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ECONOMIC FLUCTUATIONS: OUTLIERS
IN MACROECONOMIC TIMES SERIES

by

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Research Paper

Federal Reserve Bank of Dallas

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Outliers in Macroeconomic Time Series

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I. Introduction

What is the nature of fluctuations in macroeconomic time series? Despite a growing literature on asymmetry and nonlinearity, most analyses of macroeconomic time series still employ linear models that assume (either implicitly or explicitly) Gaussian innovations.¹ Indeed, reflecting this Gaussian orientation, most stylized facts for macroeconomic time series have been computed using only the first two moments of the data.² However, anomalies may be present in macroeconomic time series that are ignored and left unexplained by traditional linear models. One such anomaly is the possibility that there are large but infrequent shocks to macroeconomic time series. Events such as oil shocks, wars, natural disasters, and changes in policy regimes are examples of relatively infrequently occurring events that may have important effects on macroeconomic time series.

The purpose of this paper is to determine the prevalence and nature of large shocks in macroeconomic time series. We attempt to establish the frequency, timing, and persistence of large shocks and whether they are important contributors to the variation in macroeconomic time series. Furthermore, we attempt to match these shocks with identifiable economic events. With these objectives in mind, we search for outliers in fifteen macroeconomic time series.

We find significant evidence in favor of the "large shock" hypothesis in post-World War II quarterly data. Not only is there evidence of large shocks in each of the fifteen macroeconomic time series examined, but in some of the series these shocks account for a substantial proportion of the total variance in the series--more than 50 percent for aggregate wage and price inflation and nearly 40 percent for consumption expenditure, M1 and M2

growth.

In addition, three basic patterns emerge in the identified outliers. First, many of the identified outliers seem to be associated with business cycles, in particular turning points and recessions. Second, there appears to be a clustering of outliers within series and across series--outliers tend to be bunched up over time and series tend to have outliers on the same date. Third, outlier dates in output and employment series do not overlap substantially with outlier dates in nominal price series such as the GNP deflator. That is, there appears to be a dichotomy between outlier behavior of real versus nominal macroeconomic time series.

Our results support and extend the evidence found by Blanchard and Watson (1986), who examined this large shock/small shock hypothesis within the context of a structural vector autogression (VAR) that included aggregate prices, output, money, and a fiscal policy variable. They found excess kurtosis in the residuals of their VAR and, hence, argued that this is consistent with large infrequent shocks. However, they had difficulty linking large residuals from this VAR to economic events and suggested that large, infrequent shocks do not dominate business cycle fluctuations.

Our analysis differs from Blanchard and Watson in several ways. We use the outlier identification procedure of Tsay (1988) to determine the date and type of outliers in the data we examine. The advantages of this procedure are that it does not depend on a priori information about when outliers have occurred, and, it is quite flexible in modeling the dynamic effects of outliers. Because it is a univariate method, we analyze each series separately. While univariate analysis by its nature limits the kinds of interesting economic interactions that can be uncovered, it does allow us to

examine many more series more flexibly than is possible in a multivariate framework.

In addition to explaining the excess kurtosis and/or skewness found in the fifteen time series, controlling for outliers eliminates much of the evidence of nonlinearity in many of the time series examined. This points to a link between identified outliers and possible nonlinearity in the time series. While generalized autoregressive conditionally heteroscedastic (GARCH) variance models are also capable of capturing the nonlinearity in many of the time series, GARCH models do not offer as rich an explanation of non-Gaussian behavior in many macroeconomic time series as the outlier model does. Apparently, important anomalies go unexplained by GARCH specifications, leaving standardized residuals that are frequently non-Gaussian and that appear to contain outliers.

The remainder of the paper is organized as follows. In Section II we provide a formal description of the large shock hypothesis. We discuss the outlier search procedure in Section III. In Section IV we present empirical results for fifteen macroeconomic time series. We also attempt to link the outlier dates with identifiable economic events. In Section V we examine the linkage between outliers and evidence of nonlinearity. We conclude in Section VI.

II. Outlier Model

We begin by positing a univariate time series model in which there are two components: a regular component and an outlier component. The idea is that there are extraordinary, infrequently occurring events or shocks that have large, dramatic effects on time series. Again, economic examples of

these types of shocks might include the effects of an oil shock, a change in policy regime, the effects of a war or natural disaster, etc. These extraordinary shocks are orthogonal to shocks in the regular component and need not have the same dynamic effect on the time series as a regular shock.

To formalize this notion, consider the following outlier model described in Tsay (1988). Let

$$Y_t = B_\omega(L)\omega_t I_t + B_a(L)a_t, \quad (1)$$

where $\omega_t I_t$ is an outlier variable, and a_t is a Gaussian variate with zero mean and variance σ_a^2 . Let $B_a(L)a_t$ be the moving average representation of an ARMA(p,q) where $B_a(L) = \theta(L)/\phi(L)$. $\theta(L)$ is a lag polynomial of order q, and $\phi(L)$ is a lag polynomial of order p. One can think of $B_a(L)a_t$ as the regular component of the time series Y_t ; that is, in the absence of extraordinary large shocks or outliers, $B_a(L)a_t$ is the moving average representation of Y_t .

The variable $\omega_t I_t$ is the outlier variable. I_t is an indicator variable that takes the value of zero when no outlier is present and is one in the presence of a large shock. ω_t is the size of the outlier. $B_\omega(L)$ represents the effect that the outlier has on Y_t . If $B_\omega(L) = 1$, then ω_t is an additive outlier (AO); this outlier has only a one period effect on the series. If $B_\omega(L) = B_a(L)$, then ω_t is an innovative outlier (IO); this outlier has the same dynamics as the regular component. If $B_\omega(L) = 1/(1-L)$, then ω_t is a level shift outlier (LS). Level shifts have a permanent effect on the time series; in effect, they permanently shift the mean of the series.³ Thus, these outlier types are distinguished by the persistence they have on the time series with the additive outlier having the least persistence and the level shift with the most persistence.

If outliers occur randomly--for example, if they are determined by a

Bernoulli distribution where $\text{Prob}[I_t = 1] = \lambda$ and $\text{Prob}[I_t = 0] = 1-\lambda$ --the unconditional distribution of $\omega_t I_t$ will not be normal. If the conditional distribution of ω_t is $N(0, \sigma_\omega^2)$, then for the Bernoulli distribution case the unconditional variance of $\omega_t I_t$ is

$$\text{Var}(\omega_t I_t) = \lambda \sigma_\omega^2, \quad (2)$$

but it will display excess kurtosis of

$$b_2(\omega_t I_t) - 3 = 3(1-\lambda)/\lambda, \quad (3)$$

where for a random variable, say X_t , $b_2(X_t) = E(X_t^4)/E(X_t^2)^2$. The more infrequent the outlier shocks are (the smaller λ), the greater excess kurtosis there is in ω_t and, hence, Y_t .⁴

Skewness is likely to be present also. If ω_t has a nonzero mean, then $\omega_t I_t$ and Y_t will exhibit skewness.⁵ Even if the unconditional mean of ω_t is zero, nonzero sample third moments are likely to occur in relatively small samples with only a few outliers. Thus, skewness is also likely to be present if outliers have occurred.

III. Searching for Outliers

If the date and type of the outlier is known, one can model the irregular component in the form of an intervention model (Box and Tiao (1975)) in which estimates of ω_t can be obtained from the coefficients on the intervention dummies. In our empirical work below, once possible outliers/level shifts have been identified, we use an intervention model to estimate ω_t . However, before we can estimate this intervention model, we must determine the timing of outliers.

To determine the existence of outliers, we use the outlier detection method described by Tsay (1988). Define $y_t = (\phi(L)/\theta(L))Y_t$, which is

equivalent to the ARMA residuals under the null hypothesis of no outliers. Define $\pi(L) = 1 - \pi_1 L - \pi_2 L^2 - \dots = \phi(L)/\theta(L)$ and $\eta(L) = 1 - \eta_1 L - \eta_2 L^2 - \dots = \pi(L)/(1-L)$. Tsay suggests the following test statistics for the various types of outliers:

$$\lambda_{IO,t} = y_t/\sigma_a,$$

$$\lambda_{AO,t} = \rho_{A,t}^2 (y_t - \sum_{i=1}^{T-t} \pi_i y_{t+i}) / (\rho_{A,t} \sigma_a), \text{ and}$$

$$\lambda_{LS,t} = \rho_{L,t}^2 (y_t - \sum_{i=1}^{T-t} \eta_i y_{t+i}) / (\rho_{L,t} \sigma_a),$$

where $\rho_{A,t}^2 = (1 + \sum_{i=1}^{T-t} \pi_i^2)^{-1}$, $\rho_{L,t}^2 = (1 + \sum_{i=1}^{T-t} \eta_i^2)^{-1}$, σ_a^2 is the variance of a_t , and T is the sample size. Let $\lambda_{\max} = \max(\lambda_{IO,\max}, \lambda_{AO,\max}, \lambda_{LS,\max})$, where $\lambda_{j,\max} = \max_{1 \leq t \leq T} \{|\lambda_{j,t}|\}$ $j = IO, AO, LS$. If the λ_{\max} statistic exceeds a given critical value, then an outlier has occurred. In this application, we choose a critical value of three; roughly, only shocks greater than three standard deviations are considered as outliers.

Tsay suggests a sequential algorithm for identifying outliers. First, estimate an ARMA model and extract the residuals and the residual variance. Second, search for outliers in the residuals using the statistics described above. If an outlier is found, remove the effect of the outlier and recalculate the residuals and residual variance. Continue searching and adjusting until no more outliers are indicated. Reestimate the ARMA model using the adjusted series and extract the residuals. Once again, search for outliers. Stop the algorithm when no additional outliers are found.

Note that the initially estimated ARMA model is the correct specification of the regular dynamics ($B_a(L)$) under the null hypothesis of no outliers. If, however, outliers are found, then the initial ARMA model for the regular component will be misspecified. Unfortunately, misspecification of the initial ARMA model can lead to missidentification of outliers. In

particular, series in which a level shift outlier is present will exhibit a high degree of serial correlation regardless of the regular dynamics. In this case, the initial ARMA model for the regular dynamics implies greater serial correlation than is in fact the case and, therefore, the residuals from this model will not reflect the true nature of the outlier. Balke (1991) has shown that an outlier search where the initial ARMA model is estimated often misidentifies level shifts as innovative outliers or misses the level shifts altogether.⁶

Therefore, in order to control for this type of misspecification, we use a modification to the Tsay procedure suggested by Balke (1991). Here, in addition to conducting an outlier search as in Tsay, we also conduct an outlier search in which the initial ARMA model is specified as an ARMA(0,0). This ARMA model is less likely to misidentify level shifts as innovative outliers than is the case when estimating an initial ARMA model. The problem with beginning the outlier search with an ARMA(0,0) is that there is a tendency to identify spurious level shifts when there is substantial serial correlation in the regular component.

If a level shift is indicated in the course of the ARMA(0,0) search, then we use the results from both outlier searches in our identification of outlier dates. Once outlier dates and types have been identified, we estimate an intervention model using dummy variables to model the outlier effects. To lessen the possibility of spurious outliers or level shifts, we stepwise eliminate intervention dummies with a t-statistic whose absolute value is less than a prespecified critical value--dropping the intervention dummy with the lowest t-statistic at each step. As in the outlier searches, we use a critical value of three for our stepwise elimination.⁷

The dynamic structure of the various outlier types imposes restrictions on the intervention model. For example, for the case where the model is given by

$$y_t = c_0 + (1-L)^{-1}\omega_{LS}I_{LS,t} + \omega_{AO}I_{AO,t} + (1-\phi L)^{-1}\omega_{IO}I_{IO,t} + (1-\phi L)^{-1}a_t,$$

where c_0 is a constant term, $I_{j,t} = 1$ ($j = IO, AO, LS$) if an outlier is identified to have occurred at time t and 0 otherwise. Rewriting this equation yields

$$y_t = c_0' + (1-\phi L)(1-L)^{-1}\omega_{LS}I_{LS,t} + (1-\phi L)\omega_{AO}I_{AO,t} + \omega_{IO}I_{IO,t} + \phi y_{t-1} + a_t.$$

Nonlinear least squares is used to estimate the above equation so that the restrictions implied by the different outlier types can be imposed during estimation.

IV. Empirical Analysis

In this section we examine fifteen quarterly macroeconomic time series spanning 1947Q1-1990Q2 to determine whether outliers are present in these series.⁸ The output series include real GNP, real consumption, real fixed investment (which includes residential as well as business investment), and industrial production. We also examine civilian noninstitutional employment and the unemployment rate as well as labor productivity in manufacturing. The price series we examine are the GNP deflator, the consumer price index (CPI), nominal compensation per hour in manufacturing, the Standard and Poors 500 stock price index, as well as yields on AAA bonds. We also examine the monetary base, M1, and M2.⁹ We use growth rates (log first differences) for all the series except for the unemployment rate, which is analyzed in first differences. After differencing, an autoregressive model was estimated for each series. Autoregressive lags were added until there was no evidence of

linear serial correlation in the residuals. Because some analysts have suggested that the growth rate of money contains a time trend (Stock and Watson (1989)), we included a time trend for the money growth series.

Table 1 displays the ARI specifications of all the variables.¹⁰ The residuals from autoregressive models indicate that all but two of the series--the money base and the unemployment rate--show significant (at the 5% level) evidence of excess kurtosis. Many of the series also show significant skewness.¹¹ Clearly, the assumption of Gaussian errors is not appropriate.

Consistent with the evidence of excess kurtosis, we detect evidence of outliers in all the series. These outliers can explain a substantial proportion of the volatility in some of the time series. For most of the series, outliers explain a little less than 20 percent of the volatility in the series.¹² However, large shocks explain more than 50 percent of the volatility in the GNP deflator, the CPI, and nominal compensation while both consumption, M1, and M2 outliers account for nearly 40 percent of the variance of those series. Furthermore, once the outliers have been removed, we find no evidence of significant excess kurtosis or skewness in any of the series, except unemployment. Thus, it appears that the large shock hypothesis is statistically plausible.

Before examining the identified outliers in detail, it may be useful to illustrate the procedure by which the final intervention models are determined. Table 2 provides an illustration of how the combine/reduce procedure suggested by Balke (1991) works. Table 2 shows the steps taken between the identification of the initial intervention model by the outlier searches and the final intervention model for the nominal compensation series. The compensation series is a nice example since all three types of

outliers are present and several steps must be taken before the final intervention model is chosen. Because we used a critical value of three in both the outlier search and the stepwise reduction of the intervention model, a couple of outliers (the level shift in 1968Q1 and the additive outlier in 1982Q1) are eliminated even though they have t statistics well over two. Most of the series examined in this paper require far fewer steps to arrive at the final specification than is the case for compensation.¹³

In order to discern patterns in the identified outliers, we present the outlier results in two ways. Table 3 describes the outliers found for each series, the type and size of the outlier as well as the date at which it occurred. In addition, we also try to link the date of each outlier to an economic phenomenon or event that occurred at or near that date. For example, real GNP experienced an innovative outlier in 1950Q1 and an additive outlier in 1980Q2. The first quarter of 1950 corresponds to the first full quarter of a recovery while 1980Q2 corresponds to the trough of the 1980 recession. Table 4 organizes the same information in a different way, presenting the outlier dates in chronological order and listing the series that have outliers on that date. By listing the outlier dates in chronological order, it is easier to show the patterns of outliers across time as well as determine which series experience outliers at the same date.

An examination of Tables 3 and 4 suggest a few rough patterns that appear to exist among the outliers and that are linked with identifiable economic events. First, many of the identified outliers seem to be associated with business cycles, in particular recessions or early in the recovery. Second, there appears to be a clustering of outliers within series and across series. Third, the number and type of outliers for the real

output and employment series are substantially different from those of the nominal price series.

In many respects, the pattern of outliers we found are similar to the "large shocks" identified by Blanchard and Watson. Based on an examination of a four variable VAR that included real GNP, the GNP deflator, M1 and a fiscal policy variable, they also found that "large shocks" were common during turning points and recessions and tended to be clustered across time and series. Because our criterion for identifying a large shock was more restrictive than Blanchard and Watson, we found fewer large shocks per series. However, when aggregating across series so that we compare the dates listed in Table 3 to the dates found in Blanchard and Watson, many of the same dates show up in both analyses.

In the following three subsections we examine further the three basic patterns of outliers previously mentioned.

Business Cycles and Outliers

Well over half of the outliers in the output and employment series are associated with business cycles, in particular, recessions. Both outliers in the real GNP series are associated with turning points in the business cycle. Aside from the Korean War outliers, all of the outliers in consumption are associated with recessions. Real fixed investment, industrial production, labor productivity, employment, unemployment rates all have outliers associated with recessions or turning points. In addition, several nominal series experience outliers associated with the business cycle. Interestingly, four of the five outliers associated with M2 occur during the first quarter of business cycle expansions.

However, not every business cycle or recession is represented (no outliers are present from the 1970 recession) nor do all the series experience outliers at the same date or even during the same recession. In fact, the 1980Q2 recession is the only common outlier date for real GNP, fixed investment, and consumption. Perhaps, the relatively short but steep recession in 1980 makes it easier for the outlier identification procedure to classify this recession as an outlier.

Overall, the fact that turning points in the business cycle and recessions feature prominently in the identified outlier dates suggests that post war U.S. business cycle behavior is inconsistent with linear gaussian models. Even a linear, Gaussian, multivariate framework is unlikely to explain this outlier behavior since linear aggregation of different Gaussian random variables is still Gaussian.¹⁴ We must look elsewhere to model business cycles. Furthermore, the fact that each business cycle is captured in different ways by the outliers suggests a multi-causal approach to business cycles--as Blanchard and Watson (1986) suggest, all business cycles are not alike.

Clustering of Outliers

Several series show a clustering of outliers across time. For example, the GNP deflator and the CPI show numerous outliers in the late 1940s and early 1950s and relatively fewer outliers in the rest of the sample. Similarly, more than half the outliers for M1 occur in the three year period between 1979Q3 and 1982Q4. This reflects the well-documented increase in the volatility of M1 money growth over this period. This period coincides with a change in monetary policy operating procedures as well as being a period of financial innovation and deregulation. This clustering of outliers across

time would be symptomatic of series with ARCH variance processes.

There is also a clustering of outliers across series. The clustering of outliers across series suggests that there may be common sources for these groups of outliers. The GNP deflator, the CPI, and compensation show evidence of level shifts at or near the same time: late 1967/early 1968, early 1973, and 1982Q4.¹⁵ These level shifts are associated with the Vietnam war expansion, the acceleration of inflation of the early-to-mid 1970s, and the Volcker disinflation. The first two level shifts are more likely picking up the general acceleration in inflation during those time periods rather than the direct effect of a particular shock or event. All three level shifts are in a sense reflecting changes in inflationary regimes that occurred during these time periods.

Several dates have more than two outliers. For example, six series have outliers in 1982Q4--GNP deflator, CPI, compensation, stock prices, AAA bond yields, and M1. Two additional dates have at least three outliers associated with them: real GNP, consumption, fixed investment, and M1 have outliers in 1980Q2; employment, unemployment, and industrial production have outliers at 1975Q1. The recessions of 1957-58, 1974-75, and 1980 contain multiple outliers. Similarly, numerous outliers, both real and nominal, are present during the Korean War.

Real versus Nominal Outliers

The pattern of outliers in the real output and employment series is substantially different from that of the nominal price series. The real output series (GNP, consumption, fixed investment, industrial production, etc.) and the employment series tend to have fewer outliers and the importance of these outliers is substantially less than the nominal price

series (the GNP deflator, the CPI, and nominal compensation). Furthermore, the timing and type of the real series outliers is different from that of the nominal series outliers. Outliers in real series tend to be associated with business cycles and are all temporary; there are no large permanent shifts in the growth rates of these series. The nominal price series, except for the Korean War outliers, exhibit almost no overlap with the dates of the outliers for the real series.

Furthermore, the growth rates of the price series do exhibit level shift outliers, or permanent large shocks. These level shifts reflect changes in the average inflation rate that occurred at or around these dates. Yet, these level shifts do not coincide with outliers (temporary or permanent) in any of the real series. These results suggest a dichotomy between the real output series and the aggregate price series and that fluctuations in these series may have different sources.¹⁶

Miscellaneous Outliers

In addition to the general patterns discussed above, there are some interesting outliers among the individual series. The GNP deflator experiences an innovative outlier in 1974Q3 which reflects the lifting of the Nixon-era wage and price controls. The 1986Q2 outlier in the CPI reflects the decline in energy prices that occurred during 1986. The outlier search and intervention model for AAA yields indicates an innovative outlier in 1979Q4 and additive outliers in 1980Q1 and 1982Q4. These outliers reflect the increased volatility and magnitudes of interest rates during this period. Steel strikes in 1952 and 1959 show up as outliers in fixed investment in 1952Q3 and industrial production in 1959Q3.

The growth rate of stock prices shows outliers in 1957Q4, 1974Q3,

1982Q4, and 1987Q4. Three of the four outliers occur during recessions and two (1974Q3 and 1982Q4) coincide with outliers in the inflation rate (GNP deflator). The 1987Q4 outlier reflects the stock market crash of October 1987. Friedman and Laibson (1989) using different techniques decompose stock market returns into ordinary and extraordinary components. They also find four "large shocks" occurring in 1962Q2, 1970Q2, 1974Q3 and 1987Q4. While the standardized residuals from the our "large shock" intervention model are relatively large in 1962Q2 and 1970Q2 (-2.68 and -2.23 respectively), they are not large enough to classified as outliers.

Finally, the outlier results suggest a puzzle with respect to the relationship between consumption and real GNP. Outliers in the consumption series explain nearly 40 percent of the variation in the growth rate of consumption. In addition, five of the six consumption outliers are negative. Of the negative outliers, three are associated with recessions. The preponderance of negative outliers for consumption is consistent with the finding of Dynarski and Sheffrin (1986), who found substantial negative skewness in consumption. Because controlling for these outliers appears to eliminate the skewness in the consumption residuals, our analysis suggests that the source of the consumption asymmetry is primarily due to large negative responses of consumption during recessions and during the Korean War.¹⁷ The 1950Q3 outlier in consumption coincides with the outbreak of the Korean War which began in June 1950. The boom in consumption in 1950Q3 was followed by a negative outlier in 1950Q4. The consumption boom during 1950Q3 may have been caused by consumer purchases in anticipation of wartime shortages that were present during World War II. Consumers, having made large purchases initially (especially durables), may have cut back on

additional purchases, hence the negative outliers in 1950Q4 and 1951Q2.

V. Outliers and Nonlinearity

It is clear that linear, univariate models with Gaussian innovations do not adequately characterize many commonly used macroeconomic time series. On the other hand, the large shock/outlier models adequately characterizes the data; the residuals of the large shock models exhibit very little excess kurtosis and skewness and, furthermore, the outliers often correspond to identifiable economic events. However, it may be possible that evidence of outliers may reflect the presence of deeper nonlinearities. Indeed, as we have suggested above, a clustering of outliers across time would be consistent with ARCH variance processes. Chaotic or other nonlinear behavior in time series such as that examined by Brock and Sayers (1988) may produce outlier type behavior in simple linear models. To examine these possibilities, we test for GARCH and general nonlinearity in the residuals of the basic ARI model and in the residuals of the outlier adjusted model. If GARCH is indicated, we calculate the standardized residuals from the GARCH model and examine whether there remains excess kurtosis and skewness as well as any residual nonlinearity.

Table 5 summarizes the tests for GARCH and general nonlinearity. A GARCH(p,q) model for the variance was specified based on the autocorrelations and partial autocorrelations of the squared residuals. The ARI-GARCH models were then estimated via maximum likelihood and tested by a Likelihood Ratio test.¹⁸ To determine whether GARCH type processes were present in the residuals of the outlier/intervention model, we examined the Ljung-Box Q statistic for general autocorrelation in the squared residuals and conducted

Lagrange Multiplier test as in Engle (1982) for particular GARCH(0,q) models.¹⁹ To test for general nonlinearity, we use the Brock, Dechert, Scheinkman (1987) (BDS) statistic. Because the BDS tests were not always conclusive, in our summary of the tests we categorize the test for nonlinearity as either rejecting linearity (Y), failing to reject linearity (N), or providing mixed results (M).

The residuals from the basic linear autoregressive model for most of the series show some evidence of either GARCH or nonlinearity. After fitting the outlier model, the evidence of GARCH and nonlinearity in many of these series disappears.²⁰ The primary exceptions are the unemployment rate, the CPI, and AAA bond yields. For the CPI, the nonlinearity is primarily due to relatively large innovations during the Korean War--again, symptomatic of GARCH. For M2, fitting the outlier model actually increased the significance of the GARCH test.

Of course, as we suggested above, clusters of outliers may reflect GARCH behavior and vice versa. For example, the fact that most of the temporary outliers for inflation occur in the late 1940s and early 1950s or that M1 outliers occurred during 1979Q3 - 1982Q4 may be evidence of GARCH. Furthermore, GARCH was equally adept at explaining the nonlinearity in many of the series. However, even after estimating GARCH models, almost all the series still showed significant excess kurtosis and/or skewness. GARCH models are in some sense a parsimonious characterization of the large shock hypothesis, but the presence of significant excess kurtosis and skewness suggests that the random outlier model for most of these series may still be a better characterization of the data than GARCH.²¹ Indeed, for many series it is as if a large shock initiates the GARCH process.

The fact that controlling for outliers lessens the evidence of nonlinearities raises several issues. It could be possible that the world is indeed linear but subject to infrequent, large shocks. This causes possible misspecifications in linear, Gaussian models, and, consequently, the residuals from these models show evidence of nonlinearity. On the other hand, it is possible that there are indeed nonlinearities that the linear outlier model captures as outliers. There are numerous other nonlinear models which we did not examine, such as Hamilton's (1989) Markov regime switching model and threshold autoregression models (Tong (1983)), that may capture the excess skewness and kurtosis as well as the nonlinearities in the data. Indeed, because the timing of many of the outliers coincided with identifiable economic events such as recessions, this suggests that business cycles could be modeled as nonlinear processes.

V. Concluding Remarks

We have shown that within the context of linear autoregressive models there is significant evidence that large, infrequent shocks are an important source of variability in many macroeconomic time series. In addition, the estimated outliers account for nearly all of the excess kurtosis and skewness present in the data and are capable of explaining much of the nonlinearity present in the data. Furthermore, several patterns emerge from the outlier analysis: many of the identified outliers seem to be associated with turning points in the business cycle; outliers tend to be clustered both within series and across series; and there appears to be a dichotomy between real and nominal series with respect to large shocks. Because so many of the outliers are associated with recessions, our analysis implies that linear,

Gaussian models of the business cycle are not appropriate.

Our analysis also suggests several extensions. The fact that many series have outliers at the same date suggests that a multivariate outlier analysis may prove useful in shedding additional light on the source of these outliers. Additionally, the link present between outliers in linear models and evidence of nonlinearity in some of the series warrants further investigation.

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Notes

1. There is a growing literature on nonlinear models for macroeconomic time series. The following studies found evidence of nonlinearity: Hinich and Patterson (1985) and Scheinkman and LeBaron (1989a) for stock price data, Hsieh (1989) for exchange rate data, and Brock and Sayers (1988) for industrial production and employment series. Examinations of asymmetry include Neftci (1984) (see, however, Sichel (1989)), DeLong and Summers (1986), and Falk (1986). The Markov regime switching model of Hamilton (1989) is another example of a nonlinear time series model. There is also the large literature on autoregressive conditional heteroscedasticity (ARCH) and numerous extensions of ARCH. Prominent papers in this literature include Engle (1982), Weiss (1984), Bollerslev (1986), and more recently Nelson (1991).
2. See, for example, the stylized fact discussion at the beginning of the Blanchard and Fischer (1988).
3. Rappoport and Reichlin (1989) and Balke and Fomby (1991) discuss shifting trends in terms of infrequent permanent shocks. See Perron (1989), Chen and Tiao (1990), and Balke and Fomby (1991) for discussions of the effect of level shifts on Dickey-Fuller tests, ARIMA models, and measures of persistence.
4. More general Markov models, such as a two-state Markov model with persistence (for example, Hamilton (1989)), will also imply excess kurtosis.
5. When the conditional mean of ω_t is nonzero [$(\omega_t|I_t=1 \sim N(\mu, \sigma_\omega^2))$], the unconditional third central moment is $E(\omega_t I_t - \lambda\mu)^3 = 3\lambda(1-\lambda)\sigma_\omega^2 + (\lambda - 3\lambda^2 + 2\lambda^3)\mu^3$.
6. See Balke (1991) for a Monte Carlo examination of the sensitivity of outlier search methods to initial ARMA specifications.
7. In a previous version of this paper we used a critical value of two. For most of the series, the final intervention model was very similar to those presented below.
8. The unemployment and employment data span 1948Q1 to 1990Q2.
9. We used seasonally adjusted data for industrial production, productivity, employment, unemployment, CPI, compensation, and the money measures because these series are most often examined in a seasonally adjusted form. The use of seasonally adjusted data probably makes it even more difficult to uncover outliers.
10. Tables that detail the basic ARI model and outlier/intervention model results are available upon request.

11. Under the null hypothesis of normality or zero excess kurtosis ($b_2(\epsilon_t) = 3$), the statistic $T^{1/2}(b_2 - 3)/(24)^{1/2} \sim AN(0,1)$, where b_2 is the measure of kurtosis. For skewness, the statistic $T^{1/2}(m_3/(m_2)^{3/2})/(6)^{1/2} \sim AN(0,1)$, where m_3 is the sample third central moment and m_2 is the sample variance.
12. The proportion of the total variance explained by outliers is calculated by comparing the variance of the raw series (in most cases this is the growth rate of the series) and variance of the outlier component. In terms of the model described by equation (1), this proportion is:

$$\text{Var}[(B_\omega(L)\omega_t I_t] / \text{Var}[Y_t].$$
13. The specifications of the final intervention models for the other series are presented in the supplementary tables.
14. In fact, aggregation, both over time or across series, will tend to obscure infrequent shocks or outliers.
15. While the final specification of compensation did not contain a level shift in 1967/68, as Table 2 suggested, there was some evidence of a level shift at this time--its t-stat was 2.62 which failed to meet the prespecified critical value.
16. King, Plosser, Stock, and Watson (1991) suggest a similar dichotomy in the long-run trend components. Here the dichotomy shows up with respect to large shocks, in particular, the level shift shocks.
17. If consumption is disaggregated into consumption of durables and consumption of nondurables and services, most of this excess asymmetry and most of the outliers are coming from consumer durables.
18. See the supplementary tables for presentation of the tests for GARCH and the maximum likelihood estimates of the ARI-GARCH model.
19. Since we were only interested in establishing the presence of GARCH type behavior in the residuals of the outlier/intervention model, we did not estimate the full intervention-GARCH model by maximum likelihood.
20. Scheinkman and LeBaron (1989b) also found that controlling for unusual periods reduced the evidence of nonlinearity in real GNP. In their examination of nonlinearity in real GNP data over the period 1872-1986, they included dummy variables to account for the Great Depression (1930-39) and World War II (1940-45).
21. While we do not conduct a formal search for outliers in the ARI/ARCH model, casual inspection of the standardized residuals from the ARCH model suggest that significant outliers remain.

Table 1

Variable	Basic ARI Model				Outlier Model				Variance Explained ^e
	Model	SEE	Kurt. ^a	Skew. ^b	Present	SEE	Kurt. ^c	Skew. ^d	
Real GNP	(2,1)	0.0100	3.9*	0.03	yes	0.0093	2.8	-0.11	14%
Consumption	(2,1)	0.0078	5.6*	-0.59*	yes	0.0060	2.6	-0.03	38%
Fixed Investment	(1,1)	0.0253	4.7*	-0.16	yes	0.0226	3.0	0.13	13%
Indust. Prod.	(4,1)	0.0188	5.0*	-0.49*	yes	0.0167	3.7	0.14	19%
Productivity (Man)	(4,1)	0.0104	4.1*	-0.55*	yes	0.0094	3.2	-0.05	19%
Unemployment rate	(2,1)	0.3330	3.7	0.44*	yes	0.3242	3.7	0.40*	6%
Employment	(1,1)	0.0050	5.1*	-0.44*	yes	0.0044	3.3	-0.23	21%
GNP deflator	(2,1)	0.0056	5.2*	0.08	yes	0.0038	2.9	-0.28	78%
CPI	(4,1)	0.0052	8.8*	-0.83*	yes	0.0038	3.3	0.11	61%
Compensation per hour (manuf.)	(3,1)	0.0069	5.9*	1.06*	yes	0.0052	3.1	0.28	55%
AAA bond yields	(2,1)	0.0394	4.2*	-0.14	yes	0.0353	3.1	0.03	22%
Stock prices	(1,1)	0.0560	5.9*	-0.50*	yes	0.0476	3.2	0.29	26%
Money Base ^f	(3,1)	0.0048	3.0	-0.25	yes	0.0047	3.2	-0.17	21%
M1 ^f	(1,1)	0.0078	10.8*	0.85*	yes	0.0050	3.4	0.02	41%
M2 ^f	(1,1)	0.0058	6.1*	0.92*	yes	0.0046	2.9	0.18	40%

* Significant at the 5 percent level.

^a Kurtosis in the ARI model residuals.

^b Skewness in the ARI model residuals.

^c Kurtosis in the residuals after adjusting for outliers.

^d Skewness in the residuals after adjusting for outliers.

^e Proportion of variance (in percent) attributable to outliers.

^f Includes a linear time trend.

Table 2

Example of Stepwise Reduction of Intervention Model: Nominal Compensation

ARMA specification before outlier search: ARMA(3,0)

Outliers and level shifts identified by an ARMA(3,0) search:

AO1948Q1, AO1949Q4, IO1950Q4, AO1952Q4, AO1982Q1, AO1990Q2

Outliers and level shifts identified by an ARMA(0,0) search:

AO1948Q1, AO1949Q4, AO1950Q4, LS1953Q1, IO1956Q2, LS1968Q1, LS1973Q1,
LS1982Q3, LS1982Q4, AO1990Q2

Intervention variables included in the initial intervention model:

AO1948Q1, AO1949Q4, AO1950Q4, IO1950Q4, AO1952Q4, LS1953Q1, IO1956Q2,
LS1968Q1, LS1973Q1, AO1982Q1, LS1982Q3, LS1982Q4, AO1990Q2

Intervention variables eliminated:

Iteration	Variable	t-statistic
1	AO1950Q4	0.71
2	AO1990Q2	0.77
3	LS1982Q3	-1.46
4	LS1953Q1	-1.47
5	LS1968Q1	2.62
6	AO1982Q1	2.68

Final intervention model:

$$\begin{aligned} \Delta \text{comp}_t = & 0.006 + 0.350 \Delta \text{comp}_{t-1} + 0.031 \Delta \text{comp}_{t-2} + 0.094 \Delta \text{comp}_{t-3} \\ & (4.97) \quad (4.99) \quad (0.44) \quad (1.37) \\ & + 0.025 \text{AO1948Q1}_t - 0.020 \text{AO1949Q4}_t + 0.034 \text{IO1950Q4}_t \\ & (4.81) \quad (-4.00) \quad (6.31) \\ & + 0.016 \text{AO1952Q4}_t + 0.017 \text{IO1956Q2}_t + 0.011 \text{LS1973Q1}_t \\ & (3.18) \quad (3.24) \quad (6.33) \\ & - 0.014 \text{LS1982Q4}_t \\ & (-6.50) \end{aligned}$$

$$R^2 = 0.635 \quad \text{SEE} = 0.0052$$

t-statistics are in parenthesis.

Table 3

Outlier Dates and Magnitudes by Variable

Variable	Type	Date	Size	S.E.	Events
GNP Growth	IO	1950Q1	0.039	0.009	1st quarter of recovery trough
	AO	1980Q2	-0.031	0.009	
Consumption	IO	1950Q3	0.027	0.006	Korean War
	IO	1950Q4	-0.035	0.007	Korean War
	AO	1951Q2	-0.022	0.006	Korean War
	AO	1958Q1	-0.019	0.006	trough
	AO	1974Q4	-0.023	0.006	recession
	AO	1980Q2	-0.028	0.006	trough
Fixed Investment	AO	1952Q3	-0.088	0.019	effect of steel strike trough
	AO	1980Q2	-0.102	0.019	
Industrial Production	IO	1959Q3	-0.059	0.017	steel strike
	AO	1960Q1	0.046	0.014	peak
	IO	1975Q1	-0.079	0.017	trough
Labor Productivity (Manufacturing)	IO	1957Q4	-0.033	0.009	1st quarter of recession steel strike 1st quarter of recession
	IO	1959Q2	-0.040	0.010	
	AO	1974Q1	-0.032	0.009	
Unemployment Rate	IO	1975Q1	1.045	0.330	trough
Employment Growth	IO	1950Q2	0.016	0.004	2nd quarter of recovery Korean War recession trough
	AO	1953Q1	0.019	0.004	
	IO	1958Q1	-0.014	0.004	
	AO	1975Q1	-0.013	0.004	
Inflation (GNP Deflator)	AO	1947Q4	0.015	0.004	Post-World War II inflation
	LS	1948Q4	-0.017	0.003	end of post-WWII inflation/expansion
	IO	1950Q3	0.021	0.004	Korean War
	AO	1951Q1	0.017	0.004	Korean War
	AO	1952Q4	0.012	0.004	Korean War
	AO	1954Q1	0.014	0.004	Korean War disarmament
	LS	1967Q4	0.009	0.002	Vietnam War build up
	LS	1973Q2	0.006	0.002	Nixon expansion/food price shocks
	IO	1974Q3	0.015	0.004	end of wage and price controls
	LS	1982Q4	-0.010	0.002	Volcker disinflation, trough

Table 3 continued

Inflation (CPI)	IO	1948Q4	-0.020	0.004	end of post-WWII inflation
	AO	1951Q1	0.023	0.003	Korean War
	AO	1951Q3	-0.017	0.003	Korean War, price controls
	LS	1967Q3	0.007	0.002	Vietnam War
	LS	1973Q1	0.009	0.002	Nixon expansion/food price shocks
	LS	1982Q4	-0.011	0.002	Volcker disinflation, trough
	AO	1986Q2	-0.011	0.002	reduction in energy prices
Compensation per hour (Manufacturing)	AO	1948Q1	0.025	0.005	Post-WWII inflation
	AO	1949Q4	-0.020	0.005	trough
	IO	1950Q4	0.034	0.005	Korean War
	AO	1952Q4	0.016	0.005	Korean War
	IO	1956Q2	0.017	0.005	
	LS	1973Q1	0.011	0.002	Nixon expansion/food price shocks
	LS	1982Q4	-0.014	0.002	Volcker disinflation, trough
AAA Yields Growth	IO	1979Q4	0.125	0.036	oil shock, peak
	AO	1980Q1	0.101	0.037	oil shock, 1st quarter of recession
	AO	1982Q4	-0.141	0.037	Volcker disinflation, trough
SP500 Index Growth	AO	1957Q4	-0.138	0.045	1st quarter of recession
	IO	1974Q3	-0.176	0.048	end of wage and price controls
	IO	1982Q4	0.170	0.048	Volcker disinflation, trough
	AO	1987Q4	-0.261	0.045	October 1987 stock market crash
Money Base Growth	LS	1963Q1	0.006	0.002	
	LS	1987Q3	-0.008	0.003	
M1 Growth	AO	1959Q1	-0.016	0.004	
	IO	1959Q4	-0.016	0.005	quarter before peak
	AO	1979Q3	0.019	0.004	quarter before peak
	IO	1980Q2	-0.029	0.005	"Monetarist experiment", trough
	IO	1980Q3	0.048	0.005	"Monetarist experiment", 1st quarter of recovery
	AO	1981Q1	-0.016	0.004	"Monetarist experiment"
	AO	1982Q2	-0.014	0.004	"Monetarist experiment", recession
	IO	1982Q4	0.019	0.005	Volcker disinflation, trough
	LS	1985Q1	0.012	0.003	
	LS	1987Q3	-0.022	0.003	
	AO	1989Q2	-0.015	0.005	
M2 Growth	AO	1958Q2	0.014	0.005	1st quarter of recovery
	IO	1975Q2	0.017	0.005	1st quarter of recovery
	AO	1980Q3	0.017	0.004	1st quarter of recovery
	AO	1983Q1	0.025	0.004	1st quarter of recovery
	LS	1985Q2	-0.014	0.003	

Table 4

Outlier Dates in Chronological Order

Date	Variable	Events
1947Q4	GNP deflator	post-WWII expansion
1948Q1	compensation	post-WWII expansion
1948Q4	GNP deflator(-), CPI(-)	recession, 1st quarter
1949Q4	compensation(-)	recession, trough
1950Q1	real GNP	expansion, 1st quarter
1950Q2	employment	expansion, 2nd quarter
1950Q3	consumption, GNP deflator	Korean War
1950Q4	consumption(-), compensation	Korean War
1951Q1	GNP deflator, CPI	Korean War
1951Q2	consumption(-)	Korean War
1951Q3	CPI(-)	Korean War, price controls
1952Q3	fixed investment(-)	Korean War, steel strike
1952Q4	GNP deflator, compensation	Korean War
1953Q1	employment	Korean War, quarter before peak
1954Q1	GNP deflator	Korean War disarmament
1956Q2	compensation	
1957Q4	stock prices(-), productivity(-)	recession
1958Q1	consumption(-), employment(-)	recession, trough
1958Q2	M2	1st quarter of recovery
1959Q1	M1(-)	
1959Q2	productivity(-)	steel strike
1959Q3	industrial production(-)	steel strike
1959Q4	M1(-)	
1960Q1	industrial production	peak
1963Q1	money base	
1967Q3	CPI	Vietnam War expansion
1967Q4	GNP deflator	Vietnam War expansion
1973Q1	CPI, compensation	Nixon expansion
1973Q2	GNP deflator	Nixon expansion
1974Q1	productivity(-)	recession, 1st quarter
1974Q3	GNP deflator, stock prices(-)	recession, end of price controls
1974Q4	consumption(-)	recession
1975Q1	employment(-), unemployment(-), industrial production(-)	recession, trough
1975Q2	M2	1st quarter of recovery
1979Q3	M1	
1979Q4	AAA bond yields	peak
1980Q1	AAA bond yields	recession
1980Q2	real GNP(-), consumption(-), fixed investment(-), M1(-)	recession, trough
1980Q3	M1, M2	1st quarter of recovery
1981Q1	M1(-)	
1982Q2	M1(-)	recession

Table 4 continued

1982Q4	GNP deflator(-), CPI(-), compensation(-), stock prices, M1, AAA bond yields(-)	Volcker disinflation, recession, trough
1983Q1	M2	1st quarter of recovery
1985Q1	M1	
1985Q2	M2(-)	
1986Q2	CPI(-)	reduction in energy prices
1987Q3	money base(-), M1(-)	
1987Q4	stock prices(-)	October crash
1989Q2	M1(-)	

Note: (-) indicates that the outlier or level shift was negative.

Table 5

Summary of Tests for ARCH and Nonlinearity

Variable	Basic ARI Model		After GARCH			Outlier Model	
	GARCH ^a	Nonlin ^b	Kurt ^c	Skew ^d	Nonlin ^e	GARCH ^f	Nonlin ^g
Real GNP	none	N	-	-	-	none	N
Consumption	GARCH(1,1)	N	Y	N	M	none	N
Fixed Investment	GARCH(0,1)	M	Y	Y	N	none	N
Indust. Prod.	GARCH(0,1)	Y	Y	Y	N	none	N
Productivity (Man)	GARCH(1,1)	Y	Y	Y	N	none	N
Unemployment rate	GARCH(1,3)	Y	Y	Y	M	Yes	Y
Employment	GARCH(0,1)	M	N	N	M	none	M
GNP deflator	GARCH(0,4)	Y	Y	N	N	none	N
CPI	GARCH(1,1)	Y	Y	Y	N	Yes	Y
Compensation per hour (manuf.)	GARCH(0,3)	Y	N	Y	N	none	N
AAA bond yields	GARCH(0,2)	Y	Y	N	N	Yes	Y
Stock prices	none	N	-	-	-	none	N
Money Base	none	N	-	-	-	none	N
M1	GARCH(1,1)	N	Y	N	M	Yes	N
M2	none	Y	-	-	-	Yes	N

Y - strong evidence (significant at 5 percent level).

N - weak evidence.

M - mixed evidence.

^a Evidence of GARCH at the 5 percent level in the ARI model residuals.

^b Evidence of nonlinearity in the ARI model residuals using BDS statistics.

^c Excess kurtosis in the standardized residuals in GARCH model (5 percent level).

^d Excess skewness in the standardized residuals in GARCH model (5 percent level).

^e Evidence of nonlinearity in the standardized residuals in GARCH model using BDS statistics.

^f Evidence of GARCH at the 5 percent level in the outlier model residuals.

^g Evidence of nonlinearity in the outlier model residuals using BDS statistics.

Supplementary Tables for
Large Shocks, Small Shocks, and Economic Fluctuations:
Outliers in Macroeconomic Time Series

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General Notes:

We employ two tests for GARCH. The first is the Lagrange Multiplier (LM) test for GARCH(0,q) versus the null of homoscedasticity. This is the LM test described by Engle (1982). The test statistic is given by $T \cdot R^2$ where the R^2 term is from squared residuals regression and T is the number of observations. This test is asymptotically distributed χ^2 with q degrees of freedom. We also use the Likelihood Ratio (LR) test for GARCH(p,q) versus the null of homoscedasticity. We use the LR test for testing for GARCH in the residuals of the basic ARI model since we are also interested in estimating via maximum likelihood a particular GARCH model. We use the LM test to test for GARCH(0,q) in the intervention model since all we wish to do is determine whether ARCH type behavior is still present the residuals of the intervention model.

The BDS statistics are calculated using the residuals of the Basic and Outlier models and the standardized residuals of the ARCH model. Under the null of linearity the asymptotic distribution of the BDS statistics is $N(0,1)$. The first row corresponds to the BDS statistics with "e" set equal to one standard deviation of the residuals. The second row corresponds to an "e" set equal to 0.5 standard deviation.

Table A1
ARI and Intervention Models for real GNP

Basic ARI Model

$$\Delta \text{gnp}_t = 0.004 + 0.323 \Delta \text{gnp}_{t-1} + 0.132 \Delta \text{gnp}_{t-2}$$

(0.001) (0.077) (0.077)

SEE = 0.0100, kurtosis = 3.910, skewness = 0.028

First ten autocorrelations of squared residuals:

0.08 0.11 0.04 0.10 -0.05 -0.00 -0.01 0.01 0.01 -0.04

Ljung-Box Q for squared residuals: Q(12) = 6.273 (p = 0.9017)

LR Test for GARCH(0,1) = 1.888 (p = 0.169)

LR Test for GARCH(1,1) = 5.798 (p = 0.0551)

Test for nonlinearity in the residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	0.96	1.82	2.33
BDS(e=0.5)	0.46	0.64	0.03

Outlier/Intervention Model

$$\Delta \text{gnp}_t = 0.004 + 0.377 \Delta \text{gnp}_{t-1} + 0.113 \Delta \text{gnp}_{t-2}$$

(0.001) (0.074) (0.074)

$$+ 0.039 \text{IO1950Q1}_t - 0.030 \text{AO1980Q2}_t$$

(0.009) (0.009)

SEE = 0.0093, kurtosis = 2.801, skewness = -0.111

First ten autocorrelations of squared residuals:

0.09 0.08 0.16 0.02 -0.02 -0.00 -0.08 0.04 0.10 0.09

Ljung-Box Q for squared residuals: Q(12) = 13.119 (p = 0.3605)

LM Test for GARCH(0,3) = 6.011 (p = 0.1111)

Test for nonlinearity in the residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	0.44	1.01	1.51
BDS(e=0.5)	-0.47	0.31	-0.19

Notes: Data are in logarithms. Sample period is 1947Q4-1990Q2.

$\text{IOdate}_t = 1$ if $t = \text{date}$, $\text{AOdate}_t = (1 - 0.377L - 0.113L^2)\text{IOdate}_t$
0 otherwise

Table A2
ARI and Intervention Models for Consumption

Basic ARI Model

$$\Delta c_t = 0.006 + 0.047 \Delta c_{t-1} + 0.218 \Delta c_{t-2}$$

(0.001) (0.075) (0.075)

SEE = 0.0078, kurtosis = 5.635, skewness = -0.585

First ten autocorrelations of squared residuals:

0.23 0.17 0.12 -0.01 -0.02 0.01 -0.06 0.02 -0.02 -0.04

Ljung-Box Q for squared residuals: Q(12) = 18.483 (p = 0.1019)

LR Test for GARCH(0,1) = 8.671 (p = 0.0032)

LR Test for GARCH(1,1) = 13.663 (p = 0.0011)

Test for nonlinearity in the residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-0.55	0.05	0.72
BDS(e=0.5)	-1.07	-0.61	-1.16

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta c_t = 0.005 + 0.118 \Delta c_{t-1} + 0.224 \Delta c_{t-2} + \epsilon_t$$

(0.001) (0.102) (0.088)

$$\sigma_t^2 = 0.0002 + 0.490 \sigma_{t-1}^2 + 0.190 \epsilon_{t-1}^2$$

(0.0001) (0.212) (0.080)

Excess kurtosis and skewness in the standardized residuals:

kurtosis = 4.173, skewness = -0.335

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-1.87	-1.82	-1.39
BDS(e=0.5)	-2.30	-1.65	-1.47

Outlier/Intervention Model

$$\Delta c_t = 0.005 + 0.144 \Delta c_{t-1} + 0.238 \Delta c_{t-2} + 0.027 I01950Q3_t$$

(0.001) (0.073) (0.071) (0.006)

$$- 0.035 I01950Q4_t - 0.022 A01951Q2_t - 0.019 A01958Q1_t$$

(0.007) (0.006) (0.006)

$$- 0.023 A01974Q4_t - 0.028 A01980Q2_t$$

(0.006) (0.006)

SEE = 0.0060, kurtosis = 2.568, skewness = -0.033

Table A2 (continued)
Consumption

First ten autocorrelations of squared residuals:

-0.10 -0.00 0.12 -0.05 0.04 -0.14 -0.05 0.03 0.03 -0.01

Ljung-Box Q for squared residuals: $Q(12) = 10.057$ ($p = 0.6109$)

LM Test for GARCH(0,3) = 4.017 ($p = 0.2597$)

Test for nonlinearity in the residuals: BDS statistics

Embedding dimension	2	3	4
BDS($\epsilon=1.0$)	-1.71	-1.63	-1.55
BDS($\epsilon=0.5$)	-1.98	-2.02	-1.79

Notes: Data are in logarithms. Sample period is 1947Q4-1990Q2.

$I0date_t = 1$ if $t = \text{date}$
0 otherwise

$A0date_t = (1 - .144L - 0.238L^2) I0date_t$

Table A3
ARI and Intervention Models for Fixed Investment

Basis ARI Model

$$\Delta i_t = 0.004 + 0.471 \Delta i_{t-1}, \quad \text{SEE} = 0.0253$$

(0.002) (0.068) kurtosis = 4.709, skewness = -0.156

First ten autocorrelations of squared residuals:
0.16 0.00 -0.07 -0.09 -0.05 0.01 0.05 0.10 0.05 0.11

Ljung-Box Q for squared residuals: Q(12) = 13.767 (p = 0.3159)
LR Test for GARCH(0,1) = 4.9271 (p = 0.0264)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	1.96	1.71	1.17
BDS(e=0.5)	1.98	1.32	0.40

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta i_t = 0.004 + 0.493 \Delta i_{t-1} + \epsilon_t, \quad \sigma_t^2 = 0.0005 + 0.137 \epsilon_{t-1}^2$$

(0.002) (0.071) (0.0001) (0.009)

Kurtosis and skewness in the standardized residuals:
kurtosis = 4.793, skewness = -0.379

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-0.26	-0.26	-0.60
BDS(e=0.5)	0.35	0.31	-0.01

Outlier/Intervention Model

$$\Delta i_t = 0.004 + 0.547 \Delta i_{t-1} - 0.088 A01952Q3_t - 0.102 A01980Q2_t$$

(0.002) (0.064) (0.019) (0.019)

SEE = 0.0226, kurtosis = 3.016, skewness = 0.133

First ten autocorrelations of squared residuals:
0.05 0.05 0.00 -0.03 0.02 0.04 0.05 0.01 0.04 0.12

Ljung-Box Q for squared residuals: Q(12) = 5.104 (p = 0.954)
LM Test for GARCH(0,1) = 0.367 (p = 0.5449)

BDS statistics for nonlinearity in the residuals

Embedding dimension-	2	3	4
BDS(e=1.0)	0.88	0.76	0.30
BDS(e=0.5)	0.45	0.73	0.11

Notes: Data are in logarithms. Sample period is 1947Q3-1990Q2.
IOdate_t = 1 if t = date, A0date_t = (1 - 0.547L) IOdate_t
0 otherwise

Table A4
ARI and Intervention Models for Industrial Production

Basic ARI Model

$$\Delta ip_t = 0.008 + 0.604 \Delta ip_{t-1} - 0.317 \Delta ip_{t-2} + 0.258 \Delta ip_{t-3} - 0.356 \Delta ip_{t-4}$$

(0.002) (0.073) (0.084) (0.084) (0.073)

SEE = 0.0188, kurtosis = 5.006, skewness = -0.491

First ten autocorrelations of squared residuals:
0.21 0.08 0.07 0.15 0.06 0.00 0.11 0.02 0.08 -0.05

Ljung-Box Q for squared residuals: Q(12) = 18.119 (p = 0.1121)
LR Test for GARCH(0,1) = 13.080 (p = 0.0003)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	3.21	3.58	3.98
BDS(e=0.5)	2.79	2.54	3.73

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta ip_t = 0.009 + 0.561 \Delta ip_{t-1} - 0.264 \Delta ip_{t-2} + 0.168 \Delta ip_{t-3} - 0.316 \Delta ip_{t-4}$$

(0.002) (0.087) (0.065) (0.065) (0.068)

$$\sigma_t^2 = 0.00024 + 0.316 \epsilon_{t-1}^2$$

(0.00004) (0.126)

Kurtosis and skewness in the standardized residuals:
kurtosis = 4.256, skewness = -0.384

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-0.61	-0.22	-0.46
BDS(e=0.5)	-0.81	-0.21	1.15

Outlier/Intervention Model

$$\Delta ip_t = 0.008 + 0.590 \Delta ip_{t-1} - 0.264 \Delta ip_{t-2} + 0.257 \Delta ip_{t-3} - 0.372 \Delta ip_{t-4}$$

(0.002) (0.068) (0.081) (0.082) (0.069)

$$- 0.059 IO1959Q3_t + 0.046 AO1960Q1_t - 0.079 IO1975Q1_t$$

(0.017) (0.014) (0.017)

SEE = 0.0167, kurtosis = 3.744, skewness = 0.140

First ten autocorrelations of squared residuals:
0.05 0.06 0.16 0.09 0.11 -0.02 0.04 0.22 -0.01 -0.05

Ljung-Box Q for squared residuals: Q(12) = 22.288 (p = 0.0344)
LM Test for GARCH(0,3) = 4.964 (p = 0.1744)

Table A4 (continued)
Industrial Production

BDS statistics for nonlinearity in the residuals

Embedding dimension-	2	3	4
BDS($\epsilon=1.0$)	0.88	0.76	0.30
BDS($\epsilon=0.5$)	0.45	0.73	0.11

Notes: Data are in logarithms. Sample period is 1948Q2-1990Q2.

$$I\text{Odate}_t = \begin{cases} 1 & \text{if } t = \text{date} \\ 0 & \text{otherwise} \end{cases}$$

$$A\text{Odate}_t = (1 - .590L + 0.264L^2 - 0.257L^3 + 0.372L^4) I\text{Odate}_t$$

Table A5
ARI and Intervention Models for Labor Productivity
(Output per hour - Manufacturing)

Basic ARI Model

$$\begin{aligned} \Delta \text{prod}_t = & 0.007 + 0.300 \Delta \text{prod}_{t-1} - 0.233 \Delta \text{prod}_{t-2} + 0.167 \Delta \text{prod}_{t-3} \\ & (0.001) \quad (0.076) \quad (0.079) \quad (0.079) \\ & - 0.208 \Delta \text{prod}_{t-4} \\ & (0.076) \end{aligned}$$

SEE = 0.0104, kurtosis = 4.144, skewness = -0.546

First ten autocorrelations of squared residuals:
0.09 0.04 0.09 0.13 0.07 0.03 0.26 -0.06 -0.00 0.01

Ljung-Box Q for squared residuals: Q(12) = 19.680 (p = 0.0734)

LR Test for GARCH(1,1) = 9.353 (p = 0.0093)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	1.12	2.32	2.86
BDS(e=0.5)	2.22	3.10	3.15

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\begin{aligned} \Delta \text{prod}_t = & 0.006 + 0.291 \Delta \text{prod}_{t-1} - 0.151 \Delta \text{prod}_{t-2} + 0.144 \Delta \text{prod}_{t-3} \\ & (0.001) \quad (0.100) \quad (0.098) \quad (0.095) \\ & - 0.177 \Delta \text{prod}_{t-4} + \epsilon_t \\ & (0.077) \end{aligned}$$

$$\begin{aligned} \sigma^2_t = & 0.000011 + 0.749 \sigma^2_{t-1} + 0.145 \epsilon^2_{t-1} \\ & (0.000009) \quad (0.126) \quad (0.084) \end{aligned}$$

Kurtosis and skewness in the standardized residuals:
kurtosis = 4.066, skewness = -0.511

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-1.27	-1.02	-1.14
BDS(e=0.5)	-0.81	-0.21	1.15

Outlier/Intervention Model

$$\begin{aligned} \Delta \text{prod}_t = & 0.007 + 0.312 \Delta \text{prod}_{t-1} - 0.240 \Delta \text{prod}_{t-2} + 0.187 \Delta \text{prod}_{t-3} \\ & (0.001) \quad (0.072) \quad (0.073) \quad (0.073) \\ & - 0.167 \Delta \text{prod}_{t-4} - 0.033 \text{IO1957Q4}_t - 0.040 \text{IO1959Q2}_t \\ & (0.072) \quad (0.009) \quad (0.010) \\ & - 0.032 \text{AO1974Q1}_t \\ & (0.009) \end{aligned}$$

Table 5 (continued)
Labor Productivity

SEE = 0.0094, kurtosis = 3.155, skewness = -0.052

First ten autocorrelations of squared residuals:

-0.01 0.15 -0.04 0.04 -0.06 0.00 0.00 0.03 0.06 0.00

Ljung-Box Q for squared residuals: $Q(12) = 7.972$ ($p = 0.7873$)

LM Test for GARCH(0,2) = 3.668 ($p = 0.1598$)

BDS statistics for nonlinearity in the residuals

Embedding dimension-	2	3	4
BDS($\epsilon=1.0$)	0.88	0.76	0.30
BDS($\epsilon=0.5$)	0.45	0.73	0.11

Notes: Data are in logarithms. Sample period is 1948Q2-1990Q2.

$IOdate_t = 1$ if $t = \text{date}$
0 otherwise

$AOdate_t = (1 - 0.312L + 0.240L^2 - 0.187L^3 + 0.167L^4) IOdate_t$

Table A6
ARI and Intervention Models for Unemployment Rate

Basic ARI Model

$$\Delta un_t = 0.040 + 0.788 \Delta un_{t-1} - 0.247 \Delta un_{t-2}$$

(0.026) (0.076) (0.076)

SEE = 0.3330, kurtosis = 3.708, skewness = 0.440

First ten autocorrelations of squared residuals:
0.23 0.11 0.20 0.15 0.08 0.09 0.02 0.07 -0.03 -0.10

Ljung-Box Q for squared residuals: Q(12) = 27.479 (p = 0.0066)
LR Test for GARCH(1,3) = 36.053 (p = 0.0000)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	3.77	5.80	7.96
BDS(e=0.5)	4.94	7.78	12.00

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta un_t = -0.017 + 0.757 \Delta un_{t-1} - 0.166 \Delta un_{t-2} + \epsilon_t$$

(0.024) (0.096) (0.092)

$$\sigma_t^2 = 0.013 + 0.434 \sigma_{t-1}^2 + 0.290 \epsilon_{t-1}^2 - 0.074 \epsilon_{t-2}^2 + 0.250 \epsilon_{t-3}^2$$

(0.007) (0.251) (0.127) (0.143) (0.193)

Kurtosis and skewness in the standardized residuals:
kurtosis = 4.033, skewness = 0.763

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	1.31	1.85	2.25
BDS(e=0.5)	0.37	2.07	5.00

Outlier/Intervention Model

$$\Delta un_t = -0.020 + 0.751 \Delta un_{t-1} - 0.238 \Delta un_{t-2} + 1.045 I01975Q1_t$$

(0.025) (0.075) (0.074) (0.330)

SEE = 0.324, kurtosis = 3.680, skewness = 0.403

First ten autocorrelations of squared residuals:
0.18 0.06 0.25 0.13 0.12 0.13 0.05 0.10 -0.00 -0.07

Ljung-Box Q for squared residuals: Q(12) = 28.250 (p = 0.0051)
LM Test for GARCH(0,4) = 15.626 (p = 0.0036)

Table A6 (continued)
Unemployment

BDS statistics for nonlinearity in the residuals

Embedding dimension-	2	3	4
BDS($\epsilon=1.0$)	2.72	4.77	7.01
BDS($\epsilon=0.5$)	4.03	7.30	11.65

Notes: Data are in levels. Sample period is 1948Q3-1990Q2.

$I Odate_t = 1$ if $t = \text{date}$
0 otherwise

$A Odate_t = (1 - 0.751L + 0.238L^2) I Odate_t$

Table A7
ARI and Intervention Models for Employment

Basic ARI Model

$$\Delta em_t = 0.002 + 0.498 \Delta em_{t-1}$$

(0.0005) (0.067)

SEE = 0.0050, kurtosis = 5.071, skewness = -0.437

First ten autocorrelations of squared residuals:
0.18 0.10 -0.04 0.08 -0.01 -0.05 -0.07 -0.01 0.04 0.06

Ljung-Box Q for squared residuals: Q(12) = 18.962 (p = 0.0894)
LR Test for GARCH(0,1) = 16.464 (p = 0.0000)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	2.07	3.09	3.35
BDS(e=0.5)	0.56	2.10	1.36

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta em_t = 0.002 + 0.624 \Delta em_{t-1} + \epsilon_t$$

(0.0005) (0.072)

$$\sigma_t^2 = 0.000055 + 0.431 \epsilon_{t-1}^2$$

(0.000002) (0.135)

Kurtosis and skewness in the standardized residuals:
kurtosis = 3.421, skewness = -0.147

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	0.15	1.27	1.40
BDS(e=0.5)	0.94	2.94	3.53

Outlier/Intervention Model

$$\Delta em_t = 0.0019 + 0.536 \Delta em_{t-1} + 0.016 IO1950Q2_t$$

(0.0004) (0.062) (0.004)

$$- 0.019 AO1953Q1_t - 0.014 IO1958Q1_t - 0.013 AO1975Q1_t$$

(0.004) (0.004) (0.004)

SEE = 0.0044, kurtosis = 3.250, skewness = -0.232

First ten autocorrelations of squared residuals:
0.03 0.11 -0.04 0.10 0.03 -0.03 0.02 -0.04 -0.03 0.01

Ljung-Box Q for squared residuals: Q(12) = 18.962 (p = 0.0894)
LM Test for GARCH(0,2) = 2.150 (p = 0.3414)

Table A7 (continued)
Employment

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS($\epsilon=1.0$)	2.07	3.09	3.35
BDS($\epsilon=0.5$)	0.56	2.10	1.36

Notes: Data are in logarithms. Sample period is 1947Q3-1990Q2.

$I0date_t = 1$ if $t = \text{date}$
0 otherwise

$A0date_t = (1 - 0.536L) I0date_t$

Table A8
ARI and Intervention Models for GNP Deflator

Basic ARI Model

$$\Delta p_t = 0.003 + 0.453 \Delta p_{t-1} + 0.298 \Delta p_{t-2}$$

(0.001) (0.074) (0.074)

SEE = 0.0056, kurtosis = 5.224, skewness = 0.078

First ten autocorrelations of squared residuals:
0.19 0.07 0.16 0.23 0.09 0.12 0.17 0.01 0.15 0.29

Ljung-Box Q for squared residuals: Q(12) = 76.165 (p = 0.0000)
LR Test for GARCH(0,4) = 23.168 (p = 0.0005)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	3.32	4.62	5.18
BDS(e=0.5)	3.29	3.76	4.70

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta p_t = 0.00088 + 0.522 \Delta p_{t-1} + 0.375 \Delta p_{t-2} + \epsilon_t$$

(0.00058) (0.077) (0.082)

$$\sigma_t^2 = 0.000010 + 0.077 \epsilon_{t-1}^2 + 0.199 \epsilon_{t-2}^2 + 0.150 \epsilon_{t-3}^2 + 0.265 \epsilon_{t-4}^2$$

(0.000003) (0.083) (0.131) (0.103) (0.106)

Kurtosis and skewness in the standardized residuals:
kurtosis = 4.299, skewness = 0.273

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	0.22	0.25	0.33
BDS(e=0.5)	-1.08	-0.74	-0.68

Outlier/Intervention Model

$$\Delta p_t = 0.011 + 0.284 \Delta p_{t-1} + 0.195 \Delta p_{t-2} + 0.015 A01947Q4_t$$

(0.002) (0.072) (0.073) (0.004)

$$- 0.017 LS1948Q4_t + 0.021 I01950Q3_t + 0.017 A01951Q1_t$$

(0.003) (0.004) (0.004)

$$+ 0.012 A01952Q4_t + 0.014 A01954Q1_t + 0.009 LS1967Q4_t$$

(0.004) (0.004) (0.002)

$$+ 0.006 LS1973Q2_t + 0.015 I01974Q3_t - 0.010 LS1982Q4_t$$

(0.002) (0.004) (0.002)

SEE = 0.0038, kurtosis = 2.855, skewness = -0.282

Table A8 (continued)
GNP Deflator

First ten autocorrelations of squared residuals:

0.04 -0.03 0.03 -0.09 -0.01 -0.09 -0.07 0.02 0.23 0.10

Ljung-Box Q for squared residuals: $Q(12) = 16.744$ ($p = 0.1595$)

LM Test for GARCH(0,4) = 2.129 ($p = 0.7120$)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS($\epsilon=1.0$)	0.51	-0.05	0.21
BDS($\epsilon=0.5$)	-0.20	0.05	-0.34

Notes: Data are in logarithms. Sample period is 1947Q4-1990Q2

$I_{Odate_t} = 1$ if $t = date$
0 otherwise

$A_{Odate_t} = (1 - .284L - 0.195L^2) I_{Odate_t}$

$L_{Sdate_t} = (1 - .284L - 0.195L^2) L_{SDUM_t}$, $L_{SDUM_t} = 1$ $t \geq date$
0 otherwise

Table A9
ARI and Intervention Models for Consumer Price Index (CPI)

Basis ARI Model

$$\Delta \text{cpi}_t = 0.0019 + 0.728\Delta \text{cpi}_{t-1} - 0.053\Delta \text{cpi}_{t-2} + 0.423\Delta \text{cpi}_{t-3} - 0.292\Delta \text{cpi}_{t-4}$$

(0.0006) (0.075) (0.089) (0.089) (0.075)

SEE = 0.0052, kurtosis = 8.777, skewness = -0.830

First ten autocorrelations of squared residuals:
0.35 0.06 0.07 0.06 0.01 0.02 0.01 -0.03 0.20 0.33

Ljung-Box Q for squared residuals: Q(12) = 50.380 (p = 0.0000)
LR Test for GARCH(0,1) = 31.230 (p = 0.0000)
LR Test for GARCH(1,1) = 42.232 (p = 0.0000)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	4.82	5.78	6.28
BDS(e=0.5)	5.95	7.67	8.52

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta \text{cpi}_t = 0.011 + 0.661\Delta \text{cpi}_{t-1} + 0.118\Delta \text{cpi}_{t-2} + 0.380\Delta \text{cpi}_{t-3} - 0.276\Delta \text{cpi}_{t-4}$$

(0.005) (0.093) (0.084) (0.098) (0.076)

$$\sigma_t^2 = 0.0000027 + 0.529 \sigma_{t-1}^2 + 0.429 \epsilon_{t-1}^2$$

(0.0000015) (0.119) (0.131)

Kurtosis and skewness in the standardized residuals:
kurtosis = 5.483, skewness = -0.641

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	0.54	0.52	0.37
BDS(e=0.5)	1.77	1.49	0.20

Outlier/Intervention Model

$$\Delta \text{cpi}_t = 0.002 + 0.658\Delta \text{cpi}_{t-1} - 0.092\Delta \text{cpi}_{t-2} + 0.368\Delta \text{cpi}_{t-3} - 0.311\Delta \text{cpi}_{t-4}$$

(0.001) (0.070) (0.081) (0.082) (0.069)

$$- 0.020 \text{IO1948Q4}_t + 0.023 \text{AO1951Q1}_t - 0.012 \text{AO1951Q3}_t$$

(0.004) (0.003) (0.003)

$$+ 0.007 \text{LS1967Q3}_t + 0.009 \text{LS1973Q1}_t - 0.011 \text{LS1982Q4}_t$$

(0.002) (0.002) (0.002)

$$- 0.011 \text{AO1986Q2}_t$$

(0.002)

SEE = 0.0038, kurtosis = 3.310, skewness = 0.110

Table A9 (continued)
Consumer Price Index

First ten autocorrelations of squared residuals:

0.28 0.15 0.04 0.09 0.18 0.06 0.02 -0.09 -0.02 0.08

Ljung-Box Q for squared residuals: $Q(12) = 28.459$ ($p = 0.0047$)

LM Test for GARCH(0,1) = 13.531 ($p = 0.0002$)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS($\epsilon=1.0$)	3.78	4.87	4.68
BDS($\epsilon=0.5$)	3.32	5.57	4.26

Notes: Data are in logarithms. Sample period is 1948Q2-1990Q2.

$IOdate_t = 1$ if $t = \text{date}$
0 otherwise

$AOdate_t = (1 - 0.658L + 0.092L^2 - 0.368L^3 + 0.311L^4) IOdate_t$

$LSdate_t = (1 - 0.658L + 0.092L^2 - 0.368L^3$
 $+ 0.311L^4) LSDum_t$, $LSDUM_t = 1$ if $t \geq \text{date}$
0 otherwise

Table A10

ARI and Intervention Models for Compensation per Hour (Manufacturing)

Basic ARI Model

$$\Delta \text{comp}_t = 0.004 + 0.377 \Delta \text{comp}_{t-1} + 0.147 \Delta \text{comp}_{t-2} + 0.201 \Delta \text{comp}_{t-3}$$

(0.001) (0.075) (0.079) (0.074)

SEE = 0.0069, kurtosis = 5.863, skewness = 1.055

First ten autocorrelations of squared residuals:

0.05 0.02 0.25 0.11 0.10 0.02 0.08 0.24 -0.05 -0.02

Ljung-Box Q for squared residuals: Q(12) = 49.573 (p = 0.0000)

LR Test for GARCH(0,3) = 22.274 (p = 0.0001)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	1.95	1.89	2.15
BDS(e=0.5)	1.65	2.14	1.91

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta \text{comp}_t = 0.0036 + 0.454 \Delta \text{comp}_{t-1} + 0.206 \Delta \text{comp}_{t-2} + 0.088 \Delta \text{comp}_{t-3}$$

(0.0011) (0.090) (0.087) (0.079)

$$\sigma_t^2 = 0.000022 + 0.312 \epsilon_{t-1}^2 - 0.012 \epsilon_{t-2}^2 + 0.184 \epsilon_{t-3}^2$$

(0.000005) (0.137) (0.080) (0.075)

Kurtosis and skewness in the standardized residuals:

kurtosis = 3.184 skewness = 0.470

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-0.76	-0.74	-0.93
BDS(e=0.5)	-1.37	-0.55	0.72

Outlier/Intervention Model

$$\Delta \text{comp}_t = 0.006 + 0.350 \Delta \text{comp}_{t-1} + 0.031 \Delta \text{comp}_{t-2} + 0.094 \Delta \text{comp}_{t-3}$$

(0.001) (0.070) (0.071) (0.068)

$$+ 0.025 \text{AO1948Q1}_t - 0.020 \text{AO1949Q4}_t + 0.034 \text{IO1950Q4}_t$$

(0.005) (0.005) (0.005)

$$+ 0.016 \text{AO1952Q4}_t + 0.017 \text{IO1956Q2}_t + 0.011 \text{LS1973Q1}_t$$

(0.005) (0.005) (0.002)

$$- 0.014 \text{LS1982Q4}_t$$

(0.002)

SEE = 0.0052, kurtosis = 3.125, skewness = 0.280

Table A10 (continued)
 Compensation per Hour

First ten autocorrelations of squared residuals:

-0.08 0.07 0.05 -0.03 0.07 -0.09 -0.02 -0.01 -0.06 -0.08

Ljung-Box Q for squared residuals: $Q(12) = 7.193$ ($p = 0.8446$)

LM Test for GARCH(0,1) = 0.974 ($p = 0.3238$)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS($\epsilon=1.0$)	-0.73	-0.79	-0.20
BDS($\epsilon=0.5$)	-1.26	-0.70	0.92

Notes: Data are in logarithms. Sample period is 1948Q1-1990Q2.

$I\text{Odate}_t = 1$ if $t = \text{date}$
 0 otherwise

$A\text{Odate}_t = (1 - 0.350L - 0.031L^2 - 0.094L^3) I\text{Odate}_t$

$L\text{Sdate}_t = (1 - 0.350L - 0.031L^2 - 0.094L^3) L\text{SDUM}_t$,

$L\text{SDUM}_t = 1$ if $t \geq \text{date}$
 0 otherwise

Table A11
ARI and Intervention Models for Yields on AAA Bonds

Basic ARI Model

$$\Delta r_{aaa_t} = 0.0075 + 0.334 \Delta r_{aaa_{t-1}} - 0.095 \Delta r_{aaa_{t-2}}$$

(0.0039) (0.077) (0.077)

SEE = 0.0394, kurtosis = 4.238, skewness = -0.137

First ten autocorrelations of squared residuals:
0.24 0.27 0.14 0.05 0.03 0.06 0.02 0.11 0.00 0.21

Ljung-Box Q for squared residuals: Q(12) = 46.302 (p = 0.0000)
LR Test for GARCH(0,2) = 18.470 (p = 0.0001)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	3.79	5.13	7.15
BDS(e=0.5)	5.09	7.61	13.32

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta r_{aaa_t} = 0.0077 + 0.384 \Delta r_{aaa_{t-1}} - 0.084 \Delta r_{aaa_{t-2}}$$

(0.0029) (0.090) (0.082)

$$\sigma_t^2 = 0.00079 + 0.338 \epsilon_{t-1}^2 + 0.186 \epsilon_{t-2}^2$$

(0.00013) (0.198) (0.143)

Kurtosis and skewness in the standardized residuals:
kurtosis = 4.626 skewness = 0.304

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-0.34	-0.11	1.59
BDS(e=0.5)	1.13	0.95	3.59

Outlier/Intervention Model

$$\Delta r_{aaa_t} = 0.0053 + 0.353 \Delta r_{aaa_{t-1}} - 0.143 \Delta r_{aaa_{t-2}}$$

(0.0028) (0.077) (0.076)

$$+ 0.127 IO1979Q4_t + 0.124 AO1980Q1_t - 0.138 AO1982Q4_t$$

(0.036) (0.034) (0.033)

SEE = 0.0353, kurtosis = 3.088, skewness = 0.025

First ten autocorrelations of squared residuals:
0.13 0.12 0.18 0.05 0.18 -0.04 0.09 0.10 -0.01 -0.02

Ljung-Box Q for squared residuals: Q(12) = 21.457 (p = 0.0440)
LM Test for GARCH(0,3) = 9.214 (p = 0.0266)
LM Test for GARCH(0,5) = 13.706 (p = 0.0176)

Table A11 (continued)
AAA Bond Yields

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	3.47	4.68	6.72
BDS(e=0.5)	4.67	7.06	11.86

Notes: Data are in logarithms. Sample period is 1947Q4-1990Q2.

$I0date_t = 1$ if $t = \text{date}$
0 otherwise

$A0date_t = (1 - 0.353L + 0.143L^2) I0date_t$

Table A12
ARI and Intervention Models for Nominal Stock Prices

Basic ARI Model

$$\Delta sp_t = 0.013 + 0.294 \Delta sp_{t-1}$$

(0.005) (0.073)

SEE = 0.0560, kurtosis = 5.911, skewness = -0.503

First ten autocorrelations of squared residuals:
-0.02 0.03 0.09 -0.04 -0.01 -0.05 0.00 -0.09 -0.01 -0.06

Ljung-Box Q for squared residuals: Q(12) = 5.312 (p = 0.9467)
LR Test for GARCH(0,1) = 0.1216 (p = 0.9687)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	-0.37	0.65	1.34
BDS(e=0.5)	-0.86	1.19	0.97

Outlier/Intervention Model

$$\Delta sp_t = 0.014 + 0.344 \Delta sp_{t-1} - 0.138 A01957Q4_t$$

(0.004) (0.067) (0.045)

$$- 0.176 IO1974Q3_t + 0.170 IO1982Q4_t - 0.261 A01987Q4_t$$

(0.048) (0.048) (0.045)

SEE = 0.0476, kurtosis = 3.244, skewness = 0.288

First ten autocorrelations of squared residuals:
-0.05 0.07 0.00 0.07 0.06 -0.13 0.01 -0.05 -0.06 -0.03

Ljung-Box Q for squared residuals: Q(12) = 7.541 (p = 0.8199)
LM Test for GARCH(0,1) = 0.396 (p = 0.5294)

BDS statistics for nonlinearity in the residuals

Embedding dimension-	2	3	4
BDS(e=1.0)	-0.85	0.70	1.17
BDS(e=0.5)	-0.00	2.21	1.61

Notes: Data are in logarithms. Sample period is 1947Q3-1990Q2.

$IOdate_t = 1$ if $t = \text{date}$
0 otherwise

$AOdate_t = (1 - 0.344L) IOdate_t$

Table A13
ARI and Intervention Models for Money Base

Basic ARI Model

$$\Delta mb_t = 0.0005 + 0.000037 t + 0.468 \Delta mb_{t-1} - 0.059 \Delta mb_{t-2} + 0.294 \Delta mb_{t-3}$$

(0.0007) (0.000013) (0.075) (0.083) (0.076)

SEE = 0.0048, kurtosis = 2.972, skewness = -0.250

First ten autocorrelations of squared residuals:
-0.04 0.18 0.00 -0.00 -0.05 0.13 0.09 0.04 -0.07 0.05

Ljung-Box Q for squared residuals: Q(12) = 13.161 (p = 0.3574)
LR Test for GARCH(0,2) = 3.313 (p = 0.1908)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	0.83	1.73	1.77
BDS(e=0.5)	0.07	0.18	1.15

Outlier/Intervention Model

$$\Delta mb_t = 0.0005 + 0.000025 t + 0.424 \Delta mb_{t-1} - 0.090 \Delta mb_{t-2}$$

(0.0008) (0.000012) (0.075) (0.082)

$$+ 0.266 \Delta mb_{t-3} + 0.008 \text{LS1961Q4}_t$$

(0.077) (0.003)

SEE = 0.0047, kurtosis = 3.152, skewness = -0.165

First ten autocorrelations of squared residuals:
-0.03 0.17 0.01 -0.00 -0.07 0.12 0.08 0.02 0.00 0.05

Ljung-Box Q for squared residuals: Q(12) = 13.161 (p = 0.3574)
LM Test for GARCH(0,2) = 5.354 (p = 0.0688)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	0.83	1.73	1.77
BDS(e=0.5)	0.07	0.18	1.15

Notes: Data are in logarithms. Sample period is 1948Q1-1990Q2.

$$\text{Iodate}_t = 1 \text{ if } t = \text{date}$$

$$0 \text{ otherwise}$$

$$\text{AOdate}_t = (1 - 0.424L + 0.090L^2 - 0.266L^3) \text{Iodate}_t$$

$$\text{LSdate}_t = (1 - 0.424L + 0.090L^2 - 0.266L^3) \text{LSDUM}_t,$$

$$\text{LSDUM}_t = 1 \text{ if } t \geq \text{date}$$

$$0 \text{ otherwise}$$

Table A14
ARI and Intervention Models for M1

Basic ARI Model

$$\Delta M1_t = 0.0014 + 0.000054 t + 0.466 \Delta M1_{t-1}$$

(0.0012) (0.000014) (0.068)

SEE = 0.0078, kurtosis = 10.847, skewness = 0.846

First ten autocorrelations of squared residuals:
0.27 0.10 0.12 0.06 0.00 -0.02 0.05 0.01 0.11 0.03

Ljung-Box Q for squared residuals: Q(12) = 20.557 (p = 0.0573)

LR Test for GARCH(0,1) = 31.333 (p = 0.0000)

LR Test for GARCH(1,1) = 53.041 (p = 0.0000)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	1.51	2.01	2.71
BDS(e=0.5)	0.88	1.52	1.78

Maximum likelihood estimates of GARCH Model for Basic ARI Model

$$\Delta M1_t = 0.0004 + 0.000050 t + 0.601 \Delta M1_{t-1} + \epsilon_t$$

(0.0009) (0.000012) (0.076)

$$\sigma^2_t = 0.000006 + 0.616 \sigma^2_{t-1} + 0.298 \epsilon^2_{t-1}$$

(0.000003) (0.128) (0.090)

Kurtosis and skewness in the standardized residuals:
kurtosis = 4.007 skewness = 0.068

Examination of nonlinearity in the standardized residuals: BDS statistics

Embedding dimension	2	3	4
BDS(e=1.0)	-1.56	-1.59	-1.05
BDS(e=0.5)	-2.23	-2.28	-2.25

Outlier/Intervention Model

$$\Delta M1_t = 0.0014 + 0.000048 t + 0.509 \Delta M1_{t-1}$$

(0.0008) (0.000012) (0.056)

$$- 0.016 A01959Q1_t - 0.016 I01959Q4_t + 0.019 A01979Q3_t$$

(0.004) (0.005) (0.004)

$$- 0.029 I01980Q2_t - 0.048 I01980Q3_t - 0.016 A01981Q1_t$$

(0.005) (0.005) (0.005)

$$- 0.014 A01982Q2_t + 0.019 I01982Q4_t + 0.012 LS1985Q1_t$$

(0.004) (0.005) (0.003)

$$- 0.022 LS1987Q3_t - 0.015 A01989Q2_t$$

(0.004) (0.005)

Table A14 (continued)
M1

SEE = 0.0050, kurtosis = 3.424, skewness = 0.018

First ten autocorrelations of squared residuals:

-0.03 0.08 0.06 0.22 -0.05 0.05 0.18 0.06 -0.08 0.02

Ljung-Box Q for squared residuals: $Q(12) = 19.496$ ($p = 0.0772$)

LM Test for GARCH(0,4) = 10.704 ($p = 0.0301$)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS($\epsilon=1.0$)	-0.37	0.14	1.00
BDS($\epsilon=0.5$)	0.24	0.17	0.39

Notes: Data are in logarithms. Sample period is 1947Q3-1990Q2.

$I\text{Odate}_t = 1$ if $t = \text{date}$
0 otherwise

$A\text{Odate}_t = (1 - 0.509L) I\text{Odate}_t$

$L\text{Sdate}_t = (1 - 0.509L) L\text{SDUM}_t$, $L\text{SDUM}_t = 1 \geq \text{date}$
0 otherwise

Table A15
ARI and Intervention Models for M2

Basic ARI Model

$$\Delta M2_t = 0.0026 + 0.000026 t + 0.695 \Delta M2_{t-1}$$

(0.0010) (0.000010) (0.057)

SEE = 0.0058, kurtosis = 6.124, skewness = 0.919

First ten autocorrelations of squared residuals:
0.15 -0.01 0.02 -0.06 -0.03 0.01 0.01 0.04 0.08 0.26

Ljung-Box Q for squared residuals: Q(12) = 20.198 (p = 0.0634)

LR Test for GARCH(0,1) = 3.630 (p = 0.0567)

LR Test for GARCH(1,1) = 4.108 (p = 0.1282)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	2.24	2.37	2.73
BDS(e=0.5)	2.78	4.31	5.41

Outlier/Intervention Model

$$\Delta M2_t = 0.0015 + 0.00004 t + 0.701 \Delta M2_{t-1}$$

(0.0008) (0.00001) (0.053)

$$+ 0.014 I01958Q2_t + 0.017 I01975Q2_t + 0.017 A01980Q3_t$$

(0.005) (0.005) (0.004)

$$+ 0.025 A01983Q1_t - 0.014 LS1985Q2_t$$

(0.004) (0.003)

SEE = 0.0046, kurtosis = 2.935, skewness = 0.181

First ten autocorrelations of squared residuals:
-0.04 -0.00 0.21 0.06 0.11 -0.05 0.05 0.08 0.04 -0.06

Ljung-Box Q for squared residuals: Q(12) = 16.154 (p = 0.1843)

LM Test for GARCH(0,3) = 8.132 (p = 0.0434)

BDS statistics for nonlinearity in the residuals

Embedding dimension	2	3	4
BDS(e=1.0)	0.15	-0.14	0.43
BDS(e=0.5)	-0.81	-0.79	0.52

Notes: Data are in logarithms. Sample period is 1947Q3-1990Q2.

I0date_t = 1 if t = date
0 otherwise

A0date_t = (1 - 0.701L) I0date_t

LSdate_t = (1 - 0.701L) LSDUM_t, LSDUM_t = 1 if t ≥ date
0 otherwise

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