Federal Reserve Bank of Dallas

presents

RESEARCH PAPER

No. 9311

Real Effects of Money and Welfare Costs of Inflation in an Endogenously Growing Economy with Transactions Costs

by

Ping Wang, Research Department
Federal Reserve Bank of Dallas

and

Chong K. Yip
Georgia State University

March 1993

REAL EFFECTS OF MONEY AND WELFARE COSTS OF INFLATION IN AN ENDOGENOUSLY GROWING ECONOMY WITH TRANSACTIONS COSTS

by

Ping Wang
Pennsylvania State University
Federal Reserve Bank of Dallas

and

Chong K. Yip
Georgia State University

First Draft: October 1991

Current Version: March 1993

Abstract: This paper studies the real effects of anticipated inflation in a monetary endogenous growth model where money is introduced via a transactions cost technology. Through a reduction in real balances per unit of consumption, an increase in the money growth rate raises transaction time and lowers the endogenous labor-augmenting technical progress, thus suppressing the growth rate of the economy. The main driving force of this non-superneutral result is that money affects the engine of growth directly. Quantitatively, the adverse non-superneutral effect on economic growth is unsubstantial, but the welfare loss of anticipated inflation is not negligible.

JEL Classification: O42.

Acknowledgement: We have benefited from comments and suggestions on an earlier version by Shaghil Ahmed, John Boyd III, Tom Cunningham, Marco Espinosa, Robert Feenstra, John Fender, Gregory Huffman, Barry Ickes, Jorge Martinez, Theodore Palivos, John Rogers, Carol Scotese, Alan Stockman, Ellis Tallman and Mark Wynne, as well as participants of the 1991 Midwest Mathematical Economics Conference in Indianapolis. Any remaining errors are, of course, our own responsibility.

Address For Correspondence: Ping Wang, Research Department, Federal Reserve Bank of Dallas, 2200 N. Pearl Street, Dallas, TX 75201.

1. INTRODUCTION

This paper studies the long-run interactions between real and monetary sectors within an endogenous growth framework.[1] We first examine how anticipated inflation may affect the engine of economic growth and quantitatively assess how important such a long-run growth effect will be. We then extend the methodology of Cooley and Hansen (1989) to compute the resulting welfare costs of the inflation tax.

Whether money matters has been controversial ever since the Milton Friedman-Walter Heller debate. The traditional Phillips-curve approach argues that moderate inflation may be welfare-improving because it can reduce the unemployment rate in the short run. This idea has been recently challenged by the believers of the zero-inflation policy, who emphasize inflation is always costly to social welfare. Apart from the above-mentioned lively controversies, the theoretical studies of money and growth have also been disputing about whether inflation is conducive or detrimental to economic growth.[2]

Traditionally, the literature analyzes the steady-state properties of the system, particularly the impact of an increase in the growth rate of money on the steady-state <u>levels</u> of (per capita) consumption and output. We

---

[1] Recently, the generation of endogenous growth without depending upon exogenous changes in population or technology has become one of the central issues in growth theory. The endogenous evolution of human capital is a major force, among others, driving economic growth. See Lucas (1988), Rebelo (1988) and Romer (1989b).

[2] The asset-substitution model of Tobin (1965) concludes that anticipated inflation promotes capital accumulation and output, while the cash-in-advance model of Stockman (1981) produces the reverse result. Using the money-in-the-utility-function approach, Sidrauski (1967) finds that money is superneutral. For an elaboration on this issue within the exogenous growth framework, the reader is referred to Dornbusch and Frenkel (1973) and Wang and Yip (1992b).

depart from this conventional wisdom by introducing money into an endogenous growth framework. We investigate the effects of anticipated inflation on the growth rates of consumption, output, real balances, and capital accumulation. Moreover, we provide a complete characterization of the transitional dynamics, which is, to our knowledge, the first attempt at such an endeavor in the area of money and endogenous growth.

Generally speaking, there are several alternatives to introduce money into an optimizing dynamic general-equilibrium model: money-in-the-utility-function (MIUF), money-in-the-production-function (MIPF), cash-in-advance (CIA) and transactions costs (TC) models. Given the homogeneity property of the utility/production function in the endogenous growth literature, the MIUF/MIPF approach will make monetary growth have no direct effect on the steady-state growth rate of the real economy in the absence of distortionary taxes [see Roubini and Sala-i-Martin (1992)/Gylfason (1991) and Wang and Yip (1992a)]. In a more complex setting, the MIUF approach may generate non-superneutral results and the real effects of monetary growth need not be adverse [see van der Ploeg and Alogoskoufis (1992)]. On the other hand, the CIA model of Rebelo (1988) generates a constant velocity of money (often equals to unity), which consequently excludes a possibly important channel of the "real effects" of anticipated inflation through changes in velocity. For generalization of this previous work using a CIA-like constraint, the reader is referred to Gomme (1991), Jones and Manuelli (1991), Mino (1991), Ireland (1992) and Marquis and Reffett (1992).

In contrast to these studies, the present work introduces money into the economy via a transactions cost technology. Following Drazen (1979), we postulate transactions time as a function of the ratio of real money balances to consumption, through which the engine of economic growth - human capital evolution - will be affected by the rate of money growth. The advantage of this framework is that the underlying transitional dynamics is well-defined and manageable using standard simulation techniques. It therefore enables us to quantitatively assess the real effects of money and the welfare costs of inflation, both at the long-run balanced-growth equilibrium and along the transition path where accumulated short-run variations are accounted for.

The main findings of the paper are as follows. First, a higher growth rate of money reduces the steady-state growth rates of per capita consumption, output, real money balances and capital accumulation. Second, contrary to standard beliefs, the effect of anticipated inflation on the income velocity of money is ambiguous both in the steady state and in transition. This is due to the presence of a negative effect via the endogenous growth rate, opposing to the conventional one. Third, since the real rate of return to capital depends on the endogenous growth rate of the economy, money growth creates an adverse effect on the real interest rate. This allows for a less-than-one-to-one adjustment of the nominal interest rate to anticipated inflation, consistent with Irving Fisher's conjecture and Summers' (1983) empirical evidence. Fourth, by performing calibration exercise, we find that the effect of higher money growth is to increase the rate of inflation almost proportionately. This implies that money growth has a quantitatively unsubstantial effect on the growth rates of consumption, output and factor inputs. In other words, money is in essence superneutral in terms of its impact on economic growth rates. This conclusion therefore corroborates the empirical findings of Christiano and Ljungqvist (1988) and

Barro (1990a).[3]

Finally, the welfare cost of anticipated inflation is computed from the lifetime utility of the representative agent along the the whole transition path and found to be not negligible. Specifically, our calibration results show that even for moderate inflation (say about 10 percent) the welfare cost is about 3.6 percent of GNP. This is much higher than the findings of the existing literature that the magnitude of the welfare cost is between 0.3 and 0.5 percent of GNP [e.g., see Fischer (1981), Lucas (1981), and Cooley and Hansen (1989, 1991)]. This is because these studies have focused only on the steady state effects rather than the cumulative effects along the whole transition path. Thus, if we allow money to affect the engine of growth, then anticipated inflation can have serious dynamic distortions on the macro economy. This study on the welfare loss of the inflation tax may therefore enhance our understanding and clarify the nature of the debate between Phillips-curve economists and zero-inflation policymakers.

The organization of the paper is as follows. In section 2, we develop the basic monetary endogenous growth model where money is introduced via a transactions costs technology. Section 3 characterizes the balanced-growth equilibrium, while section 4 analyzes the transitional dynamics. In section 5, we calibrate the model to assess the effects of anticipated inflation quantitatively. Section 6 examines the welfare costs of the inflation tax, and section 7 concludes the paper.

---

[3] Christiano and Ljungqvist (1988) find that the rate of money growth fails to Granger-cause the growth rate of output, although the level of money stock appears to strongly Granger-cause the level of output. Barro (1990a) reports a negative but weak relationship between the inflation rate and the growth rate of real per capita GDP for the period between 1970 and 1985 in a cross section of 117 countries.

2. THE MODEL

Consider a continuous-time, representative-agent, perfect-foresight model of neoclassical monetary growth, in which both physical and human capital are endogenously determined. For simplicity, leisure is assumed inelastic.[4] The representative consumer is interested in maximizing his/her lifetime utility, W, which is given by

$$W = \int_0^\infty U(c(t))\, e^{-\rho t}\, dt,$$

where $\rho > 0$ is the consumer's (constant) rate of time preferences and c denotes (per capita) consumption.

Two constraints are faced by the consumer in the utility maximization problem. The first one is a Sidrauski (1967)-like budget constraint

$$c(t) + \dot{k}(t) + \dot{m}(t) = F(k(t),L(t)) - nk(t) - (n+\pi(t))m(t) + \tau(t) \qquad (1)$$

where k and m are (per capita) physical capital and real money balances respectively, τ is the (per capita) lump-sum transfer from the government and π and n are the rates of inflation and population growth respectively. The major difference between (1) and the standard budget constraint in the monetary growth literature is that effective labor input, L, is embodied with an endogenous productivity factor. Specifically, L is defined as the product of labor productivity, h, and the fraction of non-leisure time allocated to production, ℓ, i.e., $L(t) = h(t)\ell(t)$. Notice that h is a Harrod-neutral technological factor which can be interpreted as financial or knowledge-based innovation, or as the human capital skill level. We follow Lucas (1988) by

---

[4] This is a common assumption in endogenous growth models, such as Lucas (1988). Nevertheless, leisure time has to be constant along a balanced growth path. Thus endogenizing leisure would not affect any of our results qualitatively within the balanced growth framework.

maintaining the latter interpretation throughout this paper.

The second constraint is the law of motion of the Harrod-neutral technical innovation, h,

$$\dot{h}(t) = \phi[1 - \ell(t) - s(m(t)/c(t))]h(t), \tag{2}$$

where ϕ denotes the maximal rate of human capital accumulation. Note that (2) is similar to Lucas' (1988) human capital evolution equation except for one major modification - the inclusion of the transactions-time term, s. Specifically, s is a C^2 function that represents the transactions effort and is postulated to be a function of the ratio of real balances to consumption [see Drazen (1979)], satisfying the conditions $s' < 0$, $s'' > 0$, $\lim_{m/c\to 0} s(m/c) = 1$ and $\lim_{m/c\to 1} s(m/c) = a < 1$. In words, holding money enables the representative consumer to economize on the resources that are necessary for carrying out transactions. The marginal return to holding money, $-s'$, is positive and diminishing. When real money balances are enough to accommodate consumption transactions (i.e., $m/c \geq 1$), shopping time is minimized at a.

Thus the problem of the representative agent is to maximize lifetime utility, W, subject to the constraints (1) and (2). In order to obtain a closed-form solution, we assume that the utility function, U, exhibits constant-relative-risk-aversion and that the production function, F, takes the Cobb-Douglas form.[5] Specifically, we have $U(c) = c^{1-\alpha}/(1 - \alpha)$ and $F(k, L) = Ak^{\gamma}L^{1-\gamma}$, where $\alpha^{-1} > 0$ is the intertemporal elasticity of substitution, γ is the capital income share and A is a constant scaling factor.

Let $\mu > 0$ be the (constant) rate of money growth. Under money market

---

[5] These functional forms are common in the endogenous growth literature, see Lucas (1988) and Barro (1990b).

equilibrium, we can express real money transfer as $\tau(t) = \mu m(t)$ and the real balances evolution as

$$\dot{m}(t)/m(t) = \mu - \pi(t) - n. \tag{3}$$

Thus, the budget constraint (1) can be rewritten as

$$c(t) + \dot{k}(t) = F(k(t), L(t)) - nk(t), \tag{4}$$

which is, in effect, the goods market equilibrium condition.

3. BALANCED GROWTH ANALYSIS

In this section, we perform a balanced growth analysis to solve for an optimal endogenous monetary growth equilibrium. Within our analytical framework, there is an equivalence between the centralized and the competitive solution. We therefore characterize the competitive equilibrium for the economy described above as a set of paths $\{c(t), \ell(t), k(t), h(t), m(t)\}$ which solve the following optimization problem:

$$\max W = \int_0^\infty U(c(t))\, e^{-\rho t}\, dt, \tag{P}$$

subject to the constraints (1) and (2), the slack variable identity, $z = \dot{m}$, the nonnegativity constraints of $c(t)$, $k(t)$, $h(t)$, $m(t)$ and $\ell(t) \in [0, 1]$. Further, we specify a balanced-growth competitive equilibrium as a set of paths $\{c(t), \ell(t), k(t), h(t), m(t)\}$ that solve the optimization (P) for some initial conditions $k(0) = k_0$, $h(0) = h_0$ and $M(0) = M_0$ (the nominal money balance), such that $c(t)$, $k(t)$, $h(t)$ and $m(t)$ grow at constant rates, and $\ell(t)$ is constant.

We next provide the necessary and sufficient conditions for the existence of an interior balanced-growth competitive equilibrium. This is given in the following proposition.[6]

Proposition 1: *Under our assumptions on the utility function U(.), the production F(.,.) and the transactions cost function s(.), the necessary conditions for the existence of an interior balanced-growth competitive equilibrium path are:*

$$c^{-\alpha} = \lambda_1 - \lambda_2\phi hs'm/c^2 \tag{5}$$

$$\lambda_1F_L = \lambda_2\phi \tag{6}$$

$$\lambda_1 = \lambda_3 \tag{7}$$

$$\dot{\lambda}_1 = \rho\lambda_1 - (F_k - n)\lambda_1 \tag{8}$$

$$\dot{\lambda}_2 = \rho\lambda_2 - \lambda_1F_L\ell - \lambda_2\phi(1-\ell-s) \tag{9}$$

$$\dot{\lambda}_3 = \rho\lambda_3 + \lambda_1(n+\pi) + \lambda_2\phi hs'/c, \tag{10}$$

together with (1) and (2). The transversality conditions of k, h and m are $\lim_{t\to\infty} e^{-\rho t}\lambda_i(t)q(t) = 0$, *where* λ_1, λ_2 *and* λ_3 *denote costate variables associated with (1), (2) and the slack variable identity, and* $q = k, h$ *and* m *for* $i = 1, 2$ *and* 3, *respectively.*

Unlike traditional Ramsey growth models, an additional term, $-\lambda_2\phi hs'm/c^2$, appears on the right hand side of the intertemporal consumption efficiency condition, (5). This term represents an additional cost of consumption due to the induced transactions cost. Equation (6) determines efficient allocation of labor effort between the production of goods and accumulation of human capital, while equation (7) indicates that the (shadow)

---

[6] All the proofs are presented in the appendix.

prices of the two stores of values (physical capital and money) are equal. Equation (8) is the standard Euler equation of physical capital in neoclassical growth models. From (9), the intertemporal pricing of human capital (λ_2) depends positively on the rate of time preference, but negatively on the marginal product of effective labor and the fraction of non-leisure time devoted to human capital accumulation. Finally, the third term on the right hand side of (10) captures the additional benefit of holding money due to a reduction in transactions costs.

Denote θ as the constant growth rate of per capita consumption, i.e., $\theta = \dot{c}/c$. Then the following lemma summarizes the properties of a balanced-growth competitive equilibrium.

Lemma 1: *Along a balanced-growth competitive equilibrium path, both working time (ℓ) and transaction effort (s) are constant. Consumption, physical capital, human capital, effective labor input and real money balances are all growing at the same rate, i.e.,* $\dot{c}/c = \dot{k}/k = \dot{y}/y = \dot{L}/L = \dot{m}/m = \theta$. *Moreover, the costate variables,* λ_i*'s (defined in proposition 1), are also growing at a common rate, i.e.,* $\dot{\lambda}_1/\lambda_1 = \dot{\lambda}_2/\lambda_2 = \dot{\lambda}_3/\lambda_3 = -\alpha\theta$.

We next provide a characterization of the real and nominal interest rates in the model. This will allow us to examine issues of the Fisher-Summers results on the adjustment of the nominal interest rate to anticipated inflation in the section below.

Lemma 2: *The real interest rate (r) is positively related to the common economic growth rate (θ), and the nominal interest rate (i) is positively related tO both the common economic growth rate (θ) and the inflation rate (π). Specifically, we have*

$$r = \rho + n + \alpha\theta \tag{11}$$

and $$i = \rho + n + \alpha\theta + \pi = -F_L s' h/c = -[(1-\gamma)F_k s']/[\gamma\ell(c/k)]. \tag{12}$$

From (12), a higher marginal transactions cost ($-s'$) leads to a higher rate of nominal interest, while a larger capital income share (γ) reduces the nominal interest rate.

We next solve for the common growth rate (θ) for an interior balanced-growth competitive equilibrium. Due to the complexity of the system, we obtain θ as a solution from a four-by-four equation system. The determination of the economic growth rate is given in the following lemma.

<u>Lemma</u> <u>3</u>: *In an interior balanced-growth competitive equilibrium, the solutions of the endogenous variables $\{\theta,\ c/k,\ m/c$ and $\ell\}$ satisfy the following equations:*

$$\alpha\theta = \phi[1 - s(m/c)] - \rho, \tag{13}$$

$$-[(1-\gamma)(\rho + n + \alpha\theta)s'(m/c)]/[\gamma\ell(c/k)] = \rho + \mu + (\alpha - 1)\theta, \tag{14}$$

$$\rho + (1 - \gamma)n + (\alpha - \gamma)\theta = \gamma(c/k), \tag{15}$$

$$\theta = \phi[1 - \ell - s(m/c)]. \tag{16}$$

From (13), an improvement in transactions efficiency (due to an increase in m/c and hence a reduction in s) promotes economic growth. Then, given the common rate of economic growth, (16) implies that a reduction in transactions costs resulting from a higher real balances to consumption ratio encourages a reallocation of non-leisure time to goods production. Also notably, Hall (1988) found that the intertemporal elasticity of substitution (α^{-1}) is empirically much less than unity. One can therefore expect that $\alpha > \gamma$ (since $\gamma < 1$) and that, from (15), there is a positive relation between θ and c/k.

We now perform a comparative-static analysis of the balanced-growth competitive equilibrium. Given constant parameters, n, ϕ, γ and α, we can totally differentiate (13) - (16) to examine the effects of changes in the money growth rate on the steady-state values of θ, c/k, m/c and ℓ. The results are given in the following proposition.

<u>Proposition</u> <u>2</u>: *Under the assumption that a higher rate of time preference will suppress economic growth, i.e., $d\theta/d\rho < 0$ and that $\alpha > \gamma$, a higher rate of money growth reduces the economic growth rate, θ, the real balances-consumption ratio, m/c, the consumption-physical capital ratio, c/k, and the work effort, ℓ.*[7]

The interpretations of the comparative statics are intuitive. An increase in the rate of monetary expansion lowers the real balances-consumption ratio along a balanced growth path. Under the transactions cost technology, this then leads to an increase in transactions effort, thus reducing the rate of the labor-augmenting technical progress and retarding the rate of growth of economic aggregates. Since the consumption substitution effect generally dominates the production substitution effect (as represented by the inequality $\alpha > \gamma$), the reduction in consumption due to an increase in the money growth rate outweighs the induced reduction in physical capital, resulting in a lower c/k ratio.

---

[7] The purpose of invoking the condition that $d\theta/d\rho < 0$ is to assure that det(D) < 0 (see the proof of proposition 2 in the appendix for more details). Of course, alternative ways of signing det(D) (such as stability analysis) are possible. We prefer to use $d\theta/d\rho < 0$ since such a condition has been well-documented in growth theory [e.g., see Lucas (1988)].

Finally, using proposition 2, the rate of inflation can be shown to rise more than proportionately in response to higher money growth. This then implies that there is only a partial adjustment of nominal interest rates to anticipated inflation which corroborates Fisher's assertion and Summers' (1983) empirical finding. We summarize these results in the following corollary.

Corollary 1: *Given $\alpha > 1$, we have*

$$d\pi/d\mu = 1 - d\theta/d\mu > 1,$$

and

$$di/d\mu = 1 + (\alpha - 1)(d\theta/d\mu) < 1.$$

4. TRANSITIONAL DYNAMICS

In the context of our endogenous monetary growth model, human capital accumulation serves as the main engine of growth. To study transitional dynamics, it is more convenient to transform all growing variables in units of the common growth component, h. We first rewrite the optimization problem of the representative agent as follows.

Lemma 4: *Define $\tilde{c} = c/h$, $\tilde{k} = k/h$, $\tilde{m} = m/h$ and $\tilde{\tau} = \tau/h$. Then the optimization problem of the representative agent described in section 2 above can be rewritten as*

$$\max_{\tilde{c},\tilde{z},\tilde{m},\tilde{k},\ell,\theta} \int_0^\infty \frac{\tilde{c}^{1-\alpha}}{(1-\alpha)[\rho - (1-\alpha)\theta]} e^{-\Delta} d\Delta \qquad (P')$$

subject to

$$\tilde{c} + [\rho-(1-\alpha)\theta](d\tilde{k}/d\Delta+\tilde{z})= F(\tilde{k},\ell) - (n+\theta)\tilde{k} - (n+\pi+\theta)\tilde{m} + \tilde{\tau} \qquad (17)$$

and

$$\theta = \phi[1 - \ell - s(\tilde{m}/\tilde{c})], \qquad (18)$$

where $\Delta(t) = \int_0^t[\rho - (1-\alpha)\theta(v)]dv$, $\tilde{z} = d\tilde{m}/d\Delta$, *and* $h(0)$ *is normalized to be unity. The invertibility of* Δ *is ensured if* $\rho - (1-\alpha)\theta(t) > 0$ *for all* t.

We next characterize the dynamics of the economy in a three-by-three system in terms of $\{\tilde{c}, \tilde{k}, \tilde{m}\}$ in the following lemma.

<u>Lemma 5</u>: *The dynamics of the transformed system is governed by the following equation system in terms of* $\{\tilde{c}, \tilde{k}, \tilde{m}\}$:

$$\dot{\tilde{c}}/\tilde{c} = -(1/\alpha)(\rho + n + \alpha\theta - F_k). \tag{19}$$

$$\dot{\tilde{m}}/\tilde{m} = \mu - \pi - n - \theta. \tag{20}$$

$$\dot{\tilde{k}} = F(\tilde{k}, \ell) - (n + \theta)\tilde{k} - \tilde{c}. \tag{21}$$

It is difficult to establish mathematically the stability property of the system of (19) - (21) due to its analytic complexity. However, using simulation technique with a fairly wide range of plausible parameter values described in section 5 below, the dynamic system of (19) - (21) has one negative and two positive characteristic roots and is, therefore, saddle-path stable. The phase diagram of the complete system is presented in Figure 1, where E represents a steady-state equilibrium point and E_1, E_2 and E_3 are its corresponding projections on the two-dimensional $(\tilde{c}, \tilde{k})$, $(\tilde{k}, \tilde{m})$ and $(\tilde{m}, \tilde{c})$ planes, respectively. Point B denotes the instantaneous position after the increase in the money growth rate while E′ is the new steady state.

A complete illustration of the transitional dynamics of the m/c, c/k and m/k ratios is presented in Figure 2a. At instant t_1, an expansionary monetary growth policy is imposed. As a consequence, it can be shown that all three ratios jump up instantaneously and decrease eventually to a permanently lower level. As illustrated in Figure 2b, work effort (ℓ) also responses to the money growth rate in a similar fashion. Finally, both the instantaneous and long-run effects of an increase in the money growth rate on economic growth (θ) can be shown to be negative. In transition, the movement of the economic growth rate may be depicted in Figure 2c. We summarize the transitional dynamics in the following proposition.

Proposition 3: *On impact, a higher rate of money growth increases work effort, the real balances-consumption ratio, the consumption-physical capital ratio and the real balances-physical capital ratio, but it reduces the economic growth rate. In transition, all these variables decrease eventually, converging to the new balanced-growth path.*

Before concluding this section, it is worth discussing the effects of anticipated inflation on the income velocity of money. Coventionally, the income velocity of money is found to depend positively on the monetary growth rate. However, in our model, we find that this relationship is in general ambiguous both in the steady state and in transition. Define the income velocity of money as $v \equiv F(k, L)/m$. Using (4), we have

$$v = c/m + (k/m)(\theta + n). \qquad (22)$$

Thus, anticipated inflation affects velocity through three channels: c/m, k/m and θ. Notice that the changes in c/m and k/m ratios represent the intertemporal substitution effect and the asset substitution effect, respectively. These effects constitute the traditional positive effect of anticipated inflation on velocity in the steady state. However, within our endogenous growth framework, there is a negative effect of money growth on velocity via the economic growth rate. Therefore, the net long-run effect on velocity cannot be determined unambiguously. We summarize the findings in the following corollary.

Corollary 2: *The effects of anticipated inflation on the income velocity of money is ambiguous both in the steady state and in transition.*

5. QUANTITATIVE EFFECTS OF MONEY GROWTH

In this section, we quantitatively assess the importance of the theoretically derived effects of monetary growth. We first choose plausible parameter values consistent with U.S. data and then compute the magnitude the real effects of the money growth rate in the steady state. We also provide a sensitivity analysis by perturbing the main parameters to test the robustness of our quantitative conclusions.

We first assume the rate of time preferences (ρ) to be 0.03 [Davies and Whalley (1991)]. Following Lucas (1988), we choose the capital income share (γ) and the population growth rate (n) as 0.25 and 0.013, respectively. Based on Hall (1988), α is usually above unity and, in particular, we take it to be 2.5 (i.e., the intertemporal elasticity is 0.4).

We next calculate the mean growth rates of (per capita) output, (per capita) money stock and the average inflation rate using U.S. data over Denison's sample period (1909-57) to obtain θ = 0.014, μ = 0.049 and π = 0.022 respectively. Similarly, we compute the average money-income and consumption-output ratios which are 0.149 and 0.75, respectively. Following Romer (1989a), we assume the capital-output ratio to be 3.9.

Substituting the above values into (11), we get F_k = 0.078 which seems reasonable. Taking the fraction of non-leisure time devoted to goods production as 0.75,[8] we get $\phi = 0.068$ and $s = 0.04412$ from (13) and (16). In order to calculate the effect of the money growth rate on economic growth, we also need to know the values of s' and s''. This requires a specification of the transactions cost function: $s = a(m/c)^{1-\varepsilon}$, for $m/c < 1$; $s = a$, for $m/c \geq 1$, where $\varepsilon > 1$ (by convexity) and a is a positive constant. Based on the above parameter values, we have $a = 0.02817$ and $\varepsilon = 1.2776$, which complete the parameterization of the model: $\rho = 0.03$, $\gamma = 0.25$, $n = 0.013$, $\alpha = 2.5$, $a = 0.02817$, $\varepsilon = 1.2776$ and $\mu = 0.049$.

Now we can calculate the determinant, det (D), which is about 0.1637. Therefore, the effects of an increase in the money growth rate on economic growth and other endogenous variables are given as follows:

<u>Result 1</u>: *The quantitative effects of money growth are given by*

$$d\theta/d\mu = -0.002568,\ d\pi/d\mu = 1.002568,$$

$$d(m/c)/d\mu = -1.5316,\ d(c/k)/d\mu = -0.02311,$$

$$di/d\mu = 0.9961,\ ds/d\mu = 0.0944,\ d\ell/d\mu = -0.05664.$$

Although a higher money growth rate retards the common growth rate of consumption, output, physical and human capital, the calibration result shows that such an effect is very marginal and that the money growth rate and the inflation rate move close to proportionately in the long run. Hence, money is in essence superneutral in terms of its impact on economic growth rates. Further, a percentage increase in the money growth rate reduces the real

---

[8] Lucas (1988) estimated that $\ell = 0.82$ in the absence of transaction costs. Therefore, assuming $\ell = 0.75$ in the presence of the transaction costs seems reasonable. For the benchmark case presented below, it characterizes a weekly non-leisure time allocation of 50, 15 and 2 hours, respectively, to goods production, human capital evolution and shopping. Further, as discussed in section V below, our calibration results are robust to the selection of ℓ.

balances-consumption ratio by approximately 1.5%. However, the drop in this ratio only creates a very small positive effect on the transactions cost, s, which is about 0.09%. This effect will be further dampened by a factor of ϕ/α = 0.027, which explains why the impact of μ on θ is very marginal. Given the negligible magnitude of $d\theta/d\mu$, the real rate of interest is almost unaffected by changes in the money growth rate. So the adjustment of the nominal interest rate to anticipated inflation is nearly complete (i.e., $di/d\mu$ = $0.9961 \approx 1$).

We next perform a sensitivity analysis with respect to the following parameters: ρ, γ, n, α, a, ε, ϕ and μ. Starting from the original balanced growth equilibrium, we perturb the initial parameter values by considering widely used alternative values for ρ, α and γ and by changing n, a, ε, ϕ and μ within a 20% range. The implied values of $d\theta/d\mu$ and $di/d\mu$ are summarized in Table 1.

Two aspects of the results deserve comments. First, the magnitude of the adverse effect of monetary expansion on economic growth is more sensitive to changes in α, ρ and ϕ. With a higher rate of maximal human capital growth (higher ϕ), a higher rate of time preferences (higher ρ), or a lower intertemporal elasticity of substitution (higher α), the magnitude of $d\theta/d\mu$ increases unambiguously. Second, within a reasonable range, selections of alternative values for other parameters never generate an substantial adverse effect from monetary expansion. Thus it seems robust to conclude:

Result 2: *Money is in essence superneutral in terms of its impact on the growth rates of economic aggregates.*

6. WELFARE COSTS OF ANTICIPATED INFLATION

This section measure explicitly the welfare costs of anticipated inflation. Following the methodology of Cooley and Hansen (1989), we measure the welfare costs by comparing different balanced growth equilibria associated with different growth rates of the money supply. There is, however, one crucial difference between our calculation and Cooley and Hansen's. We compute the welfare cost of anticipated inflation from the cumulative lifetime utility of the representative agent along the whole transition path rather than just focusing on the steady state alone.

Under the constant-relative-risk-aversion specification of the instantaneous utility function, the lifetime utility of the representative agent can be expressed as

$$W = \frac{1}{\rho(\alpha - 1)} + \frac{c(0)^{1-\alpha}}{1 - \alpha} \frac{1}{\rho + (\alpha-1)\theta}, \tag{23}$$

where c(0) denotes the initial level of consumption. Thus, the money growth rate affects individual's welfare via two channels: the (endogenous) initial consumption level, c(0), and the (endogenous) economic growth rate, θ. The welfare costs of monetary growth is studied numerically using the parameter values specified in section 5 and are reported in Table 2.

It is apparent from the theoretical construct that increasing money holdings will not be welfare-improving as it is sufficient to accommodate all transactions. Specifically, for any further reduction of money growth below -5.25% (at which m/c = 1), the economic growth rate and other endogenous variables remain unchanged because the transactions cost is minimized at a. Thus μ = -5.25% can be called an optimal money growth rate.

We then compute the welfare costs for cases when the money growth rate is higher than the critical first-best level (μ = -0.0525). Table 2 shows that under the average money growth rate of 4.9% in the U.S., there is a welfare loss of 7.8% approximately. When money supply grows at 400%, the welfare cost will be around 20%.

In order to compare our results with the finding of the existing literature, we first convert the welfare loss into an equivalent measure in terms of percentage changes of initial real consumption [$\Delta C(0)/C(0)$]. We then present the results in terms of percentage of initial real output [$\Delta C(0)/Y(0)$] by simply multiplying the consumption-output ratio of 0.75 to $\Delta C(0)/C(0)$. For moderate inflation (say about 10%), the existing literature finds that the magnitude of the welfare cost is between 0.3 and 0.5 percent of GNP. For instance, Cooley and Hansen (1989) find that to be about 0.4 percent in their calibration, while Fischer (1981) and Lucas (1981) obtain an estimate of 0.3 and 0.45 percent, respectively, by approximating the area under a money demand function within a partial equilibrium framework.[9] In our transactions cost economy, we can compute the welfare cost of inflation with exogenous growth, which is about 1.7 percent of GNP.[10] Due to distortions in production, money growth creates a larger welfare loss in our model.[11] Moreover, when we account for the adverse effects of anticipated inflation on

---

[9] Notice that in a subsequent paper of theirs, Cooley and Hansen (1991) find that the computed welfare cost of the inflation tax is doubled in the presence of other distorting taxes (such as capital and labor income taxes).

[10] This number is obtained from column 3 of Table 2: 0.75x(1.029-1.005)/1.029 = 0.017.

[11] In our model, money growth affects transactions costs and distorts time allocation, thus creating direct production inefficiency. This type of production distortions is usually absent in the existing literature on examining the welfare costs of anticipated inflation. For instance, Cooley and Hansen (1989) focuses on the distortion in consumption between cash and credit goods. There is no production distortion because their cash-in-advance constraint does not apply to investment goods.

the endogenous growth rate, the welfare costs increase to 3.6 percent of GNP.[12] Although money is "almost superneutral", the adverse effect of money growth on economic welfare is not small.

Result 3: *Even for moderate inflation, the welfare costs is not negligible.*

We obtain the non-negligible welfare costs of moderate inflation mainly because we take into consideration of the cumulative effects along the whole transition path in our calculation. An increase in anticipated inflation, by reducing the real money balances and hence raising transaction costs, retards economic growth by slowing down the accumulation process of human capital. Since the engine of growth usually plays an important role in welfare analysis in endogenous growth models [see Barro (1991) for a discussion], even moderate inflation may lead to high welfare loss by suppressing the accumulation of human capital. Such a channel has been absent in the existing literature of exogenous growth and thus previous studies may have underestimated the welfare costs of anticipated inflation.

7. FURTHER DISCUSSION

This paper studies the economic effects of the rate of money growth in a tractable transactions cost model within an endogenous growth framework. We find that an increase in the rate of monetary expansion retards the rate of growth of macroeconomic aggregates. Money is, in general, non-superneutral and there is a negative relation between the real interest rate and the anticipated rate of inflation. Since the enhancement of labor productivity requires time input, anticipated inflation can affect economic growth via the

[12] This number is obtained from column 6 of Table 2: 0.75x0.0483 = 0.036.

reallocation of time. By reducing the real money balances to consumption ratio, an increase in the rate of money growth raises transactions time and retards the endogenous labor-augmenting technical progress, thus resulting in a lower economic growth rate. Notably, money has a direct impact on the engine of economic growth through the transactions cost technology. Applying standard calibration techniques, we quantitatively assess the non-superneutral effect of monetary growth and find it very small under a wide range of plausible parameter values. Nevertheless, the welfare cost associated with higher money growth is not negligible.

Although theoretically the adjustment of the nominal interest rate to inflation expectations is partial, the calibrated coefficient of adjustment turns out to be 0.9961, which is very close to unity. This contrasts with conventional empirical findings in which the adjustment coefficient is found to be far below one [e.g., see Summers (1983)]. Thus, there is a possible bias in our calibration results. Nevertheless, this bias can be corrected by slightly extending the present theoretical framework. First, if one considers endogenous fertility choice, then an increase in the money growth rate raises transactions costs and hence decreases the amount of time available for child rearing. This reduces the population growth rate (n) and so a lower nominal interest rate will be obtained using (12). Second, analogous to the Uzawa (1965) specification, we can postulate the effective rate of time preference to be a positive function of the consumption growth rate, θ. Then an increase in the money growth rate suppresses economic growth and lowers the effective time preference rate, thus further decreasing the nominal interest rate. Finally, we can reduce the upward bias of the adjustment of nominal interest rates by introducing taxes, a conjecture also given by Summers (1983, p.225).

For instance, given the government spending and deficit structure, an increase in the money growth rate can decrease the capital tax rate by replacing factor tax financing with money financing. This then reduces the real rate of interest, according to the modified golden rule, and generates a lower nominal interest rate.

REFERENCES

Barro, R., "Comment" on R, Dornbusch, F. Sturzenegger, and H. Wolf, "Extreme Inflation: Dynamics and Stabilization," *Brooking Papers on Economic Activity*, 2 (1990a), 68-75.

---------, "Government Spending in a Simple Model of Economic Growth," *Journal of Political Economy*, 98 (1990b), S103-S125.

---------, "Economic Growth in a Cross Section of Countries," *Quarterly Journal of Economics*, 105 (1991), pp. 407-443.

Christiano, L. and L. Ljungqvist, "Money Does Granger-Cause Output in the Bivariate Money-Output Relation," *Journal of Monetary Economics*, 22 (1988), 217-235.

Cooley, T. and G. Hansen, "The Inflation Tax in a Real Business Cycle Model," *American Economic Review*, 79 (1989), 733-748.

---------, "The Welfare Costs of Moderate Inflations," *Journal of Money, Credit, and Banking*, 23 (1991), 483-503.

Davies, J. and J. Whalley, "Taxes and Capital Formation: How Important is Human Capital?" in *National Saving and Economic Performance*, B.D. Bernheim and J.B. Shoven ed., NBER: University of Chicago Press, 1991.

Denison, E., *The Sources of Economic Growth in the United States*, Committee for Economic Development, New York, 1961.

Dornbusch, R. and J. Frenkel, "Inflation and Growth: Alternative Approaches," *Journal of Money, Credit and Banking*, 5 (1973), 141-56.

Drazen, A., "The Optimal Rate of Inflation Revised," *Journal of Monetary Economics*, 5 (1979), 231-48.

Fischer, S., "Towards an Understanding of The Costs of Inflation," in *Carnegie-Rochester Conference on Public Policy*, K. Brunner and A. Meltzer, eds., 15 (1981), 5-42.

Gomme, P., "Money and Growth Revisited," Institute for Empirical Macroeconomics Discussion Paper No. 55, Minneapolis, MN, November 1991.

Gylfason, T., "Endogenous Growth and Inflation," Institute for International Economics Studies Seminar Paper No. 502, Stockholm University, Stockholm, Sweden, October 1991.

Hall, R., "Intertemporal Substitution in Consumption," *Journal of Political Economy*, 96 (1988), 339-57.

Ireland, P., "Money and Growth: An Alternative Approach," unpublished manuscript, Federal Reserve Bank of Richmond, Richmond, VA, April 1992.

Jones, L. and R. Manuelli, "Growth and the Effects of Inflation," unpublished manuscript, Northwestern University, Evanston, IL and Stanford University, Stanford, CA, December 1991.

Marquis, M. and K. Reffett, "Endogenous Growth with Liquidity-Constrained Human Capital Acquisition," unpublished manuscript, Florida State University, Tallahassee, FL, 1992.

Mino, K., "Money and Endogenous Growth in a Cash-In-Advance Economy," unpublished manuscript, Tohoku University, Japan, 1991.

Lucas, R., "Discussion of Towards an Understanding of The Costs of Inflation: II," in *Carnegie-Rochester Conference on Public Policy*, K. Brunner and A. Meltzer, eds., 15 (1981), 43-52.

---------, "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 23 (1988), 3-42.

Rebelo, S., "Essays on Growth and Business Cycles," unpublished Ph.D. thesis, University of Rochester, 1988.

Romer, P., "Capital Accumulation in the Theory of Long-Run Growth," in: R. Barro, ed., *Modern Business Cycle Theory*, Harvard University Press, Cambridge, MA, 1989a.

---------, "Human Capital and Growth: Theory and Evidence," NBER Working Paper No. 3173, Cambridge, MA, November 1989b.

Roubini, Nouriel and Xavier Sala-i-Martin, "A Growth Model of Inflation, Tax Evasion, and Financial Repression," NBER Working Paper No. 4062, Cambridge, MA, May 1992.

Sidrauski, M., "Rational Choice and Patterns of Growth in a Monetary Economy," *American Economic Review: Papers and Proceedings*, 57 (1967), 534-44.

Stockman, A., "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy," *Journal of Monetary Economics*, 8 (1981), 387-93.

Summers, Lawrence H., "The Non-Adjustment of Nominal Interest Rates: A Study of the Fisher Effect," in: James Tobin, ed., *Macroeconomics, Prices and Quantities*, Washington: The Brookings Institution, 1983.

Tobin, J., "Money and Economic Growth," *Econometrica*, 33 (1965), 671-84.

Uzawa, H., "Time Preference, the Consumption Function, and Optimum Asset Holdings," *Value, Capital, and Growth: Papers in Honour of Sir John Hicks*, edited by J.N. Wolfe. Aldine Press, Chicago, 1968.

van der Ploeg, F. and G. Alogoskoufis, "Money and Endogenous Growth," unpublished manuscript, University of Amsterdam, Amsterdam, The

Netherlands, December 1992.

Wang, P. and C. Yip, "Examining the Long-Run Effect of Money on Economic Growth," *Journal of Macroeconomics*, 14 (1992a), 359-69.

--------, "Alternative Approaches to Money and Growth," *Journal of Money, Credit and Banking*, 24 (1992b), 553-62.

APPENDIX

Proof of Proposition 1: The current-value Hamiltonian ($\mathcal{H}$) for the optimization problem can be specified as

$$\mathcal{H} = U(c) + \lambda_1[F(k, L) - nk - (n+\pi)m + \tau - c - z] + \lambda_2[\phi(1 - \ell - s)h] + \lambda_3 z.$$

Then a direct application of Pontryagin's Maximum Principle yields the above first-order necessary conditions. Q.E.D.

Proof of Lemma 1: First, notice that (8) implies

$$F_k = \rho + n - \dot{\lambda}_1/\lambda_1, \tag{A1}$$

which is constant along a balanced growth path. This together with $F_k/\gamma = F/k$ and (4) imply $\dot{c}/c = \dot{k}/k = \dot{y}/y = \dot{L}/L = \theta$. Next, given a fixed endowment of time, an immediate consequence of (2) is that $(\ell + s)$ has to be constant along a balanced growth path. From (6) and (9), we get

$$\dot{\lambda}_2/\lambda_2 = \rho - \phi(1 - s). \tag{A2}$$

Thus s is constant on a balanced growth path and so is ℓ. We now have $\dot{L}/L = \dot{h}/h = \theta$. Further, the fact that s is constant implies a constant ratio of real balances to consumption: $\dot{c}/c = \dot{m}/m = \theta$. From (6), (7) and the constant marginal product of effective labor (because F_k is constant), we have

$$\dot{\lambda}_1/\lambda_1 = \dot{\lambda}_2/\lambda_2 = \dot{\lambda}_3/\lambda_3. \tag{A3}$$

Using (5) and (6), we obtain

$$c^{-\alpha} = \lambda_1(1 - F_L s' hm/c^2). \tag{A4}$$

Since c, m and h are growing at the same rate, the term in bracket on the right hand side of (A4) has to be constant under a balanced growth equilibrium. Total differentiation of (A4) then yields

$$\dot{\lambda}_1/\lambda_1 = -\alpha\theta, \tag{A5}$$

which completes the proof. Q.E.D.

<u>Proof</u> <u>of</u> <u>Lemma</u> <u>2</u>: Combining (A1) and (A5), we have

$$F_k = \rho + n + \alpha\theta$$

which combines with the factor market equilibrium condition that $r = F_k$ yield (11). From the Fisher equation, $i = F_k + \pi$, and (6), (7), (10), (A3) and (A5), we then get (12). Q.E.D.

<u>Proof</u> <u>of</u> <u>Lemma</u> <u>3</u>: Combining (A2), (A3) and (A5), we have (13). Next, (3), (A3), (A5), (12) and (13) together imply (14). From (4), (A3) and (15), as well as the fact that $F_k/\gamma = F/k$, we get (15). Finally, we use (2) and the common growth rate condition to obtain (16). Q.E.D.

<u>Proof</u> <u>of</u> <u>Proposition</u> <u>2</u>: Total differentiation of (13) - (16) gives

$$D \begin{bmatrix} d\theta \\ d(c/k) \\ d(m/c) \\ d\ell \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} d\mu + \begin{bmatrix} -1 \\ 1 + \Gamma s'/F_k \\ -1/\gamma \\ 0 \end{bmatrix} d\rho$$

where

$$D = \begin{bmatrix} \alpha & 0 & \phi s' & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} \\ (\alpha-\gamma)/\gamma & -1 & 0 & 0 \\ 1 & 0 & \phi s' & \phi \end{bmatrix}$$

with $a_{21} = -\Gamma\alpha s'/F_k + 1 - \alpha$, $a_{22} = \Gamma s'/(c/k) < 0$, $a_{23} = -\Gamma s'' < 0$, $a_{24} = \Gamma s'/\ell < 0$ and $\Gamma = [(1-\gamma)F_k]/[\gamma\ell(c/k)] > 0$. Straightforward derivations yield

$$d\theta/d\rho = \{\phi a_{23} - \phi s' a_{24} + \phi^2 s' [1 - \Gamma s'(n+\theta)/(cF_k/k)]\}/[\det(D)].$$

Given that $d\theta/d\rho < 0$ and that the numerator in the expression of $d\theta/d\rho$ is unambiguously negative, we can conclude that $\det(D) > 0$. The comparative-static results are then obtained: $d\theta/d\mu = \phi^2 s'/[\det(D)] < 0$, $d(m/c)/d\mu = -\phi\alpha/[\det(D)] < 0$, $d(c/k)/d\mu = [(\alpha - \gamma)/\gamma](d\theta/d\mu) < 0$ and $d\ell/d\mu = [(\alpha - 1)/\phi](d\theta/d\mu) < 0$. Q.E.D.

Proof of Corollary 1: Differentiating of (3) and (12) with respect to μ and substituting $d\theta/d\mu$ from proposition 2 yield the above results. Q.E.D.

Proof of Lemma 4: After dividing all the endogenous variables by the growth component, $h(t)$, it can be easily seen that the effective discount factor, Δ, will depend on the endogenous growth rate, θ. We then apply the Uzawa (1968) transformation to obtain the modified problem (P'). Q.E.D.

Proof of Lemma 5: Straightforward application of Pontryagin's Maximum Principle to (P') allows us to derive the first-order conditions in terms of Δ. By transforming all expression in terms of t, we obtain

$$(\dot{\tilde{m}} + \dot{\tilde{k}}) = F(\tilde{k}, \ell) - (n+\theta)\tilde{k} - (n+\pi+\theta)\tilde{m} + \tilde{\tau} - \tilde{c} \tag{A6}$$

$$(1-\alpha)(\dot{\tilde{m}} + \dot{\tilde{k}}) = -\tilde{c} + F_\ell s'\tilde{m}/\tilde{c} + [\rho - (1-\alpha)\theta](\tilde{m}+\tilde{k}+F_\ell/\phi) \tag{A7}$$

$$F_k + \pi = -F_\ell s'/\tilde{c}. \tag{A8}$$

Notice that (A6) and (A7) together imply

$$[\rho - (1-\alpha)\theta]F_\ell/\phi + F_\ell s'\tilde{m}/\tilde{c} + [\rho + (1-\alpha)(n + \pi)]\tilde{m} \tag{A9}$$
$$+ [\rho + (1-\alpha)n]\tilde{k} - (1-\alpha)F(\tilde{k}, \ell) - \alpha\tilde{c} = (1-\alpha)\tilde{\tau}.$$

Now, (18), (A8) and (A9) form a decomposable sub-system through which θ, π and ℓ can all be expressed as functions of $\tilde{c}$, $\tilde{k}$ and $\tilde{m}$. Next, straightforward manipulation of the first-order conditions gives the Keynes-Ramsey rule equation, (19). Using the definition, $\tilde{\tau} = \mu\tilde{m}$, together with the money market equilibrium condition, we obtain (20). Finally, substituting (20) into (A6) then yields (21). Q.E.D.

<u>Proof of Proposition 3</u>: To derive the instantaneous effects of an increase in μ, we first assume normality of time allocation; that is, a once-and-for-all improvement in the transaction-time technology (e.g., a one-time financial or transportation innovation) is assumed to increase both working and schooling. Differentiating the sub-system formed by (18), (A8) and (A9), we get

$$\theta = \theta(\underset{?}{\tilde{c}},\ \underset{-}{\tilde{k}},\ \underset{?}{\tilde{m}};\ \underset{-}{\mu}) \tag{A10}$$

$$\pi = \pi(\underset{+}{\tilde{c}},\ \underset{-}{\tilde{k}},\ \underset{?}{\tilde{m}};\ \underset{-}{\mu}) \tag{A11}$$

$$\ell = \ell(\underset{-}{\tilde{c}},\ \underset{+}{\tilde{k}},\ \underset{?}{\tilde{m}};\ \underset{+}{\mu}). \tag{A12}$$

On impact, $\tilde{k}$ is fixed. From (21), (A10) and (A12), $\tilde{c}$ must be higher to keep $\dot{\tilde{k}} > 0$ and thus the c/k ratio rises as μ increases. Then, using (20), (A10) and (A11), we have $\dot{\tilde{m}} > 0$ and so the m/k ratio rises as μ increases. Next, from (19) and (20), we have $(\dot{m}/m - \dot{c}/c) = \mu - \pi - F_k/\alpha + (\rho + n)/\alpha$. It can be shown that $|d\pi/d\mu| > |dF_k/d\mu|$ and so, given $\alpha > 1$, the m/c ratio rises as μ increases. Moreover, (A12) implies ℓ goes up as μ rises. From Proposition 2, all these variables decline eventually from the instantaneous position to the new steady state. Finally, (A10) and proposition 2 imply that both instantaneous and long-run effects of higher money growth on economic growth are negative. From (18), the money growth rate affects economic growth through the work effort and transaction time (s). Given the responses of m/c ratio and ℓ to an increase in μ, the work-effort and transaction-time effects must work in opposite directions. Thus, the movement of the economic growth rate is ambiguous. Q.E.D.

<u>Proof of Corollary 2</u>: On impact, proposition 3 implies that c/m, k/m and θ all decrease when the money growth rate rises and so v drops instantaneously. In the steady state, both c/m and k/m ratios rise while θ declines and so the effect on velocity is ambiguous. In transition, this ambiguity is also present. Q.E.D.

Table 1

Sensitivity Analysis

	$d\theta/d\mu$	$di/d\mu$
Benchmark Case*	-0.002568	0.9961
Alternative Values		
ρ = 0.015	-0.002090	0.9969
ρ = 0.04	-0.002917	0.9956
γ = 0.28	-0.002644	0.9960
γ = 0.33	-0.002785	0.9958
n = 0.0104	-0.002584	0.9961
n = 0.0156	-0.002553	0.9962
α = 4	-0.003026	0.9909
α = 10	-0.003524	0.9683
a = 0.02254	-0.002591	0.9961
a = 0.03380	-0.002546	0.9962
ε = 1.0221	-0.002619	0.9961
ε = 1.5331	-0.002554	0.9962
ϕ = 0.0544	-0.001610	0.9976
ϕ = 0.0816	-0.003819	0.9943
μ = 0.0392	-0.002319	0.9965
μ = 0.0588	-0.002817	0.9958

*In the benchmark case, the values of ρ, γ, n, α, a, ε, ϕ and μ are 0.03, 0.25, 0.013, 2.5, 0.02817, 1.2776, 0.068 and 0.049, respectively.

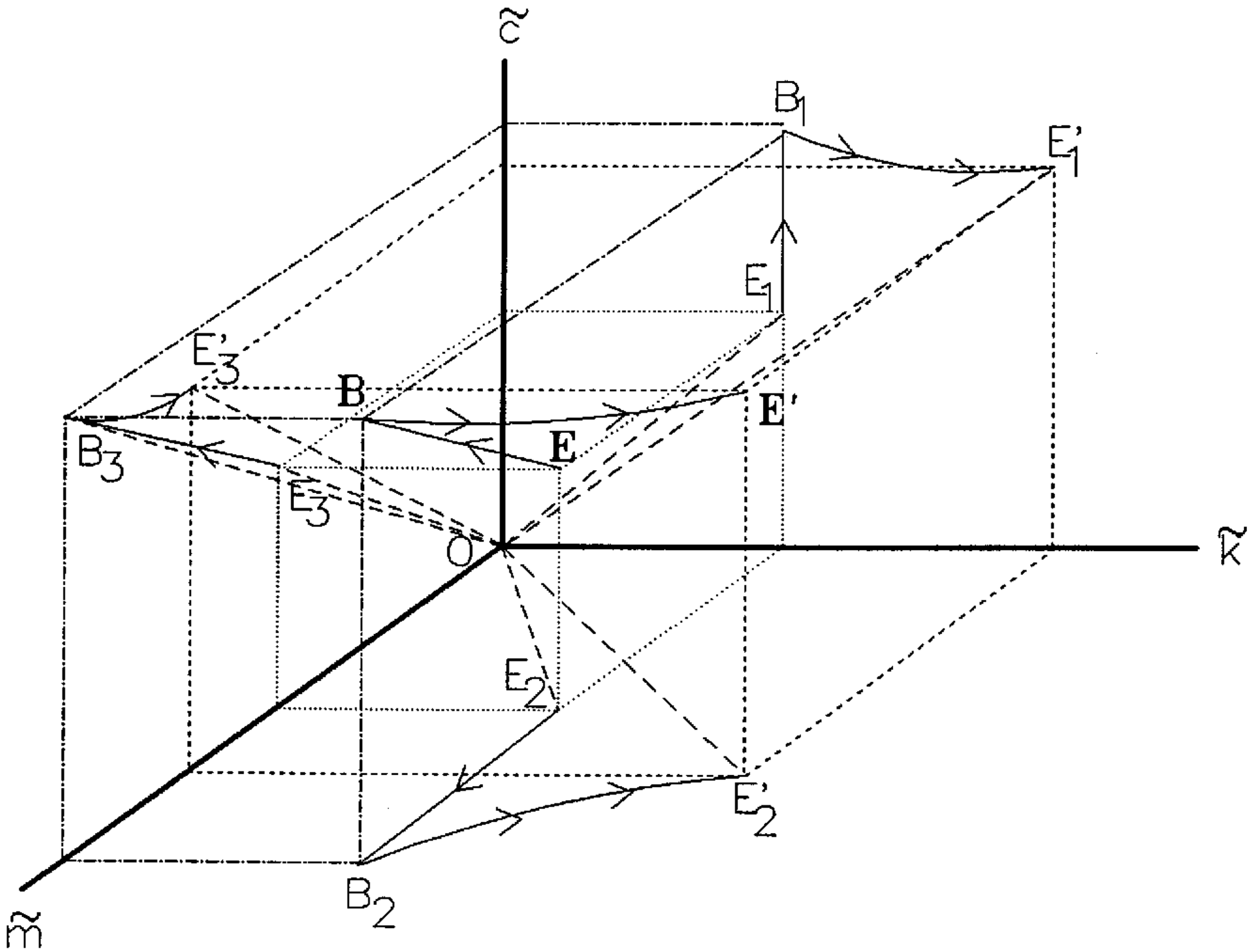
Table 2

Steady-State Solutions and Welfare Costs with Various Money Growth Rates

Money Growth Rate (%) (μ)	Economic Growth Rate (%) (θ)	Initial Consumption-human capital ratio $c(0)$	Welfare Level (W)	($\Delta W/W$)*	Change in Welfare (%) $\left[\frac{\Delta c(0)}{c(0)}\right]$	$\Delta\theta$
400	1.231	0.981	8.067	-19.76	-10.88	-0.571
200	1.270	0.986	8.345	-17.00	-9.361	-0.492
100	1.304	0.991	8.585	-14.61	-8.044	-0.422
50	1.332	0.997	8.778	-12.69	-6.988	-0.367
10	1.377	1.005	9.172	-8.770	-4.830	-0.254
4.9	1.389	1.008	9.273	-7.766	-4.278	-0.225
0	1.407	1.011	9.397	-6.531	-3.597	-0.189
-1.0	1.412	1.012	9.434	-6.158	-3.392	-0.178
-2.0	1.418	1.014	9.480	-5.708	-3.144	-0.165
-4.0	1.438	1.017	9.620	-4.316	-2.378	-0.125
-5.0	1.464	1.022	9.803	-2.495	-1.375	-0.072
-5.2	1.484	1.026	9.939	-1.146	-0.628	-0.033
-5.25	1.501	1.029	10.05	0	0	0

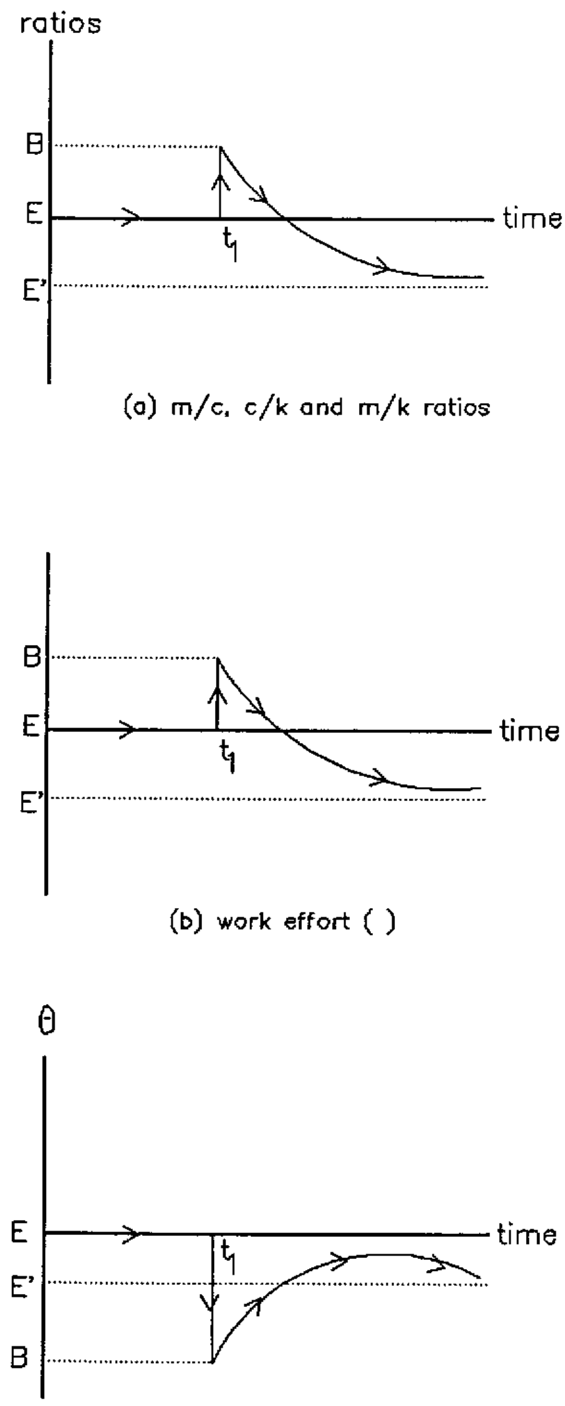
* $\Delta W/W$ is calculated based on percentage changes from the initial welfare level associated with the optimal money growth rate (μ = -5.25%) which is 10.054.

FIGURE 1



Steady State and Transitional Dynamics

FIGURE 2



(a) m/c, c/k and m/k ratios

(b) work effort ( )

(c) economic growth rate (θ)

RESEARCH PAPERS OF THE RESEARCH DEPARTMENT
FEDERAL RESERVE BANK OF DALLAS

Available, at no charge, from the Research Department
Federal Reserve Bank of Dallas, Station K
Dallas, Texas 75222

9201 Are Deep Recessions Followed by Strong Recoveries? (Mark A. Wynne and Nathan S. Balke)

9202 The Case of the "Missing M2" (John V. Duca)

9203 Immigrant Links to the Home Country: Implications for Trade, Welfare and Factor Rewards (David M. Gould)

9204 Does Aggregate Output Have a Unit Root? (Mark A. Wynne)

9205 Inflation and Its Variability: A Note (Kenneth M. Emery)

9206 Budget Constrained Frontier Measures of Fiscal Equality and Efficiency in Schooling (Shawna Grosskopf, Kathy Hayes, Lori Taylor, William Weber)

9207 The Effects of Credit Availability, Nonbank Competition, and Tax Reform on Bank Consumer Lending (John V. Duca and Bonnie Garrett)

9208 On the Future Erosion of the North American Free Trade Agreement (William C. Gruben)

9209 Threshold Cointegration (Nathan S. Balke and Thomas B. Fomby)

9210 Cointegration and Tests of a Classical Model of Inflation in Argentina, Bolivia, Brazil, Mexico, and Peru (Raúl Anibal Feliz and John H. Welch)

9211 Nominal Feedback Rules for Monetary Policy: Some Comments (Evan F. Koenig)

9212 The Analysis of Fiscal Policy in Neoclassical Models[1] (Mark Wynne)

9213 Measuring the Value of School Quality (Lori Taylor)

9214 Forecasting Turning Points: Is a Two-State Characterization of the Business Cycle Appropriate? (Kenneth M. Emery & Evan F. Koenig)

9215 Energy Security: A Comparison of Protectionist Policies (Mine K. Yücel and Carol Dahl)

9216 An Analysis of the Impact of Two Fiscal Policies on the Behavior of a Dynamic Asset Market (Gregory W. Huffman)

9301 Human Capital Externalities, Trade, and Economic Growth (David Gould and Roy J. Ruffin)

9302 The New Face of Latin America: Financial Flows, Markets, and Institutions in the 1990s (John Welch)

9303 A General Two Sector Model of Endogenous Growth with Human and Physical Capital (Eric Bond, Ping Wang, and Chong K. Yip)

9304 The Political Economy of School Reform (S. Grosskopf, K. Hayes, L. Taylor, and W. Weber)

9305 Money, Output, and Income Velocity (Theodore Palivos and Ping Wang)

9306 Constructing an Alternative Measure of Changes in Reserve Requirement Ratios (Joseph H. Haslag and Scott E. Hein)

9307 Money Demand and Relative Prices During Episodes of Hyperinflation (Ellis W. Tallman and Ping Wang)

9308 On Quantity Theory Restrictions and the Signalling Value of the Money Multiplier (Joseph Haslag)

9309 The Algebra of Price Stability (Nathan S. Balke and Kenneth M. Emery)

9310 Does It Matter How Monetary Policy is Implemented? (Joseph H. Haslag and Scott E. Hein)

9311 Real Effects of Money and Welfare Costs of Inflation in an Endogenously Growing Economy with Transactions Costs (Ping Wang and Chong Yip)