SEIGNIORAGE IN A NEOCLASSICAL ECONOMY: SOME COMPUTATIONAL RESULTS

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Abstract

In this paper, we consider a government that executes a permanent open market sale. The government is forced to eventually use money creation to pay for the debt's expenses, choosing between changing either the money growth rate (the inflation-tax rate) or the reserve requirement ratio (the inflation-tax base). We first derive conditions under which each of the two second-best alternative policies are feasible in an economy with neoclassical production. Armed with these conditions, we ask the following question: which monetary policy action is better in a welfare sense? With neoclassical production, monetary policy potentially has long-run effects on the capital stock and the marginal product of capital. The curvature of the production function is crucial. The computational experiments show, somewhat surprisingly, that a permanent increase in government bonds is financed by either lower reserve requirements or faster money growth. Accordingly, steady-state welfare for all generations is higher under the reserve-requirement policy.

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1 Introduction

Governments routinely use seigniorage to finance some part of their expenditures. This fact has attracted researchers; indeed, there is a substantial literature devoted to the study of money growth and its role in government finance.[1]

The purpose of this paper is to address a simple question: in a closed economy, given that a government must raise seigniorage revenue, which tool — raising reserve requirements or increasing the money growth rate — is the preferred one? Seigniorage revenue is the product of a tax base and a tax rate. Therefore, our primary question can be restated as follows: would agents prefer that the government choose a policy action aimed directly at increasing the tax base, as the reserve requirement does, or prefer a policy aimed directly at increasing the tax rate, as the money growth rate does?[2]

To that end, we use a simple general equilibrium model of overlapping generations with production. Bank deposits are the most desirable store of value for agents. The deposits overcome a friction that prevents small savers from acquiring capital or holding government bonds directly. Fiat money is valued solely because banks are required to hold it. These required reserves constitute the seigniorage tax base, while the seigniorage tax rate is positively related to the rate of currency creation. The government initiates a permanent open market sale, permanently increasing the quantity of one-period, default-free bonds right at the start of time. We focus on equilibria where the promised real return on these exceed the real growth rate of the economy thereby precluding any revenue generation from bond seignorage. In such a setting, interest obligations will, in steady state, exceed the revenue from increased bond sales, requiring the government to rely on currency seigniorage to make up the revenue shortfall. Our central question is this: if the government is benevolent, which instrument — reserve requirements *or* money growth — should it use to generate this

[1]To list but a few of these papers, see Friedman (1971), Auernheimer (1974), Brock (1989), Easterly, Mauro, and Schmidt-Hebbel (1995) and Chari, Christiano, and Kehoe (1996).

[2]In a related paper, Freeman (1987) derives the optimal steady-state seigniorage policy, consisting of both tools. The reserve requirement and money growth rate are selected so that the inflation tax is effectively a lump-sum tax. Here, we focus on policies from among the set of second-best choices. Put differently, we seek an answer to the following question: if a government must raise a certain amount of revenue from seignorage, and if it is constrained to use *either* one of two "second-best" instruments, reserve requirements or money creation, which should it use? Freeman is silent on the issue of whether changes in the level of government expenditure *can* be financed using the only tool at a time, and if so, what restrictions get accordingly imposed on the policy space. Another difference between Freeman's analysis and ours is that we use a standard neoclassical concave production technology compared with his use of a linear production technology.

revenue?[3] To the best of our knowledge, no one has examined this financing decision from the perspective of a horserace between these alternative monetary policy tools.

Production adds some very interesting things to the model economy. As always, there is a Laffer-curve tension present. Because fiat money is rate-of-return dominated, an increase in the money growth rate or the reserve requirement, for instance, will reduce the gross real return on deposits. So, while the monetary policy action would increase the amount of real seigniorage revenue, agents have an incentive to reduce the quantity of deposits they hold. The overall impact is ambiguous. In contrast to endowment economies, or those with linear production technologies, neoclassical production introduces the possibility that the gross real return on capital is now sensitive to monetary policy actions. The implications of this are discussed below.

Because we are comparing two nonnested policy actions, our results are stated in the form of sufficient conditions. Even with concave utility functions and production technologies, it is difficult to derive unambiguous answers to our primary question. Further complications arise because this setup is consistent with the existence of multiple long-run equilibria (see Azariadis, 1993). Therefore, to highlight what is going on in the model economy, we compute the change in each monetary policy variable that is necessary to fund the permanent open market sale. In doing so, different factors operating on long-run seigniorage are captured.

After a complete description of the model economy, we start by deriving restrictions on the policy parameter space. It is important to identify whether conditions such as feasibility and rate-of-return dominance restrict the policy parameter space. Not surprisingly, the presence of neoclassical production technologies does put some nontrivial restrictions on the policy parameter space. Armed with this knowledge, we conduct the computational experiments, using common functional forms — log utility and Cobb-Douglas production — so that the interior equilibrium is unique. The results from these computational experiments are somewhat surprising; specifically, for most of the parameter settings we consider, reserve requirements *decline* in order to finance a permanent increase in government bonds. These results owe mostly to the large, positive relationship between reserve requirements and the gross real return on deposits. A higher reserve requirement shifts greater weight in the bank's portfolio to a lower-yielding asset — fiat money. However, an increase in the reserve requirement also crowds out enough capital so that the return on deposits increases despite the increased holdings of fiat money balances. In short, the reserve-requirement policy is on the "wrong side" of the Laffer curve. For the money-growth policy, the computational experiments indicate that faster money growth finances the permanent increase in government bonds. Put differently, a "unpleasant mone-

[3] Of course, reserve requirements have a natural upper bound of 100%. In effect, then, we are assuming that the revenue needed to be raised by the government is "small enough" that it is actually feasible to raise it solely via increases in the reserve requirement.

tarist arithmetic" type result is observed. However, when monetary policy includes a reserve requirement action, the results are "pleasant" primarily owing to the fact that the reserve requirement is a blunt instrument. Small changes in the reserve requirement have large impacts on the gross real return to capital. In contrast, if the technology is linear, the reserve requirement is less blunt because the scope of reserve-requirement effects is limited to the direct effect on the size of real fiat money balances and through the equilibrium saving allocation.

The results from the computational experiments shed light on which monetary policy action is preferred by agents. Because steady-state welfare is inversely related to both the reserve requirement and money growth rate, it is important to know the direction and magnitude of change in each policy variable that is necessary to keep the government's budget constraint satisfied. Based on the numerical analyses, the permanent increase in government bonds is financed by either *lower* reserve requirements or *faster* money growth. Hence, steady-state welfare is higher under the reserve-requirement policy. In addition, the reserve-requirement policy action is constructed so that the initial old's deposit endowment is constant; consequently, they prefer the reserve-requirement policy to the money growth policy as well.[4]

The results therefore indicate that the policy-tool horserace depends crucially on the concavity of the production technology. This conclusion stems from the impact that changes in reserve requirements have on the gross real return on deposits or more specifically, on how responsive is the equilibrium quantity of capital to changes in reserve requirements. Based on the numerical results, the equilibrium quantity of capital is relatively unresponsive to changes in money growth rates.

The rest of the paper is organized as follows. The economic environment is described in Section 2. In Section 3, a monetary competitive equilibrium is defined and characterized. Section 4 describes endogenous restrictions on the policy space. The principal experiment in this paper is a permanent open market sale of government bonds. The financing decision is explored in Section 5. The welfare comparison is derived in Section 6 while Section 7 concludes.

2 The model

The model is a modified version of Diamond's (1965) economy. There is an infinite sequence of periods indexed by $t = 1, 2, 3, \ldots$ Agents live for two periods. At each date t, two generations coexist — those in the first period of life (hereafter, the young) and those in the second period of life (hereafter the old). N people are born each period $t \geq 1$. At $t = 1$, there are N people in the economy who live only one period

[4] Specifically, fiat money is added so that the value of the initial old's real money is held constant. Without the additional fiat money balances, a decline in reserve requirements would reduce the value of the initial old's fiat money balances.

(hereafter the initial old).

Each agent is endowed with one unit of work time only when young. This is supplied inelastically to competitive labor markets, receiving w_t units of the single good. The agent allocates these goods between consumption and bank deposits. The proceeds of the bank deposits are then used to acquire second period consumption.

Preferences are identical for all agents born at date $t \geq 1$. Let c_i denote consumption in the i^{th} period of an agent's life. The utility function, $B(.)$, represented by

$$B(c_{1t}, c_{2t+1}) = U(c_{1t}) + V(c_{2t+1}) \tag{1}$$

is twice continuously differentiable; moreover, U', $V' > 0$, $U'' \leq 0$, $V'' < 0$, and $U'(0) = V'(0) = \infty$ hold. Then, the agent's problem is to maximize (1) subject to $c_{1t} \leq w_t - d_t$ and $c_{2t+1} \leq r_{t+1}d_t$ where d denotes the quantity of goods deposited with the bank, and r is the gross real return on the deposits.

Firms operate in a competitive environment, taking the rental rate on capital, q, wages, w, and the price of the consumption good, p, as given. All firms combine labor and capital using a common production technology to produce the final good. Let k denote the capital-labor ratio. Then, the production function in intensive form is defined by $f(k)$ where f is *CRS*, $f(0) = 0$, $f'' < 0 < f'$, and Inada conditions hold. Because the firm does not own capital, the firm's problem is equivalent to solving a sequence of one-period problems. As such, a firm's program is to maximize one-period profits subject to a resource constraint, written in per-young-person terms as $c_{1t} + c_{2t} + k_t \leq f(k_{t-1}) - g_t$, where g denotes the quantity of government purchases per young person.

In this economy, banks competitively provide a simple pooling function along the lines described in Bryant and Wallace (1980). We assume that capital and government bonds are "illiquid" in the sense that they are created/issued in large minimum denominations, denoted by κ where $\kappa > w_t \; \forall t$.[5] In such a setting, it is natural to think that banks arise to provide this simple intermediation function, namely, they accumulate deposits of small savers and acquire capital (on their behalf). Bank deposits are one-period contracts, guaranteeing r_{t+1} units for each unit of the consumption good deposited. Correspondingly, capital acquired at date t is rented to firms. Rental payments are received at date $t+1$. Capital completely depreciates after one period.

Let m denote the quantity of fiat money per young person, p denote the quantity of money per unit of the consumption good (the price level), and $z \equiv \frac{m}{p}$ denote real money balances. We assume that the bank faces a standard reserve requirement[6] on its deposits represented by

$$z_t \geq \varphi d_t. \tag{2}$$

[5]Henceforth, we focus on equilibria in which κ is "large enough" to ensure that banks continue to have a purpose for existence.

[6]Such a reserve requirement is commonly employed; see Wallace (1984), Freeman (1987), Bhattacharya, Guzman, and Smith (1998), and Espinosa and Russell (1998) for example.

Here, the bank accepts deposits and acquires assets. The returns from its various asset holdings are used to pay the interest obligations on the deposit contracts. The bank operates in a competitive environment, taking the return to capital and government bonds, the return on deposits, and the price level as given. The quantity of bank deposits is the outcome of bank's optimization program. Specifically, the bank chooses the level of deposits to maximize the present value of profits over an infinite horizon. The bank rents capital to a firm at date t, receiving payment at date $t+1$. The final output is available the following period. In the absence of uncertainty, the bank's problem can be represented as a sequence of identical one-period problems (with quantities represented per young person). Formally, the static one-period maximand is

$$qk_{t-1} + x_t b_{t-1} + \frac{m_{t-1} + h_t}{p_t} - r_t d_{t-1} \tag{3}$$

where x is the gross real return on government bonds, and h denotes a lump-sum transfer (per young person) distributed to the banks at the beginning of the period. The bank's problem is to maximize (3) subject to

$$k_{t-1} + b_{t-1} + \frac{m_{t-1}}{p_{t-1}} \leq d_{t-1}, \tag{4}$$

and

$$\varphi d_{t-1} \leq \frac{m_{t-1}}{p_{t-1}} \tag{5}$$

where (4) is the bank's balance-sheet constraint, and (5) is the reserve requirement.

In order to complete the description of the environment, we now describe the activities of the government. The government purchases g_t units of the consumption good and costlessly transforms these one-for-one into units of the "government good" which are useless to agents. The government finances these purchases through sale of indexed, pure discount government bonds and through the printing of fiat money. Government bonds are perfect substitutes for private capital. In our setting, it is useful to imagine that the government acquires the goods directly from the bank, paying $h_t \equiv m_t - m_{t-1}$ units of fiat money (measured per young person).[7] Formally, the government's date-t budget constraint is

$$g_t + x_t b_{t-1} \leq \frac{m_t - m_{t-1}}{p_t} + b_t \tag{6}$$

Lastly, we assume that the ratio of the government's two types of paper is set exogenously. More specifically, let the bond-money ratio be denoted by $\mu_t \equiv \frac{b_t}{z_t}$.

[7]This setup may appear odd, but it is equivalent to one in which the agent is required to hold a fraction of her portfolio as fiat money, receives a payment of newly printed money for some of her consumption good.

Because the reserve requirement binds, fixing μ automatically pins down the bond-deposit ratio, that is, $\mu_t \varphi_t d_t = \eta_t d_t$, where $\mu_t \varphi_t \equiv \eta_t$ is the bond-deposit ratio.

3 A monetary equilibrium

We begin by defining a competitive equilibrium for our economy and then proceed to characterize it. A perfect-foresight competitive equilibrium is defined as a sequence of allocations $\{c_{1t}\}_{t=1}^{\infty}$ and $\{c_{2t}\}_{t=1}^{\infty}$; prices: $\{r_t\}_{t=1}^{\infty}, \{x_t\}_{t=1}^{\infty}, \{q_t\}_{t=1}^{\infty}, \{p_t\}_{t=1}^{\infty}, \{w_t\}_{t=1}^{\infty}$; and policy variables: $\{m_t\}_{t=1}^{\infty}, \{\varphi_t\}_{t=1}^{\infty}, \{\eta_t\}_{t=1}^{\infty}, \{g_t\}_{t=1}^{\infty}$ such that

(i) taking prices and policy variables as given, the agent's allocations $\{c_{1t}\}_{t=1}^{\infty}$ and $\{c_{2t}\}_{t=1}^{\infty}$ solve the agent's maximization problem;

(ii) taking prices and policy variables as given, the bank's choice of $\{d_t\}_{t=1}^{\infty}$maximizes its profit function;

(iii) taking prices and policy variables as given, the firm's choice of $\{k_t\}_{t=1}^{\infty}$ maximizes its profit function;

(iv) the goods market clears;

(v) the government's budget constraint is satisfied.

The first-order condition for the agent's program is

$$U'(w_t - d_t) = r_{t+1} V'(r_{t+1} d_t). \tag{7}$$

This implicitly defines a deposit function, $d(w, r)$ for a given w and r. The first-order condition for the bank's problem is

$$q_t(1 - \eta_t - \varphi_t) + \eta_t x_t + \varphi_t \frac{p_{t-1}}{p_t} = r_t, \tag{8}$$

and for the firm's problem are

$$f'(k_{t-1}) = q_t, \tag{9}$$

and

$$f(k_{t-1}) - k_{t-1} f'(k_{t-1}) = w_t. \tag{10}$$

Since government bonds and private capital are perfect substitutes, $f'(.) = x$ holds. Consequently, (8) reduces to $(1-\varphi_t)q_t + \varphi_t \frac{p_{t-1}}{p_t} = r_t$. The bank's first-order condition, therefore, says that the return to deposits is just a weighted average of returns to the bank's three assets — private capital, government bonds, and fiat money. We focus on equilibria where $\frac{p_{t-1}}{p_t} < q_t$ holds for all t. With capital dominating fiat money in rate-of-return, the reserve requirement binds implying that (2) holds as an equality. Consequently, as is well-known, the reserve requirement drives a wedge between the return to capital and the return to deposits.

Under a binding reserve requirement and for a given bond-money ratio, it is possible to use the goods market clearing condition to write an expression for the capital stock as a function of deposits, namely,

$$k_t = [1 - \varphi_t(1 + \mu_t)]d_t. \tag{11}$$

It is clear that $k_t > 0$ is satisfied if

$$0 < \varphi_t < \frac{1}{1 + \mu_t}$$

or alternatively if the bond-money ratio satisfies

$$\mu_t < \frac{1}{\varphi_t} - 1.$$

These two inequalities identify the interaction between the reserve ratio and the bond-money ratio. If either is "too large" in the sense of violating either inequality, the government's policy settings would be such that physical capital is effectively crowded out of the bank's portfolio. The upshot is that, together these two equations restrict the policy space.

It is also possible to write the government budget constraint as a function of the capital stock (and correspondingly of deposits). Use the reserve requirement and rearrange to obtain

$$g_t = \left[\frac{\varphi_t(1 + \mu_t)}{1 - \varphi_t(1 + \mu_t)}\right] k_{t+1} - \left[\frac{\varphi_t(\frac{p_t}{p_{t+1}} + q_t)}{1 - \varphi_t(1 + \mu_t)}\right] k_t \tag{12}$$

Equation (12) states that the government can acquire g of the date-t goods using a combination of the reserve requirement, rate of price inflation, and the bond-money ratio. A steady state version of (12) will serve as the basis for assessing whether a permanent open market sale can be afforded by either changing reserve requirements or the money growth rate.

Finally, it is important to write down the law of motion for the capital-labor ratio:

$$k_t = [1 - \varphi_t(1 + \mu_t)]d\{w(k_{t-1}), r(k_{t-1})\}$$

Time-invariant solutions to (11) constitute stationary equilibria of this economy. As is well understood, our economy admits (possibly) multiple stationary (and nonstationary) solutions.[8] Henceforth, we focus only on stationary equilibria.

[8] A discussion is available in Galor and Ryder (1989) and Azariadis (1993).

4 Some Parameter Restrictions

As mentioned above, our analysis focuses on stationary equilibria. Thus, the sequence of prices and policy variables, except for the money stock and the price level, will be held constant over time.

With a constant level of deposits, the reserve requirement expression implies that the real value of money balances will be constant. Rewrite the expression to obtain $m_t = p_t z_t$. Update this expression one period, yielding the following quantity theory equation: $\frac{p_{t+1}}{p_t} = \frac{m_{t+1}}{m_t}$. For purposes of this paper, let the money growth rate be expressed as a constant; that is, $m_t = \theta m_{t-1}, \theta > 1$. Then, $\frac{p_{t+1}}{p_t} = \theta$.

The government budget constraint places several restrictions on the policy parameters. To see this, note that a stationary version of the government's budget constraint (see (12)) is

$$\frac{g}{k} = \frac{\varphi}{1-\phi}\left[\frac{\theta-1}{\theta} - (q-1)\,\mu\right] \tag{13}$$

where $\phi \equiv \varphi(1+\mu) < 1$. Take the steady-state level of capital as given along with the size of government spending and the bond-money ratio. Then (13) dictates the size of the money growth rate and the reserve requirement that is necessary to finance this.

Equation (13) is useful for identifying restrictions on the steady-state size of government. Measured as the ratio of government spending to capital, rate-of-return dominance puts a lower bound on the size of government, while the government budget constraint imposes an upper bound on the size of government. The boundaries on government spending, the money growth rate and the reserve requirement are presented in the following proposition.

Proposition 1 *Let $\theta > 1$. In steady states, taking money growth rates, reserve requirements, and the bond money-ratio as given, the size of the government relative to the capital stock must satisfy the following condition:*

$$\frac{\phi - 2\varphi q}{1-\phi} < \frac{g}{k} < \frac{\phi - \varphi q}{1-\phi}.$$

Alternatively, taking reserve requirements, government spending, and the bond-money ratio as given, the money growth rate must satisfy:

$$1 - q(2-\mu) < \frac{\theta-1}{\theta} < 1 - q(1-\mu).$$

Finally, taking the money growth rate, government spending, and bond-money ratio as given, the restriction on the reserve requirement is

$$\frac{\varphi}{1+2q}\left[\frac{\theta-1}{\theta} - (q-1)\mu\right]^{-1} < \varphi < \frac{\varphi}{1+q}\left[\frac{\theta-1}{\theta} - (q-1)\mu\right]^{-1}.$$

Proof. The upper bound on the size of government spending comes directly from (13). Note that the bracketed term is positive if q is not "too large", or more precisely, if $q < 1 + \frac{\theta - 1}{\theta\mu}$ holds. With a constant-returns-to-scale production technology, $\frac{g}{k}$ is positive if the capital stock is large enough to satisfy the restriction on q. The other inequality is formed so that capital (and bonds) dominate fiat money in terms of rate of return; in short, when $q > \theta^{-1}$. Use the government budget constraint to solve for the money growth rate as a function of $\frac{g}{k}$. Then, rate-of-return dominance implies that

$$q > \frac{\phi}{\varphi} - q - \frac{1-\Delta}{\varphi}\frac{g}{k}$$

After rearranging, this condition can be rewritten as

$$\frac{g}{k} > \frac{\phi - 2\varphi q}{1-\phi}.$$

Thus, the size of government relative to the capital stock must be large enough so that the return to capital dominates the return to fiat money. The other inequalities are derived from the restriction on the $\frac{g}{k}$ ratio, solving for either the money growth rate or reserve ratio.[9] ■

Proposition 1 identifies the restrictions that apply to the parameter space when the government has a variety of tools at its disposal and the intensive form of the production function is concave. Next, we turn to the basic policy experiments.

5 Policy Experiments and Financing Results

We now turn our attention to the issue of whether a certain amount of revenue *can* be raised using one of the two tools, and if so, what restrictions get endogenously imposed on the policy space as a result. To examine this issue, it is necessary to find the change in reserve requirements or money growth rates that are required to satisfy the government's budget constraint. To foreshadow, our computational experiments indicate that reserve requirements must *decline* and money growth rates must *increase* in order to finance a permanent increase in government bonds.

5.1 Changing the reserve requirement

Consider a situation in which the government permanently raises the quantity of government bonds outstanding. With $q \leq 1$, it is possible that the government could potentially finance the bond issue by earning the revenues from the initial bond sale.[10] Of course, given our current stated purpose, we are not interested in such scenarios.

[9] In both cases, we use (13) to substitute out the ratio of government spending to capital.

[10] Espinosa and Russell (1998) examine a model in which bond seigniorage is a source of revenue.

Following Sargent and Wallace (1981), we henceforth focus on cases in which $q > 1$ holds. In such a setting, interest obligations on outstanding bonds will, in steady state, exceed the revenue from increased new bond sales, requiring the government to depend on currency seigniorage to make up the revenue shortfall.

The change in the quantity of government bonds immediately affects the bond-to-deposit ratio. Recall that $\mu\varphi \equiv \eta$. We restrict our attention to a monetary policy action in which the bond-money ratio is held constant, that is, $\mu = \frac{b}{z} = \frac{\hat{b}}{\hat{z}}$, where hats above variables are used to denote the levels after the policy change has been implemented.[11] Thus, the increase in government bonds is exactly matched by the increase in real money balances. To further identify the experiment, we follow Auernheimer (1974) and Freeman and Haslag (1996), and fix the price level by adding the appropriate amount of money to the economy at date $t = 1$. The intuition for this coordinated policy action is straightforward. A change in the reserve ratio will, in general, affect the value of the initial old's fiat money balances. Hence, to avoid the implicit transfer to the initial old, we fix the price level, thus maintaining the value of their money balances.[12]

The following proposition is a sufficient condition, depicting when a permanent increase in the quantity of government bonds outstanding can be financed by an increase in the reserve requirement ratio. Given the Laffer curve tension, the implication is that reserve requirements would have to decrease in order to satisfy the government budget constraint.

Proposition 2 *A permanent increase in the quantity of government bonds, an open market sale, can be financed by raising reserve requirements*[13] *if the steady state level*

[11] It is straightforward to show that the increase in reserve requirements is consistent with a policy in which the price level is held constant, i.e., $\hat{z} - z = \frac{1}{p}(\hat{m} - m)$.

[12] Unlike Freeman and Haslag (1996), however, we are assuming that goods bought with the increase in fiat money are government liabilities. In their paper, the increase in fiat money was used to purchase interest-bearing assets, which in turn affected the steady-state government budget constraint.

[13] Of course, there is a natural upper and lower bound for φ, which is unity and zero, respectively. This in turn implies an upper bound on the size of the new bond issue that can be financed using reserve requirements. In fact, using (14), it is easy to check that such an upper bound is described by

$$\hat{b} \leq \frac{\hat{d} - \varphi d(1 - \theta^{-1}) + b(q - 1)}{\hat{q} - 1} \equiv b_{\max}.$$

We are of course assuming that this and the lower-bound condition are satisfied. Moreover, we know from Freeman (1987) that the optimal reserve requirement is between zero and one. More specifically, if the reserve requirement is above the $g/f(k)$ ratio, one would lower the reserve ratio until $\varphi = g/(f(k)$ and let money grow at an infinite rate. Our results takes the money growth as finite and given.

of capital satisfies the following:

$$1 \leq [f'(k)]^{-1}\left[1+\frac{b(\hat{q}-q)}{b-\hat{b}}+\frac{1-\theta^{-1}}{\mu}\right]. \tag{14}$$

Proof. From (13), an increase in reserve requirements will pay for a permanent increase in government bonds if

$$(\hat{\psi}\hat{d}-\psi d)\left(1-\theta^{-1}\right) \geq \hat{b}(\hat{q}-1)-b(q-1).$$

In words, the gain in real seigniorage revenue raised through higher reserve requirements is at least as great as the increase in net interest obligations associated with the increase in government bonds. After some algebra, this expression reduces to

$$\frac{b\left(\hat{q}-q\right)}{b-\hat{b}}+\frac{1-\theta^{-1}}{\mu}+1 \geq q.$$

After substituting for q on the right-hand-side of the expression, one obtains the sufficient condition in Proposition 2. ■

Proposition 2 identifies the capital stock that is necessary for an increase in the reserve requirement ratio to finance the permanent open market sale. To see this, rewrite (14) in the following way: let $k^{\spadesuit}$ denote the size of the capital stock such that the expression in Proposition 2 holds with equality, i.e.,

$$1=\left[f'(k^{\spadesuit})\right]^{-1}\left[1+\frac{b(\hat{q}-q)}{b-\hat{b}}+\frac{1-\theta^{-1}}{\mu}\right] \tag{15}$$

holds. Note however that, in general, $k^{\spadesuit}$ may not be unique.

5.2 Remarks

Our equilibrium concept requires that the government budget constraint be satisfied. The condition in Proposition 2 therefore asks whether there is an increase in the reserve ratio that is sufficient to generate enough revenue to fund the increased net interest obligations associated with the increase in bonds. Note however that an increase in the reserve ratio does *not* necessarily result in an increase in real seigniorage revenue; in other words, it is not clear whether an increase or a decrease in the reserve requirement is needed to finance a permanent open market sale. In a direct sense, therefore, the condition presented in Proposition 2 makes a statement about which side of the Laffer curve the economy is on. It is easy to see this Laffer-curve tension by recognizing the differing signs of the terms inside the bracket of equation (14). Notably, since it is an open market sale, $\hat{b} > b$ holds, implying that the second term inside the bracket is negative. With $\theta > 1$, the third term is positive. So, the economy is on the right side of the Laffer curve, provided that the sufficient condition in Proposition 2 is satisfied.

The intuition is straightforward. For a given level of savings, an increase in government bonds and reserve requirements will reduce the quantity of physical capital, resulting in a higher gross real return to capital. It is not clear that the increase in real money balances brought about by a change in the reserve requirement — the seigniorage tax base — would be sufficient to pay for this increased interest expense. Further complicating matters is the fact that the effect of an increase in the reserve requirement on total savings is ambiguous.

The proposition does however offer some clear guidelines for assessing the role that other policy variables play in satisfying the feasibility condition. For instance, consider the role of the money growth rate. One can see that money growth directly enters the third term inside the brackets of equation (15). Here, $k^{\spadesuit}$ is inversely related to movements in the money growth rate. So, if the financing experiment is rerun with faster money growth, an increase in the reserve requirement will finance the permanent open market operation with a smaller capital stock. The idea is that for a given tax base, faster money growth corresponds to a higher seigniorage tax rate. Thus, an increase in reserve requirement will generate enough real seigniorage revenue with a smaller physical capital stock.

Next, consider, the effect of the government's net interest expense relative to the size of the open market operation. This is the second term inside the bracket; henceforth, we refer to it as the interest-expense ratio. Because $b < \hat{b}$, the interest-expense ratio is negative. As the interest-expense ratio increases, in absolute value terms, $k^{\spadesuit}$ increases. In words, an increase in the reserve requirement will finance the permanent open market sale with a larger capital stock in the face of a larger government interest expense ratio. In effect, it takes a larger seigniorage tax base for the increase in reserve requirements to fund a larger increase in government interest payments.

5.2.1 Numerical Analysis

Generically, this model economy will have more than one steady state level of the capital stock.[14] Consequently, the effect on the equilibrium return on deposits is ambiguous. In order to make the intuition of the previous exercise precise, we now describe an example with additively-separable log preferences. With log-utility, it is apparent that (a) the steady state capital stock is unique, and (b) the equilibrium level of savings is independent of the return on deposits. For us, this means that the capital stock declines in response to this open market sale-reserve requirement action. With

[14] Our model economy is identical to the one studied in Diamond (1965) if $\varphi = 0$ obtains. As is well known, such economies generically admit multiple steady-state equilibria. See Galor and Ryder (1989) for details. This multiplicity of equilibria is preserved for all "interior" values of φ.

a smaller capital stock, the rate of return on capital, and correspondingly, government bonds, must rise. As such, an increase in the reserve ratio does not guarantee that the open market sale is paid for by higher reserve ratios. The Laffer curve tension comes from pitting the higher net interest expense, arising from a larger number of outstanding bonds and a higher interest rate, *versus,* a larger seigniorage tax base.

What change in reserve requirements will satisfy the steady state government budget constraint? A simple numerical example should help to shed light on the conditions presented in Proposition 2. For the purposes of this exercise, preferences are captured by log utility and the production technology is Cobb-Douglas. The appendix contains the details and derivations of the expressions used in the numerical exercise. Unless stated otherwise, the parameters of the model economy are as follows: $\theta = 1.214$, $\beta = 0.95$, $\eta = 0.05$.

Figure 1 plots the change in reserve requirement that is necessary to finance a permanent increase in the bond-money ratio, holding the level of government purchases constant. The change in reserve requirements is computed for different initial values, letting the reserve requirement range from 1% to 95% and for four alternative values of the capital share parameter, $\alpha = [0.1, 0.35, 0.6, 0.9]$. Based on Figure 1, $d\varphi$ is generally negative, implying that an increase in the bond-money ratio is financed by a decrease in the reserve requirement. In other words, the model economy is on the wrong side of the Laffer curve. Upon closer inspection, note that gross real return on deposits is positively related to changes in the reserve requirement. With a concave production technology, the decline in the steady-state capital stock declines as the reserve requirement rises, raising the marginal product of capital. For this numerical example, the sensitivity of the marginal product of capital more than offsets the greater weight placed on the lower-yielding asset — fiat money. The upshot is that a permanent increase in the bond-money ratio can only be financed by decreases in the reserve requirement.[15]

In addition, we consider some experiments in which the time rate of preference and the bond-deposit ratio parameters vary. Our aim is verify that the results presented in Figure 1 are robust to parameter variation. For these computational experiments, we fix $\alpha = 0.35$. Figure 2 recomputes the change in the reserve ratio under different values of the time rate of preference, $\beta = 0.1, 0.5, 0.75, 0.99$. Figure 3 plots the change in reserve requirements for alternative values of the bond-deposit ratio; $\eta = 0.01, 0.1$.[16] Together Figures 2 and 3 indicate that reserve requirement must decline to pay for a permanent open market sale, corroborating the results presented in Figure 1. Overall, the results from this computational experiment indicate that is difficult to find a part

[15] In each of the bottom two panels of Figure 1, there is a sharp spike in the $d\varphi$ as the sign switches from negative to positive. The steady-state capital stock is becoming less sensitive to changes in the reserve requirement, causing $d\varphi$ to switch signs.

[16] Larger values of the bond-deposit ratio yield complex solutions for the change in reserve requirements.

of the parameter space in which this model economy is on the upward-sloping portion of its reserve-requirement Laffer curve.

It is quite straightforward to show the role that neoclassical production plays in these numerical findings. To that end, let the model economy's environment be altered to be consistent with the introduction of a linear technology: $f(k) = xk$.[17] Preferences are not changed. Thus, the chief difference between this model economy and the one with neoclassical production properties is the curvature in the production frontier. Figure 4 plots the change in the reserve requirement necessary to finance the permanent open market sale in this model economy; as is clear from the figure, the change in the reserve requirement required is positive. In Figure 4, preferences are such that changes in the gross real return on capital do not affect the equilibrium quantity of deposits. Here, a permanent increase in government bonds implies that net interest expenses must be paid for by raising seigniorage revenue. Capital is crowded out by the open market sale, but there is no change in the marginal product of capital. With fixed savings, the inflation-tax base rises unambiguously with an increase in the reserve requirement ratio. In contrast, with concave production, the crowding out of capital raises the marginal product of capital. As Figures 1-3 show, the curvature present in the Cobb-Douglas specification is enough to require a *reduction* in the reserve requirement to pay for the open market sale. With a lower reserve requirement, the inflation-tax base is made smaller, but at the benefit of a smaller net interest expense on the government debt.

5.3 Changing the money growth rate

Next, consider a case in which the government uses faster money growth to finance a permanent open market sale. Here, the reserve ratio is held constant. The impact of the increase in government bonds together with the change in the inflation rate will induce an endogenous change in the steady-state bond-money ratio. We now state a result analogous to Proposition 2.

Proposition 3 *A permanent increase in government bonds can be financed by increasing the money growth rate if the steady state level of capital satisfies the following*

$$k \geq \frac{(1-\phi)\left[\hat{z}(\hat{q}-1)\hat{\mu}-(1-\theta^{-1})\right]}{\varphi\left[(q-1)\mu-(1-\theta^{-1})\right]} \equiv k^{\dagger} \tag{16}$$

Proof. From the government budget constraint, feasibility requires that

$$\varphi\left[\hat{d}(1-\hat{\theta}^{-1})-d(1-\theta^{-1})\right] \geq \hat{b}(\hat{q}-1)-b(q-1)$$

[17] Of course, further modifications are also required. Most importantly, suppose that agents receive an endowment of y goods when young, and nothing when old.

After this statement, it is simply a series of substitutions that yield the sufficient condition in terms of the capital stock. ■

The condition is analogous to the reserve-ratio policy. In words, the condition states that the capital stock must be large enough to ensure that an increase in the money growth rate will yield enough funds to the pay for the additional net interest obligations.

5.3.1 Remarks

As before, the interaction of the other policy parameters is noteworthy. Specifically, consider a case in which the finance experiment is redone with a higher reserve ratio. Then, it is easy to see that $k^{\dagger}$ declines as the reserve requirement rises. Thus, an increase in the money growth rate will fund the permanent open market sale with a smaller capital stock. With a larger reserve requirement, an increase in the money growth rate will finance a permanent open market sale with a smaller capital stock. The intuition is straightforward. For a given capital stock, the size of the seigniorage tax base is larger with a greater reserve requirement. Hence, an increase in the money growth rate generates more real seigniorage revenue; in short, the money growth rate policy has more bang for the buck when the reserve requirement is higher.

The chief aim of this section is to identify conditions under which a permanent open market sale of government bonds could be financed by either an increase in the reserve requirement ratio or an increase in the money growth rate. As such, the starting point (necessary condition) for these feasibility conditions is that the economy is on the "good side" of the Laffer curve.

5.3.2 Numerical Analysis

We conduct numerical analyses similar to those presented for the reserve-requirement policy. Figure 5 plots the change in the money growth rate that satisfies the steady state government budget constraint. The functional forms and parameter settings are identical to those used in the reserve-requirement analyses. In contrast to the reserve-requirement policy, the money growth rate *increases* in order to finance the permanent increase in government bonds. In short, the money growth policy is on the good side of the Laffer curve. The primary difference between the two numerical analyses is that the reserve requirement has a large, positive effect on the gross real return on deposits whereas the latter is negatively related to changes in the money growth rate for the parameter settings considered here.

6 Welfare analysis

Presumably, what is important is that the government choose that policy action — changing reserve ratios or changing money growth rates — that yields the highest steady-state utility. Accordingly, we assess the welfare implications of the two alternative monetary policy actions. We are comparing steady-state welfare, so that the government budget constraint is satisfied.

In steady state, welfare of the future generations may be written as a function of both the reserve requirement and money growth rate. Formally,

$$B(\varphi,\theta) = U\left[w(\varphi,\theta) - d(\varphi,\theta)\right] + V\left[r(\varphi,\theta)d(\varphi,\theta)\right] \tag{17}$$

The following proposition spells out a sufficient condition for welfare comparison.

Proposition 4 *A permanent open market sale accompanied by an increase in the reserve requirement is the more preferred policy action, deemed by steady state utility, if*

$$w_\varphi U'(.) + r_\varphi d(.)V'(.) > w_\theta U'(.) + r_\theta d(.)V'(.) \tag{18}$$

Proof. The proof is straightforward. Differentiate (17) with respect to the reserve requirement and also with respect to the money growth rate. Note that $-U'(.) + rV'(.) = 0$ holds by (7). ■

6.1 Remarks

It is important to establish the relationship between steady state utility and the monetary policy variables. To that end, note that

$$w_\varphi = w'(k)k_\varphi,$$

$$r_\varphi = \left[(1-\varphi)f''(k)k_\varphi - f'(k) + \frac{1}{\theta}\right],$$

and

$$r_\theta = -\frac{\varphi}{\theta^2} < 0$$

where the subscript refers to the monetary-policy variable, φ or θ, the derivative is taken with respect to. Using these, it is possible to rewrite (18) as

$$w'(k)k_\varphi U'(.) + \left[(1-\varphi)f''(k)k_\varphi - f'(k) + \frac{1}{\theta}\right]d(.)V'(.) > w'(k)k_\theta U'(.) - \frac{\varphi}{\theta^2}d(.)V'(.) \tag{19}$$

Consider the results from the numerical analyses in which lifetime utility is logarithmic and the production technology is Cobb-Douglas. In this specific log-utility example, $k_\theta = 0$; the steady-state wage rate is invariant to changes in the money growth rate. Then the r.h.s of (19) is just $-\frac{\varphi}{\theta^2}d(.)V'(.) < 0$. [18]

The intermediate step is to identify the change in reserve requirements or money growth rate that would support the expenses associated with an increase in the bond-deposit ratio. In our simple numerical exercise, reserve requirements would generally decline and the money growth rate increase, in order to finance the permanent increase in government bonds. A fall in φ leads to a rise in k or -$k_\varphi > 0$ and hence, $w_\varphi = w'(k)(-k_\varphi) > 0$. Recall also that $f'(k) > \frac{1}{\theta}$ holds. To disentangle the effect of a fall in φ on the gross real return on deposits, r, it is useful to split r up into its components, the return to capital, $f'(k)$ with its portfolio weight, $(1 - \varphi)$, and the return to currency, $\frac{1}{\theta}$, with its portfolio weight, φ. A fall in φ obviously reduces the portfolio weight on the dominated asset (money). At the same time, a fall in φ raises the portfolio weight on capital, and via its effect on the equilibrium capital stock, reduces the marginal product of capital (the real return on capital). It is then apparent that the sign of r_φ is in general ambiguous. If however, $(1 - \varphi)f''(k)k_\varphi > \frac{1}{\theta} - f'(k)$ obtains, then a fall in φ is accompanied by a rise in r. Then, it is clear that following a decline in φ, the l.h.s of (19) is clearly positive. In words, a decrease in the reserve requirement ratio results in an increase in steady-state welfare. In contrast, for the specific functional forms considered here, an increase in the money growth rate results in lower steady-state welfare.

Next, plug in the results from the intermediate step; that is, the numerical exercise for the government financing step. For this special case, the inequality in equation (18) holds. Put differently, if comparisons are made on the basis of steady-state utility, and if equilibrium savings is totally unresponsive to the real interest rate, a permanent open market sale accompanied by an decrease in the reserve requirement is preferred to a policy action requiring an increase in the money growth rate by all generations $t \geq 1$.

In addition, the reserve-requirement policy is preferred by members of the initial old generation. The combination of the reserve-requirement action and the money supply change ensures that the price-level path is not affected. Hence, the initial old's deposits are not devalued by the reserve-requirement policy. With the money-growth policy, prices increase at a faster rate, taxing the deposits, at least the part held in the form of fiat money balances. Thus, the purchasing power of the initial old's deposits is reduced. As such, the initial old enjoy greater consumption under the reserve-requirement policy than under the money-growth policy. Consequently, they too prefer the former over the latter.

[18] That steady-state welfare is inversely related to the money growth rate with log utility and Cobb-Douglas production is well-known.

7 Concluding remarks

In this paper, we attempt an answer to the following question: if a benevolent government is forced to raise revenue from seigniorage, which instrument — changing the reserve requirement or raising the money growth — should it choose? Overall, our analysis may thus be interpreted as providing monetary authorities with some guidance as to its options when the tool-kit includes only two "second-best" tools.

With general sets of preferences and general neoclassical production technologies it is impossible to find a policy that strictly dominates the other. With sufficient structure imposed on preferences, technology, and policy parameters — more specifically, with log utility and Cobb-Douglas production — there is a unique steady state. We then conduct computational experiments to assess the steady state impact on welfare of the two alternative policies. The results of the computational experiments are somewhat surprising. Given a permanent open market sale, the steady-state government budget constraint is satisfied by *decreasing* reserve requirements or increasing the money growth rate. In other words, for a large part of the parameter space, the model economy is on the "bad side" of the steady-state Laffer curve in the reserve-requirement -real-seigniorage-revenue space. The numerical results also indicate that the reserve-requirement is a rather blunt instrument in the sense it has large impacts on the steady-state level of capital, even with log utility. Interestingly, the equilibrium gross return on deposits is positively associated with the reserve requirement ratio. Consequently, to fund the open market sale, the reserve requirement must fall. The money-growth experiments show that faster money growth has a muted impact on the gross real return on deposits. Hence, real seigniorage revenue is positively related to money growth for the parameter settings we consider. Plugging the results from the government finance experiments, it is straightforward to show that steady-state utility is greater under the reserve-requirement policy than under the money-growth policy. Moreover, the reserve-requirement policy is setup in such a way that the initial old also prefer it to the money-growth policy.

As such, our results add to the long list of unpleasant-monetarist-arithmetic results. Our main contribution is to consider the reserve-requirement policy in addition to the money-growth policy. Sargent and Wallace (1981) focus on the financing result, showing that faster money growth would be needed to finance a permanent open-market sale. Our computational experiments are in line with the unpleasant monetarist arithmetic. However, in this general equilibrium setup, there may be a pleasant monetary policy result. Judging by the welfare impact, the permanent open-market sale can be financed by lower reserve requirements. In this model economy, welfare increases with lower reserve requirements and a finite, constant money growth rate.

It is in order that several caveats be recorded. First, our analysis has been restricted to comparisons based on steady-state utility. It is possible to imagine other criteria of comparison which would yield vastly different results. Indeed, an obvious extension to our research would be to assess welfare in a stochastic version of this model economy in which transition dynamics could matter. Second, the environment we study is one where the fiscal authorities dominate the monetary authorities and dictate to them, the entire current and future path of fiscal activity. Many modern central banks, by virtue of constitutional protection, do not find themselves in this subservient position. Thus, the guidance offered here may not be all that useful to those monetary policymakers.

Appendix

Here, we derive the expressions to compute changes in the reserve requirement ratio that will satisfy the government budget constraint. Utility is represented as $B(c_{1t}, c_{2t+1}) = \ln c_{1t} + \beta \ln c_{2t+1}$ where β denotes the time-rate of preference. The production technology is Cobb-Douglas, written in its intensive form as $f(k) = k^{\alpha}$, where α is the capital-share parameter. We focus on steady states. It is easy to verify that the steady-state solution is

$$c_1 = w\left[1 - \frac{\beta}{1+\beta}\right].$$

Since $d = w - c_1$, it is straightforward to show that $d = w\left[\frac{\beta}{1+\beta}\right]$. Hence, substituting $(1-\alpha)k^{\alpha}$ for w, into (11), yields the following time-invariant representation

$$\frac{k}{1-\varphi(1+\mu)} = \frac{\beta}{1+\beta}(1-\alpha)k^{\alpha}. \tag{A.1}$$

Then, the steady state solution for the capital-labor ratio is

$$k^* = \left[\frac{\beta}{1+\beta}(1-\alpha)(1-\varphi-\eta)\right]^{\frac{1}{1-\alpha}}. \tag{A.2}$$

The steady-state government budget constraint (using (12)) may be written as

$$g = (k^*)^{\alpha}\left\{1 - \frac{1-\alpha}{1+\beta}[1+\beta r(k^*)]\right\} \tag{A.3}$$

where

$$r(k^*) = (1-\varphi)(1-\alpha)(k^*)^{\alpha-1} + \frac{\varphi}{\theta}.$$

Substitute (A.2) into (A.3). To compute the change in the monetary policy parameter, totally differentiate (A.3), setting $dg = 0$. For the reserve requirement policy, set $d\theta = 0$, yielding

$$d\varphi = -\frac{\omega_0\omega_1 h_{\eta} + \omega_2\omega_3 l_{\eta}}{\omega_0\omega_1 h_{\varphi} + \omega_2\omega_3 l_{\varphi}} d\eta$$

where

$$\omega_0 \equiv (1-\alpha)(k^*)^{\alpha-1},$$

$$\omega_1 \equiv \left\{1 - \frac{1-\alpha}{1+\beta}[1+\beta r(k^*)\right\},$$

$$\omega_2 \equiv (k^*)^\alpha,$$

$$\omega_3 \equiv -\left[\frac{(1-\alpha)\beta}{1+\beta},\right]$$

$$h_i \equiv \frac{dk^*}{di},\ for\ i = \eta, \varphi,$$

and

$$l_i \equiv \frac{dr(k^*)}{di}, for\ i = \eta, \varphi.$$

Similarly, the expression for the change in the money growth rate is

$$d\theta = -\frac{\omega_0\omega_1 h_\eta + \omega_2\omega_3 l_\eta}{\omega_2\omega_3 l_\theta} d\eta$$

where $l_\theta \equiv \frac{dr(k^*)}{d\theta}$.

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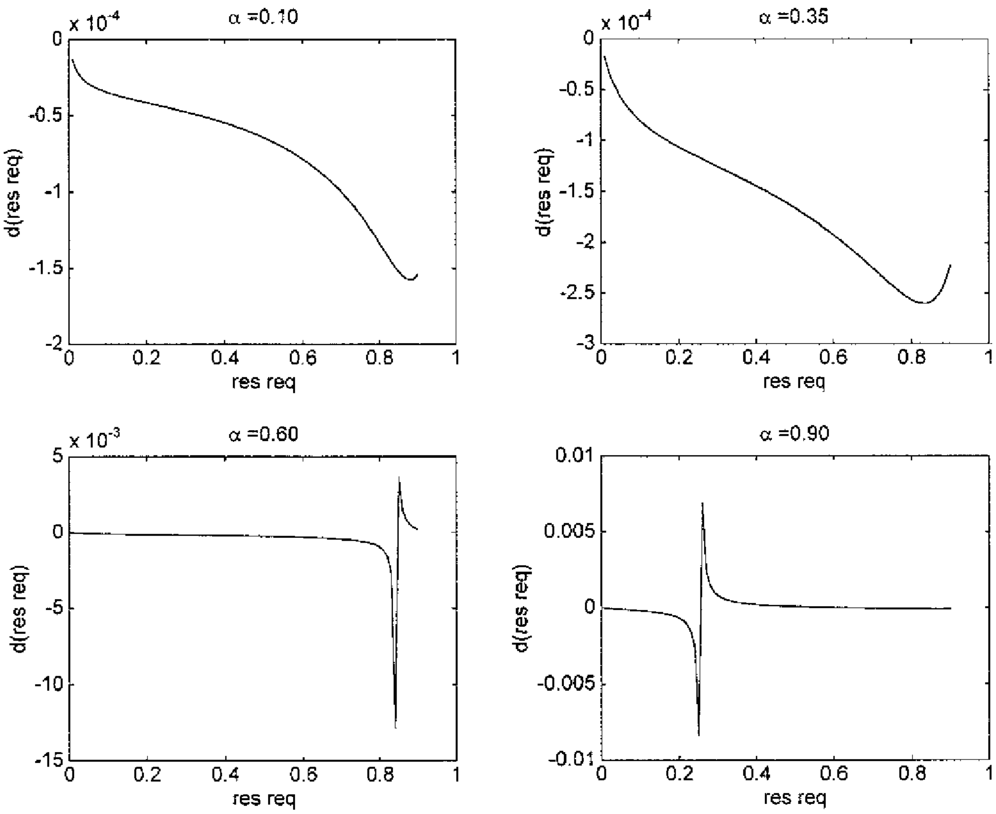
Figure 1



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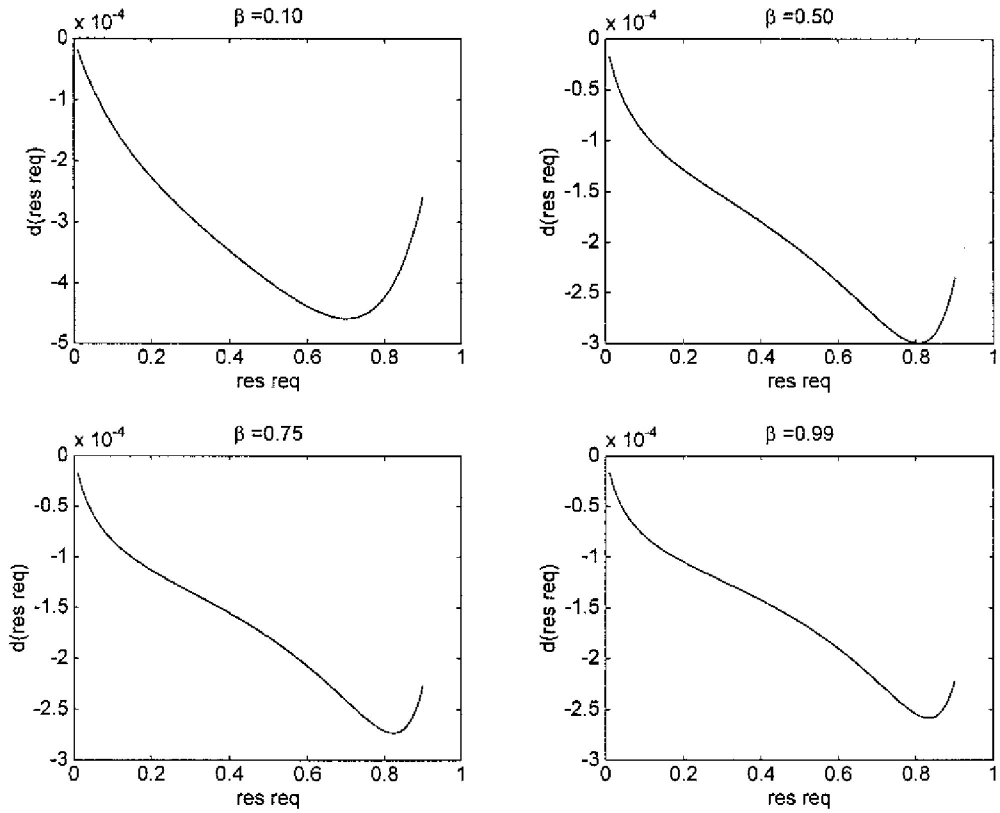
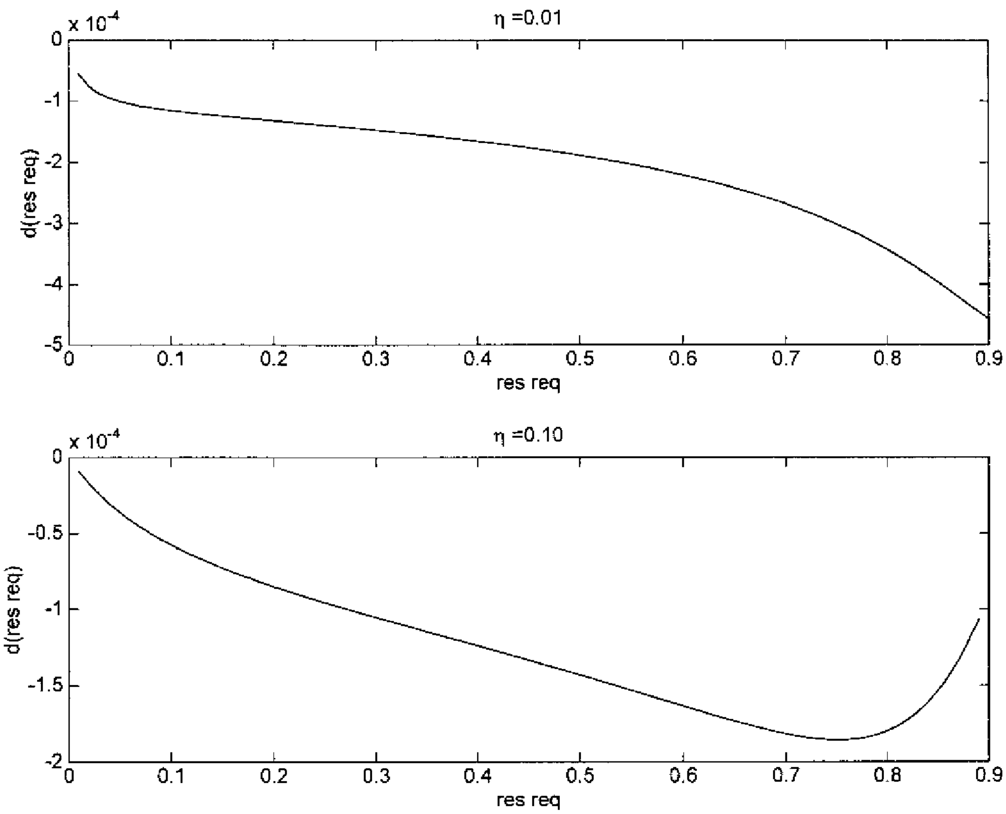
x 10^-4
d(res req)
res req
α =0.35
x 10^-4
d(res req)
res req
α =0.60
x 10^-3
d(res req)
res req
α =0.90
d(res req)
res req

Figure 2



β =0.10
x 10^-4
d(res req)
res req
β =0.50
x 10^-4
d(res req)
res req
β =0.75
x 10^-4
d(res req)
res req
β =0.99
x 10^-4
d(res req)
res req

Figure 3


η =0.01
x 10^-4
0
-1
-2
-3
-4
-5
d(res req)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
res req
η =0.10
x 10^-4
0
-0.5
-1
-1.5
-2
d(res req)
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0.7
0.8
0.9
res req

Figure 4

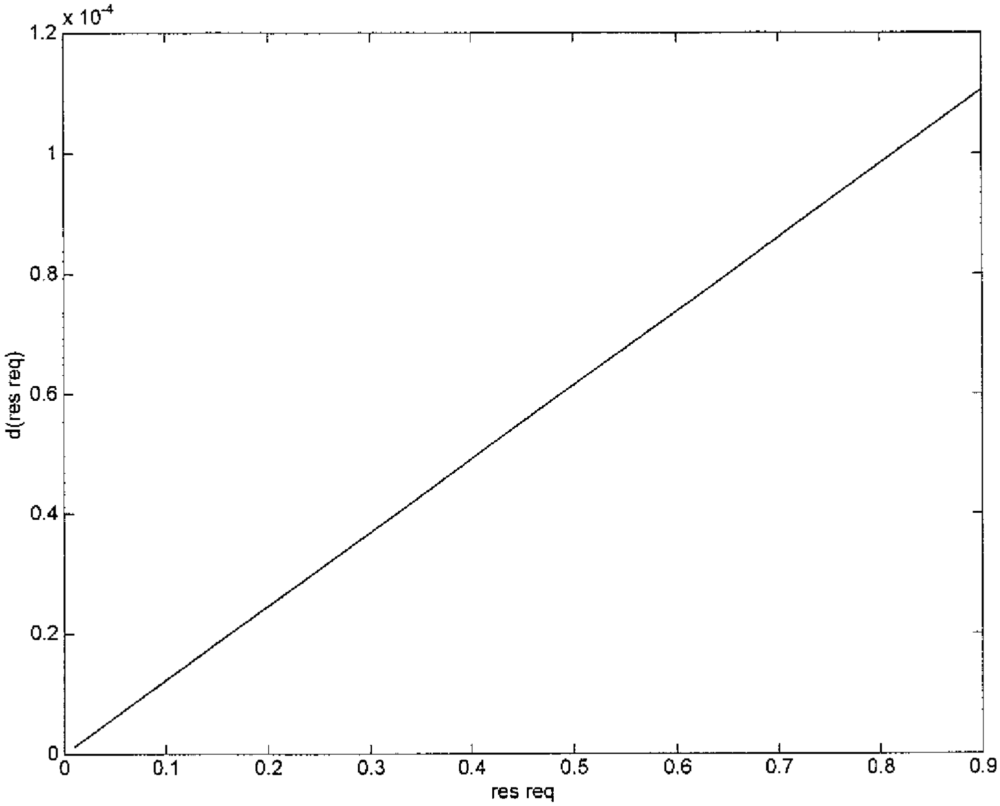

Figure 5