Fiscal Policy in More General Equilibrium

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*The views expressed in this article are solely those of the authors and should not be attributed to the Federal Reserve Bank of Dallas or to the Federal Reserve System.

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Abstract

In this paper we examine the sensitivity of existing results in the equilibrium analysis of fiscal policy to assumptions about the slope of the long-run supply curve of capital. In the 'standard' model, based on the neoclassical growth model, the long-run supply of capital is perfectly elastic at the representative agent's fixed rate of time preference. This assumption is shown to have strong implications for the effects of government consumption purchases on output, employment, interest rates and other macroeconomic variables. We explore the implications of relaxing this assumption in a more general model that allows for flexible time preference. We show that the multiplier effect of permanent changes in government purchases on output is enhanced, primarily as a result of increased capital accumulation. In an interesting Keynesian twist, private consumption may in fact *rise* in response to increased government purchases.

1 Introduction

Perfectly elastic or perfectly inelastic supply or demand curves have much to recommend them. Equilibrium analysis which would otherwise be fraught with ambiguity yields forth sharp predictions when one assumes either demand or supply are either perfectly elastic or inelastic. Nonetheless, this is not the way we typically teach equilibrium analysis nor, in most circumstances, perform it. Neoclassical macroeconomics is an exception to this rule. More generally, capital accumulation models in which a representative agent maximizes the standard additively-separable, fixed-discount-factor utility function—to which class most equilibrium business cycle models belong—imply a long run supply curve for capital which is perfectly elastic at the agent's fixed rate of time preference.

This property of the what we will refer to as the 'standard' model is, and has been, well-known and well-criticized, even by users of the standard model.[1] The present paper should not be seen as a general condemnation of that model. All modelling must strike some balance between tractability and realism; it is because reality is so intractable that we have need for models at all.

However, the question of exactly where and when this assumption ceases to be innocuous—*i.e.*, for what sorts of experiments it is or isn't a harmless simplification—has been given surprisingly short shrift.[2] A number of instances where this assumption could be important suggest themselves, one instance in particular being the recent literature on the dynamic general equilibrium effects of fiscal policy. The way to interpret the present exercise—or at least our intent—is as a 'robustness check' of the standard model with particular regard to this branch of the neoclassical macroeconomics literature.

The remainder of the paper is organized as follows. In section 2 we summarize some recent results in the equilibrium analysis of fiscal policy, as well as demonstrate some properties of the standard model which will be important for our subsequent analysis. Section 3 of the paper lays out a more general framework, which allows for a non-zero slope to capital's long-run supply. The section begins with a description of the type of preferences we will use, and then sets about solving the more general model and characterizing its equilibrium in terms of efficiency conditions.

Our analysis of fiscal policy proper is then undertaken from two perspectives, a 'comparative steady state' analysis—exploring the two models' differing responses to truly permanent changes in government purchases—and a quantitative, numerical analysis of the effects of both transitory and persistent changes on the models' complete dynamical

[1] In *partial* equilibrium, it implies an 'all or nothing' type of behavioral response—when faced with constant interest rates, agents wish to hold either no capital or an infinite amount. In *general* equilibrium, unless all agents share the same common discount factor, all capital ends up in the hands of the most patient agent; when agents share the same discount factor, the long-run distribution of capital holdings across agents is indeterminate. See, for example, Becker [6].

[2] A recent exception is a paper of Gomme and Greenwood [14], which utilizes an endogenous time preference specification similar to ours in a real business cycle model. These sorts of preferences have also, quite naturally, shown up in the open economy macro literature, where for a small open economy fixity of time preference implies an indeterminacy in the economy's long-run debt position. The need to get away from fixed rates of time preference is here very clear and has been addressed, for example, by Mendoza [23].

systems.

The steady state analysis, in section 4, is particularly useful for developing the intuition of what makes models with flexible time preference 'different.' We begin by demonstrating that permanent changes in purchases can have long-run output effects even absent elastic labor supply, a result impossible in the standard model. The output effect here is fully attributable to capital accumulation. Putting elastic labor supply back into the model, this 'capital accumulation effect' is shown to account for much of the difference in the sizes of the output effects which the fixed- and flexible-discount-factor models generate. In particular, under reasonable parameter values, introducing flexible time preference in a manner consistent with an upward-sloping long-run capital supply curve can generate much *larger* output effects—'multipliers'—than the standard model, while keeping the employment effect basically the same. Also, permanent changes in government purchases, even when financed through lump-sum taxes, give rise to long-run interest rate effects in the more general model. In a somewhat Keynesian twist, steady-state consumption may actually *rise* in response to a permanent increase in purchases, depending on the value assigned to a parameter which gauges the responsiveness of time preference to changes in consumption and leisure. The agent is, nonetheless, worse off as a result.

Section 5 contains our analysis of the effects of both transitory and persistent changes in government purchases on output, employment, investment and so forth in both the short and long runs. In particular, following the numerical solution techniques outlined in [20], we approximate the models' dynamics in a linear fashion and report responses of the approximate dynamical systems to deviations in purchases which display different degrees of persistence. We find that the results of Baxter and King [5] and Aiyagari, Christiano and Eichenbaum [1]—that transitory shocks to purchases yield smaller output effects than persistent shocks—continue to obtain even in our more general framework. In the case of transitory shocks, we find that the impact effects on employment, consumption and output are much larger, and the impact effect on investment much smaller, in the flexible-time preference model, though the propagation is significantly weaker—the transition back to the steady state is quite rapid. The same is true for the responses of the real wage and the real interest rate. In the case of persistent—in fact 'nearly permanent'—shocks, the effects at impact on all quantity and price variables are qualitatively the same across the two models, but much larger in the more general model. Subsequent to impact, the differing responses of the two models is accounted for largely by the 'capital accumulation effect' which arose in our steady state analysis.

2 The equilibrium approach to fiscal policy

The 'equilibrium approach to fiscal policy' analyzes the effects of government purchases, distortionary taxes, government financing rules, conscription, *etc.* within explicit models of dynamic general equilibrium, emphasizing the supply-side responses of capital and labor to various policies, rather than the usual Keynesian demand-side response of consumption and income.[3] Like much of the equilibrium business cycle literature, the equilibrium approach to fiscal policy utilizes the standard neoclassical growth model, fleshed out, of course, to include taxes, government purchases, productive government investment and so forth. The recent papers of Aiyagari, Christiano and Eichenbaum [1] and Baxter and King [5] provide a good summary of the main results thus far in the 'equilibrium approach.'

Aiyagari, Christiano and Eichenbaum have shown analytically, and Baxter and King *via* numerical simulations, that the basic neoclassical stochastic growth model augmented to incorporate government purchases yields results on the magnitudes of effects on output of persistent and transitory changes in purchases which contradict the arguments given by Barro [3] and Hall [16]. In particular, persistent changes in government purchases are shown to have a larger effect on output than transitory changes. Numerically, both Baxter and King and Aiyagari, Christiano and Eichenbaum have shown that for some parametrizations of the model, for persistent changes in government spending, one does get true spending 'multipliers' in the sense of unit changes in spending leading to greater-than-unit changes in output.[4] Baxter and King have also shown that the government's financing decision is, in some cases, more important than the resource costs of government purchases, and that productive government investment can have large effects on private-sector output and investment.

At a purely *analytical* level, these results of course rely on the fact that the preferences which the authors specify are of the standard time-additively separable, fixed-discount-factor variety. This is true of many results. At an *intuitive* level, though, the fixity of the rate of time prefer-

[3]Early work in this area was done by Aschauer [2] and Barro [4], though these analyses utilized models either without elastic labor supply [4] or without capital [2]. Much more complete treatments are given by Aiyagari, Christiano and Eichenbaum [1] and, especially, Baxter and King [5], who consider a thorough list of fiscal policy experiments.

[4]Throughout, we use the terms 'multiplier' and 'multiplier effects' to denote cases where increases in government spending yield greater than one-for-one increases in output. Cases where changes in government spending lead to *any* changes in output, not necessarily more than one-for-one, we refer to simply as 'output effects.'

ence *should* be important simply because the implied long-run supply of capital pins down the steady-state real interest rate, to which the model must return subsequent to any shock to purchases.

That a fixed rate of time preference sharpens the derivation of multiplier effects for permanent changes in government purchases is apparent enough from simply considering the deterministic steady state of the basic neoclassical growth model augmented to incorporate government purchases.[5] Since much of the basic framework will be used throughout our analysis, it's worth stating that framework formally at the outset. Here and below, F is a constant-returns-to-scale production function with capital, k, and labor, n, as inputs. Expected lifetime utility, with the rate of time preference fixed, is given by

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t, 1-n_t)\right\} \tag{1}$$

where c is consumption and $1-n$ is leisure, the agent's per-period endowment of discretionary time having been normalized to unity. Each period the economy faces a resource constraint of the form

$$c_t + k_{t+1} = F(k_t, n_t) + (1-\delta)\,k_t - g_t, \tag{2}$$

where δ is the depreciation rate of capital and g denotes government purchases. We abstract from distortionary taxation in order to focus solely on the effects of government purchases as a pure drain on output.[6] Further, optima and equilibria will continue to coincide under lump-sum financing, so we may treat the equilibrium as the solution to an optimal growth problem—maximizing (1) subject to (2) at each date—given a stochastic process for government purchases.

The deterministic steady state of this model—setting $g_t = g$ for all t—is described by three equations. The three equations are a labor-market-clearing equation—*i.e.*, the intratemporal efficiency condition equating the marginal rate of substitution between consumption and leisure to the real wage; a capital-market-clearing equation—the steady state version of the consumption Euler equation; and the steady state version of the resource constraint:[7]

$$\frac{D_2u(c, 1-n)}{D_1u(c, 1-n)} = D_2F(k, n) \tag{3}$$

[5]This is essentially the tack taken by King in [19].

[6]Baxter and King [5], however, have shown that the presence of distortionary taxation has important implications within the standard model, and this would no doubt be true in our model as well. The presence of distortionary taxes also renders important the question of financing.

[7]Here and elsewhere, the notation $D_i f$ denotes the ith partial derivative of the function f, and $D_{ij} f$ the jth partial derivative of $D_i f$.

$$1/\beta = D_1 F(k,n) + 1 - \delta \tag{4}$$

$$F(k,n) - \delta k = c + g. \tag{5}$$

With F assumed to exhibit constant returns to scale, equilibrium in the capital market—equation (4)—pins down the long-run capital-labor ratio independent of g. [See Figure 1] Consequently, the right hand side of (3) is fixed as well—in essence labor *demand* is rendered perfectly elastic at a fixed real wage, independent of g as well. Let $z \doteq k/n$ and $f(z) = F(z,1) - \delta z$. Let z^* denote the capital-labor ratio determined by the capital market clearing condition. The question of whether multipliers exist boils down to calculating the derivative of the function $n(g)$ defined implicitly by

$$\frac{D_2 u(nf(z^*) - g, 1-n)}{D_1 u(nf(z^*) - g, 1-n)} = f(z^*) - z^* f'(z^*)\,.$$

A multiplier will exist whenever $n'(g) > 1/f(z^*)$.[8] It is straightforward to show that there are specifications of u, F, β, δ and g yielding this result. Given that we have relatively more confidence, empirically, in what the last four primitives on this list should look like than we do in regard to u, the existence proposition would typically be stated as "long-run multipliers will exist if leisure is sufficiently income-elastic."

What's going on here can be visualized in a simple graph with consumption on one axis and leisure on the other. [See Figure 2] Given that the capital-labor ratio has been determined in the capital market, the long run equilibrium occurs at the intersection of two curves in consumption-leisure space. One curve is simply the 'income expansion path' of $u(c,l)$ when the wage rate is given by $w(z^*) = f(z^*) - z^* f'(z^*)$—*i.e.*, the collection of all pairs (c,l) with $D_2 u(c,l)/D_1 u(c,l) = w(z^*)$. The other curve represents the locus of feasible consumption-leisure pairs given the capital-labor ratio z^*. It is the downward-sloping straight line determined by the equation $c = (1-l)\,f(z^*) - g$. Permanent changes in g induce parallel shifts in this 'budget line', and the magnitude of the resulting changes in leisure—equivalently, labor—depend on the slopes of the 'income expansion path' and 'budget line' near the equilibrium. [See Figure 3]

Since the capital-labor ratio is fixed, any change in n is implicitly accompanied by an equal-proportioned change in k. Also, changes in output are proportional to changes in labor as well, and steady state consumption clearly falls. Obviously, in such a model, permanent—*i.e.*, steady state—changes in g have no interest rate effects.

[8]Since, with $y = nf(z^*)$, $dy/dg = n'(g)\,f(z^*)$.

The natural experiment to conduct would then be to demonstrate—given accepted parametrizations of F, β and g—exactly *how* 'income-elastic' leisure has to be. In percentage terms, since a one percent change in leisure yields a $-(1-n)/n$ percent change in labor, a moderate responsiveness of leisure can yield a large responsiveness of labor.[9] One would then ask whether the set of numbers which are "sufficient" overlap with the set of numbers which are "plausible."[10] Our contention is not that this is an unreasonable way to approach the problem; rather, it is that what is "sufficient" no doubt depends on the rather special assumption of a perfectly elastic long-run supply of capital. The aim of the present research is to explore the implications of relaxing that assumption. We might then consider whether the "sufficient" and the "plausible" grow nearer together or farther apart as capital's long-run supply becomes less than perfectly elastic.

3 A More General General Equilibrium Model

Epstein [11] has shown that members of the class of stationary recursive utility functions consistent with expected utility must take the form

$$U(\{c_t\}) = E\left\{\sum_{t=0}^{\infty} u(c_t)\exp\left[-\sum_{s=0}^{t-1} v(c_s)\right]\right\}$$

in the case of a single consumption good. Intuitively, and in theory, if labor supply is elastic, there's no reason why this cannot be modified to

$$U(\{c_t, n_t\}) = E\left\{\sum_{t=0}^{\infty} u(c_t, n_t)\exp\left[-\sum_{s=0}^{t-1} v(c_s, n_s)\right]\right\}.$$

For reasons of parsimony and tractability, we adopt the following specification:

$$U(\{c_t, n_t\}) = E\left\{\sum_{t=0}^{\infty} B_t u(c_t, 1-n_t)\right\} \tag{6}$$

where

$$B_t = \prod_{s=0}^{t-1} \beta[u(c_s, 1-n_s)]. \tag{7}$$

That is, the utility discount factor β which an agent applies each period to next-period's utility depends on how much *utility* he or she received

[9] Following King, *op. cit.*, estimates of the long-run fraction of discretionary hours devoted to labor range variously from two-tenths to one-third, implying $(1-n)/n$ in the range of two to four.

[10] For example, Pencavel [25].

this period, rather than on c_t and n_t directly. It's probably clearer here to look at the aggregator representation:

$$U_t = u(c_t, 1-n_t) + \beta[u(c_t, 1-n_t)]\, E_t\tilde{U}_{t+1}. \tag{8}$$

This differs from the time-additive case only in the dependence of β on current utility u. We say this form is suggested for reasons of 'parsimony' because simply saying $\beta = \beta(c, 1-n)$ would imply having to specify numbers like $D_{12}\beta(c, 1-n)$, whereas the form we propose only requires elasticities like $u\beta'(u)\,/\beta(u)$ and $u\beta''(u)\,/\beta'(u)$ to be specified. Furthermore, as will become clear, this form of flexible discounting does not alter the agent's intratemporal consumption-leisure choice.

As in the standard model described above, when we restrict our attention to government spending financed through lump-sum taxes, optima and equilibria continue to coincide in the more general model under mild convexity, monotonicity and interiority assumptions. Characterizing equilibria, then, amounts to characterizing solutions to the problem of maximizing (6) subject to the sequence of resource constraints (2) given k_0 and a stochastic process for g_t.

There are a number of ways to think about solving this problem from an analytical standpoint. The choice we make here is to set the problem in a standard discrete-time optimal control framework, using a technique introduced by Obstfeld [24] in a continuous-time context. Pick any fixed β between zero and one. The utility function can then be written as

$$\sum_{t=0}^{\infty} \beta^t x_t u\,(c_t, 1-n_t)$$

where

$$x_t = (1/\beta)^t \prod_{s=0}^{t-1} \beta\,[u\,(c_s, 1-n_s)]\,.$$

Importantly, x_t can be incorporated as an added state variable, and its definition can be written as the state-transition equation

$$x_{t+1} = \frac{\beta\,[u\,(c_t, 1-n_t)]}{\beta} x_t \tag{9}$$

together with the initial condition $x_0 = 1$. The complete problem is then given by

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t x_t u\,(c_t, 1-n_t) \right\} \tag{10}$$

subject to the state-transition constraints for x_t, equation (9), and for capital k_t:

$$k_{t+1} = F\,(k_t, 1-n_t) + (1-\delta)\,k_t - g_t - c_t \tag{11}$$

plus initial conditions, k_0 given and $x_0 = 1$.

Letting λ denote the costate for capital and μ the costate for x, the first order conditions for this problem are:

$$x_t D_1 u_t \times \left\{ 1 + \mu_t \frac{\beta'(u_t)}{\beta} \right\} = \lambda_t \tag{12}$$

$$x_t D_2 u_t \times \left\{ 1 + \mu_t \frac{\beta'(u_t)}{\beta} \right\} = \lambda_t D_2 F_t \tag{13}$$

$$\lambda_t = \beta E_t \lambda_{t+1} \left\{ D_1 F_{t+1} + 1 - \delta \right\} \tag{14}$$

$$\mu_t = \beta E_t \left\{ u_{t+1} + \beta(u_{t+1}) \mu_{t+1} \right\} \tag{15}$$

as well as the two original transition equations, (9) and (11).

In principle, the first two equations (12) and (13) generate solutions for the controls, c_t and n_t, as functions of k_t, x_t, λ_t and μ_t. Plugging these into the four state and costate equations, and specifying a transition for the forcing variable g, yields a five-variable first order stochastic difference equation system, which completely summarizes the dynamics of the model given initial conditions and transversality conditions.

Note that the steady state values of the quantities k, c and n—given a constant g—are independent of the choice of the number β. So, a natural way to proceed is to first calculate steady state $\bar{c}$ and $\bar{n}$, and then set the number β equal to $\beta[u(\bar{c}, 1 - \bar{n})]$. Alternatively, if the steady state value of the discount factor is a parameter we wish to impose in quantitative experiments, we would choose the *number* β equal to this parameter, and parametrize the *function* $\beta(\cdot)$ to guarantee consistency with this choice. The latter method is in fact the one we will adopt in our numerical simulations.

In our quantitative analysis, as should be clear from the first order conditions above, linearization yields coefficients involving elasticities of $\beta(u)$—$u\beta'(u)/\beta(u)$, which we will subsequently denote by ν_u, and $u\beta''(u)/\beta'(u)$, which we denote by ν_{uu}. The fixed-discount-factor case can then be recovered by setting both of these parameters equal to zero. The results we obtain—and in fact the stability of the dynamic system—when ν_u and ν_{uu} are non-zero will depend on both the sizes and signs of these parameters. The appendix discusses stability restrictions on these and other parameters, though at this point it's worthwhile to discuss at least one important choice which we make—the sign of $\beta'(u)$. Since utility is increasing in consumption and leisure, which are in turn increasing in income, we are in fact asking a well-worn question—dating back to Fisher [15] and Hayek [17]—as to whether impatience increases or decreases with income.

The case of $\beta' > 0$—so that increases in within-period-utility bring the discount factor closer to one—can be thought of as reflecting some idea that the more happiness I receive today, the more 'patient' I become with respect to future happiness. Conversely, $\beta' < 0$ corresponds to the equally arguable notion that the more happiness I receive today, the less I care about future installments of happiness.

Perhaps the most compelling case—offered originally by Hayek, subsequently formalized by Epstein [11], Lucas and Stokey [22] and others—is that $\beta' < 0$ guarantees long-run stability in the one-sector model. This is most easily seen by abstracting for a moment from labor supply. If labor supply were inelastic, then one could think of $1/\beta\left[u\left(f\left(k\right)-k\right)\right]$ as representing the long-run supply curve for capital. Then, at least for values of k with $f\left(k\right)-k$ increasing, $\beta' < 0$ corresponds to an upward-sloping long-run supply curve. This, in fact, is the assumption we maintain throughout our analysis.[11]

4 Long-run Output Effects

To get a feel for the impact of flexible time preference, it's worth initially considering the deterministic steady state of a model otherwise identical to ours, but with *inelastic* labor supply.[12] That is, suppose utility is defined only over consumption, and output depends only on capital, or equivalently capital per worker. Again let f denote the neoclassical production function in 'per worker' form, net of depreciation—*i.e.*, $f\left(k\right)=F\left(k,1\right)-\delta k$. When time preference is fixed, the steady state is determined by the familiar conditions

$$c=f(k)-g$$

and

$$\beta f'(k)=1.$$

Obviously, capital—hence output—is independent of g, and consequently any change in g is exactly offset by a change in consumption, c. There

[11]Lawrence [21], using PSID data, finds evidence that subjective discount factors *rise* with labor income, though it's not clear what implication this has for our assumption of $\beta' < 0$. In particular, individual rates of time preference in her specification are assumed to be independent of individual consumption. The Euler equations which she uses to obtain her estimates are thus identical to the ones the standard model would generate, except in that the β's are allowed to differ across individuals. Further, as a little algebra applied to the impulses responses we later report will show, the discount factor and labor *income* are positively related in the experiments we conduct as well.

[12]Devereux [8] was the first to address the effects of fiscal policy in the context of a model such as this with flexible time preference. However, his primary focus was on the response of interest rates to temporary and permanent spending shocks.

are no output effects of permanent changes in g in this setting.

Now, consider what happens when time preference is endogenous. Let β depend on $u(c)$. The steady state is now determined by the resource constraint, given above, and a new version of the Euler equation—

$$\beta[u(c)]\,f'(k) = 1.$$

In this case, changes in g are not offset one-for-one by changes in consumption, and in fact when $\beta' < 0$ the steady state capital stock and steady-state output *rise*—though output rises less than one-for-one.[13] Plug the resource constraint $c = f(k) - g$ into the Euler equation and differentiate to obtain

$$\frac{dk}{dg} = \frac{\beta' u' f'}{\beta' u' [f']^2 + \beta f''} = \frac{|\beta'|\, u' f'}{|\beta'|\, u' [f']^2 + \beta\, |f''|} > 0.$$

Since output depends only on k, the associated change in steady state output is given by

$$\frac{dy}{dg} = f' \frac{dk}{dg} = \frac{|\beta'|\, u' [f']^2}{|\beta'|\, u' [f']^2 + \beta\, |f''|},$$

which is positive, but clearly less than one. Nonetheless, there *is* an output effect, deriving solely from increased capital, or capital per worker, which is not the case in the fixed-time-preference model. That is, even with inelastic labor supply, the mere introduction of an upward-sloping long-run supply of capital gives rise to steady-state output effects of government spending.

What we'll see subsequently is that in the full model—incorporating a labor-leisure choice—flexibility of time preference adds a 'capital accumulation effect' to the standard income effect on labor supply to yield larger multipliers than the standard model. That is, for reasonable parameter values, the *employment* effect of a permanent change in government spending is approximately the same whether time preference is fixed or flexible. But, in the case of flexible time preference, a positive effect on the *capital-labor ratio* leads to larger effects of government spending on *output* than occur within the standard model.

The deterministic steady state of the full model, now allowing for elastic labor supply, is still relatively simple—at least compared to the various formulations of the full dynamical system which we will discuss below. It is determined by three conditions which yield steady values for c, k and n. They are (1) the steady state resource constraint $c =$

[13] As discussed above, the condition $\beta' < 0$ guarantees an *upward* slope of the long-run capital supply curve.

$F(k,n)-\delta k-g$, plus (2) the intratemporal efficiency condition governing the labor market—

$$\frac{D_2u(c,1-n)}{D_1u(c,1-n)}=D_2F(k,n)$$

—and (3) the Euler equation governing the capital market—

$$1=\beta[u(c,1-n)]\left\{D_1F(k,n)+1-\delta\right\}.$$

The fact that the intratemporal efficiency condition is unchanged from the fixed-discount-factor case is a consequence of adopting a rate of time preference which depends on current-period utility, rather than consumption and leisure directly. This fact is easily seen from the first-order conditions (12) and (13) above.

It's instructive to cast this equilibrium in terms similar to those used above in discussing the fixed-discount-factor case—namely, thinking in terms of consumption, labor (or leisure) and the capital-labor ratio. Suppose that the capital-labor ratio, z, is given. Then, the intratemporal efficiency condition again defines an 'income expansion path' consisting of pairs (c,l) such that $D_2u/D_1u=D_2F(z,1)=f(z)-zf'(z)$. The resource constraint again determines a downward-sloping straight line given by $c=\{F(z,1)-\delta z\}n-g=f(z)(1-l)-g$. The intersection of the two curves yields choices of consumption and leisure given z and g—call them $c(z,g)$ and $l(z,g)$. [Just as in Figure 2] The key feature of the *fixed*-discount factor case—as we've already seen—is that the equilibrium value of z is determined independent of g by:

$$f'(z^*)=1/\beta.$$

The effect in that case on l, say, of a change in g is simply the direct effect embodied in the definition of $l(z^*,g)$—which again amounts to moving along the *fixed* income expansion path. Now, however, z is not determined so simply and independently. In a slight abuse of notation, let $u(z,g)\equiv u[c(z,g),l(z,g)]$—that is, $u(z,g)$ is the value of utility consistent with the resource constraint and intratemporal efficiency *given* values for both z and g. The equilibrium value of z in the flexible-discount-factor case is then determined by the capital market condition

$$f'(z)=1/\beta[u(z,g)].$$

The solution to this equation—assuming one exists—will give the steady state capital-labor ratio as a function of g—$z(g)$, say. [See Figure 4] Going back to the 'intratemporal' picture yields up $c(g)=c(z(g),g)$ and $l(g)=l(z(g),g)$.

Now, one can show under standard assumptions that $u(z,g)$ is increasing in z, for a given value of g, and decreasing in g, for a given value of z. If we assume that $\beta' < 0$, $1/\beta[u(z,g)]$ defines an upward-sloping long-run supply curve for capital—actually for z—in the space with z on the horizontal axis and the real interest rate on the vertical axis. Given what we've said about the dependence of $u(z,g)$ on g, and β on u, the supply schedule will shift out—*i.e.*, down and to the right—in response to an increase in g. [See Figure 5]

Now, suppose the economy is in a steady state, given a constant level of purchases g. A permanent increase in purchases from g to $g + \Delta g$, say, will impact simultaneously on the steady-state values of c, n and z. Heuristically, though, it's instructive to view the change in the equilibrium through 'partial equilibrium' glasses—and in terms of our two diagrams characterizing the consumption-leisure choice given the capital-labor ratio and the long-run capital market. Given the original steady state value of z, an increase in government spending impacts on the consumption-leisure choice by shifting downward in parallel fashion the 'budget line' in the consumption-leisure diagram—just as in the fixed-discount-factor model. [See Figure 6] This has the effect of lowering consumption and leisure—*i.e.*, increasing labor—as well as lowering the steady state flow of utility u. In the fixed-discount-factor case, this would be the end of the story, but here the change in u impacts on discounting and hence the capital market. The long-run supply of capital shifts out, leading to a lower steady state interest rate and a higher capital-labor ratio. [Again as in Figure 5] The increased capital-labor ratio in turn impacts on the consumption-leisure choice, affecting both the 'income expansion path'—rotating it upward—and the 'budget line'—increasing its slope and vertical intercept. The contribution of this second adjustment is clearly positive with respect to consumption—relative to the initial 'fixed-z' movement—and ambiguous with respect to leisure. Allowing the capital-labor ratio to adjust can mean either more or less leisure taken in the steady state, relative to the initial fixed-z effect. If we think of the fixed-z effect as the new steady state of the fixed-discount-factor model, then allowing for a flexible discount factor implies an employment effect which can be greater than, less than, or equal to the fixed-discount-factor employment effect.

Suppose the shifts in the 'budget line' and 'income expansion path' engendered by the adjustment of z lead to roughly the same level of steady-state employment as was the case when z was held fixed. Is it then the case that the steady-state *output* effect should be the same in either case? The answer is no, since when the capital-labor ratio changes, movements in output are no longer proportional to movements in labor hours—and here, recalling the outward shift of capital's long-run supply, we have an increase in the capital-labor ratio. Thus, even when introducing flexibility of the discount factor engenders no difference in steady state *employment* effects, effects on *output* are always magnified, relative to the fixed-discount-factor case, by the accumulation of additional steady-state capital.

This scenario is, roughly speaking, exactly what plays itself out when the model is evaluated numerically, given standard parameter values. Precisely, given values for things like factor income shares, expenditure shares, the steady-state interest rate and parameters of u at an original steady state, the changes in c, n and z in response to a small change in g can be written as functions of a parameter—the elasticity of $\beta(u)$—to which the slope of capital's long run supply is proportional. For a wide range of values for this parameter—which we denote ν_u—the employment effect of a given change in steady state g varies slightly, in fact falling, while the output effect increases rather dramatically as ν_u moves further away from zero, which corresponds to the fixed-discount-factor case. The enhanced output effects are due almost entirely to increases in the capital-labor ratio.

To be concrete, suppose that u is given by

$$u(c,l) = \frac{\left[c\, l^{\theta}\right]^{1-\sigma}}{1-\sigma}$$

for $\sigma > 0$, $\sigma \neq 1$. Stability conditions given in Epstein [11] require that $u < 0$, or $\sigma > 1$, which we assume. Other restrictions on σ and $\beta(u)$ are discussed in an appendix. We will assume also that $F(k,n) = k^{1-\alpha} n^{\alpha}$.

Differentiation of the capital-market clearing condition yields

$$-\nu_u\,(\sigma-1)\,\hat{c} + \nu_u\,(\sigma-1)\,\theta \frac{n}{1-n}\hat{n} - \alpha\left[1-\beta\,(1-\delta)\right]\hat{z} = 0,$$

where ν_u denotes the elasticity of $\beta(u)$; α is labor's share of national income; and a hat over a variable denotes percentage deviation from steady state. The condition $\beta' < 0$, in conjunction with other conditions on β and u, guarantees an upward slope to the long-run capital supply curve. Together with the restriction $u < 0$, this implies that $\nu_u > 0$ as is $\nu_u\,(\sigma - 1)$.

Note that when ν_u equals zero, the expression reduces to $\hat{z} = 0$, which reflects the fact, mentioned earlier, that the capital-labor ratio is fixed in the long run in the constant-discount-factor case.

Differentiation of the intratemporal efficiency condition yields:

$$\hat{c} + \frac{n}{1-n}\hat{n} - (1-\alpha)\,\hat{z} = 0.$$

As one would expect, this expression is as in the standard model.

Similarly, the 'hat' version of the resource constraint is also as in the basic model:

$$-s_c\hat{c} + (1 - s_i)\,\hat{n} + [(1 - \alpha) - s_i]\,\hat{z} = s_g\hat{g},$$

where the subscripted s's denote steady-state output shares of consumption, government spending and investment.

Note that ν_u, the elasticity of $\beta(u)$ enters only into the capital-market equation, and even there it is actually $\nu_u\,(\sigma - 1)$ which matters. Let this product be denoted by ξ. Once we specify values for steady state β, α, δ, n, θ, and the shares s_c, s_i and s_g, we can derive solutions for $\hat{c}/\hat{g}$, $\hat{n}/\hat{g}$ and so forth as functions of ξ. Setting $\xi = 0$ recovers the fixed discount factor case. Given solutions for $\hat{z}/\hat{g}$ and $\hat{n}/\hat{g}$, one can also obtain expressions for $\hat{y}/\hat{g}$, which is simply $\hat{n}/\hat{g} + (1 - \alpha)\,\hat{z}/\hat{g}$, and the 'multiplier' dy/dg, which is simply $(1/s_g)\,\hat{y}/\hat{g}$.

Following standard procedure—and in order to maintain comparability with other results—we set $\alpha = .58$ and, following Baxter and King [5], $\beta = .94$. The parameter θ is set, given the other parameter values, so that $n = .20$ is chosen by the agent in the steady state. We choose the empirically plausible value of $s_g = .20$ for government's share.

This leaves the remaining output share parameters—s_c and s_i—and the depreciation rate δ to be specified. Obviously since the shares must sum to one, and $s_g = .20$ has already been chosen, only one output share remains free to be chosen. When the standard model is 'calibrated' rather than estimated, the usual procedure is to impose α, β and δ, and let investment's share take on whatever value is necessary to be consistent with the model's steady state. The standard choice for δ is 10% per annum. Together with $\alpha = .58$ and $\beta = .94$, this implies a steady-state investment share of slightly less than 26%, which seems to us quite high.[14] If we instead impose investment's share to be a more plausible 15%, together with $\alpha = .58$ and $\beta = .94$, the depreciation rate implied by the model is a much smaller 3.55% per annum.[15] Which of the two approaches we choose actually has a significant impact on the results, as will be seen below.

Results of these exercises as ξ—*i.e.*, $\nu_u\,(\sigma - 1)$—varies over $(0, 1]$ are reported in Table 1. The top half is for a 'realistic' investment share

[14] Furthermore, as a simple measurement matter, of the four parameters, the rate of depreciation is certainly the most problematic. An informal survey of macroeconomists confirms this. Respondents were shown the list of four parameters and asked to name the one in which they had the least confidence in measuring empirically. Four out of (the) five macroeconomists questioned answered δ.

[15] Hercowitz [18], using Canadian national accounts data, obtained an estimate of δ of about 5% per annum.

(.15), but a 'low' depreciation rate (.0355); the bottom for 'standard' depreciation (.10), and consequently 'high' investment share (.2564). The interpretation of the tables is that for each variable x, what's recorded in that row are the steady-state elasticities $\hat{x}/\hat{g}$ for each value of ξ running across the top row—so, for example, in the first table, a one percent permanent increase in g when $\xi = 0$ engenders a permanent .1976% increase in labor effort.

5 The Effects of Transitory and Persistent Changes in Government Purchases

We report impulse responses of the full dynamical system to an increase in government spending under various assumptions as to the persistence of the disturbance, the steady-share of output devoted to investment, and the curvature of utility. The results are obtained by treating the problem in the discrete time optimal control manner outlined in section 3, then following the linearization techniques set forth in King, Plosser and Rebelo [20].

For all of our simulations, we maintain a 'core' set of parameters: labor's share of national income, $\alpha = .58$; government's steady state share of national output, $s_g = .20$; and the steady state discount factor, $\beta = .94$.

We continue to maintain the momentary utility function u of the form

$$u(c, l) = \frac{\left[cl^{\theta}\right]^{1-\sigma}}{1-\sigma}$$

where we again take $\sigma > 1$. For all of the results we report, we set $\sigma = 1.5$. In all cases we set the parameter θ to guarantee that $n = .20$ in the steady state.

For the full dynamics, both the parameter ν_u and ν_{uu}—the elasticity of $\beta'(u)$—need to be specified. To that end, we adopt a particular functional form for β—

$$\beta(u) = 1 - e^{\eta u}$$

—where $\eta > 0$. We set the value of η to be consistent with the initial steady-state discount factor $\beta = .94$ and the steady state level of utility u.

For the most part, we also adopt the 'realistic investment share, low depreciation' parameters described in section 4—that is, we impose investment's share of output to be fifteen percent, implying a depreciation of 3.55% per annum. For comparison, we report some impulse responses for 'standard depreciation' of ten percent per annum and investment's share correspondingly 25.64%.

The process for government spending is assumed, in percentage deviations from steady state, to follow an $AR(1)$ process, with AR parameter ρ. We illustrate the effects of a shock to government purchases under three different assumptions about its persistence. The first is a purely temporary shock with $\rho = 0$. The second is a permanent shock, which is mimicked by setting ρ arbitrarily close to one (we set $\rho = .9999$). The third is an intermediate case with $\rho = .94$, which is the estimated value reported by Burnside, Eichenbaum and Rebelo [7].

All impulse responses are for a 1% shock to g, and plot the corresponding paths of $\hat{c}$, $\hat{n}$, $\hat{k}$, *etc.* The horizontal scales in all cases are in years.

The first set of six pictures, Figure 7, records the responses of consumption, effort, the capital stock, output, the interest rate and the real wage for the case of a purely transitory ($\rho = 0$) shock to purchases, under $\sigma = 1.5$ and $s_i = .15$ ($\delta = .0355$). The two paths in the picture of each variable are that variable's response under flexible time preference—in all cases the 'x' line—and fixed time preference—the 'o' line.

The main features one observes in these responses are that, first of all, flexible time preference of the sort we have specified yields a *qualitatively* similar response for five of the six variables as obtains in the fixed time preference case, with output a slight exception in its transition back to steady state. At impact, in both cases consumption and investment (not shown) fall, while effort, and hence—because capital is initially fixed—output, rise. The real wage falls at impact, and the real interest rate rises. In the second and subsequent periods, capital in both cases is below its steady state level, owing to the smaller investment in the impact period. At this point, the transitional dynamics of both models dictate that effort and investment should be high, and consumption low, relative to their steady states until the systems converge back to their original positions. The subsequent paths of output differ in that, in the fixed time preference case, output falls below its steady state in period two, and smoothly rises back up, while in the flexible time preference case, output falls to a level slightly above its steady state, and smoothly falls the rest of the way in subsequent periods.

Quantitatively, the flexible time preference responses show much larger effects at impact on consumption, effort and output than fixed time preference responses. The same can be said for the at-impact responses of the real interest rate and the real wage. Accordingly, the response at impact of investment is smaller in the flexible case, and in the subsequent period the capital stock is nearer to its steady state value than under fixed time preference. Since the model's transitional dynamics from an initially low capital stock take over at this point, and since capital is not quite so far out of line with its steady state value, the flexible-time-preference responses show much less propagation of the shock than do the fixed-time-preference responses.

The greater at-impact responses of consumption and effort—as well as the smaller response of investment—have a simple diagrammatic explanation in terms of the consumption-leisure-investment choice which the representative agent faces at impact. *Given* the level of investment optimal *prior* to the shock, the transitory increase in g has the effect of a parallel shift down in today's consumption-leisure possibilities set. Consumption decreases and labor effort increases. *But*, the originally optimal level of investment is no longer optimal. If we view investment as chosen to equate its marginal cost—the marginal utility of consumption—with its marginal benefit—the discounted expected marginal value of capital—then we've had an upward shift in the marginal cost schedule. In the fixed-discount-factor case, that's all that occurs—consequently, investment is reduced somewhat from its previously optimal level, and the initial negative effects on consumption and leisure checked somewhat. But, with flexible time preference, the increase in the marginal cost of investment is accompanied by an increase in its *marginal benefit*—since the expected marginal value of capital is discounted less as today's utility falls. Consequently, the adjustment in investment is smaller—so investment falls by less in the flexible time preference case—and hence the 'correction' of the initial effects on consumption and effort lessened.

The next set of six pictures—Figure 8—shows, for the same variables and parameter values, responses to a 'permanent' ($\rho = .9999$) shock to purchases. As one would expect, for both flexible and fixed time preference, the effects at impact on consumption, effort and output are much larger now—for example, under fixed time preference, the impact multiplier on output is about .75 in the permanent case versus about .10 in the purely transitory case.[16] There is also now a positive effect on investment, as the increase in the marginal cost of investment is accompanied now in both fixed and flexible cases by a large increase in the marginal benefit of investment—if the shock is going to be around for awhile, the marginal value of extra capital for those periods is high. With flexible time preference, however, we again get a substantial added boost on the marginal benefit side due to the change in discounting. Consequently, the at-impact responses of all variables are larger under flexible time preference than under fixed time preference. This difference is particularly noticeable in investment—where the difference is by more than a

[16]Recall that with $s_g = .20$, the multiplier dy/dg is five times the elasticity $\hat{y}/\hat{g}$. Since $\hat{g}_1 = 1$ in our experiments, the impact multipliers are five times the values of $\hat{y}_1$ recorded in Figures 7.4 and 8.4.

factor of five—and in output—where the impact multiplier is now well over 1.5.

After impact, the dynamics reflect the transitions of the variable to their 'new steady states.' As our comparative steady state analysis showed, the difference between fixed and flexible time preference in this regard is dominated by the desire to greatly increase steady-state capital.

The paths of the interest rate and real wage under flexible time preference are precisely what one would expect given the movements in labor effort and capital—after large impact effects, both quickly settle to their new steady states, the interest rate lower, the real wage higher.

The third set of pictures—Figure 9—illustrate the effect of a shock to purchases when the persistence parameter is chosen to match postwar US data, $\rho = .94$, as estimated by Burnside, Eichenbaum and Rebelo [7]. The responses of the key aggregates are now dramatically different depending on whether the rate of time preference is fixed or flexible. Starting with the response of consumption, note that whereas in the fixed time preference case consumption is persistently below its steady-state level following the shock, with flexible time preference consumption actually exceeds its steady-state level by a modest amount for a while. This is possible because of the persistently greater response of output following the innovation to government spending, which is in turn primarily attributable to the response of capital. Effort also increases by more in the fixed time preference case than in the flexible time preference case, but it is the qualitative difference in the response of capital in each case that plays the key role in the response of output. Under fixed time preference, the shock to government purchases is smoothed in part by running down the capital stock. Under flexible time preference, households accumulate capital to smooth out the effect of the shock. This qualitative difference in the response of capital also explains the differences in the response of the real interest rates and real wages.

We also ran impulse responses under alternative assumptions on σ, the utility curvature parameter, and δ, the depreciation rate, though we will only breifly describe the results of those execises here.

With near-logarithmic utility ($\sigma = 1.001$) the impact, transitional and long-run effects of both transitory and permanent shocks are to all intents and purposes identical under both fixed and flexible time preference specifications. This is not surprising, given that the coefficients of the linearized system depend only on $\nu_u\,(1-\sigma)$ and $\nu_{uu}\,(1-\sigma)$, and not ν_u and ν_{uu} directly. Beyond simply looking at the equations, however, we have yet to formulate a good intuitive explanation—perhaps in terms of countervailing income and substitution effects—of why this should be the case.

What happens if we set $\delta = 0.1$ instead of 0.0355? For temporary shocks, the primary consequence of choosing a higher depreciation rate is to exacerbate the impact effects of the shocks. For permanent shocks, the higher depreciation rate makes little difference for the response of consumption. The impact effect on effort is slightly stronger under the assumption of fixed time preference. There is also a noticeable difference in the long-run response of effort, with the flexible time preference model yielding a larger long-run response. The paths of output become qualitatively more similar with high depreciation, and the long-run responses are enhanced.

6 Conclusions

The manner in which the spending decisions of governments affect the aggregate economy is one of the central questions in macroeconomics. In this paper we have extended the existing literature on the equilibrium approach to fiscal policy to allow for endogenous time preference, thereby generating an upward-sloping long-run supply curve for capital. This contrasts with the existing analyses which assume a perfectly elastic long-run supply curve for capital at the representative agent's rate of time preference. We showed that generalizing the analysis in this manner enhances the output effects of persistent changes in government purchases. The reason for this is the enhanced effect on capital accumulation of permanent changes in wealth. In a surprising Keynesian twist, we showed that it is possible for steady state consumption to increase in response to a permanent change in government purchases. This is in direct contrast to the standard model with fixed time preference, where consumption must always fall in response to increased government purchases.

The analysis in this paper was conducted under the assumption that all government purchases were financed by lump-sum taxes. An obvious extension, which we are currently pursuing, allows for distortionary taxes on labor and capital. A number of other areas for future research also suggest themselves. For example, is it possible to improve the performance of existing dynamic general equilibrium models in terms of their ability to explain cyclical movements in aggregate activity by altering preferences to allow for endogenous time preference.

Finally, it is important to be clear about what is sacrificed in moving to a model with endogenous time preference. In relaxing the assumption of a fixed discount factor, there are many directions one could move in. What's more, in relaxing the fixity of time preference, one faces 'tradeoffs' along several dimensions. First of all, recursivity and stationarity—implying time-consistency and amenability to dynamic programming—

need not necessarily be maintained, though the tractability afforded by recursive, stationary preferences is costly to forego. Likewise, should the preferences be consistent with the expected utility hypothesis? Numerous arguments have been made for moving away from the von Neumann-Morgenstern framework—for example, Epstein and Zin [12], Farmer [13] and Weil [26], to cite but a few. In the interest of deviating as little as possible from the standard model, so as not to cloud our conclusions in a multiplicity of alterations, we opted to maintain consistency with expected utility. Finally, should preferences be consistent with non-stochastic balanced growth? This is a feature of the standard model, when momentary utility is taken to be homogeneous of a fixed degree or logarithmically homogeneous in consumption. We would like to preserve this feature, but as one can see from inspection of Epstein's form for expected-utility-consistent stationary, recursive preferences, this will only be possible if the discount factor is fixed.[17] Apparently, the only intersection of these sets of preferences—stationary and recursive, consistent with expected utility and consistent with balanced growth—is the standard time-additive utility function, with homogeneous or logarithmic momentary utility.[18]

7 Appendix: Restrictions on ν_u, ν_{uu} and σ

One can consult the papers of Epstein [11], Mendoza [23] or Obstfeld [24] for conditions on utility and discounting which guarantee long-run stability in capital accumulation models with flexible time preference of the sort considered above. Putting aside some of the more technical aspects of these conditions, the basic idea is to guarantee that the long-run capital supply curve slopes up—though this is clearly not a *necessary* condition. If the discount factor depends on consumption, and consumption is increasing in steady-state capital, then the discount factor should be decreasing in consumption. The same can be said if the discount factor depends on consumption and leisure, and these are increasing in capital—the discount factor should be decreasing in consumption and leisure.

In our model, these conditions translate into restrictions on the three parameters ν_u, ν_{uu} and σ. The parameter σ figures prominently since our form of utility lets u 'intermediate' the effect of consumption and leisure on discounting.

Epstein and Obstfeld consider models with utility defined only over

[17]In order to be consistent with balanced growth, the intertemporal marginal rate of substitution in consumption must be independent of the scale of consumption.

[18]This conjecture is based on results in [10], [11] and [9].

consumption. Recalling here that

$$u(c,l) = \frac{\left[c\,l^{\theta}\right]^{1-\sigma}}{1-\sigma},$$

and following Mendoza, we may state these conditions with respect to the 'composite' good cl^{θ}, which will be increasing in the level of steady-state capital. Let $v(r) = r^{1-\sigma}/(1-\sigma)$. We then require, with respect to v, $v < 0$; $v' > 0$; and $\ln[-v]$ convex. This will be the case if, as we've assumed throughout, $\sigma > 1$.

With respect to discounting, let $\phi(r) = -\ln\beta[v(r)]$. We require, in addition to the obvious $\phi > 0$, that: $\phi' > 0$; $\phi'' < 0$; and $\exp[\phi(r)]\,v'(r)$ nonincreasing. A little algebra reveals that these conditions translate into the following restrictions on ν_u, ν_{uu} and σ:

$$\nu_u > 0,$$

$$\nu_{uu} \geq \nu_u - \frac{\sigma}{\sigma-1},$$

and

$$\nu_u < \frac{\sigma}{\sigma-1}.$$

These three conditions define, for a given $\sigma > 1$, a simple region in (ν_u, ν_{uu})-space. The size of the region increases as σ approaches one. In particular, for any ν_u we'd like to consider, there's a maximum feasible choice of σ.

The choice of more specific functional forms for β can impose sharper restrictions. For example, the form

$$\beta(u) = 1 - e^{\eta u} \qquad (\eta > 0)$$

has the further property that $(1-\beta)\,\nu_{uu} + \beta\nu_u = 0$. Thus, once we select a value of β for the steady state, we've restricted ourselves to a smaller subset of the collection of feasible ν-pairs. For any ν_u there is again a maximum feasible σ.

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FIGURE 1
LONG-RUN CAPITAL MARKET, STANDARD MODEL

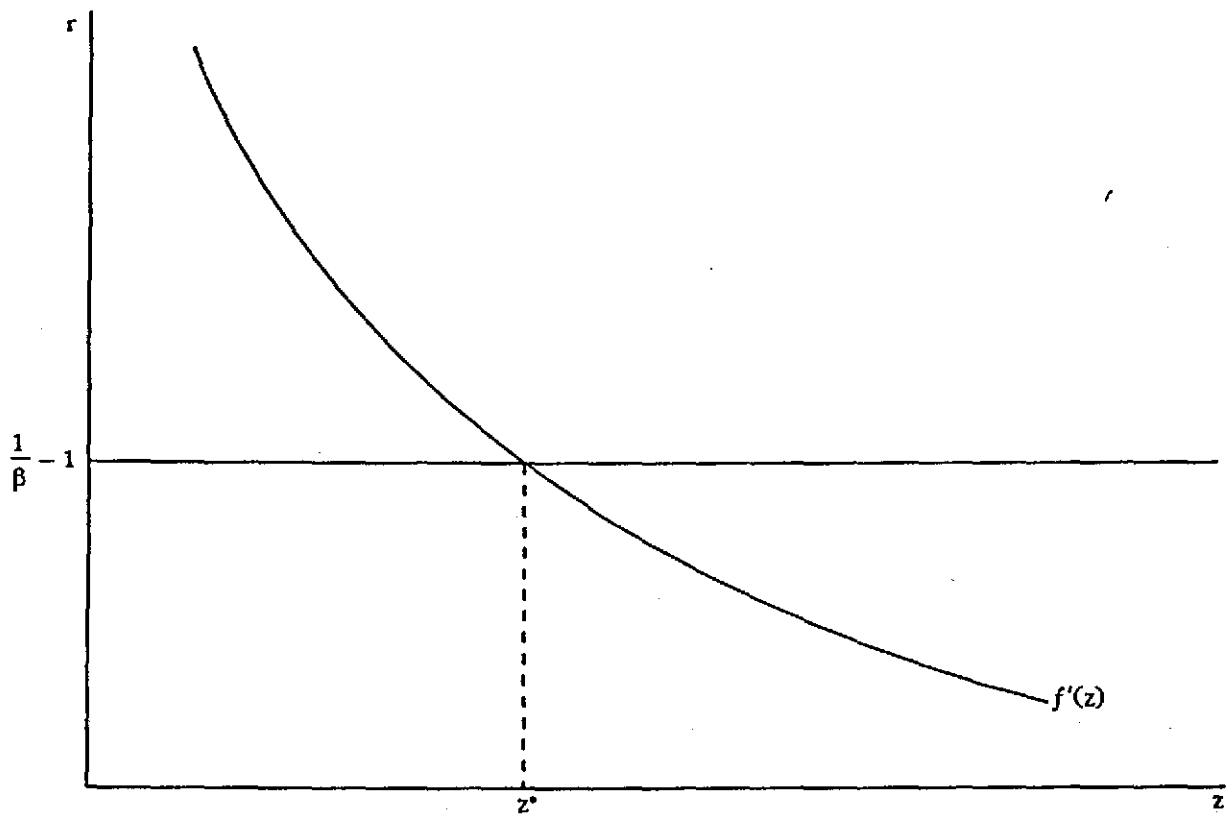


FIGURE 2
LONG-RUN CONSUMPTION-LEISURE CHOICE, STANDARD MODEL

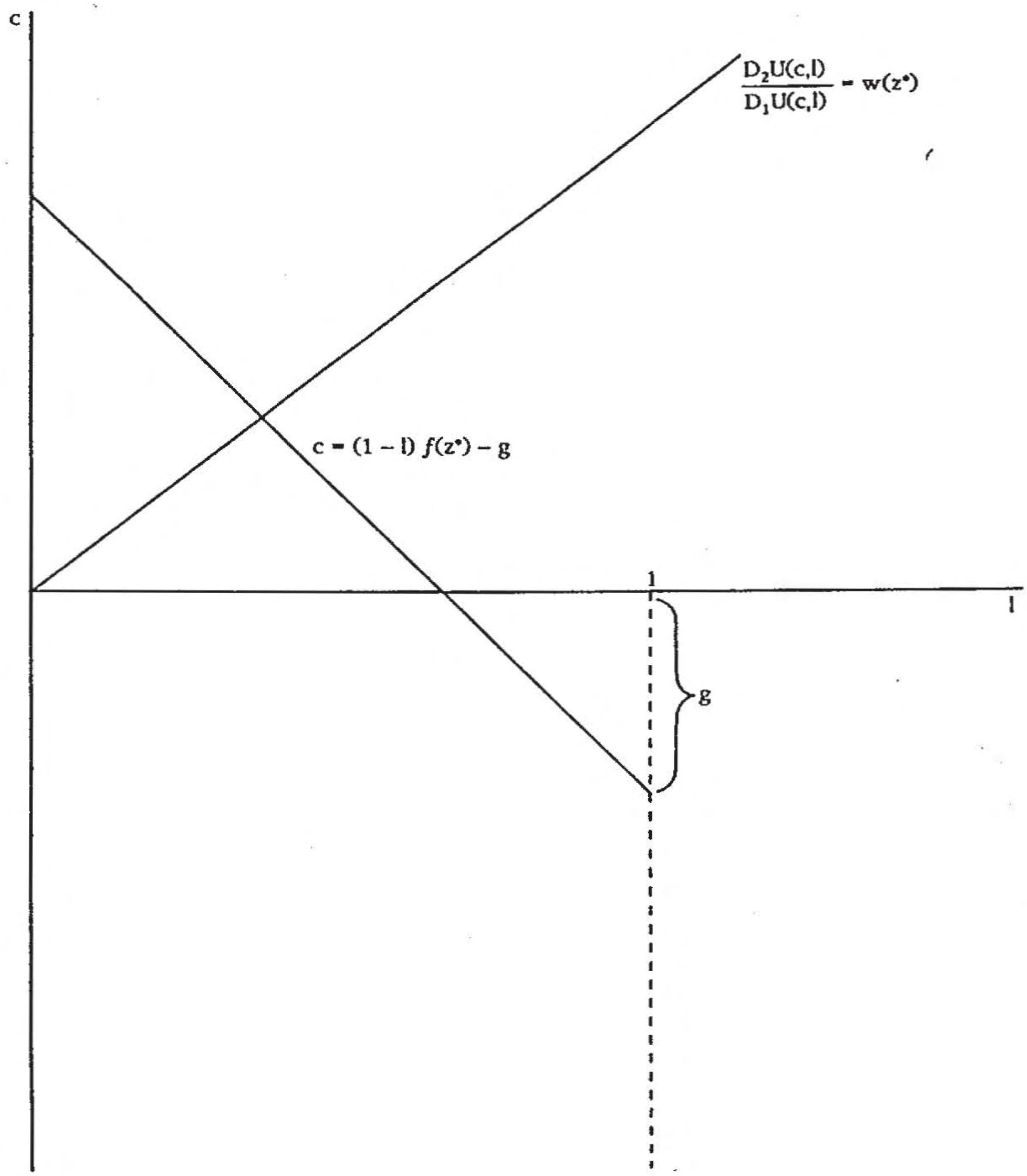


FIGURE 3
LONG-RUN EMPLOYMENT EFFECT OF $\Delta g > 0$, STANDARD MODEL

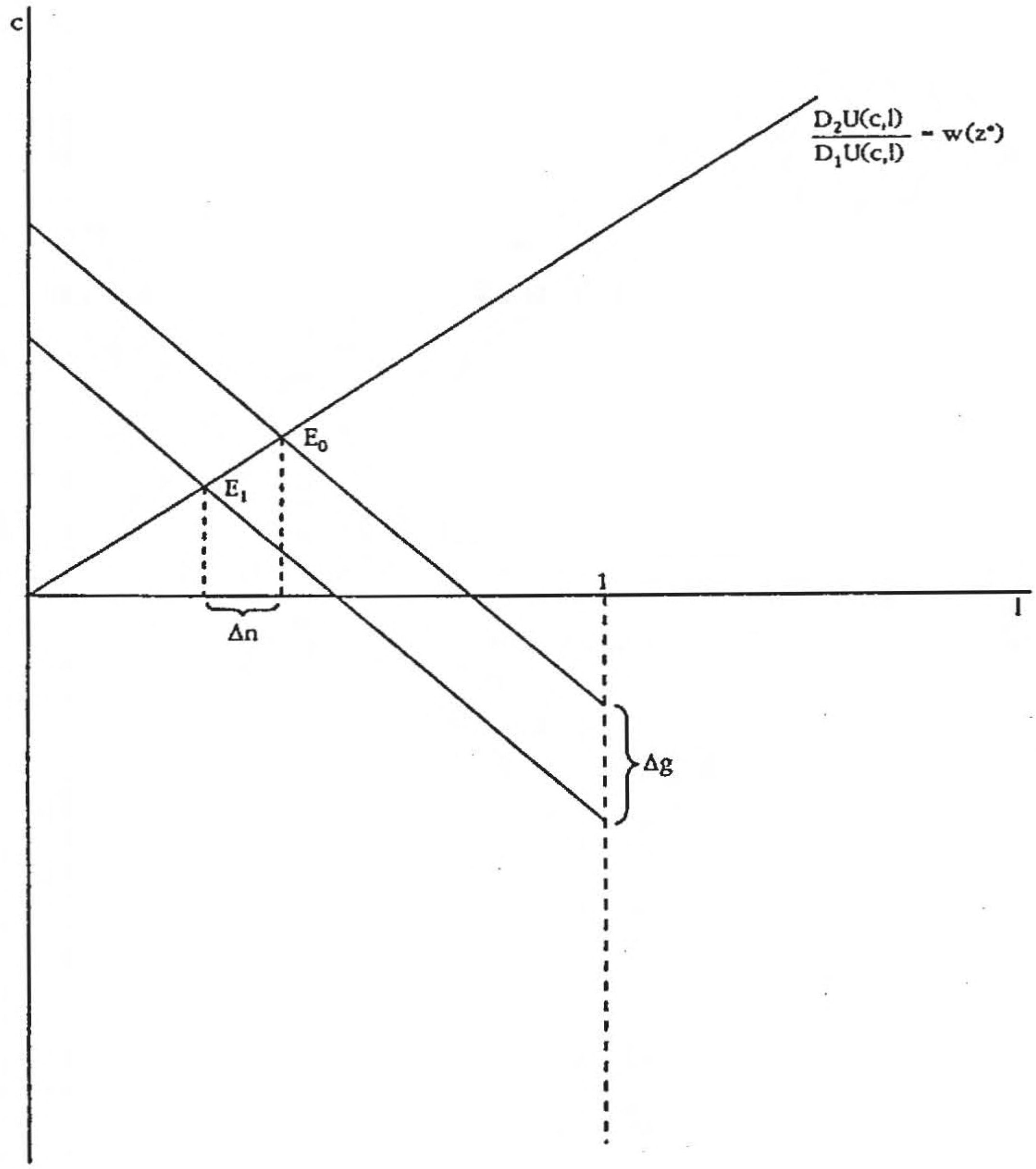


FIGURE 4
LONG-RUN CAPITAL MARKET, FLEXIBLE β

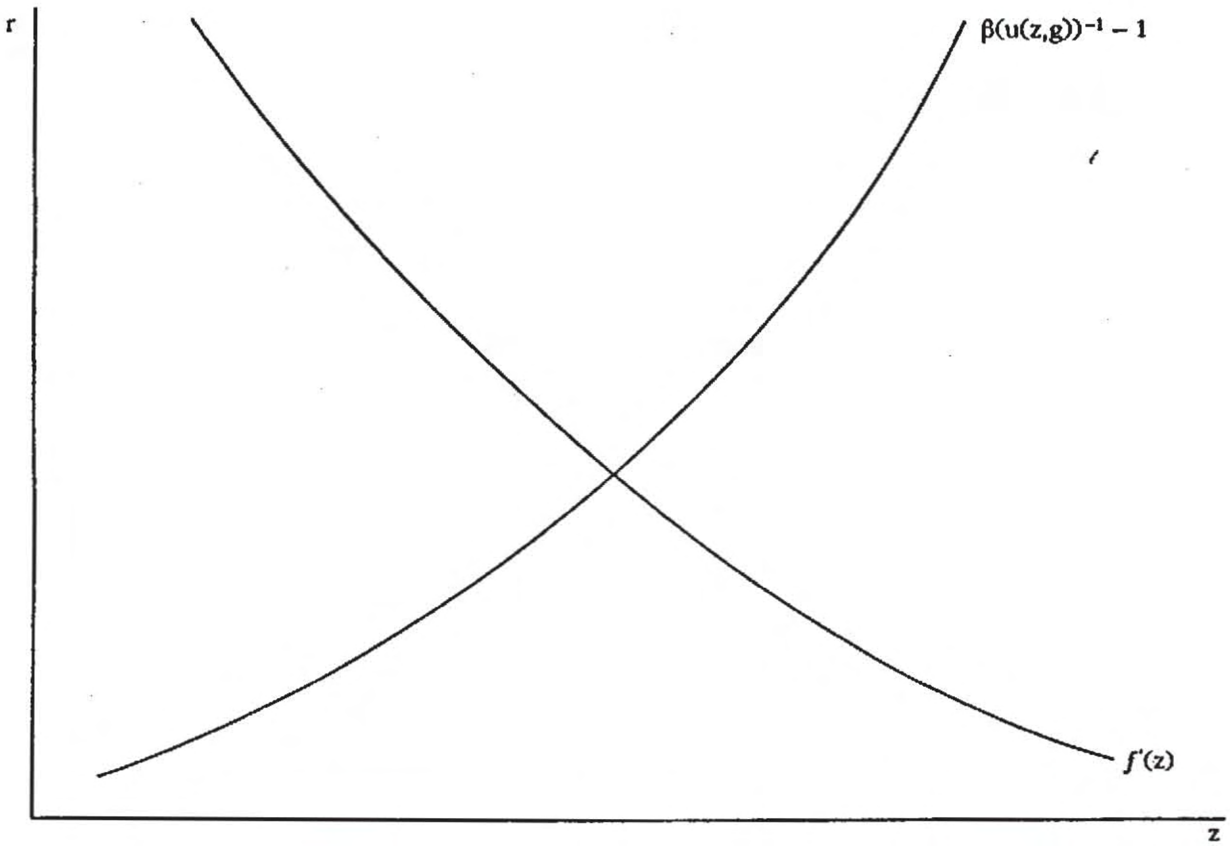


FIGURE 5
EFFECT ON LONG-RUN CAPITAL MARKET OF $\Delta g > 0$, FLEXIBLE β

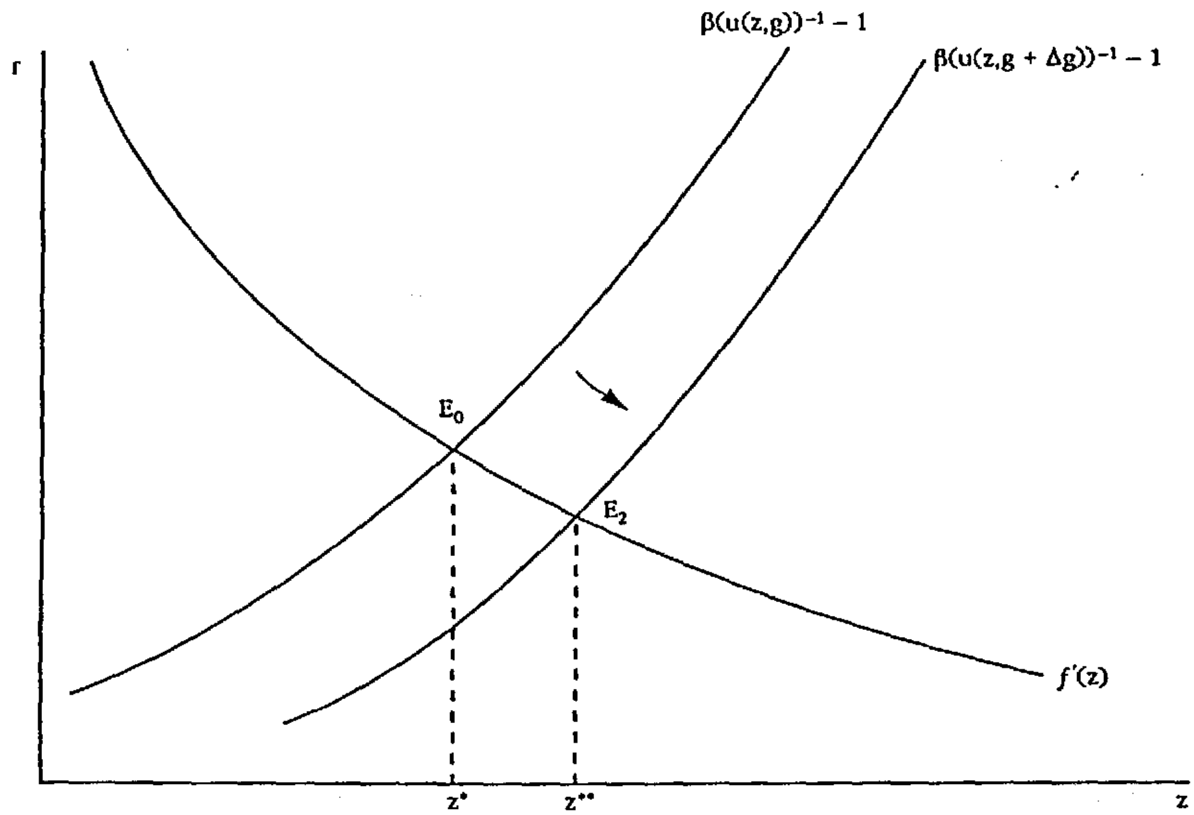


FIGURE 6
IMPACT OF HIGHER z ON LONG-RUN CONSUMPTION-LEISURE CHOICE

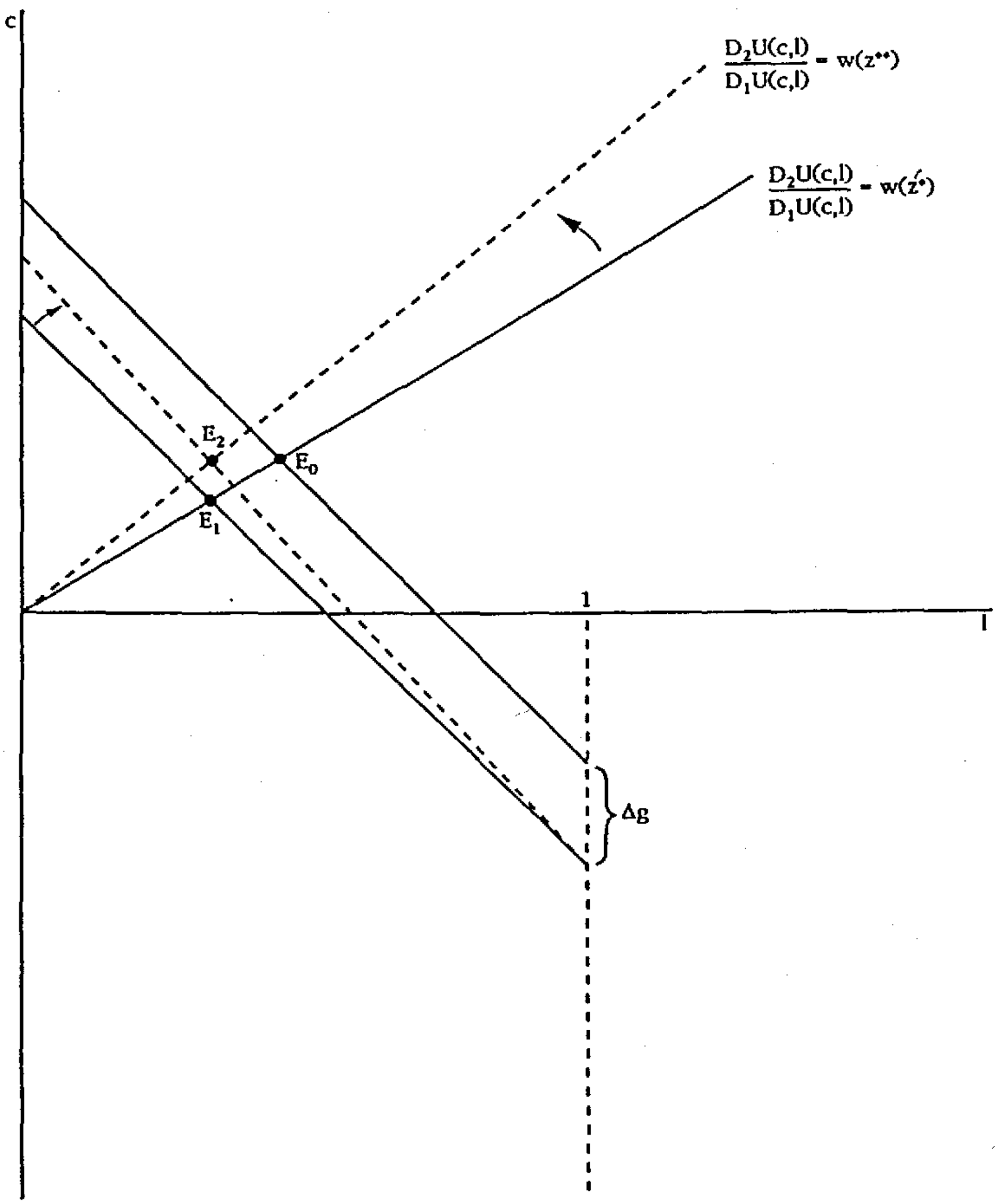


TABLE 1

LONG-RUN ELASTICITIES FOR VARIOUS VALUES OF $\xi = \nu(\sigma - 1)$

REALISTIC INVESTMENT SHARE, LOW DEPRECIATION

$\xi =$	0.00	0.01	0.05	0.10	0.25	0.75	1.00
n	.1976	.1977	.1980	.1983	.1986	.1989	.1990
c	−.0493	−.0331	.0138	.0495	.1001	.1442	.1516
z	.0000	.0387	.1506	.2356	.3565	.4617	.4758
k	.1976	.2364	.3486	.4339	.5551	.6606	.6784
y	.1976	.2140	.2613	.2973	.3483	.3928	.3988
dy/dg	.9880	1.0698	1.3606	1.4863	1.7417	1.9641	1.9942

LONG-RUN ELASTICITIES FOR VARIOUS VALUES OF $\xi = \nu_u(\sigma - 1)$

STANDARD DEPRECIATION, HIGH INVESTMENT SHARE

$\xi =$	0.00	0.01	0.05	0.10	0.25	0.75	1.00
n	.2275	.2298	.2379	.2456	.2598	.2772	.2808
c	−.0568	−.0438	.0002	.0421	.1200	.2152	.2348
z	.0000	.0324	.1420	.2463	.4403	.6774	.7263
k	.2275	.2622	.3799	.4919	.7001	.9546	1.0071
y	.2275	.2434	.2975	.3490	.4447	.5617	.5858
dy/dg	1.1375	1.2170	1.4877	1.7452	2.2236	2.8085	2.9292

FIGURE 7
EFFECT OF A TEMPORARY CHANGE IN g

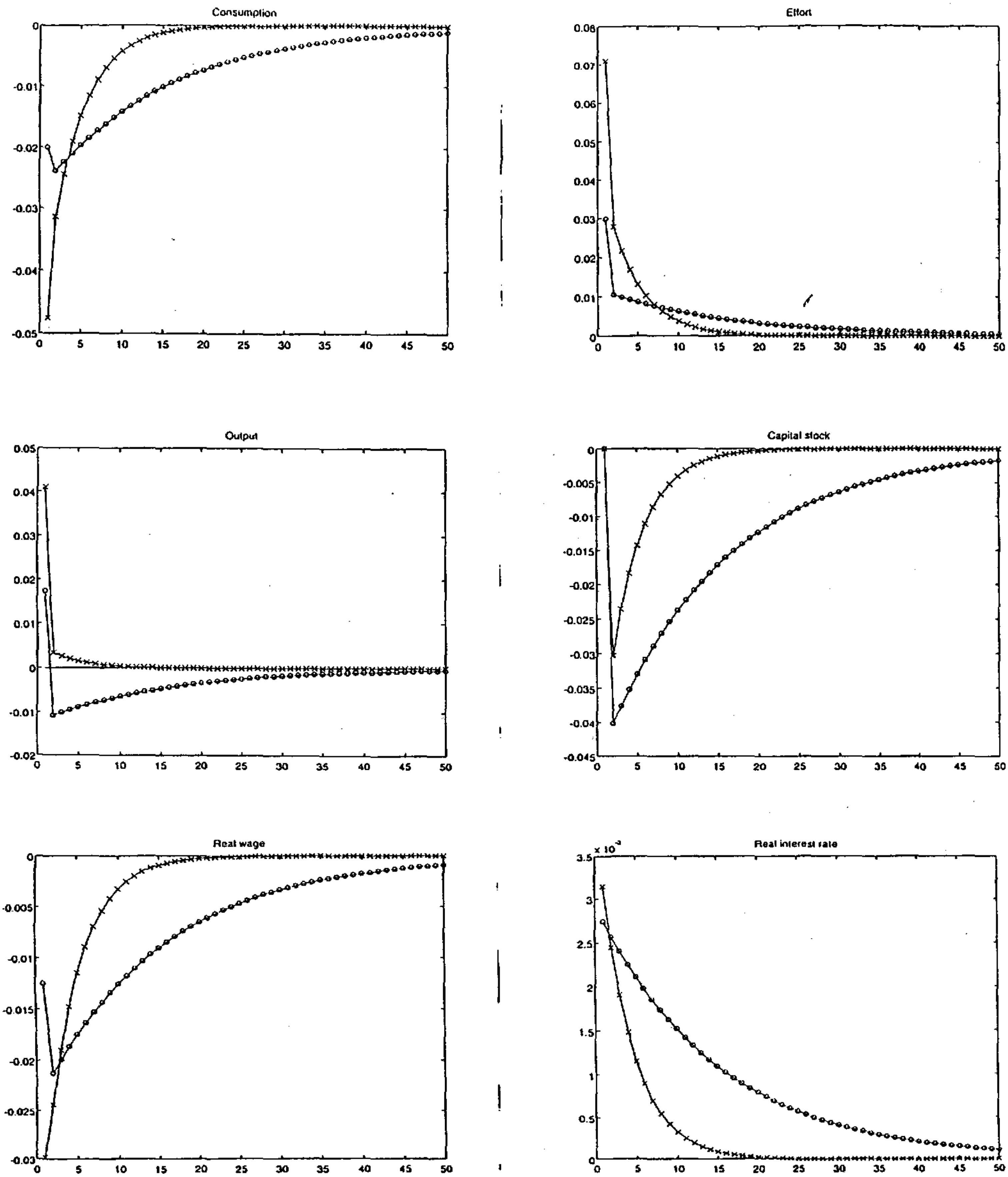


FIGURE 8
EFFECT OF A 'PERMANENT' CHANGE IN g

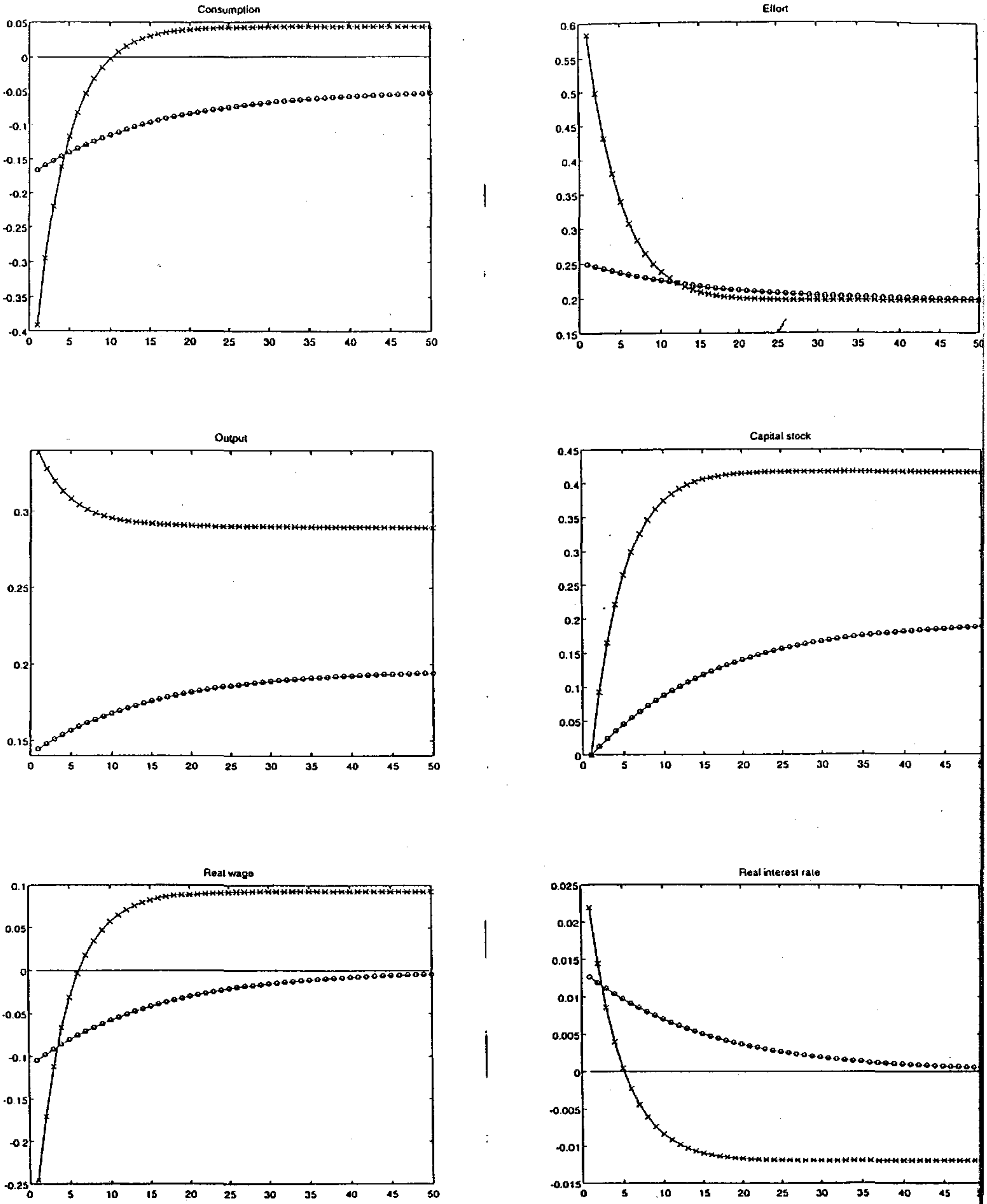
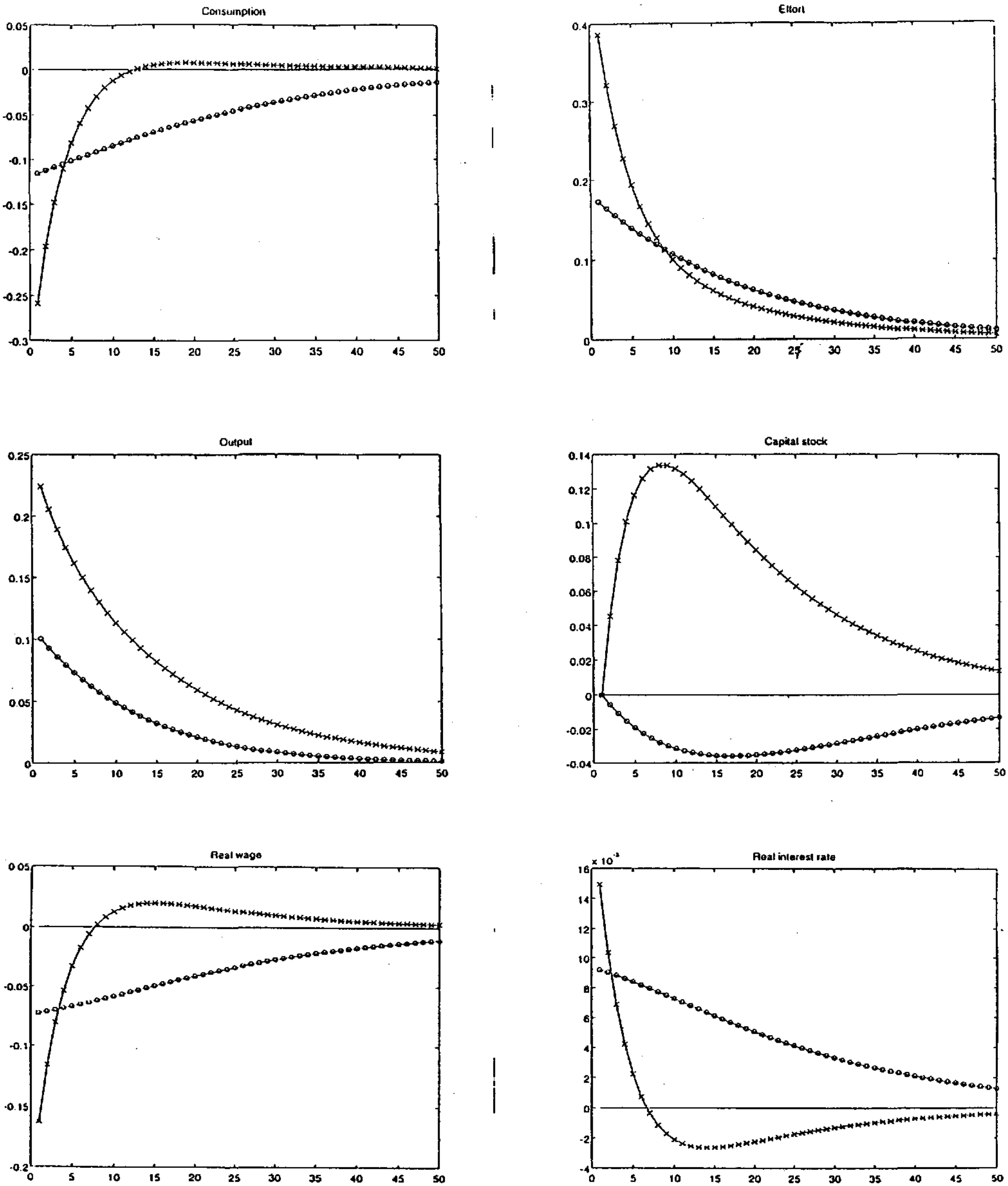


FIGURE 9
EFFECT OF AN INTERMEDIATE ($\rho = .94$)CHANGE IN g



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