HYPERINFLATIONS AND MORAL HAZARD IN THE APPROPRIATION OF SEIGNIORAGE: AN EMPIRICAL IMPLEMENTATION WITH A CALIBRATION APPROACH

Carlos E. Zarazaga

November 1995

RESEARCH DEPARTMENT

WORKING PAPER

95-17

Federal Reserve Bank of Dallas

HYPERINFLATIONS AND MORAL HAZARD IN THE APPROPRIATION OF SEIGNIORAGE: AN EMPIRICAL IMPLEMENTATION WITH A CALIBRATION APPROACH

Carlos E. Zarazaga
Federal Reserve Bank of Dallas
November 1995

The views expressed here are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Dallas or of the Federal Reserve System.

The author acknowledges valuable comments from Toni Braun, Sally Burke, V.V. Chari, Harold Cole, John Duca, Edward Green, Gregory Huffman, Joe Haslag, Finn Kydland, Mariano Tommasi, Neil Wallace, Warren Weber and especially, Satyajit Chatterjee, and Víctor Rios-Rull.

The author benefited as well from discussions with participants at the 1991 Western Economic Association Meetings, the 1992 Annual Congress of the Society for Economic Dynamics and Control, at the 1993 Latin American Meeting of the Econometric Society, at the Winter 1994 Econometric Society Meetings and at seminars at the Federal Reserve Bank of Minneapolis and at the Wharton School of Business.

Special thanks are given to Christopher Sims, without whose support and encouragement this paper would have never been written. All remaining errors are, of course, mine.

This research was partially financed by a Sloan Grant for Open Economy Macroeconomics and the Heller Fellowship for Studies in Public Economics.

ABSTRACT

The paper studies the "megainflationary" experiences of several countries during the 1980s and 1990s. Such experiences have been characterized by the cyclical succession of short-lived extreme inflation rates (100-400% *a month*) amidst protracted periods of much lower, although still quite high by international standards, inflation rates (5-30% *a month*).

The paper links formally this "roller coaster" pattern of the inflation rates to particular institutional arrangements. This is accomplished with a game theoretic model in which several units compete for seigniorage under conditions of imperfect monitoring. In the interpretation of the model the "megainflationary" periods play the same role as the price wars in the models of oligopolistic competition under imperfect monitoring developed in the industrial organization literature. That is, the "megainflations" occur with positive probability as part of a self-enforcing mechanism that restrains the greedy behavior of the units competing for seigniorage, and makes it possible to support relatively protracted periods of much more moderate (although still quite high by international standards) inflation rates than would be possible otherwise.

The empirical relevance of the hypothesis was explored with the aid of a calibration exercise that delivered satisfactory results in terms of the ability of the model to mimic chronic inflation rates and megainflationary rates within the ranges actually observed in "megainflationary" economies.

Interesting enough, the simulations show that the model can reconcile the "conventional wisdom" explanation that high inflation episodes are simply the result of high real fiscal deficits with "Laffer curve" interpretations inspired by evidence indicating that real seigniorage does not increase (and even declines) when the inflation rate reaches its peak. The model is consistent as well with some other evidence, such as cross-country studies suggesting a positive correlation between the average level of inflation and its variability, or such as that often used to favor "crises of confidence" interpretations of the extreme inflation episodes, even if in the model only "fundamentals" are involved. The model also offers insights into the nature of the "lack of credibility" that has haunted several stabilization plans implemented in megainflationary economies in the past, and into the reasons for their ultimate failure, despite a promising start.

"A very slight cause which escapes our notice determines a considerable effect which we cannot fail to see, and then we say that this effect is due to chance"

-Poincaré.

1. INTRODUCTION

The 1980s and 1990s have witnessed one of the most amazing sequences of extreme inflation episodes since the 1920s. Perú has the dubious honor of exhibiting the record monthly inflation for that decade: 396% in August of 1990. Close runners-up were Argentina, with 197% in July of 1989, and Bolivia, with 182% in February of 1985. Brazil, with 81.3% in March of 1990, was another contender in this race. Yugoslavia and Poland have also experienced inflation rates above 50% a month in the 1990s. Zaire had an inflation rate of 250% in October of 1993.

As striking as the intensity of these inflationary explosions is their recurrent pattern. For example, the record monthly inflation rates reported above were succeeded by a rate of 95.5% in March of 1990 for the case of Argentina and preceded by an inflation rate of 114% in September of 1988 and October of 1991 for the cases of Peru and Zaire, respectively (see Figures 1 and 2). In what follows we will refer to these extreme inflation episodes as "megainflations."[1]

There is yet another important regularity: most of these megainflations took place in economies suffering "chronic inflations," that is, in economies that experienced long periods during which the inflation rates fluctuated around *monthly* rates of 5% - 30% which, even if quite high by international standards, were still much lower than those prevailing during the megainflationary episodes.

Up to now, however, "chronic inflations" and recurrent megainflations have been regarded by most researchers as completely different phenomena, caused by inherently different factors or policy regimes and, therefore, economically and theoretically unrelated to one another.[2] Not surprisingly, this has resulted in

---

[1]This term has been used by several authors (see Cardoso (1991)), to differentiate the extreme inflations considered in this paper from the "hyperinflations" that took place between the two World Wars, which Dornbusch, Sturzenegger, and Wolf (1990) and Kiguel and Liviatan (1992a) argue were different phenomena. The model of this paper might help to verify that conjecture.

[2]On the one hand, the megainflations have often been interpreted in the light of Sargent and Wallace (1987) --SW hereafter-- rational expectations model of the interwar hyperinflations (see Dornbusch et al., (1990), and Azariadis (1993, p. 454)). On the other hand, the stubborn, in some views self-sustained inertia of the "chronic inflation" periods has been explained by "adaptive expectations" models (see Dornbusch et al., (1990), Bruno (1990) and, for a summary of those views, Sargent (1982)).

two largely divorced bodies of research, founded many times on behavioral assumptions that are difficult to reconcile.[3]

At the same time, the unusual historical string of recurrent megainflations hitting economies that, up to then, had been characterized only by pathological "chronic inflations" strongly hints at the existence of an intimate, yet uncovered link between "chronic inflations" and "megainflations." The main contribution of this paper is to propose an alternative, novel interpretation of "chronic inflations" and "megainflations" that formally unravels that link.

That is, we provide a model in which the megainflations and their alternation with "chronic inflation" periods are explained with the same rationale and microfoundations. This is accomplished with a game-theoretic dynamic model in which several policymakers with conflicting objectives "compete" for seigniorage in conditions of *imperfect* monitoring. The parallel of this environment with that of the imperfect information repeated games developed in the industrial organization literature (such as those by Porter (1983), Green and Porter (1984) [GP hereafter], and Abreu, Pearce, and Stachetti (1986, 1990) [APS hereafter]) immediately suggests the main conjecture explored in this paper: that the megainflations play the same role as the "price wars" in those models. We will argue in the paper that this interpretation makes it possible to explain several puzzles and regularities often mentioned in the high inflation literature:

First, our model explains why megainflations occur, even if it is perfectly known that *all* agents' welfare will decrease during those episodes and, *simultaneously*, why the inflation rates during the "chronic inflation" periods, even if more subdued, may still remain "stuck" at inefficiently high levels.

Second, due to our proposed link between "chronic-inflations-cum-megainflations" processes and certain informational and institutional features of the environment, the model explains why these phenomena are present in some economies and not in others, even if the presence of multiple policymakers with conflicting objectives is eventually a problem common to all economies.[4]

Third, the model rationalizes the long-standing claim that revenues from the

---

[3]Cagan (1956) himself was inadvertently caught in this dilemma between mutually inconsistent behavioral assumptions. Although his model and arguments were built on the assumption of "adaptive expectations," he had to appeal to "forward-looking" expectations of currency reform (page 55) when trying to explain why his econometric estimations could not account for the recovery of real money balances in the last months of the hyperinflations that he studied.

[4]That is, in contrast with Heymann, Navajas, and Warnes (1991), Aizenman (1992) and Mondino, Sturzenegger, and Tommasi (1992), the presence of several policymakers with conflicting objectives is not sufficient for our results.

inflationary tax have declined at extreme inflation rates, even if our specification does not produce (in steady state) a "Laffer curve" for seigniorage.[5]

Fourth, the model is consistent with cross-country studies (Logue and Willett (1976), Foster (1978), and Bléjer (1979)) that report a positive correlation between the average level of inflation and its variability.

Fifth, the model explains why the evidence might suggest "crisis of confidence" (or "sunspots") interpretations of extreme inflation episodes, even if only "fundamentals" are involved.

Sixth, the model can explain the root of the "lack of credibility" that has haunted, after a promising start, several stabilization plans implemented in the past in "chronic-inflation-cum-megainflation" economies.

To our knowledge, the model presented in this paper is the first general equilibrium rational expectations model to accommodate *simultaneously* and within the same theoretical framework all of the puzzles and regularities mentioned above *even if* optimal (conditional on the structure of information) strategies are allowed.[6]

There is also an important difference between the methodological approach of this paper and that of previous related studies. In this paper, the policymakers' payoffs are not defined "ad-hoc," as has been the common practice in previous studies of inflation with a game-theoretic approach, but are derived explicitly from the optimization problem faced by the typical constituent represented by each policymaker. Because with this approach real money balances (a state variable) appear explicitly in the policymakers' payoff functions, the merely repeated game of GP is transformed into a truly dynamic one, a departure that raised technical issues in the formulation and solution of the model that were absent in GP and that are of some independent interest, as is our particular implementation of the dynamic programming approach developed by APS. It is important to emphasize that

---

[5]Such a curve has played a critical role in alternative interpretations of high inflation experiences, such as those inspired by SW (see footnote 22.)

[6]Mondino et al. (1992) obtain cyclical fluctuations of inflation, but their results disappear when trigger strategies are allowed. In addition, inflation in their model is "too cyclical:" it is always falling or rising and it never settles into the long spells of "chronic inflation" that it is one of our goals to explain. A similar problem affects Marcet and Sargent (1989), who introduce learning in a non-rational expectations version of SW. Much more successful is a modification of Marcet-Sargent by Marcet and Nicolini (1995). Their model, however, is still within the non-rational expectations paradigm and relies on the presence of a Laffer curve and on the exogenous imposition of unanticipated anti-inflation programs, none of which are necessary for our results.

we followed our methodology not for the sake of technical virtuosity, but because such methodology imposes more discipline on the theory and its predictions than the standard approach, a feature that enhances the scope for empirical refutability of the model.[7] In fact, because of this approach, we were able to take some modest steps towards an empirical assessment of the model. Our numerical experiments will suggest that the model can potentially replicate the predicted "roller coaster" pattern of the inflation rates within the ranges actually observed in some historical experiences. This should be regarded as an important feature of our approach because it is difficult to deliver inflation rates of the intensity observed in the data as an efficient outcome in models constructed following more conventional paradigms.[8]

The rest of the paper is organized as follows: Section 2 briefly summarizes some empirical regularities of "megainflationary" economies. Section 3 formally lays out the model. Section 4 solves the model for different equilibrium concepts and presents a heuristic interpretation of "chronic-inflation-cum-megainflations" in light of a particular class of equilibria of the model. Section 5 explores the main qualitative and quantitative properties of the equilibria of the model with the aid of a calibration exercise. Section 6 concludes.

2. EMPIRICAL EVIDENCE

2.1 - Empirical Evidence: The Time Series

The most important regularities observed in the behavior of time series in economies that have been subject to "megainflations" are:

---

[7] As Baxter (1988) has pointed out: "... *if the objective function the policymaker is maximizing is that of society or a representative agent, then one is not free to specify behavioral equations for private agents, such as an expenditure function, separately from the policymaker's dynamic maximization problem*...(otherwise) *there exists the suspicion that, given an arbitrary empirical observation, there exists a set of policymakers' objective functions and a game that will "rationalize" this observation*...(in which case the claim) *that you can get anything you want from these models seems warranted.*"

[8] In fact, economists, social scientists, and politicians have been befuddled by the inability to explain megainflations within the paradigm of a benevolent and monolithic policymaker maximizing some welfare function. "Populist" government explanations contending that such intense inflation rates resulted from subsidies attempting to capture electoral votes must confront the fact that extreme inflation episodes seem to have been very unpopular, since almost all of them have ended up with the incumbents being ousted from office (either by popular vote or by less democratic methods). Given the widespread practice of indexation in these economies, it is doubtful that such extreme inflation rates succeeded in reducing the real value of government debt. Finally, the megainflations have been associated invariably with dramatic falls in output, so it would be difficult to explain them within the "inflation-unemployment" trade-off tradition (and thus, by imperfect information models such as that by Canzoneri, 1985. See subsection 5.2)

i) The already reported pattern of short-lived and intense inflationary explosions (50% *a month* and above) amid relatively longer periods of lower but still high by international standards inflation rates fluctuating around 5%-30% *a month* (see Figures 1 and 2).

ii) The normalized, unconditional variance (coefficient of variation) of the inflation rates is much higher than the international average (e.g., the coefficient of variation for Argentina is four times higher than that for the United States), suggesting that the inflationary processes under study are qualitatively different from a mere "blown-up" version of those in low inflation countries.

iii) Megainflations are usually followed by a "restoration of confidence" period along which real money balances recover at the pace of progressively lower inflation expectations. However, this process stalls once inflation reaches its historical "chronic" level, as if economic agents had suddenly become skeptical about the possibility of further permanent reductions of the inflation rate. The nature and sources of this sudden "lack of credibility" in stabilization programs that inspired considerable confidence in the early stages of their implementation remain largely unknown, despite the research effort on this important economic policy subject.[9]

iv) The marginal revenues from the inflationary tax seem to have been negative during the megainflationary outbursts. Usually, this has been interpreted as indicating the presence of a "Laffer curve" for seigniorage.[10]

v) Granger causality tests have found a double causality pattern, with inflation Granger causing money and money Granger causing inflation.[11] This suggests the presence of a "feedback" mechanism from inflation to money creation that any successful theory of megainflations should be able to account for.

2.2 - Empirical Evidence: Institutions and Structure of Information

Equally important for the purpose of this paper are several features of the

---

[9] An excellent presentation of these credibility issues can be found in Calvo and Végh (1995). See also Calvo (1986), Dornbusch (1991), Guidotti and Végh (1992), and Kiguel and Liviatan (1992b).

[10] See Easterly, Mauro, and Schmidt-Hebbel (1992) and Phylaktis and Taylor (1993) for the cases of Brazil and Peru, Melnick (1990) for the case of Argentina, and Edwards for the cases of Chile and Zaire. Section 5 will provide, however, a note of caution on how to interpret those results and why they seem to contradict Eckstein and Leiderman (1992) and Zarazaga (1994), who found no conclusive evidence in favor of the Laffer curve hypothesis.

[11] For the case of Peru, see Pizarro Ríos (1993). For some of the other countries, this pattern is suggested by the results reported in Table 8 of Dornbusch et al. (1990).

institutions and the budgetary process that are common to the economies that have suffered megainflations:[12]

i) A ramshackle tax-structure and a poorly functioning tax-collection system.

ii) The inability to market government debt in significant volume. This has the implication that the government cannot smooth transitory disturbances to its cash-flow deficit, but must monetize them as they come.[13]

iii) No systematic and comprehensive budget process to allocate planned expenditures and to control the actual ones. Instead, the government budget is constantly revised without parliamentary legitimization, or validated only ex-post by a mere "rubber-stamping" process, amid the pressures of various political groups attempting to capture a larger share of the de facto appropriations.

iv) Both a consequence and a cause of iii, the public sector accounting system is very fuzzy. The data are poor, unreliable, or simply nonexistent. More often than not it is impossible to determine how government appropriations have been apportioned between the federal and local governments and among and within different government services, agencies, and publicly owned industries.[14]

This last piece of evidence, which will play a crucial role in our imperfect monitoring assumption, is forcefully and aptly summarized by the following description of the situation of the Bolivian fiscal data at the time of that country's megainflation in 1985:

> *Surprisingly, it is difficult even four years in retrospect, to uncover precisely the causes for this jump in money creation...The problem with nailing down a culprit lies with the disarray of Bolivian fiscal data during this period. The following kind of problems inhibit a clear assessment of the fiscal situation...*(Jeffrey Sachs, "The Bolivian Hyperinflation and Stabilization," Working Paper 2073, National Bureau of Economic Research, 1986).

A more detailed documentation and discussion of the evidence just summarized in this Section can be found in Appendix A of Zarazaga (1993). In any case, what is

---

[12]Part of this summary has been borrowed from Leijonhufvud (1991).

[13]In fact, in most "megainflationary" economies the megainflations occurred when the public debt was already in arrears and the government was therefore unable to borrow.

[14]As explained in Appendix A of Zarazaga (1993), or in more detail in Bléjer and Cheasty (1993), the problem goes beyond the already serious one of lack of formal statistics, because typically these economies have large public sectors that make it more difficult to detect accounting "schemes" that hide transfers or subsidies in accounts in which they cannot be easily identified as such, least quantified.

important about this evidence is that it reveals that the economies that have experienced "chronic-inflation-cum-megainflations" have shared monetary and fiscal institutions, as well as a structure of information with respect to the economic operations of the public sector, that have made it difficult, if not impossible, to establish with certainty how the private resources appropriated by the government (through, for example, the inflationary tax) have been apportioned between different uses, jurisdictions, and constituencies.

3. THE MODEL

3.1.1 - Institutions and Monetary and Fiscal Policies

In this section, we lay out an abstract representation of the economies described in the previous section. We will capture the presence of multiple policymakers discussed in 2.2.iii with an "islands economy."

There are N islands. Each island has a political agent or administrator with the political power to create money to finance the provision of an "island-specific" good enjoyed only by the representative consumer of the island he represents. We will index the N islands' political agents as policymaker 1, policymaker 2, etc.

The rate of expansion of the money supply induced by the policymaker of island i at period t will be denoted δ_t^i , $\delta_t^i \in [0,\infty)$, where i = 1,...,N. Adding up the δ's across islands gives $\sum_{i=1}^{N} \delta_t^i$, the rate of expansion of the money supply applied to finance "island-specific" goods, which will be referred to as "subsidy induced rate of expansion" of the money supply (SIRE hereafter). This SIRE will be endogenously determined as a result of the maximization problem solved by the islands' political agents to be presented in Section 4.

Although the islands are "separated" with respect to the "island-specific" good, they share a common currency, and a common market for all the transactions in private goods and assets. The operation of those markets requires the support of several political and economic institutions to which we'll refer somewhat metaphorically as "agencies of the confederation of islands." The expenditures of those agencies, which will be referred to in what follows as government consumption and denoted g_t, does not enter into the utility function of the representative consumer of any island.[15]

It will be assumed that the agencies of the confederation finance their activities entirely with money creation. Furthermore, it will be assumed that the

---

[15]This assumption is not essential. A process $\{g_t\}$ such as the one described in footnote 32 could be introduced into the utility function without altering the qualitative nature of our results.

rate of expansion of the money supply jointly induced by those agencies in the process of financing their activities can be represented by the single stochastic process $\{\xi_t\}$, with distribution function F_t, density $f_t(.)$, and support $[\underline{\xi}, \bar{\xi}]$, $\underline{\xi} \geq 0$.[16] More heuristically, the confederation agencies behave in the aggregate *as if* there were an N+1-th policymaker drawing a rate of growth of the money supply ξ_t from a density $f_t(.)$ each period. This will be the other source of expansion of the money supply in the model and will be referred to as the "government consumption induced rate of expansion" of the money supply (GCIRE hereafter), to differentiate it from the SIRE described earlier.

Summarizing, the overall rate of growth of the nominal money supply, θ_t, will be given by:

$$\theta_t \equiv \sum_{i=1}^{N} \delta_t^i + \xi_t \qquad \textbf{[1]}$$

where ξ_t is a realization of the exogenously given process $\{\xi_t\}$ and the δ_t^i's will be endogenously determined in the way to be described in Section 4.[17]

For the specification of the model presented in this paper, the δ_t^i's will be non-stochastic variables,[18] thoughtfully chosen by policymakers 1 through N so as to maximize the welfare of the islands they represent. Therefore, the only source of randomness in **[1]** will be the "impartial" N+1-th policymaker,[19] who mechanically expands the money supply each period at the stochastic rate ξ_t drawn from the distribution f_t. In keeping the parallel with GP, this policymaker will introduce

---

[16]The distribution must have a lower bound (namely, 0%) because in the model there are no instruments (such as open market or discount window operations) with which to contract the money supply. While an upper bound may not seem to emerge as naturally, without such a restriction it would eventually be possible to find some $\{\xi_t\}$ process capable of reproducing any given inflationary path. The interesting exercise is that of generating extreme inflation rates out of a model in which the process $\{\xi_t\}$ is not allowed to be freely specified. This is somewhat in the spirit of the real business cycle theory, in which the stochastic properties of the Solow residuals are not freely specified to explain cyclical fluctuations.

[17]The above formulation implicitly assumes that the Central Bank behaves passively, that is, that it acts mostly as an automatic teller, issuing as much fiat money as requested by the different fiscal authorities (namely, the confederation agencies and the islands). This assumption guarantees that our results do not depend on exogenous restrictions on the degree of independence of the Central Bank from the islands.

[18]We will be ruling out, therefore, mixed strategies.

[19]This policymaker is impartial in the sense that the money he creates goes to finance government consumption that, unlike spending in some "island-specific" good, does not enter into the utility function of the representative consumer of any particular island.

a noise in the money supply that, with the structure of information described in the next subsection, will play the same role as the shock to the demand in GP.

Notice that the stochastic component ξ_t in [1] is analogous to the stochastic process assumed for the whole money supply in Lucas (1972).[20] As in that model, there is no claim here that this stochastic scheme for financing the confederation agencies' spending is "optimal" in any sense.[21] Rather, the objective of this paper is to show how, in the presence of an informational friction, a particular policy can deliver outcomes (cyclical megainflations in our case) very different from those conventional wisdom would have envisioned.[22]

3.1.2 - Structure of Information

The critical assumption will be that the economic agents of our model economy can observe the overall rate of growth of the money supply, θ_t,[23] but not which part of that rate of growth has been due to the SIRE and which part to the GCIRE. This seems to be a natural assumption in light of the evidence presented in subsection 2.2.iv.

To be more precise, consider the following identity relating consumption by the agencies of the confederation and level of provision of "island-specific" goods to total seigniorage and rate of growth of the money supply:

$$G_t \equiv \eta_i g_t^i + \sum_{j \neq i}^{N} \eta_j g_t^j + g_t \sum_{i=1}^{N} \eta_i \equiv \left(\delta_t^i + \sum_{j \neq i}^{N} \delta_t^j + \xi_t \right) \frac{M_{t-1}}{P_t} \equiv \frac{\theta_t}{1 + \theta_t} \frac{M_t}{P_t} \qquad [2]$$

where G_t is the total real resources captured by the government sector (islands' political agents and confederation agencies), η_i is the number of households in island i, g_t^i denotes island i per household real consumption of "island-specific" good i at time t, g_t is the per capita amount of real resources used up by all the confederation agencies (government consumption), M_t is the total money supply at period t, P_t is the price of the private good, and the rest of the symbols are as

---

[20]The use of money supply shocks such as ξ_t is standard as well in the literature studying the real effects of monetary policies (see, for example, Barro (1977, 1978), Chari, Christiano, and Eichenbaum (1994), Christiano and Eichenbaum (1992), and Leeper and Gordon (1994)).

[21]Although see footnote 56, when discussing the calibration of the model.

[22]This is in the same spirit of Sargent and Wallace (1987) who showed how the apparently innocuous rule of financing a constant level of government with the inflationary tax, in combination with rational expectations and a nonlinearity (a "Laffer" curve for seigniorage), could drive the economy to an "inefficient" hyperinflationary trap, in the sense that in SW's model there is a lower inflation rate that would have produced the same amount of seigniorage in real terms.

[23]Here our model departs from the above mentioned study by Lucas.

defined before. At time t, the representative consumer of island i, i = 1, ..., N; knows δ^i_t, θ_t, M_t, P_t, g^i_t, and the distribution of the process $\{\xi_t\}$, but not the particular realization ξ_t.

More precisely, rewrite [1] as

$$\theta_t - \delta^i_t = \sum_{j\neq i}^{N} \delta^j_t + \xi_t \qquad [3]$$

Although at time t the representative consumer of island i knows θ_t and δ^i_t, the unobservability of ξ_t will not allow him to verify directly how the "residual" growth of money supply, that is, the right hand side of [3], is apportioned between the GCIRE originated in the confederation agencies and the SIRE originated in the other islands. That is, the political agent of island i (and the consumers he represents) cannot perfectly monitor the other islands' political agents' actions.[24]

An heuristic interpretation of this assumption is that policymakers 1 through N (the political agents of the islands) can "mimic" the identity of one or more of the confederation agencies.[25] That is, the political agent of a particular island could expand the money supply to finance a higher than agreed (in a sense that will become clear later) level of provision of his "island-specific" good and attempt to deceive the political agents of the other islands into believing that the corresponding expansion was necessary to finance the operation of the agencies of the confederation.[26]

It is now time to discuss the behavior of the representative consumer populating each island, the welfare of which is precisely what the political agent of each island will try to maximize.

---

[24]Notice that, in contrast with Ruge-Murcia (1995), we do allow agents to observe G_t, the overall government real expenditures.

[25]For example, financial institutions could obtain a "hidden" subsidy by successfully inducing the Central Bank to overvalue assets used as collateral in discount window operations (this seems to have been a common practice among the several state-owned banks present in Brazil and Argentina.) Likewise, government owned utilities could artificially increase the compensation of their workers beyond labor productivity gains by diverting funds that should have been used for maintenance or depreciation allowances. The insufficiency of the collateral or depreciation funds would be very difficult to detect, or even to prove. For more examples, see the references in footnote 14.

[26]Notice that this setup implies that in our model, in contrast with Aizenman (1993), information about the true causes of the expansion of the money supply observed in any period cannot be credibly revealed with any lag.

3.1.3 - Preferences

The N islands are populated by infinitely lived households.

The preferences of the representative agent of each island are assumed to be represented by the utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \ln m_t^i + (1-\gamma) \ln c_t^i + \mathbf{W}(g_t^i) \right] \qquad \textbf{[4]}$$

where the superscript i denotes the representative household of island i; the subscript t indexes time period, m represents real money balances; c, real consumption of the private good purchased in the market by each household; and g^i, real consumption of an "island-specific" good, enjoyed only by the members of island i, not bought in the market but supplied to them directly in the manner explained below.[27]

The function **W**, characterizing the preferences for the "island-specific" good, will be assumed to be twice continuously differentiable, strictly increasing and strictly concave in its argument.

3.1.4 - Endowments

Each household j of island i is endowed with y_j^i units of the private good. Households in a particular island are alike; therefore, the subindex j will be dropped hereafter. The time path of this endowment will be assumed to be deterministic across time as well as across households.

For simplicity it is assumed that each island has a constant share λ^i of the total endowment at any point in time. Aggregate endowment is normalized to 1.

3.1.5 - Production and Technology

This is an exchange economy, so total output equals total endowments.

The technologies transforming the private goods into government consumption and "island-specific" goods are linear and deterministic. For simplicity, it will be assumed that this transformation takes place on a one-to-one basis.

3.2 - Consumer's Problem

Each consumer faces the problem of maximizing **[4]**, with respect to $\{m_t^i\}$ and $\{c^i\}$, subject to:

---

[27]Money was introduced in the utility function for pragmatic and empirical reasons. On the one hand, the introduction of money in the utility function along with the particular functional form used here greatly simplified the computation of the equilibria of the model. On the other hand, this specification has been tested and found empirically relevant in econometric estimations of the money demand in high inflation countries (see Eckstein and Leiderman (1992) and Zarazaga (1994)). A seemingly different alternative, a Cash-in-Advance (CIA) constraint, is not only a special case of MUF but also it does not seem to be able to capture important features of the data (see Hodrick, Kocherlakota, and Lucas (1991)).

$$c_t^i + m_t^i = y_t^i + m_{t-1}^i \frac{P_{t-1}}{P_t} \qquad [5]$$

Note that in this formulation, each household takes the level of provision of "island-specific" goods as given.

In solving this problem, we assume that households know at t all variables dated t and earlier that are not private information of other agents. Thus, the agents observe θ_t at the beginning of the period, before committing to any portfolio or consumption allocation.[28]

The corresponding necessary and sufficient conditions for utility maximization generate the following consumer's decision rules:

$$c_t^i = y_t^i + \frac{M_{t-1}^i}{P_{t-1}} \frac{P_{t-1}}{P_t} - \frac{M_t^i}{P_t} \qquad [6]$$

$$\frac{\gamma}{1-\gamma} \frac{c_t^i}{m_t^i} + \beta\, \mathbb{E}_t \frac{c_t^i}{c_{t+1}^i} \frac{P_t}{P_{t+1}} = 1 \qquad [7]$$

Since households are alike, except maybe for a time invariant constant of proportionality in their endowments, aggregating **[7]** over households[29] allows us to derive the following first order stochastic difference equation for the price level:

$$\frac{\gamma}{1-\gamma} \frac{1}{M_t^d} + \beta\, \mathbb{E}_t \frac{1}{P_{t+1}\; c_{t+1}} = \frac{1}{P_t\, c_t} \qquad [8]$$

where M_t^d is the amount of nominal balances demanded in the aggregate and c_t is aggregate real consumption.

Substituting in **[8]** the money market equilibrium condition $M_t = M_t^d$, the stochastic price level sequence $\{P_t\}_0^\infty$ can be characterized by the linear stochastic difference equation:

---

[28]This timing is not essential at a conceptual level but makes it possible to solve explicitly for the price level and greatly simplifies the computation of the model.

[29]This aggregation is possible because, as proved in Proposition 1 in Appendix A, the overall rate of growth of the money supply θ_t, which is publicly observed, will be a *sufficient statistic* for the money supply process when the stochastic process $\{\xi_t\}$ satisfies some conditions. That is, private information will be irrelevant in computing mathematical expectations.

$$\frac{\gamma}{1-\gamma}\frac{1}{M_t} = \mathbb{E}_t(1 - \beta L^{-1})\frac{1}{P_t c_t}$$

where L^{-1} is the forward shift operator and $\mathbb{E}_t$ denotes the mathematical expectations conditional on the information set at time t.

After substituting [6] aggregated over households in the above expression and some algebra, this equation has the solution:[30]

$$P_t = \frac{M_t}{y_t}\left[\frac{1-\gamma}{\gamma}\frac{1}{\mathbb{E}_t \sum_{h=0}^{\infty} \beta^h \prod_{k=1}^{h}\left[\frac{1}{1+\theta_{t+k}}\right]} + \frac{\theta_t}{1+\theta_t}\right] \qquad [9]$$

where $\prod$ is the multiplication operator. We will also adopt throughout this paper the convention that in the above expression $\prod_{k=1}^{h}\left[\frac{1}{1+\theta_{t+k}}\right] \equiv 1$ for $h < k$.

It is easy to verify that [9] implicitly defines a demand for real money balances that in steady state does not deliver a "Laffer curve" for seigniorage. This is important because the model will systematically generate observations whose casual inspection will suggest the presence of such a curve.

The optimal choices of $\{c_t^i\}_0^\infty$, $\{m_t^i\}_0^\infty$ by the representative consumer of island i will be given by equation [9], along with the equations:

$$c_t^i = \lambda^i\left[y_t - \frac{\theta_t}{1+\theta_t}\frac{M_t}{P_t}\right] \qquad [10]$$

$$\frac{M_t^i}{P_t} = \lambda^i \frac{M_t}{P_t} \qquad [11]$$

where the first equation was derived from the consumer's budget constraint and the second one from the money market equilibrium condition, and where λ^i is the fraction of aggregate endowments on island i.

Upon replacing [9] into [10] and [11], it can be verified that consumption and real money balances of the representative agent of island i will be solely a function of $\{\theta_t\}$.[31]

Given that the inflationary tax will be the only source of government revenues in the model economy, the characterization of the real allocations will be

---

[30] Only solutions without speculative bubbles are considered.

[31] Again, this result obtains from the conditions in Proposition 1.

completed by the identities:

$$g_t \sum_{i=1}^{N} \eta_i \equiv \frac{\xi_t}{1+\theta_t} \frac{M_t}{P_t}$$

$$\eta_i\, g_t^i \equiv \frac{\delta_t^i}{1+\theta_t} \frac{M_t}{P_t} \qquad [12]$$

Notice that the real resources, or seigniorage, captured both by the islands and by the confederation agencies, is endogenously determined in the model, depending on the particular realization ξ_t (and, therefore of M_t/P_t).[32]

In interpreting the above equations, it is important to keep in mind that the representative consumer of each island takes g_t^i as given: it is his political agent who, through the choice of δ_t^i's, influences the level of provision of this "island-specific" good. A natural question, then, is how $\{\delta_t^i\}$ is determined. This is the subject of the next subsection.

3.3 - Political Agent's Payoff

The single-period payoff function of the political agent of island i is obtained by replacing **[9]** into **[10]**, **[11]**, and **[12]** and by replacing the result of these substitutions in turn into **[4]**. It is easy to verify that the single-period payoff derived in this way, denoted by Π_{t+1}^i, will depend on θ_{t+1}, δ_{t+1}^i, and I_{t+1}^i, where I^i represents all the variables contained in the information set at t+1, that is[33]

$$\Pi_{t+1}^i\left[\delta_{t+1}^i,\ \theta_{t+1}\left(\delta_{t+1}^i, \sum_{j\neq i}\delta_{t+1}^i + \xi_{t+1}\right),\ \mathbb{E}\left[\Theta^{t+2} \,\middle|\, I_{t+1}^i\right]\right] \qquad [13]$$

where

---

[32]An alternative modeling strategy would be to assume an exogenously given $\{g_t\}$ and that the technology transforming the endowment into g_t is subject to shocks unobservable to the islands. Such alternative specification is left for future research. It is not clear, however, that such an assumption will be a better representation of reality: real government investment (which would seem closely related to the type of activities that in the abstraction of the model would be performed by the confederation agencies) typically declines during the extreme inflation periods. In addition, the exogeneity of $\{g_t\}$ would preclude the derivation of the closed form solution for the political agents' payoffs function **[13]**, which in turn would make the analysis of the model with numerical methods much more burdensome and much less transparent, if at all feasible. In any case, we conjecture that the qualitative results under this alternative assumption will not differ much from the ones obtained in this paper.

[33]We refer to the period t+1 rather than to the period t single-period payoff for notational convenience, since due to the timing of decisions, only payoffs from t+1 will be involved in the political agents' maximization problems discussed below.

$$\mathbb{E}\left[\Theta^{t+2} \middle| I^{i}_{t+1}\right] \equiv \mathbb{E}_{t+1} \sum_{h=0}^{\infty} \beta^{h} \prod_{k=1}^{h} \left[\frac{1}{1+\theta_{t+1+k}} \right] \qquad [14]$$

where the convention indicated in equation [9] applies when $h < k$.[34]

The objective of each political agent will be to maximize the expected present value of his payoff function by the appropriate choice of the δ^{i}_{t}'s. Notice the fundamental tension built into [13] by comparing it with the representative agent's utility function from which it was derived (expression [4]): the political agent has incentives to increase δ^{i}_{t}, since this (ceteris paribus) increases the provision of the "island-specific" good which, through **W**, increases utility as well. But this gain will be at the expense of reducing the provision of private goods (private consumption and real money balances) to all agents, including those of its own island. The political agent then faces the non-trivial maximization problem of expanding δ^{i}_{t} only up to the point at which the marginal gains in utility from an increased provision of the "island-specific" good outweigh the losses from reduced consumption and money balances of the members of his own island. In doing so, the political agent of island i does not internalize the costs (reduced consumption of both private and "island-specific" good and lower money balances) that the increase in δ^{i}_{t} imposes on the other islands. The failure to internalize such costs and not time inconsistency (as in Canzoneri (1985)) is the source of the inefficiency of the equilibria that will be presented below.

4. SOLUTION OF THE MODEL

We limit attention to time-consistent, sequentially rational Nash equilibria, that is, to self-enforcing equilibria in which decisions are made on a period-by-period basis.[35]

The presence of an endogenous state variable (real money balances) in this paper considerably complicates the implementation of GP-type trigger strategies. The problem is that in the imperfect information environment of our model, island i's private information on δ^{i}_{t} might be useful in forecasting the future rate of growth of the money supply. In that case, the mathematical expectations in [7] could potentially differ across islands, invalidating the aggregation procedure through

---

[34] Again, the observation in footnote 29 explains the omission of the superscript i, that is, any reference to private information, in the right hand side of [14].

[35] This restriction seems only natural in the environments studied in the model, characterized by the lack of institutions capable of enforcing any commitments (including the Constitution.) Heuristically, this means that each island will be represented by a succession of political agents, none of which is bound by any commitments eventually made by his predecessors.

which the payoff function [13] was obtained.[36]

We get around the complication above by requiring that the stochastic process $\{\xi_t\}$ be uncorrelated with variables that are not publicly observable and by limiting attention to equilibria in the class of public equilibria, that is, to equilibria in which each political agent ignores his private information (e.g., his island's past δ^i's) in choosing his actions.[37]

A formal discussion of these ideas, a rigorous mathematical representation and definition of our equilibrium concept, as well as their implementation with a dynamic programming approach in the spirit of APS involves some heavy notation and technical concepts that might prove burdensome for the uninterested reader. For that reason, we defer presentation of that material to Appendix B. In this section we focus, instead, in providing a more intuitive characterization and heuristic interpretation of the equilibria consider in the paper.

To that end, it will be convenient to keep in mind the following assumptions:

1) Timing of decisions: the political agents of all islands and the confederation agencies make their request for the issuance of new money to the Central Bank (which formally handles them) *simultaneously* at the beginning of each period,[38] *before* the representative consumers make their consumption and portfolio allocation decisions.

2) Strategy Space: $\delta^i_t \in [0, \infty)$ for all t and i = 1, 2, ..., N.

3) Distribution of ξ_t: For analytical tractability and computational simplicity, in this paper $\{\xi_t\}$ is restricted to the class of identically and independently distributed processes.[39]

4) Definition: The public history of the rate of the growth of the money supply up to time t, that is, the finite sequence $\{\theta_k\}_{k=0}^{t}$, will be denoted h_t. Denote by

---

[36]For a concrete example illustrating these potential difficulties, see the discussion following Proposition 1 in Appendix A.

[37]It is under those conditions that we derive the result in Proposition 1, Appendix A, that θ_t is a sufficient statistic for the mathematical expectations [14]. Intuitively, this result emerges because in our set up the conditions above make it possible to abstract from distributional issues. In particular, the choice of δ^h_t for all h = 1, 2, ..., N and t does not affect island's h share λ^h in total private consumption and total real money balances.

[38]The structure of information described in subsection 3.1.2 is not invalidated by this procedure: recall that the islands' political agents can mimic confederation agencies, so the Central Bank is still unable to tell whether a particular request should be classified as GCIRE or as SIRE.

[39]Note that this is just a subset of the wider class of processes admitted in Proposition 1.

Σ_t the set of all possible public histories h_t.

4.1 - Trigger Strategy Equilibria

We will start by discussing sequentially rational trigger strategy stationary Nash equilibria because the "simpler" myopic stationary Nash equilibria (the equivalent, in this dynamic setting, of the "single-shot" Nash equilibria of repeated games) are just a particular case of the latter.

In this paper, the continuation payoffs of the trigger strategy equilibria will be generated by strategies of the Green-Porter type. Those strategies take a very simple form: the agents "cooperate" and moderate their demands for inflationary financing of the "island-specific" good until the realization of θ_t exceeds a trigger value $\tilde{\theta}$. After that, they revert to "myopic" behavior for Q periods, after which cooperation is resumed until $\theta_{t*} > \tilde{\theta}$ for $t* > t + Q$ and so on. In what follows we'll refer to these trigger strategy equilibria as QNE.

In general, GP trigger strategies generate a multiplicity of equilibria, since there are no restrictions (other than $\tilde{\theta} > 0$ and Q a positive natural number) on the choice of a pair $(\tilde{\theta}, Q)$ and each of such pairs could generate a different set of QNE, an issue addressed later in Section 5.

As explained in detail in Appendix B, the trigger strategies above induce a stationary partition $\sum$ of the set of all possible public histories h_t into Q+2 subsets or "states of nature" as follows:

$$h_{t+1} \in \begin{cases} H_{CL} & \text{if } \left(h_t \in H_{CL} \text{ or } h_t \in H_{N_Q}\right) \text{ and } \theta_{t+1} \leq \tilde{\theta} \\ H_{CH} & \text{if } \left(h_t \in H_{CL} \text{ or } h_t \in H_{N_Q}\right) \text{ and } \theta_{t+1} > \tilde{\theta} \\ H_{N_1} & \text{if } h_t \in H_{CH} \\ H_{N_m} & \text{if } h_t \in H_{N_{m-1}} \qquad m = 2, 3, \dots, Q \end{cases} \qquad [15]$$

where H_r, r = CL, CH, N_1, ..., N_Q, is a "state of nature" or subset of $\sum$.

The initial history h_0 is arbitrarily assigned the H_{CL} state, i.e., $h_0 \in H_{CL}$.

The state H_{CL} is that in which δ^i_{t+1} is at the "cooperative" level (δ^i_{t+1} was chosen at t and the history h_t up to t had been one of cooperation) and the low realization of the signal θ_{t+1} instructs the political agents to cooperate again next period. The state H_{CH} is that in which δ_{t+1} is still set at the cooperative level,[40] but in which a high realization ξ_{t+1} "pushes" the signal θ_{t+1} beyond the

[40]Recall that δ^i_{t+1} was chosen in the previous period, when history H_{CL} instructed

trigger threshold $\tilde{\theta}$ and therefore instructs the players to revert to "non-cooperative" behavior, or to a punishment phase starting at t+2 and for the following Q periods.

Notice the incentives to cooperate in the state H_{CL} built into the strategies above: in that state the political agent of island i will refrain from increasing δ^i_{t+1} above the "cooperative" level, say δ^i_C, for fear that this will trigger a punishment phase with higher probability, since

$$\delta^i_{t+1} > \delta^i_C \Longrightarrow \Pr\left(\left(\delta^i_{t+1} + \sum_{j\neq i} \delta^j_C + \xi_{t+1}\right) > \tilde{\theta}\right) > \Pr\left(\left(\delta^i_C + \sum_{j\neq i} \delta^j_C + \xi_{t+1}\right) > \tilde{\theta}\right).$$

Once cooperation is broken, however (that is, once history H_{CH} is realized), the transition between states is deterministic for the next Q periods: there is only one possible successor state to states H_{CH} and H_{N_m}, m = 1, 2,..., Q-1, regardless of a particular island's political agent action at t+1, and this is reflected in the simpler structure of [15] for those states.

At this point, it is important to remark a difference between our QNE and those of GP. The trigger strategy equilibria of GP prescribe just two actions: one for the cooperative phase and another one for all of the punishment phase. Our QNE instead will prescribe Q+1 actions: a cooperative one (in states H_{CL} and H_{CH}) and Q different ones (one for each stage) in the punishment phase. The state variable, as shown rigorously in Appendix B, is responsible for this result. Intuitively, the reason is that the single period payoff [13] is not, in contrast with GP, an invariant function of current actions along the different stages of the punishment phase: since the conditional expectations in [13] are not necessarily the same in those stages, the optimal actions are not the same either.

Summarizing, we conjecture that the optimal N(Q+1) actions, that is, the optimal δ^i_{t+1} of island's i political agent at each phase of the game, will be given by a function of the previous history h_t or state of nature as follows:

$$\delta^i_{t+1} = \begin{cases} \delta^i_C & \text{if } h_t \in H_{CL} \text{ or } h_t \in H_{N_Q} \\ \delta^i_{N_1} & \text{if } h_t \in H_{CH} \\ \delta^i_{N_m} & \text{if } h_t \in H_{N_{m-1}} \qquad m = 2, 3, \ldots, Q \end{cases} \qquad [16]$$

In Appendix A we show how an equilibrium vector [16] can be found as a solution

---

the players to keep cooperating in the next period.

to a dynamic programming problem in N(Q+1) equations and N(Q+1) unknowns.[41]

Notice that the equivalent for this dynamic game of the standard single-shot Nash equilibrium for static games can be found as a particular case of QNE by making the transition function [15] flat, that is, by not distinguishing between histories of "cooperation" and histories of "non-cooperation." In that case, the same δ^i_{t+1} will be chosen period after period, independently of past histories or states. We will refer to those equilibria as Myopic Nash Equilibria (MNE hereafter) and denote by δ^i_{MNE} the same repeated action characterizing each of those equilibria.

We are now in position of providing a heuristic interpretation of the equilibria of the model and in particular, of how the QNE can potentially capture the "chronic-inflation-cum-megainflations" phenomenon that motivated this paper.

4.2 - Interpretation of the Equilibria of the Model

The interpretation of the MNE is rather straightforward, since it is analogous to the infinite repetition of a one-shot Nash equilibrium in repeated games. In these equilibria, the islands' political agents simply limit themselves to myopically maximize next period's expected utility. That is, because they do not fully internalize the costs of higher inflation (see subsection 3.3), they set δ^i_{t+1} at the inefficiently high level δ_{MNE} period after period. As a result of the permanently high expansion of the money supply, these equilibria exhibit, therefore, permanently high inflation.

The main hypothesis of this paper, however, is captured by the dynamics of inflation in the QNE. In the QNE, the political agents of the islands cooperate by setting δ^i_{t+1} at the relatively low level δ^i_C as long as the overall rate of growth of the money supply in the current period, θ_t, has not exceeded the critical threshold $\tilde{\theta}$, that is, as long as history is described by the event H_{CL} defined in [15]. As a result, inflation remains relatively subdued (although, by the reasons we discuss later, potentially above the fully cooperative one) during relatively protracted periods of "chronic inflation." However, the political agents revert for Q periods to a non-cooperative mode if the history up to t has been one of cooperation (H_{CL}) but $\theta_t > \tilde{\theta}$, that is, if the public history H_{CH} defined in [15] occurs. During this reversionary phase, each island's political agents expands the money supply above the cooperative level in an attempt to increase the utility of his constituents by capturing a larger share of the inflationary tax and using it to finance a higher level of provision of the "island-specific" good. As a result,

---

[41]There are Q+2 states but only Q+1 actions because recall that two of the states (H_{CH} and H_{CL}) share the same action δ^i_C.

inflation is higher (eventually, much higher) than during the preceding chronic inflation (or cooperative) period, which is precisely what suggested our interpretation that the occasional megainflationary episodes observed in the data are nothing but a manifestation of the reversionary phase of a QNE.

Note that the reversionary phase can be interpreted as a "spend more" on "island-specific" goods phase, very much in the same way that Green-Porter price wars could be described as a "produce more" phase. Despite the higher level of provision of the "island-specific" good, the representative consumers experience utility losses during this phase, for the same reasons that the higher level of production during GP price wars results in lower profits. In the case of GP, the fall in the price more than offsets the additional revenues from the increased quantity produced by each firm. In the case of this model, the utility losses associated with the lower real money balances more than offset the gains in utility from a higher provision of the island-specific good during the reversionary phase.

It is important to emphasize that although the islands' political agents are "punished" with a lower payoff during the inflationary war, the interpretation of the reversionary phase as a punishment for deviations from cooperation would be inappropriate. Just like the price wars in GP, the inflationary wars in this model do not occur to punish deviations, but to prevent them.[42] Therefore, also like the price wars in GP, the *ex-post* inefficient megainflations are part of an *ex-ante* efficient (relative to the environment) self-enforcing mechanism.

Incidentally, notice that the interpretation of the reversionary phase as a "spend more" in "island-specific" good phase carries some of the flavor of conventional theories of hyperinflation, which maintain that such experiences are simply the result of attempting to finance excessive fiscal deficits. At the same time, however, the model challenges this "conventional wisdom" because the model can potentially deliver QNE in which the lowest fiscal deficits (in real terms)

---

[42]For the reader unfamiliar with GP, the subtle mechanism behind the equilibria described above is analogous to the one at work routinely in baseball. Whenever a pitcher throws a "bean" ball, the batter is awarded a run to first base. We can presume that this penalty deters pitchers from throwing "bean balls" on purpose. But it is not easy to control a ball launched at 90 miles per hour, so pitchers will occasionally hit batters anyway just by accident. But suppressing the penalty on that ground would create perverse incentives for pitchers to throw bean balls on purpose. Analogously, not implementing the punishment in our model on the grounds that θ_t occasionally exceeds $\tilde{\theta}_t$ because of an unusually high ξ_t and not because some political agent deviated from cooperation (i.e. $\delta^i_t > \delta^i_C$) would ignore that it is precisely the effective implementation of the punishment what keeps the political agents from deviating.

are contemporaneous with the highest inflation rates, an outcome that seems to be more in line with the observations that led some researchers to dismiss more conventional explanations of extreme inflations and to entertain instead "Laffer curve" explanations of these phenomena. Section 5 will offer a possible explanation for this paradox.

5. NUMERICAL ANALYSIS OF THE MODEL

This section is devoted to characterize numerically the equilibria of the model.[43]

The main objective of the computational experiments below is to describe the dynamic path of the variables of interest and identify those features of the equilibria of the model that seem to be consistent with our interpretation of the stylized facts presented in Section 2.

Although the main motivation of the computational experiments was a qualitative analysis of the model, we were also interested in a preliminary assessment of the potential empirical relevance of the model. To that end, and to the extent possible, the parameters in the simulations have been pinned down, in the spirit of the calibration tradition, to values suggested by information and estimates contained in other available independent studies.

The quantitative results turned out to be encouraging, but given the limitations encountered in the implementation of the calibration, we regard those results more as a useful guide for future research than as a conclusive confirmation of the hypothesis explored in this paper.

5.1 - Calibration of the parameters of the model

The calibration approach to parameter value selection imposed some quantitative discipline in the computational experiments by reducing the number of free parameters.

Some of our assumptions, however, limited the actual economies that could be used for that purpose. In particular, the assumption that $\{\xi_t\}$ is i.i.d. limited those exercises to economies whose money supply growth process $\{\theta_t\}$ could be characterized as empirically close to an i.i.d process. Brazil was found to meet this requirement and therefore the parameter values were calibrated to the extent possible to the economy of Brazil.[44] Unfortunately, Brazil is precisely one of the

---

[43]The difficulties that usually preclude the derivation of analytical results are present in this model: the objective functions of the policymakers (the islands' political agents in this case) do not inherit the concavity properties of the utility function of the respective representative consumer; there are endogenous state variables; and the strategy space is not bounded.

[44]We reached this conclusion from a series of simple univariate autoregressions

countries for which the inflation data seem to have been more severely tainted by the presence of frequent and somewhat effective price controls during the sample period. This will limit to some extent the ability to compare the inflation rates of the actual and simulated economies.

5.1.1 Parameters of the Utility Function

Several empirical studies were useful in pinning down the parameter values of the utility function **[4]**.

Using stock returns, Reinhart and Végh (1994) found that the quarterly subjective discount factor for Brazil lies most likely in the range 0.984-0.993, which corresponds to a monthly discount factor between 0.996 and 0.998. For the simulations β was set to the average of these two values, that is, $\beta = 0.997$.

We were unable to find empirical studies establishing the value of parameter γ in **[4]** for the particular case of Brazil.[45] As a rough guide, we used the parameter values for Argentina in the study by this author mentioned in footnote 27. The estimates for the logarithmic specification in that study suggested that γ is in the range of .029 to to .036. The parameter γ was set to the average of those two values, that is, $\gamma = 0.032$.

For the utility from the "island-specific" good the following generic functional form was adopted:

$$W(g_t^i) = \zeta \, \frac{\left(g_t^i + \chi\right)^{1-\rho} - 1}{1-\rho}$$

where $\gamma > 0$, $\zeta > 0$, $\rho > 0$, $\chi \geq 0$.[46]

The specification above implies that the intertemporal elasticity of substitution (IES) for the "island-specific" good is not necessarily equal to 1, the IES for private consumption and real money balances implied by the logarithmic specification adopted for those arguments of the utility function **[4]**. The reason for this asymmetry is that, unlike the case for real money balances and private consumption, neither tractability nor empirical considerations dictated the

---

on seasonally adjusted rates of growth of different nominal monetary aggregates. For details on the sample used, see footnote 54.

[45] The representation of preferences for real money balances and private consumption with a logarithmic function was already justified in subsection 3.1.3.

[46] The motivation for the introduction of the parameter χ was to not exogenously force strictly positive δ_t^i's for all parameterizations of the model. In the absence of χ, δ_t^i could never be 0 because in that case $g_t^i = 0$ and the marginal utility from the "island-specific" good would be infinitely negative, an outcome incompatible with utility maximization by the islands' political agents.

assumption of an IES equal to one for the "island-specific" good.

One problem for the calibration exercise, however, is that quantitative studies pinning down the value of ρ will be impossible if our assumption about the structure of information is empirically relevant: in that case the necessary information on δ_t^i (or equivalently, g_t^i) will not be available. Lacking other relevant evidence, we exploited the fact that g_t^i would definitely be classified as a non-tradable good in the study by Ostry and Reinhart (1992), who estimated the IES in a model of consumption with tradable and non-tradable goods. The range for the IES in that study was found to be 0.37-0.43 for Latin American countries. This suggested a range of 2.3 to 2.7 for ρ (the reciprocal of the IES). Consistent with our previous criteria, ρ was set to the average of those two values, that is, $\rho = 2.5$.

The total absence of studies providing direct or indirect evidence on the range of plausible values for the parameters χ and ζ made the task of underpinning them particularly difficult. One possible way to restrict the value of those parameters is to assume that the economy being calibrated would be able to reproduce the inflation rates observed in low inflation countries, such as the OECD countries or United States, if it operated under the same conditions prevailing in those economies, that is, under perfect information and under no financing of government consumption with money creation (that is, with $\bar{\xi} = 0$).

Under these conditions trigger strategies can support efficient outcomes[47] and it is therefore possible to interpret the inflation rates observed in those economies as the solution outcome of a joint utility maximization problem solved by the political agents of the islands when $\bar{\xi} = 0$.[48]

We set $\chi = 0.02$, $\zeta = 0.000255$ and $N = 2$ because for these parameters values[49] the

[47]For the reasons given in subsection 5.2, full cooperation cannot typically be enforced in our imperfect information environment unless $\bar{\xi} = 0$.

[48]This procedure its open to the criticism that in this way the parameter values χ and ζ are not really set independently, but are functions of the other parameters of the model. Notice, however, that the calibration of χ and ζ with this approach will still be independent of the punishment phase parameters $\tilde{\theta}$ and Q and of the parameterization of $\{\xi_t\}$ for high inflation economies.

[49]Lacking any definite evidence about N, the decision to set N = 2 was arbitrary and made on the grounds that this is the minimum number of islands or political agents for which it makes sense to interpret observed inflation rates as outcomes of some "seigniorage game." In any case, this number also seems to adequately capture the institutional feature that in several of the high inflation countries under study typically two organizations, one representing trade unions and another one representing business associations, channel the demands and political pressures of various interest groups. The political clout of these two organizations is apparent in the fact governments of those countries have rarely

joint utility maximization problem specified above is solved by $\delta^1_{FC} = \delta^2_{FC} = 0.013$.[50] This full cooperation (FC) solution implies a steady-state monthly inflation rate of 0.026% or, equivalently, an annual rate of about 3.12%, which is approximately the average annual inflation rate in the period 1960-90 for West Germany. In other words, under the conditions and parameter values above the calibrated economy would be able to generate inflation rates in the same range as a country whose reputation for fighting inflation and not financing its fiscal deficits with money creation would not be subject to much dispute.

5.1.2 Parameterization of the punishment phase

Since in principle there is a continuum of QNE (one or more for each pair ($\tilde{\theta}$, Q), we need to narrow the number of empirically relevant ones by calibrating the probability of reversion and the length of the punishment phase.

Figures 1 and 2 suggest that Argentina and Peru each experienced, roughly, 2 megainflations in the lapse of 96 months.[51] Given the i.i.d. assumption, this frequency suggests a probability of reversion of 2%, or equivalently, a threshold $\tilde{\xi} = 0.1355$ for the value beyond which realization of ξ_t will trigger an "inflationary war."[52]

For the calibration of the punishment phase, we took into account the model's prediction that θ_t should be above average during that phase. A casual inspection of the data over the sample period (see footnote 54) shows M1 growing above average from September 1990 to May 1991. Identifying the first of these months with the state H_{CH} suggests Q = 8.

5.1.3 - Parameterization of the i.i.d. process $\{\xi_t\}$

The assumption that $\xi_t \in [0, \bar{\xi}]$ leads almost naturally to consider the family of beta distributions, denoted B(α1, α2).[53] Accordingly, it is necessary to calibrate

---

implemented "income policies" without having previously reached "prices and wages pacts" with just those two organizations.

[50]We chose the symmetric solution because later on we'll focus our attention on symmetric QNE. For the same token, we set $\eta_i = 1$ for all i.

[51]We preferred not to consider the number of megainflations that occurred in Brazil in a similar period because of the potential distortions in reported inflation mentioned in subsection 5.1.

[52]Given the assumptions on the distribution function, $\tilde{\xi}$ can be inferred by solving the equation $\Pr\left(\xi_t > \tilde{\xi}\right) = 0.02$. This in turn determines, through **[1]**, the observable threshold $\tilde{\theta}$ implicit in any QNE.

[53]This family is defined in the interval [0,1] and possesses the attractive feature of its versatility (a beta density function can be hump-shaped or U-shaped, symmetric or skewed to the right or to the left, etc.)

the "shape" parameters $\alpha 1$ and $\alpha 2$ and, given that we do not require $\bar{\xi} = 1$, also the "scale" parameter $\bar{\xi}$.

The calibration is complicated by the unobservability of ξ_t. Fortunately, this difficulty can be overcome for the shape parameters because, as identity [1] suggests, the "shape" of the unobservable ξ_t is inherited by the observable θ_t and can, therefore, be determined independently of the δ_C^i's, which merely shifts the distribution by the factor $\sum \delta_C^i$. Therefore, we assumed that the empirical counterpart of $\{\theta_t\}$ in Brazil was the monthly rate of growth of nominal M1 between July 1985 and December 1992. The parameters $\alpha 1$ and $\alpha 2$ were computed from this empirical distribution by applying the simple method of moments.[54] The resulting values were $\alpha 1 = 1.703240909$ and $\alpha 2 = 3.253909252$.

The assumption $\underline{\xi} = 0$ might be used as the identifying assumption in calibrating the "scale" parameter $\bar{\xi}$.[55] Unfortunately, the presence of some negative realizations for θ_t in the sample data suggests that in reality $\underline{\xi}$ is likely less than zero and lies, therefore, in a region of the parameter space the model cannot accommodate for the reasons given in footnote 16. Setting $\bar{\xi} = \bar{\theta} - \underline{\theta}$ seems therefore empirically inadvisable in this context, because it would result in an unrealistically large scale parameter $\bar{\xi}$. To eliminate this bias, $\bar{\xi}$ was calibrated instead to a value such that the average monthly rate of growth of the money supply during the cooperation periods of the simulated model matches its counterpart in the data, the average monthly M1 growth during the chronic inflation periods identified in the previous section.

Again, the procedure above is open to the criticism that in calibration exercises the parameters should not be calibrated to the phenomenon being studied. But the megainflations are an essential part of the phenomenon studied in this paper and $\bar{\xi}$ is not being calibrated to match that aspect of the data. In other words, none of the parameters has been chosen (with premeditated malice) to

---

[54]The data were seasonally adjusted with dummies for the months of January and December to take into account the seasonal effects of the Christmas Holidays and the beginning of the peak-tourist season. The period September 1990 - May 1991 was also excluded because if it corresponds to the "punishment phase" as conjectured in 5.1.2, then the θ_t's during this period are not drawn from the same distribution but from one that shifts according to the changes in the equilibrium values of $\delta^i_{N_r}$, $r = 1, 2, \ldots, Q$ during that phase. Also excluded from the sample were three observations that appeared to be outliers: March and December 1986 and April 1987.

[55]From [1], $\bar{\xi} - \underline{\xi} = \bar{\theta} - \underline{\theta}$. Without an identifying assumption on $\underline{\xi}$ (such as $\underline{\xi} = 0$), $\underline{\xi}$ and $\bar{\xi}$ cannot be uniquely determined.

reproduce the observed megainflationary episodes, a fact that should be born in mind when evaluating the ability of the model to replicate them.

We found that with all the other parameter values set as above, $\bar{\xi}$ = 0.174625332 generates the average inflation rate of 15% a month observed in Brazil during the chronic inflation periods in the sample (see footnote 54).

Summarizing, we calibrated the i.i.d process $\{\xi_t\}$ as a beta distribution B(1.703240909, 3.253909252) defined in the interval [0, 0.174625332].[56]

5.2 - Numerical Characterization of the Equilibria of the Model

Table 1 reports the solution vector [16] for the *symmetric* QNE (QSNE hereafter) corresponding to the parameterization of the previous section. The table presents as well the expected value (conditional on the stage of the game) of the main variables of interest corresponding to those equilibria along with their empirical counterparts as identified in the previous section for the case of Brazil.

It is important to mention that the actual inflation rate displayed in Table 1 (and plotted in Figure 3) corresponds to the monthly depreciation of the exchange rate in the black market instead of to the rate of increase in any of the available price indexes (such the Consumer Price Index used in Figure 1.) This criterion was adopted because the black market exchange rate is more likely to be a better proxy of the true inflationary process than price indexes contaminated, as already mentioned in subsection 5.1, by the several forms of price controls that prevailed in Brazil during the sample period used for the calibration.

The main quantitative and qualitative features of the model, as suggested by Table 1, (the respective solution vector is reported in Table 2 are as follows:

1) The numerical simulation of the calibrated economy is consistent with the interpretation of "chronic-inflation-cum-megainflations" offered in subsection 4.2. The computed QSNE display relatively protracted "cooperative phases" during which inflation rates fluctuate around an average rate of 15% a month, suddenly interrupted by megainflationary explosions during which inflation rates jump, on average, to 162% *a month*. For purposes of comparison, a typical realization of the inflation rate in the calibrated QSNE has been plotted along with the actual inflation rates (measured as indicated above) in Figure 3.

The simulated inflation rates are in line with the empirical evidence reported

---

[56]This distribution implies an average GCIRE of 6% a month. This implies that the GCIRE explains 6 percentage points of the average the inflation rate at each stage. Although this induced monthly inflation may seem excessively high, it is still within the range of inflation rates that some authors have found to be "optimal" in a different context (see, for example, Braun (1994)).

in 2.1.i.[57] The average monthly inflation of 15% during the cooperative phase matches the one corresponding to the chronic inflation periods in Brazil, but this of course is a trivial consequence of our calibration procedure. What is really interesting is that the extreme inflation rates (162% a month, on average) of the simulated QSNE are close to those observed in Argentina, Bolivia, Peru, and Zaire. However, such extreme inflation rates are twice higher than those observed in Brazil, the actual economy used for the calibration. By this extreme inflation criteria, the model economy therefore overshoots. But it performs extraordinarily well by the alternative measure of the average monthly inflation rate during the reversionary phase: between the state H_{CH} and state H_{N_8} that average is 43% for the simulated QSNE and 41% for the actual economy. The fact that the simulated inflation rates are on target for the average inflation rate during the reversionary phase suggests that the model is empirically plausible and that its discrepancies with the actual data (overshooting in stage H_{CH} and undershooting along the punishment stages)[58] can potentially be corrected in future versions with a different timing of decisions and/or structure of trigger strategies.[59]

It is important to emphasize once more that the source of the extreme inflation rates delivered by the model is the conflict between different islands (e.g interest groups) and not the tension between the optimal and the time inconsistent plan exploited by Canzoneri (1985). This qualitative difference impinges quantitatively in the model as well. The highest inflation rate in Canzoneri corresponds to the time consistent myopic equilibrium. That solution would

---

[57]Some of the authors mentioned in footnote 6 would dispute this characterization on the grounds that the megainflations appear to have been preceded by a period of progressively accelerating inflation rates when inflation is measured by official price indexes. However, this pattern is much less apparent when inflation is measured by proxies (such as the black market exchange rate) less likely to have been contaminated by the effects of the several forms of price controls implemented in several occasions in the economies under study. (The potential importance of these distortions for the case of Brazil is suggested by the different pattern of the inflation rates when the two alternative measures of inflation in Figure 1 and Figure 2 are used). In any case, whether a true feature of the data or an artifact of price controls, the alleged pattern of rising inflation preceding the megainflationary spikes can potentially be accommodated in future versions of our model by adding to the i.i.d. process $\{\xi_t\}$ a component correlated with past publicly observed variables, such as past inflation rates or rates of growth of the money supply.

[58]These discrepancies must be judged with caution, however, because the inflation rate has not been measured directly but through a proxy.

[59]Alternative trigger strategies are the "two-tail" trigger strategies proposed by APS, in which one threshold triggers the punishment and another one the return to cooperation.

correspond to our MNE with N = 1, which, for the other parameters as before, gives δ_{MNE} = 0.064 and, therefore, a monthly inflation rate of about 12.4% a month, well below the inflation rates of our model economy during the reversionary phase.

There are two other features of the simulated QSNE that are relevant for the discussion that follows in **2)** and **3)** below on the ability of the model to account for the empirical regularities reported in 2.1.iii and 2.1.iv.

One is that the sizable jump of the inflation rate in the "credibility breakdown" period (the state H_{CH}) is explained mostly by a sharp decline in the demand for real money balances (a 53% fall in Table 1) rather than by the comparatively negligible average increase of the money supply in that state (just 8 percentages points above the average θ_t in the cooperative or chronic inflation phase, according to Table 1).

The other relevant feature is that, somewhat unexpectedly, in the simulated QSNE of Table 1 real money balances start recovering from the low levels attained at the state H_{CH} well before the reversionary phase is over. The dynamics of inflation and real money balances just described is intuitively appealing: it suggests that confidence is "lost" in the H_{CH} state and then progressively restored as the punishment approaches its end.[60]

2) The two features mentioned in the last paragraph play a critical role in the ability of the model to explain several puzzles often mentioned in the high inflation literature, such as why megainflationary episodes are often attributed to "crises of confidence" phenomena when, in our interpretation, only fundamentals are involved, why a brief "restoration of confidence" seems to have accompanied the early stages of stabilization programs that typically have followed those megainflations, and why a "lack of credibility" in such programs seems to have appeared later on.

In some interpretations, the megainflations have been the outcome of "self -fulfilling" prophecies or, more rigorously, "sunspot" equilibria.[61] This view has frequently been based on the observation that the megainflationary outbursts were not preceded, in general, by expansions of the money supply substantially above the rate that had been observed in previous periods in which the inflation rate had not jumped so dramatically. Absent a change in "fundamentals" strong enough to

---

[60]Notice that the autocorrelation exhibited by real money balances and inflation during this reversionary mode is endogenously induced by the model out of an i.i.d $\{\xi_t\}$ process. The model prediction of a different autocorrelation pattern in the chronic inflation (none in our model) and in the megainflationary period could be of interest for future empirical studies.

[61]For a typical exposition of that interpretation, see Azariadis (1993, p. 454).

warrant such an "out of proportion" inflationary reaction, the conclusion of "expectationally driven" megainflations seems almost unavoidable. Furthermore, these interpretations argue that it was only after the inflationary outburst that the money supply started to grow considerably faster, and so it was inflation that caused money creation and not the other way around. The evidence on a distinct Granger causality pattern from inflation to money creation reported in Subsection 2.1 gives further support to this interpretation. These same interpretations, however, have usually failed to explain the symmetric "restoration of confidence" that has reportedly been observed after the megainflations, when real money balances recover in a way that cannot be explained by the current inflation rate, as if driven by the expectations of an imminent period of stabilization.

The model presented here captures endogenously and within the same theoretical framework both the "crisis of confidence" and "restoration of confidence" periods without the need for relying on unexplained extrinsic beliefs: only fundamentals are involved. As explained in 1) above, at the "credibility breakdown" state H_{CH} the inflation rate jumps substantially even if the contemporaneous rate of growth of the money supply is only slightly higher than usual. The inflationary outburst is "expectationally driven" indeed, but not due to unexplained extrinsic beliefs, as in the "sunspot" equilibria models, but to "fundamentals": even if only slightly above previous realizations of the rate of growth of the money supply that were inconsequential in the past, the informational content of the signal changes dramatically once it has exceeded the trigger threshold $\tilde{\theta}$. Thus, the model unravels the "slight cause" that escaped our notice and triggered the dramatic chain of events that we could not fail to see, to borrow from Poincaré's quotation at the beginning of the paper.

As is apparent from Table 1, the high rates of growth of the money supply of the reversionary phase do not prevent real money balances from recovering steadily along this phase because expectations of future inflation gradually decline as the resumption of "cooperation" gets closer and closer.[62] Once "cooperation" is reestablished, however, the probability of a megainflation in the immediately following period is no longer zero, and inflation expectations get "stuck" at the "chronic inflation" level. Alternatively, the "restoration of confidence" stalls in what could appear as a sudden "lack of credibility" in the ability of the government to permanently reduce the inflation rate below that "chronic inflation"

[62]Flood and Garber (1980) and Drazen and Helpman (1990) have been able to deliver a similar result by exogenously introducing an expected switch to a lower inflation regime at a more or less arbitrary stage of the hyperinflationary period. In this paper, the rationale for the regime switch is entirely endogenous.

level. Our interpretation, therefore, accounts for the evidence summarized in 2.1.iii.

3) The simulations confirm that the model can generate observations that, with a naive reading, could be interpreted as suggesting a Laffer curve for seigniorage even if such "Laffer curve" is not generated by the demand for real money balances in the usual sense, as discussed in Subsection 3.2. This feature of the model is consistent with the evidence provided in Subsection 2.1.iv.

Figure 4 plots the inflation rates for the typical realization of the simulated QSNE represented in Figure 3 against the real seigniorage corresponding to that realization. A cursory interpretation of this plot could indeed suggest the presence of a Laffer curve. This Laffer-curve effect, however, as indicated in subsection 4.2, is an illusion created by the "shift" of the demand for money at different stages of the game, rather than a genuine outcome generated by movements along a static "hump shaped" seigniorage function.[63]

An heuristic insight of this Laffer-curve paradox can be gained with the aid of Figure 5, in which a generic seigniorage function (and the underlying demand function for real money balances) for each state of the game is labeled with the corresponding expectations.

As Figure 5 suggests, it's not only the fall in real money balances in state H_{CH} described in 1) above what accounts for this Laffer-curve illusion: according to Table 1 the dynamics of the QSNE induces a negative correlation between inflation and seigniorage in the immediately following stages of the punishment phase. Notice that total seigniorage is, on average, higher during the punishment phase than during the cooperative phase, validating our interpretation in subsection 4.2 of that phase as a "spend more" on "island-specific" goods phase.

4) The simulated QSNE replicates the dynamic pattern of acceleration of the inflation rates preceding accelerations in the rate of growth of the money supply observed in the data (see Table 1.) This feature of the model may account for the Granger causality pattern from inflation to money creation reported in 2.1.iv.[64]

5) The great variability of the inflation rates between the "cooperative phase"

---

[63]Note also that in the light of SW's model the plot in Figure 4 could misleadingly suggest that observed high inflations are the outcome of an inflationary path converging to the "slippery" side of the Laffer curve, when such interpretation is unwarranted in the context of our model. The empirical evidence reported in footnote 10 should perhaps be re-examined in light of this insight.

[64]Of course, this is a result that many rational expectations model with anticipated switches of regime will be able to deliver, but it is interesting that it arises in the context of a self-enforcing mechanism the was not designed to explain this aspect of the data.

(or "chronic inflation" period) and the "punishment phase" can account for the positive correlation between average inflation and variability of inflation in the cross-country studies mentioned in the introduction. Foster's (1978) remark that this relationship could be a spurious one attributed to inter-country differences "in economic and governmental structure" is particularly relevant, because the model suggests not only that that is indeed the case but it also identifies the source of the institutional differences eventually responsible for the reported correlation.

6) As the results reported above are based on simulating a particular equilibrium, a natural question is whether they hold for the other QSNE as well. For the reasons given in footnote 43, an answer to this question is limited by the ability to characterize the model only with numerical methods. Several other numerical experiments were carried out with that purpose. Limitations of space prevents us from presenting here more than a brief summary of the relevant findings. The reader is referred to Zarazaga (1993, Appendix B) for more details.

All the computed QSNE in the experiments shared the qualitative properties described above. At the quantitative level, in general the average megainflationary spike increased and the average chronic inflation decreased monotonically with Q (the length of the punishment phase.)

For some parameterizations it was not possible to find any equilibrium. However, when we found MSNE, we also found QSNE for any length of Q. The opposite is not true: there were parameterizations for which we could find QSNE but not MSNE. For the latter cases it was not possible, therefore, to compute QSNE with irreversible punishment (that is, with $Q = \infty$). Interesting enough, one such parameterization was the one corresponding to the simulated QSNE for the calibrated economy reported in Table 1. In other words, the alternation of chronic inflation periods with occasional megainflations seems to be a necessary feature of *all* the equilibria of the calibrated economy.[65] An implication of this result is that, unlike the result in Porter (1983) would suggest, the optimal QSNE for the calibrated economy could not prescribe a permanent reversion to a MSNE.[66]

---

[65]These cases caution against the use of "bounded rationality" arguments favoring the selection of the "simpler" MSNE over the QSNE: application of this selection criteria to those cases would have the unappealing consequence of eliminating the only equilibria that exist.

[66]The results above are unfamiliar by conventional repeated game standards: the non-existence of single-shot Nash equilibria (the analogue of the MSNE of our dynamic setting) would have prevented the existence of GP trigger strategy equilibria (the analogue of our QSNE). The presence of a state variable, real money balances, alters that result by transforming our game in a dynamic, rather

Some parameterizations produced QSNE with extremely low probability of reversion. A typical realization of such equilibria will display a long chronic inflation period, uninterrupted by megainflationary reversions that remain virtually dormant in the equilibrium path. The implications of this result is that the model can eventually be used to study economies seemingly characterized only by "chronic inflation," with the interpretation that such economies are in reality "chronic-inflation-cum-megainflations" economies in which megainflations have not occurred *yet*.[67]

To close this point, it is important to remark that for those parameterizations for which we were able to compute equilibria, we found just one MSNE, as well as just one QSNE for each pair $(\bar{\theta}, Q)$. That the continuum of QSNE can eventually be indexed by the pair $(\bar{\theta}, Q)$ is of little consolation, since there still remains the issue of which of those equilibria will be selected and how the political agents and the atomistic consumers they represent come to coordinate in a particular one. Rather than providing a lengthy list of refinement equilibrium concepts that could be applied to our model, we limit ourselves to point out that the problem of multiplicity of equilibria is not peculiar to our game theoretic approach but one that bedevils almost all the models in the high inflation literature, including those with a rational expectations competitive equilibrium approach such as SW.

7) Finally, several numerical experiments not reported here suggest, as anticipated in the introduction, that the more moderate average inflation rate of the chronic inflation periods is still inefficiently high in *all* QSNE. In other words, it seems that no QSNE can support, during the cooperative phase, an average

---

than repeated one. Intuitively, the dynamics of the model is such that expectations of high inflation in the future increase the benefits of increasing δ^i_t today. The return to a low inflation stage prescribed by QSNE with temporary reversions moderates inflation expectations and keeps them "anchored" to a region in which QSNE exist. Equilibria that do not prescribe a switch to a low inflation stage in the future remove that anchor, sending the economy to a divergent path of ever rising inflation expectations and δ^i_t's, thus explaining our failure to find MSNE and permanent reversion QSNE.

[67]This interpretation is valid only to the extent that those economies share with the ones that actually experienced megainflations the institutional and informational arrangements captured in this model and critical to its results, as described in subsection 2.2. Since that seems to be the case (see Zarazaga, 1993, Appendix A) for several East European countries and independent and autonomous republics that were part of the now disintegrated Soviet Union (such as Lithuania, Russia, and Ukraine), the model can be potentially useful to study the recent inflationary experience of those countries. Although admittedly this interpretation might stretch too much the implications of the model, it is consistent with the phenomenon known as "the peso problem" (see Lizondo (1983), Obstfeld (1986), and Hodrick (1987, pp.24 and 155)).

inflation rate as low as the fully cooperative (efficient) one that it is possible to obtain under perfect monitoring.

The conjecture above, is based on the inefficiency results established by Porter (1983) and Radner, Myerson, and Maskin (1986), who showed that trigger strategies could not enforce perfect cooperation (fully efficient outcomes) in their imperfect monitoring games.[68] Although our game is dynamic and not merely repeated as theirs, it shares with those models the feature critical for their results: the distribution of the signal (θ_t in our model) depends on the sum of the actions of the individual players ($\sum_i \delta_t^i$ in our model), and therefore deviations by different players are undistinguishable from one another. Unfortunately, a rigorous analytical verification of this conjecture is problematic in this model for the reasons given in footnote 43.

6. CONCLUSION

The last two decades have witnessed inflationary phenomena with a very peculiar feature: extraordinarily intense and recurrent short-lived inflation rates (100% - 400% *a month*) have alternated in a number of countries with relatively protracted periods of more moderate - although still quite high by international standards - inflation rates (5% - 30% *a month*).

We have argued that available theories cannot explain these "chronic inflation" and "megainflationary" periods except by appealing to mutually inconsistent behavioral hypotheses. We have proposed therefore an alternative game-theoretic interpretation, in which several policymakers with conflicting objectives "compete" for seigniorage in conditions of imperfect monitoring.

We have shown how in the trigger strategy equilibria of the model (the only ones that exist in some circumstances) the chronic inflation and the extreme inflation periods emerge endogenously as different stages of an altogether anomalous "chronic-inflation-cum-megainflations" process, in which the megainflations play the same role as the price wars in related models of the industrial organization literature. Thus, our interpretation makes it possible to answer simultaneously and within the same general equilibrium, rational expectations theoretical framework two questions that have intrigued economists and statesmen alike and that have inspired a considerable volume of research: why do megainflations occur, even though every consumer's welfare is lower during these episodes? And why do the inflation rates during the "chronic inflation" periods exhibit a troublesome, stubborn inertia that prevents them from falling to more efficient levels?

---

[68] Canzoneri (1985) can support fully efficient outcomes despite imperfect monitoring because in his model $\beta = 1$ and the result above holds only for $\beta < 1$.

Our model is in the same "Sorcerer's Apprentice" spirit of Sargent and Wallace, in the sense that the attempt to finance government consumption with just a "little" bit of inflation unleashes hidden inflationary forces far more intense and disruptive than perhaps intended.[69] But while their results are driven by the particular mechanics induced by a non-linearity, ours are driven by pathological monetary and budgetary institutions that prevent the monitoring of the different policymakers determining fiscal and monetary policies.

Because the model highlights the role of institutional features in the genesis of "chronic-inflation-cum-megainflations," it can explain why these inflationary processes are observed in some economies and not in others, even if the multiplicity of policymakers with conflicting objectives is a problem that eventually pervades all economies.

We have explored the dynamics of the model by numerical simulation and shown how, even in its most crude and simple version, it can generate "roller coaster" inflation rates that are quantitatively within the range observed in some actual economies. We have also shown that it can reproduce some other reported regularities as well, such as the positive correlation between the level of inflation and its variability found in several cross-country studies, the sharp fall in real money balances preceding substantial increases of the money supply that might suggest "crisis of confidence" interpretations of the megainflationary episodes, and the systematic declines in revenues from the inflationary tax (seigniorage) at extreme inflation rates that could suggest the presence of a "Laffer curve" for seigniorage, even if there isn't one in our model.

We have argued that the last finding is particularly interesting because although our interpretation is within the "conventional wisdom" view that high inflations ultimately reflect high fiscal deficits, it generates observations that seemingly challenge that view and might misleadingly suggest "Laffer curve" interpretations of extreme inflation episodes instead.

In light of the proposed connection between institutions and the dynamics of inflation, the model makes it possible to advance some conjectures on the fate and credibility of past or future stabilization plans in "chronic-inflation-cum-megainflations" economies. In particular, the model suggests that institutional reforms enhancing the ability to monitor the public sector are essential for the credibility and success of stabilization plans trying to bring down the inflation rate to international standards in high inflation economies.

---

[69] See footnote 22 and discussion preceding it.

REFERENCES

Abreu, D., D. Pearce and E. Stachetti (1986): "Optimal Cartel Equilibria with Imperfect Monitoring," *Journal of Economic Theory* 39: 251-69.

———————————————— (1990): "Toward a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica* 58: 1041-64.

Aizenman, J. (1992): "Competitive Externalities and Optimal Seignorage Segmentation," *Journal of Money, Credit and Banking* 1: 61-71.

Aizenman, J. (1993): "Soft Budget Constraints, Taxes, and the Incentive to Cooperate," *International Economic Review* 4: 819-32.

Azariadis, C. (1993): "Intertemporal Macroeconomics," Cambridge, Massachusetts: Blackwell Publishers.

Barro, R. J. (1977): "Unanticipated Money Growth and Unemployment in the United States," *American Economic Review* 67: 101-15

Barro, R. J. (1978): "Unanticipated Money, Output, and the Price Level in the United States," *Journal of Political Economy* 86: 549-80.

Baxter, M (1988): "Toward an Empirical Assessment of Game-Theoretic Models of Policymaking. A Comment," *Carnegie-Rochester Series in Economic Policy* 28: 141-52.

Bléjer, Mario I. (1979): "Inflation Variability in Latin America: A Note on the Time Series Evidence," *Economic Letters* 2: 337-41.

Bléjer, Mario I. and A. Cheasty (1993): "The Deficit as an Indicator of Government Solvency: Changes in Public Sector Net Worth," in Bléjer, Mario I. and A. Cheasty (eds.), *How to Measure the Fiscal Deficit: Analytical and Methodological Issues*. International Monetary Fund, Washington, D.C.

Braun, Anton R. (1994): "How Large is the Optimal Inflation Tax? *Journal of Monetary Economics* 34: 201-14

Bruno, M. (1990): "High Inflation and the Nominal Anchors of an Open Economy," Working Paper No. 3518. National Bureau of Economic Research.

Bruno, M. and S. Fischer (1990): "Seigniorage, Operating Rules and the High Inflation Trap," *Quarterly Journal of Economics* 105: 353-74.

Cagan, P. (1956): "The Monetary Dynamics of Hyperinflation," in M. Friedman (ed.) *Studies in the Quantity Theory of Money*. Chicago: University of Chicago Press.

Calvo, G. (1986): "Temporary Stabilization: Predetermined Exchange Rates," *Journal of Political Economy* 94: 1319-29.

Calvo, G. A. and C. A. Végh (1995): "Credibility and the Dynamics of Stabilization Policy: A Basic Framework," in C. A. Sims (editor), *Advances in Econometrics, Sixth World Congress*, Cambridge University Press, pp. 377-420.

Canzoneri, M.B. (1985): "Monetary Policy Games and the Role of Private Information," *American Economic Review* 75: 1056-70.

Cardoso, E. (1991): "From Inertia to Megainflation: Brazil in the 1980's," M. Bruno, S. Fischer, E. Helpman, and N. Liviatan (eds.), *Lessons of Economic Stabilization and Its Aftermath,* MIT Press, pp. 283-313.

Chari, V. V., L. J. Christiano, and M. Eichenbaum (1994): "Inside Money, Outside Money, and Short Term Interest Rates," September, mimeo.

Chari, V.V. and P. Kehoe (1990): "Sustainable Plans," *Journal of Political Economy* 98: 783-802.

Christiano, L. J. and M. Eichenbaum (1992): "Liquidity Effects, Monetary Policy, and the Business Cycle," Working Paper No. 4129, National Bureau of Economic Research.

Dornbusch, R. (1991): "Credibility and Stabilization," *Quarterly Journal of Economics*, 61, 837-50.

Dornbusch, R., F. Sturzenegger, and H. Wolf (1990): "Extreme Inflation: Dynamics and Stabilization," *Brookings Papers on Economic Activity* 2: 1-84.

Drazen, A., and E. Helpman, (1990): "Inflationary Consequences of Anticipated Macroeconomic Policies," *Review of Economic Studies* 57: 147-64.

Easterly, W., P. Mauro, and K. Schmidt-Hebbel (1995): "Money Demand and Seigniorage-Maximizing Inflation," *Journal of Money, Credit, and Banking* 27: 583-603

Eckstein, Z. and L. Leiderman (1992): "Seignorage and the Welfare Cost of Inflation: Evidence from an Intertemporal Model of Money and Consumption," *Journal of Monetary Economics* 29: 389-410.

Edwards, S. and G. Tabellini (1991): "Political Instability, Political Weakness, and Inflation: an Empirical Analysis," Working Paper 3721, National Bureau of Economic Research.

Flood, R. P., and P. M. Garber (1980): "An Economic Theory of Monetary Reform," *Journal of Political Economy* 11: 24-58.

Foster, E. (1978): "The Variability of Inflation," *Review of Economics and Statistics* 60: 346-50.

Green, E. and R. Porter (1984): "Noncooperative Collusion under Imperfect Private Information," *Econometrica* 52: 87-100.

Guidotti, P. and C. Végh (1992): "Losing Credibility: The Stabilization Blues," Manuscript, Unpublished.

Heymann, D., F. Navajas and I. Warnes (1991): "Conflicto Distributivo y Deficit Fiscal: Algunos Juegos Inflacionarios," *El Trimestre Económico* 58: 101-37.

Hodrick, R. J. (1987): ""The Empirical Evidence on the Efficiency of Forward and Futures Foreign Exchange Markets," Harwood Academic Publishers.

Hodrick, Robert J., N. Kocherlakota and D. Lucas (1991): "The Variability of Velocity in Cash-in-Advance Models," *Journal of Political Economy* 99: 358-84.

Kiguel, M. and N. Liviatan (1992a): "Stopping Three Big Inflations," Unpublished, Washington: World Bank.

——————————— (1992b): "The Business Cycle Associated with Exchange-Rate-Based Stabilization," *The World Bank Economic Review* 6: 279-305.

Leeper, E. and D. B. Gordon (1994): "The Dynamic Impacts of Monetary Policy: An Exercise in Tentative Identification," *Journal of Political Economy* 102: 1228-47.

Leijonhufvud, A (1991): "High Inflations and Contemporary Monetary Theory," Working Paper 638, Department of Economics, University of California, Los Angeles.

Lizondo, J. S. (1983): "Foreign Exchange Future Prices Under Fixed Exchange Rates," *Journal of International Economics* 14: 69-84.

Logue, D. E. and T. D. Willett (1976): "A Note on the Relation Between the Rate and Variability of Inflation," *Economica* 43: 151-8.

Lucas, Robert E., Jr. (1972): "Expectations and The Neutrality of Money," *Journal of Economic Theory* 4: 103-24.

Marcet, A. and J. P. Nicolini (1995): "Convergence of Learning Rules and Recurrent Hyperinflations," Universitat Pompeu Fabra and Universidad Torcuato Di Tella, March, mimeo

Marcet, A. and T.J. Sargent (1989): "Least-squares Learning and the Dynamics of Hyperinflation," in W. Barnett, J. Geweke, and K. Shell (eds.), *Economic Complexity: Chaos, Sunspots, Bubbles and Nonlinearities. Proceedings from the 4th International Symposium in Economic Theory and Econometrics*, Cambridge University Press, pp. 119-37.

Melnick, R. (1990): "The Demand for Money in Argentina 1978-1987: Before and After the Austral Program," *Journal of Business and Economic Statistics* 8: 427-34.

Mondino, G., F. Sturzenegger and M. Tommasi (1992): "Recurrent High Inflation and Stabilization: A Dynamic Game," Working Paper 678, Department of Economics, University of California, Los Angeles.

Obstfeld, M. (1986): "Peso Problems, Bubbles, and Risk in the Empirical Assessment of Exchange-Rate Behavior," in *Financial Risk: Theory, Evidence and Implications*, St. Louis Federal Reserve Bank.

Ostry, J. D. and C. M. Reinhart (1992): "Private Saving and Terms of Trade Shocks," IMF Staff Papers 39: 495-517

Phylaktis, K. and Mark P. Taylor (1993): "Money Demand, the Cagan Model and the Inflation Tax: Some Latin American Evidence," *The Review of Economics and Statistics* 75: 32-37.

Pizarro Rios, J.V. (1993): "The Links Between Money and Prices: Causality Tests for the Peruvian and Chilean Case," Cahier 93.11, Department of Econometrics, University of Geneva.

Porter, R. (1983): "Optimal Cartel Trigger Price Strategies," *Journal of Economic Theory* 29: 313-338.

Prasnikar, J and Z. Pregl (1991): "Economic Development in Yugoslavia and Prospect for the Future," *American Economic Review, Papers and Proceedings of the 103rd Annual Meeting of the American Economic Association*, Vol. 81, 2, 191-95.

Radner, R., R. Myerson and E. Maskin (1986): "An Example of a Repeated Partnership Game with Discounting and with Uniformly Inefficient Equilibria," *Review of Economic Studies* 53: 59-69.

Reinhart, C. M. and C. A. Végh (1994): "Intertemporal Consumption Substitution and Inflation Stabilization," International Monetary Fund, June, mimeo.

Rolnick, A., B. Smith, and W. Weber (1993): "The Origins of the Monetary Union in the United States," Working Paper 9212, Cornell University, Center for Analytic Economics.

Ruge-Murcia, F. J. (1995): "Credibility and Changes in Policy Regime," *Journal of Political Economy* 103: 176-208.

Sachs, J. (1986): "The Bolivian Hyperinflation and Stabilization," Working Paper No. 2073, National Bureau of Economic Research.

——————— (1982): "The Ends of Four Big Inflations," in Robert E. Hall, (ed.), *Inflation: Causes and Effects*, Chicago: University of Chicago Press.

Sargent, T. J. and N. Wallace (1987): "Inflation and the Government Budget Constraint," in A. Razin and E. Sadka (eds.), *Economic Policy in Theory and Practice*. Macmillan.

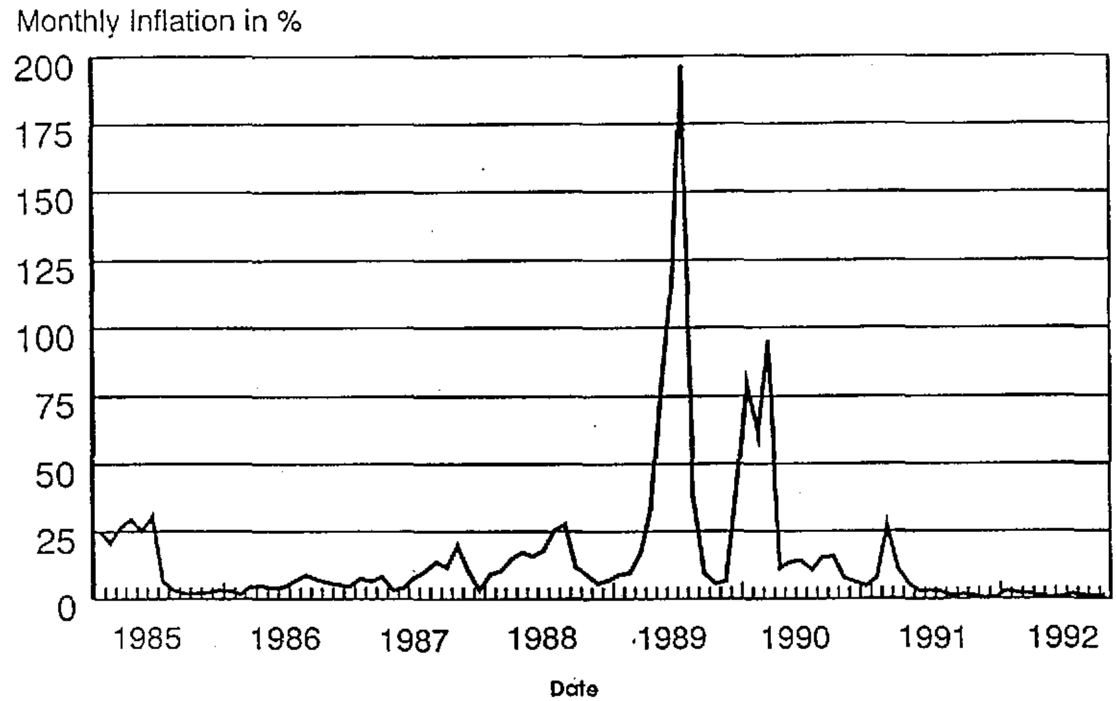
Zarazaga, C. (1993): "Hyperinflations, Institutions, and Moral Hazard in the Appropriation of Seigniorage," University of Minnesota, Ph.D. dissertation.

Zarazaga, C. (1994): "Is There a Laffer Curve for the Inflationary Tax?" Working Paper (forthcoming), Federal Reserve Bank of Philadelphia.

FIGURE 1

Inflation Rate for Argentina, 1985:1 - 1992:10

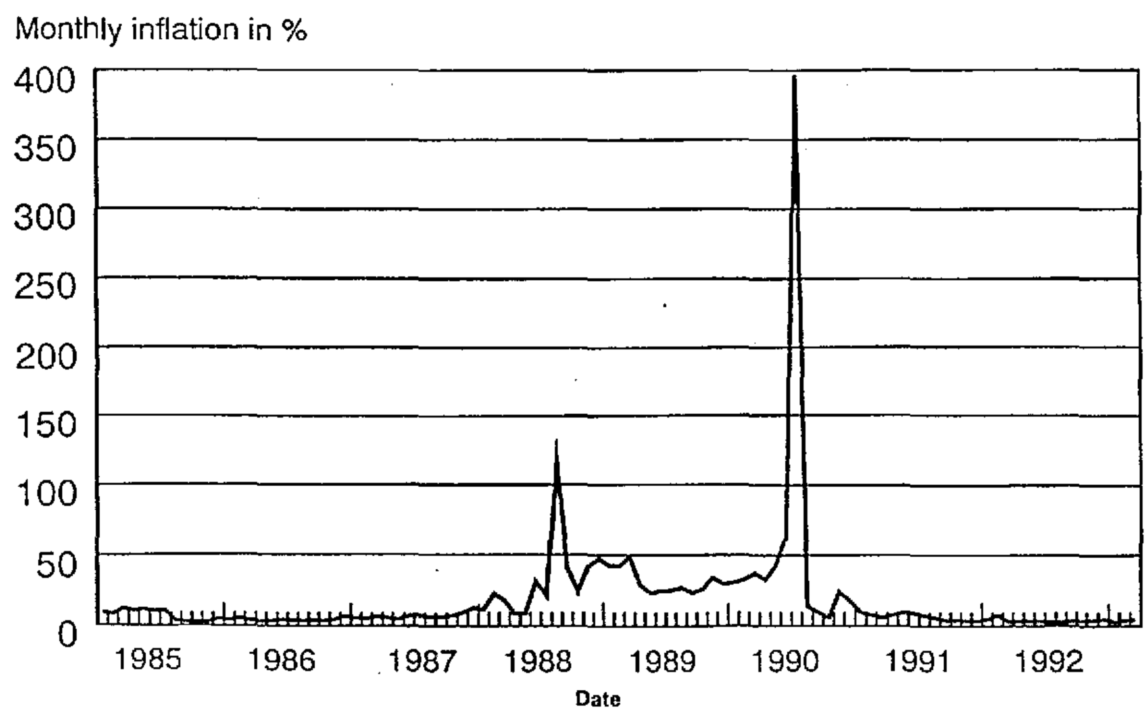
(as measured by the Consumer Price Index for Buenos Aires)



Source: Instituto Nacional de Estadística y Censos

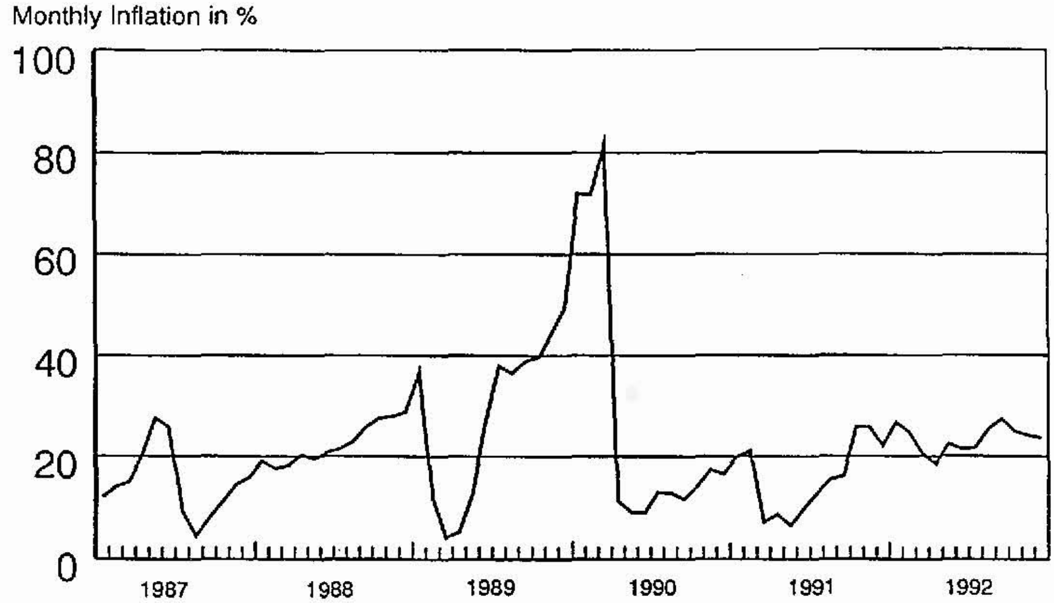
Inflation Rate for Peru, 1985:02 - 1992:12

(as measured by the Consumer Price Index)



Source: Banco Central de Reserva del Peru

FIGURE 2
Inflation Rate for Brazil, 1987:1-1992:12
(as measured by the Indice Gral. do Precos - Disp. Interna)



Source: Conjuntura Economica, Fundacao Getulio Vargas, Rio de Janeiro, Brazil.

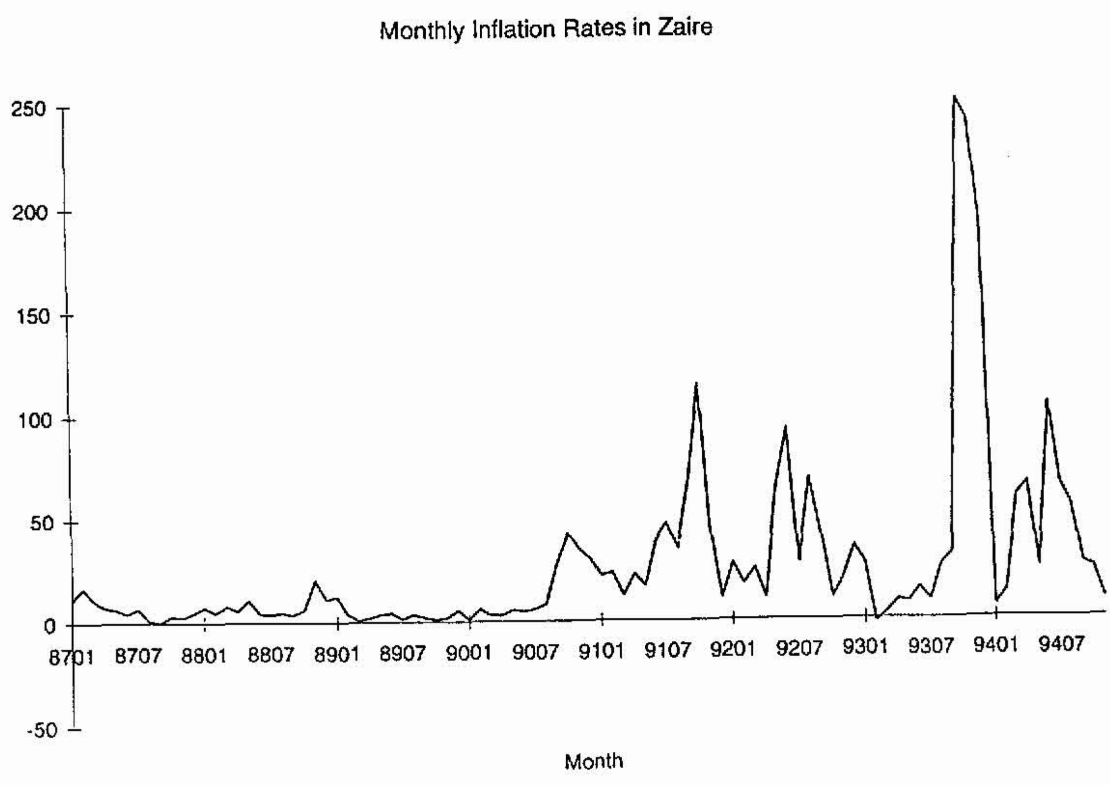


FIGURE 3

Inflation in Actual and Simulated Economies

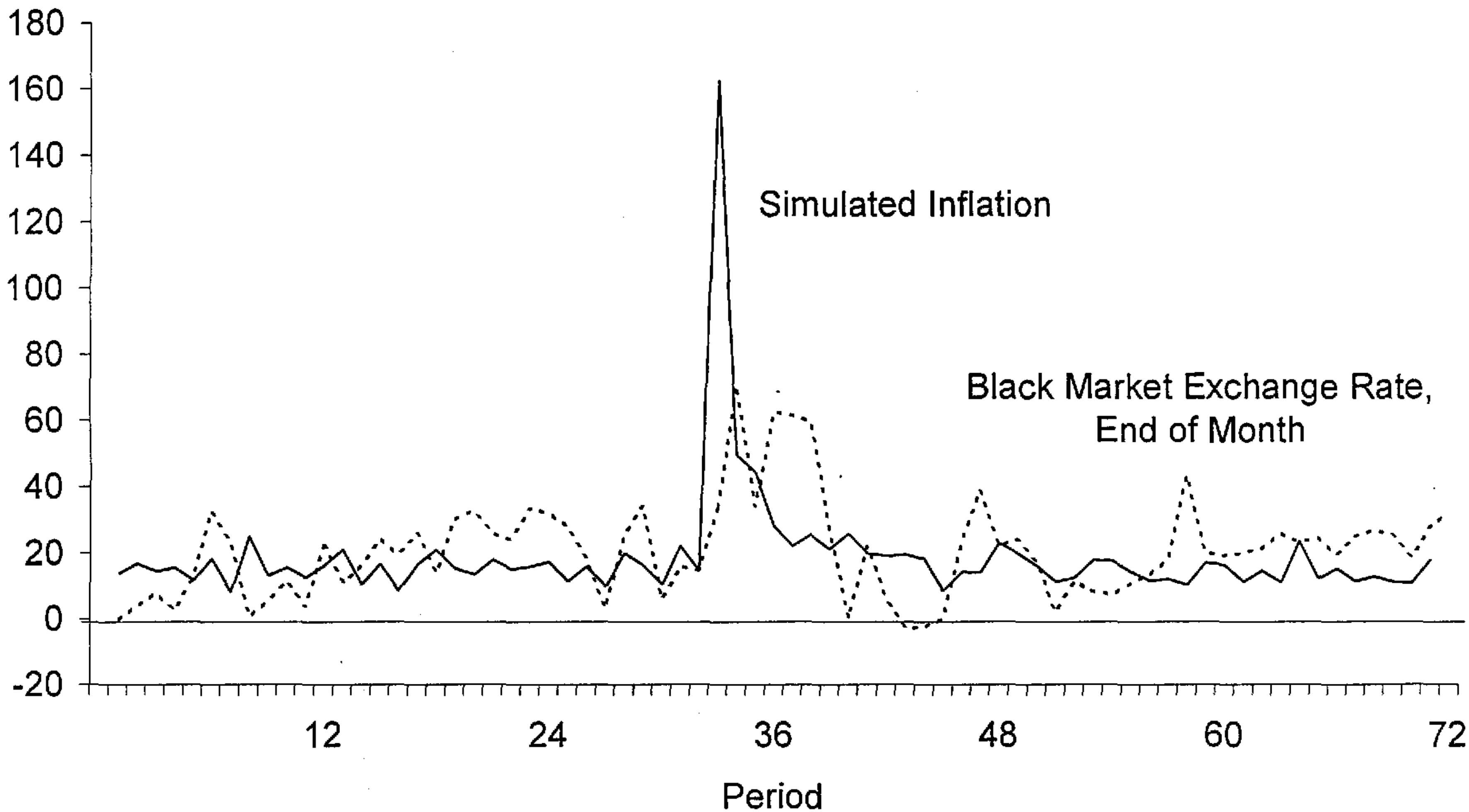


TABLE 1[1]

SIMULATION RESULTS

	SIMULATED ECONOMY[2]				ACTUAL[3] ECONOMY	
Stage	inflation (% per month)	money growth (% per month)	M/P (% GDP)	seigniorage (% GDP)	inflation (% per month) (4)	money growth (% per month) (5)
H_{CL}	14.74(5)	14.72	22.90	2.92	15.64	14.82
H_{CH}	162.37(5)	23.39	10.77	2.04	33.6	36.42
H_{N_1}	46.46	62.62	11.96	4.60	70.5	36.18
H_{N_2}	35.41	51.59	13.39	4.55	33.8	40.93
H_{N_3}	30.51	44.35	14.81	4.54	62.4	69.81
H_{N_4}	26.80	39.11	16.25	4.56	61.5	35.86
H_{N_5}	23.85	35.07	17.72	4.59	59.5	91.87
H_{N_6}	21.43	31.84	19.24	4.64	27.2	194.07
H_{N_7}	19.40	29.18	20.82	4.69	0.0	35.55
H_{N_8}	17.65	26.94	22.47	4.75	22.1	36.02

NOTES:

(1) The complete solution vector $\sigma(H)$ is reported in Table 2.

(2) : Figures correspond to expected values conditional on the stage of the game except when indicated.

(3) : Figures for H_{CL} correspond to the average of the period July 1985 - December 1992, excluding the megainflationary period September 1990 - May 1991. Figures for stages H_{N_1} through H_{N_8} correspond, respectively, to the 8 actual monthly observations from the latter period.

(4) : Correspond to variations of the exchange rate in the black market at the end of each month.

(5) : Correspond to average inflation rates when the preceding state was H_{CL}. Inflation rates are slightly lower when the preceding state was H_{N_8}.

TABLE 2

CALIBRATED TRIGGER STRATEGY SYMMETRIC NASH EQUILIBRIA (QSNE)

PARAMETER VALUES AND SOLUTION VECTOR

Parameters of the utility function

$\beta = 0.997$
$\gamma = 0.032$
$\rho = 2.5$
$\chi = 0.02$
$\zeta = 0.000255$
$N = 2$
$\eta_i = 1, \ i = 1, \ 2.$

Parameters of the punishment phase

Probability of reversion: 2% $\Rightarrow$ $\tilde{\xi} = 0.1355$
$Q = 8$

Parameters of the distribution function

$\xi_t \sim B(1.703240909, \ 3.253909252)$
$\bar{\xi} = 0.174625332$

Solution Vector

$\sigma(H_{CL}) = \delta^i_C = 0.04353758139294602175$2

$\sigma(H_{CH}) = \delta^i_{N_1} = 0.13518590355101711697$

$\sigma(H_{N_1}) = \delta^i_{N_2} = 0.12614208964207129782$

$\sigma(H_{N_2}) = \delta^i_{N_3} = 0.11795696227646480791$

$\sigma(H_{N_3}) = \delta^i_{N_4} = 0.11049560242620476425$

$\sigma(H_{N_4}) = \delta^i_{N_5} = 0.10365340057764844961$

$\sigma(H_{N_5}) = \delta^i_{N_6} = 0.09734775326506449333$3

$\sigma(H_{N_6}) = \delta^i_{N_7} = 0.09151237708696562112$0

$\sigma(H_{N_7}) = \delta^i_{N_8} = 0.08609331560144578565$6

FIGURE 4

Seigniorage and Inflation in the Model Economy

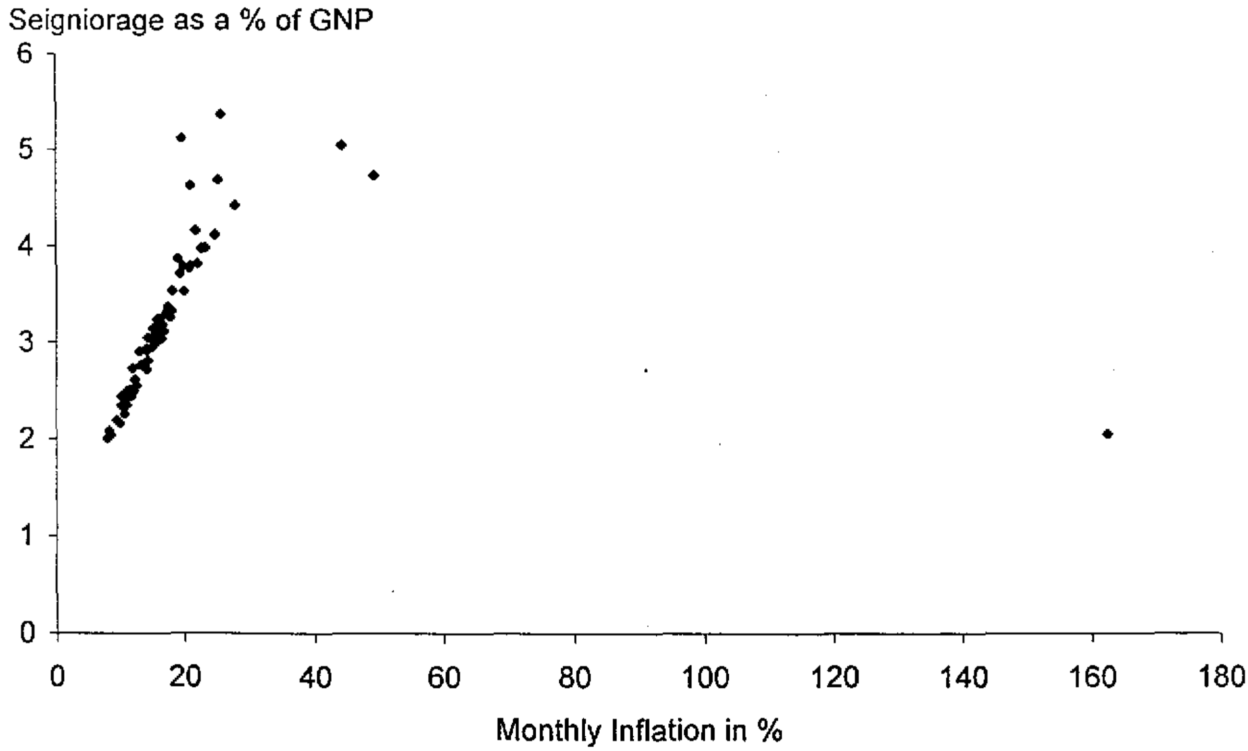
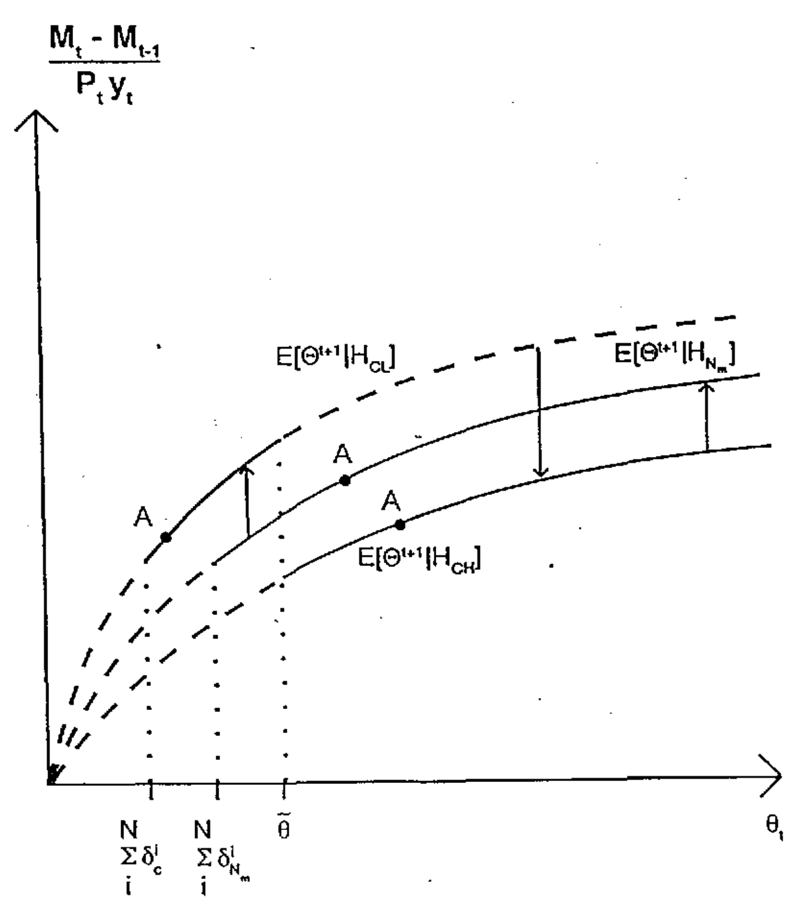


FIGURE 5



APPENDIX A

I.A - Definitions and Notation

This section introduces the notation and definitions that will be used throughout this Appendix and the next

Let $i = 1, 2, \ldots, N$ index the political agents and representative consumer of island i.

Let $S^i \subseteq [0,\infty)$ denote the set of actions available to the political agent of island i. In particular, S^i is the set that specifies the space on which δ^i_t, the rate of expansion of the money supply induced by island's i political agent, may lie. As stated in the text, it will be assumed that $\delta^i_t \in [0, \infty)$.[1]

Define the vector of period t actions by $\delta_t \equiv (\delta^1_t, \ldots, \delta^{N-1}_t, \delta^N_t)$.

Let the total rate of growth of the money supply up to time t be denoted by h_t, that is $h_t \equiv \{\theta_1, \theta_2, \ldots, \theta_t\}$.

Let $\{\xi_t\}$ be an i.i.d. stochastic process with constant support $[\underline{\xi}, \bar{\xi}]$, distribution function F(.), and density function f(.)[2] As suggested by **[1]** in the text, for each t, the vector of choices δ_t and the unobservable process ξ_t induce a probability measure on θ_t. The political agent and representative consumer of island i will eventually parameterize such a measure (if it exists, see footnote 1 of this Appendix) by public histories and their privately observed histories up to time t, that is, by $I^i_t \equiv (h^i_t, h_t)$, where I^i_t is the information set of the representative consumer of island i at period t and h^i_t his privately observed history at t, that is, $h^i_t \equiv \{\delta^i_1, \delta^i_2, \ldots, \delta^i_t\}$. Let $\Omega_t \subseteq \mathbb{R}^+$ denote the set on which θ_t will take on values at each t. Given the assumptions on $\{\xi_t\}$ and δ_t, the support of the signal θ_t at each t will be $\left[\sum \delta^i_t + \underline{\xi}, \sum \delta^i_t + \bar{\xi}\right]$. Let the corresponding probability distribution of θ_t be denoted by $\psi_t(\ . \ ,\delta^i_t; I^i_{t-1})$ with density ϑ_t.[3]

Strategies will be defined below as functions mapping *publicly* observed histories into actions. That formal definition will require, therefore, that we map each possible history of the overall rate of growth of the money supply, that is, each h_t, into a set Σ_t of *public histories* or *publicly observed states* with typical element H_t. That is, the set Σ_t will represent a partition of all possible histories h_t, so that each possible history h_t will be fully characterized by one

---

[1]Notice that the strategy space is not compact, and therefore, the existence of equilibria will not be guaranteed with this formulation of the model, as already explained in subsection 5.2.

[2]That is, we specialize the distribution function in subsection 3.1.1 to be time invariant.

[3]Note that in this formulation distribution of the signal does not have the constant support assumed in APS.

and only one element of Σ_t.[4]

A strategy will generate a sequence of transition functions Γ_t mapping the previous publicly observed state H_t and the current realization of the money supply θ_t into current publicly observed states or histories. Of course, each strategy may generate a different transition function. Since we will be dealing only with stationary (i.e., time-independent) strategies, the time subindexes will become a nuisance and Γ will define a recursive Markovian succession between states in an ergodic set Σ, that is, $H \in \Sigma$ and

$$\Gamma : \Sigma \times \Omega_t \longrightarrow \Sigma \qquad \textbf{[1.B]}$$

In the equilibria that will be discussed below, the transition probability function between states will then be induced by that of θ_t, that is, by ψ.[5]

An island political agents will have a *pure public strategy* if his actions δ^i_{t+1} in each period t = 1, 2, ..., can be represented as a function of only *publicly observed* outcomes H_t, $H_t \in \Sigma_t$, that is, if for each $t \geq 1$, the strategy σ^i_t is given by the (Lebesgue) measurable function $\sigma^i_t : \Sigma_t \longrightarrow S^i$.

Let $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_t, \sigma_{t+1}, \dots)$ where $\sigma_t = (\sigma^1_t, \sigma^2_t, \dots, \sigma^N_t)$ for every $t \geq 1$. Then a public strategy profile σ can be written $\sigma = (\sigma^1, \dots, \sigma^N)$, where $\sigma^i = \left(\sigma^i_1(H_0), \sigma^i_2(H_1), \dots, \sigma^i_t(H_{t-1}), \sigma^i_{t+1}(H_t), \dots\right)$, i = 1, 2, ..., N.

Given a history H_t and a strategy profile σ, the expression $\sigma|_{H_t}$ will denote the strategy profile induced by σ after the t-period publicly observed history H_t.

The time subindexes will be dropped from the above definitions of the strategies when the latter are stationary.

II.A - Proposition 1

Let ξ_{t+1} be a random variable that is a function of an independent and identically distributed stochastic process and, possibly, of past *publicly observed* variables. Assume that the islands' political agents have strategies that can be represented in terms of pure public stationary strategies as defined above. Then the publicly observed history up to t+1 is a sufficient statistic for the conditional expectations of the rate of growth of the money supply from time t+2 on.

Heuristically, the proposition says that deviations (from some agreed pattern of play) eventually known only to the deviant will not matter *per se*, but through their effect on publicly observed signals, which implies that $\mathbb{E}\left[\Theta^{t+2} | I^i_{t+1}\right] = \mathbb{E}\left[\Theta^{t+2} | h_{t+1}\right]$. The payoff **[13]** in the text therefore will be well defined.

---

[4]The elements of Σ_t can be, for example, the labels H_C and H_{NC}, indicating that the history of the growth rates of the money supply up to t has been one of cooperation or non-cooperation, respectively.

[5]The transition probability function turns out to be the same for all agents because, as shown below, the probability distribution ψ does not depend on any private history h^i_t.

Proof:

By definition

$$\mathbb{E}\left[\Theta^{t+2}\middle| h_{t+1},\delta^{i}_{t+1}\right] \equiv \mathbb{E}\left[\sum_{h=0}^{\infty}\beta^{h}\left[\prod_{k=1}^{h}\frac{1}{1+\theta_{t+1+k}}\right]\middle| h_{t+1},\ \delta^{i}_{t+1}\right] \equiv$$

$$\equiv \mathbb{E}\left[\sum_{h=0}^{\infty}\beta^{h}\left[\prod_{k=1}^{h}\frac{1}{1+\xi_{t+k+1}+\sum\delta^{i}_{t+k+1}}\right]\middle| h_{t+1},\ \delta^{i}_{t+1}\right]$$

where the convention adopted in equation **[9]** in the text holds when $h < k$.

Since the δ's from t+2 on are taken as given, that is, chosen according to the strategy vector σ that depends only on public histories, and since by assumption ξ_{t+k+1} is a function only of previously publicly observed histories and some i.i.d. process μ, along a time consistent equilibrium path the above expression should be equal to:

$$\mathbb{E}\left[\sum_{h=0}^{\infty}\beta^{h}\left[\prod_{k=1}^{h}\frac{1}{1+\xi(h_{t+k},\mu_{t+k+1})+\sum_{i}\sigma^{i}(h_{t+k})}\right]\middle| h_{t+1},\ \delta^{i}_{t+1}\right] \qquad \textbf{[1.A]}$$

Note that δ^{i}_{t+1} does not help to predict μ_{t+k+1}, $k \geq 1$, since by assumption, $\{\mu_t\}$ is an i.i.d. process. Since $\theta_{t+2} = 1 + \xi(h_{t+1},\ \mu_{t+2}) + \sum \delta^{i}_{t+2}(h_{t+1})$, this implies that δ^{i}_{t+1} does not provide any useful information in forecasting the realization of the signal at t+2. Since according to **[15]** in the paper the transition between histories h_{t+1} and h_{t+2} is governed by θ_{t+2} and h_{t+1} alone, this in turn implies that δ^{i}_{t+1} is not helpful in predicting h_{t+2} either. But then, given that μ_{t+3} is uncorrelated with δ^{i}_{t+1}, δ^{i}_{t+1} does not have any predictive power in forecasting θ_{t+3}, and thus, h_{t+3}. Applying this argument recursively shows that δ^{i}_{t+1} does not provide any information about the process $\left\{\theta_{t+k+1}\right\}_{k=1}^{\infty}$ in addition to that already contained in h_{t+1}, that is, h_{t+1} is a sufficient statistic for the conditional expectations **[1.A]**. An argument entirely analogous to that used for δ^{i}_{t+1} will show that no previous δ^{i}_{t*}, $t^{*} < t + 1$, helps to predict future θ_t's and, therefore, that h^{i}_{t+1} does not help to predict $\left\{\theta_{t+k+1}\right\}_{k=1}^{\infty}$. This implies $\mathbb{E}\left[\Theta^{t+2}\middle| I^{i}_{t+1}\right] \equiv$

$\mathbb{E}\left[\Theta^{t+2}\middle| h^{i}_{t+1},\ h_{t+1}\right] = \mathbb{E}\left[\Theta^{t+2}\middle| h_{t+1}\right]$. **Q.E.D.**

Remarks

Note that the process $\{\xi_t\}$ given by $\xi_t = c + \rho_1\theta_{t-1} + \mu_t$ satisfies the conditions of the above Proposition, but not $\xi_t = c + \rho_2\xi_{t-1} + \mu_t$. Note also that if $\{\xi_t\}$ is strictly exogenous, ρ_1 in the first case should be set equal to 0. We have considered the general case, however, because $\{\xi_t\}$ will be a function of θ_t (and maybe other publicly observable variables) in a model where $\{g_t\}$ is an exogenous process.

To see why the Proposition would not hold for the second case, suppose that at a particular date, t+1, the strategy specified $\delta_{t+1} = (\delta_c, \delta_c, \ldots, \delta_c)$, but political agent i actually set $\delta^i_{t+1} = \delta_c + \Delta$. Then, denoting with $\mathbb{E}(r)$ the forecast by the representative consumer of island r and assuming for simplicity that $\mathbb{E}(\mu) = 0$, the island i representative consumer's forecast of ξ_{t+2} at time t+1 (after θ_{t+1} is realized) will be given by $\mathbb{E}(i)\left[\xi_{t+2}\middle|\theta_{t+1},\delta^i_{t+1}\right] = c + \rho_2(\theta_{t+1} - N\delta_c - \Delta)$, while for $j\neq i$, $\mathbb{E}(j)\left[\xi_{t+2}\middle|\theta_{t+1}, \delta^j_{t+1}\right] = c + \rho_2(\theta_{t+1} - N\delta_c)$. This implies that the island's i representative consumer will infer the true ξ_{t+1} while the consumers of the other islands will not, and therefore that forecasts of ξ_{t+2} (and of θ_{t+2}) based on private information will differ between agents. That is, δ^i_{t+1} will contain useful information to the representative consumers of island i in forecasting θ_{t+1} in addition to that already revealed in h_{t+1}, which implies that in that case the publicly observed history h_{t+1} would not be a sufficient statistic for the money supply from t+2 on.

APPENDIX B

DEFINITION AND COMPUTATION OF THE EQUILIBRIA OF THE MODEL

This Appendix presents the more technical material involved in the formal definition of our equilibrium concept and in the computation of the equilibria of the model.

II.B - COMPUTATION OF THE EQUILIBRIA: A DYNAMIC PROGRAMMING APPROACH

II.1.B Some Preliminaries

In this section, we develop a dynamic programming approach that will be useful for defining and computing the equilibria of our model.

As stated in the text, only stationary (that is, time-independent, although not necessarily history-independent) pure public strategies will be considered. Within this class, we'll study only time-consistent, sequentially rational, Nash equilibria.

Sequential rationality requires proving that a given strategy is a Nash equilibrium for *any* history, including histories following eventual deviations from prescribed actions. We already explained in Section 4 of the text and in the example in the remarks following the proof of Proposition 1 in Appendix A how the usual difficulties for finding such equilibria in repeated games in the presence of private information are further complicated in our model by the fact that expectations about future actions appear directly as an argument in the single-period payoff, a feature typically absent in repeated games (such as GP and APS) without endogenous state variables.

We also indicated in section 4 of the text that the messy consequences of private information can be avoided in our model when h_t is a sufficient statistic for the computation of mathematical expectations, a result that turns out to hold under the assumptions of Proposition 1 in Appendix A. In what follows we make use of that result to formalize our equilibrium concept.

In that case, inspection of the single-period payoff **[13]** in the text hints that the optimal action, that is, the optimal δ^i_{t+1} chosen by island's i political agent at the beginning of t+1, can be represented by a function (or correspondence) of the publicly observed history alone.[6]

To verify this conjecture, let H be the publicly observed state at t, and assume that there is a function $\mathbf{V}^i: \Sigma \longrightarrow \mathbb{R}^+$ that gives the expected value of the game to island's i political agent i for every possible state $H \in \Sigma$. Taking into account **[1.B]**, this means that the continuation value of the game when history H', $H' \in \Sigma$, succeeds H will be given by $\mathbf{V}^i\left(\Gamma\left(H,\ \theta_{t+1}\left(\delta^i_{t+1},\ \delta^{-i}_{t+1},\ \xi_{t+1}\right)\right)\right)$ where, following the standard notation, δ^{-i}_{t+1} denotes actions of islands' political agents other than i at t+1. In the spirit of APS, we will impose the restriction that this continuation value must belong to the set of equilibrium payoffs as well, in the sense that it should be equal to the discounted present value of the game if it were started at the successor of state H and the islands' political agents decisions were sequentially optimal for that and all possible subsequent histories.

Thus, the solution to island's i political agent's decision problem at t+1, any $t \geq 0$, will be given by the solution to the following functional equation:

$$\mathbf{V}^i(H) = \max_{\delta^i_{t+1}} \left\{ \mathbb{E}\left[\Pi_{t+1}\left[\delta^i_{t+1}, \theta_{t+1}\left(\delta^i_{t+1}, \sigma^{-i}(H)\right), \mathbb{E}\left(\Theta^{t+2} \middle| \Gamma\left(H, \theta_{t+1}\left(\delta^i_{t+1}, \sigma^{-i}(H)\right)\right)\right)\right] + \right.\right.$$

$$\left.\left. \beta\ \mathbf{V}^i\left(\Gamma\left(H, \theta_{t+1}\left(\delta^i_{t+1}, \sigma^{-i}(H)\right)\right)\right) \right] \middle| H \right\} \qquad \textbf{[3.B]}$$

where H denotes a generic element of Σ, and where the single-period payoff **[13]** in the text has been rewritten exploiting the fact that, by Proposition 1, $I^i_{t+1} \equiv (h_{t+1},\ h^i_{t+1})$ can be replaced by h_{t+1}, the fact that h_{t+1} belongs to some $H \in \Sigma$, and the fact that, by **[1.B]**, $H_{t+1} = \Gamma(H_t,\ \theta_{t+1})$.

In interpreting **[3.B]** recall that, according to **[1.B]** and the timing of decisions explained in section 4 of the text, the particular realization of H_{t+1} will depend on the past history H_t (known at the beginning of t+1) and on the realization of θ_{t+1} that will emerge *after* the political agents have made their decisions on δ^i_{t+1} and *before* the representative consumers of all islands have made

[6] More explicitly, when all other agents (including the political agents that will represent island i from t+2 onwards) use *pure public* strategies and when the representative consumers of all islands form their expectations accordingly, that is, based only on publicly available information.

their consumption and portfolio allocations. Note that the dependence of θ_{t+1} on ξ_{t+1} has not been explicitly indicated for ease of notation but that the dependence of the former on δ^i_{t+1} has been made explicit to make clear that the choice of the action δ^i_{t+1} has two consequences: it affects not only the single-period payoff at t+1 but also potentially the transition between states and, therefore, the continuation value of the game. Notice that in solving problem **[3.B]**, the political agent takes as given the strategies of all other political agents (including his successors in representing island i) as well as the conditional expectations $\mathbb{E}\left[\Theta^{t+2}|H'\right]$. We will replace the latter with the notation $e_{\Theta^{t+2}}\left(\Gamma(H,\ \theta_{t+1})\right)$ when wanting to emphasize that these expectations are taken parametrically, although, of course, the value of these two expressions should be equal in equilibrium.

Notice that past actions δ_t do not appear anywhere in the t+1 single-period payoff,[7] the first term of the right-hand side of **[3.B]**. Thus, if the function $\mathbf{V}^i$ were known,[8] we could define a function σ^i(or a correspondence, if the solution to **[3.B]** were not unique), σ^i: $\Sigma \longrightarrow S^i$ as follows: for each H, H $\in \Sigma$, assign $\overset{*}{\delta}{}^i_{t+1}$, the value that attains the maximum in **[3.B]**, to $\sigma^i(H)$. Thus σ^i will completely describe the solution to the decision problem of the political agent of island i for any history H, without reference to any private information. Only publicly observed outcomes will be involved, as conjectured.

In the following section we discuss how use the basic structure provided by **[3.B]** to find time-consistent sequentially rational Nash equilibria with GP trigger strategies.

II.2.B - Definition of the Equilibrium Concept

II.2.1.B - Trigger Strategy Equilibria

These equilibria induce "cooperation" by rewarding the political agents when the realized history indicates that it is highly probable they have been cooperating and by punishing them otherwise. This can be accomplished by making the continuation payoffs dependent on past history and, therefore, on the actions of the political agents, since, as suggested by **[1]** in the text, these actions condition the realization of θ_t governing the transition (through **[1.B]**) from "cooperative" outcomes to "noncooperative" outcomes.

In this paper, such continuation payoffs are generated with the GP trigger strategy described in subsection 4.1 of the text. Under the assumption in the text that $\{\xi_t\}$ is i.i.d., those strategies will generate a transition function Γ formally defined by replacing "$h_{t+1}\ \in$" in the left hand side of expression **[15]** in the text with "$\Gamma(H,\ \theta)$ =".

We conjecture that the continuation values corresponding to the transition function thus defined will take the following form:

---

[7]See footnote 13 of this Appendix.

[8]See footnote 1 of Appendix A.

$$V^{i}\left(\Gamma(H,\theta_{t+1})\right) = \begin{cases} V^{i}_{C} & \text{if} \quad \Gamma(H,\, \theta_{t+1}) = H_{CL} \text{ (or } N_{N_Q}\text{)} \\ V^{i}_{N_1} & \text{if} \quad \Gamma(H,\, \theta_{t+1}) = H_{CH} \\ V^{i}_{N_m} & \text{if} \quad \Gamma(H,\, \theta_{t+1}) = H_{N_{m-1}} \qquad m= 2,\, 3,\, \dots,\, Q \end{cases} \qquad \textbf{[4.B]}$$

With these specifications, each island's political agent faces a stationary Q+1-stage Markov dynamic programming and problem **[3.B]** for $\beta \in (0,1)$ can be characterized by the following Q+1 functional equations:

For $H = H_{CL}$,

$$V^{i}(H_{CL}) = \max_{\delta^{i}_{t+1}} \left\{ \mathbb{E}\left[\Pi^{i}_{t+1}\left[\delta^{i}_{t+1},\theta_{t+1}\left(\delta^{i}_{t+1},\sigma^{-i}(H_{CL})\right),{}^{e}\Theta^{t+2}\left(\Gamma(H_{CL},\theta_{t+1})\right)\right]\middle| H_{CL}\right] + \right.$$

$$\left. \beta \Pr\left(\theta_{t+1}\left(\delta^{i}_{t+1},\sigma^{-i}(H_{CL})\right) \le \tilde{\theta}\right) V^{i}_{C} + \beta \Pr\left(\theta_{t+1}\left(\delta^{i}_{t+1},\sigma^{-i}(H_{CL})\right) > \tilde{\theta}\right) V^{i}_{N_1} \right\} \qquad \textbf{[5.B]}$$

For $H = H_{CH}$ (taking into account that $\Gamma(H_{CH},\theta_{t+1}) = H_{N_1}$),

$$V^{i}(H_{CH}) = \max_{\delta^{i}_{t+1}} \left\{ \mathbb{E}\left[\Pi^{i}_{t+1}\left[\delta^{i}_{t+1},\, \theta_{t+1}\left(\delta^{i}_{t+1},\sigma^{-i}(H_{CH})\right),\ {}^{e}\Theta^{t+2}(H_{N_1})\right] \middle| H_{CH}\right] + \beta V^{i}_{N_2} \right\} \qquad \textbf{[6.B]}$$

Similarly, for the Q-1 states H_{N_m} , $m = 1, 2,\dots,Q-1$:

$$V^{i}(H_{N_m}) = \max_{\delta^{i}_{t+1}} \left\{ \mathbb{E}\left[\Pi^{i}_{t+1}\left[\delta^{i}_{t+1},\theta_{t+1}\left(\delta^{i}_{t+1},\sigma^{-i}(H_{N_m})\right),{}^{e}\Theta^{t+2}(H_{N_{m+1}})\right] \middle| H_{N_m}\right] + \beta V^{i}_{N_{m+2}} \right\} \qquad \textbf{[7.B]}$$

with $V^{i}_{N_{m+2}}$ replaced by V^{i}_{C} when $m = Q - 1$

Finally, when $H = H_{N_Q}$, the political agents return to cooperate at t+1, so the problem is exactly equivalent to that when $H = H_{CL}$.

Notice that, as anticipated in subsection 4.1 of the text, there is no reason to suppose that the conditional expectations appearing inside the single-period payoffs of problems **[5.B]** through **[7.B]** will be equal across all the Q stages of the punishment phase. Therefore, the δ^{i}_{t+1}'s may differ across the different stages of the punishment phase, in contrast with GP, in which they are necessarily equal.[9] These considerations make it only natural to conjecture the solution [16] proposed in the text which can now be more formally represented as:

$$\sigma^i(H) = \begin{cases} \delta^i_C & \text{if } H = H_{CL} \text{ or } H = H_{N_Q} \\ \delta^i_{N_1} & \text{if } H = H_{CH} \\ \delta^i_{N_m} & \text{if } H = H_{N_{m-1}} \qquad m = 2, 3, \ldots, Q \end{cases} \qquad \textbf{[8.B]}$$

We are now in a position to define more formally the equilibria under trigger strategies. Without loss of generality, we will limit the analysis to the symmetric case.[10]

Definition

A stationary strategy vector σ is a *time-consistent, sequentially rational, symmetric Nash equilibrium in fixed length punishment pure public trigger strategies* (QSNE hereafter) if it satisfies all the conditions below:

1) $\sigma^i(H) = \sigma^j(H)$ for all i, j; j≠i; and for all $H \in \Sigma$
2) Expectations are rational, i.e.:

$$\mathbb{E}\left[\Theta^{t+2}\left(\sigma|_{H'}\right)\right] = {}^e\Theta^{t+2}(H') \qquad \text{for all } H' \in \Sigma$$

3) The strategy profile **[8.B]** solves the maximization problems **[5.B]**, **[6.B]**, and **[7.B]**.
4) The continuation values in **[4.B]** satisfy:

$$V_C = V(H_{CL}) = V(H_{N_Q}); \quad V_{N_1} = V(H_{CH}); \quad V_{N_m} = V(H_{N_{m-1}}) \quad m = 2, 3, \ldots, Q. \quad \blacksquare$$

Condition 3 states that subjective expectations must equal mathematical expectations. We have introduced the notation $\mathbb{E}\left[\Theta^{t+2}\left(\sigma|_{H'}\right)\right]$ to indicate that those expectations are calculated over the possible paths of the money supply generated by the strategy profile induced by σ in the subtree following H'.

Condition 4 imposes the restriction that the continuation values after any history H must belong to the set of equilibrium payoffs.[11] Therefore, a critical issue in computing these equilibria is to find the values in **[4.B]** that satisfy this condition, an issue we address in the next section.

Notice that **[15]** in the text (or its more elegant representation using the transition function as explained above) encompasses all possible histories h_t. It

---

[9] In fact, that these expectations can be different across stages of the punishment phase is what explains our result that for some parameter values trigger strategy equilibria can exist even if "myopic" ones do not (see subSection 5.2).

[10] The asymmetric case will consist of N versions of the equations **[5.B]** to **[7.B]**, one for each island.

[11] That is, the continuation values in **[4.B]** belong, in APS terminology, to the self-generating set of equilibrium payoffs **V**. In the terminology of Chari and Kehoe (1990), this is equivalent to asserting that the political agents must have "sustainable plans."

should be obvious therefore that our dynamic programming formulation guarantees the sequential rationality of the equilibria defined above.

In any case, for the sake of clarity, assume that $H_t = H_{CL}$ and that a realization of $\theta_{t+1} > \tilde{\theta}$ that has prior probability 0 in the equilibrium path occurs.[12] Will the strategy profile **[8.B]** still be a time-consistent Nash equilibrium for the remaining of the game? The answer is yes, provided the deviation does not induce a revision of representative consumers' beliefs about how the successors of period t+1 political agents will behave in the future, that is, provided that the representative consumers still expect the political agents representing the islands from t+2 on to stick to the strategy profile **[8.B]**.[13] In that case, h_{t+1} will be simply "classified" as H_{CH} and the game will proceed as dictated by **[8.B]**. In other words, at t+2 the successors of period t+1 political agents will face again problem **[6.B]** (with the time indexes shifted forward 1 period).[14] Afterwards, the political agents will face, in sequence, problems **[7.B]**, **[5.B]**, **[6.B]**, and so on. In other words, after the deviation the political agents

---

[12]This can happen in our environment because, unlike in GP, the support of $\langle\xi\rangle$ is compact. For example, suppose that "by mistake" the political agent of island i sets $\delta^i_{t+1} = \delta^i_C + \bar{\xi}$. In that case $\theta_{t+1} = \sum_i \delta^i_C + \bar{\xi} + \xi_{t+1}$, which is greater than $\sum_i \delta^i_C + \bar{\xi}$, the upper bound for θ_{t+1} (see section I.A of Appendix A) if all political agents had conformed to the strategy $\sigma(H_{CL}) = \delta^i_C$ when $H_t = H_{CL}$. An alternative formulation that will make deviations "undetectable" is to assume, as in GP, that the support of ξ_t is unbounded, that is, to assume $\xi_t \in [0, \infty)$. We did not adopt this formulation because it was unnecessary in our context and, more importantly, because it is empirically unattractive: the few parametric distributions defined in that interval, such as the exponential distribution, are not as versatile as the beta distribution assumed in the text and are, therefore, more difficult "to fit" to the data (a related problem is that the sample will not contain observations corresponding to the very low probability events that typically characterize the right tail of the density function of such distributions).

[13]Notice that the revision of $\mathbb{E}\left[\Theta^{t+2}\left(\sigma|_{H'}\right)\right]$ is the only channel through which past δ^i_t's can affect future single-period payoffs because the intratemporal and intertemporal separability of the "island-specific" good guarantees that any effects of past δ^i_t's on the utility from this good at t do not carry over to the future.

[14]In our formulation all that matters is that $\theta_{t+1} > \tilde{\theta}$. The amount by which θ_{t+1} exceeds $\tilde{\theta}$ does not affect the continuation payoffs if the representative consumers don't think it does. This is because, by the reasons given in the previous footnote, a deviation can impinge "physically" on the next single-period payoff only through its effects on expectations. If the representative consumers interpret deviation as accidental and not as induced by strategic considerations, the unintended error contains no information about the likely future play, and therefore the representative consumers of all islands will have no reason to set the conditional expectations ${}^e\Theta^{t+2}(H_{N_1})$ to a value other than $\mathbb{E}\left[\Theta^{t+2}\left(\sigma|_{H_{N_1}}\right)\right]$, that is, the expectations corresponding to the equilibrium path generated by **[8.B]** after state H_{N_1}.

will face exactly the same sequence of problems **[5.B]** to **[7.B]** for which **[8.B]** was a solution, which implies that the strategies **[8.B]** still define a time consistent Nash equilibrium for the remaining of the game.[1] A similar argument applies to deviations at any other phase of the game.

II.2.2.B - "Myopic" Nash equilibria

As explained in subsection 4.1 of the text, these equilibria do not distinguish between histories of "cooperation" and "noncooperation", and is therefore the single-shot analogue of repeated games.

Definition

A stationary strategy vector σ is a *myopic time-consistent sequentially rational symmetric pure public strategies Nash equilibrium* (**MSNE** hereafter) if it satisfies all the conditions of the **QSNE**, with condition 1) replaced by $\sigma(H) = \delta^{i}_{MSNE}$, all H, and condition 4) replaced by $V^{i}(H) = V^{i}_{MSNE}$, all H.

II.3.B - Numerical Computation of the Equilibria

The numerical implementation of the equilibrium just defined requires then finding the strategy vector δ^{i}_{r} and the continuation values V_{r}, r = C, N_1, ..., N_Q, that solve the problems **[5.B]** through **[7.B]** above. A constructive algorithm for solving this type of problems is discussed in APS. That task is relatively simple, however, when attention is restricted to GP trigger strategies, because the "myopic" nature of the problem in the Q stages of the punishment phase readily suggests that the continuation values $\mathbf{V}_{N_m}$, m = 1, 2,...,Q can be computed from:

$$\mathbf{V}_{N_m} = \sum_{r=m}^{Q} \mathbb{E}\left\{\beta^{r-m}\, \Pi^{i}_{t+1+r-m}\left[\delta^{i}_{N_r},\, \theta_{t+1+r-m}\left(\delta^{i}_{N_r}, \delta^{-i}_{N_r}\right),\, \mathbb{E}\left[\Theta^{t+2+r-m}\left(\sigma \middle| H_{N_r}\right)\right]\right] \middle| H_{N_{m-1}}\right\} +$$

$$+ \beta^{Q-(m-1)}\mathbf{V}_{C} \qquad \textbf{[9.B]}$$

where m = 1, 2, 3,...,Q and $H_{N_{m-1}} = H_{CH}$ when m = 1.

The analogous expression for the continuation value $\mathbf{V}_C$ is less obvious and can be found by proceeding along the lines of Porter (1983), based on the conjecture that in any stationary equilibrium, $\mathbf{V}_C = \mathbf{V}(H_{CL})$. Replacing the above expression for $\mathbf{V}_{N_m}$ in **[5.B]** along with this conjecture yields, after some algebra:

$$\mathbf{V}_C \equiv \mathbf{V}(H_{CL}) =$$

$$\frac{RQ}{1-\beta^{Q}} + \frac{1-\beta}{\left(1-\beta F(\tilde{\theta}|\delta^{i}_{C})\right)(1-\beta^{Q}) + \beta^{Q}(1-\beta)}\left(EUC_{Q}(\delta^{i}_{C}) - RQ\right) \qquad \textbf{[10.B]}$$

---

[1]Notice that this implies that deviations are unprofitable, so it would be warranted to interpret such deviations if they occurred (they do not in equilibrium) just as accidents that do not justify, therefore, the revision of the expectations for the continuation of the game.

where

$$\mathrm{EUC}_Q(\delta_C^i) \equiv \frac{1 - \beta^Q}{1 - \beta}\ \mathrm{EUC}(\delta_C^i)$$

$$\mathrm{EUC}(\delta_C^i) \equiv \mathbb{E}_t^{\xi}\left[\Pi_{t+1}\left[\delta_C^i,\ \theta_{t+1}\left(\delta_C^i,\ \delta_C^{-i}\right), \mathbb{E}\left[\Theta^{t+2}\left(\sigma\,\middle|_{H'}\right)\right]\right] \middle\|\ H_{CL}\right]$$

$$\mathrm{RQ} \equiv \sum_{h=1}^{Q} \mathbb{E}_t^{\xi}\left[\beta^{h-1}\Pi_{t+h}^i\left[\delta_{N_h}^i,\ \theta_{t+h}\left(\delta_{N_h}^i, \delta_{N_h}^{-i}\right),\ \mathbb{E}\left[\Theta^{t+1+h}\left(\sigma\,\middle|_{H_{N_h}}\right)\right]\right] \middle\|\ H_{CH}\right]$$

where the function F(.) is the distribution function defined in Section 4 of the paper and $F(\tilde{\theta}\,|\,\delta_C^i) \equiv \Pr\left(\theta_{t+1}(\delta_C^i,\ \sigma^{-1}(H_{CL})) \leq \tilde{\theta}\right)$ is a shorthand for $F(\tilde{\theta} - \delta_C^i - \sum_{j\neq i} \delta_C^j)$.

Note that EUC_Q is simply the discounted value of the same single-period utility payoff in the cooperative stage if that payoff were received during Q successive periods, while RQ is the discounted sum of the Q different single-period payoffs of the reversionary phase. Thus, the expression for $\mathbf{V}_C$ in **[10.B]** above says that the expected discounted payoff of the game as of the cooperative stage H_{CL} equals the expected discounted utility of a game in which the reversionary phase is immediately and indefinitely succeeded by another one (the first term in **[10.B]**), plus the gains in payoff (appropriately discounted) resulting when cooperation instead of punishment is in place (the second term in **[10.B]**). Note that equation **[10.B]** does not have exactly the same form than in Porter (1983) or GP because, unlike in those models, in this one the values of the single-period payoffs corresponding to each stage of the punishment phase can be different from one another.

APPENDIX C (*)

This Appendix presents the more technical material involved in the formulation and computation of the model. It is divided into three sections. Section I.C discusses how the equilibria can be computed with a first order conditions (Kuhn-Tucker) approach. Section II.C explains the method for computing the conditional expectations appearing in the delegates' single-period payoffs. Section III.C gives details on the algorithm and software used in the numerical implementation of the solution.

I.C - COMPUTATION OF THE EQUILIBRIA WITH A FIRST ORDER CONDITIONS APPROACH

In this section we indicate how to exploit the continuity and (twice) differentiability of the delegate's payoff to pursue the computation of the equilibria with a first order conditions (Kuhn-Tucker) approach.

If the symmetric strategy σ defines a QSNE equilibrium, then δ^i_C optimizes the expected payoff of the game when $H = H_{CL}$ and therefore equation **[10.B]** of Appendix B must satisfy:

$$V^i(H_{CL}) = \frac{RQ}{1-\beta^Q} + \frac{1-\beta}{\left[1-\beta F\left(\tilde{\theta}\,|\,\sigma^i(H_{CL})\right)\right]\left(1-\beta^Q\right)+\beta^Q(1-\beta)}\left[EUC_Q\left(\sigma^i(H_{CL})\right) - RQ\right] =$$

$$= \frac{RQ}{1-\beta^Q} + \frac{1-\beta}{\left[1-\beta F\left(\tilde{\theta}\,|\,\delta^i_C\right)\right]\left(1-\beta^Q\right)+\beta^Q(1-\beta)}\left[EUC_Q(\delta^i_C) - RQ\right] >$$

$$> \frac{RQ}{1-\beta^Q} + \frac{1-\beta}{\left[1-\beta F\left(\tilde{\theta}\,|\,\delta^i\right)\right]\left(1-\beta^Q\right)+\beta^Q(1-\beta)}\left[EUC_Q(\delta^i) - RQ\right] \qquad \textbf{[1.C]}$$

for all $\delta^i \in S_i$, $i = 1, 2, \ldots, N$; where all the symbols are as in Appendix A and $F\left(\tilde{\theta}\,|\,\delta^i\right)$ is a shorthand for $F\left(\tilde{\theta} - \delta^i - \sum_{j\neq i}\sigma^j(H_{CL})\right)$.

(*) Corresponds to Appendix C of Zarazaga (1993).

If $\delta^i_C > 0$, the inequality in **[1.C]** implies that

$$\frac{\partial EUC}{\partial \delta^i_C} - \frac{\beta(1-\beta)f\left(\tilde{\theta}\,|\,\delta^i_C\right)}{\left[1-\beta F\left(\tilde{\theta}\,|\,\delta^i_C\right)\right]\left(1-\beta^Q\right)+\beta^Q(1-\beta)}\left[EUC_Q(\delta^i_C) - RQ\right] = 0 \qquad \textbf{[2.1.C]}$$

where the left hand side is the derivative of the expression following the inequality sign in **[1.C]**, evaluated at $\delta^i = \delta^i_C$, taking into account that $\partial F\left(\tilde{\theta}\,|\,\delta^i\right)/\partial\delta^i \equiv -f\left(\tilde{\theta}\,|\,\delta^i\right)$.

Inequalities such as **[1.C]** should hold in equilibrium at any stage of the game, and therefore expression **[9.B]** of Appendix B readily suggests that, for the process $\{\xi_t\}$ of the paper, the following Q first order necessary conditions must also be satisfied at the optimal strategy vector σ:

$$\frac{\partial \mathbb{E}^{\xi}_t\left[\Pi^i_{t+1}\left[\delta^i_{N_m}, \theta_{t+1}\left(\delta^i_{N_m}, \delta^{-i}_{N_m}\right), \mathbb{E}^{\xi}\left[\Theta^{t+2}\left(\sigma\,|_{H_{N_m}}\right)\right]\right] \,\middle|\, H_{N_{m-1}}\right]}{\partial \delta^i_{N_m}} = 0 \qquad \textbf{[2.2.C]}$$

$$m = 1, 2, \ldots, Q$$

for all $\delta^i \in S_i$, $i = 1, 2, \ldots, N$.

The first order necessary conditions for an interior solution **[2.C]** define a system of Q+1 equations in Q+1 unknowns that, provided such a solution exists, can be solved in principle numerically. Given the difficulties mentioned in Section 5 of the paper, however, there is no guarantee that all or any of those solutions will correspond to a global maximum, so the issue of whether a particular one also solves the dynamic programming problems **[5.B]-[7.B]** of Appendix B must be explored by direct evaluation of the payoffs at all the strategy vectors σ satisfying **[2.C]**.

Before concluding this section, we would like to remark that equation **[2.1.C]** is typical of repeated games with trigger strategies. In particular, it is entirely analogous to the corresponding expression derived by Porter (1983) and Green and Porter (1984), except for the fact that the Q single-period payoffs added into RQ (see subsection II.3.B of Appendix B) are different from each other. It has the standard interpretation that in equilibrium the gains from myopic deviations, given by the first term in the left hand side, must be exactly offset by the (appropriately discounted) marginal losses (represented by $EUC_Q - RQ$) from triggering reversionary episodes with higher probability (represented by $f(\tilde{\theta}\,|\,\delta^i_C)$) given by the second term in the left hand side.

II.C - COMPUTATION OF EXPECTATIONS

It should be clear from the previous section that a critical step in the computation of the equilibria will be the evaluation of the conditional expectations appearing in the single-period payoff [13] of the text, that is, of expressions of the form:

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = H\right] \equiv \mathbb{E}\left[\sum_{h=0}^{\infty} \beta^h \left[\prod_{k=1}^{h} \frac{1}{1 + \xi(H_{t+k}, \mu_{t+k+1}) + \sum_i \sigma^i(H_{t+k})} \right] \middle| H_{t+1} = H \right] \qquad [3.C]$$

for all possible histories H, $H \in \Sigma$.

Most of this section is devoted to explaining the steps that, under the i.i.d. assumption for $\{\xi_t\}$, allowed us to obtain easily computable formulas for these expectations. At the end we will briefly indicate how such formulas were actually used in evaluating the expected value at t of t+1 single-period payoffs (or of functions of them).

The basic linear structure of the expectations revealed by definition [3.C] and the Markovian, recursive nature of the game suggests that an analytical expression for them can be obtained inductively by repeated application of the law of iterated expectations. That is, using the convention of subsection 3.3 of the paper when h < k, we can rewrite definition [3.C] as

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1}\right] \equiv 1 + \beta\, \mathbb{E}\left[\frac{1}{1+\theta_{t+2}} \middle| H_{t+1}\right] + \beta^2\, \mathbb{E}\left[\frac{1}{1+\theta_{t+2}} \frac{1}{1+\theta_{t+3}} \middle| H_{t+1}\right] +$$

$$+ \beta^3\, \mathbb{E}\left[\frac{1}{1+\theta_{t+2}} \frac{1}{1+\theta_{t+3}} \frac{1}{1+\theta_{t+4}} \middle| H_{t+1}\right] + \ldots \qquad [4.C]$$

and then apply the law of iterated expectations to each of these components. The introduction of some notation is necessary before proceeding along these lines.

We start by discussing the computation of [4.C] under trigger strategy equilibria. Recall from Section I.A of Appendix A that the distribution of θ_t does not have constant support, but shifts at different stages of the game along any QSNE equilibrium path. From [1], [15], [16] in the text, and the fact that $\xi \in [\underline{\xi}, \overline{\xi}]$, it is easy to verify that that support will be given by:

$$
\theta \in \begin{cases}
\left[\sum_i^N \delta_C^i + \underline{\xi},\ \tilde{\theta}\right] \equiv \Omega_{CL} & \text{if } \ H_{t+1} = H_{CL} \\
\left[\tilde{\theta},\ \sum_i^N \delta_C^i + \bar{\xi}\right] \equiv \Omega_{CH} & \text{if } \ H_{t+1} = H_{CH} \\
\left[\sum_i^N \delta_{N_h}^i + \underline{\xi}\ ,\ \sum_i^N \delta_{N_h}^i + \bar{\xi}\right] \equiv \Omega_{N_h} & \text{if } H_{t+1} = H_{N_h} \qquad h = 1, 2, \ldots, Q.
\end{cases}
$$

where $\tilde{\theta}$ is the trigger value for the rate of growth of the money supply.

Taking into account identity **[1]** in the text and that f(.) is the density function of the i.i.d. process $\{\xi_t\}$ defined in Section 4 of the paper, denote:

$$
I_{CL} \equiv \int_{\Omega_{CL}} \frac{1}{1 + \theta_{t+2}} f\left(\theta_{t+2} - \sum_i^N \delta_C^i\right) d\xi
$$

$$
I_{CH} \equiv \int_{\Omega_{CH}} \frac{1}{1 + \theta_{t+2}} f\left(\theta_{t+2} - \sum_i^N \delta_C^j\right) d\xi \qquad \textbf{[5.C]}
$$

$$
I_C \equiv \int_{\Omega_{CL} \cup \Omega_{CH}} \frac{1}{1 + \theta_{t+2}} f\left(\theta_{t+2} - \sum_i^N \delta_C^i\right) d\xi \equiv I_{CL} + I_{CH}
$$

$$
I_{N_h} \equiv \int_{\Omega_{N_h}} \frac{1}{1 + \theta_{t+2}} f\left(\theta_{t+2} - \sum_i^N \delta_{N_h}^i\right) d\xi \qquad h = 1, 2, \ldots, Q.
$$

In defining the above integrals, we have taken into account the fact that the delegates take the actions after t+1 as given and determined by the proposed equilibrium strategy σ.

We now apply the law of iterated expectations to each of the terms in the right hand side of **[4.C]**, starting with the case $H_{t+1} = H_{CL}$. For the second term, we have:

$$\mathbb{E}\left[\frac{1}{1+\theta_{t+2}} \middle| H_{t+1} = H_{CL}\right] = \int_{\Omega_{CH} \cup \Omega_{CL}} \frac{1}{1+\theta_{t+2}} f\left(\theta_{t+2} - \sum_{i}^{N} \delta_{c}^{i}\right) d\xi \equiv I_{c} \qquad \textbf{[6.1.C]}$$

The third term in **[4.C]** can be written:

$$\mathbb{E}\left[\frac{1}{1+\theta_{t+2}} \frac{1}{1+\theta_{t+3}} \middle| H_{t+1} = H_{CL}\right] = \mathbb{E}\left[\frac{1}{1+\theta_{t+2}} \mathbb{E}\left[\frac{1}{1+\theta_{t+3}} \middle| H_{t+2}\right] \middle| H_{t+1} = H_{CL}\right] =$$

$$= \int_{\Omega_{CL} \cup \Omega_{CH}} \frac{1}{1+\theta_{t+2}} \mathbb{E}\left[\frac{1}{1+\theta_{t+3}} \middle| H_{t+2}\right] f(\xi)\, d\xi =$$

$$= \int_{\Omega_{CL}} \frac{1}{1+\theta_{t+2}} \mathbb{E}\left[\frac{1}{1+\theta_{t+3}} \middle| H_{t+2} = H_{CL}\right] f(\xi)\, d\xi +$$

$$+ \int_{\Omega_{CH}} \frac{1}{1+\theta_{t+2}} \mathbb{E}\left[\frac{1}{1+\theta_{t+3}} \middle| H_{t+2} = H_{CH}\right] f(\xi)\, d\xi =$$

where the last equality follows from the transition between H_{t+1} and H_{t+2} dictated by **[15]** in the text. Taking into account that the i.i.d. assumption implies that the conditional expectations under the integral signs are independent of the particular realization of ξ_{t+2} within the respective limits of integration, and considering again **[15]** in the text and **[5.C]** above, the last expression is equal to

$$= I_C \int_{\Omega_{CL}} \frac{1}{1+\theta_{t+2}} f(\xi)\, d\xi + I_{N_1} \int_{\Omega_{CH}} \frac{1}{1+\theta_{t+2}} f(\xi)\, d\xi = I_C\, I_{CL} + I_{N_1} I_{CH} \qquad \textbf{[6.2.C]}$$

Applying an entirely analogous procedure to the fourth term in **[4.C]** we get

$$\mathbb{E}\left[\frac{1}{1+\theta_{t+2}}\,\frac{1}{1+\theta_{t+3}}\,\frac{1}{1+\theta_{t+4}}\,\middle|\, H_{t+1} = H_{CL}\right] =$$

$$= I_C (I_{CL})^2 + I_{CH}\, I_{CL}\, I_{N_1} + I_{CH}\, I_{N_1}\, I_{N_2} \qquad \textbf{[6.3.C]}$$

Inspection of expressions **[6.C]** suggests that reiterated application of this procedure to each of the remaining infinite terms in **[4.C]** will allow us to express them as products and sums of the simple integrals in **[5.C]**.

In practice, this method gets unwieldy, and it is better to define a transition matrix **Z** and derive the expressions for the conditional expectations by successive multiplications of that matrix. That is, define the matrix **Z** as:

	H_{CL}	H_{CH}	H_{N_1}	H_{N_2}	. . . .	H_{N_Q}
H_{CL}	I_{CL}	I_{CH}	0	0	0	0
H_{CH}	0	0	I_{N_1}	0	0	0
H_{N_1}	0	0	0	I_{N_2}	0	0
H_{N_2}	0	0	0	0	I_{N_3} 0	0
.						
.						
.						
.						
$H_{N_{Q-1}}$	0	0	0	0	0 0 . . .	I_{N_Q}
H_{N_Q}	I_{CL}	I_{CH}	0	0	0 0 . . .	0

Note that the sum of the elements of the first row of **Z** gives I_C, which is precisely **[6.1.C]**.

Multiplication of **Z** by itself would show that the sum of the terms in the first row of $\mathbf{Z}^2$ equals $I_{CL}\ I_C + I_{CH}\ I_{N_1}$, which is precisely the expression found in **[6.2.C]**.

Likewise, the sum of the terms in the first row of $\mathbf{Z}^3$ gives $I_C\ (I_{CL})^2 + I_{CH}\ I_{CL}\ I_{N_1} + I_{CH}\ I_{N_1} I_{N_2}$, which is the expression for **[6.3.C]**.

Proceeding in this fashion infinitely many times will give expressions entirely analogous to those in **[6.C]** for each of the infinite terms in the right hand side of **[4.C]**.

In general, each of those terms (except for the first) can be written:

$$\mathbb{E}\left[\prod_{m=1}^{T} \frac{1}{1+\theta_{t+1+m}} \,\middle|\, H_{t+1} = H_r\right] = \sum_{s=1}^{Q+2} \mathbf{z}(\mathrm{T})_{rs}$$

where H_r denotes the history represented by the first row of **Z** and $\mathbf{z}(\tau)_{rs}$ denotes the r–th row and s–th column of the matrix $\mathbf{Z}^{\tau}$. This implies that it is possible to represent **[3.C]** as follows:

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = H_r\right] \equiv 1 + \sum_{\tau=0}^{\infty} \sum_{s=1}^{Q+2} \mathbf{z}(\tau)_{rs} \qquad \textbf{[7.C]}$$

Expression **[7.C]** is of no practical use in the form above because it involves the task of evaluating numerically an infinite number of terms, which of course could not be accomplished in finite time. However, the recursive, stationary nature of the problem induces a recursive pattern as well in the terms in the right hand side of **[7.C]** that can be exploited to derive a compact, easily computable formula for that infinite sum. Thus, the only loss of accuracy in evaluating **[7.C]** (or **[3.C]**) will come from approximation errors in the numerical integration of the simple integrals in **[5.C]**, which can be made arbitrarily small by the appropriate choice of grid.

For example, after grouping terms in the right hand side of **[7.C]** according to their recurrent pattern, and after considerable algebra, the conditional expectations **[3.C]** when $H_{t+1} = H_{CH}$ are given by:

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = H_{CH}\right] =$$

$$1 + \beta I_{N_1} + \beta^2 I_{N_1} I_{N_2} + \beta^3 I_{N_1} I_{N_2} I_{N_3} + \dots + \beta^Q I_{N_1} I_{N_2} I_{N_3} \dots I_{N_{Q-1}} I_{N_Q} +$$

$$+ \beta^{Q+1} I_{N_1} I_{N_2} I_{N_3} \dots I_{N_{Q-1}} I_{N_Q} \frac{I_C + I_{CH} * IQ}{1 - \beta\left(I_{CL} + \beta^Q I_{CH} I_{N_1} I_{N_2} I_{N_3} \dots I_{N_{Q-1}} I_{N_Q}\right)} \qquad \textbf{[8.C]}$$

where

$$IQ = \beta I_{N_1} + \beta^2 I_{N_1} I_{N_2} + \beta^3 I_{N_1} I_{N_2} I_{N_3} + \dots + \beta^Q I_{N_1} I_{N_2} I_{N_3} \dots I_{N_{Q-1}} I_{N_Q}$$

An entirely analogous procedure allows us to derive the conditional expectations for the remaining histories. An inductive argument will show that those conditional expectations are Markovian (as ought to be expected given the nature of the problem) and, whenever $H \neq H_{CL}$, are linked by the formula:

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = \Gamma(H, \theta_{t+1})\right] = \frac{\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = H\right] - 1}{\beta \, I_{\Gamma(H, \theta_{t+1})}} \qquad \text{[9.C]}$$

where Γ is the transition function defined in **[15]** in the text and $I_{\Gamma(H, \theta_{t+1})}$ denotes the integral in **[5.C]** whose subindex corresponds to the history $\Gamma(H, \theta_{t+1})$ that succeeds history H.

Thus, once **[8.C]** has been evaluated, that is, once the expectations corresponding to history H_{CH} have been evaluated, the conditional expectations for the remaining histories can be easily computed from **[9.C]**. For illustration purposes, we fully spell out the conditional expectations that would obtain following this procedure for the case Q = 2.

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = H_{CH}\right] = 1 + \beta \, I_{N_1} + \beta^2 \, I_{N_1} I_{N_2} + \beta^3 \, I_{N_1} I_{N_2} \frac{I_C + I_{CH}\left(\beta \, I_{N_1} + \beta^2 \, I_{N_1} I_{N_2}\right)}{1 - \beta\left(I_{CL} + \beta^2 \, I_{CH} I_{N_1} I_{N_2}\right)}$$

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = H_{N_1}\right] = 1 + \beta \, I_{N_2} + \beta^2 \, I_{N_2} \frac{I_C + I_{CH}\left(\beta \, I_{N_1} + \beta^2 \, I_{N_1} I_{N_2}\right)}{1 - \beta\left(I_{CL} + \beta^2 \, I_{CH} I_{N_1} I_{N_2}\right)}$$

$$\mathbb{E}\left[\Theta^{t+2} \middle| H_{t+1} = H_{N_2} \text{ or } H_{CL}\right] = 1 + \beta \frac{I_C + I_{CH}\left(\beta \, I_{N_1} + \beta^2 \, I_{N_1} I_{N_2}\right)}{1 - \beta\left(I_{CL} + \beta^2 \, I_{CH} I_{N_1} I_{N_2}\right)}$$

Notice that for the case of MSNE, that is, when $\sigma^{i}(H) = \delta_{MSNE}$ and $Q = \infty$, we would have $I_{CL} = 0$ and $I_{CH} = I_{N_1} = I_{N_2} = \ldots = I_N$, so that **[8.C]** (and **[9.C]**) will collapse to

$$\mathbb{E}\left[\Theta^{t+2} \,|\, H_{t+1} = H\right] = \frac{1}{1 - \beta\, I_N}$$

which is exactly the same expression that could have been derived directly from **[3.C]**.

Once the conditional expectations for all possible public histories have been evaluated, it is rather straightforward to compute the expected value at t of the t+1 single-period payoffs (or of its derivatives, as needed in **[2.C]**). To see this, follow the steps indicated in subsection 3.3 and, for the case in which $H_t = H_{CL}$, write down **[13]** in the text explicitly as:

$$\mathbb{E}\left[\Pi_{t+1} \,|\, H_t = H_{CL}\right] = \ln \lambda^i\, y_{t+1} + \mathbb{E}\left[\left[(1-\gamma)\ \ln \frac{1-\gamma}{\gamma} \frac{1}{\mathbb{E}\left[\Theta^{t+2} \,|\, H_{t+1}\right]} + \right.\right.$$

$$+ \ln \frac{1}{\dfrac{1-\gamma}{\gamma} \dfrac{1}{\mathbb{E}\left[\Theta^{t+2} \,|\, H_{t+1}\right]} + \dfrac{\theta_{t+1}}{1+\theta_{t+1}}} +$$

$$\left.\left. + \mathbf{W}\left(\frac{\delta^i_{t+1}}{1+\theta_{t+1}}\, y_{t+1} \frac{1}{\dfrac{1-\gamma}{\gamma} \dfrac{1}{\mathbb{E}\left[\Theta^{t+2} \,|\, H_{t+1}\right]} + \dfrac{\theta_{t+1}}{1+\theta_{t+1}}}\right)\right] \,\right|\, H_t = H_{CL}\right]$$

Taking into account **[15]** in the text, this is equal to

$$
= \ln \lambda^{i} y_{t+1} + \int_{\underline{\xi}}^{\tilde{\theta}-\delta^{i}_{t+1}-\sum_{j\neq i}\delta^{j}_{C}} \left[(1-\gamma) \ln \frac{1-\gamma}{\gamma} \frac{1}{\mathbb{E}\left[\Theta^{t+2} \middle| H_{CL}\right]} + \right.
$$

$$
+ \ln \frac{1}{\dfrac{1-\gamma}{\gamma} \dfrac{1}{\mathbb{E}\left[\Theta^{t+2} \middle| H_{CL}\right]} + \dfrac{\delta^{i}_{t+1} + \sum_{j\neq i}\delta^{j}_{C} + \xi_{t+1}}{1 + \delta^{i}_{t+1} + \sum_{j\neq i}\delta^{j}_{C} + \xi_{t+1}}} +
$$

$$
\left. + W\left(\frac{\delta^{i}_{t+1}}{1 + \delta^{i}_{t+1} + \sum_{j\neq i}\delta^{j}_{C} + \xi_{t+1}} \; \frac{y_{t+1}}{\dfrac{1-\gamma}{\gamma} \dfrac{1}{\mathbb{E}\left[\Theta^{t+2} \middle| H_{CL}\right]} + \dfrac{\delta^{i}_{t+1} + \sum_{j\neq i}\delta^{j}_{C} + \xi_{t+1}}{1 + \delta^{i}_{t+1} + \sum_{j\neq i}\delta^{j}_{C} + \xi_{t+1}}} \right) \right] dF(\xi) +
$$

$$+ \int_{\tilde{\theta} - \delta^i_{t+1} - \sum_{j \neq i} \delta^j_C}^{\bar{\xi}} \left[(1-\gamma) \ln \frac{1-\gamma}{\gamma} \frac{1}{\mathbb{E}\left[\Theta^{t+2} \middle| H_{CH}\right]} + \right.$$

$$+ \ln \frac{1}{\dfrac{1-\gamma}{\gamma} \dfrac{1}{\mathbb{E}\left[\Theta^{t+2} \middle| H_{CH}\right]} + \dfrac{\delta^i_{t+1} + \sum_{j \neq i} \delta^j_C + \xi_{t+1}}{1 + \delta^i_{t+1} + \sum_{j \neq i} \delta^j_C + \xi_{t+1}}} +$$

$$\left. + \mathbf{W} \left(\frac{\delta^i_{t+1}}{1 + \delta^i_{t+1} + \sum_{j \neq i} \delta^j_C + \xi_{t+1}} \frac{y_{t+1}}{\dfrac{1-\gamma}{\gamma} \dfrac{1}{\mathbb{E}\left[\Theta^{t+2} \middle| H_{CH}\right]} + \dfrac{\delta^i_{t+1} + \sum_{j \neq i} \delta^j_C + \xi_{t+1}}{1 + \delta^i_{t+1} + \sum_{j \neq i} \delta^j_C + \xi_{t+1}}} \right) \right] dF(\xi) \qquad [10.C]$$

Notice that δ^i_{t+1} enters not only into the realized single-period payoff for each possible successor history of H_{CL} but also into the limits of integration because, as ought to be expected from [15] in the text, it affects the transition probability between histories through its effect on the distribution of θ_{t+1}. Of course, this dependence was taken into account in the numerical implementation of the first order conditions [2.C].

The expressions for the other possible histories are entirely analogous to [10.C], although of a simpler structure given that they have only one possible successor history.

III.C - ALGORITHM

The computation of equilibria was accomplished by solving the system of nonlinear equations [2.C], with the terms involving expectations of the form [3.C] evaluated according to the recursive formulas [8.C] and [9.C]. The relevant second order conditions were checked for each solution of [2.C] in order to verify that such solution constituted indeed a local maximum rather than a saddle point. We also considered the possibility of corner solutions by checking the relevant Kuhn-Tucker conditions (all solutions turned out to be interior in the experiments reported in the text.)

The computations required numerical integration at several stages: in computing the single integrals [5.C] and in evaluating the expected value of payoffs and their derivatives in [2.C]. For that purpose, we used the subroutine QAGD of QUADPACK available in the public domain library NETLIB. The algorithm uses quadrature methods and is described with detail in "Quadpack, A Subroutine Package for Automatic Integration," by R. Piessens, E.deDoncker-Kapenga, C. Überhuber, and D. Kahaner, Springer-Verlag, 1983.

The subroutine employed in solving the nonlinear system [2.C] was HYBRD from MINPACK, prepared for the Argonne National Laboratory by Burton S. Garbow, Kenneth E. Hillstrom, and Jorge J. Moré, and available in the public domain library NETLIB. The computational details of the algorithm can be found in "User Guide for Minpack-1," by Moré, Garbow, and Hillstrom, Argonne National Laboratory, Argonne, Illinois.

The above subroutines were combined in the following algorithm:

Step 1) Set the initial vector δ_0 to some arbitrary value.

Step 2) Evaluate the conditional expectations [3.C] at δ_0 and replace these values in [2.C].

Step 3) Evaluate the FONC's [2.C] at δ_0 by performing the corresponding numerical integrations.

Step 4) Approximate numerically the Jacobian of the system [2.C].

Step 5) Using the information from 3) and 4), select a new vector δ and restart the process from 1) with the new δ replacing δ_0 until the following convergence criteria is satisfied:

$$\Delta \leq XTOL^* \|\delta\|$$

where Δ is the step bound, $\| \cdot \|$ is the Euclidean norm, and XTOL is the relative error desired in the approximate solution. That is, termination occurs when the relative error between two consecutive iterations is at most XTOL.

All the computations were carried out in double precision. XTOL was set to the standard square root of the machine precision. The step bound is set internally in the subroutine HYBRD so as to guarantee that the absolute value of the functions corresponding to the FONC effectively decrease in each iteration. For further details consult the reference above ("User Guide for MINPACK-1" by Moré, Garbow, and Hillstrom).

In order to verify that the solution corresponded to a local maximum in a fairly wide neighborhood of the solution to the system **[2.C]**, a grid procedure was followed. The left hand side of the FONC **[2.C]** corresponding to delegate i for history H_r was evaluated in the neighborhood of δ^i_r, where δ^i_r is the proposed optimal action at history H_r, with the other delegates' actions for that history set at the proposed solution, that is, $\delta^{-i} = \sigma(H_r)$ and the conditional expectations of the form **[3.C]** evaluated at δ, that is, at the proposed solution vector for all histories. That is, the left hand sides of the FONC's **[2.C]** were evaluated taking as given the current stage choices of δ by the other coalitions and future choices of δ by all coalitions.[1] In all cases, the left hand sides of each of the equations in **[2.C]** were decreasing in δ^i, with δ^i set at each of the endpoints of the 10,000 subintervals in which the interval [0, 10E4] was divided. This verified that the solution satisfied the second order conditions for a local maximum in that interval as well.

Finally, the robustness of the solutions was checked by restarting the algorithm from different initial points. In all cases in which convergence was attained, it was to the same solution. Moreover, only one solution was found for each of the numerical experiments reported in the text.

---

[1]Recall that in time-consistent stationary Nash equilibria the decisions are made sequentially.

RESEARCH PAPERS OF THE RESEARCH DEPARTMENT
FEDERAL RESERVE BANK OF DALLAS

Available, at no charge, from the Research Department
Federal Reserve Bank of Dallas, P. O. Box 655906
Dallas, Texas 75265-5906

Please check the titles of the Research Papers you would like to receive:

___ 9201 Are Deep Recessions Followed by Strong Recoveries? (Mark A. Wynne and Nathan S. Balke)
___ 9202 The Case of the "Missing M2" (John V. Duca)
___ 9203 Immigrant Links to the Home Country: Implications for Trade, Welfare and Factor Rewards (David M. Gould)
___ 9204 Does Aggregate Output Have a Unit Root? (Mark A. Wynne)
___ 9205 Inflation and Its Variability: A Note (Kenneth M. Emery)
___ 9206 Budget Constrained Frontier Measures of Fiscal Equality and Efficiency in Schooling (Shawna Grosskopf, Kathy Hayes, Lori L. Taylor, William Weber)
___ 9207 The Effects of Credit Availability, Nonbank Competition, and Tax Reform on Bank Consumer Lending (John V. Duca and Bonnie Garrett)
___ 9208 On the Future Erosion of the North American Free Trade Agreement (William C. Gruben)
___ 9209 Threshold Cointegration (Nathan S. Balke and Thomas B. Fomby)
___ 9210 Cointegration and Tests of a Classical Model of Inflation in Argentina, Bolivia, Brazil, Mexico, and Peru (Raul Anibal Feliz and John H. Welch)
___ 9211 Nominal Feedback Rules for Monetary Policy: Some Comments (Evan F. Koenig)
___ 9212 The Analysis of Fiscal Policy in Neoclassical Models[1] (Mark Wynne)
___ 9213 Measuring the Value of School Quality (Lori Taylor)
___ 9214 Forecasting Turning Points: Is a Two-State Characterization of the Business Cycle Appropriate? (Kenneth M. Emery & Evan F. Koenig)
___ 9215 Energy Security: A Comparison of Protectionist Policies (Mine K. Yücel and Carol Dahl)
___ 9216 An Analysis of the Impact of Two Fiscal Policies on the Behavior of a Dynamic Asset Market (Gregory W. Huffman)
___ 9301 Human Capital Externalities, Trade, and Economic Growth (David Gould and Roy J. Ruffin)
___ 9302 The New Face of Latin America: Financial Flows, Markets, and Institutions in the 1990s (John Welch)
___ 9303 A General Two Sector Model of Endogenous Growth with Human and Physical Capital (Eric Bond, Ping Wang, and Chong K. Yip)
___ 9304 The Political Economy of School Reform (S. Grosskopf, K. Hayes, L. Taylor, and W. Weber)
___ 9305 Money, Output, and Income Velocity (Theodore Palivos and Ping Wang)
___ 9306 Constructing an Alternative Measure of Changes in Reserve Requirement Ratios (Joseph H. Haslag and Scott E. Hein)
___ 9307 Money Demand and Relative Prices During Episodes of Hyperinflation (Ellis W. Tallman and Ping Wang)
___ 9308 On Quantity Theory Restrictions and the Signalling Value of the Money Multiplier (Joseph Haslag)
___ 9309 The Algebra of Price Stability (Nathan S. Balke and Kenneth M. Emery)
___ 9310 Does It Matter How Monetary Policy is Implemented? (Joseph H. Haslag and Scott Hein)
___ 9311 Real Effects of Money and Welfare Costs of Inflation in an Endogenously Growing Economy with Transactions Costs (Ping Wang and Chong K. Yip)
___ 9312 Borrowing Constraints, Household Debt, and Racial Discrimination in Loan Markets (John V. Duca and Stuart Rosenthal)
___ 9313 Default Risk, Dollarization, and Currency Substitution in Mexico (William Gruben and John Welch)
___ 9314 Technological Unemployment (W. Michael Cox)
___ 9315 Output, Inflation, and Stabilization in a Small Open Economy: Evidence from Mexico (John H. Rogers and Ping Wang)
___ 9316 Price Stabilization, Output Stabilization and Coordinated Monetary Policy Actions (Joseph H. Haslag)
___ 9317 An Alternative Neo-Classical Growth Model with Closed-Form Decision Rules (Gregory W. Huffman)
___ 9318 Why the Composite Index of Leading Indicators Doesn't Lead (Evan F. Koenig and Kenneth M. Emery)

___ 9319 Allocative Inefficiency and Local Government: Evidence Rejecting the Tiebout Hypothesis (Lori L. Taylor)
___ 9320 The Output Effects of Government Consumption: A Note (Mark A. Wynne)
___ 9321 Should Bond Funds be Included in M2? (John V. Duca)
___ 9322 Recessions and Recoveries in Real Business Cycle Models: Do Real Business Cycle Models Generate Cyclical Behavior? (Mark A. Wynne)
___ 9323* Retaliation, Liberalization, and Trade Wars: The Political Economy of Nonstrategic Trade Policy (David M. Gould and Graeme L. Woodbridge)
___ 9324 A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics (Eric W. Bond, Ping Wang,and Chong K. Yip)
___ 9325 Growth and Equity with Endogenous Human Capital: Taiwan's Economic Miracle Revisited (Maw-Lin Lee, Ben-Chieh Liu, and Ping Wang)
___ 9326 Clearinghouse Banks and Banknote Over-issue (Scott Freeman)
___ 9327 Coal, Natural Gas and Oil Markets after World War II: What's Old, What's New? (Mine K. Yücel and Shengyi Guo)
___ 9328 On the Optimality of Interest-Bearing Reserves in Economies of Overlapping Generations (Scott Freeman and Joseph Haslag)
___ 9329* Retaliation, Liberalization, and Trade Wars: The Political Economy of Nonstrategic Trade Policy (David M. Gould and Graeme L. Woodbridge) (Reprint of 9323 in error)
___ 9330 On the Existence of Nonoptimal Equilibria in Dynamic Stochastic Economies (Jeremy Greenwood and Gregory W. Huffman)
___ 9331 The Credibility and Performance of Unilateral Target Zones: A Comparison of the Mexican and Chilean Cases (Raul A. Feliz and John H. Welch)
___ 9332 Endogenous Growth and International Trade (Roy J. Ruffin)
___ 9333 Wealth Effects, Heterogeneity and Dynamic Fiscal Policy (Zsolt Becsi)
___ 9334 The Inefficiency of Seigniorage from Required Reserves (Scott Freeman)
___ 9335 Problems of Testing Fiscal Solvency in High Inflation Economies: Evidence from Argentina, Brazil, and Mexico (John H. Welch)
___ 9336 Income Taxes as Reciprocal Tariffs (W. Michael Cox, David M. Gould, and Roy J. Ruffin)
___ 9337 Assessing the Economic Cost of Unilateral Oil Conservation (Stephen P.A. Brown and Hillard G. Huntington)
___ 9338 Exchange Rate Uncertainty and Economic Growth in Latin America (Darryl McLeod and John H. Welch)
___ 9339 Searching for a Stable M2-Demand Equation (Evan F. Koenig)
___ 9340 A Survey of Measurement Biases in Price Indexes (Mark A. Wynne and Fiona Sigalla)
___ 9341 Are Net Discount Rates Stationary?: Some Further Evidence (Joseph H. Haslag, Michael Nieswiadomy, and D. J. Slottje)
___ 9342 On the Fluctuations Induced by Majority Voting (Gregory W. Huffman)
___ 9401 Adding Bond Funds to M2 in the P-Star Model of Inflation (Zsolt Becsi and John Duca)
___ 9402 Capacity Utilization and the Evolution of Manufacturing Output: A Closer Look at the "Bounce-Back Effect" (Evan F. Koenig)
___ 9403 The Disappearing January Blip and Other State Employment Mysteries (Frank Berger and Keith R. Phillips)
___ 9404 Energy Policy: Does it Achieve its Intended Goals? (Mine Yücel and Shengyi Guo)
___ 9405 Protecting Social Interest in Free Invention (Stephen P.A. Brown and William C. Gruben)
___ 9406 The Dynamics of Recoveries (Nathan S. Balke and Mark A. Wynne)
___ 9407 Fiscal Policy in More General Equilibriium (Jim Dolman and Mark Wynne)
___ 9408 On the Political Economy of School Deregulation (Shawna Grosskopf, Kathy Hayes, Lori Taylor, and William Weber)
___ 9409 The Role of Intellectual Property Rights in Economic Growth (David M. Gould and William C. Gruben)
___ 9410 U.S. Banks, Competition, and the Mexican Banking System: How Much Will NAFTA Matter? (William C. Gruben, John H. Welch and Jeffery W. Gunther)
___ 9411 Monetary Base Rules: The Currency Caveat (R. W. Hafer, Joseph H. Haslag, andScott E. Hein)
___ 9412 The Information Content of the Paper-Bill Spread (Kenneth M. Emery)
___ 9413 The Role of Tax Policy in the Boom/Bust Cycle of the Texas Construction Sector (D'Ann Petersen, Keith Phillips and Mine Yücel)
___ 9414 The P* Model of Inflation, Revisited (Evan F. Koenig)
___ 9415 The Effects of Monetary Policy in a Model with Reserve Requirements (Joseph H. Haslag)

___	9501	An Equilibrium Analysis of Central Bank Independence and Inflation (Gregory W. Huffman)
___	9502	Inflation and Intermediation in a Model with Endogenous Growth (Joseph H. Haslag)
___	9503	Country-Bashing Tariffs: Do Bilateral Trade Deficits Matter? (W. Michael Cox and Roy J. Ruffin)
___	9504	Building a Regional Forecasting Model Utilizing Long-Term Relationships and Short-Term Indicators (Keith R. Phillips and Chih-Ping Chang)
___	9505	Building Trade Barriers and Knocking Them Down: The Political Economy of Unilateral Trade Liberalizations (David M. Gould and Graeme L. Woodbridge)
___	9506	On Competition and School Efficiency (Shawna Grosskopf, Kathy Hayes, Lori L. Taylor and William L. Weber)
___	9507	Alternative Methods of Corporate Control in Commercial Banks (Stephen Prowse)
___	9508	The Role of Intratemporal Adjustment Costs in a Multi-Sector Economy (Gregory W. Huffman and Mark A. Wynne)
___	9509	Are Deep Recessions Followed By Strong Recoveries? Results for the G-7 Countries (Nathan S. Balke and Mark A. Wynne)
___	9510	Oil Prices and Inflation (Stephen P.A. Brown, David B. Oppedahl and Mine K. Yücel)
___	9511	A Comparison of Alternative Monetary Environments (Joseph H. Haslag))
___	9512	Regulatory Changes and Housing Coefficients (John V. Duca)
___	9513	The Interest Sensitivity of GDP and Accurate Reg Q Measures (John V. Duca)
___	9514	Credit Availability, Bank Consumer Lending, and Consumer Durables (John V. Duca and Bonnie Garrett)
___	9515	Monetary Policy, Banking, and Growth (Joseph H. Haslag)
___	9516	The Stock Market and Monetary Policy: The Role of Macroeconomic States (Chih-Ping Chang and Huan Zhang)
___	9517	Hyperinflations and Moral Hazard in the Appropriation of Seigniorage: An Empirical Implementation With A Calibration Approach (Carlos E. Zarazaga)

Name:	**Organization:**
Address:	**City, State and Zip Code:**
Please add me to your mailing list to receive future Research Papers: _____ **Yes** ____ **No**	

Research Papers Presented at the
1994 Texas Conference on Monetary Economics
April 23-24, 1994
held at the Federal Reserve Bank of Dallas, Dallas, Texas

Available, at no charge, from the Research Department
Federal Reserve Bank of Dallas, P. O. Box 655906
Dallas, Texas 75265-5906

Please check the titles of the Research Papers you would like to receive:

____ 1 A Sticky-Price Manifesto (Laurence Ball and N. Gregory Mankiw)

____ 2 Sequential Markets and the Suboptimality of the Friedman Rule (Stephen D. Williamson)

____ 3 Sources of Real Exchange Rate Fluctuations: How Important Are Nominal Shocks? (Richard Clarida and Jordi Gali)

____ 4 On Leading Indicators: Getting It Straight (Mark A. Thoma and Jo Anna Gray)

____ 5 The Effects of Monetary Policy Shocks: Evidence From the Flow of Funds (Lawrence J. Christiano, Martin Eichenbaum and Charles Evans)

Name:	Organization:
Address:	City, State and Zip Code:
Please add me to your mailing list to receive future Research Papers:	______ Yes ______ No