



**ENDOGENOUS TAX DETERMINATION
AND THE DISTRIBUTION OF WEALTH**

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by

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ABSTRACT

In this paper a dynamic model is constructed in which labor and capital taxes are determined endogenously through majority voting. The wealth distribution of the economy is shown to influence the voting behavior, and hence the equilibrium levels of the tax rates, which in turn affect the future distribution of wealth. It is shown that the economy exhibits a unique dynamic behavior. Because the tax rates are endogenously determined, asset prices, wealth distribution, and the tax rates can display persistent fluctuations or cycles, in reaction to exogenous disturbances, or even due to initial conditions. "Tax smoothing" does not necessarily appear to naturally arise in such an environment. The features in the model that can produce these fluctuations are studied in detail.

INTRODUCTION

In this paper a dynamic equilibrium model is constructed in which agents *vote* each period on the desired capital and labor taxes that are to be implemented in order to finance a given level of government consumption. Based on their different wealth levels and planning horizons, agents will have distinctive preferences concerning these tax rates. An interesting dynamic behavior arises in cross sectional wealth levels, as well as in the time-series paths of asset prices and tax rates. It is shown that temporary exogenous disturbances can have not only persistent effects, but also a permanent impact on a variety of endogenous variables. Exogenous disturbances, as well as initial conditions are shown to produce cycles in the wealth distribution, asset prices, as well as in the endogenously determined tax rates. This is important because there is a paucity of models, with endogenous policy formulation, in which policies implemented in one period influence the economy in such a manner as to alter the *future* course of policy formulation as well.

There has been a tremendous literature written on both the normative and positive aspects of taxation. The normative aspects usually relate to how tax policy can be formulated so as to maximize some sort of welfare criterion, or minimizing the tax burden of agents, or possibly maximizing growth. For example, one might ask the question as to what are the appropriate distortional tax rates that should be implemented in order to retire a given amount of government debt. Or, alternatively one might ask whether a capital-gains tax would have a deleterious effect on the growth rate of the economy. The positive analyses of the literature might examine the impact of actual taxes on welfare, or various endogenous variables. A case in point would be whether the actual levels of capital and labor taxation exacerbate, rather than ameliorate, the cyclical fluctuations of aggregates over the course of the business cycle.¹

Until very recently, however, what had been missing from most of these analyses of tax policy is a theory of how taxes are actually determined. Presumably in most market economies the policy-makers make decisions based on what they perceive to be the collective desires of a diverse group of agents in the economy.

Government policies may then influence the distribution of wealth across the economy. This distribution then influences the manner in which future policy variables are determined, which in turn affects the levels of future policy variables through the policy-making mechanism. Such mechanisms are usually absent from traditional analyses of the affect of government policy. It is the thesis of this paper that it is important to understand how the wealth dynamics generated by an economy help to influence the rules determining public policy parameters, rather than vice versa. Furthermore, this inquiry is also conducted within the context of a simple dynamic general equilibrium framework. This is important because there have been comparatively few papers that have incorporated endogenous optimal voting decisions into dynamic equilibrium models with utility-maximizing agents, presumably because of the complicated nature of the problem.

There has been a great deal of work done recently in which some government policies are determined endogenously. Tabellini (1991) studies the behavior of government debt in an economy in which policies are determined by majority rule. Tabellini and Alesina (1990) study an economy in which agents vote on the composition of government spending. They then characterize the factors that influence the size of the budget deficit. Tabellini and Persson (1990) provide a comprehensive guide to how credibility and political issues can influence the determination of macroeconomic policy. Alesina (1988) and Perotti (1992) provide a detailed set of references to this growing theoretical as well as applied literature. What is missing from much of the existing literature, and what is the main point of the present paper, is an explanation of how policies implemented in one period transform the distribution of wealth in such a manner as to also influence the policies that are chosen in future periods.²

There have been some recent attempts to address the issue of how government policies might be determined endogenously in an explicitly dynamic environment, as is done here. In some of this work, such as Alesina and Rodrik (1994), Bertola (1993), Cooley and Soares (1994), voting is permitted only in the initial period on the policies that are going to be implemented in the future.

Perhaps the work most closely related to the present approach is that of Krusell, and Ríos-Rull (1993a), and Krusell, Quadrini, and Ríos-Rull (1994). In these models the relative wealth of the median voter can potentially change, and the median voter can determine the equilibrium policies in each period. These papers employ a population consisting of infinitely-lived agents, and the median voter is always the same agent. However, the model described below is one of overlapping generations, and consequently is one of the few such frameworks in which the median voter must change each period. Therefore agents must determine how their decisions will influence the future decisions of future generations.

There is a further empirical motivation for utilizing the approach to the determination of policy parameters studied in this paper. One might be *very* hard-pressed to derive an argument for the optimality of the current observed government tax and spending structures based on the solution to some optimal welfare problem, so the reason for such a policy structure may well lie with the political nature in which policy parameters are formulated. One might suggest that it is no coincidence that in the U.S. there has recently been a growing interest in cutting capital taxation, not to mention running higher government budget deficits, and spending more money on social programs for the elderly, at the same time that there is a growing population of elderly citizens. Obviously, these people have a strong incentive to participate in the political process in such a way as to encourage policy-makers to divert more resources in their own direction, and possibly also cut tax rates on their main sources of income. For the elderly, this is more likely to be capital income. Although this particular issue is not addressed in this paper, this observation serves as a motivation for studying or modelling how the economic agents can influence the policy-making mechanism.

A closely related motivation is that in observed economies, the median voter might appear to change from election to election. More recently in the U.S. the median voter is usually older than he or she was in the previous election cycle due to the aging of the baby-boom generation. Yet in much of this economic literature there is no such dynamic "hand-off" from one median voter to another in successive

elections. This has serious repercussions for how policy-makers choose to handle questions related to Medicare and Social Security. Therefore, an important ingredient may be missing from these models. Yet there is usually a difficulty encountered in analyzing this particular issue. It is generally an onerous task to investigate how policies chosen by a median voter in one period will influence the behavior of other median voters in some future period. The present paper is an attempt to study this issue, as well as the analytical and computational challenges that it presents.

The remainder of this paper is organized as follows. In the next section the physical and political structure, as well as the nature of the equilibrium of this very simple economy is described in detail. The economy is populated by a sequence of overlapping generations, each of which lives for three periods only. Agents work in their first two periods of life, and hold capital into the last two periods of life. Each period the government must finance a fixed level of real consumption for itself. Each period agents vote on the appropriate levels of capital and labor taxation, while playing a Nash game against future generations. It is assumed that the majority of the voters determine the levels of these parameters. In Section III a series of examples are presented. It is shown how temporary exogenous disturbances can have persistent and permanent effects on the equilibrium, and on the endogenous policy variables. It is shown that there can exist multiple steady-state equilibria, which depend on the initial conditions of the economy. The implications for the wealth distribution, and asset prices are studied. Also, it is shown that the tax rates can appear to fluctuate *dramatically* in the model, with the agents voting to use the labor tax, *or* the capital tax, but apparently never both simultaneously. This provides some motivation for why one might expect to observe the antithesis of the usual tax smoothing behavior. Section IV contains some final remarks.

THE ECONOMIC ENVIRONMENT AND THE EQUILIBRIUM

The economy is one in which time is discrete and is indexed by $t = 1, 2, \dots$. Each period there is a generation of agents who enter the economy, and are present there for three periods. The size of each generation is normalized to unity. An agent who enters the economy in period t will be said to be a member of generation t , and is present in the economy in periods t , $t+1$, and $t+2$. Agents have perfect foresight concerning the future. Each member of generation t wishes to consume some of the single consumption good in period $t+1$, and $t+2$. That is to say, they do not consume in the first period of life. Such agents have one unit of labor effort to supply inelastically in period t and in $t+1$, and which will produce $w_{1,t}$ and $w_{2,t+1}$ units of the consumption good in periods t and $t+1$ respectively. These wages are measured in units of the consumption good.

There is productive capital in the economy which produces a dividend in units of the consumption good each period. The capital stock may be thought of as land since it does not depreciate, and cannot be augmented. Agents who are members of generation t will wish to consume in future periods, and can do so by purchasing productive capital in period t at a price P_t . For each unit of capital held by the agent, they then receive a dividend of d_t units of the consumption good in period $t+1$, and can then sell some of their capital or buy more capital in period $t+1$ at a price P_{t+1} .

A member of generation t has a utility function that will be described as follows

$$\ln(c_{2,t+1}) + \beta \ln(c_{3,t+2}), \quad (1)$$

where $c_{s,t}$ represents consumption by an agent in period t who is currently in period s of his life ($s = 2, 3$), and $\beta > 0$ is the discount factor. In period t each member of generation t supplies their labor inelastically. The agent has his labor income taxed at a rate τ^l_t . The agent will then purchase capital with the remaining income, so that the period t budget constraint for such an agent is then

$$P_t x_{2,t+1} = w_{1,t} (1 - \tau_t^l). \quad (2)$$

Here $x_{s,t+1}$ represents the number of units of capital purchased in period t by an agent, who is currently in period $(s-1)$ of his life, and which is then taken into period $t+1$.

A member of generation t who enters period $t+1$ with $x_{2,t+1}$ units of capital then collects a dividend, in units of the consumption good, in the amount of d_{t+1} per unit of capital. Furthermore capital can then be purchased or sold at a price of P_{t+1} . However, the total return to holding capital - dividend and price of capital - is taxed at a rate of τ_{t+1}^k .³ The agent also supplies his unit of labor, and collects wage income of $w_{2,t+1}$ and pays taxes on this income. The agent will then wish to take some capital into the final period of his lifetime, and the amount of this capital will be denoted by $x_{3,t+2}$. The budget constraint for such an agent in the second period of his planning horizon is then written as follows

$$c_{2,t+1} = (P_{t+1} + d_{t+1})(1 - \tau_{t+1}^k)x_{2,t+1} + w_{2,t+1}(1 - \tau_{t+1}^l) - P_{t+1}x_{3,t+2}. \quad (3)$$

In the final period of this agent's life he brings $x_{3,t+2}$ units of capital into the period. The agent then collects the dividend (d_{t+2}) on the capital, and sells his stock of capital at a price P_{t+2} . The agent must pay the capital tax at a rate τ_{t+2}^k on the total return to capital, and consumes the remaining proceeds. The budget constraint for the agent in this period can then be written as

$$c_{3,t+2} = (P_{t+2} + d_{t+2})(1 - \tau_{t+2}^k)x_{3,t+2}. \quad (4)$$

It will be assumed that the available amount of capital is fixed. This amount is normalized to be equal unity. Then the market clearing condition for the capital market in every period t will then be written as

$$x_{2,t+1} + x_{3,t+1} = 1. \quad (5)$$

Each period there is a certain amount of real government consumption g_t that must be financed through taxing labor and/or capital income. This government

expenditure provides no utility to agents.

To preserve the simplicity of the environment, it will also be assumed that the exogenous sequence $\{w_{1,t}, w_{2,t}, d_t, g_t\}_{t=1}^{\infty}$ is strictly positive and is known with certainty $\forall t$.

Parenthetically, it should be noted that at date $t=1$, there exist the members of generation 0, and -1. At the beginning of this period these agents hold the aggregate capital stock of one unit (i.e. $x_{0,1} + x_{-1,1} = 1$). In period 1 the members of generation (-1) supply all their capital to maximize their period 1 consumption. In this same period the members of generation 0 face budget constraint (3), and constraint (4) in the following period. These agents then maximize their utility function subject to these constraints.

Now, the tax rates that appear in the budget constraints are yet to be determined. The mechanism that sets these parameters is now described. It is assumed that at the beginning of every period t , the members of generation $t-2$, $t-1$, and t vote on the size of the tax rates (τ_t^k, τ_t^l) , which are restricted to be non-negative. After the tax rates are then determined, the agents maximize utility subject to their budget constraints while acting as price takers, and taking as given the behavior of other agents, including the behavior of future generations.

It should be apparent that the members of the old generation will prefer the labor income tax since their sole source of income derives from capital. It should also be apparent now that in any period the members of the young generation will always prefer a capital income tax to a labor income tax, since their sole source of income is labor income. Hence, the decision as to what the tax rates will actually be is assumed to be determined solely by the members of middle-aged generation. More will be said about this in the Appendix. It should also be noted that it is assumed that agents are unable to commit to future voting strategies. Voting outcomes are determined sequentially and are not tied to past voting behavior, except through the state variable $(x_{2,t})$.

Obviously the middle-aged agents must balance costs of both types of taxation. In particular, they dislike capital income taxation because they hold some

capital. But they also dislike labor taxation for two reasons. First, a labor tax is also a tax on their second period labor income, and so hurts them directly. Secondly, this latter tax lowers the labor income of the young and middle-aged agents, and thereby lowers the equilibrium price of capital, and hence lowers the return to holding capital.

Along with these latter consequences are other indirect effects of the taxation methods. As will be made obvious below, both these taxes have the impact of lowering the price of capital from what it would otherwise be if there were no taxes. This may be a fortunate effect from the point of view of a middle-aged agent since this may allow them to purchase *more* capital at the reduced price and thereby raise their consumption in the last period of their life. Yet another effect is that a change in the tax rates, while an agent is middle aged, will influence the amount of capital taken by his generation into the last period of life. This has the potentially unfavorable affect of lowering the rate of return to capital, since these agents will be subsequently supplying their capital inelastically, which helps lower the price and rate of return. Lastly, the agent must also consider how changes in the tax rates will alter the future distribution of assets and thereby influence the voting behavior future generations.

There is still one additional factor that the middle-aged agents must also take into consideration. In each period t the government must finance a level of expenditures g_t , and the government revenue from capital and labor sources must be sufficient to finance this expenditure level. The middle-aged agents must also take this into consideration when formulating their voting strategy. This means that the consideration of a marginally lower capital tax rate must then necessarily imply a marginally higher labor tax. Agents are assumed to take *all* of these effects into consideration when formulating their optimal voting strategy.

To illuminate this discussion, it may help to proceed with the solution of the agents' optimization problem. In particular note first that in period t the agents who are members of generation t have a trivial sort of behavior, described by equation (2), in that they buy as much capital as their after-tax labor income will permit.

Members of generation t-2 consume all their after tax capital earnings, and so their decision bears no more discussion. The interesting problem is then posed by analyzing the decision problem of a member of generation t-1. Now consider this optimization problem from the point of view of such an agent after the tax rates in period t have been set. Then such an agent maximizes the following objective function

$$\ln(c_{2,t}) + \beta \ln(c_{3,t+1}), \quad (6)$$

subject to the constraints

$$c_{2,t} = (P_t + d_t)(1 - \tau_t^k)x_{2,t} + w_{2,t}(1 - \tau_t^\ell) - P_t x_{3,t} \quad (7)$$

$$c_{3,t+1} = (P_{t+1} + d_{t+1})(1 - \tau_{t+1}^k)x_{3,t+1}. \quad (8)$$

It is easily seen that the solution to this problem is of the following form

$$P_t x_{3,t} = \left(\frac{\beta}{1+\beta} \right) \left[(P_t + d_t)(1 - \tau_t^k)x_{2,t} + w_{2,t}(1 - \tau_t^\ell) \right], \quad (9)$$

$$c_{2,t} = \left(\frac{1}{1+\beta} \right) \left[(P_t + d_t)(1 - \tau_t^k)x_{2,t} + w_{2,t}(1 - \tau_t^\ell) \right]. \quad (10)$$

Now by substituting equations (2) and (9) into (5), the equilibrium price of capital can then be derived as follows

$$P_t = \left[\frac{w_{1,t}(1 - \tau_t^\ell) + \left[d_t(1 - \tau_t^k)x_{2,t} + w_{2,t}(1 - \tau_t^\ell) \right] \left(\frac{\beta}{1+\beta} \right)}{1 - x_{2,t} \left(\frac{\beta}{1+\beta} \right) (1 - \tau_t^k)} \right]. \quad (11)$$

There are several important features of this pricing equation to note. First, observe that the *distribution* of capital across the population influences the price of capital. The more capital that is held by the middle-aged generation ($x_{2,t}$), as opposed to agents who are in the last period of their life, the higher the price of capital. This

should make sense since the agents who are in the third period of life will be *supplying all their capital inelastically*, and the more capital they have, the lower will be the equilibrium price of capital. Consequently, the more capital held by members of the middle-aged generation, the higher will be the price of capital. Secondly, note that the higher the labor income tax the lower will be the price of capital since the members of generation t cannot afford to purchase as much capital. Lastly, the higher is the capital income tax the lower is the price of capital as well. It is also the case that the impact that changes in τ_t^ℓ or τ_t^k have on P_t will depend on the values of $w_{1,t}$, $w_{2,t}$, d_t , and $x_{2,t}$.

The government is restricted to balancing its budget each period so that it must implement labor and capital tax rates each period to finance its expenditures. Hence its budget constraint is written as follows

$$g_t = \tau_t^\ell (w_{1,t} + w_{2,t}) + \tau_t^k (P_t + d_t). \quad (12)$$

Now a substitution of the optimal decision rules, (4) and (10), for a member of generation $t-1$ back into their utility function (6) produces the following version of an indirect utility function

$$\ln \left[\left(\frac{1}{1 + \beta} \right) \left((P_t + d_t)(1 - \tau_t^k)x_{2,t} + (1 - \tau_t^\ell)w_{2,t} \right) \right] + \beta \ln \left[(P_{t+1} + d_{t+1})(1 - \tau_{t+1}^k)x_{3,t+1} \right]. \quad (13)$$

This is the indirect utility function that the agents, who are middle-aged in period t , seek to maximize. However, these agents also realize that their choice of taxes will influence the price of capital in the present period (P_t), the future price of capital (P_{t+1}), as well as their future asset holdings ($x_{3,t+1}$). Since $x_{3,t+1}$ is determined from equations (5) and (9), and P_t and P_{t+1} are determined by versions of equation (11), by making these substitutions into equation (13), and then after conducting a mammoth amount of algebra, this indirect utility function can then be rewritten as follows.

$$\begin{aligned}
V_t(\tau_t^k, \tau_t^\ell) = & (1 + \beta) \ln \left[x_{2,t}(1 - \tau_t^k) [w_{1,t}(1 - \tau_t^\ell) + d_t] + w_{2,t}(1 - \tau_t^\ell) \right] - \ln \left[1 - x_{2,t}(1 - \tau_t^k) \left(\frac{\beta}{1 + \beta} \right) \right] \\
& - \beta \ln \left((1 - \tau_t^\ell) [w_{1,t}(1 + \beta) + \beta w_{2,t}] + x_{2,t}(1 - \tau_t^k) \beta d_t - \beta (1 - \tau_{t+1}^k) (1 - \tau_t^\ell) w_{1,t} \left[1 - x_{2,t}(1 - \tau_t^k) \left(\frac{\beta}{1 + \beta} \right) \right] \right) \\
& \beta \ln \left[w_{1,t+1}(1 - \tau_{t+1}^\ell) + \left(\frac{\beta}{1 + \beta} \right) w_{2,t+1}(1 - \tau_{t+1}^\ell) + d_{t+1} \right] + \beta \ln(1 - \tau_{t+1}^k)
\end{aligned} \tag{14}$$

This is the indirect utility function faced by an agent, who is a member of generation $t-1$, at the beginning of period t , and it reflects the optimal savings behavior for such an agent, given any specified level for the tax parameters (τ_t^k, τ_t^ℓ) , as well as other variables including the tax rates in the last period of the agent's life $(\tau_{t+1}^k, \tau_{t+1}^\ell)$. The variables $(w_{1,t+1}, w_{2,t+1}, d_{t+1})$ appear because they influence the price of capital in period $t+1$, and thereby influence the welfare of an agent who is a member of generation $t-1$.

Two types of equilibria will be considered. The first is one in which the middle-aged agent at date t takes the values of $(\tau_{t+1}^k, \tau_{t+1}^\ell)$ as given when making their decisions. For lack of a better name, this might be referred to as *myopic voting*. In this instance the future taxes $(\tau_{t+1}^k, \tau_{t+1}^\ell)$ can influence the decisions and welfare of agents at date t , and the levels of these taxes are taken into account. However these agents ignore how their current policy decisions influence future tax rates, since these are taken to be fixed parameters.

Perhaps a more satisfactory approach is to permit agents to take this feature into account, although it significantly complicates the solution of the model. This is the primary equilibrium under study and will be referred to as *optimal voting*. In this instance it is apparent that the taxes chosen in period t , (τ_t^k, τ_t^ℓ) , will influence the wealth distribution in the subsequent period, $(x_{2,t+1})$, and thereby influence the choices of $(\tau_{t+1}^k, \tau_{t+1}^\ell)$ that are made in the subsequent period. The middle-aged agents at date t then assume there is a dual mapping $\tau_{t+1}^k = g_k(x_{2,t+1})$, and $\tau_{t+1}^\ell = g_\ell(x_{2,t+1})$. These agents must then essentially use this mapping to substitute these functions for τ_{t+1}^k and τ_{t+1}^ℓ in equation (14). Additionally, they must then take into

account how the choices of τ^k_t and τ^ℓ_t influence the value of $x_{2,t+1}$. All of this is a long-winded way of stating that the objective function given by equation (14) is even more complicated than it might already appear in the instance in which agents take all these features into account.

This is the point at which it is perhaps the easiest to see the difference between the present analysis, and that of some other papers in this area. In the present model, agents must make decisions each period regarding how their voting decision will influence the future distribution of capital, and consequently how this distribution will influence the future voting decisions of *other agents* in the economy. Voting occurs each period, not just in the initial period, and the median voter is not the same agent each period.

To accurately describe the nature of the equilibrium under study, it will be useful to define a *future* at date t to be the following: $\mathcal{F}_t \equiv \{w_{1,s}, w_{2,s}, d_s, g_s\}_{s=t}^\infty$. Since the strategies of agents at date t can be influenced by strategies employed by future generations, it seems only natural that the proper state-space representation for the strategies of agents at date t must then include \mathcal{F}_t since this shows how the strategies must be a function of exogenous variables realized in the present and future. Additionally, as explained above, the level of $x_{2,t}$ is also a state variable at date t . It would then be perhaps more accurate to write the value function that is to be maximized, given by equation (14), as $V(\tau^k_{t+1}, \tau^\ell_{t+1}; x_{2,t}, \mathcal{F}_t)$. Then the resulting *maximized value* of this function could be written as $V^*(x_{2,t}, \mathcal{F}_t)$, since it is then dependent on these two arguments.

Therefore, the strategies described below will consist of functions $\pi_\ell(\bullet)$ and $\pi_k(\bullet)$ such that $\pi_i: [0,1] \times \mathbb{R}_+^{4\infty} \rightarrow [0,1]$ for $i = k, \ell$. These functions, which represent the voting strategies of an agent at date t , can then be written as $\tau^k_{t+1} = \pi_k(x_{2,t}, \mathcal{F}_t)$ and $\tau^\ell_{t+1} = \pi_\ell(x_{2,t}, \mathcal{F}_t)$.⁴ Embedded in this description is a notion of symmetry since all agents are assumed to employ the same decision strategies.⁵ Details of the computation and characterization of the equilibria are presented in the Appendix.

In any case, it is assumed that at the beginning of each period the agents, in

choosing the tax rates, are playing a Nash game against future generations. In the case of the myopic voting, the middle-aged agents take future tax rates as given when choosing their optimal tax rates. However, they take into account how these tax rates will influence the present and future prices of capital, and asset holdings.^{6,7} However, the main equilibrium under study is the optimal voting scheme where middle-aged agents know that their decisions will influence the future distribution of wealth and therefore the subsequent voting behavior.

Finally, to make this discussion precise, it is worthwhile to proceed with the following formal definition of the equilibrium under study when agents vote optimally.

Definition: A *Symmetric Nash Competitive Equilibrium* for this perfect foresight economy is a collection of non-negative sequences $\{d_t, g_t, P_t, w_{1,t}, w_{2,t}, x_{1,t}, x_{2,t}, c_{1,t}, c_{2,t}, \tau_t^k, \tau_t^\ell\}_{t=1}^\infty$, such that for $t \geq 1$, the following conditions are satisfied.

i) For members of generation $t-1$, given the realizations of $(\mathcal{F}_t, x_{2,t})$, and the functions which determine the voting behavior of future generations, $\tau_{t+1}^k = \pi_k(x_{2,t+1}, \mathcal{F}_{t+1})$ and $\tau_{t+1}^\ell = \pi_\ell(x_{2,t+1}, \mathcal{F}_{t+1})$, the period t taxes (τ_t^k, τ_t^ℓ) are chosen to maximize the value function $V_t(\tau_t^k, \tau_t^\ell)$, as given by equation (14), subject to the government's budget constraint (12). Furthermore, the following consistency conditions are satisfied: $\tau_t^k = \pi_k(x_{2,t}, \mathcal{F}_t)$, $\tau_t^\ell = \pi_\ell(x_{2,t}, \mathcal{F}_t)$.

ii) Given tax rates (τ_t^k, τ_t^ℓ) , and the price of capital P_t , the quantities $(x_{1,t}, x_{2,t}, c_{1,t}, c_{2,t})$ maximize the utility function (6), subject to the budget constraints. This implies the decision rules (2), (4), (9), and (10) are satisfied.

iii) The government budget constraint (12) holds for each period.

iv) Equation (5) holds, so that there is equilibrium in the capital market. This implies that the price of capital (P_t) is given by equation (11).

It should also be noted that an exogenous constraint that is being imposed is that the tax rates (τ_t^k, τ_t^ℓ) are restricted to being non-negative. This is an ad-hoc restriction, but one that permits the behavior of the model to be somewhat more

interesting.

The case of myopic voting can be defined in a similar manner to that above. In this instance the middle-aged agent at date t takes the tax rates, τ_{t+1}^k and τ_{t+1}^ℓ , as given when making his decisions.

Now equation (14) is a formidable and intimidating expression. Rather than attempting to gain insights directly from this equation, it will be more enlightening to look at a series of examples to obtain a feel for the nature of the equilibria of the economy.

SOME SAMPLE ECONOMIES

Much of the work in the optimal taxation literature has the implication that the "optimal" level of distortional taxation is that which minimizes the social deadweight loss. This usually gives rise to optimal tax rates in which all commodities are taxed to some degree so that the *marginal* social costs from all forms of taxation are equated. This would be the case if one followed the Ramsey tax rules. This might be referred to as a tax-smoothing argument.⁸ It is then of interest to see if, in the context of the above-specified framework with agents voting on the optimal levels of taxation, the resulting tax levels would display such properties, in the sense that agents will choose to have positive taxation on both labor and capital.

As it happens, and as will be shown, for many of the examples that will be presented below, this result does not obtain. In particular, middle-aged agents always prefer to have capital taxation *or* labor taxation, *but not both*. Another way of putting this is to say that the value function displayed in equation (14), does not appear to be concave in the tax rates (τ_v^k, τ_v^ℓ) , which would help to produce an interior optimum for these variables. The reason for this will be explained in the first example. This is an illuminating result in that it shows a potential avenue through which there might be a divergence between the actual observed taxation rates, and those derived from the solution from some optimal planning problem.

Example #1: This example illustrates the potential fluctuations that exist in the model. In particular, there exists two equilibria which depend upon the initial conditions of the economy. One displays limit cycles of two periods in length, while the other does not.

The parameter values are chosen as follows: $w_{1,t}=8$, $w_{2,t}=g_t=7$, $d_t=5$, and $\beta=1$, $\forall t$. Two different initial conditions are chosen for capital holdings of the initial middle-aged agents.⁹ One is $x_{2,1}=.430$, while the other is $x_{2,1}=.431$. There is no other exogenous uncertainty in the economy. Figure 1 illustrates the resulting paths for the price of capital in each case. The solid line shows the behavior for the price of capital when $x_{2,1}=.431$ while the dashed line is the price of capital when $x_{2,1}=.430$. Obviously, the solid line converges relatively quickly to a constant steady state while the dashed line displays cycles. Figure 2 shows the resulting paths for the capital holdings by the *middle-aged agents* for the same example. Again the cycles appear in this variable as well, for the case where $x_{2,1}=.430$.

Figure 3 illustrates the behavior of the tax rates that are observed in the cyclic equilibrium. The dashed line is the path for the capital income tax, while the solid line is the path for the labor income tax. In this non-cyclic equilibrium the agents choose to use only the capital tax.

What is happening in this example is that when $x_{2,1}=.430$, the initial middle-aged agents begin with relatively little capital, and consequently vote to tax capital heavily in the first period of the economy, while choosing to not tax labor at all. Consequently, they have comparatively little capital in period 2 because they began with little in the previous period, and also because the young agents in the previous period did not have their labor income taxed, and could then afford to purchase plenty of capital. In period 2, the new middle-aged agents then have plenty of capital, relative to the initial middle-aged in the previous period, and they do the reverse: they vote to tax only labor and not capital at all. This pattern of behavior then repeats itself every two periods. There is no convergence of any of the decision variables, nor for the price of capital, or the distribution of capital.

For the case where $x_{2,1}=.431$, the initial middle-aged agents again initially

choose to tax capital in the first period. However, because they initially hold a sufficiently large amount of capital, relative to the case where $x_{2,1}=.430$, the subsequent middle-aged generation in period 2 holds less capital, and therefore they choose to tax capital in period 2. In comparison, when $x_{2,1}=.431$, the middle-aged generation in *period 2* chooses to tax capital instead of labor since they hold less capital than their second period counterpart would hold when $x_{2,1}=.430$. Consequently when $x_{2,1}=.431$, the labor tax is never chosen, and capital tax is the only method chosen to finance government consumption.

As noted above, it is also of interest to see if the agents in this economy voluntarily choose to implement some variant of what may be called a tax-smoothing policy. As Figure 3 shows, in the cyclic equilibrium, the solutions observed for the tax rates tend to be of the "bang-bang" variety, with the middle-aged agents choosing *either* labor *or* capital income tax, but not both.

It should be noted that there appears to a good reason why the middle-aged agents dislike imposing both capital and labor taxation simultaneously. There exists a form of double taxation in this instance because labor taxation lowers the price of capital, and lowers the return to capital, while the capital taxation also has a similar impact, and lowers the after tax return to capital. Imposing both taxes simultaneously appears to compound the effects. Agents in this environment want to minimize their tax burden by essentially forcing other agents to bear the cost of paying taxes. This implies making either the youngest or oldest agents bear the brunt of paying taxes.

Figure 4 illustrates the transition dynamics for middle-aged capital holdings in this example. The horizontal axis measures the quantity of capital held by the middle-aged in period t ($x_{2,t}$), while the horizontal axis measures the same variable in the following period ($x_{2,t+1}$). The upward-sloping line in the figure is a 45 degree line which helps pinpoint the stationary equilibria. The downward-sloping line with a break describes the transitional dynamics of asset holdings. As can be seen in the diagram, there are many equilibria, depending on what the level of the initial asset holdings. However, there appear to be only two limiting equilibria. One has a

constant steady-state for capital holdings equal to .5346. The second equilibria is the one which displays cycles, with $x_{2,t}$ alternating between .4016 and .5763.

In any period, for any capital holdings ($x_{2,t}$) less than .567 the middle-aged agents will choose to use only capital taxation to finance government spending. Alternatively, for a capital holdings greater than .567 the middle-aged agents will choose only labor taxation. That is, the upper branch of this line (to the left of this diagram) reflects the amount of capital held by middle-aged agents next period when the capital tax is imposed, while the lower branch describes the amount of capital held when the labor tax is enacted. For capital holdings just equal to .567, the middle-aged agent will be indifferent between the two types of taxation.¹⁰

There is an interesting dynamic behavior in this example, under the assumption that the endowments, dividends, and government spending are constant. If the initial middle-aged agents hold a capital stock less than .430, or greater than .567, then the economy ultimately converges to the cyclical equilibrium. On the other hand, if the initial middle-aged agents have capital holdings between .430 and .567, then the economy ultimately converges to the non-cyclic equilibrium. However, both the cyclic and non-cyclic equilibria are locally stable.

Despite the fact that there is an equilibrium in which all endogenous variables display cycles, this equilibrium is indeed stationary, since the variables do not depend on time, once the capital holdings at the beginning of the period are known. Also, in contrast with other models which produce cycles, the present framework does not employ a backward-bending supply curve for saving, as a function of the interest rate, and neither does it make use of any externality, except to the extent that the voting scheme can be interpreted as one.

It is also of interest to compare the tax rates that arise in these equilibria with those that would result from the solution to some sort of social planning problem. Consider this same economy in which all endowments and dividends are constant in all time periods. It is straightforward to set up a social planning problem which maximizes the welfare of a typical "representative agent" who could be a member of any generation, and who maximizes utility subject to the budget and resource

constraints described above. The planning problem is then to choose these tax rates so as to maximize the utility of a representative agent. Obviously, this assumes treating all agents in all generations identically.¹¹ In this case, for this example it turns out that the solution is to set the capital income tax rate to zero in each period. The reason for this is relatively straightforward. Agents prefer a higher rate of return to saving since this enables them to obtain a higher level of utility from consumption in the final period, and this is accomplished by having a low level of capital taxation.

It is not obvious that this result obtains for all such economies. It is clear from equation (11) that labor taxation also influences the rate of return to holding capital, and it may be the case that in some instances the solution to this social planning problem may entail some capital taxation. Even if this were the case, it should not be too surprising. Capital is not an accumulative factor here, so taxing capital does not influence the growth rate of output or consumption.

It is also of interest to compare the equilibria that can potentially arise for this economy with those that arise with myopic voting, which is the case in which the middle-aged agent at date t takes the tax rates $(\tau_{t+1}^k, \tau_{t+1}^\ell)$ of date $t+1$ as given. That is, the agent takes into account how his tax choices (τ_t^k, τ_t^ℓ) at date t influence the distribution of wealth $(x_{2,t+1})$, but ignores how this influences the tax rates $(\tau_{t+1}^k, \tau_{t+1}^\ell)$ in that period. This assumption greatly simplifies the computation of the equilibria since there is a complicated strategic avenue that can be streamlined. It turns out that for this example the equilibria that arise are identical to those in which this assumption is not made, and agents consider the full impact of their tax decisions on subsequent variables. That is, the equilibria are identical.

The reason why these might be identical is that in the myopic voting equilibrium, the optimal tax rules are piece-wise linear or "flat", and of the following form

$$\tau_i^e = \begin{cases} .467 & \text{if } x_{2,t} < .567 \\ 0.0 & \text{otherwise} \end{cases} \quad (15)$$

Consequently, when evaluated at either $x_{2,t} = .4016$, or $x_{2,t} = .5763$, the derivative of this function is zero. If one were then to take the derivative of equation (14) with respect to the tax rates, this derivative would involve terms such as $(\partial \tau_{i+1}^e / \partial x_{2,t+1})(\partial x_{2,t+1} / \partial \tau_i^e)$. These terms would be zero along the equilibrium path. A similar argument could be made for the capital tax rate, but in this instance the tax rate is "almost piecewise flat."¹² Therefore the strategic interaction that would be present in which the agent would consider how the choice of his tax rate would fully influence the determination of future taxes, appears to amount to a second order effect at best.

A word of caution should be issued however, as it is not always the case that these equilibria are identical. As will be shown in the subsequent example, as the level of government spending rises, the more likely it will be that the transition relation of Figure 4 will not cross the 45 degree line. In the case of optimal voting there does not appear to be an easily characterizable equilibrium. In the case of myopic voting, this means there is no non-cyclic equilibrium.

Additionally, it is of interest to compare the equilibria for this economy to those that arise if the middle-aged agents at date $t=1$ do the voting for *all future time periods*. That is, in the first period these agents choose a set of tax rates they wish to see imposed for *all future time periods*. This is of interest since this is sometimes a simplifying assumption made in the literature which permits the characterization of models that might otherwise be unmanageable (Alesina and Rodrik (1994), Bertola (1993), and Cooley and Soares (1994)).

Figure 5 is an attempt to make this comparison, although it should be interpreted with caution. The horizontal axis is the capital holdings of middle-aged agents in the first period ($x_{2,1}$) and the vertical axis is the capital holdings of middle-aged agents in period 2 ($x_{2,2}$). The solid line is merely an extension of the transition

diagram of Figure 4, and therefore represents the transition of capital holdings when agents vote optimally each period. The dashed line is the transition diagram when there is voting only in the initial period. For $x_{2,t} > .567$, or $x_{2,t} \leq .178$, the dashed and solid lines are identical. When optimal voting takes place each period, if $x_{2,1} > .567$, the labor tax is chosen, and otherwise capital taxation is imposed. For the case in which voting takes place only in the initial period the transition relation is continuous, and the labor tax is chosen by the initial period *for all future periods* if $x_{2,1} > .191$. For $.178 < x_{2,1} < .191$, the policies that are chosen consist of positive capital and labor taxes. Hence this indicates the differences that are likely to appear in models where agents can only vote in an initial period, as opposed to voting in every period. This appears to point to how distinctive these equilibria can be since the equilibrium tax rates can be quite different.

Example #2: The point of this example is to illustrate the manner in which the equilibrium of the model may change in reaction to a fully anticipated exogenous shock to the labor endowment of young agents. Again there exists two equilibria for this economy, and if the exogenous shock is sufficiently large the economy will move from the non-cyclic equilibrium to the cyclical one.

The parameter values are chosen as follows: $w_{1,10}=10.4$, and otherwise $w_{1,t}=8$, $w_{2,t}=7$, $d_t=g_t=5$, $\beta=1 \forall t$, and $x_{2,1}=.4$. That is, the economy has no exogenous disturbances until period 10 when the young agents receive a temporarily high endowment shock.

Figure 6 shows the resulting behavior for the capital holdings of the middle-aged agents. Starting from $x_{2,1}=.535$, the capital holdings have converged to a constant, and capital is always taxed while labor is not. However, the exogenous disturbance is sufficiently large that from period 10 onward, the economy is in a cyclical equilibrium with every second generation holding relatively large quantities of capital. The resulting behavior for the price of capital is shown in Figure 7. Again, this price is converging to a constant until period 10, when it then exhibits cycles as well.

The path for the equilibrium tax rates displays a similar behavior. Until period 10, the agents choose to only use the capital tax, with the labor tax always being zero. After period 10, the agents choose to alternate these taxes, choosing the labor tax only in even periods and the capital tax in odd periods.

It should be noted that the exogenous shock in this example has been "large enough" to cause the economy to take a path onto the cyclical equilibrium. For smaller disturbances the economy might only gravitate back to the non-cyclic equilibrium after several periods. Additionally, changing the levels for some of the parameters or variables may alter this behavior. For example, for the same identical economy but with a slightly different discount factor, this same exogenous disturbance may result in merely a return to the non-cyclic equilibrium. This type of disturbance is "more likely" to result in a move to the cyclic equilibrium, the closer is the "break" in the transition relation (see Figure 4) to the 45 degree line. It is also possible that an economy that is currently following the cyclic equilibrium may move to the non-cyclic equilibrium in reaction to an exogenous disturbance.

This is a good point to note that it is indeed the endogenous voting mechanism that is generating the cycles in this economy. It is easy to show that with constant tax rates, an endowment shock of the type described above would generate relatively mild fluctuation in asset holdings and the price of capital for a few periods, but the economy would then return to the non-cyclic steady-state.

It should also be noted that examples can also be constructed in which a temporarily unusually high or low level of government consumption is also capable of generating these types of cycles.

Example #3: This example is presented to illustrate how the transition dynamics for capital holdings is affected by the level of government spending. Consider two alternative economies, one which is identical to that in the first example with $w_{1,t}=8$, $w_{2,t}=g_t=7$, $d_t=5$, $\forall t$ and $\beta=1$. The second is identical except that $g_t = 5$. Clearly, in this new case the tax rates must be lower to finance the lower level of government consumption. The lines in Figure 8 describing transitional dynamics

display a smaller "break" in this case in which $g_t = 5$ since the taxes, when imposed, have a much smaller impact on the future capital holdings since the taxes are lower. If government consumption were zero, then there would be no break in the transitional dynamics line at all since all taxes would be zero. This illustrates why there are no such cycles when $g_t = 0$. What this seems to imply is that, roughly speaking, the lower is the level of government spending, the more likely it will be that the economy will attain the non-cyclic equilibrium.

It should also be noted that to produce a cyclic equilibrium, it is necessary that the "break" in the transition relation be "near" the 45 degree line. If it takes place far enough away from this line, there will be no cyclic equilibrium.

An exogenous constraint imposed on this problem is to only permit the tax rates to be non-negative. Permitting some negative taxation has an effect similar to that of increasing the level of government consumption, and thereby potentially increase the likelihood that there will be a cyclical equilibrium. Permitting negative taxes only increases the ability of middle-aged agents to extract resources from other generations and this is what can lead to the existence of the cyclical equilibrium.

Example #4: This example illustrates how the model behaves for different discount factors. This example is identical to the example #1 with the exception that the discount factor is lower. For both economies the parameters are $w_{1,t}=8$, $w_{2,t}=g_t=7$, $d_t=5$, $\forall t$. For one economy $\beta = 1.0$, while for the other $\beta = .80$.

To obtain an understanding of how changing the discount factor influences the equilibrium it may be useful to look at Figure 9, where the transitional dynamics for asset holdings is illustrated for different discount factors. Increasing (decreasing) the discount factor tends to move this function toward (away from) the origin. The reason appears to be that increasing β results in increased saving and therefore a higher price of capital. This in turn means that young agents cannot afford to purchase as much capital, and so this results in a lower level of $x_{2,t}$.

It should be stated that not all configurations of this economy exhibit the cycles displayed in these examples. To produce these cycles requires the right kind

of balance between the ratio of capital to labor income on the one hand, and the discount factor on the other hand. In particular, for a given discount factor (β), if the agent's labor income is too high, relative to the amount of capital income, then agents may always prefer to have labor taxed. Conversely, holding the discount factor constant, if there is very little aggregate labor income, then agents may then always prefer capital taxation. Alternatively, holding constant the levels of endowments and dividends, the agent is more likely to prefer the capital (labor) tax, the lower (higher) is the discount factor since this makes future consumption less (more) important. It should be noted, however, that experimentation has revealed that although not all economies exhibit these fluctuations, neither are the economies that do exhibit this behavior "knife-edge," or extremely special cases. A wide range of economies can be seen to display these features.

It would also possible to incorporate an endogenous labor decision in the agents second period of life. However, in some cases this effect only serves to exacerbate the effects described above.¹³ This is because a fall in the agent's wage will then encourage them to work less, which will then lower their wage income even further.

Example #5: This example is presented to show how the transition dynamics for capital holdings is affected by the composition of aggregate output into its components of labor and capital income. Consider two alternative economies, one which is identical to that in the first example with $w_{1,t}=8$, $w_{2,t}=g_t=7$, $d_t=5$, $\forall t$, and $\beta=1$. The second is identical except that $w_{1,t}=12$. Figure 10 shows how the transitional dynamics for capital holdings behaves in the two cases. Loosely speaking, increasing the agent's labor income has the effect of making capital income relatively less scarce than it would otherwise be. Consequently, agents are more likely to want to choose the labor tax, and more likely to choose the capital tax. In Figure 10, the break in the line describing the transitional dynamics of asset holdings is moved to the left when the amount of steady-state labor income increases. Because of this the non-cyclic equilibria exists in both of these cases, but

when $w_{1,t}=12$ this equilibrium has only labor income being taxed. In the equilibrium in which $w_{1,t}=8$, the equilibrium has only capital being taxed.

FURTHER REMARKS

Barro (1979) presents some normative reasons why, in a dynamic environment, governments should "smooth" the tax rates so as to minimize the burden of taxes.¹⁴ The present positive analysis illustrates why atomistic agents, behaving in a privately optimal manner, would choose to have a tax structure which would appear to *cause* some fluctuations in endogenous variables. In particular, this analysis points to where the potential divergences might arise among positive and normative tax analyses. The public finance literature is replete with research showing the ways in which observed tax rates may differ from the "optimal" tax policies, for reasons that are usually left unexplained. Nevertheless, presumably an arguable view is that society has arrived at its current tax policies by agents making optimal choices when choosing political representatives who will make policy choices for that will affect society. It is also our task to understand how and why these choices are made.

In some sense, the results of this paper support those found in Krusell, Quadrini, and Ríos-Rull (1994), and Krusell and Ríos-Rull (1993a). These models show the radical metamorphosis that an economy can undertake once policy determination is made endogenous. Krusell and Ríos-Rull show how a redistribution of small amount of capital from one group of agents to the others can result in a large change in the steady-state levels of consumption, output and capital.

In attempting to compare the behavior of the model with that in the data, it must be noted that agents in the model are using the available policy tools to maximize their welfare. Here the available policy tools are the tax rates on capital and labor. More generally, one would think that there are many more policy tools that agents could employ, such as a whole plethora of government spending and taxation schemes that could be utilized, which might obscure the fluctuations in the tax rates alone.

As with any analysis, the present paper leaves many questions unanswered. Here assumptions were placed on how policy variables were determined, but alternative mechanisms could also be employed. It was exogenously imposed that government revenue is derived from the taxation of labor or (gross) capital income. It would be better if it could be shown that such a policy mechanism is "optimal" relative to a set of potential mechanisms. This remains a formidable topic for future research.

The present model has a fixed capital stock. It would be enlightening to know how this type of endogenous policy determination would influence the level of endogenous investment and output. Presumably higher capital tax rates would deter capital accumulation, and influence the wealth distribution in the future.¹⁵ Huffman (1993) has already studied this issue within the context of a simple model in which agents vote myopically, in the sense described above. Most of the results shown above still obtain when capital accumulation is incorporated. In particular, the agents still choose tax rates of the "bang-bang" variety. It is also possible to show that if agents are permitted to vote on the size of the inflation tax in any period in order to finance government spending, then they might choose a volatile path for inflation, as this is just another tax.

Additionally, it is also of interest to know how the results presented above would change if a different utility function were employed. Preliminary work in this area indicates that the existing dynamics still are present with other utility functions, but that other dynamics are also present. By changing the elasticity of substitution of consumption between periods, it appears that the slope of the line describing the transition dynamics of asset holdings (e.g. Figure 4) can be made steeper or flatter. In particular, if this line is made sufficiently steep then the non-cyclic steady-state equilibrium can be made unstable. However, this topic would appear to present even more computational or analytical challenges, since the logarithmic utility function appears to considerably simplify the analysis.

One change in the utility function which would likely radically change the results would be to include bequests by making agents care about subsequent

generations. This would destroy the desire of agents to choose policies and actions that would *hurt* the subsequent generations. In fact, this would likely make the economy much more similar to that of Krusell and Ríos-Rull (1993a), who analyze a representative agent economy.

It is possible that this approach could also be utilized to explain why government spending would be increased at some times and not others. Rather than just saying that this government spending is wasteful, instead this activity might be undertaken due to the fact that there would be a significantly large constituency that benefits from such expenditures. Additionally, it may be possible to use a similar model to explain the level of the deficit that the government may run. Possibly the aforementioned features of the economy could cause government spending to display an unstable response to a temporary disturbance to the economy.

The forgoing analysis raises obvious questions concerning the manner in which our economic policy-making institutions are designed. Do we choose to have institutions in which citizens can potentially exert unremitting or day-to-day control of government policy based on their own private self interests? Or, on the other hand, do we choose to have institutions which set out policy according to some relatively fixed rules that cannot be easily changed based on the whimsy or vocal protests of groups of citizens?¹⁶ Should we choose to have constitutional amendments prohibiting certain types of taxation, as there effectively is now in the U.S. with the poll tax, or at least put some restrictions on the amount of this taxation? To gain an increased understanding of the factors that influence the determination of policy parameters, it would appear that one cannot avoid writing down formal models and investigating how agent's preferences over various policies can change the equilibrium.

APPENDIX

First of all, from the discussion above it should be apparent that the equilibrium choices tax rates can potentially depend on the future levels of the exogenous variables, which can be written as $\mathcal{F}_t \equiv \{w_{1,s}, w_{2,s}, d_s, g_s\}_{s=t}^{\infty}$. However, it will be convenient to suppress this dependence for notational convenience. It is then necessary to find functions $g_k(\bullet)$ and $g_\ell(\bullet)$ that characterize the equilibrium voting behavior such that $\tau_{t+1}^k = g_k(x_{2,t+1})$, and $\tau_{t+1}^\ell = g_\ell(x_{2,t+1})$. Of course, it is easily shown from equations (2), (5), (9), (11), and (12) that the tax rates chosen in period t (τ_t^k, τ_t^ℓ) influence the level resulting level of $x_{2,t+1}$. Let $x_{2,t+1} = H(\tau_t^k, \tau_t^\ell, x_{2,t})$ denote this relationship, which can be written out in a closed-form but cumbersome expression. Keep in mind that the function $H(\bullet)$ is known, but the functions $g_k(\bullet)$ and $g_\ell(\bullet)$ are to be determined. The value function of equation (14) can then be re-written as follows:

$$\begin{aligned}
 V_t(\tau_t^k, \tau_t^\ell) &= (1 + \beta) \ln \left[x_{2,t} (1 - \tau_t^k) [w_{1,t} (1 - \tau_t^k) + d_t] + w_{2,t} (1 - \tau_t^k) \right] - \ln \left[1 - x_{2,t} (1 - \tau_t^k) \left(\frac{\beta}{1 + \beta} \right) \right] \\
 &- \beta \ln \left((1 - \tau_t^k) [w_{1,t} (1 + \beta) + \beta w_{2,t}] + x_{2,t} (1 - \tau_t^k) \beta d_t - \beta (1 - g_k[H(\tau_t^k, \tau_t^\ell, x_{2,t})]) (1 - \tau_t^k) w_{1,t} \left[1 - x_{2,t} (1 - \tau_t^k) \left(\frac{\beta}{1 + \beta} \right) \right] \right) \quad (A1) \\
 &\beta \ln \left[w_{1,t+1} + \left(\frac{\beta}{1 + \beta} \right) w_{2,t+1} \right] (1 - g_\ell[H(\tau_t^k, \tau_t^\ell, x_{2,t})]) + d_{t+1} \Big] + \beta \ln (1 - g_k[H(\tau_t^k, \tau_t^\ell, x_{2,t})])
 \end{aligned}$$

Characterizing the tax rates then involves finding a pair of functions, $g_k(\bullet)$ and $g_\ell(\bullet)$, which maximize the function given by equation (A1).¹⁷ In particular, it is necessary to find functions such that when agents expect future votes to be determined by such a function, then they will also choose to behave in the same manner. This ensures that agents are taking into account all the equilibrium effects of various available policies on present and future prices, tax rates, and capital allocations. In practice, a guess for these functions is obtained from solving the simpler myopic voting problem. Then the method of successive approximations is applied by calculating the optimal response of an agent given that future generations

are expected to follow behavior described by the initial guess. The voting behavior of this agent is then used as a new guess for the functions $g_k(\bullet)$ and $g_e(\bullet)$. This method is then repeatedly applied until a fixed-point is obtained.

After the decisions have been derived, various other issues can be addressed. It is easy to see that the old agents would always prefer the labor tax. It seems entirely plausible that young agents in any period would always prefer a capital tax since they own no capital. However, intuitive plausibility rarely constitutes a proof, and in this case with so many complicated avenues for a voting agent to consider, this may or may not be the case. Fortunately, in these economies once the decision rules are constructed, it is possible to check that indeed the young agents do prefer the labor tax to be set to zero.

The computations for this economy were conducted by partitioning the choice space for $x_{2,t}$ into a grid of 1800 points, and restricting the agents' asset choices to lie in this space. Similarly, the choice space for the capital and labor tax rates is similarly partitioned. Increasing the number of points in this grid beyond 1800 did not change the results in any experiments that were conducted. Each iteration of the procedure for computing the tax rates would take approximately 6 hours on a Pentium machine. The main stumbling-block to obtaining quicker computations is in *inverting* the functions $g_k(\bullet)$ and $g_e(\bullet)$ to obtain the function $H(\bullet)$. Presumably this process could be accelerated by utilizing some sort of approximation scheme, but then this would bring into question the accuracy of such a method. For some economies, the results were identical when there were only 200 points in the grid and the computations were obviously much faster in this instance. Computing the equilibrium for the myopic voting scheme is *much* quicker, and produces the same results in some instances as the optimal voting scheme. Therefore, it was used as an initial guess for the voting in the procedure described above.

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FOOTNOTES

1. On the normative side, Lucas (1990) describes why the desired tax on capital should be zero. In a positive and normative analysis, Barro (1979) shows why the government may wish to "smooth" the levels of distortional taxation over time so as to minimize the deadweight loss from the taxation.

2. Much of this existing literature contains analyses of models which are finite horizon economies. The model studied in the present paper has an infinite horizon, and as such permits an analysis of how the endogenous variables evolve over time in reaction to various disturbances. Alesina and Spear (1988) use the overlapping generations model to construct a model of electoral competition. Boldrin (1993) uses a three-period overlapping generations framework to analyze the impact that public school financing has on the accumulation has on human capital. Krusell, Quadrini, and Ríos-Rull (1994), and Krusell and Ríos-Rull (1993a, 1993b) also provide a very interesting analysis of the impact of endogenous policy formulation within a dynamic infinite-horizon environment.

As noted by Alesina (1988), much of the extant literature is rather descriptive, and not cast within the context of a general equilibrium optimizing framework. The present paper, however, does fall into this category.

3. It could alternatively be assumed that only the dividend was taxable but this would not alter the central qualitative nature of the results. It would, however, change some of the quantitative results presented in the next section since it would lower the tax base for the capital tax rate.

4. Of course, the optimal strategies may in fact be characterized by a correspondence and, as will be shown below, in practice some states appear to have utility-maximizing strategies that are not unique. In this discussion however, it is assumed that the voting strategies are determined by a selection from this correspondence.

5. Also, the equilibrium has a Markov structure which is somewhat unusual in that one state variable is the value of \mathcal{F}_t , which describes the future values of endowments, dividends, and government consumption.

6. Agents also take into account how the taxes imposed in the present period will influence *future* asset holdings of the next generation, (since this is just one minus the amount the current middle-aged generation will choose to hold) and hence how these asset holdings will affect the future price of capital.
7. It may be that there would be other equilibria as well with more complex forms of strategic interaction, but the present approach would seem to preserve a sufficient degree of simplicity and tractability, given the complexity of the dynamics in this infinite horizon economy.
8. That is, this is meant to be an intratemporal argument: tax rates in a given period should be set so as to minimize this tax burden. However, as argued by Barro (1979), obviously the same argument has been used to conclude that taxes should also be set intertemporally to minimize this burden as well.
9. For this example, the capital-share of total income, equal to $(1/4)$, is similar to that in the data. Nevertheless, it is unclear as to what the appropriate level should be of some parameters within this overlapping models.
10. In this instance, the agent will be indifferent between complete capital taxation or labor taxation, and hence the optimal strategy is a correspondence.
11. However, the welfare of the initial old and middle-aged generations are ignored so as to consider only the generations who enter the economy after period $t=1$. Taking the initial generations into account would likely alter this conclusion.
12. The reason it is not quite piece-wise flat is that the price of capital is endogenous.
13. This would depend on the relative strengths of the wealth and substitution effects. If the latter dominated the former then the economy is likely to display the features described above since a labor tax would then reduce the work effort and further reduce labor income.
14. The environment considered here has the government running a balanced budget each period, whereas Barro is concerned with setting taxes optimally when the government

can have positive levels of debt outstanding.

15. Hence, this majority voting scheme may be another propagation mechanism whereby temporary technology disturbances would influence future levels of output, although it seems unlikely that these could be used to explain the high-frequency business cycle movements in aggregate time-series.

16. This has everything to do with how central banks are structured in different economies. Some countries, such as the Germany choose to have relatively independent central banks that are supposed to focus primarily on producing price stability. Other countries choose to have central banks that are much less independent of the executive or legislative branches of government, and are more susceptible to political pressure.

This is also related to how government institutions at different levels are designed. For example, what policy forces should be vested in the Federal Government of a country, and which powers should be possessed by the local governments?

17. In fact it is necessary to find only one such function since the other is determined jointly by equations (11) and (12).

Figure 1

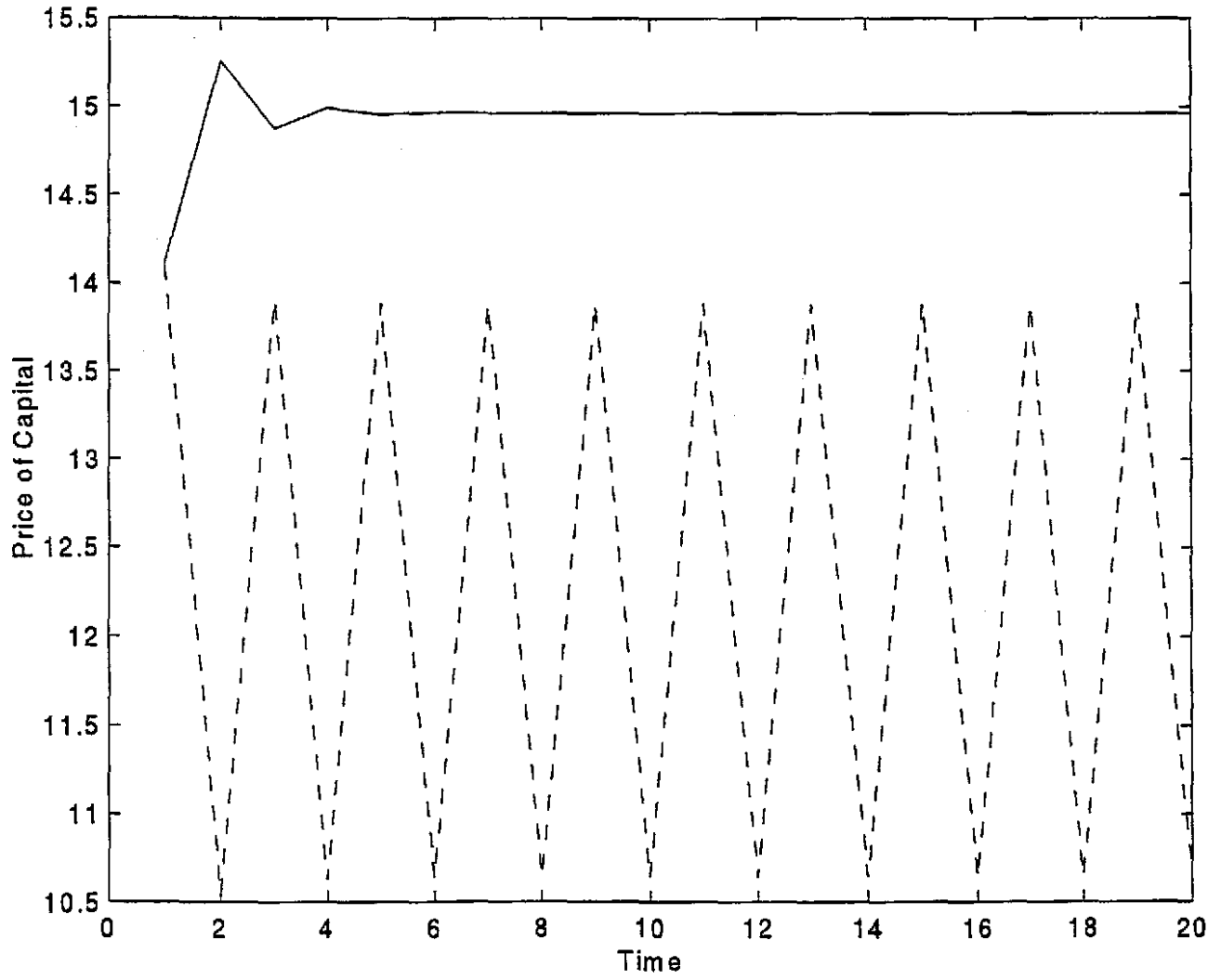


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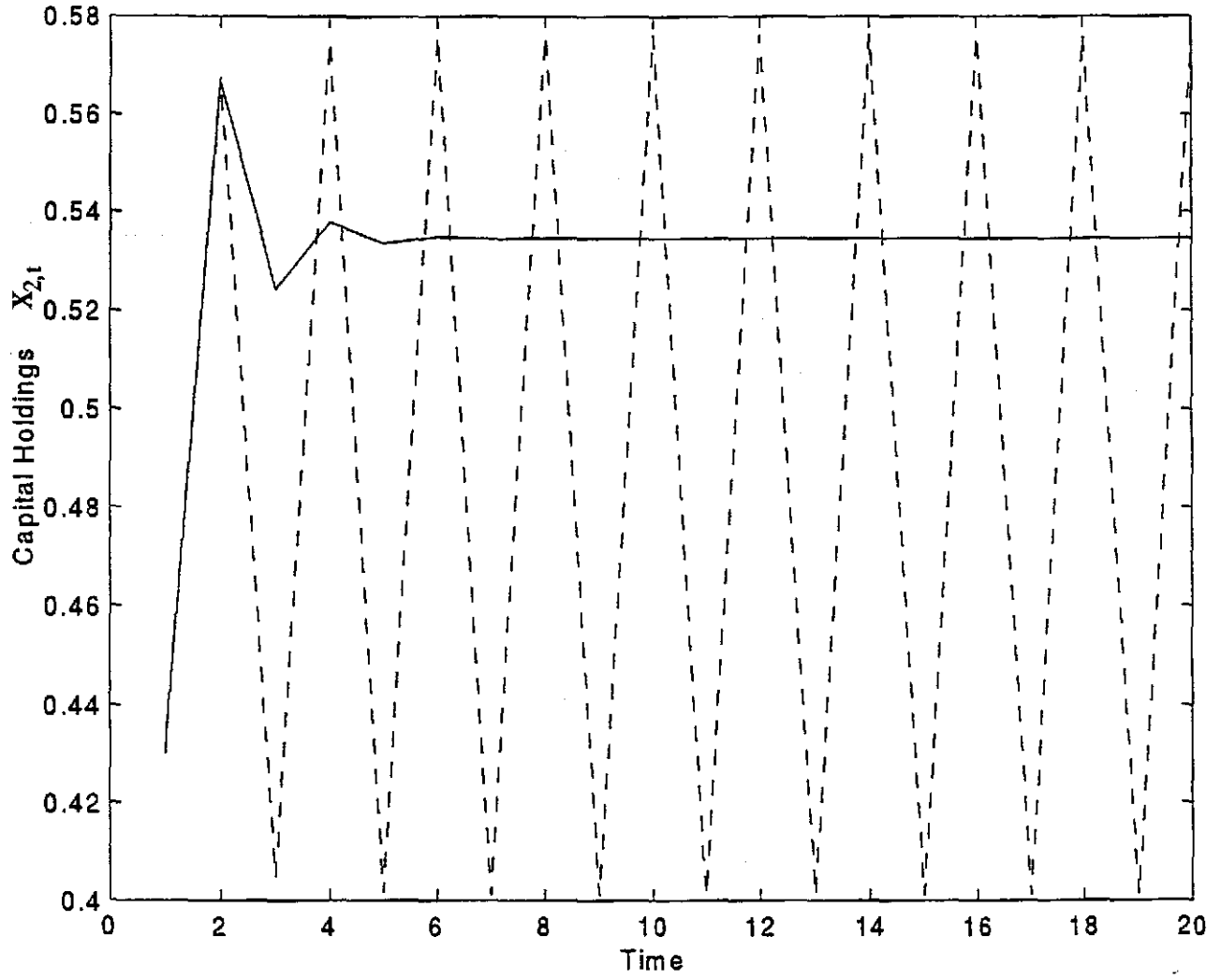


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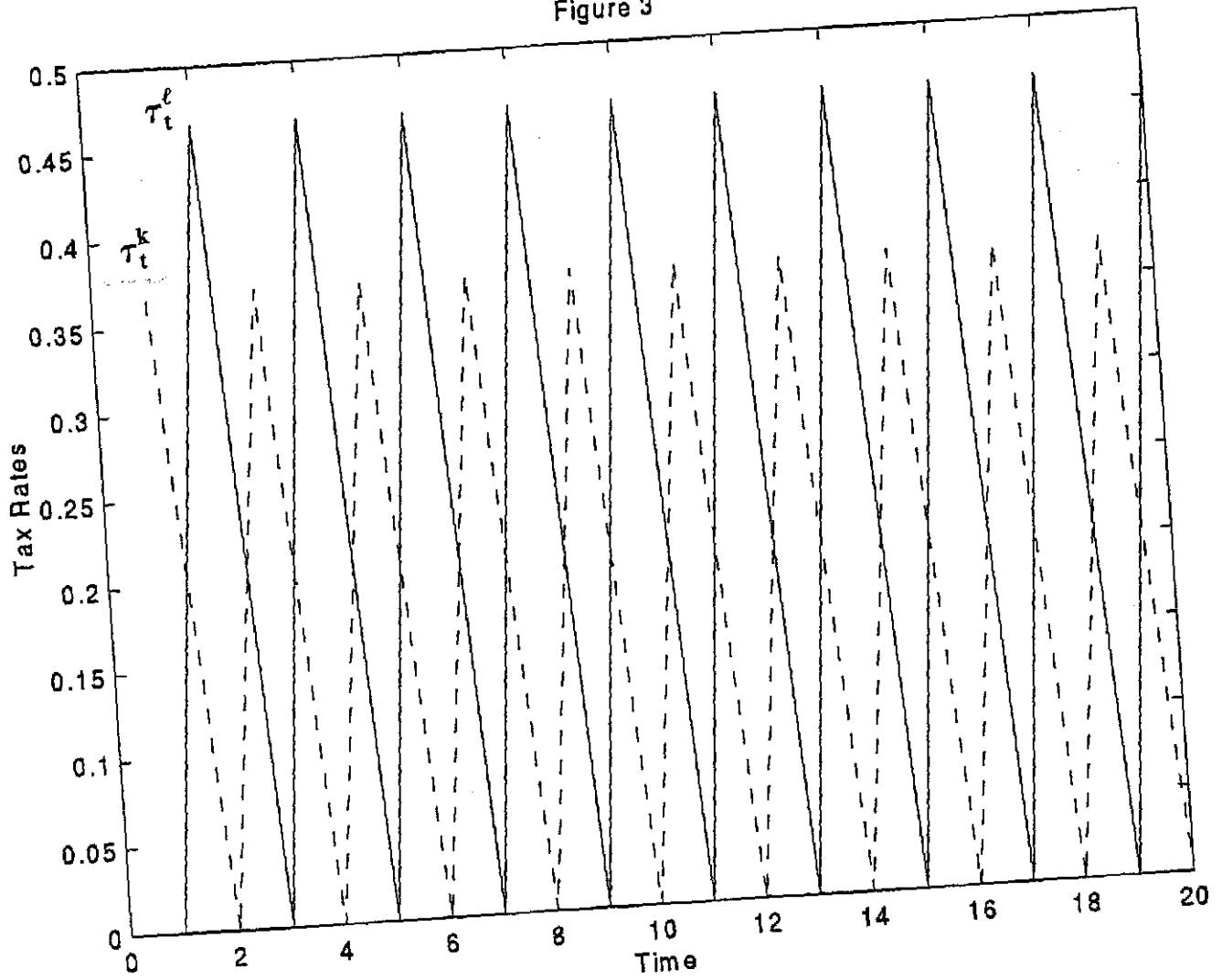


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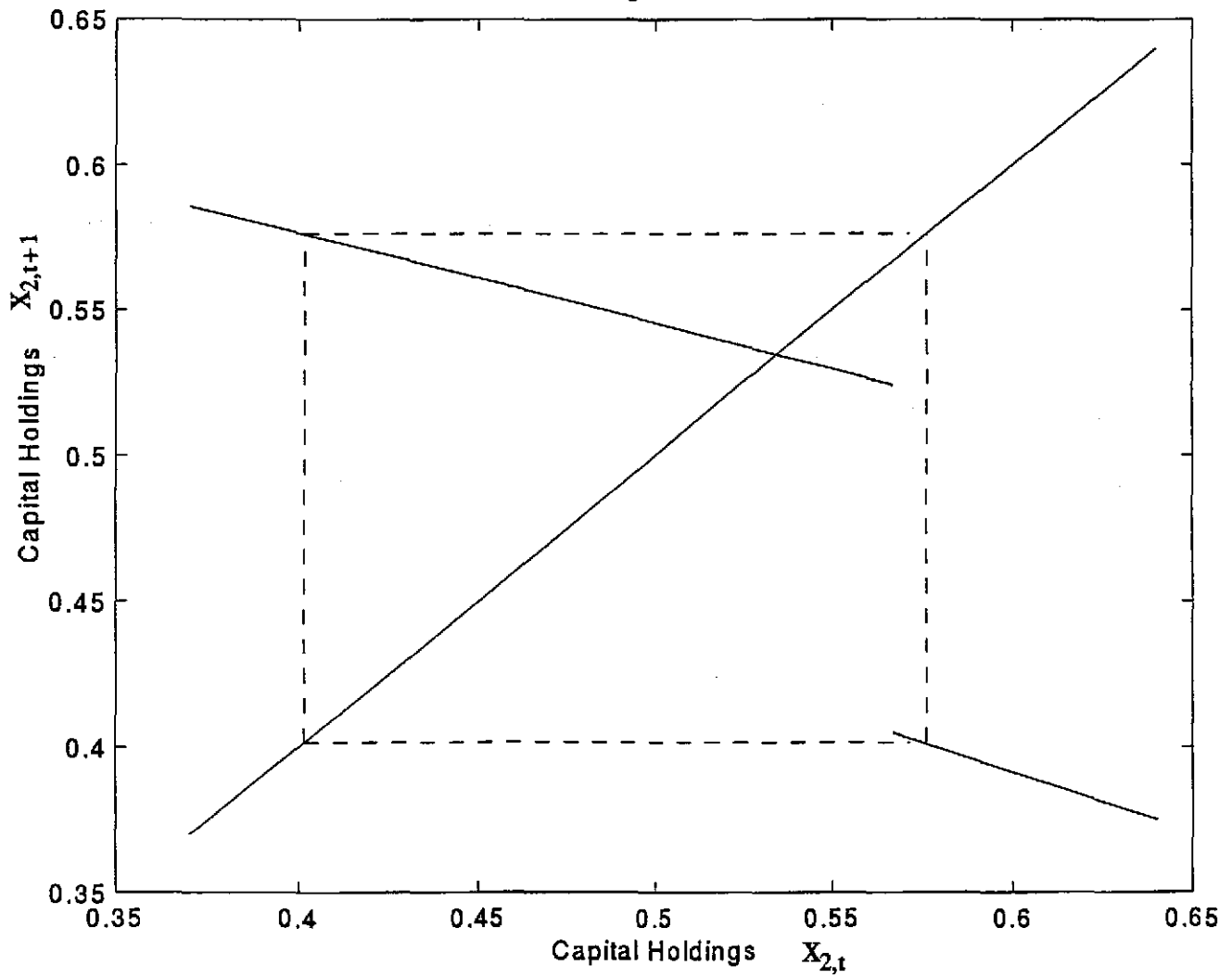


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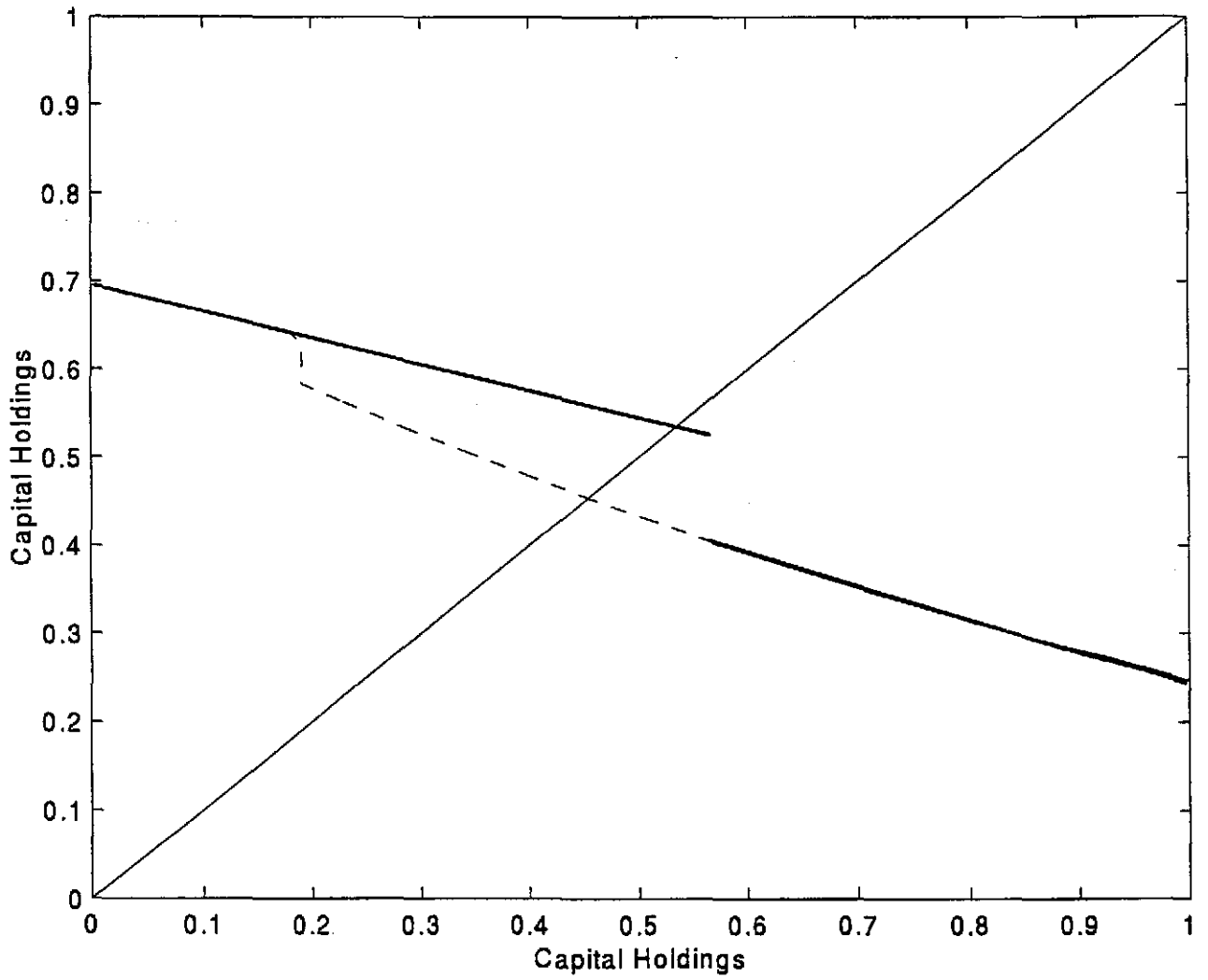


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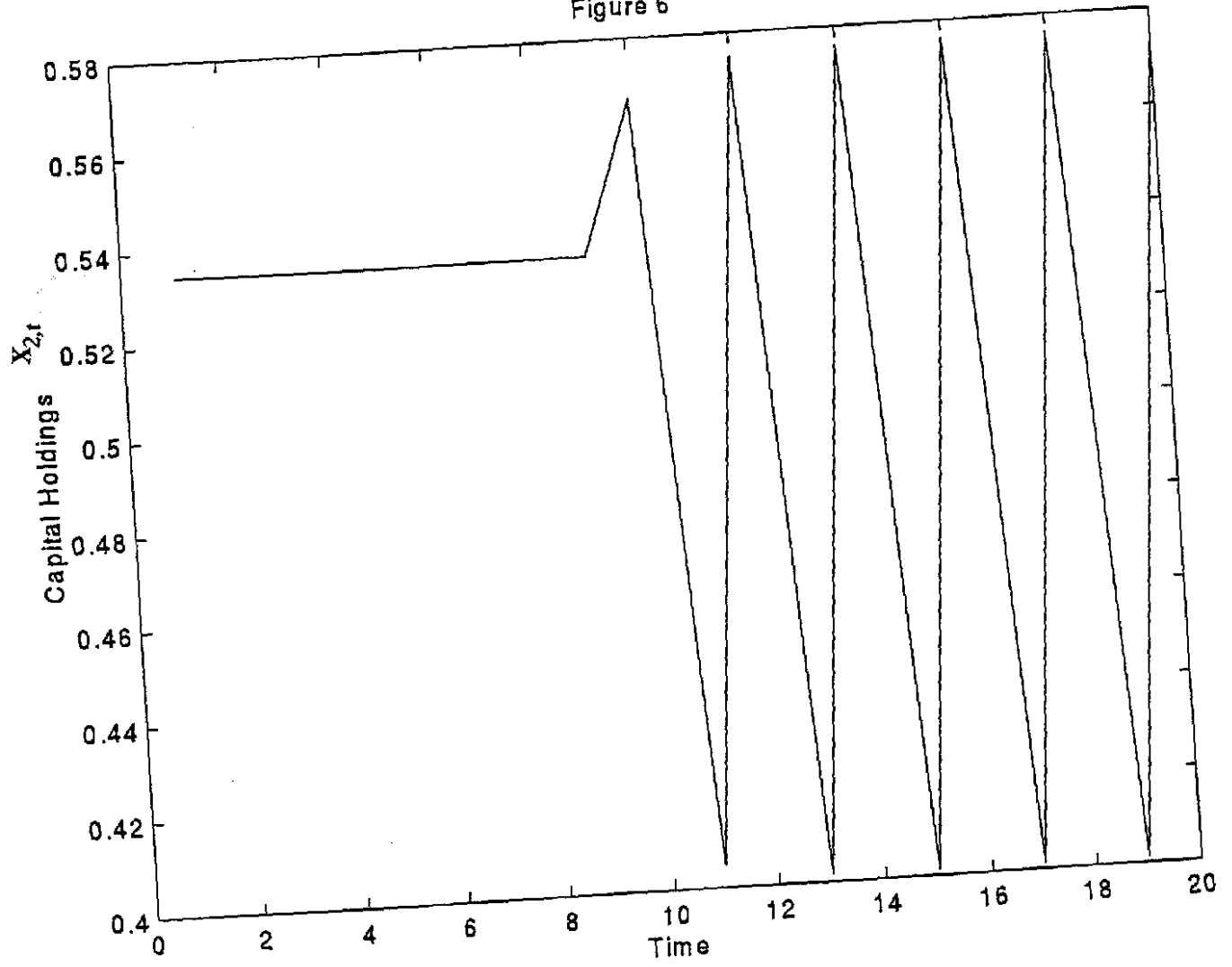


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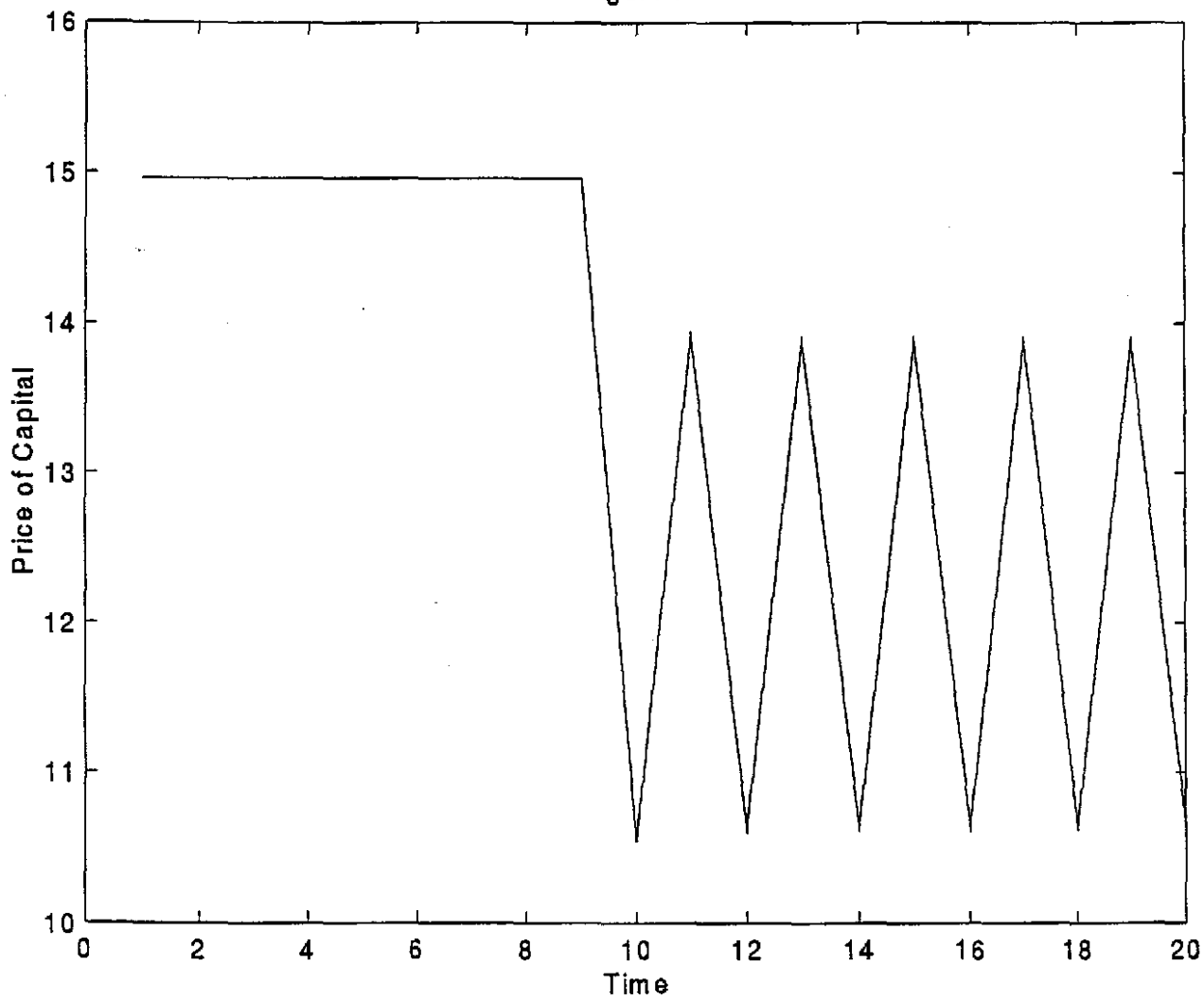


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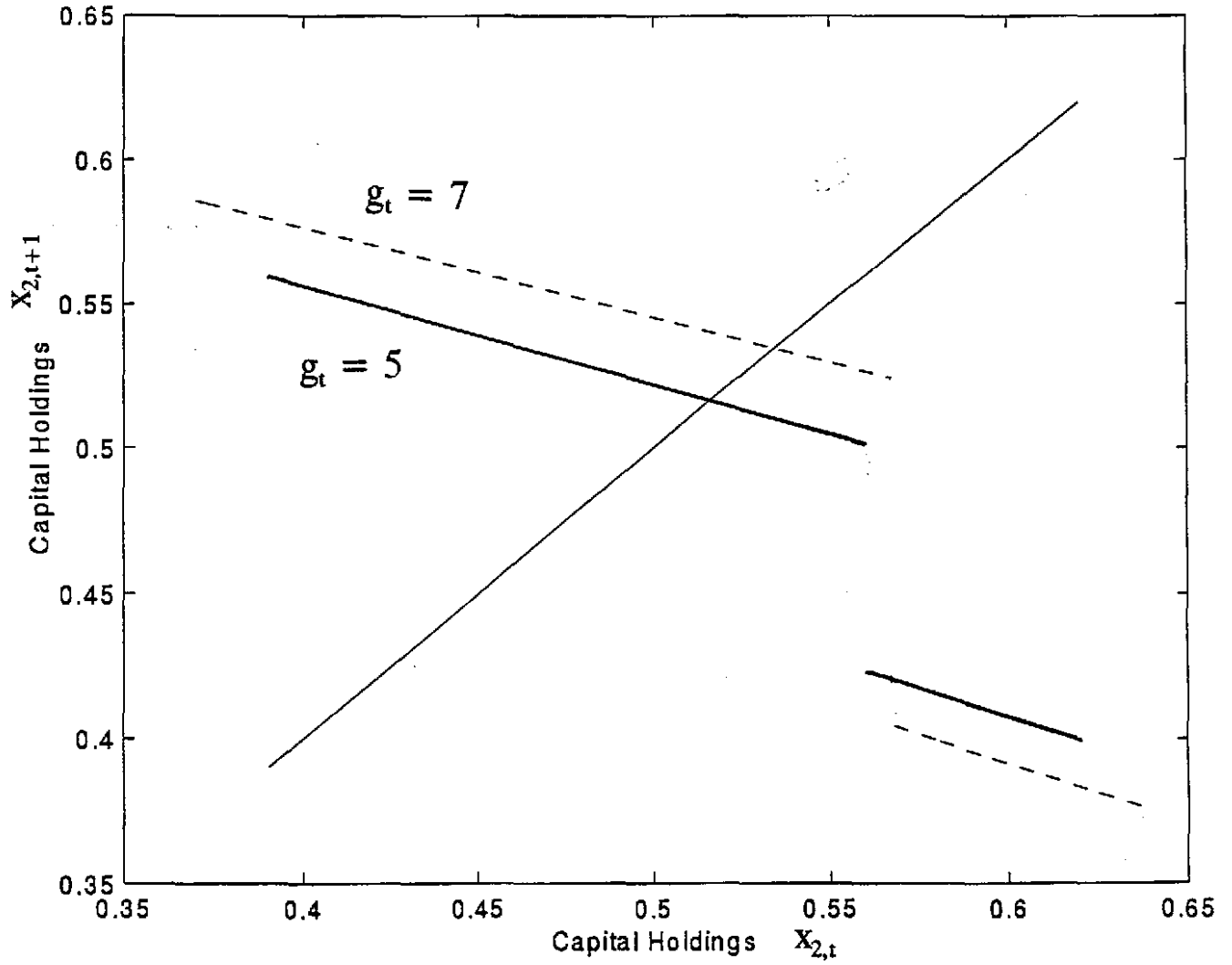


Figure 9

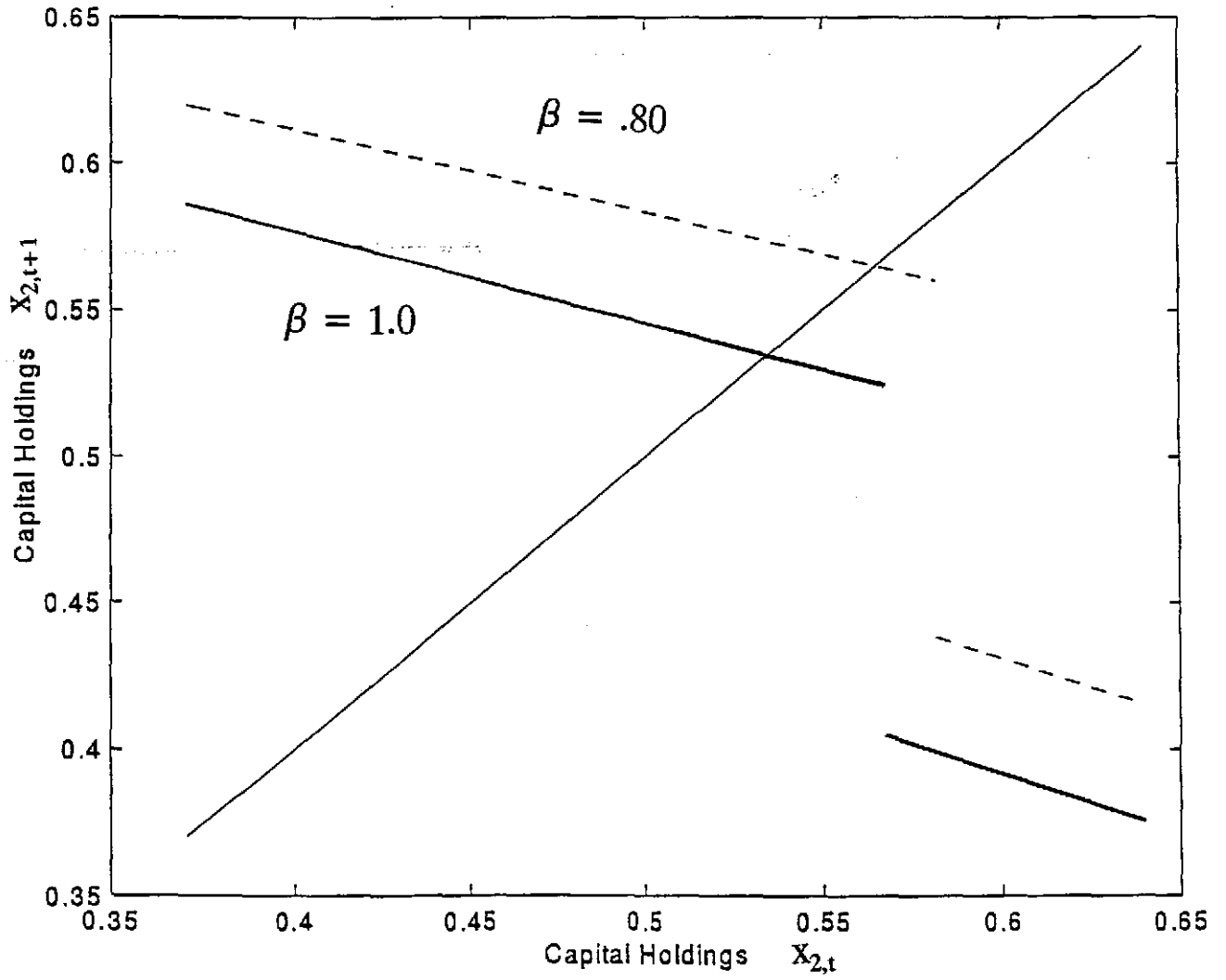
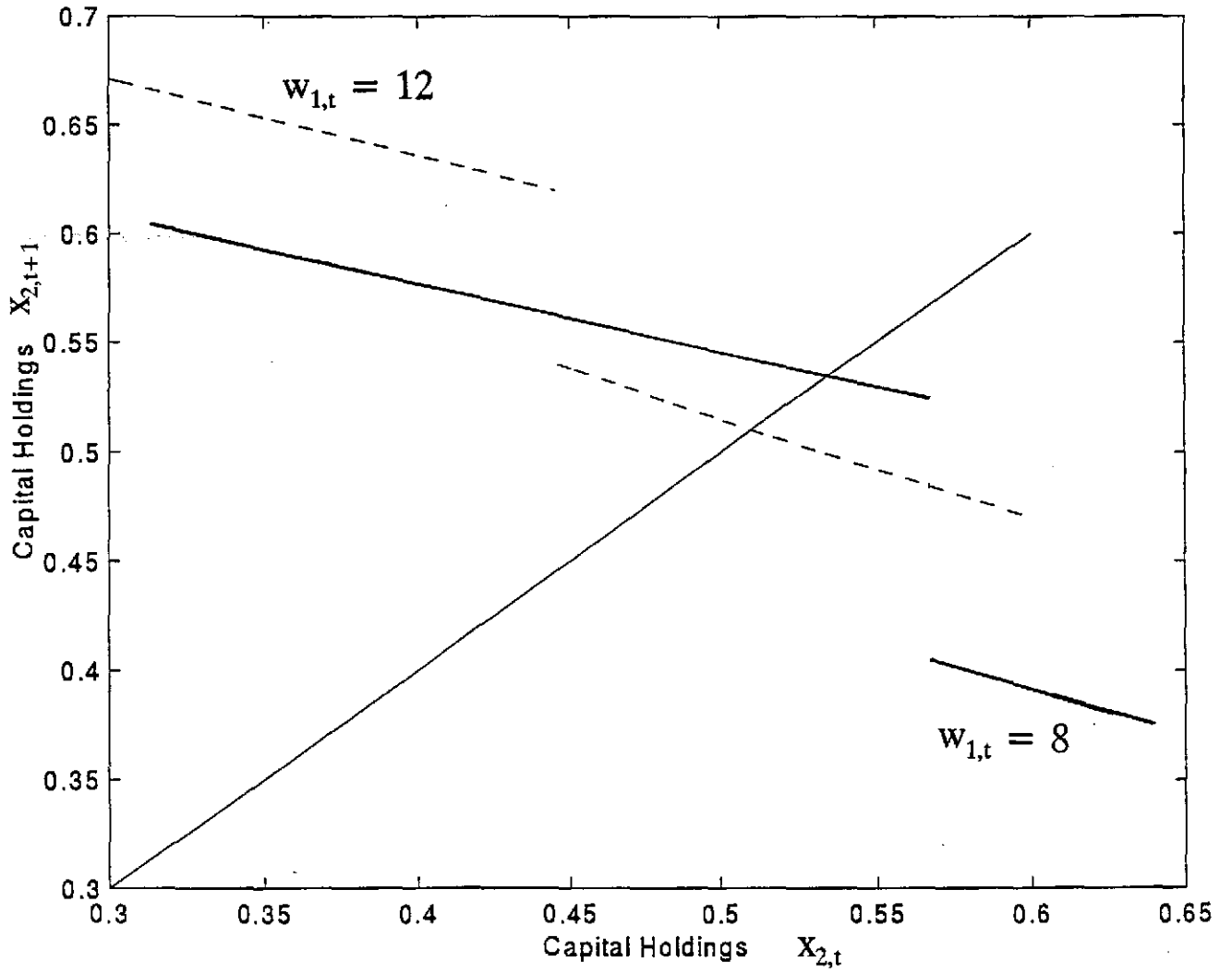


Figure 10



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