks

The purpose of this section was to uncover the crucial element in the model economy that was responsible for generating our central result of shared financing responsibility. We showed that, ceteris paribus, the finiteness of the lifetimes of our agents (and the inherent market incompleteness in the overlapping generations setup) was neither a necessary nor a sufficient

[24] Here, the government uses fiat money in exchange for units of the consumption good.

precondition. In the same vein, it was not qualitatively important whether government expenditures got rebated (in the form of a lump-sum old-age transfer) or whether they were purely without purpose. As such, it is clear that the distortion caused by the binding reserve requirement and the restriction that the money stock cannot contract are *necessary* conditions to obtaining our main result. Without these two distortions, the solution to the government's problem in our model economy would coincide with the fixed money stock Pareto efficient policy exactly as in Wallace (1980).

6 Concluding remarks

In this paper, we explore the quantitative properties of a simple general equilibrium model in which two-period lived agents get an old-age transfer from their government. The government makes a decision about how much of this transfer should come from lump-sum taxes on the young. The alternative means of finance is seigniorage.

At first glance, it might appear that the answer is obvious: the government should raise the revenue from the non-distortionary lump-sum tax. Somewhat surprisingly, it turns out that for a nontrivial part of the parameter space, agents in the model economy would prefer that some (if not all) the revenue comes from seigniorage. In our analysis, a reserve requirement distorts the only means of saving – deposits held with a financial intermediary. In such an environment, it is possible that the loss in first-period disposable income due to the lump-sum tax outweighs the potential benefit of having the return on deposits relatively unaffected. As such, agents may prefer that the return on savings takes the hit as opposed to their young-age disposable income. This would imply that the government would then choose to raise some of the revenue from money creation. We show that two conditions are crucial to obtaining the shared financing responsibility result: the coexistence of a reserve requirement and the restriction that the money supply cannot contract. When these two necessary conditions are satisfied, lump-sum taxes become so inextricably linked to the distortionary inflation tax that even when lump-sum taxes are used exclusively, the agent's consumption-saving decision gets distorted. This is not a feature of the standard textbook experiment. In other words, lifetime welfare *can* be higher with distortionary taxes even though lump-sum taxes are available.

It bears emphasis here that the current paper does not contribute to the vast literature on the "optimal inflation tax". In that literature, the focus is on the social planner's problem and hence, the goal is to spell out the Pareto efficient monetary policy. In fact, the Pareto efficient policy in our setup is still identical to the one in Wallace (1980). Here, the government takes as *given* the "policy reaction functions" of the agents, and then decides on a policy that maximizes a representative agent's welfare. We simply show that there are competitive equilibria which correspond to welfare maxima and where the government does not let the money stock contract.

The current paper has been silent as to *why* a government would take the reserve requirement and the restriction on the money-supply process as given. An interesting direction for future work would be to study whether benevolent governments would endogenously accept these restrictions.

A Appendix A

In this appendix, we explore certain properties of the law of motion for real balances with a view to proving the existence of a unique valid steady state equilibrium in the model economy. We begin by writing down the law of motion for real money balances. Let z denote the demand for real money balances or $z = m/p$. Then,

$$z_t = \gamma d_t \tag{A.1}$$

Substitute for deposits, using the equilibrium decision rule:

$$d_t = \frac{y - \phi a}{2} - \frac{a}{2r_t} \tag{A.2}$$

where

$$r_t = (1-\gamma)x + \gamma\left(\frac{p_t}{p_{t+1}}\right). \tag{A.3}$$

To derive the equilibrium expression for $\left(\frac{p_t}{p_{t+1}}\right)$, note that seigniorage equals $(1-\phi)a$, i.e.,

$$\frac{m_{t+1} - m_t}{p_{t+1}} = (1-\phi)a$$

Using (A.1) in the above expression, we get

$$\left(\frac{p_t}{p_{t+1}}\right) = \frac{z_{t+1} - (1-\phi)a}{z_t}. \tag{A.4}$$

Combining (A.1)-(A.4), we get the law of motion for equilibrium real balances in this model economy. More specifically,

$$\begin{aligned} z_t &= \gamma d_t = \gamma\left[\frac{y-\phi a}{2} - \frac{a}{2r_t}\right] = \gamma\left[\frac{y-\phi a}{2} - \frac{a}{2\left\{(1-\gamma)x + \gamma\left(\frac{p_t}{p_{t+1}}\right)\right\}}\right] \\ &= \gamma\left[\frac{y-\phi a}{2} - \frac{a}{2\left\{(1-\gamma)x + \gamma\left[\frac{z_{t+1}-(1-\phi)a}{z_t}\right]\right\}}\right] \end{aligned}$$

Then,

$$2z_t = \gamma\left[(y-\phi a) - \frac{a}{\left\{(1-\gamma)x + \gamma\left[\frac{z_{t+1}-(1-\phi)a}{z_t}\right]\right\}}\right]$$

which is a first-order difference equation in z. Simplifying, we get

$$\gamma(y-\phi a)-2z_t=\frac{a\gamma}{\left\{(1-\gamma)x+\gamma\left[\frac{z_{t+1}-(1-\phi)a}{z_t}\right]\right\}}>0 \tag{A.4$'$}$$

where the inequality follows from (A.4). Some more routine algebra yields

$$z_{t+1}=\Phi(z_t)\equiv\frac{a}{\frac{\gamma(y-\phi a)}{z_t}-2}-\frac{(1-\gamma)}{\gamma}xz_t+(1-\phi)a. \tag{A.5}$$

All legitimate equilibria z_t must also satisfy the following restrictions:[25]

$$\begin{aligned} z_t &> 0 \\ x &> \left(\frac{p_t}{p_{t+1}}\right)=\frac{z_{t+1}-(1-\phi)a}{z_t} \\ \left(\frac{p_t}{p_{t+1}}\right) &= \frac{z_{t+1}-(1-\phi)a}{z_t}>0 \end{aligned} \tag{**}$$

Straightforward differentiation of (A.5) yields

$$\frac{dz_{t+1}}{dz_t}=\frac{a\gamma(y-\phi a)}{[\gamma(y-\phi a)-2z_t]^2}-\frac{(1-\gamma)}{\gamma}x. \tag{A.6}$$

It is also easy to verify that

$$\frac{d^2z_{t+1}}{dz_t^2}=\frac{4a\gamma(y-\phi a)}{[\gamma(y-\phi a)-2z]^3}>0$$

where the inequality follows from $(A.4)'$. Thus $\Phi(z_t)$ is strictly concave in z_t. In other words, the $z_{t+1}=\Phi(z_t)$ locus *cannot* be $\cap$ shaped.

As is clear from (A.6), $\frac{dz_{t+1}}{dz_t}$ is of ambiguous sign. In particular, the possibility arises that $\frac{dz_{t+1}}{dz_t}$ might be negative for some range of z and positive for some other range; in which case, $z_{t+1}=\Phi(z_t)$ would be non-monotonic.

Claim 3 *Define*

$$A_0=\gamma(y-\phi a)$$
$$A_1=\frac{(1-\gamma)}{\gamma}x$$

[25] That is, currency must be valued, storage must dominate currency in rate of return, and price levels must be positive.

$$\hat{z}_1 = \frac{A_0 + \sqrt{\frac{A_0(a-A_1)}{A_1} + (A_0)^2}}{2}$$

and

$$\hat{z}_2 = \frac{A_0 - \sqrt{\frac{A_0(a-A_1)}{A_1} + (A_0)^2}}{2}.$$

Then, $\frac{dz_{t+1}}{dz_t} = 0$ *if* $z = \hat{z}_1$ *or* $z = \hat{z}_2$.

Proof. Use (A.6) to set

$$\frac{dz_{t+1}}{dz_t} = \frac{a\gamma\,(y-\phi a)}{\left[\frac{\gamma(y-\phi a)}{z_t} - 2\right]^2}\frac{1}{z_t^2} - \frac{(1-\gamma)}{\gamma}x = 0,$$

and rewrite it as

$$\frac{A_0}{\left[\frac{A_0}{z} - 2\right]^2}\frac{1}{z^2} = A_1.$$

Straightforward rearrangement yields

$$z^2 - A_0 z - \frac{A_0(a-A_1)}{4A_1} = 0.$$

It is easy to check that the solutions to this quadratic are $z = \hat{z}_1$ or $z = \hat{z}_2$. ■

An immediate implication of Claim 3 is that it is impossible for $\frac{dz_{t+1}}{dz_t} > 0$ for all z. In other words, $z_{t+1} = \Phi(z_t)$ cannot be a monotonically increasing sequence.

In steady state, $z_{t+1} = z_t = z$. Substituting z into equation (A.5) yields,

$$z = \frac{a}{\frac{\gamma\,(y-\phi a)}{z} - 2} - \frac{(1-\gamma)}{\gamma}xz + (1-\phi)a \tag{A.7}$$

Note *all* valid steady state equilibria must satisfy the following conditions (analogous to (**)):

$$\begin{aligned} z &> 0 \qquad (*) \\ x &> \left(\frac{p_t}{p_{t+1}}\right) = 1 - \frac{(1-\phi)a}{z} \\ \left(\frac{p_t}{p_{t+1}}\right) &= 1 - \frac{(1-\phi)a}{z} > 0 \end{aligned}$$

Claim 4 *At any valid steady state* z, $\frac{dz_{t+1}}{dz_t}|_z$ *cannot be negative.*

Proof. It is possible to rewrite (A.7) as

$$\frac{(1-\gamma)}{\gamma}x = \frac{a}{\gamma\left(y-\phi a\right)-2z} + \frac{(1-\phi)a}{z} - 1 \tag{A.9}$$

That is, any candidate steady state must satisfy (A.9). Now, substitute for $\frac{(1-\gamma)}{\gamma}x$ from (A.9) into (A.6) yielding:

$$\frac{dz_{t+1}}{dz_t} = \frac{a\gamma\left(y-\phi a\right)}{\left[\gamma\left(y-\phi a\right)-2z\right]^2} - \frac{a}{\gamma\left(y-\phi a\right)-2z} - \frac{(1-\phi)a}{z} + 1$$

Next, multiply the second term on the rhs by $\frac{\gamma(y-\phi a)-2z}{\gamma(y-\phi a)-2z}$, and rearrange to get

$$\frac{dz_{t+1}}{dz_t} = \frac{2az}{\left[\gamma\left(y-\phi a\right)-2z\right]^2} \underbrace{-\frac{(1-\phi)a}{z} + 1}$$

>From (*), it follows that at any *valid* steady state, $\left(\frac{p_t}{p_{t+1}}\right) = 1 - \frac{(1-\phi)a}{z} > 0$ implying that the sum of the second and third terms is positive. The claim is verified. ■

Alternatively, there does *not* exist any valid steady states on the downward sloping part of the $z_{t+1} = \Phi(z_t)$ locus. By implication, valid steady states (if any) are to be found on the *upward* sloping portion of the locus.

The possibility remains that there are multiple steady states on the upward sloping part of the $z_{t+1} = \Phi(z_t)$ locus. To rule this out, it is sufficient to prove that at the minimum point of the $z_{t+1} = \Phi(z_t)$ locus, $z_t > z_{t+1}$. To see this, recall that the minimum point on the $z_{t+1} = \Phi(z_t)$ locus may be computed from

$$\frac{dz_{t+1}}{dz_t} = \frac{a\gamma\left(y-\phi a\right)}{\left[\gamma\left(y-\phi a\right)-2z_t\right]^2} - \frac{(1-\gamma)}{\gamma}x = 0$$

or, that the minimum point on the $z_{t+1} = \Phi(z_t)$ locus satisfies

$$\frac{a\gamma\left(y-\phi a\right)z}{\left[\gamma\left(y-\phi a\right)-2z\right]^2} = \frac{(1-\gamma)}{\gamma}xz$$

Use this in (A.5) to get

$$\begin{aligned} z_{t+1} &= \Phi(z_t) \equiv \frac{az_t}{\gamma\left(y-\phi a\right)-2z_t} - \frac{(1-\gamma)}{\gamma}xz_t + (1-\phi)a \\ &= \frac{az_t}{\gamma\left(y-\phi a\right)-2z_t} - \frac{a\gamma\left(y-\phi a\right)z_t}{\left[\gamma\left(y-\phi a\right)-2z_t\right]^2} + (1-\phi)a \end{aligned}$$

which upon rearrangement yields

$$\frac{z_{t+1}}{z_t} = \frac{a}{\gamma\left(y-\phi a\right)-2z_t}[1-\frac{a\gamma\left(y-\phi a\right)}{\gamma\left(y-\phi a\right)-2z_t}]+\frac{(1-\phi)a}{z_t}. \tag{A.10}$$

Recall (from (**)) that $\left(\frac{p_{t-1}}{p_t}\right)=\frac{z_t-(1-\phi)a}{z_{t-1}}>0$ or that

$$z_t > (1-\phi)a \tag{A.11}$$

must obtain. From (A.10), it follows that

$$\frac{z_{t+1}}{z_t}-\frac{(1-\phi)a}{z_t} = \frac{a}{\gamma\left(y-\phi a\right)-2z_t}[1-\frac{a\gamma\left(y-\phi a\right)}{\gamma\left(y-\phi a\right)-2z_t}]<0$$

or that,

$$\frac{z_{t+1}}{z_t} < \frac{(1-\phi)a}{z_t} < 1$$

where the last inequality follows from (A.11).

To summarize, the only possible configuration for the $z_{t+1} = \Phi(z_t)$ locus is the one illustrated in Figure 8. The low z steady state is invalid by Claim 4. So the unique valid steady state is the high- z steady state. Also note that by implication, all non-stationary equilibria that start to the left of the high- z steady state become invalidated after the passage of sufficient time (since they end up corresponding to equilibria on the downward sloping part of the $z_{t+1} = \Phi(z_t)$ locus).

B Appendix B

In this appendix, we follow Freeman (1993) and sketch a model identical to the current setup except that we allow agents to leave bequests to their offspring. Consider a two-period lived overlapping generations model with pure exchange identical to the one discussed in Section 2. Here, agents have "dynastic" preferences in that the utility of an agent born at t, U_t equals $u(c_{1t}) + v(c_{2t+1})$ plus the discounted utility of her child, βU_{t+1}, i.e.,

$$U_t = u(c_{1t}) + v(c_{2t+1}) + \beta U_{t+1}. \tag{B.1}$$

β is the discount factor and $\beta \in (0, 1)$. We will assume

$$1 > \beta > 0.5. \tag{B.1*}$$

As is standard, (B.1) may be rewritten as

$$U_t = \sum_{t=0}^{\infty} \beta^t \left[u(c_{1t}) + v(c_{2t+1}) \right]. \tag{B.2}$$

In the first period of her life, a typical agent receives a bequest $b \geq 0$ from her parents. In addition, she receives the endowment of y units of the consumption good. When old, she gets a transfer a from the government and potentially leaves a bequest to her child. Her budget constraints are as follows:

$$y + b_t = c_{1t} + d_t + \tau$$

and

$$r_t d_t + a = c_{2t+1} + b_{t+1}.$$

Eliminating d from these constraints, we get the intertemporal lifetime budget constraint of the agent:

$$y + b_t + \frac{a}{r_t} = c_{1t} + \frac{c_{2t+1}}{r_t} + \frac{b_{t+1}}{r_t} - \tau. \tag{B.3}$$

The agent maximizes (B.2) subject to (B.3). We confine our attention to stationary equilibria. It is easy to check that the first order conditions for an interior solution to the agent's problem (with positive bequests) yields

$$c_1 = \beta c_2, \tag{B.4}$$

$$r = \frac{1}{\beta} = (1-\gamma)x + \frac{\gamma}{\theta}, \tag{B.5}$$

along with

$$y + b + \frac{a}{r} = c_1 + \frac{c_2}{r} + \frac{b}{r} - \tau. \tag{B.6}$$

Note that (B.5) implies

$$\left(1 - \frac{1}{\theta}\right) = 1 - \frac{(1-\gamma)x - \frac{1}{\beta}}{\gamma} \tag{B.7}$$

As before,

$$(1-\phi)\, a = \gamma d \left(1 - \frac{1}{\theta}\right) \tag{B.8}$$

or, using (B.7),

$$d^* = \frac{(1-\phi)\, a}{\gamma - (1-\gamma)x + \frac{1}{\beta}}. \tag{B.9}$$

Use $\tau = \phi a$, (B.4) and (B.5) in (B.6) to get

$$b = \frac{2\beta c_2 - (y + \phi a) - a\beta}{1-\beta}. \tag{B.10}$$

Also, use (B.4) and (B.5) in (B.6) to get

$$d = \beta(c_2 + b - a). \tag{B.11}$$

Equate (B.9) and (B.11) using (B.10) to get

$$c_2 = \frac{d^*(1-\beta) + (y + \phi a) + a}{1+\beta}. \tag{B.12}$$

Using (B.12) in (B.10) would then yield a value for b. We focus on equilibria where $b > 0$.

The government chooses ϕ to maximize steady state utility, or

$$\phi = \arg\max \left(\frac{1}{1-\beta}\right) [\ln c_1(\phi) + \ln c_2(\phi)].$$

It is easy to see that maximization of steady state welfare is equivalent to maximizing c_2. Use (B.9) and (B.12) to write

$$c_2 \equiv c_2(\phi) = \frac{\frac{(1-\phi)a(1-\beta)}{\gamma-(1-\gamma)x+\frac{1}{\beta}} + (y+\phi a) + a}{1+\beta} \tag{B.12*}$$

Then,

$$\frac{\partial c_2}{\partial \phi} = \frac{a}{1+\beta}\left[1 - \left(\frac{1-\beta}{\beta}\right)\frac{1}{\gamma - (1-\gamma)x + \frac{1}{\beta}}\right] \tag{B.13}$$

An immediate implication of (B.13) is that the relationship between c_2 (and hence steady state welfare) and ϕ is monotonic. From (B.13), it readily follows that

$$\frac{\partial c_2}{\partial \phi} > 0 \qquad \text{if } x < \frac{1+\gamma}{1-\gamma}.$$

For consistency, we also need to ensure (from (B.7)) that

$$\theta = \frac{\gamma}{\frac{1}{\beta} - (1-\gamma)x} > 1$$

or that

$$\gamma + (1-\gamma)x > \frac{1}{\beta} \tag{B.14}$$

Thus, for $\frac{\partial c_2}{\partial \phi} > 0$, we need

$$\frac{1}{\beta} - \gamma < (1-\gamma)x < 1+\gamma \tag{B.15}$$

to obtain. If (B.15) obtains, then the second-best optimum is $\phi = 1$. If ϕ is indeed 1, will bequests still be positive?

To see this, use (B.12*) in (B.10) to write:

$$b = \frac{\dfrac{\dfrac{2a\beta(1-\phi)(1-\beta)}{\gamma - (1-\gamma)x + \frac{1}{\beta}} + y + \phi a + a}{1+\beta} - y - \phi a - a\beta}{1-\beta}. \tag{B.16}$$

If $\phi = 1$ holds, (B.16) reduces to

$$b = \frac{a(1-2\beta) - \beta y - a\beta^2}{(1+\beta)(1-\beta)}$$

which is *negative* if (B.1*) holds. Thus, under the assumption (B.1*), $\phi = 1$ could *not* be the optimal choice of the government. In other words, welfare is maximum at some $\phi \in [0, 1)$ if bequests are positive and (B.15) holds. In which case, the best thing the government can do is make the treasury and the central bank *share* the financing responsibility. As such, even with altruistic agents, it is still possible that the government would want to raise at least some revenue from money creation even though lump-sum taxes are available.

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Figure 1

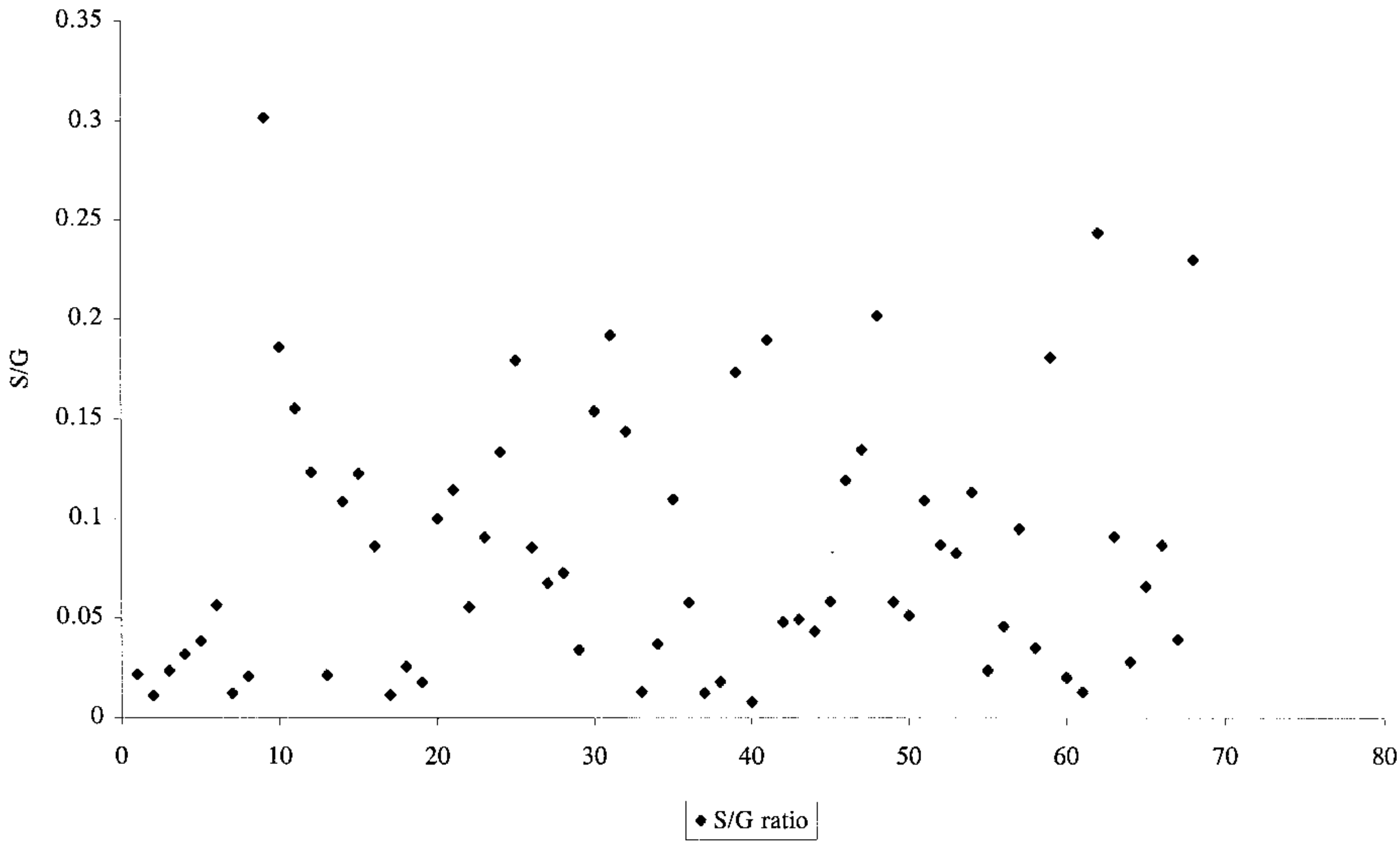


Figure 2: The steady state seigniorage Laffer curve

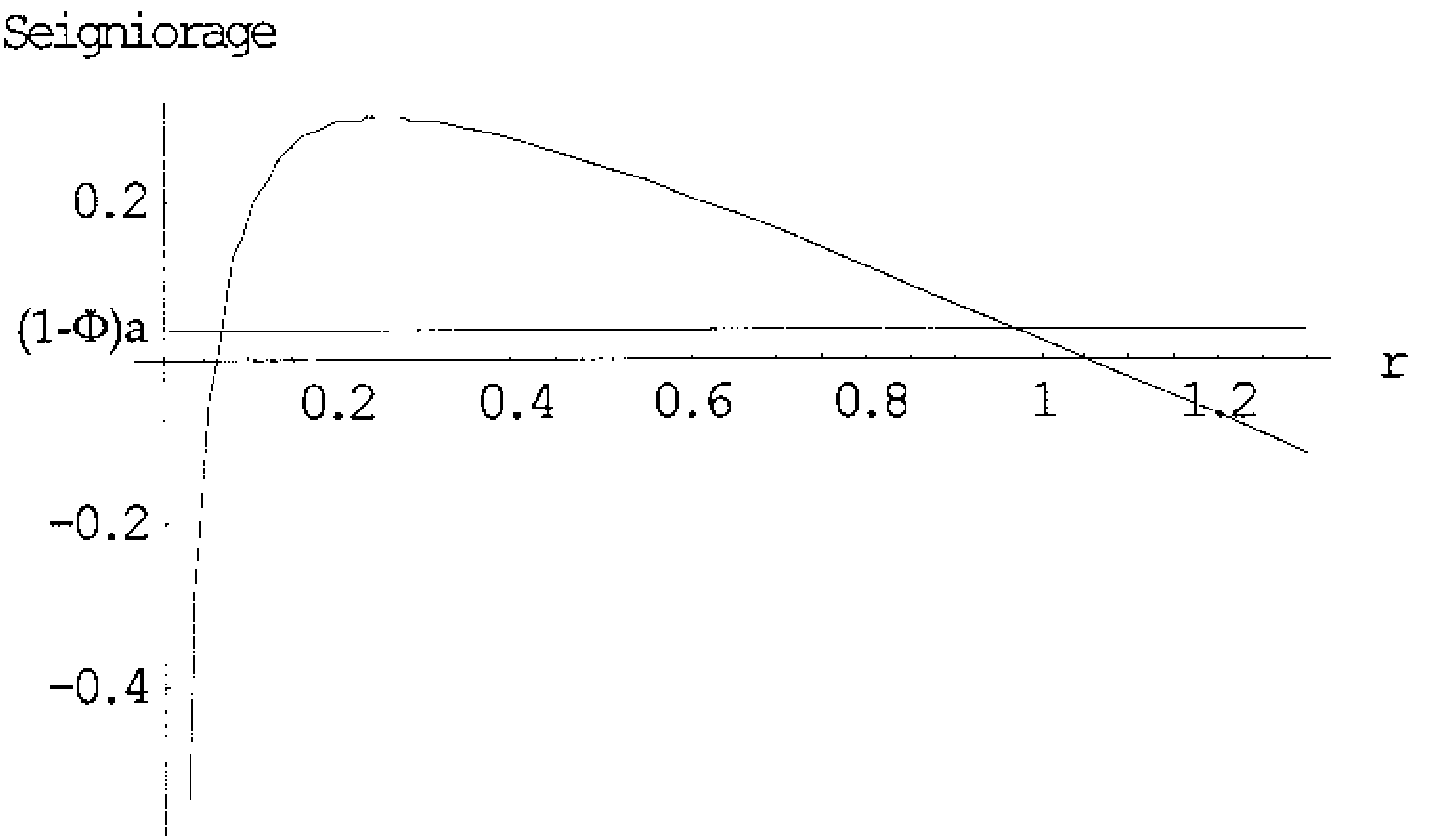


Figure 3

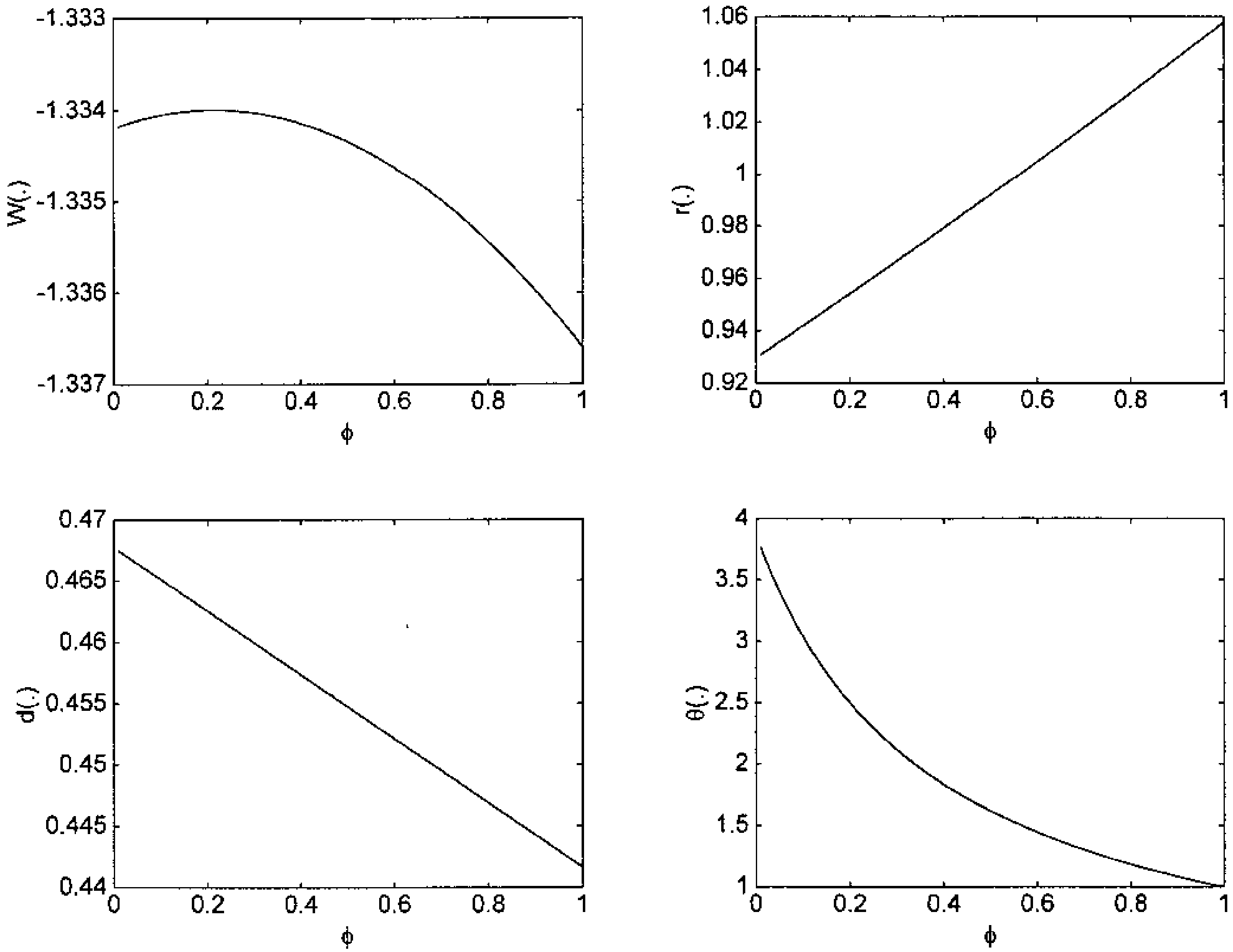
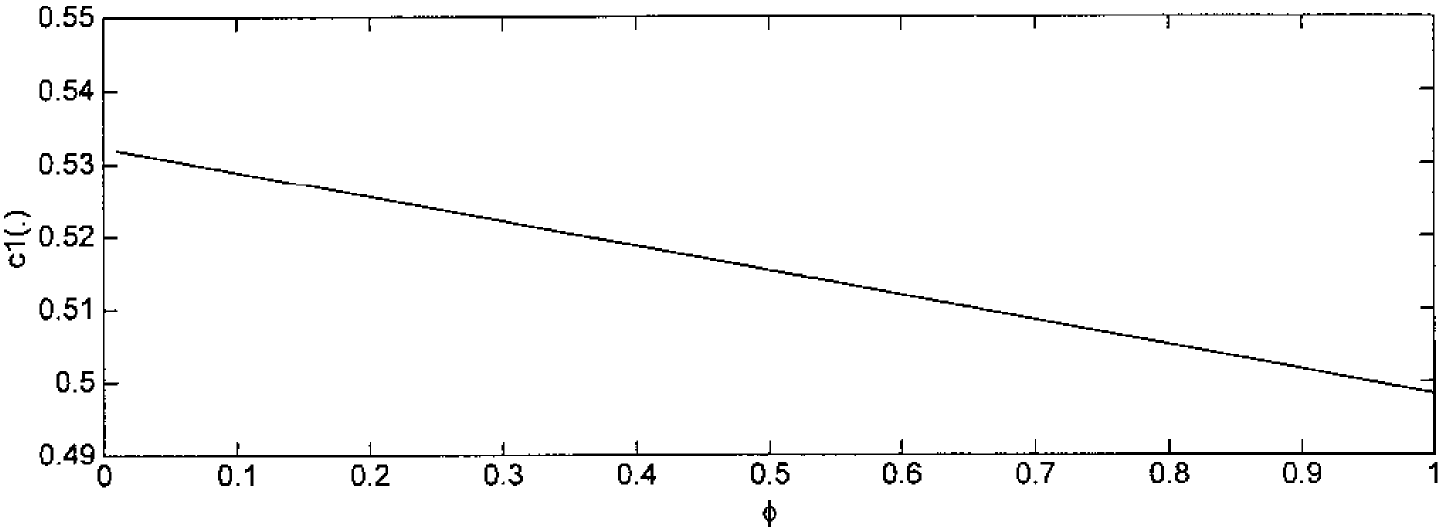
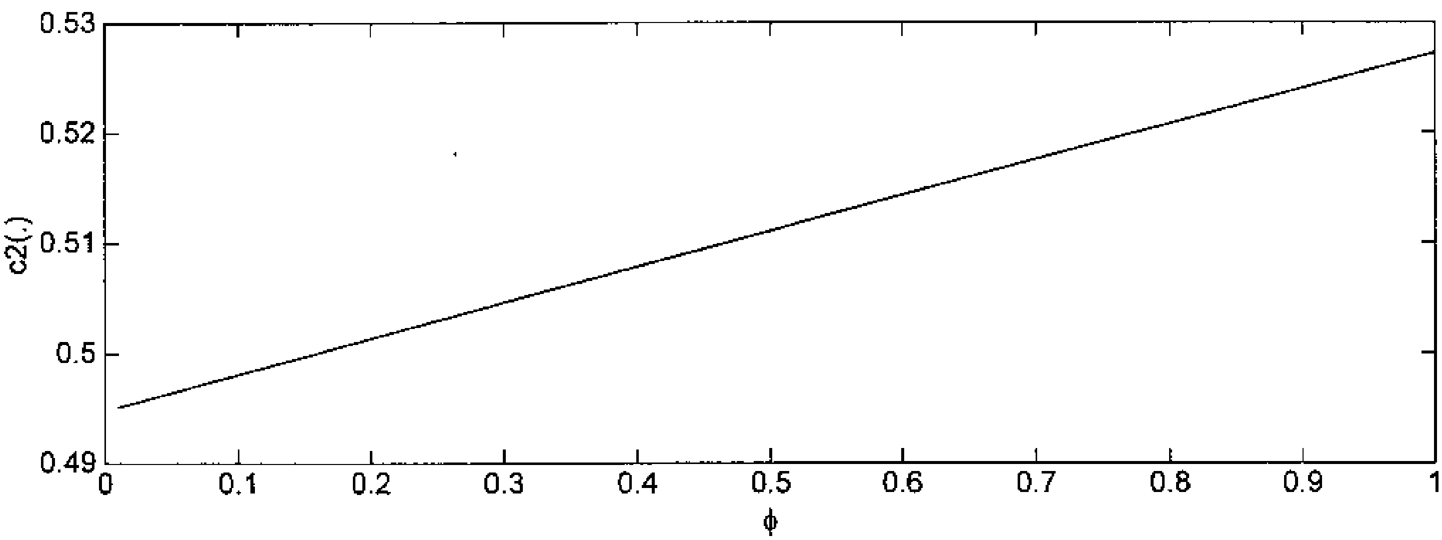

-1.333
-1.334
-1.335
-1.336
-1.337
W(.)
0 0.2 0.4 0.6 0.8 1
ϕ
1.06
1.04
1.02
1
0.98
0.96
0.94
0.92
r(.)
0 0.2 0.4 0.6 0.8 1
ϕ
0.47
0.465
0.46
0.455
0.45
0.445
0.44
d(.)
0 0.2 0.4 0.6 0.8 1
ϕ
4
3.5
3
2.5
2
1.5
1
θ(.)
0 0.2 0.4 0.6 0.8 1
ϕ

Figure 4

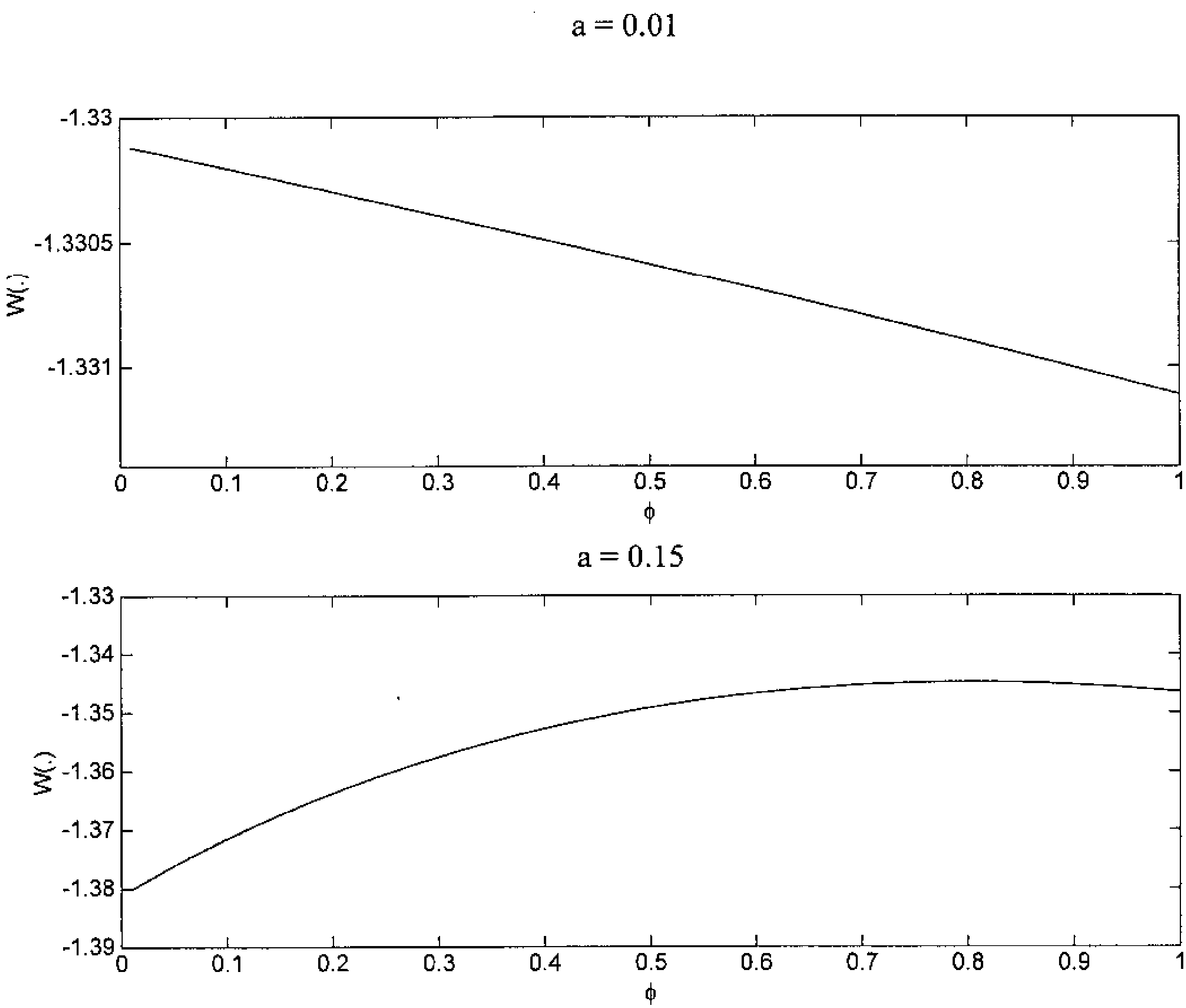


0.55
0.54
0.53
0.52
0.51
0.5
0.49
c1(.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ϕ



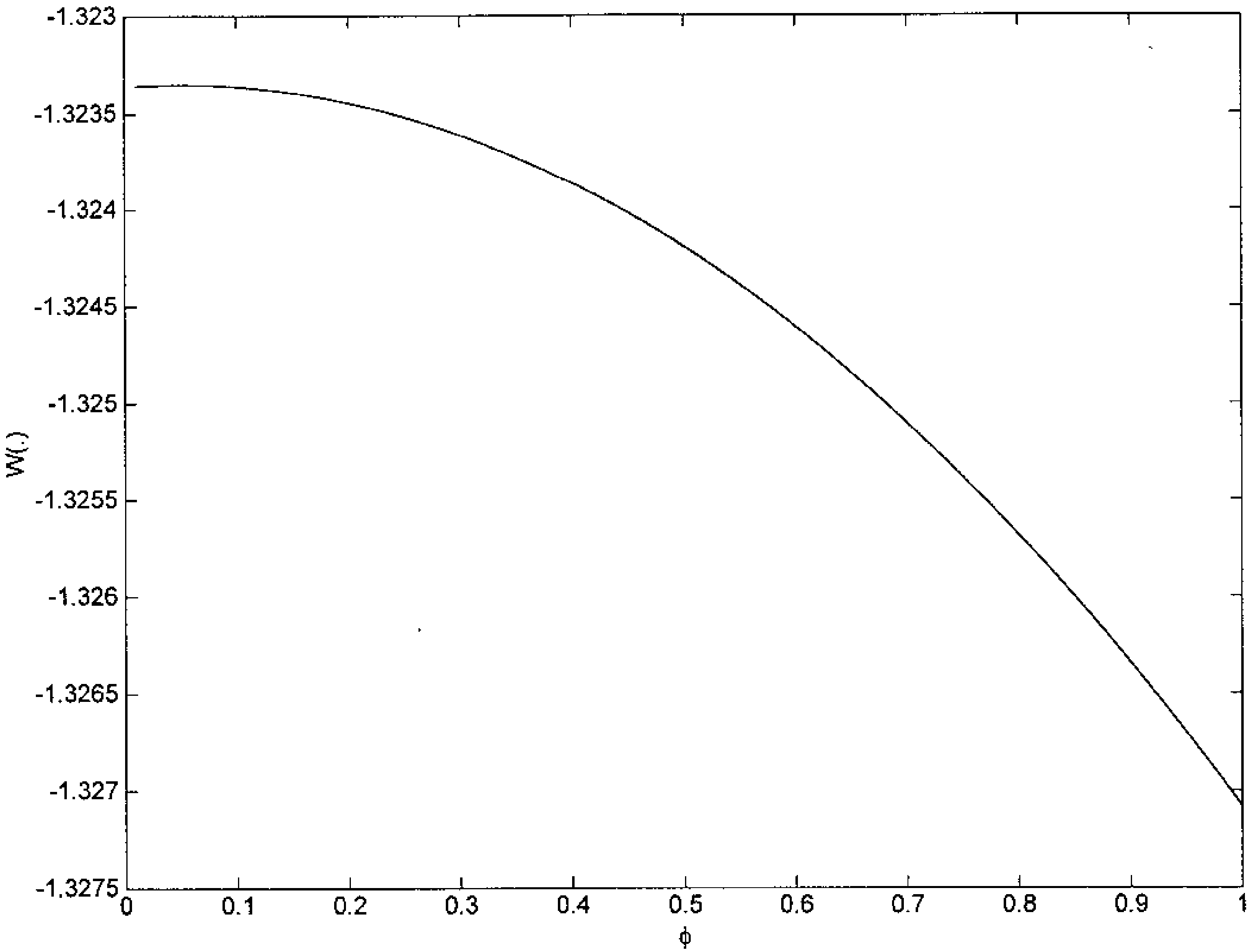
0.53
0.52
0.51
0.5
0.49
c2(.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ϕ

Figure 5



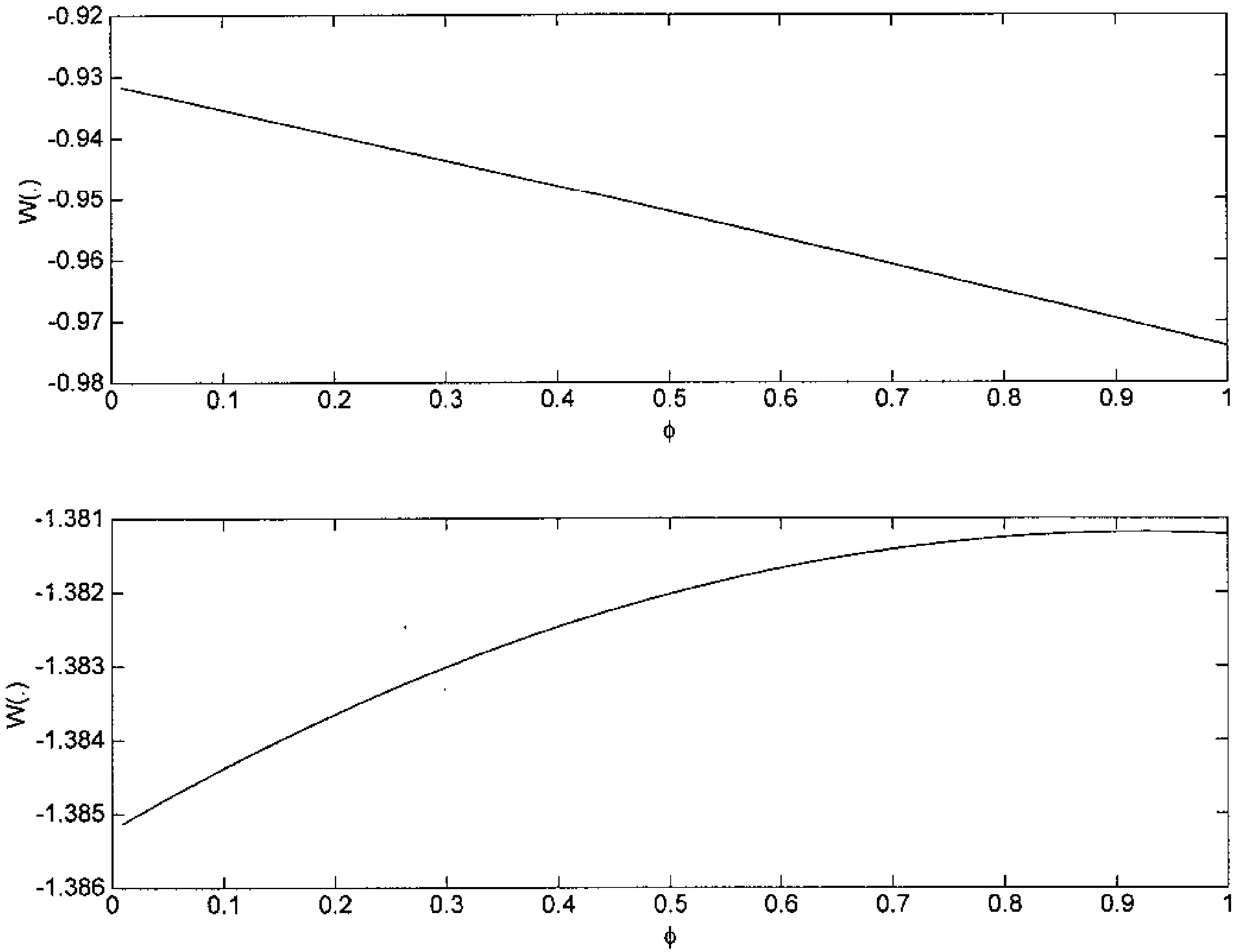
a = 0.01
-1.33
-1.3305
-1.331
W(.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
φ
a = 0.15
-1.33
-1.34
-1.35
-1.36
-1.37
-1.38
-1.39
W(.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
φ

Figure 6



-1.323
-1.3235
-1.324
-1.3245
-1.325
-1.3255
-1.326
-1.3265
-1.327
-1.3275
W(.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ϕ

Figure 7



-0.92
-0.93
-0.94
-0.95
-0.96
-0.97
-0.98
W(.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ϕ
-1.381
-1.382
-1.383
-1.384
-1.385
-1.386
W(.)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
ϕ

Figure 8: Equilibrium law of motion for real balances