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The paper presents a model of banks as clearinghouses of private debt where money is used as the means of payment. The model is then used to analyze elements of the real bills controversy. Implications of the model include: i) the private provision of banknotes or a discount window may be needed to avoid the insufficient debt clearing that results from an inflexible currency stock; ii) an uncontrolled total money stock may result in a multiplicity of equilibria including an inflationary banknote over-issue; and iii) the over-issue of banknotes is not a problem when banknotes are backed by productive assets.

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In recent years the profession has begun to explicitly model many of the functions of financial intermediaries, especially of banks. The roles of banks as monitors of investment and as providers of liquidity, for example, have been shown to be optimal responses to explicit environments of uncertainty and asymmetric information.[1] These models offer sensible explanations of the structure of bank assets and liabilities, the form of debt offered to and by banks.

In a similar fashion we wish to model in the role of banks in the clearing of private debt, debt created between two nonbank agents. One manifestation of this role, check clearing, is an obvious everyday function of banks. Another, the discounting of "real bills of exchange" for an elastic provision of currency has been the subject of heated controversy.

The debate between the real bills doctrine and the quantity theory of money has raged for more than a century. Over most of the course of the real bills debate, neither side offered a formal, fully explicit general equilibrium model whose assumptions or implications could be examined and challenged. A task of modern monetary theorists is therefore to build formal models that might sort out these conflicting claims.

A modern effort at an explicit modelling of the real bills question comes from Sargent and Wallace (1982). Their model resurrects the real bills doctrine by exposing a welfare cost of the quantity theory's separation of money from credit -- funds from money holders can not be used to satisfy the demand for credit, which generates differences in the marginal rate of intertemporal substitution faced by the holders of debt and money. This effort, while highly influential, has failed to convert many quantity theorists. Laidler (1984), for example, argues that the model of Sargent and Wallace misses key features in the

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[1] A recent survey of this literature may be found in Bhattacharya and Thakor (1991).

debate, in particular, the potential for the over-issue of banknotes in a monetary regime that follows the prescriptions of the real bills doctrine.[2]

This paper's model of the bank as a clearinghouse joins the renewal of this debate and finds reasonable grounds for elements of each view: the model displays both a useful role for the discounting of private debt as advocated by real bills adherents and a danger of an inflationary over-issue of private banknotes as quantity theorists have warned.[3] Naive applications of quantity theory restrictions fail to permit the useful discounting, which the real bills regime would permit. On the other hand naive applications of the real bills regime permit banknote over-issue, which quantity theory restrictions would prevent. In the model economy of this paper, a sophisticated application of the real bills doctrine performs the best in the presence of a productive outside asset; a sophisticated application of the quantity theory performs the best its absence.

To evaluate the conflicting policy advice of the the real bills doctrine and the quantity theory, it is necessary to model a bank's role as a clearinghouse of private debts. Such a model must feature demands for both currency and private nonbank debt. There must also be an impediment to the bilateral settling of debt. Our model displays these features in a model of spatially separated agents who trade using credit and currency. Debt is redeemed as each agent travels through a common area in which clearinghouses emerge. Moreover, in equilibrium agents choose to use currency as a medium of exchange (to make purchases at destinations away from an agent's place of origin) and as a means of payment (to settle debts).

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[2]Smith (1988) and Mourmouras and Russell (1992) have joined this debate by comparing the two regimes in the face of sunspot equilibria. Sunspots play no role in the analysis I present here – we will see that an over-issue of banknotes may be found even in stationary equilibria.

[3] Champ, Smith, and Williamson (1992) examine an elastic supply of central bank or private banknotes in response to a random demand for currency, but do not examine the possibility of banknote over-issue.

In section I, the basic model of debt-clearing is constructed and analyzed. In this basic version fiat money is the only outside asset, an assumption critical to the implications of the model. Section II compares the equilibria of the basic model under versions of real bills and quantity theory regimes. The narrowest interpretation of the quantity theory regime is shown to provide insufficient flexibility in the stock of currency – a limitation that may be overcome through temporary issues of private banknotes or the operation of a discount window by the central bank. On the other hand it is shown that the discounting of real bills, if the total money stock is uncontrolled, may lead to a multiplicity of equilibria including inflationary equilibria that result from the over-issue of private banknotes. Section III examines the implications of banknote overissue when banks are competitive. In section IV a second outside asset, productive capital, is introduced. Competitive banks that intermediate capital are shown to be free banknote over-issue. A brief conclusion is presented in section V.

I. A Model of Banks as Clearinghouses

The model

A large, even number I of islands are arranged in a circle around a central island. The islands are numbered consecutively around the circle in a clockwise direction by the index i, $i = 1, 2, 3, \ldots, I$. On each island on the circle, N agents are born in each period $t \geq 1$. Each agent is endowed at birth with y units of a non-storable good specific to its island (and with nothing when old). In the first period each island also has N agents (to be called the initial old) who live only in the first period.

Agents born on odd-numbered islands wish to consume the goods of both odd- and even-numbered islands when young and nothing when old (let us call such agents "debtors" for reasons that will soon be apparent). The utility of a debtor is given by the function $v(c_t, d_t)$. Agents born on even-numbered islands (let us call such agents "creditors") wish to consume the good of even-numbered islands when young and of even-numbered islands when old. No other consumption is desired. The utility of a creditor is given by the function $u(c_{1t}, c_{2t+1})$. Both utility functions are additively separable, increasing and concave in each argument, continuous, and continuously differentiable with indifference curves that do not cross the axes.

Each odd-numbered island is paired when young with an even-numbered island with which it may trade. When old, agents from each island travel to the central island. Agents from even-numbered islands (creditors) then continue on to an odd-numbered island where they may trade with young debtors. When old, agents do not visit the island with which it traded when young. The old agents arrive at their final destination after all travel by the young has been completed.

The sequence of travel within a period

young born and endowed

young visit neighboring islands

old visit central island

old arrive at final destination

Figure 1

On the central island live a large number of infinitely lived agents. They wish to consume the good of odd-numbered islands but have no endowment of goods themselves. Their utility is given by $\sum_{t=0}^{\infty} \beta^t e_t$ where e_t represents their consumption at t and β is a constant between zero and 1. They are endowed only with costless technologies of record-keeping and contract enforcement.

All agents are able to issue unfalsifiable IOUs that identify the issuer. A legal authority exists on each island that can enforce agreements between parties currently on the same island. This authority can also bear honest, written, transportable witness about any activity that takes place on an island.

The initial old creditors own a fixed stock of fiat money, totaling M dollars on each even-numbered island. Fiat money is non-counterfeitable, unbacked, intrinsically useless, and costlessly exchanged.[4]

Equilibrium conditions

To consume when old, creditors must bring something of value to the young debtors. Fiat money will be accepted by young of each odd-numbered island if it helps them to acquire the goods they desire. If it is accepted in equilibrium, fiat money serves as a "medium of exchange."

The young debtors wish to consume goods from even-numbered islands but own no goods valued by the young creditors that can be offered in immediate direct exchange. Nor do the debtors have any money at the time of this visit. They will later be able to sell some of their endowment for the money of the old but this money is not yet in the hands of a young debtor when it visits its neighbor. The only thing a debtor can offer creditors is a promise to pay a sum of money in the next period on the central island. The young debtor will acquire this money by selling some of its endowment to old creditors or others bringing money to the island.

In this monetary equilibrium both fiat money and debt are valued. Money serves not only as a medium of exchange but also as a means of payment, the means by which debts are cleared. Money is essential in this model for the clearing of debts and the existence of a

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[4]A fixed stock of gold or some other easily tradeable commodity money would serve just as well for most of the analysis, but the assumption of an intrinsically useless money allows us to abstract from consideration of the possible consumption of the commodity money.

credit market – without valued money, equilibrium debt equals zero. As we will see, in addition to fiat money there may another form of money.

The sequence of trades within a period

young born and endowed	young debtors purchase goods using IOUs	young debtors sell goods for money
	old visit central island where IOUs are cleared using money	old creditors trade money for the goods of young debtors

Figure 2

The budget constraints of a creditor born at t may be written in nominal terms as follows:

$$y \ \rho_t p_t \ = \ c_{1t} \rho_t p_t \ + \ l_t \tag{1.1}$$

$$l_t \ = \ c_{2t+1} p_{t+1} \tag{1.2}$$

or

$$y \ = \ c_{1t} \ + \frac{c_{2t+1}}{\rho_t} \frac{p_{t+1}}{p_t} \tag{1.3}$$

where ρ_t denotes the price of goods of an even-numbered island at t in terms of the goods of odd-numbered islands, p_t denotes the dollar price of a good on an odd-numbered island at t, and l_t denotes the nominal value at t of the creditor's loans to debtors. Because fiat money is the only outside asset, the net nominal interest rate will equal zero in this economy. The budget constraint (1.2) has already taken this into account.

Letting u_i denote the derivative of u(.,.) with respect to the i^{th} argument, the resulting first order condition for utility maximization can be written as

$$\frac{u_1}{u_2} \ = \ \rho_t \frac{p_t}{p_{t+1}} \tag{1.4}$$

The budget constraints of a debtor born at t may be written in nominal terms as follows:

$$y\,p_t = c_t\,p_t + m_t \quad (1.5)$$

$$m_t = h_t \quad (1.6)$$

$$p_t\,\rho_t\,d_t = h_t \quad (1.7)$$

where m_t denotes the debtor's nominal demand for currency, h_t denotes the nominal value at t of his indebtedness. Combined, these budget constraints yield

$$y = c_t + \rho_t\,d_t \quad (1.8)$$

The resulting first order condition for utility maximization is

$$\frac{v_c}{v_d} = \frac{1}{\rho_t} \quad (1.9)$$

Equilibrium also requires that the markets for goods, loans, and currency clear. The conditions for the clearing of the markets for the goods of even- and odd-numbered islands are respectively

$$y = c_t + c_{1t} \quad (1.10)$$

$$y = d_t + c_{2t} + e_t\,. \quad (1.11)$$

The clearing of the market for loans requires that

$$l_t = h_t \quad (1.12)$$

The clearing of the market for currency requires that on each island total currency demand equal the sum of the stocks of fiat money and perfect substitutes for fiat money, "banknotes," B_t, (if any exist)

$$Nm_t = M + B_t \quad (1.13)$$

If a loan market exists and operates ($l_t = h_t > 0$), we can combine and simplify these equilibrium conditions in the following way. The clearing of the currency market and (1.5) gives us an expression for p_t:

$$p_t = \frac{M + B_t}{N(y - c_t)}. \quad (1.14)$$

Institutions for the clearing of debt

The description of equilibrium is not yet complete. It takes as given some not-yet-specified arrangement for the clearing of debt and the issuing of banknotes. If debt markets operate, creditors will arrive at the central island with the IOUs of the debtors, while debtors arrive with currency to repay the IOUs. The nature of the transactions at the central island depends on the timing of the visits to the central island. Direct repayment of debt, repayment through a clearinghouse, and the issuance of clearinghouse debt can each represent the equilibrium institutional structure through which debts can be settled. Let us now examine some possible arrival patterns and debt clearing arrangements.

If all agents arrive simultaneously at the central island, debtors can repay their debts, directly or indirectly, using their currency balances. If it is costless to seek out the issuer of an IOU, the settling of debts may be accomplished through a direct meeting of debtor and creditor.

The settling of debts may also take place through a clearinghouse operated by agents of the central island. At the clearinghouse creditors would present the IOUs they possess in exchange for currency, debtors would present enough currency to clear their outstanding IOUs. At the conclusion of the transactions, the clearinghouse would possess neither IOUs or fiat money. A clearinghouse would be superior to the direct repayment of IOUs if and only if the technology of tracking down debtors exhibited increasing returns to scale. The intermediary in this case offers a check-clearing service but does not discount private debt because all debts are cleared simultaneously, before anyone leaves the central island.

If agents do not visit the central island simultaneously, direct repayment of IOUs is not possible, and thus a clearinghouse plays a much more useful role. Suppose, for example, that agents visit the central island in pairs drawn from non-adjacent islands, one member of the pair from an odd-numbered island, the other from an even-numbered island. Because the two members are drawn from each of the two different types of islands, the net debt of the two will sum to zero but because they come from non-adjacent islands, neither will be holding the personal debt of the other. In this case the clearinghouse can purchase the IOUs brought to it and accept fiat money in payment of the IOUs issued by the pair. The clearinghouse will record these transactions and use the fiat money payment of the debtor to pay the creditor what it is owed. At any given point in this sequence the clearinghouse holds positive balances of IOUs payable and receivable but has no net position in debt.

The role of the clearinghouse becomes even more important if the arrival of debtors and creditors is not so perfectly synchronized. Suppose that there is no overlap in the visits of debtors and creditors to the central island -- one group arrives and departs before the arrival of the other. If the first to arrive are debtors, they will deposit fiat money to make up the difference between their IOUs payable and receivable. As a result, the clearinghouse will at first accumulate positive balances of fiat money, which it will subsequently use to pay off the creditors upon their arrival.

If creditors arrive first, the clearing of debt is more complicated. Creditors wish to depart from the central island with greater balances of fiat money than they possessed upon arrival. In the other cases we have studied, creditors were paid their due with the currency balances brought by the debtors. In this case, however, debtors arrive too late for their currency to be used in payments to creditors. In this way the model displays a currency shortage to which private banks or the central bank may wish to respond.

How creditors can be paid when they arrive first is the central topic of this paper. The answer is important to central bank policy and the continuing debate between adherents of the quantity theory of money and of the real bills doctrine.

II. Providing Liquidity

In this section we examine several institutional monetary arrangements for the functioning of a clearinghouse when net creditors desire payment from the clearinghouse before the arrival of net debtors. We first examine the monetary arrangements that would be advocated by naive versions of the quantity theory and real bills doctrine in order to point out the potential of each for poor policy. Roughly speaking, we will identify arrangements permitting the free issue of notes with the real bills doctrine and arrangements with strict controls on the total stock of fiat money and its substitutes with the quantity theory. Narrow interpretations of these theories will serve as benchmarks. This does not imply that advocates of one or the other believe in these narrow interpretations. Indeed, we will see that the most sophisticated interpretations of both, featuring the wisest qualifications and caveats, will sometimes lead to equivalent equilibria.

A strict quantity theory regime

Suppose that fiat money is restricted to be the only monetary asset – the only asset that may be carried by the old from the central island to their destinations around the circle – and that the stock of fiat money is fixed.

Under this strict monetary regime the clearinghouse has no means of paying creditors should they arrive before debtors. Any agent arriving at the clearinghouse as a creditor receives nothing in exchange for the debt he would present to the clearinghouse. Anticipating this a potential creditor will have no desire to make a loan.[5] In such an autarkic equilibrium, utility is low for both creditors and debtors. Therefore, monetary arrangements that remove this constraint will be welfare-improving.

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[5]In this simple version of the model creditors facing a zero rate of return don't care whether or not they make the loan because they get no utility from consuming their endowment. If, however, creditors derive even the smallest amount of utility from the consumption of their endowment, or if a loan is at all costly to arrange, lenders facing a zero rate of return will strictly prefer not to lend.

To remove the constraint, the clearinghouse must have some means of paying creditors if they arrive before debtors. Additions to the fiat money stock or banknotes issued by the clearinghouse could be used for this purpose. Let us examine these in turn.

A discount window

Suppose that the monetary authority agrees to print enough new fiat money to enable the payment of creditors. This increase in the supply of fiat money will not be inflationary if it is only temporary -- that is, if the supply of fiat money is brought back to its original level when the debtors use fiat money to pay off their debts.

An institutional arrangement effecting this policy is a discount window. At a discount window operated by the monetary authority, a clearinghouse exchanges IOUs (which the clearinghouse has received from the creditors) for fiat money needed to pay off the creditors. Later, once the debtors have arrived at the clearinghouse to redeem their debts with fiat money balances, the clearinghouse in turn uses these fiat money balances to redeem the IOUs left with the monetary authority. By the end of the period, the liquidity problem has been circumvented with no net change in the fiat money stock.

Although the stock of fiat money fluctuates within every period in a discount window regime, there is no fluctuation in prices. The price level is determined by the equality of the demand for currency with the end-of-period fiat money stock: $Ny = M/p_t$, implying $p_t = \frac{M}{Ny}$.

A naive real bills doctrine regime

Consider now a less regulated regime, in which clearinghouses are permitted to print their own banknotes entitling the bearer of the note to one dollar, payable on the demand of the bearer. A clearinghouse may now pay off old creditors with its own money, which they may exchange for the endowment of the young debtors. The young debtors will accept clearinghouse notes as perfect substitutes for fiat money because they know they will travel to the central island in the next period, where they may redeem the notes for fiat money if they choose. Figure 3 presents the balance sheet of the clearinghouse after the arrival of the creditors, and again after the arrival of the debtors.

The Clearinghouse's Balance Sheet

after the exchanges with the net creditors

Assets		Liabilities
IOUs issued by net debtors	=	IOUs issued by net creditors
		+ banknotes

The Clearinghouse's Balance Sheet

after the exchanges with the net debtors

Assets		Liabilities
fiat money reserves	=	banknotes

Figure 3

Although this regime follows the policy prescriptions of the real bills doctrine, many aspects of the resulting equilibrium are exactly what the strictest version of quantity theory would predict. In particular, consider the link between the total money stock and the price level. The quantity theory defines the total money stock as the sum of all assets in public hands that may be readily used to make purchases. In this economy the total money stock per island at t is the sum of publicly held balances of fiat money (denote this M_t*) and privately issued banknotes (B_t).

As we found earlier, prices will be determined by the clearing of the currency market on each island,

$$p_t = \frac{M_t^* + B_t}{Ny}. \tag{2.1}$$

Notice that, in accordance with the quantity theory, the price level is strictly proportional to the total money stock, including privately issued banknotes, in public hands.

Full redemption

The effects of this regime depend on the frequency of the redemption of the clearinghouse notes. If the notes are always redeemed for fiat money in the period after their issue, the clearinghouse must hold as reserves all the fiat money it receives from debtors in order to meet the anticipated redemptions. What does this imply for the stock of currency used on each island? Publicly held currency is made up of publicly held fiat money plus banknotes. By definition, public balances of fiat money equal the total fiat money stock minus bank reserves R_t:

$$M_t^* = M - R_t \tag{2.2}$$

Bank reserves must be equal to the stock of banknotes in anticipation of their redemption:

$$R_t = B_t \tag{2.3}$$

Therefore, public currency holdings

$$M_t^* + B_t = [M - R_t] + B_t = M \tag{2.4}$$

are equal in size to the fiat money stock. Total currency in public hands has not been changed by the issue of banknotes because the banknotes in public hands simply replace an equal stock of fiat money now held out of circulation the the vault of the clearinghouse. Because the currency stock is unaffected by the extent of private banknote issue, the price level is also unaffected:

$$p_t = \frac{M_t^* + B_t}{Ny} = \frac{M}{Ny} \tag{2.5}$$

Therefore, the equilibrium under this regime is identical to that of the discount window regime -- a full provision of liquidity with no inflationary consequence.

No redemption

If the banknotes of the clearinghouse have no expiration date and thus are truly perfect substitutes for fiat money, there is no reason for agents to redeem them for fiat money. If banknotes are never redeemed, however, the reserves of the clearinghouse are never needed to meet the obligations of the clearinghouse. There is an opportunity for profit-taking here – a clearinghouse could use its fiat money reserves to purchase goods for its own consumption without ever going into default. Equivalently, if an investment opportunity were available on the central island, a clearinghouse could spend its reserves on some investment and take the interest as profits. In either case an issue of banknotes represents an expansion of the total stock of unbacked money: the privately created money is added to the total stock of currency in public hands without an offsetting subtraction of fiat money from circulation. In essence, clearinghouses are given (limited) permission to print fiat money and thus enjoy the profits from creating at no cost valued money.

The effects of this opportunity to take seigniorage profits will depend on whether or not the banking sector is competitive. In this section of the paper suppose that banks are not competitive -- there are a limited number of chartered banks prohibited from any actions to attract more business. In this case the bank receives the rents from seigniorage -- the bank's charter is literally a license to print money equal to the nominal value of the debt brought in. As a result, in each period there will be an expansion of the money stock, with the easily predictable consequence of a rising price level.

In each period clearinghouses are allowed to issue new banknotes equal to the nominal value of the debt:

$$B_t - B_{t-1} = N h_{t-1} \qquad (2.6)$$

These banknotes will be spent by creditors purchasing goods from the young on odd-numbered islands. Debtors repay their debts by bringing last period's currency stock to the clearinghouses. Notice that because the only use of currency in the model is to repay loans, the stock of currency at t-1 is equal to the nominal value of the debt at t :

$$M + B_{t-1} = N h_{t-1} \qquad (2.7)$$

Using this fact with (2.6) reveals the time path of banknotes:

$$B_t - B_{t-1} = M + B_{t-1} \qquad (2.8)$$

or $$B_t = M + 2B_{t-1} \qquad \text{with } B_0 = M. \qquad (2.9)$$

Once the clearinghouses have received the old currency balances from debtors paying their debts, they use the old currency balances to purchase goods from the young on odd-numbered islands.[6] In this way both the old currency balances owned by the clearinghouses and the new banknotes compete for goods owned by young debtors, who will need currency to pay off their debts in the next period.

Using the time path of banknotes (2.9) we can find the time path of the total stock of currency $M + B_t$:

$$M + B_t = M + [M + 2B_{t-1}] = 2[M + B_{t-1}]. \qquad (2.10)$$

The stock of currency doubles in every period as clearinghouses print up new banknotes equal in value to the nominal debt, which is equal to the previous period's stock of currency.[7] It follows from the money market clearing condition (1.14) that the price level will also double in every period in a stationary equilibrium.

Even though the clearinghouses are restricted to issuing notes only when presented with evidence of default-free debt, we see the excessive banknote creation about which the opponents of the real bills doctrine have warned.[8]

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[6]It can be verified below that $p_t/p_{t+1} < 1 < 1/\beta$, implying that the central island owners of the clearinghouses will wish to spend their seigniorage profits on current consumption.

[7]The currency stock will grow at a lesser rate if currency is held for reasons in addition to the clearing of debt. This case is taken up at the end of section III

[8]See Laidler (1984).

The multiplicity of equilibria

Between these two extremes of full redemption and no redemption lies a continuum of equilibria. Agents, being atomistic, are individually indifferent between banknote redemption rates, which determine the equilibrium rate of inflation.

Suppose for example that a constant fraction γ of all banknotes are redeemed. Then the current stock of banknotes equals the old unredeemed banknotes plus the new banknote issue. Banks are permitted to issue enough notes to cover the current nominal debt, Nh_t. The stock of banknotes is therefore given by

$$B_t \quad = \quad (1-\gamma)\,B_{t-1} \;+\; Nh_t \,. \tag{2.11}$$

In equilibrium, the current debt, Nh_t, is equal in size to old currency balances ($M - R_{t-1} + B_{t-1}$) because all currency is used to pay off the debt, implying that

$$\begin{aligned} B_t \quad &= \quad (1-\gamma)\,B_{t-1} + [M - R_{t-1} + B_{t-1}] \\ &= \quad (1-\gamma)\,B_{t-1} + [M - \gamma B_{t-1} + B_{t-1}] \\ &= \quad 2(1-\gamma)\,B_{t-1} + M \end{aligned} \tag{2.12}$$

with $B_0 = M$. It is easily verified that for $0 < \gamma \leq 1/2$, B_t grows without bound and that for $1/2 < \gamma < 1$, B_t converges to $\frac{M}{2\gamma - 1}$.

The (gross) rate of growth of nominal currency balances is, by definition,

$$\begin{aligned} &\frac{M - R_t + B_t}{M - R_{t-1} + B_{t-1}} \\ &= \quad \frac{M - \gamma B_t + B_t}{M - \gamma B_{t-1} + B_{t-1}} \end{aligned} \tag{2.13}$$

Using the expression for B_t given in (2.12), this expression can be simplified (after a few algebraic steps) to

$$2\,(1-\gamma) \quad + \quad \frac{\gamma\, M}{M + (1-\gamma)B_t} \tag{2.14}$$

As we found earlier, there is no change in the currency stock for $\gamma = 1$ (full redemption) while prices double every period for $\gamma = 0$ (no redemption). It can be verified from (2.14) that the gross rate of increase in the stock of currency is $2(1-\gamma)$ in the limit when the stock of banknotes is unbounded in the limit (*i.e.*, for $0 < \gamma \leq 1/2$) and 1 in the limit when the stock of banknotes converges to $\frac{M}{2\gamma - 1}$ (*i.e.*, for $1/2 < \gamma < 1$).

If real currency demand is constant, the (gross) inflation rate is given by the growth rate of currency balances in (2.14).

The continual additions to the stock of currency represent a seigniorage income to the agents running the clearinghouses of the central island. The additions compete with the existing currency balances with which creditors purchase goods on the odd-numbered islands. In this way inflation represents a wealth transfer from creditors to agents on the central island. Debtors are unaffected by inflation even though they are the ones who actually hold the currency balances over time. The currency balances of debtors, however, are exactly offset by their nominal IOUs, leaving them with zero net nominal assets. Since creditors own the IOUs of the debtors, which are backed by the currency stock, it is they who truly own currency balances and thus bear the cost of seigniorage.

Preventing the overissue of banknotes

To prevent the overissue of banknotes, one must ensure that they are fully backed by reserves of fiat money by the end of every period. An obvious way to do this is to require that clearinghouses hold reserves of fiat money equal to 100% of the notes issued. In this way any increase in the issuance of private currency is matched one-for-one by a decrease in the public holding of fiat money, just as it was when notes were always redeemed after a single period. Then the total stock of currency in public hands remains the same, leaving prices unaffected by the amount of privately issued banknotes. By itself, a constraint on the flow of money that permits banknote issue only in exchange for real debt is insufficient to prevent banknote overissue. As quantity theorists have argued, a constraint on the total stock of money is also necessary.

A requirement that banknotes have a limited term so that they must be redeemed for fiat money every period would have the same effect of ensuring that clearinghouses maintain fiat money balances backing their issue of notes. As we have seen, an equilibrium with a 100% rate of redemption forces banks to back their notes with fiat money reserves of 100%, thereby preventing banknote over-issue.

Notice, however, that the mere option of exchanging banknotes for fiat money on demand is not sufficient to prevent excessive note issue and rising prices, for nothing induces or forces agents to turn in their banknotes, which are viewed as perfect substitutes for fiat money. The absence of banknote redemption is not implausible. Since banknotes are a perfect substitute for fiat currency, even a small cost or bother of redemption may discourage all redemption. Knowing this, noncompetitive banks, whose seigniorage profits depend on less than full redemption, can be expected to discourage banknote redemption.

III. Competitive Banking

Because the owners of the clearinghouses earn seigniorage profits, every agent on the central island will wish to run a clearinghouse and to attract as many clients as possible. To this point in the analysis there has been no outlet through which the competitive desires of clearinghouses could be expressed. Let us rectify this omission with a look at the effects of competition, first through lump-sum gifts, then through premia on debt.

Competition through lump-sum gifts

Suppose there is free entry into banking, but banks can only offer creditors lump-sum gifts for clearing debt. Consider the naive real bills monetary regime in which banks can issue banknotes equal to the debt presented to them but are under no obligation to maintain reserves backing their banknotes. Assume that no banknotes are actually redeemed.[9] With each creditor that brings in debt to be cleared, a bank can issue an equal amount of banknotes. Because currency is used only to pay off debts, the old stock of currency also equals the nominal debt. Therefore, the banknote issue doubles the stock of currency, earning seigniorage worth half the real value of the currency stock. The real value of the currency stock is equal to the real value of the debt when currency is held only to clear debt. A competitive bank would therefore be willing to offer a gift of up to half the average real value of the debt per creditor to a creditor that brings in his entire debt. The zero-profit condition for a competitive equilibrium ensures that the gift will exactly equal the value of seigniorage profits, thus returning to the creditors the wealth lost through seigniorage.

We see from the creditors' first order maximization condition

$$\frac{u_1}{u_2} = \rho_t \frac{p_t}{p_{t+1}} \qquad (3.1)$$

that inflation directly affects the rate of return facing creditors. For any given relative price, ρ, an increase in the rate of inflation will increase c_1, which represents a decrease in the use of loans and currency.

The inflationary equilibrium will not be Pareto optimal because inflation reduces the real rate of return below the "golden rule" or "biological" interest rate (the rate of population growth). It is easily verified that the first order condition for maximizing steady-state utility, the golden rule, requires that

$$\frac{u_1}{u_2} = \frac{v_c}{v_d} \qquad (3.2)$$

which is true for a monetary equilibrium only when there is no inflation.

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[9]Any rate of redemption could be analyzed here but a rate of 0% is the easiest to study because the rate of inflation is the same in every period.

Competition through debt premia

If a bank makes positive seigniorage profits on each unit of debt brought to it, it will wish to attract even more of such debt. To do so an atomistic bank, if permitted, will pay a premium on debt brought to be cleared. The zero-profit condition of perfect competition requires that the bank pay a premium that returns seigniorage profits to creditors in exact proportion to the debt (and thus the seigniorage) they bring in. Consider the case with no banknote redemption. Let π_{t+1} denote the seigniorage profits per lender and thus the premium paid to creditors. The zero-profit condition requires that $\pi_{t+1} = l_t$. The budget constraints of the creditors in this case are

$$y \ \rho_t p_t = c_{1t} \rho_t p_t + l_t \tag{3.3}$$

$$l_t + \pi_{t+1} = 2 \, l_t = c_{2t+1} p_{t+1} \tag{3.4}$$

and the resulting first order condition is

$$\frac{u_1}{u_2} = 2\rho_t \frac{p_t}{p_{t+1}} \tag{3.5}$$

Since $\frac{p_t}{p_{t+1}} = \frac{1}{2}$ in a stationary monetary equilibrium, the first order conditions are

$$\frac{u_1}{u_2} = \rho_t = \frac{v_c}{v_d} \tag{3.6}$$

which are exactly the same as the creditors' first order condition in the absence of inflation and the same as the golden rule (3.2). Because the quantity of debt equals creditors' currency balances, seigniorage proportional to the quantity of debt is also proportional to currency balances. Therefore, inflation has no real effect on the real value of currency balances.

This superneutrality of money is not obtained when currency is held for reasons in addition to the clearing of debt. Consider, for example, a version of the model in which creditors wish to consume on even- as well as odd-numbered islands when old. As a result of this new assumption, creditors will hold currency as well as debt over time and there will be an exchange of currency for goods on all islands. Creditors selling their endowment will prefer debt to an equal nominal value of currency because of the premium on debt offered by the clearinghouse. It follows that competitive creditors will offer in turn a premium of $\$\phi_t$ for each dollar of debt at t.[10] Let $c_2{}^*{}_{t+1}$ denote the consumption of goods on an even-numbered island at t+1 of a creditor born at t and $m^*{}_t$ his currency balances. Let us consider the case of no redemption. Then a creditor seeks to maximize $u(c_{1t}, c_{2t+1}, c_2{}^*{}_{t+1})$ subject to

$$y \ \rho_t p_t = c_{1t} \rho_t p_t + (1+\phi_t) l_t + m^*{}_t \tag{3.7}$$

and $$l_t + \pi_{t+1} + m^*{}_t = c_{2t+1} p_{t+1} + c_2{}^*{}_{t+1} p_{t+1} \rho_{t+1} \tag{3.8}$$

Recall that in the case of no banknote redemption $l_t = \pi_{t+1}$ so that (3.8) can be written

$$2l_t + m^*{}_t = c_{2t+1} p_{t+1} + c_2{}^*{}_{t+1} p_{t+1} \rho_{t+1} \tag{3.9}$$

The resulting first order maximization conditions are

$$\phi = 1 \tag{3.10}$$

$$\frac{u_1}{u_2} = \rho_t \frac{p_t}{p_{t+1}} \tag{3.11}$$

$$\frac{u_1}{u_3} = \frac{\rho_t}{\rho_{t+1}} \frac{p_t}{p_{t+1}} \tag{3.12}$$

Equation (3.10) is a no-arbitrage condition requiring that the premium on debt equate the rates of return on debt and currency. As a result seigniorage revenue is passed back to debtors.

The clearing of the market for currency requires that the stock of currency equal the sum of the currency balances of creditors and debtors:

$$M + B_t = Nm_t{}^* + Nm_t \tag{3.13}$$

Let us define μ_t to be the fraction of currency balances held by debtors to repay their debts:

$$\mu_t [M + B_t] = Nm_t \tag{3.14}$$

In the absence of redemption, the change in the stock of banknotes equals the nominal value of the debt. In equilibrium this equals that fraction of the currency stock held to repay the debt:

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[10]An equivalent scheme would pay negative interest on the debt.

$$B_t - B_{t-1} = Nh_{t-1}$$

$$= \mu_{t-1}(M + B_{t-1}) \tag{3.15}$$

It follows that the total stock of currency is given by

$$M + B_t = M + B_{t-1} + \mu(M + B_{t-1})$$

$$= (1 + \mu_{t-1})(M + B_{t-1}) \tag{3.16}$$

In a stationary equilibrium the gross rate of inflation will equal the gross rate of change in the currency stock:

$$\frac{p_{t+1}}{p_t} = \frac{M + B_t}{M + B_{t-1}} = 1 + \mu \tag{3.17}$$

Substituting this into the first order conditions yields, for a stationary equilibrium,

$$\frac{u_1}{u_2} = \rho \frac{1}{1 + \mu} \tag{3.18}$$

$$\frac{u_1}{u_3} = \frac{1}{1 + \mu} \tag{3.19}$$

The real rates of return of both currency and loans fall below that of the golden rule as a result of the over-issue of currency.

In this naive real bills regime, debt becomes a license to print money. Debtors, as the creators of this license, collect the rent produced by the over-issue of currency. The premium alters incentives for debtors changing the debtors' first order condition to

$$\frac{v_c}{v_d} = \frac{\phi}{\rho} \tag{3.20}$$

If the substitution effect dominates the wealth effect, the premium on debt will encourage creditors to acquire more debt in order to earn these premia from seigniorage.

IV. Real Bills Backed by Productive Assets

A key feature of the environment permitting an overissue of banknotes is that fiat money is the only outside asset. Real Bills Doctrine advocates stressed, however, that only note issues backed by productive assets should be permitted. To evaluate this policy prescription, let us therefore adapt our model so that the assets brought to the clearinghouse are not consumption loans but titles to the output of productive assets.

Assume that on each island i, $i = 1, 2, ..., I$, there live N two-period lived agents, "creditors," living in overlapping generations. Each creditor born on island i is endowed when young with y units of the island's consumption good but with nothing when old. He wishes to consume the good of his own island when both young and old and then travels to the next island (i+1 for $i = 1, 2, ..., I$-1 and island 1 for $i = I$) whose good he also wishes to consume. Let $U(c_{1t}, c_{2t+1}, d_{t+1})$ denote his utility as a function of his consumption of his own island's good when young, his own island's good when old, and the good of the neighboring island when old, respectively.

On his way to the neighboring island he travels through the central island, which is identical to the central island of the preceding sections. The creditors visit the central island in the order of their index number i (island i arrives and departs before the arrival of island j if and only if $i < j$).

On each island live infinitely lived warehouses. A warehouse has no endowment of goods but is endowed with a storage technology that yields x of that island's goods at t+1 for each unit of the island's goods stored in period t. Assume that $x > 1$ (i.e., storage is productive). Goods can not be transported across islands. Let k^i_t denote the amount stored per creditor born at t on island i. Warehouses are known to creditors on the same island and to the agents on the central island but not to people on other islands.

With no endowment of their own, warehouses will only store goods lent to them by creditors. How can these loans can be repaid? All that the warehouses of island i can offer is a title to their nontransportable output. This output can provide for the consumption of creditors on their own island i but not the desired consumption when they travel to island i+1. The storage is desired by the creditors of island i-1. What must be arranged is a way for creditors to trade for the goods they desire.

The central island, because it is a destination common to all, may be able to provide a venue for trade. Suppose each creditor brought with him a title to some of the goods stored on his island. If all creditors visited the central island at the same time, a spot market would suffice to clear all debt. The assumed sequence of visits to the central island, however, makes things a bit more difficult. Except for the creditors of island I, every creditor arrives at the central island before any titles to the goods he desires. We must ask how then the creditors can arrange to trade for the goods they desire.

Real Bills Equilibrium

Let us begin by examining a laissez-faire regime, like that envisioned by real bills advocates, in which clearinghouses are permitted to issue notes backed by productive capital.

Suppose that just before travelling to the central island, old creditors place some goods in storage with the young of their home island. Let this storage be witnessed by the island's legal authority, which issues a title to the storage. Will the old encounter anyone in their travels who might accept the title to these stored goods in exchange for something of value to the old?

The goods just stored on island i are valued by those who will arrive at this island as old creditors in the next period (those currently young on island i -1), but the old creditors will never meet this group. There is, however, a means of indirect exchange between these two groups. Suppose that in period t the clearinghouse accepts from the old of each island i a title to storage worth $x^j k$ units of island i goods in period $t+j$. In return it offers a note that promises to pay $x^j k$ goods to the bearer in period $t+j$. The old themselves do not wish to consume after period t, but the young do and so will accept the banknote, the negotiable promise of the clearinghouse, at its present value of k goods. The clearinghouse's promise to island i is backed by the title brought from island i+2, etc. In this arrangement clearinghouses issue notes payable to the bearer (banknotes) backed by productive capital (storage). Alternatively, one could say that clearinghouses discount real bills that are backed by productive assets.

The young seek to maximize $U(c_{1t}, c_{2t+1}, d_{t+1})$ subject to the budget constraints

$$y = c_{1t} + s_t \tag{4.1}$$

$$xs_t = c_{2t+1} + d_{t+1} \tag{4.2}$$

where s_t represents the savings of a young creditor (including banknotes acquired from the previous generation) by the young at t.

The first order conditions of a stationary interior equilibrium require that

$$\frac{U_1}{U_2} = x = \frac{U_1}{U_3} \tag{4.3}$$

Is there a danger of the over-issue of banknotes under laissez-faire? The bank's budget constraint is

$$B_t + xK_{t-1} \geq r_t B_{t-1} + K_t \tag{4.4}$$

where B_t represents its outstanding stock of banknotes at t, r_t the gross interest it pays on these notes, and K_t the capital stock it owns at t. Competitive pressures in intermediation will force clearinghouses to pay the market rate of return of capital, x, on its notes. The rate of return $x > 1$ can be paid by the clearinghouses only if they maintain a stock of capital equal in value to their liabilities, their outstanding stock of banknotes ($K_t = B_t$).

A Strict Quantity Theory Equilibrium

Suppose that clearinghouses are prohibited from issuing banknotes when presented with titles to storage. This prohibition would force creditors to hold fiat money in order to consume at their neighbor's island. The budget constraints of the creditors become

$$y = c_{1t} + k_t + m_t/p_t \tag{4.5}$$

$$xk_t = c_{2t+1} \tag{4.6}$$

$$m_t/p_{t+1} = d_{t+1}$$

where k_t represents the storage on behalf of a young creditor at t, m_t his nominal fiat money balances, and p_t the price of a good in units of fiat money.

The first order conditions of a stationary interior equilibrium with a constant stock of fiat money require that

$$\frac{U_1}{U_2} = x \qquad (4.7)$$

$$\frac{U_1}{U_3} = \frac{p_t}{p_{t+1}} = 1 \qquad (4.8)$$

The aggregate capital stock is lower in the quantity theory equilibrium than under laissez-faire because fiat money rather than capital backs the banknotes used as currency. Creditors are thus offered a lower rate of return on their currency and lower steady-state utility. The quantity theory equilibrium is not Pareto inferior to laissez-faire because it is preferred by the initial generation (whose balances of fiat money have no value under laissez-faire.)

Intermediation through the central bank

The implications of the quantity theory regime for capital and welfare are greatly dependent on the means through which the government introduces its currency into the economy. Implicitly, we have assumed that the initial stock of currency has been given to the initial old. For this reason the prohibition on privately issued banknotes, which increased the demand for fiat currency, acted as a wealth transfer from later generations of creditors to the initial generation.

Suppose instead that the government sells the initial stock of currency in exchange for warehouse receipts. Such an open market operation gives the government a stock of capital equal in value to the outstanding stock of currency. This capital generates output sufficient to finance a gross real rate of return on currency equal to x, the same rate of return offered by unfettered private banks under laissez-faire and thus the same equilibrium levels of capital and utility. In sum, the intermediation conducted by private clearinghouses under laissez-faire is now accomplished by a central (government operated) bank.

V. Concluding Remarks

In this model's world of separated markets, centrally accessible banks can play a useful role in the clearing of debt. When the debt presented to banks temporarily exceeds the available means of payment, banks need the ability to issue their own means of payment (banknotes) in order to pay off this debt and function most efficiently as clearinghouses.

When an economy's means of payment is fiat money, permitting banks to issue their own unbacked currency as substitutes for fiat money gives them a license to collect seigniorage through an inflationary over-issue of private currency. This over-issue of currency may be prevented by regulations that directly or indirectly force banks to maintain reserves of fiat money equalling the outstanding stock of banknotes so that the total stock of currency in public hands does not grow with the use of banknotes.

The over-issue of private currency will not occur if banknotes pay interest and are backed by productive assets. In this case banks must maintain reserves of productive assets in order to pay interest on its notes, preventing them from earning seigniorage through the issue of unbacked notes. Regulations that require the use of unbacked fiat money instead of backed banknotes reduce equilibrium capital. Government currency backed by capital and paying the market rate of interest, however, leads to the same levels of capital and welfare as unrestricted private intermediation.

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