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ABSTRACT

A stochastic general equilibrium model is constructed in which an analysis can be conducted into the effects of various distortional government policies on the behavior of financial market variables. In particular, a tax on transactions in assets and a capital gains tax are studied separately. The effects of these policies on the equilibrium behavior of capital prices, rates of return, and the level of transaction volume are quantified. Additionally, some estimates of the welfare costs of such policies are presented. Although the model is a version of the representative agent framework with time-separable preferences, it is also shown that it can generate an endogenous distribution of wealth.

I. INTRODUCTION

In this paper a stochastic general equilibrium infinitely-lived agent model is constructed in which it is possible to analyze the impact that various government policies have on the behavior of financial market variables. In particular, an investigation is conducted into how increases in transactions costs, possibly viewed as a tax on the purchase or sale of assets, can influence the dynamic properties of the price and rate of return of capital, as well as on the equilibrium transaction volume. Additionally, within the context of a similar model an analysis is conducted to study the impact of the implementation of two versions of a capital gains tax and its effect on the financial market variables. This is an important breakthrough because there appears to be a remarkable shortage of dynamic general equilibrium models that are used to study the impact that such distortional fiscal policies have on the equilibrium behavior of financial market variables.

It has been occasionally suggested by some researchers that it might be appropriate to impose a government tax on the purchase or sale of financial assets, with the apparent goal being to deter large fluctuations in prices that are associated with large transaction volume. Since it has been empirically documented that transaction volume is positively correlated with the magnitude of price changes, it may be thought that a policy that seeks to lower the volume of transactions may also lower the volatility of prices. Similarly, the imposition of a capital gains tax has been a much discussed policy considered recently in the U.S. However, to date there has been a dearth of literature that attempts to study such issues. In particular, there have been practically no analyses that have studied the impact of such policies within the context of a stochastic general equilibrium model. It may be especially important to study these policies within such a context, since it is only within a fully articulated general equilibrium model that the full feedback effects of such a policy on all the endogenous variables, such as the price of capital and the volume of transactions, can be fully revealed. Additionally, in such models it is also possible to estimate the cost, in welfare terms, of such policies. Lastly, the model can also be used to show how the dynamics of the distribution of wealth or assets among the population can change over time.

Much of the work on general equilibrium models of asset pricing have virtually no implications for the behavior of transaction volume for these same economies. This is particularly true for models employing the representative agent paradigm [e.g. Lucas (1987)]. The model employed in this paper makes use of a version of this representative agent model to observe how the level of transaction volume can be affected by policies affecting agents investment decisions. The approach adopted in this paper has the added benefit of displaying agents that can participate in the asset market for virtually any number of periods - no matter how short or long a period of time - an apparently rather realistic property that is notably absent in most other models of asset pricing. This model can then be viewed as a contribution to the literature using the infinitely-lived agent modelling construct, so that it can be shown how this paradigm can be employed to address additional questions. In fact, the model presented in this paper has the infinitely-lived representative agent model, <u>and</u> the two-period-lived overlapping generations model as special cases.

This paper is also a contribution to the recent and growing literature that employs dynamic general equilibrium models to study the impact, both in terms of welfare and the behavior of the resulting aggregates, of various government policies. Lucas (1987) uses a representative agent model and concludes that the fluctuations in consumption arising over the course of the business cycle are not sufficiently large to justify employing government policy tools to combat the fluctuations. Cooley and Hansen (1987) use a similar framework to calculate the effects of different monetary policies that produce alternative inflation taxes. Greenwood and Huffman (1991) study the impact that alternative labor and capital income taxes have on the business cycle properties of the U.S. economy, and on the welfare of agents who live in such an economy. In the present analysis a tax on exchanges of financial assets, as well as two versions of a capital gains tax will be analyzed.[1] It will be shown how these policies affect the equilibrium behavior of the endogenous variables. Additionally, some measures of the change in agents' welfare as a result of such policies will also be presented.

The remainder of the paper is organized as follows. In Section II the economic environment without transaction costs is described. It is a version of a representative agent economy in which agents enter or leave the economy depending on a realization of a random variable. Agents can purchase or sell capital from one period to the next, and can finance their consumption activity by changing their portfolio accordingly. The agent's decision rules and the economies' equilibrium conditions are characterized. A non-stochastic version of the model is briefly studied so as to gain insight into the behavior of the steady-state distribution of wealth over time. This is important because it is usually assumed that representative-agent models populated by infinitely-lived agents are reticent on the issue of the distribution of wealth. In Section III the model is altered to incorporate a transaction cost on the purchase and sale of assets. Numerical results are presented to show how changing the level of transaction costs will affect the stochastic properties of the price of capital, the rate of return on capital, and the level of transaction volume. In Section IV the model is modified to incorporate two versions of a capital gains tax that is payable, period by period, by agents. It is shown that such a tax can influence the serial correlation properties of the price of capital. Again, numerical results are presented to show how changing the level of this tax will affect the stochastic properties of the financial market variables. Final remarks are presented in Section V.

II. ECONOMIC ENVIRONMENT

The economy is one in which time is discrete, and indexed by t=1,2,3... Initially at date t=1 there are a continuum of agents in the economy, and this population is said to be of size N. For convenience, the population size is normalized to unity. For all agents that are in the economy at date t, their preferences can be described by the following utility function

$$E\left[\sum_{s=t}^{\infty}\beta^{s}\log(c_{s})\right],$$

where $\beta \in (0, 1)$. At each date t, αN of the existing agents leave the economy, and αN "new" agents enter the economy where $\alpha \in (0, 1)$.[2] Agents know at the beginning of a period whether or not they must leave the economy at the end of that period. In any period t, the probability that an agent will have to leave the economy in the following period is α, and this probability is identical for all agents and for all periods.[3] In other words, the probability that an agent who has been present for only one period will leave, is equal to that probability for an agent who had been there for many periods.

Agents can buy and sell capital on the capital market. It is assumed that there exists an aggregate supply of one unit of capital, which produces a stochastic dividend of r_t in period t. In any period t, an agent (j) may enter the economy with x^j_t units of capital. The price of capital in this period is P_t, quoted in units of the consumption good. The agent can then decide whether to purchase more capital, or to sell some of his existing stock to finance current consumption. With this in mind, the optimization problem for an agent who was in the economy in both periods t and t+1 is to maximize the following expected utility function

$$E\left[\sum_{s=t}^{\infty} \beta^{s} \log(c_s)\right] \tag{1}$$

subject to the budget constraints for s ≥ t,

$$c_s + P_s x_{s+1} \leq (P_s + r_s) x_s. \tag{2}$$

Here the expectation operator reflects the expectation with respect to future prices and rates of return, as well as the probability (α) that the agent will leave the market. In any period s when αN agents enter the economy, they each enter with w_s units of the single consumption good as an endowment. Therefore, these agents maximize the utility function (1) subject to the following budget constraint

$$c_s + P_s x_{s+1} \leq w_s.$$

The realization of the variable w_s occurs prior to or simultaneously with the entrance of these agents into the economy. This precludes any risk-sharing agreements that could be arranged based on the realization of the level of this variable. As a benchmark, and in order to make the behavior of the model conform with that of the data, it will be assumed below that both the dividend (r_t) and endowment (w_t) are composed of the sum of a stochastic i.i.d. random variable, and a common trend element so that aggregate output exhibits growth as follows

$$\log(w_t) = t\lambda + \epsilon_t^1$$
$$\log(r_t) = t\lambda + \epsilon_t^2.$$

Lastly, for those agents who know they are spending their last period in the economy, they merely consume the value of all their assets before leaving. Therefore, their budget constraint is as follows

$$c_t \leq (P_t + r_t) x_t.$$

It should be noted that since there are a continuum of agents and the fraction α of agents will be leaving the economy, then the fraction α of aggregate capital must be sold by agents as well. This places a lower bound, but only a lower bound, on the transactions volume in any period.

Equilibrium Without Transactions Costs.

It is straightforward to solve the model described above, and characterize its behavior. The dynamic programming problem can then be cast as follows. Let $V[(P_t + r_t)x_t]$ denote the value function for an agent who enters period t with x_t units of the asset, and the price and dividend are P_t and r_t respectively. Then the dynamic programming problem faced by such an agent can be written as

$$V[(P_t + r_t)x_t] = \max\{\log(c_t) + E\beta[(1 - \alpha)V[(P_{t+1} + r_{t+1})x_{t+1}] + \alpha\log([P_{t+1} + r_{t+1}]x_{t+1})]\}, \quad (3)$$

where the maximization is subject to equation (2) and the expectation operator reflects the expectation with respect to future prices and dividends.[4] After some manipulation, it is possible to verify that the Euler equation associated with this problem is the following

$$\left[\frac{1}{(1 - s)(P_t + r_t)x_t}\right] = \left[\frac{(1-\alpha)\beta}{s(1 - s)(P_t + r_t)x_t}\right] + \left[\frac{\alpha\beta}{s(P_t + r_t)x_t}\right]$$

where

$$s = \left[\frac{P_t x_{t+1}}{(P_t + r_t)x_t}\right],$$

i.e. the savings rate. This can then be used to show that

$$s = \left[\frac{\beta}{(1 + \alpha\beta)}\right],$$

or that

$$P_t x_{t+1} = \left[\frac{\beta}{(1 + \alpha\beta)}\right][P_t + r_t]x_t. \quad (4)$$

Hence, the level of savings or investment is a decreasing function of α, the rate at which agents leave the economy, but an increasing function of β, the discount factor. The higher is α, the shorter is the expected horizon over which the agent's optimization problem takes place, and hence the less they will wish to save for future consumption. When $\alpha = 0$, the savings rate is equal to the discount factor (β), which conforms with what is known about economies in which the technology is constant returns, and preferences are logarithmic.[5]

Since there is a fixed quantity (unity) of capital in the economy, the following equilibrium condition must hold

$$\sum_{j \in \Omega_t} x^j_{t+1} + \sum_{j \in \Omega_t'} x^j_{t+1} = 1, \tag{5}$$

where Ω_t is the set of agents who will be in the economy in both periods t and t+1, while Ω_t' represents the set of agents who first entered the economy in period t, and x_t^j denotes the amount of capital chosen by agent j in period t-1, to be held until the following period. Of course, this equation merely states that the demand for capital must equal the supply.[6]

Similarly, it is easily shown that for agents who are just entering the economy, their portfolio decision is described by the following

$$P_t x_{t+1} = \left[\frac{\beta}{(1 + \alpha\beta)}\right] w_t. \tag{6}$$

Substitution of equations (4) and (6) into equation (5) then yields the following equation defining the equilibrium price of capital

$$P_t = \frac{\beta[\alpha w_t + (1 - \alpha) r_t]}{[1 + \alpha\beta - \beta(1 - \alpha)]}. \tag{7}$$

Of course, fluctuations in the variables w_t, r_t, or even if α were free to fluctuate, would produce changes in the price of capital and hence in the rate of return. Notice that a high realization of α raises the size of the denominator, and thus helps to lower the price of capital, because more capital is being supplied through those agents exiting the economy. On the other hand if $w_t > r_t$, then a high realization of α will raise the numerator and raise the price of capital, because newly entering agents have large endowments and their savings will then drive up the price of capital. Because the probability of exiting the economy (α) is the same for all agents, the wealth distribution has no impact on the price of capital. However, as will be shown below, the model is capable of generating an endogenous distribution of wealth or capital holdings.

The Non-stochastic Steady State

To gain insight into the behavior of the model, it seems best at first to shut off all sources of exogenous uncertainty. It is then possible to utilize this framework to characterize the steady-state distribution of capital holdings, and also to see how the distribution of wealth evolves over time. This can be done through the following two examples.

Example 1: Let the parameter values be as follows: $\beta = .95$, $\alpha = .10$, r = 30, w = 50, for all time periods. Furthermore, consider the extreme benchmark case in which at time t = 1, all existing capital is owned equi-proportionally by 10% of the existing agents in the economy. Since there is only one unit of aggregate capital, initially the agents who hold capital need to each hold ten units. Figure 1 then shows how the wealth distribution changes over time in this non-stochastic environment, beginning from these initial conditions. Here the horizontal axis measures, from left to right, the poorest percentage of the population, while the vertical axis measures the amount of capital held by that poorest percentage of the population. Initially at t=1 the poorest 90% of the population holds zero capital. As time evolves, the poorer agents in the population gradually get richer only because of new agents entering the economy. Existing "poor" agents do not get richer, but instead merely leave the economy. On the other hand, the very rich agents actually get richer because they are saving some of their earnings from capital, and hence become even more wealthy. Asymptotically the distribution of capital approaches the steady state distribution, which is shown by the dashed line in Figure 2.[7] Note that for visual convenience the vertical axis of Figure 2 is measured in units of the logarithm of the quantity of capital. In the steady state new entrants to the economy enter with relatively little wealth, and gradually save more over their lifetime. Hence, in Figure 2, as agents get older they are getting richer, and are moving from the left to the right in the distribution of capital holdings. It should also be noted that various parameters of the environment influence the distribution of capital holdings. To illustrate this, the solid line in Figure 2 denotes the steady-state distribution of capital holdings for the exact same economy as the dashed line, except that the dividend is r = 10. In this case the rate of return on capital is lower, and therefore agents are slower to acquire more capital over their lifetime. Consequently the distribution appears to be flatter in this second case.

Example 2: Let the parameter values be as follows: $\beta = .95$, $\alpha = .10$, r = 2, w = 50, for all time periods. Again suppose that at time t = 1, all existing capital is owned by 10% of the existing agents in the economy. Figure 3 then shows the evolution of the wealth distribution over time. In contrast to Figure 1, the rich agents who stay in the economy actually get poorer over time because they are consuming more and more of their earnings from assets. The reason for the different behavior of the wealth distribution in the second example is that the rate of return on capital is lower than in the first example. In this case the distribution of capital converges to that shown by the dashed line in Figure 4. Obviously, this example is the converse of Figure 2 because agents here are depleting their capital holdings over their lifetime. Hence new entrants to the economy enter relatively wealthy, and consequently move from the right to the left in the distribution of Figure 4. Again, it is shown in Figure 4 that raising the dividend from r=2 to r=5 means shifting the steady-state distribution from the dashed line to the solid line. In this case raising the rate of return makes the distribution "flatter."

In both of the above examples, the distribution of capital holdings moves "faster" towards the eventual steady state, the higher is parameter α. For example, if $\alpha = 1$, then all agents in the economy at time t=1 leave the next period, and the economy immediately goes to the new steady state. Similarly, the distribution of capital holdings moves "faster" towards the eventual steady state, the higher is w relative to r. In this case, new entrants can then afford to purchase more capital and thereby raise the share of capital held by new entrants. Additionally, the behavior of capital holdings is more likely to look like Figures 1 and 2, as opposed to Figure 3 and 4, the higher is the savings rate (s), and the lower is the endowment level (w) relative to that of the dividend (r). The higher is the savings rate s (or the level of β), the more saving will take place, and so the more capital the rich agents will purchase. The higher is the dividend (r) relative to the endowment (w), the higher the rate of return to holding capital, and the more capital the rich agents will purchase. These types of experiments can be conducted for stochastic and non-stochastic versions of the economy. Also, given an arbitrary initial distribution for wealth, the resulting wealth distribution for an arbitrary finite number of periods can be calculated as well.

In the stochastic economies of the following two sections, the wealth distribution will behave in a manner similar to that displayed in Figures 3 and 4, since the participants in the financial market will let their capital holdings erode to finance higher consumption levels. However, the change in the distribution of capital holdings will obviously be influenced by the rate of return on capital which, as will be shown, will be influenced by the various government policies under consideration. In the following section an increase in transaction costs, possibly viewed as a tax on transactions, will be analyzed within a version of the present model. In Section IV two versions of a capital gains tax are implemented in this model to study the impact on the equilibrium financial market variables.

These policies are implemented separately, as opposed to simultaneously, to facilitate the analysis of each policy individually.

III. EQUILIBRIUM WITH TRANSACTIONS COSTS

Now consider an environment in which there are transaction costs imposed on the purchase and sale of assets. In particular, what is considered is a constant tax on the value of the purchase and sale of assets, calculated in units of the consumption good. This could alternatively be considered a natural transaction cost imposed by the environment on the agent's behavior, rather than a government policy variable. Let θ denote the size of this transaction cost. Of course, agents still maximize the preferences given by equation (1), but their budget constraint now is changed to

$$c_t + P_t x_{t+1} \leq (P_t + r_t)x_t - \theta P_t |(x_{t+1} - x_t)|. \tag{8}$$

To facilitate the understanding of the effects of such a policy, equation (8) will be rewritten as follows

$$c_t + P_t x_{t+1} \leq (P_t + r_t)x_t - \mu P_t (x_{t+1} - x_t).$$

where μ obviously represents the parameter $\theta \geq 0$, and is determined as follows

$$\begin{aligned} \mu &= \theta, \text{ if } x_{t+1} \geq x_t \\ \mu &= -\theta, \text{ if } x_{t+1} < x_t. \end{aligned}$$

This framework permits the analysis of the effects of constant or linear taxes imposed on transaction activity, without the use of artificial non-linear (convex or linear-quadratic) approximations that have been used in other analyses.[8]

Again, the optimization problem for agents who enter the economy is to solve the maximization problem given by equation (1) subject to the following constraint[9]

$$c_t + [P_t(1 + |\mu|)]x_{t+1} \leq w_t.$$

The optimization problem for agents who are leaving the economy is to merely consume the value of their assets, as given by the following constraint

$$c_t \leq [P_t\,(1 - |\mu|) + r_t]x_t.$$

Lastly, for agents who are in the economy in both periods t and t+1, the budget constraint for the optimization problem is described by equation (3) subject to equation (8). For these agents, it is easily shown that the Euler equation associated with this problem is then of the following form

$$\left[\frac{1 + \mu}{(1 - s)(P_t + r_t)x_t}\right] = \left[\frac{(1-\alpha)\beta}{s(1 - s)(P_t + r_t)x_t}\right] + \left[\frac{\alpha\beta}{s(P_t + r_t)x_t}\right]$$

where s again is the savings rate, and is determined as follows[10]

$$s = \left[\frac{\beta}{(1 + \mu + \beta\alpha)}\right].$$

Hence if $\mu < 0$, the addition of a higher transaction cost raises the savings rate of income since this raises the cost of future consumption by raising the cost of selling capital, and the agent must then compensate for this by saving more, or dissaving less. If $\mu > 0$, the addition of transaction costs lowers the savings rate of income since this raises the cost of saving by raising the cost of purchasing more assets, and the agent will then compensate for this by saving less and consuming more. The consumption and saving decision rules respectively can then be written as follows

$$c_t = [1 - s][P_t(1 + \mu) + r_t]x_t$$

$$P_t x_{t+1} = \left[\frac{s}{1 + \mu}\right][P_t(1 + \mu) + r_t]x_t. \tag{9}$$

For agents newly entering the economy their saving decision is the following

$$P_t x_{t+1} = \left[\frac{s}{1 + |\mu|}\right]w_t. \tag{10}$$

It is then easily shown for existing agents in the economy, that equation (9) can be employed to show that the value of the capital that is traded or exchanged by the agent is of the following form

$$P_t(x_{t+1} - x_t) = \left[\frac{\beta}{(1+\mu+\alpha\beta)(1+\mu)}\right]r_t x_t - \left[\frac{1+\mu-(1-\alpha)\beta}{1+\mu+\alpha\beta}\right]P_t x_t. \tag{11}$$

It is easily seen that if $\mu > 0$, so that $x_{t+1} > x_t$, then a further slight increase in the value of μ will reduce the size of the first term on the right side of equation (11), and raise the value of the second term. Therefore, holding other things constant, a rise in μ would tend to deter further purchases of capital. Similarly, if $\mu < 0$, so that $x_{t+1} < x_t$, then a further lowering in the value of μ will raise the size of the first term on the right side of equation (11), and lower the value of the second term. Therefore, holding the price P_t constant, an increase in the value of the transaction cost parameter $|\mu|$ will lower the value of the assets that an agent will wish to purchase or sell. This is very similar to what the addition of a decreasing returns to scale adjustment cost technology to capital accumulation will do to a neoclassical growth model.

By substituting the equilibrium decision rules (9) and (10) into the market-clearing equation for capital, the equilibrium price of capital can be determined as follows. In the case in which there are transaction costs, this reduces to

$$P_t = \left[\frac{\dfrac{\beta\alpha w_t}{(1+|\mu|)} + \dfrac{\beta(1-\alpha)r_t}{(1+\mu)}}{1+\mu+\beta\alpha-\beta(1-\alpha)} \right]. \tag{12}$$

Clearly, when $\mu > 0$, this transaction cost causes the price of capital to be lower than it otherwise would be. This is because in this situation agents are accumulating capital over their lifetime (x_{t+1} - $x_t > 0$), and the higher transaction cost causes agents to restrain their asset accumulation, and this causes the price of capital to be lower. Conversely, when $\mu < 0$, the price of capital would be higher than otherwise if the two terms in equation (12) involving μ dominate the absolute value term $|\mu|$. In this case, agents are selling capital, and the higher transaction cost causes them sell less capital, and this causes the price of capital to rise. If the effects of the absolute value term $|\mu|$ dominate the other two terms in equation (12), then the transaction cost of purchasing capital by newly entering participants in the economy dominates the effect on the remaining agents, and the price of capital is lower as a result. Setting $\mu = 0$ in equation (12) gives the pricing equation when transaction costs are zero, as given by equation (7).

Lastly, it should be noted that the previous analysis indicates that there may be a sense in which introducing a tax on transactions may be beneficial within the context of other similar environments. If the tax is set such that $\mu < 0$, then a further decrease in μ, which is a rise in the tax rate, would also raise the saving rate. To the extent that it is desirable to raise the savings rate because of other distortions, and thereby possibly raise the level of the capital stock or growth rate, a transaction tax may be one way to accomplish this task [see Auerbach (1992) for a similar discussion].

Numerical Results:

It is important to obtain a feel for how the imposition of a higher transaction cost would influence the dynamic behavior of variables determined by the equilibrium behavior of the asset market. For this reason, the behavior of several such economies is simulated and characterized below. The size of a period for the model is chosen to be a quarter, and dividends are paid each quarter. Before this experiment can be conducted, specifications must be chosen for the value of certain variables. First the discount factor β must be chosen. As will be seen below, this was set between .90 and .9995, with remarkably little difference in the results.

Next, the parameter α will help determine the rate at which agents enter and leave the economy, and thereby influence the rate at which assets "turnover." The rate of turnover of financial assets in the U.S. has fluctuated a great deal during this century. Shares on the NYSE reached a local maximum of the annual rate of turnover of around 100% in 1925. Since then annual turnover has been considerably lower, averaging around 20% per year from 1940 to 1975. Since 1975 turnover has increased, reaching 40% in 1982, 73% in 1987, and falling to 52% in 1989. Therefore, it was decided that a benchmark value of the turnover rate would be 50%.[11] The parameter α then also determines the lower bound on the level of transaction volume in any period since agents who enter or leave the financial market will obviously be transacting in assets.

A benchmark value for the level of the transactions cost θ must also be chosen. Since this represents the marginal cost of purchasing or selling a financial asset, there are easily accessible sources for such information. Unfortunately, there is a wide range of values for such a parameter. Retail brokerage firms can charge commission fees that can run as high as over 3% for purchases or sales of assets (depending upon who is doing the pilfering). Firms who have seats on the actual exchanges, as well as mutual funds, who account for most of the trading on a day to day basis, can lower the marginal cost of transactions to below 1%. Therefore, merely as a benchmark, a marginal transaction cost of 1% for θ was chosen.

Next, values for the random variables w_t and r_t must be chosen, and this was done as follows. The average real rate of return on stocks from 1926 - 1982 was 9%, with a standard deviation of 21.8% [see Ibbotson and Sinquefield (1983)]. These variables are also assumed to have a common deterministic growth or trend component so that they grow at an annual rate of 2.9%, which conforms with the observed growth in real output from 1929 to 1990. The ratio of w_t to r_t helps determine the average rate of return, with the variability of both of these variables influencing the standard deviation of the rate of return. Therefore, both detrended variables were assumed to be independently log-normally distributed with a common variance, and the variance was chosen to mimic the actual variability of the rate of return. The ratio of the means of these variables was chosen to mimic the actual average rate of return on stocks. The independence assumption was employed since there appeared to be no obvious reason to presume a particular degree of positive or negative correlation between these variables.[12]

Table 1 shows the resulting impact, for four different economies, of changing the transaction cost from 1% to 3%.[13] For all four economies, the model is calibrated, using half a million observations, to mimic the behavior of observed rates of return and turnover of assets with a 1% transaction cost. This cost is then raised to 3%. As can be seen, the results do not change markedly for different values of β. The average detrended price of capital rises by over 14%, and the reason for this is as follows. The presence of the higher transaction cost dissuades asset holders from selling assets as quickly as they might otherwise wish, and therefore, their increased saving raises the price of capital. For similar reasons, the average level of transaction volume falls by over 12%. This is largely due to the fact that sales of capital by existing agents declines leading to a rise in price and a fall in transaction volume. Additionally, existing agents in the economy are deterred by the higher cost from selling as much capital. This is further illustrated in Figure 5 where an illustration of the probability distribution of transaction volume is shown to have shifted to the left because of this policy. Interestingly, the percentage standard deviation, or volatility of the detrended price of capital does not change with the increased transaction cost. The reason is that although the mean price is higher, the standard deviation is also higher by the same amount and so the volatility of prices is not dissipated by an increase in transactions costs. This is easily seen by analyzing equation (12) where it can be seen that a decrease in μ (< 0) will raise the mean price but also raise the standard deviation of the price.[14] Therefore, to the extent that these transaction costs are the effects of government taxation on transactions, such a policy will not produce a more "stable" behavior for asset prices and, in fact, may actually exacerbate the variability of prices as measured by the standard deviation of the detrended price. This experiment, which shows that the effect of the higher transaction cost is a fall in transaction volume and unchanged level of volatility of prices, also shows that the perceived high volatility of asset prices cannot be said to be "caused" by the high level of transaction volume, despite the fact that the two variables are correlated in equilibrium.

For all four benchmark economies described in Table 1, the correlation between the price of capital and transaction volume is 0.64, and the correlation between the absolute value of the change in price and transaction volume is 0.12. These positive correlations are consistent with those described in much of the financial market research, as described by Karpoff (1987), and gives more reassurance that the results of the policy experiments can be taken seriously. However, as mentioned above, despite the fact that the covariance between the price of capital and the magnitude or absolute value of price changes on the one hand, and the equilibrium transaction volume on the other, is positive, this does not imply that a policy that is designed to lower volume will also lower price variability. Another reassuring aspect to the equilibrium behavior of these models, is that the average dividend to price ratios for each of these benchmark economies is 3.9%, which is relatively close to the average from 1949 - 1990 of 4.2%.

In Table 1, the variable R_b refers to the annual rate of return on assets <u>ignoring transaction costs</u>, while R_a refers to the annual rate of return net of transactions costs. This last variable is calculated by supposing the agent purchases the asset at the beginning of one year, collects dividends during that year, and sells the asset at the end of that year and pays the required transaction fees at both purchase and sale.[15] For all four benchmark economies, when the transaction cost is $\theta = 0.01$, the average value for R_a is 6.9%. Increasing the transaction costs from 1% to 3% lowers the average value for R_b by only 0.5%, but lowers the rate of return net of transaction costs by 4.6%. The last row in Table 1 shows how the size of these transaction cost compare with the level of consumption on average. In fact, these costs appear to be rather small, being less than 0.5% in all cases since such a small fraction of the assets are traded in each period.

Table 2 presents similar results derived from raising the transaction cost from 1% to 5%. This is a very large punishment for trading assets, and has the effect of raising prices, and lowering transaction volume even further. The average annual rate of return, net of transaction costs falls to being negative (from 6.9% to -2.2%). The cost of raising the transaction cost to 5%, as a percentage of total consumption, is slightly less than twice that shown in Table 1.

Table 3 presents some measures of the welfare cost of such a policy when $\beta = .99$ and $\alpha = .0716$. The welfare measure employed is the equivalent percentage tax on an agent's initial wealth that would leave the agent with exactly the same expected welfare prior to entering the market. In this case both levels of expected utility (or value function) are calculated for the agent unconditionally (i.e. as the value before the agent has entered the economy). Also shown in Table 3 is the average amount of revenue, measured as a percentage of total consumption, that this equivalent tax would produce.[16] A comparison of Tables 1 and 3 shows the following. A transaction cost of 3% yields resources equal to 0.39% of total consumption, whereas a 2.69% initial wealth tax gives agents the same expected utility, but produces revenue on average equal to 2.53% of total consumption. In other words, it would appear that if such a transaction tax were a feasible fiscal policy tool, agents would have a strong dislike for it, and that the government could generate a given amount of revenue at a smaller dead-weight loss by employing a lump-sum or, which is the same thing in this framework, a wealth tax on agents. The reason for the agent's intense dislike of increasing the transaction cost is that agents' decisions in this framework are motivated by consumption smoothing, and transactions costs merely inhibit this behavior in each and every period and can thereby have a punishing impact on welfare.

Lastly, as shown above, the change in the transaction costs lowers the rate of return on capital and changes the manner in which the distribution of capital changes over time. The higher transaction costs also raises the effective savings rate and discourages existing agents from selling more units of capital in successive time periods. On the other hand, the lower average rate of return, encourages the existing agent to sell more capital to finance consumption. It turns out that the first effect dominates the second so that, for the experiment conducted in Figure 3, the economy with higher transaction costs would then not move as quick to the steady-state distribution of capital in the presence of the transaction tax.

IV. EQUILIBRIUM WITH A CAPITAL GAINS TAX

The model described in Section II is ideally suited to investigate the implications of implementing a version of a capital gains tax on assets. To this end, suppose that the physical environment is exactly the same as specified in Section II. Suppose further that the government imposes a tax of τ on the increase in the value of an agent's asset holdings from one period to the next. For convenience, and to gain some insight into the effects of such a policy, it is supposed temporarily that decreases in the value of an agent's asset holdings yield a tax credit or subsidy from the government. The policy of imposing taxes alone without subsidies is also considered later. Again the government uses the resulting revenue to spend on goods which are used in some independent manner.[17] Therefore, let the agent's preferences be given again by equation (1), and for existing agents their budget constraint be written as follows

$$c_t + P_t x_{t+1} \leq (P_t + r_t)x_t - \tau(P_t - P_{t-1})x_t$$

where $\tau \in [0, 1)$. This can then be rewritten as follows

$$c_t + P_t x_{t+1} \leq [P_t(1 - \tau) + r_t + \tau P_{t-1}]x_t.$$

After again setting up and solving the dynamic programming problem, it is possible to show that the agents optimal decision rule is as follows

$$P_t x_{t+1} = \left[\frac{\beta}{1 + \alpha\beta}\right][P_t(1 - \tau) + r_t + \tau P_{t-1}]x_t. \tag{13}$$

Substituting agents' decision rules into the market clearing condition (5) then produces the equilibrium price of capital

$$P_t = \frac{\beta[\alpha w_t + (1 - \alpha)(r_t + \tau P_{t-1})]}{[1 + \alpha\beta - \beta(1 - \alpha)(1 - \tau)]}. \tag{14}$$

Of course, setting $\tau = 0$, produces the previous pricing equation for capital without taxes or transaction costs, which is re-written as

$$P_t = \frac{\beta[\alpha w_t + (1 - \alpha)r_t]}{[1 + \alpha\beta - \beta(1 - \alpha)]}. \tag{7}$$

It is interesting to compare these two asset pricing equations. Introducing a capital gains tax/subsidy induces positive serial correlation into the price of capital where it need not have existed previously, and this correlation is higher, the higher is the tax. Obviously, this would also affect the serial correlation properties of rates of return as well. The reason for this is as follows. A low realization of, say, the dividend r_{t-1} in the previous period will drive down the price of capital in period t-1, and help to increase the potential capital gains tax payments of agents in the subsequent period (t) when the dividend would return to its "normal" level. Hence agents in period t will perceive this and save less because they perceive their wealth to be worth less. This lower level of saving then lowers the price of capital in period t. Hence, a lower price of capital in the previous period helps to lower the price of capital in the current period.[18]

Now it is also of interest to consider the impact of imposing a capital gains tax alone, without giving the agents a tax subsidy or rebate when the price of capital fell. Fortunately, the effect of such a policy is easy to understand once one analyzes both the cases of no tax or subsidy, and the case in which the tax/subsidy scheme is in place. For low realizations of w_t and r_t, so that the price of capital falls, the tax is not relevant ($\tau = 0$), and the price of capital is determined by equation (7). Additionally, there is obviously no serial correlation in the price induced by the tax since $\tau = 0$. Alternatively, when there are high realizations for the random variables w_t and r_t, the tax is operative ($\tau > 0$), and the price of capital is given by equation (14). Consequently, there will be serial correlation in the price induced by the tax in this case. Note as well that since the denominator of equation (14) is increasing in the tax parameter (τ), for a given percentage change in either w_t or r_t, the price of capital will respond in a larger manner to <u>decreases</u> in these variables, as opposed to increases, since the tax parameter (τ) will tend to be zero and hence the denominator larger in this instance. Thus there appears to be an asymmetric behavior to the price of capital induced by the asymmetric nature of the tax parameter ($\tau \geq 0$). Low realizations of either w_t and r_t tend to produce greater falls in the price of capital, and less serial correlation in the price as well.

Fortunately, this last version of the model is relatively straightforward to analyze once the previous version is studied. Imposing the tax alone on agents in periods when the price of capital would otherwise be above its level from the previous period causes agents to save somewhat less, but not so much as to cause the price to fall below the level from the previous period. In fact, in the experiments conducted below, existing agents in the market continue to sell assets from one period to the next, and the tax causes a small decrease in the amount of capital that agents sell.

<u>Numerical Results:</u>

Again, in this section numerical methods are employed in order to investigate the impact of this version of the capital gains policy on the equilibrium behavior of the economy. The benchmark value for the capital gains policy is arbitrarily chosen to be τ =0.0. The benchmark economy is again calibrated in a manner similar to that described in the previous section. In light of the robust results of the previous section to changes in the discount factor, in this instance β was set at .99, and the corresponding value for α was set at 0.0665. The probability distribution for w_t and r_t were chosen in exactly the same manner to that described in the previous section.

The results obtained from raising the capital gains tax/subsidy from zero to 5% and to 10% are displayed in Table 4. As can be seen, the average price of capital is lowered by a rather negligible amount by this policy. Additionally, the percentage variability of the price of capital is lowered from such a policy. The reason for this is that in the presence of the capital gains tax/subsidy, the response of the price of capital to a change in either w_t or r_t is diminished. Therefore, to the extent that various policies may be instituted to dissipate the volatility in the prices of assets, a capital gains tax/subsidy may be seen as one possible avenue to help perform this task.

Interestingly, the average level of transaction volume is marginally lowered by such a policy. However, the standard deviation of the transaction volume is dramatically increased. This is evidenced by Figure 6 which shows the probability densities for transaction volume when τ = 0.0 and 0.10. The reason for this increase in variance is as follows. By calculating the amount of capital purchased by an agent from equation (7), without any tax/subsidy, and subtracting it from the amount derived from equation (14), with the tax/subsidy, this produces the following difference

$$(s\tau)\left[\left(\frac{P_{t-1}}{P_t}\right) - 1\right].$$

Obviously this term is zero when τ = 0, but generally this amount can be positive or negative. Consider the case in which $P_t > P_{t-1}$, and so the agent feels less wealthy because of the capital gains payment. Also, because the present price of capital (P_t) is high the cost of purchasing capital is high. Both these effects make the agent purchase less capital than he otherwise would have if the capital gains tax/subsidy were not in place. Similarly when $P_t < P_{t-1}$, the agent feels more wealthy because of the capital gains subsidy that he receives, and as well the present price (P_t) is low so the cost of capital is low. Both these effects reinforce each other to make the agent purchase more capital than he otherwise would have. Hence, a rise in the level of τ will cause more variability in the amount of assets transacted while reducing the variability in the price.

Another way to think of this effect is to observe that equation (13) implies that sales of assets by existing agents are somewhat more sensitive to changes in the price P_t (the cost of capital) since the price in the previous period P_{t-1} now influences the agent's wealth. This is similar to having the supply curve of assets become more price-elastic. Consequently, it is of little surprise to find that with this changed shape of the supply curve, the price of capital becomes less volatile and the transaction volume becomes <u>more</u> volatile.

The average rate of return on capital, ignoring transaction costs, falls marginally because of the change in the capital gains tax/subsidy. Not surprisingly, the rate of return, net of transaction costs, is lowered by the capital gains tax/subsidy.[19] The fall in this rate of return may not be viewed as being too substantial, and this is due to the fact that the growth rate of prices is largely influenced by the growth rate of total consumption, which is the annual rate of 2.9%. Therefore, the amount of the tax actually paid is not substantial. Note also that the standard deviation of the rates of return also falls.

The nature of the serial correlation of the price of capital is illustrated by Table 5. This shows how this correlation is influenced by the capital gains tax/subsidy, and this influence is substantial given that both w_t and r_t are intertemporally independent.

Tables 6 and 7 show the impact on the financial market variables when the capital gains tax alone (without the subsidy) is imposed on agents. In this case the average price of capital is affected more by the policy, causing it to fall. This is due to the asymmetric behavior, described above, induced in the price of capital by the tax which causes the price to respond in a larger manner to low realizations of w_t and r_t, as opposed to high realizations. The volatility of the price of capital falls as well, but not as much as when the tax/subsidy scheme was implemented. The apparent reason for this is that when low realizations for the variables w_t and r_t occur, the price of capital falls more when $\tau = 0$. This causes the price of capital to be more variable. There is still a small fall in average transaction volume, and a dramatic increase in the variability in volume caused by the policy. The average rates of return on capital, both ignoring taxes and net of taxes, are not affected as much when the capital gains tax alone is imposed, as compared with the joint tax/subsidy scheme. Table 6 also shows the average amount of revenue that can be collected from such a policy, measured as a percentage of aggregate consumption. A 5% capital gains tax produces tax revenue on average equal to 2.1% of aggregate consumption. A doubling of the tax from 5% to 10% increases the average revenue by 71%. The last row of Table 6 also shows the equivalent tax on initial wealth that would leave the agent equally well off ex-ante, as under the relevant capital gains tax. The agent would be equally well off with either a 10% capital gains tax, or alternatively giving up 0.44% of his initial endowment upon entering the economy. The number in parenthesis in the last row of Table 6 shows the resulting average expected revenue, measured as a percentage of total consumption, that would arise from the wealth tax applied to the initial wealth of agents. This should be compared with the numbers on the second last row of Table 6. For example, a 10% capital gains tax yields the same discounted expected utility as a 0.44% initial wealth tax on agents, but the capital gains tax produces tax revenue of 3.6% of total consumption, whereas the equivalent wealth tax yields revenue equivalent to 3.08% of total consumption. The main reason why the capital gains tax may not be too punishing in welfare terms is that it also tends to lower the variability of the rate of return on capital, without lowering the average return too much, and this effect can make risk-averse agents better off. Another reason why the capital gains tax may impose less of a welfare burden than the initial endowment levy is the capital gains tax may be viewed as less deleterious since it only works when prices rise, which is when wealth is high and the marginal utility of wealth is low. In contrast, the endowment levy works irrespective of the level of wealth. One conclusion from this may be that in some instances there maybe some welfare benefit from reducing taxes on, say, capital and levying a capital gains tax to recover the resulting revenue.

Raising the capital gains tax from 5% to 10% is actually capable of making agent's (unconditionally) better off. This can be seen by noting that the equivalent wealth tax for a 10% capital gains tax is actually 0.44%, which is less than that for a 5% capital gains tax. This lower wealth tax also brings in less tax revenue as a percentage of aggregate consumption. The reason for this is that the lower variability in the rate of return on capital works to make the agents better off and offset the potential wealth effects from the capital gains tax.

A comparison of the results from Table 6 with Table 2 shows that, for example, a 5% percent capital gains tax, or the equivalent 0.48% wealth tax produces a much higher level of revenue, as a percentage of total consumption, than the resources lost through a 5% transaction cost. The reason for this is simple. These transaction costs are levied only on those capital assets which are transacted in a period, and the amount of these transactions is very small when the transaction cost is 5%. In contrast, the capital gains tax has a much broader tax base and is therefore capable of producing much more tax revenue as a percentage of total consumption.

Table 7 also shows how the implementation of the capital gains tax causes serial correlation in the price of capital. For the reasons described above, this serial correlation is less than would appear of it were a tax/subsidy scheme, and this is easily seen by comparing Table 7 with Table 5.

Lastly, it is of interest to note how the imposition of the capital gains tax influences the distribution of capital over time. The lower rate of return on capital encourages existing capital holders to sell their capital at a quicker rate, and consequently for the experiment conducted in Figure 3, the distribution of capital holdings would move quicker to that of the steady-state in the presence of the capital gains tax than it would otherwise.

V. FURTHER REMARKS

It has been the goal of this study to analyze the impact of two distortional fiscal policies on the behavior of financial market variables. The analysis has been conducted within the context of a fully articulated stochastic general equilibrium model in which agents' preferences and trading opportunities are specified, because it is only within the context of such a model that the effects of various policies can be studied while taking into account how such policies affect the equilibrium of the market. The model has been constructed in such a manner so that the fiscal policies can have an impact on the equilibrium prices and rates of return on capital. Additionally, the model has the property that distortional fiscal policies influence the dynamic properties of transaction volume, an analysis which is notably absent in much of the existing literature.

Issues related to the production of goods or capital accumulation have been ignored in this study. This is because the fiscal policies in question have their impact primarily through influencing consumer behavior. Nevertheless, because the equilibrium interest rates and asset prices are influenced by such policies, it is clear then that such policies would also influence the amount of capital accumulation within the context of model which had endogenous production. For example, the implementation of a higher transaction cost analyzed in Section III, resulted in a higher price of capital and a lower rate of return to capital. It would seem then that such a policy might lead to greater capital accumulation or investment to ameliorate the rising price of capital, which naturally reflects its scarcity.

The parameter α, which determined the rate at which agents enter and leave the economy, was set exogenously so as to mimic the observed average turnover of assets. Holding α constant in this manner imposes some discipline on the exercises conducted, so that variations in this parameter cannot influence the nature of the results. Ideally one might wish to somehow endogenize the behavior of agents who are entering or leaving he economy (see footnote 2). It simply cannot be a complete story that agents exogenously enter or leave such an economy, but instead they must do so for a reason. Presumably one reason for entering the market for financial assets is that the rate of return on financial assets is sufficiently high to induce the participation of such agents. One might expect that the presence of capital gains taxes might deter agents from participating in the market, but this would not necessarily be the case if all other assets were subject to the tax as well. Therefore, it seems sensible to take seriously the results of this paper as a first approximation.

As shown above, various fiscal policies can influence the rate at which agents wish to buy or sell assets when their goal is to maximize utility. The model does not have the property that agents are quickly buying and selling different assets in order to capture small expected gains in returns (i.e. churning). It would seem that having the government levy a transaction tax would have a strong impact on turnover done for this last reason since expected returns on different assets are unlikely to differ by an amount sufficient to offset such costs.

One might also ask about the effect of imposing these policies in a model in which agents had some other avenue such as another asset, through which wealth could be held so as to avoid paying either of the proposed taxes. Of course, in this instance the utility-based costs would certainly be less, but also the revenue raised for the government would also be less. One might reasonably believe that the relevant measure should be the utility-based cost <u>per unit of revenue raised</u>. In this case, it is not clear that in the presence of multiple assets, that this relative measure of costs would be higher or lower than the present measures.

The results of Section III were interpreted as the results of a government policy designed to tax the activity of transacting in assets. Alternatively, the opposite experiment could have been conducted of decreasing the transaction cost and interpreting this as the result of an (exogenous) financial innovation which lowers the transaction costs associated with transacting in financial assets. In this instance, it might be said that these innovations would result in lower average asset prices, higher average rates of return, and higher transaction volume.

In the models presented above an agent's behavior is motivated by consumption-smoothing, or which is the same thing - utility maximization. An obvious question is how the results would change if a different utility function were employed. Consider instead of agents having preferences described by equation (1), they were of the following relative risk-aversion variety

$$\sum_{t=1}^{\infty} \beta^t (c_t^{1-\rho}), \qquad \rho \in (1\,,\,\infty).$$

Then the logarithmic case of equation (1) should be interpreted as the case corresponding to $\rho = 1$. If instead $\rho > 1$, then it is well known that the agent finds it more important, in a welfare sense, to smooth consumption across periods. In this instance, one might expect the welfare costs of Section III are certainly to be magnified the larger is ρ. There is some evidence to suggest that agents do not substantially substitute consumption intertemporally, and perhaps the numbers presented in Section III then underestimate these costs to some degree. Similarly, if $\rho < 1$, then these costs would then be smaller.

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APPENDIX

The exact form of the value function associated with the problem given in Sections II and III is

$$V[(P_t + r_t)x] = \pi_0 + \pi_1 \log[(P_t + r_t)x],$$

where

$$\pi_1 = \left(\frac{1 + \beta\alpha}{1 - \beta(1-\alpha)}\right)$$

and

$$\pi_0 = \left[\log\left[\frac{1 + (\alpha-1)\beta}{1 + \alpha\beta}\right] + \right.$$

$$\left[\frac{\beta}{1-\beta(1 - \alpha)}\right]\left(\log\left[\frac{\beta/(1 + \mu)}{1+\alpha\beta}\right] + E_t\left[\log\left(\frac{P_{t+1} + r_{t+1}}{P_t}\right)\right]\right)\Bigg]\left[\frac{1}{1 - \beta(1 - \alpha)}\right]^{-1}$$

where $\mu = 0$ for the problem described in Section II. Similarly, for new entrants to the economy the value function is written as

$$V[w_t] = \pi_0 + \pi_1 \log[w_t],$$

where π_0 and π_1 are as defined above.

FOOTNOTES

1. It should also be noted that Auerbach (1992) presents a stimulating discussion of the effects of a capital gains tax, and also studies the effects of such a tax within the context of a three-period economy.

2. It does not have to be that the same number of agents leave and enter the economy at the same time. However, this assumption makes these disturbances idiosyncratic, and thereby does not permit this exogenous to produce aggregate shocks.

It should also be noted that the expectation operator in the preferences above reflects, among other things, the fact that in any period with probability α the agent may be leaving the economy.

3. There are many interpretations of the parameter α in this context. This could be interpreted as the rate at which agents enter and leave the capital market of this economy. In other words, $(1/\alpha)$ is the average number of time periods that agents spend in this capital market before liquidating all their assets. More generally, one might expect that an agent's decision as to leave or enter such a capital market is not an exogenous decision, and is instead motivated by their own decisions based on factors in the environment. That is, it could be that agents might leave the capital market because there exists some privately held (and not publicly observable) technology to which they alone have access, such as a housing market, which will yield a much higher rate of return than they anticipate receiving in the capital market. It would be straightforward to incorporate such a feature into the present model, but would merely serve to complicate the resulting analysis. The fact that here the same number of agents enter and leave the economy in each period merely makes the α shocks idiosyncratic and abstracts from aggregate disturbances to the number of agents participating in the capital market.

4. The exact form of the value function is shown in the Appendix.

5. Of course, when $\alpha = 1$, the model is one in which the population is one of two-period lived overlapping generations. Similarly, when $\alpha = 0$, the model is one of an infinitely-lived representative agent. The decision rule (4) conforms with what would arise from these respective models.

6. It is assumed that there cannot exist a firm that buys up all the assets and issues its own equity in an attempt to lower the potential transaction costs associated with buying or selling capital. Alternatively, it can be assumed that even such a firm's shares are subject to transaction costs, and so there would never be any need for such a firm.

It should also be noted that equation (6) implies that if in any period the purchases of assets by new entrants equals the amount sold by agents leaving the economy, then existing agents are neither purchasing or selling assets.

7. It is easy to show for the stochastic as well as the non-stochastic versions of the economy that there exists a unique distribution for the distribution of capital holdings across the population.

8. The problem with employing the usual type of convex transaction cost technologies, such as quadratic, is that the marginal cost of altering an agent's portfolio is increasing in the number of units transacted. This is counterfactual. If anything, brokers give quantity discounts so one could more accurately argue that these costs should be concave. Employing concave transaction costs is very problematic since the agents' optimization problem then ceases to have convex constraints. Nevertheless, it is hoped that the constant costs used in this paper will help to give some insight as to how various tax policies might influence the behavior of the asset market variables.

9. The value function associated with this problem is given in the Appendix.

10. The cases of $\mu > 0$, and $\mu < 0$ are considered separately here. Although the agent gets to indirectly decide whether μ is positive or negative by his decision to purchase or sell assets, he obviously does not influence the magnitude of $|\mu|$. Throughout the numerical results, the following approach is adopted. First $\mu > 0$ is postulated and it will then be investigated to see if the agent's behavior is consistent with this assumption (i.e. whether they actually purchase assets). Then $\mu < 0$ is postulated, and it is seen if the agent's behavior is consistent with this assumption. It turns out that in equilibrium for the numerical models under study, the agent's behavior is always consistent with the assumption $\mu < 0$, but never with $\mu > 0$. That is, for the proposed parameter specification, in equilibrium, if $\mu > 0$ is postulated, the agent then chooses to sell assets, which is inconsistent with $\mu > 0$. On the other hand, if $\mu < 0$ is postulated, the agent then chooses to sell assets (but of a different quantity), which is consistent with $\mu < 0$.

11. This was the value chosen by Aiyagari and Gertler (1991) as well. Of course the rate of turnover or velocity is not the same for all assets, but this serves as one empirical counterpart to the variable determined by the model. Of course, this should serve as an estimate of the upper limit of the rate of turnover. In the model agents do not trade in assets so as to change their portfolio structure, since there is really only one asset. In a world where there are many assets, taxing transactions in one asset would likely lead to increased trade in other assets in which the transactions were not taxed.

12. Furthermore, it is not clear exactly what the empirical counterparts to these variables should be.

13. It should be noted that there appears to be an inverse relationship between the required values of α and β in the table. This occurs because if the agents discount factor β is raised, this makes him care more about his future utility and he will then wish to save more in the way of assets. To generate the required level of trade in assets, the parameter α must then rise in order to encourage the agent to save less to offset the rise in the discount factor.

14. It is important to note here that the value of $\mu = -\theta$ is less than zero, and that an increase in the transaction tax amounts to a further lowering of μ. This leads existing agents to sell less of their capital holdings, and hence raises the price of capital. However, the elasticity of P_t with respect to either w_t or r_t is independent of μ. Hence the volatility of P_t that results from changes in either w_t or r_t will not depend on the level of μ.

15. Of course, the longer the agent holds the asset, the closer will be the return net of transaction costs, to the return ignoring transaction costs.

16. Note that since this initial tax does not distort any future decision, it could be interpreted as a lump-sum tax.

17. This type of tax is slightly different from what one might observe actual governments implementing. In the present context, if the price of capital rises, the agent immediately pays a tax on the increase in the value of their asset holdings. In reality, the agent might be able to postpone paying the tax until the assets had been sold. However, modelling and studying this latter policy is very problematic since there would be an explosion in the proliferation of state variables for the agent's optimization problem. It would then be necessary to keep track of each agent's assets, and when they were purchased, and at what price. Purchase of assets would then depend on how they would influence the tax liability much later in the agent's life under all states of the world. It is hoped that the approximation employed in this paper will yield some insights into the impact of other versions of capital gains taxes.

However, it can be shown that the scheme analyzed in this paper is equivalent to a tax which is paid only when assets are sold, providing the government charges/pays interest on all previous capital gains and losses throughout the agent's life. In fact, such a scheme presently does exist. Investors who purchase long-term discount bonds, for which the return is fully in the form of capital gains since there is no interest, must pay tax on the implied interest on the bond over the tax year.

18. Another interesting feature is that if the stochastic process for w_t and r_t are such that these variables individually follow martingales, then the price given by (7) will also be a martingale. However, if a capital gains tax is introduced, then the price as given by equation (14) will in general not be a martingale.

19. In this case, this rate of return is calculated as if the asset were purchased at the beginning of one year, and held for exactly four quarters and then sold and the required taxes were paid on the assets.

TABLE 1

Change the transaction cost (θ) from 1% to 3%

	β = .9995 α = .0760	β = .9900 α = .0716	β = .9500 α = .0520	β = .9000 α = .0251
% change in average price of capital	14.3%	14.5%	14.8%	15.4%
% change in average volume	-12.6%	-12.7%	-13.0%	-13.3%
% change in st. dev. of volume	-9.3%	-9.4%	-9.5%	-10.0%
change in R_b	-.5%	-.5%	-.5%	-.5%
change in st. dev. of R_b	-0.1%	-0.1%	-0.2%	0.0%
change in R_a	-4.6%	-4.6%	-4.6%	-4.6%
change in st. dev. of R_a	-0.9%	-0.9%	-1.0%	-0.9%
average transaction costs as % of total consumption	0.42%	0.39%	0.28%	0.13%

R_b refers to the annual rate of return ignoring transaction costs.

R_a refers to the annual rate of return net of transaction costs.

TABLE 2

Change the transaction cost (θ) from 1% to 5%

	$\beta = .9995$ $\alpha = .0760$	$\beta = .9900$ $\alpha = .0716$	$\beta = .9500$ $\alpha = .0520$	$\beta = .9000$ $\alpha = .0251$
% change in average price of capital	34.5%	34.6%	35.8%	37.4%
% change in average volume	-25.7%	-25.8%	-26.4%	-27.1%
% change in st. dev. of volume	-20.0%	-20.0%	-20.6%	-21.4%
change in R_b	-1.0%	-1.0%	-1.1%	-1.1%
change in st. dev. of R_b	-0.3%	-0.2%	-0.3%	-0.1%
change in R_a	-9.1%	-9.1%	-9.1%	-9.1%
change in st. dev. of R_a	-1.9%	-1.8%	-1.9%	-1.7%
average transaction costs as % of total consumption	0.70%	0.66%	0.69%	0.22%

R_b refers to the annual rate of return ignoring transaction costs.

R_a refers to the annual rate of return net of transaction costs.

TABLE 3

Different welfare costs of various transactions costs

	$\theta = .01$	$\theta = .03$	$\theta = .05$
Welfare cost* (average revenue as a percentage of total consumption)	0.92% (0.86%)	2.69% (2.53%)	4.40% (4.12%)

$\alpha = .0716$, $\beta = .99$.

* - Welfare cost is measured as the equivalent tax on initial wealth that would leave the agent equally well-off, but in an environment in which there were no transactions costs at all, resulting in a different distribution of rates of return since the transactions costs are absent.

TABLE 4

Effect of different capital gains taxes and subsidies

	τ = .05	τ = .10
% change in average Price of Capital	-.022%	-.045%
Change in the % variability of the Price of Capital	-3.1%	-4.8%
% change in average volume	-0.28%	-0.32%
% change in st. dev. of volume	300%	474%
change in average R_b	-0.84%	-1.23%
change in st. dev of R_b	-0.84%	-1.23%
change in average R_a	-1.06%	-1.60%
change in st. dev of R_a	-5.69%	-9.04%

α = .0665, β = .99.

R_b refers to the annual rate of return ignoring taxes.

R_a refers to the annual rate of return net of taxes.

TABLE 5

Serial correlation resulting from the capital gains taxes and subsidies.

	τ = 0.0	τ = 0.5	τ = 0.10
Serial correlation of the price of capital	0.0	0.242	0.392

α = .0665, β = .99.

TABLE 6

Effect of different capital gains taxes alone

	τ = .05	τ= .10
% change in average Price of Capital	-2.2%	-3.78%
Change in the % variability of the Price of Capital	-1.67%	-2.70%
% change in average volume	-0.61%	-0.95%
% change in st. dev. of volume	409%	745%
change in average R_b	-0.35%	-0.55%
change in st. dev of R_b	-2.29%	-3.86%
change in average R_a	-0.84%	-1.47%
change in st. dev of R_a	-0.84%	-1.47%
average tax revenue as a percentage of total consumption	2.1%	3.6%
equivalent wealth tax (resulting revenue)*	0.48% (3.38%)	0.44% (3.08%)

α = .0665, β = .99.

R_b refers to the annual rate of return ignoring taxes.

R_a refers to the annual rate of return net of taxes.

* This is the average tax revenue as a percentage of aggregate consumption that arises from applying the relevant tax rate to the initial wealth of all newly entering agents in the economy.

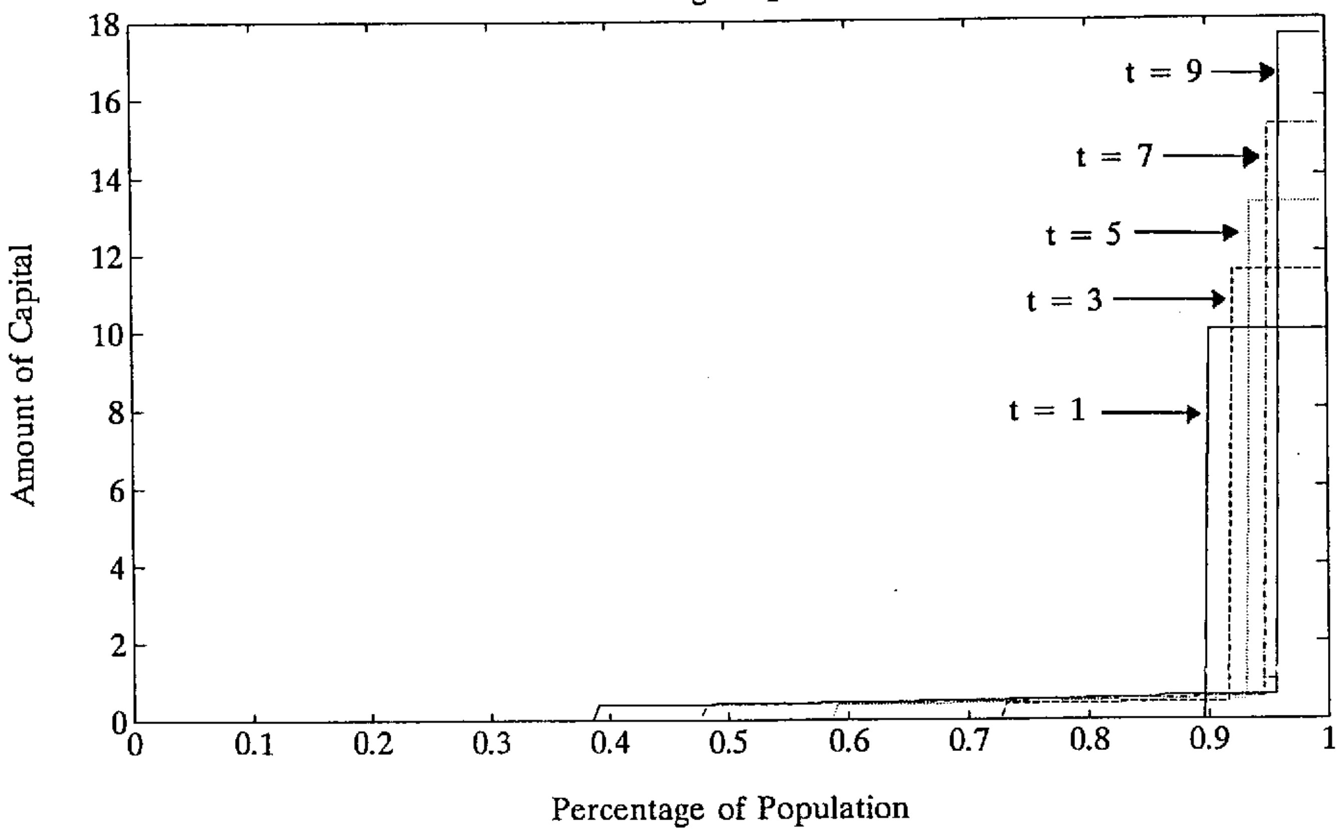
TABLE 7

Serial correlation resulting from the capital gains taxes alone

	τ = 0.0	τ = 0.05	τ = 0.10
Serial correlation of the price of capital	0.0	0.125	0.221

α = .0665, β = .99.

Figure 1



t = 9
t = 7
t = 5
t = 3
t = 1
Amount of Capital
Percentage of Population
0
2
4
6
8
10
12
14
16
18
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 2

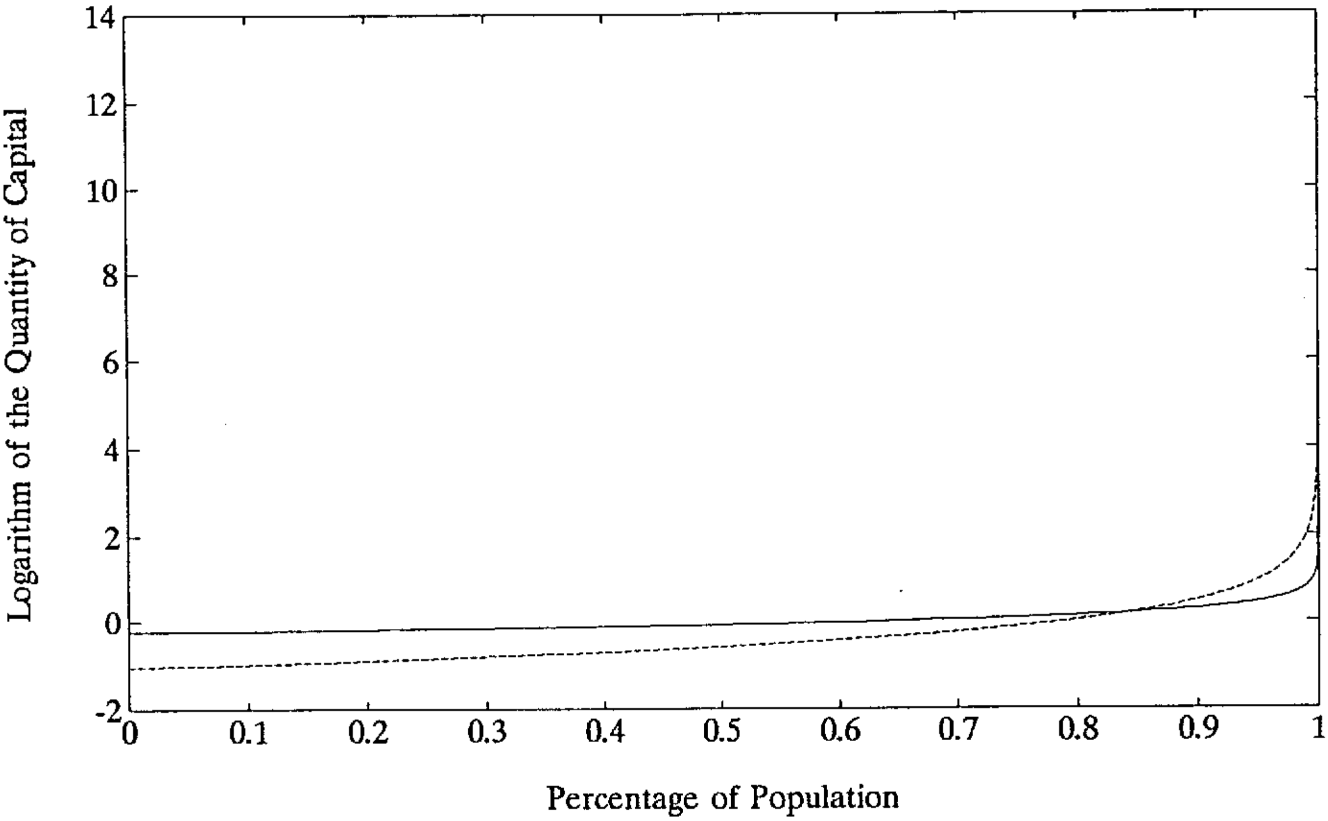
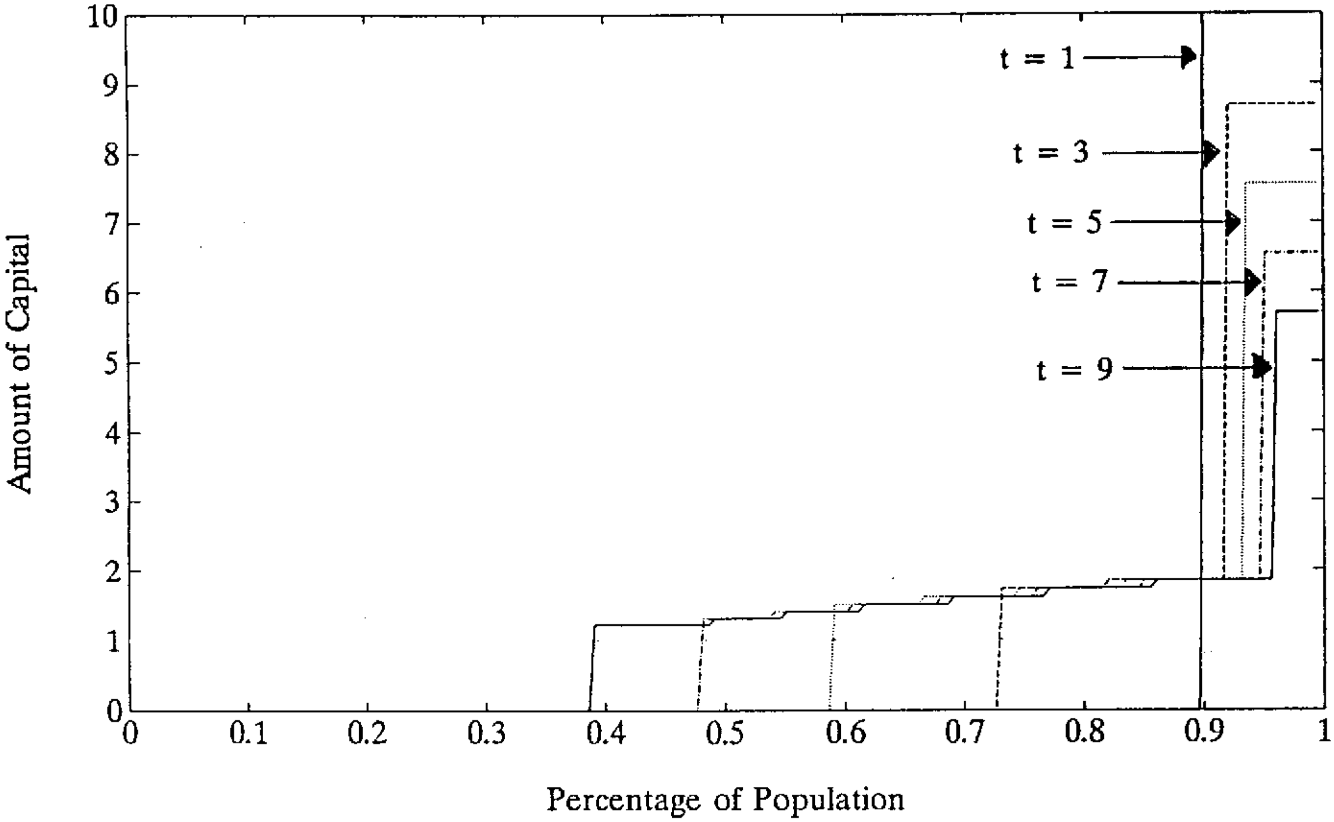
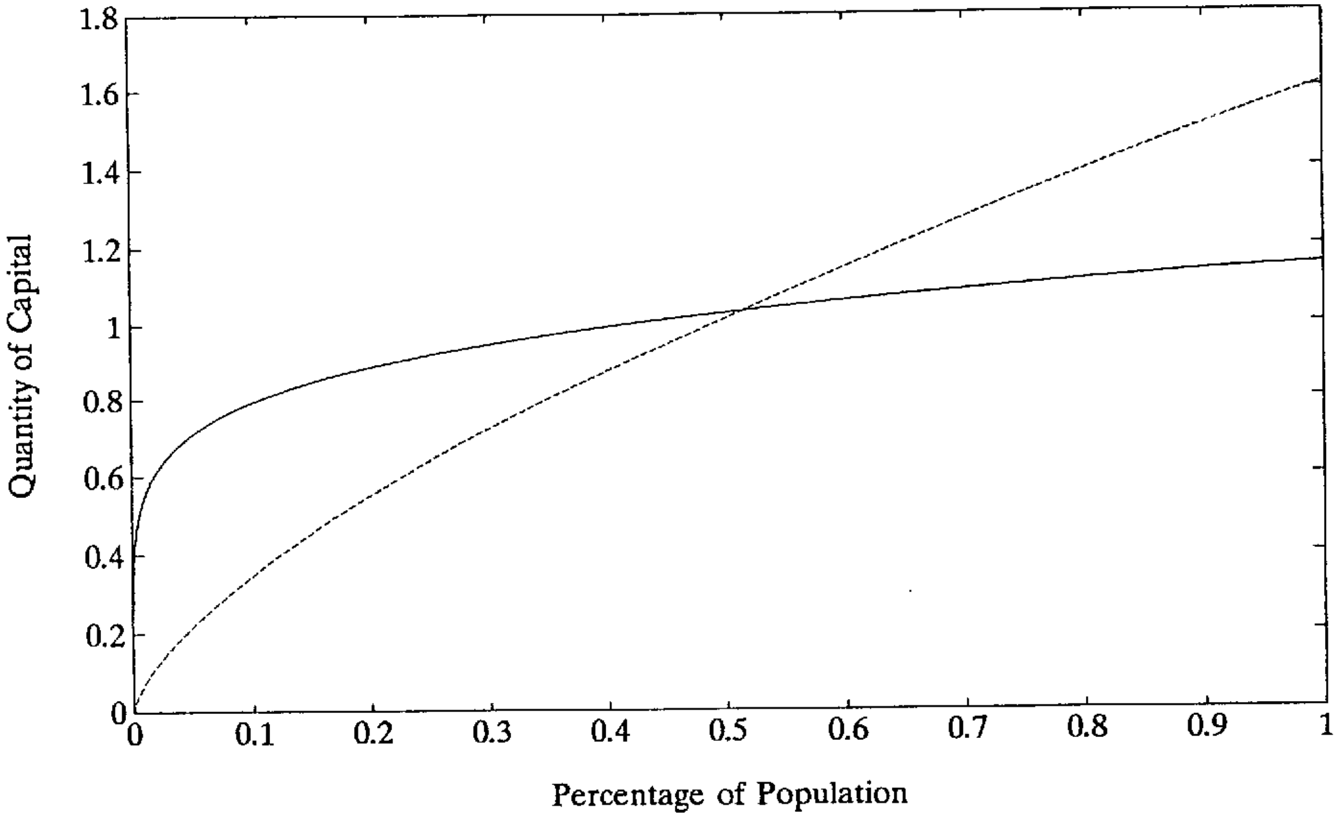

Logarithm of the Quantity of Capital
14
12
10
8
6
4
2
0
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Percentage of Population

Figure 3



t = 1
t = 3
t = 5
t = 7
t = 9
Amount of Capital
Percentage of Population
0
1
2
3
4
5
6
7
8
9
10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Figure 4



1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Quantity of Capital
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Percentage of Population

Figure 5

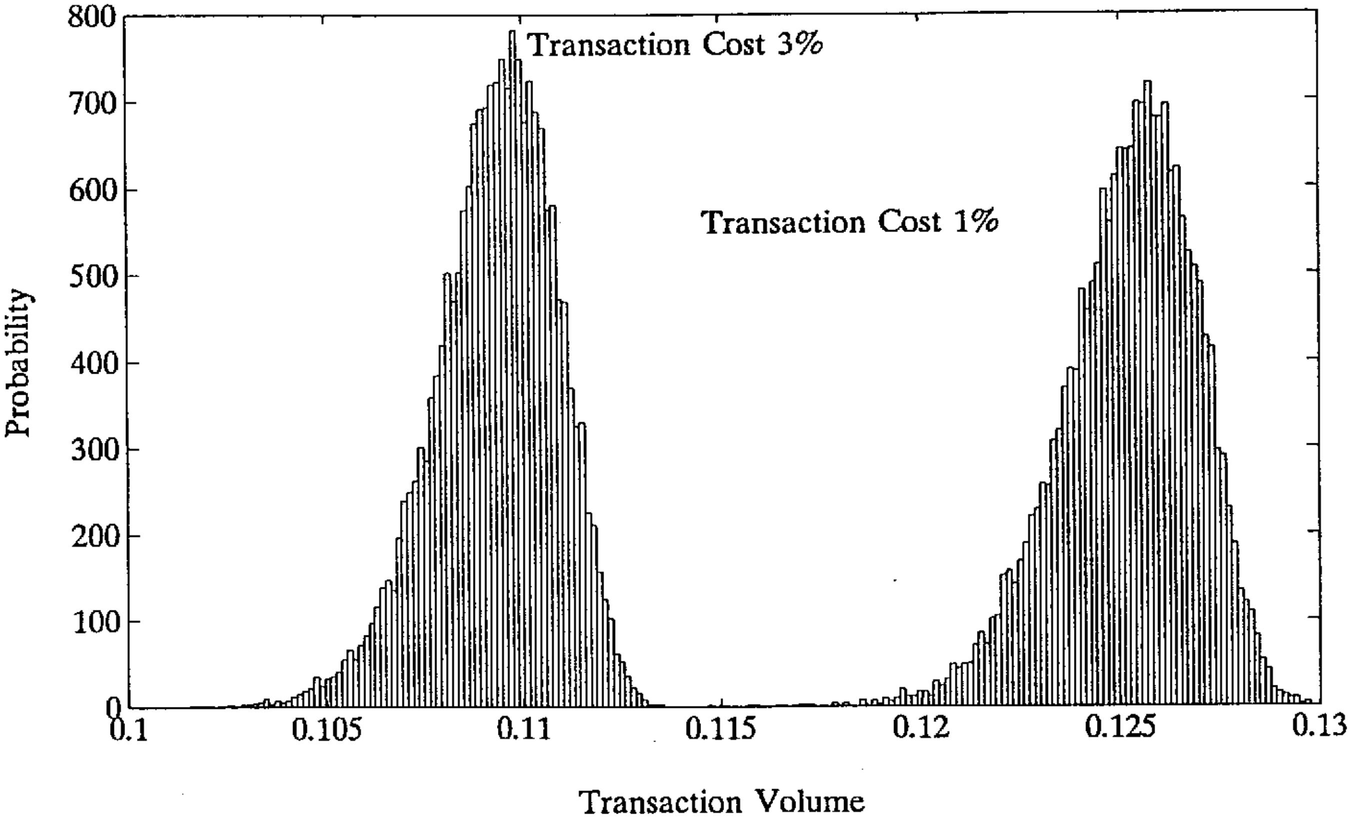
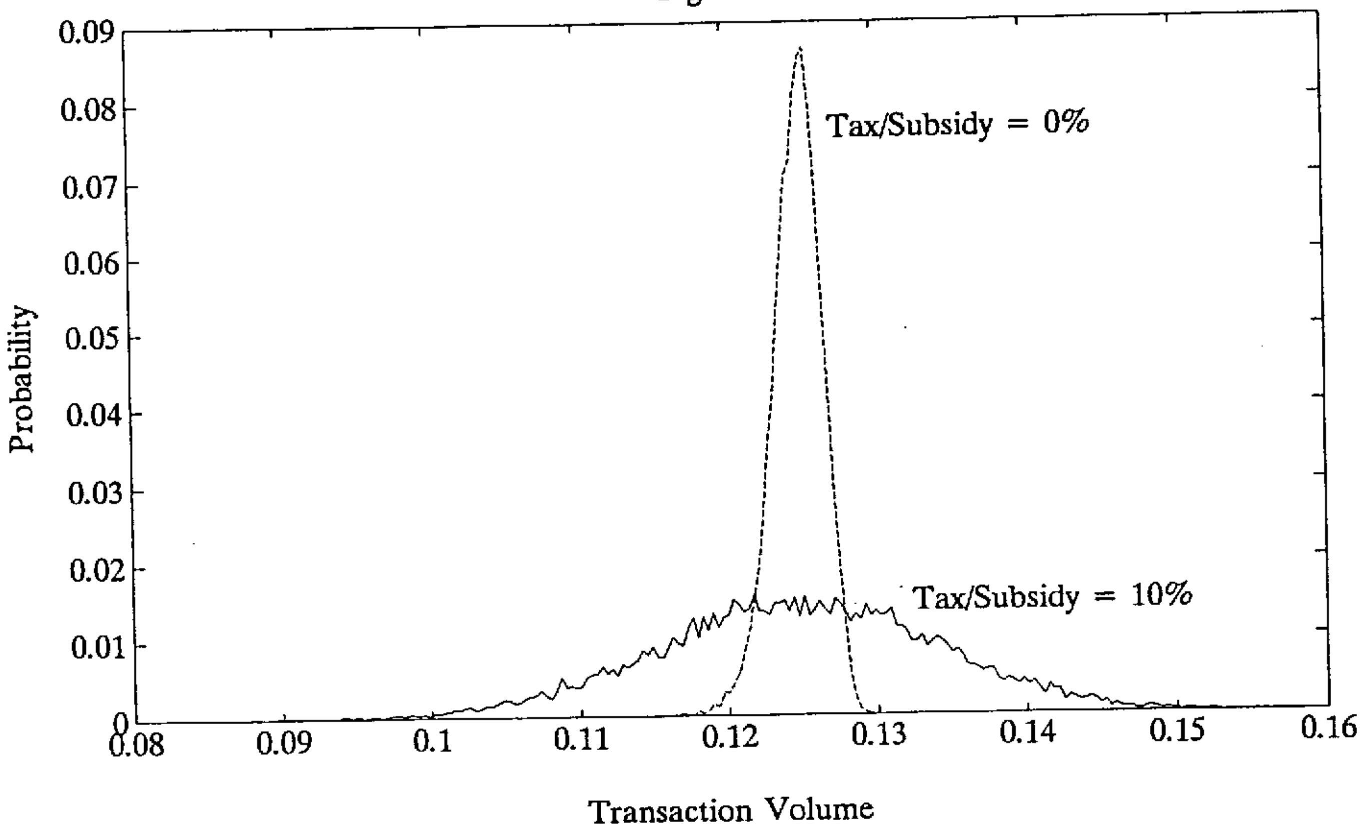

Transaction Cost 3%
Transaction Cost 1%
Probability
800
700
600
500
400
300
200
100
0
0.1
0.105
0.11
0.115
0.12
0.125
0.13
Transaction Volume

Figure 6


Tax/Subsidy = 0%
Tax/Subsidy = 10%
Probability
Transaction Volume
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0.08
0.09
0.1
0.11
0.12
0.13
0.14
0.15
0.16

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The Analysis of Fiscal Policy

in Neoclassical Models[1]

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