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RESEARCH PAPER

No. 9324

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July 1993

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First Draft: December 1991
Current Version: June 1993

JEL Classification Number: D90

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* We would like to thank John Boyd III, David Gould, Greg Huffman, Ron Jones, Bob King, Paul Romer, Karl Shell, Alan Stockman, Lori Taylor and Henry Wan, as well as seminar participants at Cornell, Penn State, and Rochester and of the 1992 ASSA Annual Meetings in New Orleans for helpful comments and suggestions on earlier drafts of the paper. Needless to say, the usual disclaimer applies.

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ABSTRACT

This paper studies a general two-sector (goods and education/learning) endogenous growth model with general constant-return-to-scale production technologies governing the evolution of human and physical capital stocks. A central feature of our analysis is the application of results from the two-sector framework of international trade to simplify the dynamics of the model. This enables us to prove analytically that the general two-sector endogenous model is saddle-path stable, regardless of the factor intensity ranking. We emphasize the role played by an intertemporal no-arbitrage condition which by equalizing the rates of return on physical and human capital, determining the dynamic adjustment of the value of human capital in the neighborhood of the balanced growth equilibrium. We provide a complete characterization of the transitional dynamics of consumption, goods and education outputs, human and physical capital inputs, the rates of factor return and the relative price of human capital investment. We also examine both short and long-run effects of changes in time preference, factor taxation and education subsidy.

I. Introduction

The neoclassical growth framework developed by Solow (1956) has played a pivotal role in analyzing the long-run macroeconomic behavior for decades. Recently, an increasing number of economists, led by the seminal work of Romer (1986), Lucas (1988) and Rebelo (1991), have turned their attention to understanding the endogenous driving forces of economic growth. Utilizing a balanced growth analysis, this new growth literature has provided useful explanation for the diverse economic development experiences and generated valuable policy prescriptions for an economy to achieve the optimal balanced growth path. Due to the nonstationary nature of the problem, however, its transitional dynamics is generally unexplored. In this paper, we completely characterize the balanced growth path and the transitional dynamics of a general two-sector model of endogenous growth with human and physical capital, and use it to provide implications for evaluating some public policy programs.

The importance of transitional dynamics has been fully recognized and emphasized by Solow (1988) in his Nobel Prize Lecture where he points out that "[t]he problem of combining long-run and short-run macroeconomics has still not been solved" (p. 310). Not only can transitional dynamics help integrating short-run movements with the long-run trend, but it is also essential for understanding the economic development process and for evaluating the dynamic effects of public policy programs. In an interesting attempt, King and Rebelo (1989) undertake a complete quantitative investigation of transitional dynamics of some widely used, exogenous, neoclassical growth models and find these models fail to interpret sustained cross-country differences in growth rates. More recently, Mulligan and Sala-i-Martin (1992) calibrate a two-sector endogenous growth model with Cobb-Douglas production technologies and find that with strictly convex production possibility frontiers, computer experimentation has not exhibit any balanced growth equilibrium which does not display saddle-path stability. This calibrated result has been confirmed analytically by some independent studies, including Caballé and Santos (1992), Xie (1992) and Benhabib and Perli (1993), using primarily the Lucas (1988) framework.[1]

Our paper extends these results in two directions. First, we extend Lucas (1988) by allowing the accumulation of human capital, through formal education or on-the-job learning [Stokey (1991) and Lucas (1993)], to depend not only on human but physical capital input, following the suggestion of Becker (1967) and Ben-Porath (1967).[2] Second, unlike previous studies, we adopt general constant-returns-to-scale production technologies, rather than relying exclusively on the Cobb-Douglas form. This allows us to take cases with factor-intensity reversals or complete specialization into account. Our model can thus be thought of as a generalization of King and Rebelo (1990) and Rebelo (1991).[3] For analytical simplicity, the present paper excludes uncompensated positive spillovers and strictly concave components of production.[4] Furthermore, since we focus primarily on characterization of the transitional dynamics, our paper can serve as a complement to the calibration studies of King and Rebelo (1990) and Mulligan and Sala-i-Martin (1992).

We utilize the two-sector model of international trade developed by Jones (1965) to simplify the dynamics of the model, thereby allowing us to obtain analytical results on transitional dynamics using general functional forms for production. We emphasize the role played by an intertemporal no-arbitrage condition which requires an equality between the net rates of return on physical and human capital, accounting for the "capital gain" of each reproducible capital stock as described in Shell, Sidrauski and Stiglitz (1969) within the Uzawa (1961,1963) two-sector exogenous growth framework. Conventionally, endogenous growth studies such as Lucas (1988) and Mulligan and Sala-i-Martin (1992) focus on quantity variables, especially the factor shares devoted to each sector. We find that centering the analysis at factor shares has generated tremendous algebraic complexity, which consequently deters us from studying the dynamics of the model analytically. Instead, our intertemporal no-arbitrage condition allows us to analyze not only the transformed state variables (as in Lucas and the following studies) but transformed control and price variables.

In this dynamic optimization problem, the relative price of education/learning output (in units of goods) will equal the value of an increment of human capital (the ratio of the respective co-state variables). Due to the two-sector nature of the technology, the wage rate and rental on capital are determined by the relative price of a unit of human capital, independent of the factor supplies. An increase in the price of education can be shown to raise the return to the factor used intensively in that sector and reduce the return to the other factor (i.e., the Stolper-Samuelson theorem). Utilizing this result we then show that the intertemporal no-arbitrage condition becomes a differential equation determining the evolution of the relative price of human capital investment and its dynamic adjustment in the neighborhood of the balanced growth equilibrium.

We show that if the education sector is labor-intensive (or, more precisely, human capital-intensive) relative to goods production, the adjustment process for the value of a unit of human capital is locally stable. If the education sector is, however, (physical) capital-intensive, the adjustment process is locally unstable. In the former case, the quantity adjustment contains the unstable force of the dynamics, whereas in the latter the price of human capital must jump instantaneously to its balanced growth value and remain so along the transition. The two-sector nature of the technology implies that an increase in the capital-labor (or physical to human capital) ratio raises production of the capital-intensive good and reduces output of the labor-intensive good at constant prices (i.e., the Rybczynski theorem). In the case where the goods sector is capital-intensive, capital accumulation results in a decrease in the output of the education sector, further increasing the capital-labor ratio. This adjustment alone would be unstable at fixed prices; however, the relative price of education output also rises along the transition path, thus enabling the adjustment process of the capital/labor ratio to converge to its balanced growth level. Therefore, for each of the factor intensity rankings, an unstable adjustment process in prices or quantities is offset by adjustment in the other variable.[5] In contrast with Mulligan and Sala-i-Martin (1992) who use calibration to study stability, we prove analytically that the general two-sector endogenous model is saddle-path stable, regardless of the factor intensity ranking.

Our framework also permits us to examine completely the transitional dynamics of consumption, outputs, human and physical capital inputs, the rates of factor return and the relative price, contrary to Lucas (1988) and other related studies which focus primarily on the two (transformed) capital inputs. Moreover, our analysis enables us to investigate parameter perturbations as well as to evaluate factor tax and subsidy policies both in the long-run balanced growth equilibrium and along the short-run transition path. In particular, we completely characterize the short- and long-run effects of alternations in time preference, factor (capital and labor) taxation and education subsidy.

Section II of the paper presents a general two-sector endogenous growth model. Section III characterizes the balanced growth equilibrium, and establishes the conditions under which a unique balanced growth path will exist. Section IV shows that the system will have a saddle path, and analyzes the transitional dynamics in the neighborhood of the balanced growth path. Section V provides some applications of the model illustrating the effects of parameter and policy changes on the balanced growth equilibrium and the transition path. Section VI concludes the paper.

II. A Two-Sector Model of Endogenous Growth

In this section we present a two-factor, two-sector model of endogenous growth in which there is a goods sector, X, and an education/learning sector, Y. In each sector, output is produced using both types of reproducible inputs, physical and human capital, under conditions of constant returns to scale, which, with proper assumptions, will lead to perpetual, balanced growth. As in Rebelo (1991), the output of the education sector raises the supply of effective labor units by adding to the stock of human capital, while the output of the goods sector may either be consumed or added to the stock of physical capital.

Let K(t) denote the aggregate stock of physical capital at time t and H(t) the aggregate stock of effective labor units (human capital). Factors are assumed to be fully mobile across sectors at a point in time, and $S_{ix}(t)$ denotes the share of the stock of factor i (i = h,k) that is allocated to sector X at time t. S_{hx} and $1-S_{hx}$ can be thought of as the fraction of time devoted to goods production and education, respectively, where the endowment of time has been normalized to unity. Under the assumption of constant returns to scale, the production technology can be specified as:

$$X = F(S_{kx} K, S_{hx} H) = S_{hx} Hf(k_x)$$

$$Y = G((1-S_{kx})K, (1-S_{hx})H) = (1-S_{hx})Hg(k_y),$$

where $k_x \equiv [(S_{kx} K)/(S_{hx} H)]$ and $k_y \equiv [(1-S_{kx})K]/[(1-S_{hx})H]$ are the capital/labor ratios in the respective sectors. The output per unit labor functions f and g are increasing and strictly concave. As in the conventional general equilibrium literature, we assume free disposal of goods and factors at any point in time.

Output in the goods sector can be either used as consumption, C(t), or added to the capital stock. Letting δ be the rate of depreciation of physical capital, the evolution of the capital stock can be expressed as:

$$\dot{K} = S_{hx}Hf(k_x) - \delta K - C. \tag{1}$$

We simplify the treatment of labor by assuming that there is a fixed population with an (inelastically supplied) stock of time endowment. The only changes in the effective labor force will result from changes in human capital per worker, which is assumed to be a perfect substitute for a change in the size of the labor force. The change in the stock of human capital will be equal to the output of the education sector less the depreciation of the existing stock of human capital,[6]

$$\dot{H} = (1-S_{hx})Hg(k_y) - \eta H. \tag{2}$$

where η is the rate of depreciation of human capital. Note that the educational process is assumed to require both physical capital (libraries, laboratories, etc) and human capital (faculty and students). By measuring the labor input in effective units, we are assuming that workers with more human capital teach more effectively and learn more quickly. This differs from the specification in Lucas (1988), where human capital accumulation requires only an input of time, and generalizes the two-sector endogenous growth model of King and Rebelo (1990) and Rebelo (1991) by considering general constant-returns-to-scale functions forms for production technologies.

The representative agent is assumed to have time-separable preferences with a constant subjective rate of time preference (ρ) and an instantaneous utility function $U(C) = C^{1-\sigma}/(1-\sigma)$. This utility function exhibits constant intertemporal elasticity of substitution; the inverse of this elasticity is denoted by σ, where $\sigma \in (0,1)\cup(1,\infty)$. It is chosen because it can yield a balanced growth path under some standard regularity conditions.[7] Given all preference and technology parameters, the representative agent's optimization problem can be specified as

$$\max_{C, S_{hx}, S_{kx}, K, H} \quad \Omega(C) = \int_0^\infty \frac{C(t)^{1-\sigma}}{1-\sigma}\, e^{-\rho t}\, dt \tag{P1}$$

subject to (1), (2), $H(0) = H_0$, $K(0) = K_0$, and non-negativity constraints on all quantities. (P1) is a standard dynamic optimization problem with control variables C, S_{hx} and S_{kx} and state variables H and K. The utility function is concave in the controls, and the technology is convex.

We will assume that the following condition, similar to that in Brock and Gale (1969), which imposes an upper bound on the maximal growth rate of the economy:

<u>Condition M</u>: *(Maximal Growth) The maximal attainable rate of consumption growth, v_{max}, satisfies* $\rho > (1-\sigma)\ v_{max}$.

This condition is more likely to be satisfied the greater is σ, which corresponds to a lower intertemporal elasticity of substitution. Under Condition M, integral in (P1) will converge and the results of Benveniste and Scheinkman (1982) can be applied to yield necessary conditions for the optimal growth path. Defining μ and λ as the costate variables associated with K and H, respectively, we can apply the Pontriyagin Maximum Principle to obtain the following conditions:

$$C^{-\sigma} - \mu = 0 \tag{3a}$$

$$(\mu f' - \lambda g')K = 0 \tag{3b}$$

$$[\mu(f - k_x f') - \lambda(g - k_y g')]H = 0 \tag{3c}$$

$$\dot{\mu} = \mu(\rho+\delta) - \mu S_{kx} f' - \lambda(1-S_{kx})g' \tag{3d}$$

$$\dot{\lambda} = \lambda(\rho+\eta) - \mu S_{hx}(f - k_x f') - \lambda(1-S_{hx})(g - k_y g') \tag{3e}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu(t)K(t) = 0 \tag{3f}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)H(t) = 0, \tag{3g}$$

together with equations (1) and (2), which can be used to characterize the solution to (P1).[8]

We will first show that the model has a balanced growth path in which K, H, and C all grow at the same rate. Along the balanced growth path, the shares of each factor allocated to the goods sector will be a constant (since both X/H and Y/H are constant), and the costate variables λ and μ grow at the same rate. Since the costate variables represent the values of a unit of human capital and physical capital respectively, we can define $p \equiv \lambda/\mu$ to be the relative price of a unit of human capital, which will be constant along the balanced growth path. After establishing the existence of a balanced growth path, we will then turn to an analysis of the transitional dynamics.

In analyzing the balanced growth path and transitional dynamics in this model, we will show that the two-factor, two-good nature of the model allows for substantial simplifications of the type that are familiar from the analysis of the Heckscher-Ohlin model of international trade theory. Specifically, this model shares the factor price equalization property of two-sector trade models that prices of productive factors (and therefore the factor proportions k_x and k_y) can be solved as functions of the relative output prices alone. This property makes it possible to analyze the dynamics of the relative price of human capital separately from the other variables in the system. Therefore, we will first analyze the behavior of p, and then turn to the evolution of the remaining variables in the system.

A. Output Prices, Factor Returns, and Steady-state Growth

In this section we solve for the relative price of human capital consistent with a balanced growth equilibrium. We then show how this price can be used to solve for the growth rate in the balanced growth equilibrium.

Equations (3b) and (3c) require that the marginal revenue products of capital and labor be equalized across sectors at each point in time, where marginal revenue products are evaluated using the price of human capital, p. Assuming that both sectors are active, these conditions can be used to establish the relationship between k_i and p (i=x,y). Let $r \equiv f'(k_x)$ denote the market rental on capital and $w \equiv f(k_x)-k_xf'(k_x)$ denote the market wage. We assume initially that there are no factor-intensity reversals, i.e. the sign of k_x - k_y is the same for all w/r. The absence of factor-intensity reversals insures that if both sectors are in operation at a given price, the capital/labor ratios in each sector are uniquely determined. Thus, we have:

Lemma 1: *If both sectors are in operation, the factor rewards and factor proportions in each sector are uniquely determined by p. The following conditions hold iff the education sector is labor- (capital-) intensive relative to the goods sector, i.e.,* $k_x > k_y$ $(k_x < k_y)$:

(a) $k_i(p)$ *is increasing (decreasing) in* p (i=x,y);

(b) w(p) *and* w(p)/p *are increasing (decreasing) in* p;

(c) r(p) *is decreasing (increasing) in* p.

Proof. Totally differentiating (3b) and (3c) yields

$$\frac{dk_x}{dp} = \frac{rk_y + w}{(k_y - k_x)f''} \quad ; \quad \frac{dk_y}{dp} = \frac{rk_x + w}{(k_y - k_x)pg''}\,, \tag{4}$$

Since the k_i's are uniquely determined in the absence of factor-intensity reversals, the relationship between price and factor rewards will also be uniquely determined by $r'(p) = f''(dk_x/dp)$ and $w'(p) = - k_x g''(dk_x/dp)$. Using the fact that f'', $g'' < 0$, (4) yields the results. Q.E.D.

Notably, (b) and (c) are a result of the Stolper-Samuelson theorem, which states that an increase in the relative price of a good causes a more than proportional increase in the reward to the factor that is used intensively in that sector, and a decrease in the reward to the other factor.

Subtracting (3d) from (3e) and utilizing Lemma 1, we obtain an expression for the evolution of p that must hold at any point in time when both sectors are in operation.

$$\frac{\dot{p}}{p} = r(p) - \frac{w(p)}{p} + \eta - \delta. \tag{5}$$

The "intertemporal no-arbitrage condition," (5), specifies the intertemporal relative-price adjustment necessary to equalize the net returns on human and physical capital. If $r_x - \delta > w_y - \eta$, the rental value of capital (net of depreciation) exceeds that on human capital, and there must be a capital gain earned on human capital investments to offset the difference in net rental values. This intertemporal adjustment in the relative price is in fact analogous to the concept of "capital gain" in the two-sector exogenous growth model of Shell, Sidrauski and Stiglitz (1969).

In a balanced growth equilibrium, the shadow values λ and μ must grow at the same rate. Therefore, the relative price in the balanced growth equilibrium will be the value of p that solves (5) with $\dot{p}/p = 0$. Note that this price will depend on the depreciation rates of the two types of capital and the marginal productivity of the factors, but not on the rate of time preference.

Differentiation of (5) yields

$$d\left(\frac{\dot{p}}{p}\right) = [r'(p) - (\frac{w'(p)}{p} - \frac{w}{p^2})]\, dp\cdot \tag{6}$$

By Lemma 1, the expression in the square bracket on the right hand side of (6) is negative iff $k_x > k_y$. This means that stability of the price adjustment process in the neighborhood of the balanced growth equilibrium requires $k_x > k_y$.

The conditions for the existence of a relative price consistent with balanced growth are illustrated in Figure 1 (a and b). The FP locus shows the values of r and w/p that are consistent with operation of both sectors for some value of p. This locus must be continuous when there are no factor-intensity reversals, because factor prices are uniquely determined by p and r is a continuous function of p. Also, FP must be downward-sloping regardless of which sector is capital-intensive, since Lemma 1 establishes that sign dr/dp = - sign d(w/p)/dp. By (5), the intersection of the FP locus with the line $r - w/p = \delta - \eta$ yields the values r and w/p that are consistent with balanced growth. We denote the value of p consistent with balanced growth by p^*, with $r^* = r(p^*)$ and $w^* = w(p^*)$. Note that since the real return differential r-w/p may be bounded above (below), it is possible that the FP line lies everywhere below (above) the $r - (w/p) = \delta-\eta$ line and no price consistent with balanced growth exists.[9] Letting $(r-w/p)_{sup}$ and $(r-w/p)_{inf}$ be the least upper bound and greatest lower bound, respectively, of the factor return differential, the following condition guarantees the existence of a p^*:

Condition FP: *(Factor Price Differential)* $(r-w/p)_{sup} \geq \delta - \eta \geq (r-w/p)_{inf}$.

Figure 1 (a,b) can also be used to illustrate the dynamics of the relative price adjustment process as determined by (5). We summarize the results in:

<u>Proposition 1</u>: *(Intertemporal Relative-Price Determination and Dynamics)*

(a) If Condition FP is satisfied, there exists a unique relative price of sector Y output, p^*, that is consistent with balanced growth.

(b) When both sectors are in operation, the intertemporal adjustment process of the relative price of human capital investment is stable (unstable), i.e., $d\dot{p}/dp < (>) \; 0$, in the neighborhood of the balanced growth equilibrium, iff the education sector is labor- (capital) intensive relative to the goods sector, $k_x > (<) \; k_y$.

Proof. (a) Under Condition FP, it is straightforward from Figure 1 (a,b) that p^* will be uniquely determined because the FP locus is downward-sloping.

(b) Notice that by examining (5), for values of r on the FP schedule shown in Figure 1 (a,b) above (below) r^*, we have $\dot{p}/p > (<) \; 0$. If $k_x > k_y$ (Figure 1a), $\dot{p} > 0$ implies $\dot{r} < 0$ and factor prices will converge to the balanced growth value p^*, implying that the intertemporal relative price adjustment is stable around p^*. Conversely, if $k_y > k_x$ (Figure 1b), $\dot{p} > 0$ implies $\dot{r} > 0$. Starting from an initial point where the return to r is above r^*, a rise in p is required to equate the return on physical and human capital. However, this rise in p raises the rental on capital relative to the wage, which widens the gap between the returns to physical and human capital (r-w/p) and thus the relative price adjustment process is unstable. Q.E.D.

Proposition 1 shows that the relative price adjustment process is stable around its unique balanced growth value if $k_x > k_y$; this adjustment process is unstable if $k_x < k_y$. In the latter case, the adjustment can only be completed if the relative price jumps instantaneously to its balanced growth level, p^*.

<u>Remark</u>: *(Factor-Intensity Reversals)* It is shown in Appendix A that the results of Proposition 1 also extend to the case in which there are factor-intensity reversals, where the factor intensity ranking is the one corresponding to the balanced growth equilibrium. The main effect of factor-intensity

reversals is to create the possibility that there may be more than one set of factor prices (w,r) and factor usages (k_x,k_y) that solve (3a) and (3b) for a given p. However, the interval (k_x^i,k_y^i) associated with the factor prices (w^i,r^i) in the i^{th} of these solutions cannot intersect the corresponding interval for any of the other solutions. Therefore, since $K/H \in [k_x,k_y]$ is required for factors to be fully employed in an equilibrium in which both sectors are active, there can be at most one solution (w,r,k_x,k_y) for a given K/H. Lemma 1 will then hold for this uniquely determined set of factor prices. It can then be shown that the factor price frontier FP in Figure 1 must be continuous and downward-sloping. These two results then yield Proposition 1 for the case of factor-intensity reversals, where the definition of relative factor intensity refers to the ranking at the balanced growth equilibria. The new element introduced by factor-intensity reversals is that there will be some values of K/H for which the process is stable, and some values for which it is unstable. □

We conclude this section by showing that the current level of p will also determine the growth rate of consumption. Differentiating condition (3a) for optimal consumption with respect to t and equating to (3d) yields

$$\dot{C} = v_C(p)C, \tag{7a}$$

$$\text{where } v_C(p) = \sigma^{-1}[r(p) - (\rho + \delta)] \equiv v. \tag{7b}$$

This condition requires that the marginal utility of consumption decline at the same rate as the shadow value of a unit of capital, $\dot{\mu}/\mu$, along the optimal path. Returns from consumption will be equalized with those from investing in capital.

<u>Lemma 2</u>: *The growth rate of consumption is decreasing (increasing) in p iff the education sector is relatively labor- (capital-) intensive, i.e.,* $k_x > (<) k_y$.

Proof. First, notice that differentiation of (7b) with respect to p yields $v_C'(p) = \sigma^{-1} r'(p)$. Then the result follows immediately from Lemma 1. Q.E.D.

Intuitively, if $k_x > k_y$, an increase in p decreases the net return on capital, making investment in physical capital less attractive. More of good X is then allocated to current consumption rather than investment.

B. Output Supply, Factor Endowments, and the Balanced Growth Path

A balanced growth path is one in which C, H, and K all grow at the common rate ν. Lemma 2 established that in order for consumption to grow at a constant rate, the relative price of sector Y output must be a constant (see Appendix B). From Proposition 1, the relative price on the balanced growth path, p^*, is uniquely determined by the intertemporal no-arbitrage condition and the requirement that the factor returns be equalized. In this section we will complete our characterization of the balanced growth path by analyzing the conditions for a constant growth rate for human and physical capital.

We will assume that the underlying production technologies are sufficiently productive to generate positive long-run growth at the price consistent with balanced growth.

Condition G: *(Nondegenerate Growth)* $r^* - \delta > \rho$.

This inequality is required to ensure that the growth rate of consumption (7a) in the balanced growth equilibrium is positive, and is analogous to the condition G in Jones and Manuelli (1990).[10]

In order to characterize the balanced growth path, it will first be useful to derive comparative statics results for the effects of p and k on the sectoral outputs, where $k \equiv K/H$ is the aggregate endowment ratio. The full employment condition for capital requires that $S_{hx}k_x(p) + (1-S_{hx})k_y(p) = k$, or $S_{hx} = (k-k_y(p))\ (k_x(p)-k_y(p))$. Scaling the outputs of each sector by the stock of human capital, we can express outputs as $x(p,k) = X/H = S_{hx}(p,k)f(k_x(p))$ and $y(p,k) = Y/H = (1-S_{hx}(p,k))g(k_y(p))$. The following lemma summarizes the comparative-statics effects of changes in prices and endowments on normalized outputs from straightforward differentiation:

<u>Lemma 3</u>: *Output supplies can be characterized as:*

$$\frac{\partial x}{\partial p}<0,\ \ \frac{\partial y}{\partial p}>0;\ \ \frac{\partial x}{\partial k} = \frac{f(k_x)}{(k_x-k_y)}\ \gtrless\ 0\ ,\ \ \frac{\partial y}{\partial k}=-\frac{g(k_y)}{(k_x-k_y)}\ \lessgtr\ 0 \text{ as } k_x \gtrless k_y .$$

The two-factor model with constant returns to scale has a concave transformation schedule, which yields the normal output supply responses. The effects of endowments on output supplies are the well known Rybczynski Theorem from international trade theory: an increase in the aggregate capital/labor ratio results in a more than proportional expansion of the output of the capital-intensive good and a decrease in the output of the labor-intensive good.

We are now ready to establish the following result:

<u>Proposition 2</u>: *(Existence and Uniqueness of a Balanced Growth Equilibrium)*

(a) If a balanced growth path exists, it exhibits a common growth rate v^* *for consumption and the two factors.*

(b) If the economy starts from an initial endowment (K^*,H^*)*, then the balanced growth path will solve the first-order conditions (3) of the dynamic optimization problem (P1) with nondegenerate growth* v^* *if Conditions M, FP, and G hold.*

(c) Under the above specified conditions, the balanced growth equilibrium is unique.

Proof. Our proof will be divided into three parts.

(a) Defining $c \equiv C/H$ and letting v_i $(i=H,K)$ denote the growth rate of factor i, equations (1) and (2) can be rewritten as

$$v_H = \dot{H}/H = y(p,k) - \eta \tag{8a}$$

$$v_K = \dot{K}/K = k^{-1}[x(p,k) - c] - \delta . \tag{8b}$$

By definition, p and v_i are constant along balanced growth paths. From (8), it is clear that constant growth rates for the capital stocks require c and k to be constant, implying that the balanced growth path is one in which C, H, and K all grow at the common rate v^*. The values of c and k consistent with balanced growth can be determined by solving (8) using $p = p^*$ and $v_H = v_K = v^*$.

(b) We next establish that solutions c^* and k^* will exist if $(w^*/p^*)-\eta > v^*$. First, note that the aggregate factor proportion in the balanced growth equilibrium, k^*, is determined by the requirement that human capital grow at rate v^*, which is positive under Condition G. In order for a balanced growth path to exist, we must have a solution of (8a) with $k \in [k_x, k_y]$. From Lemma 3, the right hand side of (8a) is linear in k since $\partial y/\partial k$ is independent of k. Lemma 3 also establishes that $\partial x/\partial k$ depends only on p, so the right hand side of (8b) is linear in k. Since $1 - S_{hx} = (k - k_x)/(k_y - k_x)$, we have $y(p^*,k_x(p^*)) = 0$ and $y(p^*, k_y(p^*)) = g(k_y)$. Evaluating (8a) at these endpoints yields $v_H(k_x) = -\eta$ and $v_H(k_y) = g(k_y) - \eta > (w^*/p^*) - \eta$. Therefore, a sufficient condition for there to exist a unique solution k^* at which $v_H = v^*$ is $(w^*/p^*) - \eta > v^*$. Since $r^*- \delta = (w^*/p^*) - \eta$, this requirement can be expressed using (7b) as $r^*- \delta = \rho + \sigma v^* > v^*$. This inequality is obviously satisfied for $\sigma > 1$. For $\sigma < 1$, Condition M can be used to rewrite the left hand side of the inequality $r^*-\delta > (1-\sigma)v_{max} + \sigma v^*$, which must exceed v^*. Therefore, there will exist a unique k^* consistent with balanced growth if Condition M is satisfied.

Given the solution for k^* from (8a), (8b) yields the effective consumption, c^*, in the balanced growth equilibrium. The condition $r^*- \delta > v^*$ is also sufficient to ensure $c^* > 0$. To establish this, note that normalized national income can be written as $x + py = w + rk$. Substituting for x and y from (8) and using the equalization of factor returns across sectors yields $c^* = (r^*- \delta - v_K^*) + (w^* - p^*(\eta + v_H^*)) = (r^*- \delta - v^*)(1+p^*)$. Since $r^*- \delta = (w^*/p^*) - \eta$, $(w^*/p^*) - \eta > v^*$ guarantees $c^* > 0$. These results have established the existence of a balanced growth path that solves conditions (3a)-(3e) of (P1). It remains to show that this solution also satisfies the transversality conditions (3f) and (3g). From (3d), μ will grow at a rate $\rho + \delta - r^*$ on the balanced growth path. Since K is growing at rate v^*, the transversality condition (3f) will be satisfied if $r^*- \delta > v^*$, which has been shown above to be guaranteed by Condition M. A similar argument establishes that $r^* - \delta > v^*$ ensures that (3g) will hold.

(c) Finally, from Proposition 1 and part (b) of this proof, we have shown that there will be a unique (p^*,k^*) consistent with non-degenerated balanced growth under Conditions M, FP and G. Thus, from Lemmas 1 and 2 and parts (a) and (b) of this proof, factor prices, factor usages, consumption and the balanced growth rate are all determined uniquely. Q.E.D.

It is noteworthy that our uniqueness result generalizes, in part, that of Xie (1992) and Benhabib and Perli (1993), in which a locally unique balanced growth equilibrium obtains in a Lucas (1988) economy without human capital externalities.[11]

IV. Transitional Dynamics

In this section we analyze the transitional dynamics of the system in the neighborhood of the balanced growth path, and investigate the role played by the relative factor intensities of the two sectors in determining the dynamics of the system.

We show in Appendix B that if the initial endowment is $k_0 \neq k^*$, then the system has a saddle path that converges to the balanced growth ratio k^*. This result is shown by noting that due to the assumptions on preferences and technology, the problem (P1) can be equivalently formulated as:

$$H_0^{1-\sigma} v(k_0) = \max_{c, S_{hx}, S_{kx}, k} \; H_0^{1-\sigma} \int_0^\infty \frac{C(t)^{1-\sigma}}{1-\sigma} e^{-\Delta(t)} \, dt \qquad \text{(P2)}$$

where $\Delta(t) \equiv [\rho + (1-\sigma)\eta]t - (1-\sigma)\,[\int_0^t (1-S_{hx})\, g(k_y)\, ds]$ and the transition rule for k is $\dot{k} = [S_{hx} f(k_x) - c - (1-S_{hx}) g(k_y) k + (\eta - \delta) k]$. Expressing the problem in the form (P2) yields two insights about the optimization problem. First, we show in Appendix B that by making use of a variant of the Uzawa transformation, (P2) can be solved as an optimal control problem with a single state variable k, and that this system has a saddle path which converges to steady-state values k^* and c^*. Moreover, it can be shown that the transversality condition for this transformed problem implies that $\dot{p} = 0$ along the saddle path. Second, since $V(K,H) = H^{1-\sigma}v(k)$, the function $V(K,H)$ is homogeneous of degree $1-\sigma$. This result will be useful in the analysis of the transitional dynamics below.

Utilizing (7) and (8), the dynamics of consumption and factor supplies can be expressed as

$$\frac{\dot{c}}{c} = \frac{\dot{C}}{C} - \frac{\dot{H}}{H} = v_C(p) - y(p,k) + \eta \tag{9}$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{H}}{H} = k^{-1}[x(p,k) - c] - y(p,k) + \eta - \delta. \tag{10}$$

Equations (5), (9) and (10) describe the evolution of the endogenous variables, p, c, and k.

The results of Lemmas 1-3 and Proposition 1 can now be used to analyze the transitional dynamics of the system. The linearized dynamic system is given by

$$\begin{bmatrix} \dot{p}/p \\ \dot{c}/c \\ \dot{k}/k \end{bmatrix} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} p - p^* \\ c - c^* \\ k - k^* \end{bmatrix} \tag{11}$$

where $a_{11} = r' - (w' - w/p)/p$, $a_{21} = -\partial y/\partial p + v_C'$, $a_{23} = -\partial y/\partial k$, $a_{31} = (\partial x/\partial p)/k - \partial y/\partial p$, $a_{32} = -1/k$, and $a_{33} = -(x-c)/k^2 + (\partial x/\partial k)/k - \partial y/\partial k$. We first establish the stability property of the dynamical system:

<u>Proposition 3</u>: *(Stability) The balanced growth equilibrium is saddle-path stable regardless of the factor intensity ranking.*

Proof. To examine stability of the above 3x3 system, we evaluate its Jacobian matrix at the balanced growth equilibrium, denoted as J_3^*. The eigenvalues of J_3^* are the solutions of its characteristic equation:

$$- \gamma^3 + Tr(J_3^*) \gamma^2 - R(J_3^*) \gamma + Det(J_3^*) = 0$$

where the trace of J_3^* is $Tr(J_3^*) \equiv a_{11} + a_{33}$, the determinant is $Det(J_3^*) \equiv - a_{11}a_{23}a_{32}$, and $R(J_3^*) \equiv a_{11}a_{33} - a_{23}a_{32}$. Utilizing Lemmas 1-3, one can easily show that (i) for $k_x < k_y$, $a_{11} > 0$, $a_{23} < 0$, $a_{32} < 0$ and $a_{33} < 0$; and, (ii) for $k_x > k_y$, $a_{11} < 0$, $a_{23} > 0$, $a_{32} < 0$, and $a_{33} > 0$. Thus, regardless of the factor intensity ranking, the determinant, $Det(J_3^*)$, must be always negative. For both cases ($k_x<k_y$ and $k_x>k_y$), a_{11} and a_{33} are of opposite sign and hence the sign of $Tr(J_3^*)$ cannot be determined; also, the term, $-R(J_3^*) + Tr(J_3^*)/Det(J_3^*)$, is ambigous in sign as well. Straightforward application of Routh Theorem implies that the characteristic equation specified above will have either one or three roots with negative real parts. The rest of this proof will be devoted to ruling out the case of all three roots with negative real parts. To see this, we utilize the block-recursive nature of the 3x3 dynamical system: its characteristic equation specified above contains a real root, $\gamma_1 = a_{11}$ (which is positive for $k_x < k_y$ and negative for $k_x > k_y$), with the remaining two roots satisfying the characteristic equation of a 2x2 subsystem (which Jacobian matrix evaluated at the balanced growth equilibrium is denoted as J_2^*):

$$\gamma^2 - Tr(J_2^*) \gamma + Det(J_2^*) = 0$$

where $Tr(J_2^*) \equiv a_{33}$ and $Det(J_2^*) \equiv - a_{23}a_{32}$. Recall that (i) for $k_x < k_y$, $a_{23} < 0$, $a_{32} < 0$ and $a_{33} < 0$; and, (ii) for $k_x > k_y$, $a_{23} > 0$, $a_{32} < 0$, and $a_{33} > 0$. So, in the former case ($k_x<k_y$), both $Tr(J_2^*)$ and $Det(J_2^*)$ are negative, while in the latter case ($k_x>k_y$), both $Tr(J_2^*)$ and $Det(J_2^*)$ are positive. Thus the remaining two roots are real and of opposite sign for $k_x < k_y$ and are with positive real parts for $k_x > k_y$. In either case, there are one root with negative real parts and two with positive real parts, implying saddle-path stability. Q.E.D.

Lemmas 1-3 and Proposition 1 show that both the price and quantity adjustment processes depend on the factor intensity ranking, so we shall examine each case separately.

<u>Case I</u>: $k_x < k_y$

As noted above, the relative price adjustment process depends only on p and is unstable in this case; so p must jump immediately to the balanced growth value, independent of the dynamic adjustment of the two quantity variables. As shown in Proposition 3 the adjustment in p is an unstable force, with the remaining two roots corresponding to the (c,k) dynamics being of opposite sign. Since p must jump instantaneously to its balanced growth value, we can focus our analysis only on the 2x2 subsystem containing the (c,k) dynamics.

The dynamics of the subsystem is illustrated in Figure 2. The $\dot{c} = 0$ locus will be vertical, since there is a unique k (given p) at which C and H grow at the same rate. In this case an increase in the capital/labor ratio reduces the relative output of good x ($a_{33} < 0$), which is a stabilizing force because it reduces the accumulation of capital relative to labor (holding c constant). The $\dot{k} = 0$ locus must then be downward sloping, since an increase in k must be accompanied by a reduction in c to keep the two factors accumulating at the same rate. Examination of Figure 2 establishes that the saddle path must be upward-sloping. For example, starting from an initial factor endowment ratio, $k_0 < k^*$, we must have $v_K > v_H$ and $v_C > v_H$ during the transition to the balanced growth path. Since the price remains constant at p^* throughout the transition, the growth rate of consumption will be constant at the steady state level v^* from (7). It is shown in Appendix B that this path is the one that satisfies the transversality condition to (P2).

The only remaining question is to rank the growth rate of capital relative to that of consumption during the transition to the balanced growth path. From (8b), the slope of the locus of values in Figure 2 along which v_K is constant will be downward-sloping for this case, since $(x-c)/k$ is decreasing in k and c. Thus, v_K must be falling as k rises along the saddle path. Since $v_K = v^*$

on the balanced growth path, $v_K > v_C = v^*$ along the transition to the balanced growth path.

Remark: *(Inactive Sectoral Production)* It should be noted that in deriving these results, we have assumed that both sectors are in operation along the optimal path. Since $p = p^*$ along the optimal path, it is straightforward to derive the range of initial values k_0 for which this assumption is satisfied. If $k_0 \in [k_x(p^*), k_y(p^*)]$, then the initial stock of capital is consistent with full employment factors and the economy will converge to k^* with both sectors in operation. If $k_0 < k_x(p^*)$, then there will be no production of Y output, implying unemployed human capital at p^*. The wage rate must then fall further below $w_x(p^*)$ to fully utilize human capital, which will require that only sector X output be produced until the aggregate k ratio rises to $k_x(p^*)$, at which point the economy will evolve as illustrated in Figure 2. Similarly, if $k_0 > k_y(p^*)$, no accumulation of capital will take place until the aggregate capital/labor ratio falls to $k_y(p^*)$. □

Case II: $k_x > k_y$

In this case, Proposition 1 implies that the price adjustment process is stable. However, as shown in Proposition 3, the dynamics of c and k at fixed prices are not stable, since an increase in k raises the output of x and reduces the output of y, leading to a further increase in k. Adjustments of the price along the optimal path will be necessary if the system is to be at least locally stable.

Notice that in the present case, p will be changing along the saddle path, which will lead to corresponding changes in the factor endowment ratio, k. We can now analyze the projected dynamics in (c,k) space only by making use of the fact that the value function is homogeneous of degree $1\text{-}\sigma$ (which is ensured as $\sigma \neq 1$). Since the costate variables are the derivatives of the value function with respect to the respective state variables, the costate variables are homogeneous of degree $-\sigma$. Therefore, $p \equiv \lambda/\mu$ is a function of k alone. Also, p(k) will be non-decreasing in k from the concavity of the value function. The dynamics in the three-dimensional representation may be illustrated as in Figure 3a.

Substituting $dp=p'(k)dk$ into (11) gives the projected dynamical system onto (c,k) space:

$$\begin{pmatrix} \dot{c}/c \\ \dot{k}/k \end{pmatrix} = \begin{pmatrix} 0 & b_{12} \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} c - c^* \\ k - k^* \end{pmatrix} \tag{12}$$

where $b_{12} = -\partial y/\partial k + [v_c'(p) - (\partial y/\partial p)]\, p'(k)$

$b_{21} = -1/k < 0$

$b_{22} = [-(x-c)/k^2 + (1/k)(\partial x/\partial k) - (\partial y/\partial k)] + [(1/k)(\partial x/\partial p) - (\partial y/\partial p)]\, p'(k).$

Since $b_{11}=0$, the projected $\dot{c} = 0$ locus must be vertical. The term b_{12} captures two conflicting forces. An increase in k reduces the output of y (at constant prices) since $\partial y/\partial k < 0$ in this case, which tends to raise c. On the other hand, the rising p associated with an increase in k raises the growth rate of consumption ($v_c' > 0$ for $k_x > k_y$) and raises output of y, both of which tend to reduce c. However, recall from Proposition 3 that the system has a saddle path, which projection onto (c,k) space implies the characteristic roots of this projected 2x2 system must have real parts of opposite sign. Thus, the determinant of the associated 2x2 Jacobian matrix evaluated at the balanced growth equilibrium, $\mathrm{Det}(J_p^*) \equiv -b_{12}b_{21}$, is negative, which, given $b_{21} < 0$, will guarantee $b_{12} < 0$.

The term b_{22} also involves conflicting forces. An increase in k will increase the relative output of good x when $k_x > k_y$. However, the rising p will reduce the relative output of good x. The fact that the system has a saddle path is not sufficient to determine which of these effects dominates. We will thus have two cases to consider. To be specific, we define $\Psi_R \equiv -(x-c)/k^2 + (1/k)(\partial x/\partial k) - (\partial y/\partial k) > 0$, representing the Rybczynski effect on relative outputs (x and y), and $\Psi_P \equiv -[(1/k)(\partial x/\partial p) - (\partial y/\partial p)]\, p'(k) < 0$, denoting the stabilizing effect of the relative price on relative outputs. When the former effect (the Rybczynski effect on relative outputs) dominates, $\mathrm{Tr}(J_2^*) \equiv b_{22} > 0$. In this case, we can easily show that the projected $\dot{k} = 0$ locus will be upward sloping. The phase diagram for this case is illustrated in Figure 3b, which shows that the projected saddle path must be upward sloping as well. If, on the contrary, the stabilizing effect of p on relative outputs dominates such that $Tr(J_2^*) \equiv b_{22} < 0$, the projected $\dot{k} = 0$ locus will now be downward-sloping, similar to that illustrated in Figure 2. Therefore, in either case we have that if $k_0 < k^*$, $\nu_K > \nu_H$ along the transition path and $\nu_C > \nu_H$. These two results are similar to those obtained for the case $k_x < k_y$.

The difference from the previous case where $k_x < k_y$ is that the rising k must cause p to rise. From Lemma 3 we know that the growth rate of consumption is a decreasing function of p when $k_x > k_y$. The growth rate of consumption will be falling along the optimal path, so $\nu_C > \nu^*$ during the transition to balanced growth. There are conflicting forces operating on the growth rates of both types of capital during the transition. The increase in k tends to raise the growth rate of physical capital and reduce the growth rate of human capital when $k_x > k_y$, but the increasing price of human capital has the opposite effect. Therefore, it will not in general be possible to rank ν_H and ν_K relative to ν^* for the transition. Similarly, it will not in general be possible to rank ν_K relative to ν_C for this case. The slope of the locus along which $\nu_K = \nu_C$ is $dc/dk = -(b_{22}-b_{12})/b_{21}$. For the case $b_{22} > 0$ in Figure 3b, this locus must be steeper than the $\dot{k} = 0$ locus, and its slope cannot be ranked in general relative to the saddle path. Similarly, for the case $b_{22} < 0$ (as in Figure 2), the locus may be upward-sloping so its slope cannot be ranked relative to the saddle path. In the case $k_x > k_y$, we obtain a wider range of possible rankings of growth rates because of the conflicting role of price effects and factor accumulation effects on the outputs of the two goods.[12]

The above results regarding dynamic adjustment can now be summarized:

Proposition 4: *(Characterization of Dynamic Adjustment) Starting from $k_0 < k^*$, then in the neighborhood of the balanced growth path we have physical capital and consumption both growing more rapidly than human capital. If $k_x < k_y$, then $\nu_K > \nu_C = \nu^* > \nu_H$; if $k_x > k_y$, then $\nu_C > \nu^*$ and* $\min(\nu_K, \nu_C) > \nu_H$.

V. Applications

In this section, we illustrate how the model can be used to analyze the long-run balanced growth and short-run dynamic effects of changes in the rate of time preference and public policy parameters.

A. Changes in Time Preference

A decrease in the subjective discount rate will make investment in both physical and human capital more attractive. Since both types of investment are affected equally, there is no effect on the intertemporal no-arbitrage condition (5) and the relative price of human capital in the balanced growth path will be unaffected by the change in the discount rate. Since p^* is unaffected and v_C^* $= (r^* - \rho - \delta)/\sigma$, a decrease in the discount rate must raise the growth rate along the balanced growth path.

Equations (9) and (10) can be used to calculate the effect of this increase in the balanced growth rate on c and k. Differentiating (9) yields $dk/dv^* = (\partial y/\partial k)^{-1}$. An increase in the growth rate requires an increase in the output of the educational sector, since all Y sector output is used for factor accumulation. An increase in y output requires an increase in k iff $k_y > k_x$ from Lemma 3. Totally differentiating (8b) and utilizing dk/dv^* yields

$$\frac{dc}{dv^*} = \left(-k\frac{\partial y}{\partial k} + \frac{\partial x}{\partial k} - \frac{x-c}{k}\right) / \frac{\partial y}{\partial k} < 0. \tag{13}$$

An increase in the growth rate requires an increase in $(x-c)/k$ in order to increase the rate of capital accumulation. If $k_y > k_x$, then $(\partial x/\partial k) < 0$ and $(\partial y/\partial k) > 0$, and the bracketed expression in (13) must be negative. An increase in the growth rate raises k and reduces x, so the only way that $(x-c)/k$ can increase is for consumption to fall. If $k_x > k_y$, by Lemma 3 we have $\partial x/\partial k > x/k > 0$ and

$\partial y/\partial k < 0$, and the bracketed expression must be positive. In this case, $dk/dv^* < 0$ and output of x must again fall. The percentage reduction in output will be greater than the percentage reduction in k, however, so that x/k falls. In order to have the growth rate increase, c/k must fall by more than x/k, which again requires a reduction in c. In summary, we have:

Proposition 5: *(Effects of Changes in Time Preference)*

A decrease in the time preference rate, ρ, raises the balanced growth rate and reduce the level of consumption per effective unit on the balanced growth path. The capital/labor ratio falls iff $k_x < k_y$.

Next, applying Figures 2 and 3, we are able to characterize the short-run transitional dynamics in response to changes in ρ. Specifically, it can readily be seen from (5), (9) and (10) that an increase in ρ will leave the $\dot{k} = 0$ and $\dot{p} = 0$ loci unaffected, but will shift the $\dot{c} = 0$ locus to the left (right) in case I (II) where $k_x < (>)\ k_y$. Straightforward comparative dynamic analysis provides us with responses of the endogenous variables, such as c, k, p, r, and w/p along the transition path. For brevity, we summarize the results in Table 1. While price variables all remain constant over time, the analysis indicates that the short-run effects of ρ on the quantity variables, c and k, depend crucially on the factor intensity ranking. Furthermore, changes in consumption need not be monotone along the transition path.

B. Changes in Public Policy

This model can also be used to study the short- and long-run effects of three representative public policy programs, capital and labor taxation and an education subsidy. Capital taxation is assumed to consist of a tax at rate τ_K on the earnings from all physical capital. Labor taxation is modelled as a tax at rate τ_H on earnings from human capital in the goods sector. The asymmetry between capital and labor taxation is that it is assumed that labor in the education sector is untaxed, i.e., the foregone earnings of labor are not taxed during the process of human capital accumulation. The education subsidy, at rate z_H, is one applied only to the labor input in the education sector.

The net tax revenues, T, collected from factor taxation and a lump-sum tax, T_0 , net of the education subsidy are assumed to be used to purchase goods which enter separably in the household utility function. The objective function for the household then remains to solve (P1) for a given public policy, subject to (2) and

$$\dot{K} = S_{hx}Hf(k_x) - T - \delta K - C \tag{1)'$$

where $T \equiv r\tau_K K + w\tau_H S_{hx}H + T_0 - wz_H(1-S_{hx})$ and w and r are the pre-tax returns to labor and capital, respectively. Equalization of after-tax returns to each factor across sectors yields

$$r \equiv f'(k_x) = pg'(k_y) \tag{14a}$$

$$w(1-\tau_H) \equiv [f(k_x)-k_x f'(k_x)](1-\tau_H) = p[g(k_y)-k_y g'(k_y)](1+z_H). \tag{14b}$$

The intertemporal arbitrage condition for this case becomes

$$\frac{\dot{p}}{p} = f'(k_x)(1-\tau_K) - [g(k_y)-k_y g'(k_y)](1+z_H) + \eta - \delta. \tag{15}$$

Thus we obtain the following results by totally differentiating (14)-(15):

<u>Proposition 6</u>: *(Balanced Growth Effects of Public Policies)*

(a) The effects of an increase in a tax on capital income are:

$$\frac{\partial k_x}{\partial \tau_K} < 0, \quad \frac{\partial k_y}{\partial \tau_K} < 0, \quad \frac{\partial p}{\partial \tau_K} \gtrless 0 \text{ as } k_y(1+z_H) \gtrless k_x(1-\tau_K)$$

(b) The effect of a tax on labor income in the goods sector are:

$$\frac{\partial k_x}{\partial \tau_H} > 0, \quad \frac{\partial k_y}{\partial \tau_H} < 0, \quad \frac{\partial p}{\partial \tau_H} < 0$$

(c) The effects of a subsidy to labor input in the education sector are:

$$\frac{\partial k_x}{\partial z_H} < 0, \quad \frac{\partial k_y}{\partial z_H} < 0, \quad \frac{\partial p}{\partial z_H} < 0 .$$

A tax on capital income raises the cost of physical capital, leading to substitution of human capital for physical capital in both sectors. The price of educational services will rise as a result of the tax on capital iff education is capital-intensive relative to goods production. Note that the definition of factor intensity relevant for determining the direction of the price change involves a comparison of $k_y(1+z_H)$ with $k_x(1-\tau_H)$, which is equivalent to a comparison of the ratio of capital to labor costs in the two sectors.

Since the tax on labor income is limited to the goods sector, it raises the cost of labor relative to capital in the goods sector, but reduces it in the education sector. This tax must reduce the relative price of education output, since the tax falls only on the goods sector. In contrast to the capital tax, the impact on price does not depend on the factor intensity ranking of the two sectors. The effect of an education subsidy is similar to that obtained for the labor income tax, except that an education subsidy encourages human capital accumulation and thus lowers the factor proportions in both sectors.

Proposition 6 can be used to calculate the effects of factor taxation on the rate of growth. Modifying (7b) with the tax/subsidy incorporated and substituting from (14a) yields the growth rate of consumption to be

$$\nu = \sigma^{-1} [r_x(k_x(\tau_K, \tau_H, z_H))(1-\tau_K) - (\rho+\delta)]. \tag{16}$$

Factor taxes and a subsidy to labor usage in the education sector affect growth through their impact on the net return to capital in the goods sector. Since labor taxation raises k_x and an education sector labor subsidy reduces k_x, a decrease in the rate of labor taxation or an increase in the education subsidy must raise the net return to capital and hence the rate of growth on the balanced growth path. A reduction in the rate of capital taxation raises the net return to capital at a given k_x, but it also increases k_x. Differentiation of (16) establishes that $d\nu/d\tau_K < 0$. Thus, lowering either types of factor taxes or raising the education subsidy will promote economic growth, contrasting with findings by Heckman's (1976) partial-equilibrium analysis.

In analogy to the transitional dynamic analysis for alternation in ρ, we can utilize Figures 2 and 3 to obtain the comparative dynamic results for changes in the public policy instruments, τ_K, τ_H and z_H. These are summarized in Table 1. There are several points worth noting. First, in response to these policy changes, consumption per effective labor unit, the relative price and factor prices may adjust instantaneously. Second, all "level" variables but consumption adjust monotonically to the long-run equilibrium and thus balanced growth analysis may be useful for evaluating the effects of policies on all these other variables, but may be biased for welfare analysis since the consumption variable is involved. Third, the factor intensity rankings again play a crucial role in determining how policy changes affect consumption, the physical to human capital ratio and the price variables, except for the case of labor taxation because it is only imposed on the goods sector without generating direct distortion on the intertemporal adjustment process of the relative price of human capital. Fourth, the correlation between consumption, capital, and output could be positive or negative, relying on the nature of the parameter or policy changes, the factor intensity ranking, as well as the timing/position in the adjustment process.

Finally, these preference and policy variations may offer additional explanation to the cross-country differences in the rates of economic growth, which rely on values of k and p, as indicated in equation (16).[13] To see this more clearly, we focus on the case of capital taxation and education subsidy, and display the results in Figure 4 their dynamic effects on the growth rates of consumption (ν_C), physical capital (ν_K) and human capital (ν_H), given standard regularity conditions presented in previous sections. One can easily find the short- and long-run dynamic responses to be very different. For instance, a reduction in the capital tax rate (τ_K) or an increase in education subsidy (z_H) may suppress economic growth in the short run, although it is unambiguous that in the longer run, these policies will always promote economic growth. Our results also lend theoretical support to the claim that public policies may account for a substantial portion of cross-country differences in economic growth.[14]

VI. Concluding Remarks

This main objective of this paper has been to develop a general two-sector model of endogenous growth with human capital and physical capital accumulation, where labor and capital are used in both production sectors. We have shown how the use of the intertemporal no-arbitrage condition yields a simplified analysis of the balanced growth path, and have shown how this technique can be applied to analyze effects of parameter and public policy changes.

Our theoretical findings have generated empirically testable implications. We have pointed out that upon exogenous structural or policy perturbations, the short- and long-run responses may be very different. Since the widely used Summers-Heston data set may well contain transitional effects from the episodes of the 1940's, our comparative dynamic results may help to disentwine the short-run transitional and long-run balanced growth effects. Moreover, due to the general two-sector nature of the model, the speed of convergence of the economy will be much slower [as found in the calibrated study of Mulligan and Sala-i-Martin (1992)], which seems to be consistent with the cross-country evidence obtained in Barro and Sala-i-Martin (1992).

We conclude with some brief comments on how the techniques in this paper can be applied to other extensions of the model. The first issue concerns the welfare analysis of taxation. We have briefly analyzed the effect of factor taxes on the rate of growth, but a more complete analysis would compare the effects of the two taxes on welfare given a government revenue objective. In addition, the model can be used to analyze other forms of taxes, such as sectoral output taxes. A second extension is to incorporate leisure time into the analysis. If leisure time enters the utility function, households must also choose the amount of human capital to be allocated to leisure activities. Such an extension does not affect the intertemporal no-arbitrage condition, but requires adjustments in the output equations to distinguish between the stock of human capital and the stock that is employed in production. Finally, the model could also be extended, at the expense of analytical generality, to allow for externalities from the stock of capital of the type analyzed by Romer (1986) and Lucas (1988).

Endnotes

1. In particular, Benhabib and Perli (1993) show that in the Lucas (1988) model with human capital externalities, one may obtain a saddle-path stable, balanced growth equilibrium if the subjective rate of time preference is greater than the maximal rate of human capital accumulation and the intertemporal elasticity of substitution is sufficiently high.

2. See also King and Rebelo (1990) and Rebelo (1991) for justifying the usefulness of a general two-sector endogenous growth model.

3.Rebelo (1991) concentrates on the solvability and characterization of the long-run balanced growth equilibrium, while King and Rebelo (1990) focus on the calibration analysis of policy effects.

4.Mulligan and Sala-i-Martin (1992) allow for uncompensated positive externalities and find that the existence of balanced growth relies crucially on a specific restriction on the Cobb-Douglas production parameters (i.e. the output elasticities of the two capital inputs and their externality factors). Jones and Manuelli (1990) include an additively separable, strictly concave component of the unified capital input into Rebelo's (1991) linear production technology.

5.There is an interesting comparison between the instability of the quantity adjustment described in this paper and that obtained in the two-sector growth model of Uzawa (1961, 1963) where only capital accumulation is endogenous. In the Uzawa model, instability may arise when the investment good is capital-intensive relative to the consumption good. This is analogous to our case in which the education sector is labor-intensive, since each factor is used intensively in its own production.

6.This formulation allows for the perpetual accumulation of human capital, which accords with the observation that ancestor's human capital skill levels can affect the descendant's through home education and intergenerational knowledge transfers; see Ehrlich and Lui (1991).

7.Differently from Rebelo (1991), we exclude the case of log-linear utility to ensure homogeneity property of the utility function, which will simplify the analysis of the dynamics of the system below. For similar treatment, see Huffman (1992, p. 1577).

8.In deriving the transversality conditions (3f) and (3g) for this problem, Benveniste and Scheinkman (1982) impose an assumption that the objective function u is non-negative. For $\sigma > 1$ this condition will not be met by (P1). However, the purpose of this assumption is to assure that the state vector and the costate variables are positive. Assumptions which accomplish this, and which are met by (P1), can be used to show that (3f) and (3g) are necessary for this problem.

9.For example, suppose the production processes in both sectors have fixed coefficients, and let a_{ij} denote the amount of factor i ($i=K,L$) required to produce a unit of output in sector j ($j=X,Y$). It can be shown that $r \in [0,1/a_{KX}]$ and $(w/p) \in [0,1/a_{HY}]$. This yields $(r-w/p)_{sup} = 1/a_{KX}$ and $(r-w/p)_{inf} = 1/a_{HY}$. On the other hand, if both sectors have Cobb-Douglas production functions, the differentials are unbounded.

10. Jones and Manuelli (1990) impose a global condition on the technology to ensure that it is possible to reach the balanced growth path starting from any initial endowment. In the two-sector model presented here, only a local condition is required because of the assumption of constant returns to scale. With constant returns to scale, it is possible to attain the balanced growth path from any initial factor endowment (K_0,H_0) by disposing freely a sufficient quantity of one of the factors until the aggregate relative endowment ratio equals the one associated with the balanced

growth equilibrium.

11. In the presence of sufficiently large spillover externalities, it is possible to have a continuum of balanced growth equilibria with positive growth rates, as found in Boldrin and Rustichini (1991), Xie (1992) and Benhabib and Perli (1993).

12. As in the previous case, this analysis holds assuming that the non-negativity constraints on factor accumulation are not binding. This is equivalent to assuming that $k \in [k_y(p), k_x(p)]$ along the optimal path. This restriction cannot be simply illustrated in (c,k) space, since p (and thus the bounds of the feasible region) are changing along the path.

13. More specifically, cross-country growth-rate differences may be due to differences in (a) preferences and technologies, (b) fiscal and development policies, and/or (c) timing/position in transition toward the long-run balanced growth path.

14. For example, Young (1992) and Lucas (1993) explain, in part, why it turned out to be successful strategy for Korea and Singapore to emphasize on tax and development policies that enhanced physical capital accumulation, and for Hong Kong and Taiwan to focus on education and trade policies that promoted human capital evolution.

APPENDIX A

Proposition 1 establishes the uniqueness of the price associated with balanced growth for the case in which there are no factor-intensity reversals (FIRs). This proof relied on the fact that when there are no reversals, w(p) and r(p) are single-valued. These functions were then used to derive the result that the FP locus must be continuous and downward-sloping. In the presence of FIRs, however, w(p) and r(p) may be correspondences and the method of proof used in the text will not apply.

In this section we show that Proposition 1 continues to hold in the presence of FIRs. This is done by showing that single-valued functions w(r) and p(r) will always exist (regardless of whether there are FIRs), and that these functions can be used to derive the properties of the FP curve.

If both sectors are in operation, perfect competition requires that

$$\Pi_x(w,r) = 1 \tag{A.1a}$$

$$\Pi_y(w,r) = p \tag{A.1b}$$

where Π_i is the unit cost function for sector i. Assuming that isoquants are strictly quasi-concave, the implicit function theorem can be applied to (A.1a) to yield w as a function of r, where $dw = -k_x dr$ using the envelope theorem. Substituting this functional relationship in (A.1b) gives p as a function of r, $dp = \Pi_{yw}(k_y - k_x)dr$, where $\Pi_{ij} \equiv \partial\Pi_i/\partial j$ $(i=x,y;\ j=r,w)$. These two relationships can then be combined to yield

$$d(w/p) = -(1/p^2)\,(w\Pi_{yw}k_y + r\Pi_{yr}k_x)\ dr \tag{A.2}$$

where use has been made of the fact that $w\Pi_{yw} + r\Pi_{yr} = p$. (A.2) indicates that the FP locus will be continuous and downward sloping, regardless of the factor intensity ranking of the sectors. Therefore, p^* will exist and be unique if condition FP is satisfied, as in the case without FIRs.

The introduction of FIRs makes the dynamics of p slightly more complicated than the cases illustrated in Figure 1. As a result of cost minimization, k_i is a single-valued, decreasing function of w/r. Since w(r)/r is decreasing in r by the above results, we can express factor proportions in each sector as a decreasing function of r, $k_i(r)$. Define $B(r) \equiv k_x(r)-k_y(r)$, and let $I_x \equiv \{r \mid B(r) > 0\}$. I_x will be the union of intervals of the real line, representing the values of r for which sector X is capital-intensive. Within each interval, r is a unique function of p, and the results of Lemma 1 apply within the interval. Similarly, I_x^c is a union of intervals indicating values of r for which sector X is labor-intensive. The result of Proposition 1 regarding the stability of the price adjustment process applies to the interval that contains r^*.

APPENDIX B

In this section we show that the problem (P1) has a saddle path which converges to the balanced growth path characterized in Proposition 1. As noted above, this is most easily shown by analyzing the problem (P2), which expresses variable in per unit human capital terms. Because the discount factor in (B.1) depends on the control variables, it is convenient to solve for the optimal policies as a function of Δ, using the fact that $d\Delta = \Gamma(t)dt$, $\Gamma(t) = \rho - (1-\sigma)v_H(t)$. This transformation turns the problem into one in which standard techniques from optimal control theory can be applied. The problem (P2) can then be restated as

$$H_0^{1-\sigma}v(k_0) = \max_{c,S_{hx},S_{kx},k} \quad H_0^{1-\sigma}\int_0^{\infty} \frac{c(t)^{1-\sigma}}{(1-\sigma)\Gamma}\, e^{-\Delta}\, d\Delta \tag{B.1}$$

$$\text{subject to } dk/d\Delta = [S_{hx}f(k_x)-c-(1-S_{hx})g(k_y)k+(\eta-\delta)k]/\Gamma.$$

To solve this problem, form the current-value Hamiltonian with costate variable, θ:

$$\Lambda(k,S_{hx},S_{kx},c,\theta) = \left[\left(c^{1-\sigma}/(1-\sigma)\right)+\theta\left(S_{hx}f(k_x)-c-(1-S_{hx})g(k_y)k+(\eta-\delta)k\right)\right]/\Gamma. \tag{B.2}$$

For $\sigma > 1$, this problem will be concave in the control variables and will satisfy the conditions of Benveniste and Scheinkman (1982). The necessary conditions are

$$c^{-\sigma} - \theta = 0 \tag{B.3a}$$

$$\theta r\left(1 + k/p\right) - \Lambda^{*}\left(1-\sigma\right)r/p = 0 \tag{B.3b}$$

$$\theta w\left(1 + k/p\right) - \Lambda^{*}\left(1-\sigma\right)w/p = 0 \tag{B.3c}$$

$$\frac{d\theta}{d\Delta} = \theta - \frac{\partial\Lambda}{\partial k} = \theta - \frac{\theta}{\Gamma}\left[S_{kx}\,r + (\eta-\delta) - \left(1-S_{hx}\right)g\left(k_y\right) - \left(1-s_{kx}\right)k\,r/p\right]$$

$$- \frac{\Lambda(1-\sigma)}{\Gamma}\left(1-S_{kx}\right)r/p \tag{B.3d}$$

$$\lim_{\Delta\to\infty} k\theta e^{-\Delta} = 0 \tag{B.3e}$$

where Λ^{*} is the value of the Hamiltonian (B.2) at its maximum.

The costate variable, θ, has the interpretation of being the value of a unit of physical capital per unit of human capital. It is useful in interpreting (B.3) to relate this variable to the variables

μ, λ, and p used in analyzing (P1). Since the costate variables are the derivatives of the value functions with respect to the respective state variables, we have $\theta = v'(k)$, $\mu = V_k$, and $\lambda = V_H$. Differentiation of the identity $H^{1-\sigma} v(k) = V(kH, H)$ yields the relations:

$$\theta = H^{\sigma}\mu \tag{B.4a}$$

$$H^{\sigma}\lambda = (-\sigma)v(k) - k\,\theta \tag{B.4b}$$

$$p = (1-\sigma)\,v(k)/\theta - k. \tag{B.4c}$$

Using (B.4), it can be seen that (B.3a-c) will be equivalent to (3a-c) if $v(k) = \Lambda^{*}$. This equality will hold since $v(k) = \int_0^{\infty} \Lambda^{*} e^{-\Delta} d\Delta - \int_0^{\infty} \theta\,(dk/d\Delta)\, e^{-\Delta} d\Delta$. Using $v'(k) = \theta$ and integrating by parts yields the desired equality.

Equation (B.3d) can be rewritten using (B.3b) as:

$$\frac{1}{\theta}\frac{d\theta}{dt} = \frac{\Gamma}{\theta}\frac{d\theta}{d\Delta} = p + \delta - r + \sigma\Big[\big(1 - S_{hx}\big)\, g\big(k_y\big) - \eta\Big]. \tag{B.5}$$

(B.5) implies that in the steady state where $d\theta/d\Delta = 0$, we have $v_H = (1\text{-}S_{hx})\, g(k_y) - \eta = (\rho+\delta\text{-}r_x)/\sigma$, which is identical to the steady state growth rates obtained using (7) and (8a) in the text.

Finally, since $\dot{k} \rightarrow 0$ and $\dot{\theta} \rightarrow 0$ in the limit, differentiation of (B.4c) establishes $\dot{p} \rightarrow 0$.

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TABLE 1
Comparative Dynamics

A. Case I: $k_x < k_y$

Increase in	Shifts in			Changes in				
	$\dot{c}$=0	$\dot{k}$=0	$\dot{p}$=0	c	k	p	r	w/p
ρ	left	none	none	(a)	-	0	0	0
τ_K	left	down	out**	- or (c)	-	+*	+*	-*
τ_H	none	down	none	-*	0	0	0	0
z_H	none	up	out**	+*	0	+*	+*	-*

B. Case II: $k_x > k_y$

Increase in	Shifts in			Changes in				
	$\dot{c}$=0	$\dot{k}$=0	$\dot{p}$=0	c	k	p	r	w/p
ρ	right	none	none	(b)	+	0	0	0
τ_K	left†	up†,‡	in	(a)	-	-	+	-
τ_H	none	down	none	-*	0	0	0	0
z_H	left	up‡	in	(a)	-	-	+	-

Notes:
(a) increase sharply instantaneously and then decrease gradually to a permanently higher level
(b) decrease instantaneously and then increase continuously but sharply to a permanently higher level
(c) increase instantaneously and then decrease continuously but sharply to a permanently lower level
\+ monotonic increase along the whole transition path
\- monotonic decrease along the whole transition path
* adjust instantaneously (i.e., jump to the new balanced growth path)
** indicates that the p=0 plane in Figure 4a shifts away from the origin
† the relative price (p) effect dominates
‡ the output effects (on x and y) dominate

FIGURE 1a

$k_x > k_y$

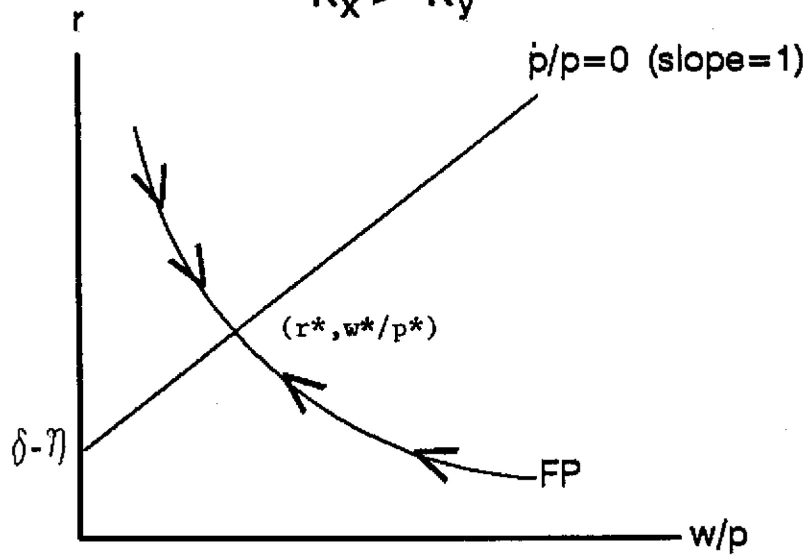


FIGURE 1b

$k_x < k_y$

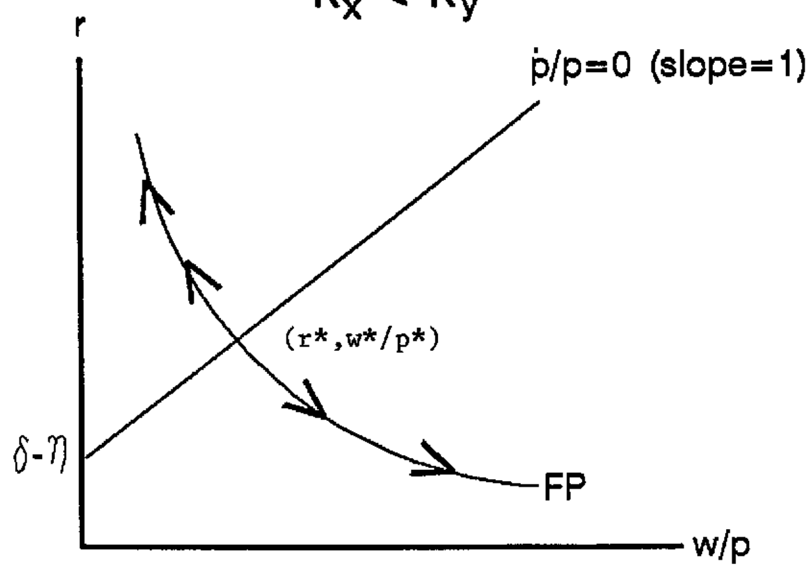


FIGURE 2
Saddle-Path Stability: $k_x < k_y$
(Projection on the c-k Space)

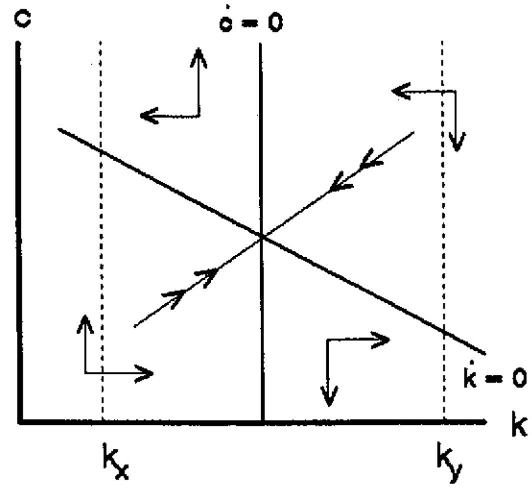


FIGURE 3a
Saddle-Path Stability: $k_x > k_y$

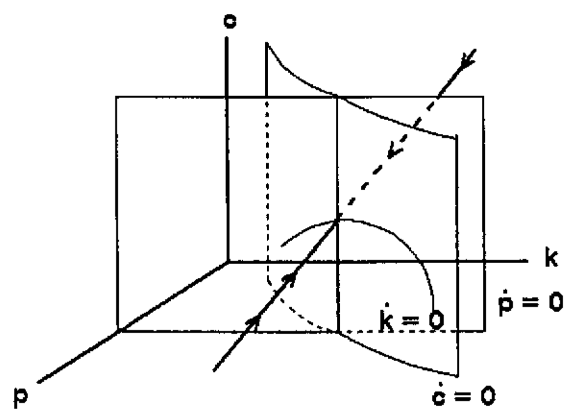


FIGURE 3b
Projection on c-k Space: $k_x > k_y$

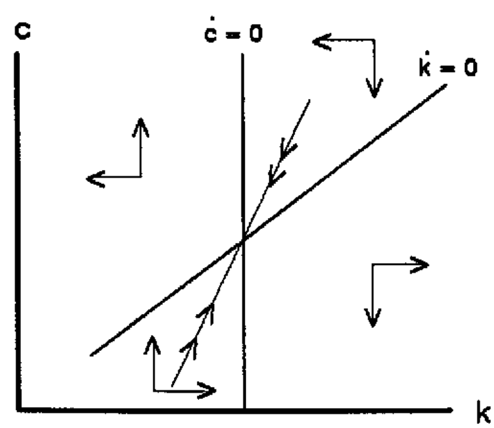
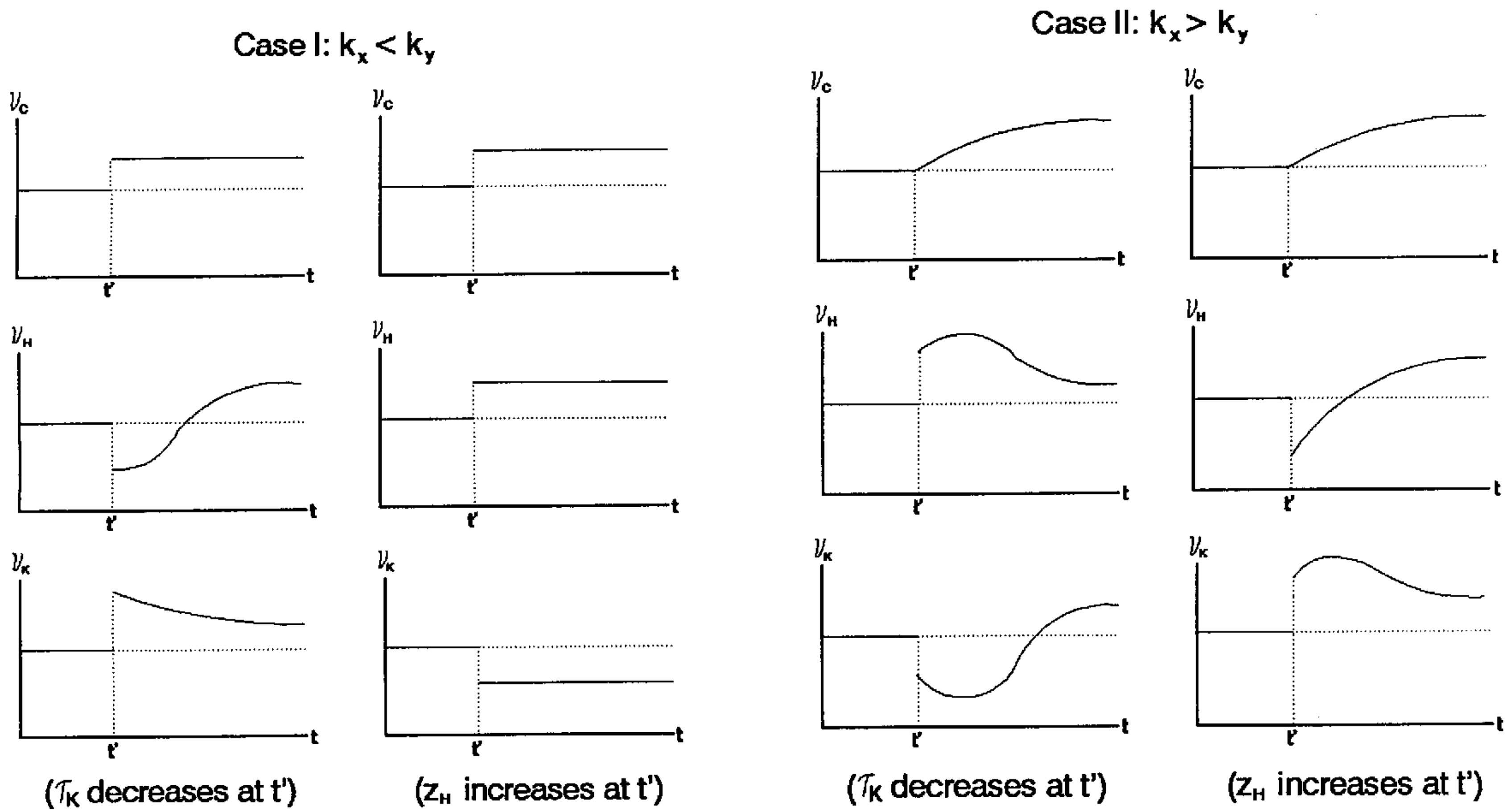


Figure 4



Case I: $k_x < k_y$
Case II: $k_x > k_y$
ν_C
ν_H
ν_K
t
t'
(τ_K decreases at t')
(z_H increases at t')
(τ_K decreases at t')
(z_H increases at t')

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