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# THE AGGREGATE EFFECTS OF TEMPORARY GOVERNMENT PURCHASES

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Federal Reserve Bank of Dallas

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## Abstract

How do changes in the level of government purchases affect the macroeconomy? This paper looks at the effects of temporary government purchases in the context of a simple dynamic general equilibrium model. The model is parameterised in a parsimonious manner and perturbed by a spending shock that captures the temporary component of government spending in the US during World War II. There is a remarkable correspondence between the movements in output, consumption and effort predicted by the model and those observed in reality.

## 1. Introduction

The manner in which the spending decisions of governments affect the level of economic activity is one of the central issues of macroeconomics. The way in which economists think about this issue has evolved over time as our understanding of the aggregate economy has grown. The neoclassical synthesis that grew out of the Keynesian revolution ascribed a key role to government purchases in stabilizing output fluctuations. This role rested on, among other things, the existence of a multiplier for government spending, so that relatively small changes in spending could offset relatively large fluctuations in output. However, events of the 1970's called into question many of the central tenets of the neoclassical synthesis and spurred the development of alternative, equilibrium models of the macroeconomy that were based explicitly on the optimizing behaviour of households and firms. Most of these models had their origin in the neoclassical growth model developed by Solow(1956), augmented to include long-lived households that make optimal decisions about consumption, work effort and saving over time. In an intertemporal framework, the distinction between changes in government purchases that are temporary and those that are permanent, as well as that between changes that are anticipated versus unanticipated, becomes important. Barro's(1981) pioneering anlaysis of the effects of government purchases on output claimed that there was no multiplier effect, and that temporary changes had larger output effects than permanent changes. However Aiyagari, Christiano and Eichenbaum(1989) have recently shown that this is not, in fact, the case in a fully specified dynamic general equilibrium model. They show that the output effects of a permanent increase in government purchases are always greater than those of a temporary increase, and that there can be an analog to the Keynesian multiplier in a neoclassical model.

The purpose of this paper is to evaluate the empirical plausibility of the predictions of the neoclassical model concerning changes in government purchases. The strategy adopted is to parameterise a relatively simple version of the model — the specification most often used to illustrate its basic properties — and then compare the movements in output, consumption, effort etc. predicted by the model in response to a shock to government purchases with actual experience. The methodology is simliar to that commonly used to evaluate the empirical content of real business cycle models, with the exception that I compare paths of variables rather than second moments generated by the model and the data. I focus on the effects of temporary changes in the level of government purchases, and in particular, one well defined episode of such temporary variation, namely World War II. The huge increases in government purchases that occurred over the course of this conflict, followed by equally large declines at its end, are the natural empirical counterparts of temporary deviations of government purchases from their steady state level in the artificial economy of the model. Furthermore, the size of the stimulus that came from the government during the war years was large enough that we can reasonably abstract from other factors that usually contribute to fluctuations in the level of economic activity.

The practice of looking at episodes of extreme variation in economic aggregates as natural experiments that enable us to discriminate between competing economic theories has its origin in the work by Friedman(1951) and Cagan(1956) on the Quantity Theory. Thus Friedman writes

> "The widespread tendency in empirical studies of economic behaviour to discard war years as "abnormal", while doubtless often justified, is, on the whole, unfortunate. The major defect of the data on which economists must rely — data generated by experience rather than deliberately contrived experiment — is the small range of variation they encompass. Experience in general proceeds smoothly and continuously. In consequence, it is difficult to disentangle systematic effects from random variation since both are of much the same order of magnitude. From this point of view, data for wartime periods are peculiarly valuable. At such times, violent changes in major economic magnitudes occur over relatively brief periods, thereby providing precisely the kind of evidence that we would like [to] get by "critical" experiments if we could conduct them. Of course, the source of the changes means that the effects in which we are interested are necessarily intertwined with others that we would eliminate from a contrived experiment. But this difficulty applies to all our data, not to data from wartime periods alone." Friedman(1951), p.612.

The empirical studies of the effects of government purchases on the level of economic activity and interest rates by Barro(1981,1987) and Ahmed(1986) have also relied heavily on wartime episodes of high military spending to obtain quantitatively significant temporary movements in government purchases.

## 2. The One Sector Neoclassical Growth Model

Consider an economy populated by a large number of identical households. The preferences of each household are defined over consumption and leisure, both of which are assumed to be normal goods. Households are infinitely lived and have perfect foresight concerning the future. Each household has access to a private production technology which transforms capital and effort into output. Current period output may be consumed, stored as capital for future production or appropriated by the government. There is no rationale for the existence of a government in this economy: I simply posit its existence and assume that it absorbs some amount of output each period, financing its purchases by lump sum taxation. Government purchases of output do not substitute for private consumption; nor do they enhance the productivity of private factors of production [1]. Growth occurs due to labor augmenting technical change which proceeds at some exogenously given deterministic rate.

The equilibrium of this economy is the solution to the following planning problem:

$$\max_{C_t, L_t} \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \tag{1}$$

subject to

$$F(K_t, X_t N_t) + (1-\delta)K_t = C_t + K_{t+1} + G_t \tag{2}$$

$$N_t + L_t \leq 1 \tag{3}$$

$$K_0 = \bar{K}_0. \tag{4}$$

All variables are in per capita terms and have the obvious interpretations. The objective function is time separable, with the point-in-time utility function assumed to be continuous and twice differentiable in $C$ and $L$. The production function is assumed to exhibit constant returns to scale in capital and effective labor. The gross rate of labor augmenting technical change is denoted by $\xi_x$, with $\xi_x = X_{t+1}/X_t$[2]. Technical change must be expressable in labor augmenting form to be consistent with the existence of a steady state. This problem differs from the standard optimal growth problem because of the inclusion of the term $G_t$ in the resource constraint. This can be interpreted as a shock to technology that leaves marginal products unchanged: I interpret it as the amount resources appropriated by the government each period[3]. The absence of this term from the utility and production functions reflects the parasitic role of government in this model[4].

[1] If desired, government purchases could be posited to enter the utility function in a separable manner, thereby giving some rationale for their existence, without altering any of what follows.

[2] Recent research has cast doubt on the ability of the neoclassical growth model to explain variations in growth rates across countries and over time — see, for example, Lucas(1988) and King and Rebelo(1989). Deterministic technical change is included in this model as the simplest way to account for the observed nonstationarity of macroeconomic time series. Repeating the analysis in this paper using an alternative model of the determinants of economic growth would provide valuable insights into the relationship between transitory shocks and long run growth.

[3] The equivalence between technology shocks and various fiscal policies was first explored by Abel and Blanchard(1983).

[4] See Barro(1987) for a discussion of how this might matter.

To analyse this problem the standard procedure is to transform it into a stationary economy. This is accomplished by dividing the model variables by the nonstationary growth variable $X$. Define $c_t = C_t/X_t$, $k_t = K_t/X_t$, $g_t = G_t/X_t$, $y_t = Y_t/X_t$, and define the Lagrangean

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^{*t} U(c_t, 1 - N_t) + \sum_{t=0}^{\infty} \Lambda_t [F(k_t, N_t) + (1-\delta)k_t - c_t - \xi_x k_{t+1} - g_t], \tag{5}$$

where $\Lambda_t$ denotes the multiplier associated with the period $t$ constraint. The first order necessary conditions characterising the solution of this problem are

$$D_1 U(c_t, 1 - N_t) = \lambda_t \tag{6}$$

$$D_2 U(c_t, 1 - N_t) = \lambda_t D_2 F(k_t, N_t) \tag{7}$$

$$\beta^* \lambda_{t+1} [(1-\delta) + D_1 F(k_{t+1}, N_{t+1})] = \xi_x \lambda_t \tag{8}$$

$$F(k_t, N_t) + (1-\delta)k_t = c_t + \xi_x k_{t+1} + g_t \tag{9}$$

and the transversality condition

$$\lim_{t \to \infty} \beta^{*t} \lambda_t k_{t+1} = 0. \tag{10}$$

$D_i$ denotes differentiation with respect to the i'th argument, $\lambda_t = \Lambda_t/(\beta^*)^t$, and $\beta^*$ is the effective rate of time preference[5].

The same set of equations characterize the equilibrium of a decentralized competitive economy with a large number of households and firms. Households choose time paths for consumption and leisure to maximise a function identical to (1) subject to a constraint that consumption and saving each period cannot exceed income from supplying labor to a competitive labor market plus interest income, all net of lump sum taxes. Firms choose time paths for investment and employment to maximise the present value of profits, with capital accumulation being financed by issuing bonds to the household sector.

Eliminating $\lambda_t$ from equations (6) and (7) we get

$$\frac{D_2 U(c_t, 1 - N_t)}{D_1 U(c_t, 1 - N_t)} = w_t = D_2 F(k_t, N_t)$$

where $w_t$ is the real wage. The equality of the marginal rate of substitution between consumption and leisure with the marginal productivity of labor describes the equilibrium of a competitive labor market.

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[5] See King, Plosser and Rebelo(1988).

Now consider equations (6) and (8). Note that $\Lambda_t$ can be interpreted as a present value price, so we can define an interest rate by $(1+r_t) = \xi_x \Lambda_t / \Lambda_{t+1}$. The equilibrium of a competitive loan market with firms and households as the sole participants is given by

$$\frac{\xi_x D_1 U(c_t, 1 - N_t)}{\beta^* D_1 U(c_{t+1}, 1 - N_{t+1})} = 1 + r_t = (1 - \delta) + D_1 F(k_{t+1}, N_{t+1}).$$

Equation (9), the resource constraint, has the interpretation of a goods market clearing condition[6].

The steady state of the economy is the solution to the nonlinear system

$$D_1 U(c, 1 - N) = \lambda \tag{11}$$

$$D_2 U(c, 1 - N) = \lambda D_2 F(k, N) \tag{12}$$

$$\beta^* [(1 - \delta) + D_1 F(k, N)] = \xi_x \tag{13}$$

$$F(k, N) = c + (\delta + \xi_x - 1)k + g, \tag{14}$$

where the absence of time subscripts is used to denote steady state values of the variables. The specifications of the utility and production functions guarantee that the steady state exists and is unique.

### 3. The Effect of Government Purchases

How does an increase in government purchases affect the equilibrium of this economy? A permanent increase in government purchases, financed by an increase in lump sum taxes, lowers household wealth. Since consumption and leisure are both normal goods, consumption falls and effort rises. The increase in the supply of effort raises the marginal productivity of capital, and thereby increases the size of the optimal steady state capital stock. During the transition to the higher capital stock the interest rate is above its long level. Consumption initially falls by more than the increase in government purchases, but grows over time and eventually settles at a level below its initial level. In the new steady state equilibrium investment is higher, as is output. In the absence of distortionary taxation, an increase in the share of output absorbed by the government is fully reflected in a decline in the share going to private consumption. The level of private consumption, however, may fall by more or less than the increase in the level of government purchases.

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[6] A more extensive discussion of the equivalence between the centrally planned and competitive economies is contained in Abel and Blanchard(1983).

A temporary increase in government purchases, on the other hand, will have a relatively small effect on household wealth. The increase in the demand for goods by the government means that output is relatively more scarce in the present than in the future. This bids up the real interest rate and induces households to consume less and work more for the duration of the period of temporarily high government purchases. Part of the increased demand by the government is met by reduced private investment, with the result that after the period of temporarily high government purchases the capital stock is below its optimal long run level. This leads to a period of high growth for the economy as the capital stock is rebuilt. When episodes of temporarily high purchases are anticipated, their impact effects on consumption and effort will be smaller. Anticipating higher taxes at some point in the future households increase their saving so as to smooth the effects of the higher taxes. This translates into increased capital accumulation at the aggregate level. The amount of smoothing that is done in response to a shock of a given size is determined by two key parameters: these are the (effective) discount rate, $\beta^*$, and the rate of depreciation of capital, $\delta$. Low values for $\beta^*$ (given the intertemporal elasticity of substitution and the exogenous rate of technical change) correspond to high pure rates of time preference, which reduce the amount of smoothing that is carried out in response to an anticipated shock. Similarly, high rates of capital depreciation make it more difficult or costly to accumulate capital in advance of periods of high levels of government spending, which also acts to reduce the amount of smoothing. These lower levels of smoothing manifest themselves in greater impact effects on consumption and effort of the higher government purchases when they occur, and smaller responses before and after the shock period.

Another important feature of this model is that it is a one good model. The amount of resources available this period that can be consumed, invested or appropriated by the government consists of output produced with capital and labor this period plus the undepreciated portion of the capital stock. This is important in understanding the response of the capital stock in this model to an anticipated increase in government spending. Capital is accumulated in anticipation of a shock to allow higher levels of production when the government is demanding a lot of output by combining it with extra labor input at such times. But part of the higher demand by the government is met by running down the capital stock to such an extent that it falls below its steady state level after the period of higher government purchases. This is optimal given the wishes of agents to smooth consumption and leisure over time.

The objective of this paper is to use this model to evaluate the effects of government induced shocks on the economy. To do so, I need to specify the parameters of tastes and technology, other characteristics of the steady state and the nature and duration of the shocks. It is to this that I now turn.

## 4. Calibration

To examine the dynamics of this model I need to assign values to the following parameters[7]: $\beta, N, \sigma_{cc}, \sigma_{cl}, \sigma_{lc}, \sigma_{ll}, \theta_n, \delta, \sigma_{KN}, \xi_x$ and $\theta_g$. The range of possible values that we can assign to these parameters is limited by the requirement that the utility and production functions satisfy standard neoclassical "niceness" restrictions. Our choice is further circumscribed by focusing only on values that in some sense might be considered representative of the U.S. economy during World War II.

I begin with the parameters of household's preferences. Choosing a value of $\beta = 0.9615$ is consistent with the earlier work of Kydland and Prescott(1982) and Hansen(1985). It is difficult to obtain empirical estimates for the various substitution elasticities of the utility function, $\sigma_{cc}$, $\sigma_{cl}$, $\sigma_{lc}$ and $\sigma_{ll}$. I opted for the logarithmic specification of utility, setting $\sigma_{cc} = \sigma_{ll} = -1$, and $\sigma_{cl} = \sigma_{lc} = 0$. This is a reasonably "neutral" benchmark specification, and is used for initial investigation of the response of the model to government purchase shocks[8]. I set the proportion of time devoted to market activities in the steady state, $N$, equal to 0.333. This is consistent with the finding that households allocate about one-third of their available time to market activities. Alternatively, if we assume that an individual can work at most 16 hours a day, 7 days a week, we come up with an estimate of 112 hours as the weekly endowment of time. From the *Handbook of Labor Statistics* we can obtain time series on Average Weekly Hours of Work in Manufacturing[9]. Normalizing these figures by the endowment of 112 and averaging over the period 1931-1940, we obtain a figure of 0.337, which is suspiciously close to one-third.

I picked a value for $\xi_x = 1.018$ based on extracting a common deterministic trend from per capita GNP, Personal Consumption Expenditures, Gross Private Investment and Government Purchases of Goods and Services over the period 1889-1986. Data for the years prior to 1929 are taken from Kendrick, *Productivity Trends in the United States*[10]. The choice of deterministic detrending is motivated by two considerations. Firstly it is the logical empirical counterpart of the deterministic technological change that induces nonstationarity in the model. Secondly there

[7] For definitions of the various parameters see the appendix.

[8] This is the specification used by Long and Plosser(1983) in their seminal study of real business cycles.

[9] Manufacturing was chosen solely on grounds of data availability. Figures for average weekly hours in the total private sector are not available before 1947.

[10] The results do not change if we use the alternative estimates of GNP developed by Balke and Gordon(1989) and Romer(1989). If we use only the official NIPA data for the period 1929-1982 we obtain an estimate of 2.472% for the average annual rate of growth of the per capita aggregates.

is the evidence from the time series properties of the key national accounts aggregates. Tables 1.A and 1.B present the sample autocorrelation functions for GNP and its key components up to five lags in both levels and first differences. The evidence in these tables suggests that these series are nonstationary. To get some idea whether the nonstationarity could be accounted for by stochastic or deterministic trends, I carried out the standard Dickey-Fuller tests for unit roots. The results of these tests are reported in Table 2. The hypothesis of a unit root (stochastic trend) is rejected at the 5% level for all series except consumption[11]. In light of this, it seems reasonable to proceed with the deterministic detrending. The unit of time is taken to be one year, so combined with the assumed value of $\beta$ this implies a steady state annual real rate of interest of 5.9%. This is similar to the figure used by King, Plosser and Rebelo(1988).

$\theta_N$ is set equal to 0.667, which is an estimate of the share of national income accruing to labor averaged over 1931-40. This is close to the type of numbers that have been found by authors looking at the share of labor over longer time periods. The elasticity of substitution between capital and labor, $\sigma_{KN}$, was set equal to one, and the depreciation rate, $\delta$, equal to 0.1.

Finally we come to specifying the characteristics of the government. The objective of this paper is to examine how well a simple neoclassical model can capture the movements in macroeconomic variables that were brought about by the large changes in government purchases during WWII. The principal requirement of the analysis is data for $\{\hat{g}\}$ that measure the deviation of government purchases in the U.S. from their steady state level (somehow defined) during the war years 1942-1945. If we look at the behaviour of time series on government purchases, and the components thereof, in the US over the course of the twentieth century, it is clear that almost all of the temporary variation is due to expansions and contractions of federal defense purchases associated with wars. This fact was exploited by Barro(1981) to simplify the decomposition of government purchases into transitory and permanent components. The permanent component of the ratio of federal defense purchases to GNP, $\bar{g}^w$, was calculated using a standard present value definition and a forecasting equation driven by a casualty rate variable and a measure of government capital. An estimate of $\hat{g}$ is then given by $(g^w - \bar{g}^w)/(\bar{g}^w + g^p)$, where $g^w$ is the ratio of federal defense purchases to GNP, and $g^p$ the ratio of federal non-defense purchases and state

[11] This contradicts the findings of Nelson and Plosser(1982) who claim to find a unit root in real per capita GNP. The difference may be due to my use of a longer sample period. Nelson and Plosser use estimates of GNP by the Commerce Department that go back to 1909. I use the estimates of GNP in Kendrick(1961) which go back to 1889 and which Romer(1988) has argued are superior to the Commerce series. The standard Dickey-Fuller test on Romer's GNP series also fails to find evidence of a unit root. Given the low statistical power of existing tests for unit roots and the conflicting nature of the evidence to date, use of a deterministic trend has the advantage of simplicity and being well understood as regards potential misspecification bias.

and local purchases to GNP. This series can be calculated from Table 2 of Barro(1981), and is plotted in Figure 1. The transitory component of government purchases is positive during the three wars that occurred during the sample period: WWII, Korea and Vietnam. The increase in government purchases during WWII is enormous compared to what took place during the other wars, thus motivating my focus on this war alone. The estimated values of $\hat{g}$ are (1.09, 2.14, 2.82, 2.58) for 1942-1945. Thus government purchases were more than twice their steady state level in 1942, more than three times in 1943 and almost four times their steady state level in 1944. So the shocks to government purchases associated with WWII were rather large. Note also the uneven size of the shock in the different war years – this will be important in interpreting the analysis that follows.

The last parameter to be set is $\theta_g$. I chose a value of 0.2051, equal to the average value over 1931-1940 of $\bar{g}^w + g^p$ as reported in Table 2 of Barro(1981). This figure seems rather high, and in fact reflects the fact that the relative price of government purchases has increased steadily over the course of the century. An alternative estimate is given by the average value of the ratio of nominal government purchases to nominal GNP over the same period. This comes out as 0.1435.

The analysis of the model proceeds by feeding in the series on the purchases shocks and looking directly at the effects on the various series. This contrasts with the more usual practice in real business cycle models of examining the implications for various moments of particular shock processes. Prescott(1986) eschews direct comparison of the paths of series generated by a model with those generated by experience as an approach to model evaluation on the basis of the sensitivity of predictions of paths to "whimsical modelling assumptions" and errors in the measurement of the shocks that drive the model that may be as large as the shocks themselves. Concern with measurement error is warranted when one is looking at a model driven by unobservable technology shocks. For the present study this is not a concern as it seems unlikely that the errors in the measurement of government purchases during WWII were as large as the change in government purchases during that period.

## 5. A Baseline Simulation

I begin by looking at the effects of four successive periods of temporarily high government purchases occurring at (model) dates 0 through 3. The higher spending in period 0 can in some sense be considered unanticipated, but that in periods 1 to 3 is perfectly foreseen. Thus, we are looking at an experiment in which households are surprised by the outbreak of a war, but as soon as it begins they know exactly its magnitude and duration. This is important in understanding how investment in period 0 responds to the higher level of purchases. If the shock only lasted one period we would expect to see investment declining. Instead it increases by some 50% to help smooth the effects of the shocks in the later periods[12]. Output and effort increase by 20% and 30% respectively. Consumption declines by just over 26% in period 0 and runs at about 21% below its long run level for the duration of the war. Output and effort remain relatively high during the first three periods of the war, declining somewhat in period 3. Output falls below its steady state level in period 4 due to the decline in the capital stock. From period 4 on all variables follow the standard path of adjustment associated with convergence to a steady state from an initial capital stock that is below the equilibrium level. Finally note the behaviour of real interest rates. Interest rates run at about 2-3 percentage points above their steady state level during the war. We can estimate the multipliers associated with the higher purchases in each period as $\theta_g^{-1}(\hat{y}_t/\hat{g}_t)$. These turn out to be 0.939, 0.517, 0.311 and 0.144 in periods 0 to 3 respectively. Clearly there is no "multiplier" effect of government purchases on output i.e. output rises less than one-for-one with the increase in purchases. This is because the basic neoclassical model acts to buffer shocks rather than to magnify them[13]. The results of this of this simulation are summarised in column 3 of Table 3 under the heading Model(0).

## 6. Comparison with US Experience in WWII

During World War II the United States experienced one of the longest periods of sustained growth in its history, exceeded only by the expansion associated with the Vietnam war and that associated with the Reagan defense program. Real GNP (in 1982 dollars) increased by 93% over 1939-1944, increasing from $716.6bn to $1380.6 bn. This translates into an annual average growth rate of 14%. Personal Consumption Expenditures increased by 16% between 1939 and 1944, going from $480.5bn to $557.1bn. Purchases of Durables increased from $35.7bn in 1939 to $46.2bn in 1941, before declining to $26.3bn in 1944 and subsequently recovering to $47.8bn in 1946. The other components of consumption purchases (Nondurables and Services) increased throughout the war. Gross Private Investment increased from $86bn in 1939 to $138.8bn in 1941. It then declined dramatically to $50.4bn in 1943 (bottoming out in this year), before increasing $56.4bn in 1944, $76.5bn in 1945 and $178.1bn in 1946. One aspect of the decline in private investment that I do not address in this paper is the fact that the federal government carried out a lot of investment in plant and machinery that was subsequently transferred to the private sector at nominal prices at the end of the war. Gorden(1969) stressed the importance of this

[12] Note that no matter how far in advance the shock is anticipated investment always increases in the first period of the war because of the uneven pattern of the shocks.

[13] See Barro(1987), p.320.

phenomenon in explaining productivity developments in the 1940's. This issue could be dealt with in the context of the neoclassical model by having the government purchase large quantities of capital goods for a while, followed by a period in which the undepreciated remainder of this stock is made available for private sector use. Data limitations preclude the pursuit of this line of inquiry at the present time. Government Purchases of Goods and Services increased from \$144.1bn to \$235.6bn over 1939-41, an increase of 63%, before peaking at \$790.8bn in 1944, a further increase over the 1941 level of 236%. Federal Government Purchases increased from \$53.8bn in 1939 to \$153.0bn in 1941 (184%), and then to \$722.5bn in 1944 (372% over 1941). State and Local Government Purchases fell from \$90.3bn in 1939 to \$82.6bn in 1941 (-9%) and then to \$68.3bn in 1944 (-17%).

How well do the predictions of the model conform with the actual experience of the US economy during WWII? The experiments in the previous section yielded predictions about the deviations of key aggregates from their steady state levels. The empirical counterpart to this can be taken to be the deviations of observed series from their trend levels. The trend levels for the principal national accounts aggregates, real GNP, real Personal Consumption Expenditures, real Gross Private Investment and real Government Purchases of Goods and Services, all in per capita terms, were calculated by fitting a common deterministic trend to them over the period 1889-1986. In Figure 2 I plot the deviations from trend in percentage terms for each series. The movements in each of the series during WWII are qualitatively the same as the predictions of the model[14]. Private consumption expenditures ran at about 10% below trend during the war, which is somewhat less than the predictions from the baseline model. Real GNP was 15% above trend in 1942, 31% in 1943, 37% in 1944 and 31% in 1945. Gross private investment did decline rather dramatically, by between 52% and 70%: the declines predicted by the model are twice this in some years. These figures are under the heading "Actual" in Table 3.

Matching the predictions of the model concerning movements in effort to the data is less straightforward. If we look at the behaviour of average weekly hours of work in manufacturing[15], we see that they were about 45 hours per week in the 1920's, fell to about 38 hours per week in the 1930's, increased to around 45 hours at the peak of WWII, before settling at about 40 hours per week in the postwar period. It is not clear whether the increase in hours worked during the war was merely a return to the pre-Depression norm or whether it constituted an increase over an already established norm of a 40 hour week [16]. If we take the 40-hour week as the trend

[14] The deviations from trend of Government Purchases are 188%, 309%, 342% and 283% in 1942-45. These figures are slightly bigger than the estimates obtained from Barro(1981).

[15] See *Handbook of Labor Statistics*.

[16] Evidence that this was in fact the case is the speed with which the 40 hour week was

level of hours worked in manufacturing during WWII, the percentage deviations in 1942-45 are 7.8%, 12.5%, 13.0% and 8.8%.

The model performs least satisfactorily when it comes to predicting movements in interest rates. Temporarily high levels of government purchases lead to increased real interest rates in the model, but it is difficult to find any evidence of this in the data. The relevant empirical counterpart of the interest rate in the model is the ex-ante real rate. Being unobservable this is a difficult variable to measure at the best of times. The usual problems associated with its measurement are compounded in this instance by the widespread use of price controls in the US during WWII and the pegging of nominal interest rates by the Fed. In Figure 3 I plot a measure of the ex-post real rate over 1889-1982 relative to its sample mean(1.673%) based on the rate of interest on prime commercial paper and the rate of inflation of the GNP deflator. We see that it is negative during the war years. This finding is robust to the use of alternative measures of the nominal interest rate and inflation rate. The substantially larger negative real rate in 1945 is due to the surge in inflation associated with the relaxation of price controls at the end of the war. Note also that Mishkin's(1981) estimates of the ex-ante real rate are negative during this period.

A well known feature of the neoclassical growth model with perfect foresight is that the impact effects of a disturbance in any period are intimately related to the degree to which the shock is anticipated[17]. By this I mean that temporarily high purchases in period 0 will have quantitatively different effects on output, consumption, etc. than will an equal sized increase at some date in the future. The key is that shocks at later dates can be buffered by capital accumulation, whereas the capital available in period 0 is predetermined. Columns 4-6 of Table 3 show the consequences of moving the start of the war back by one or more periods. Whereas Model(0) is the model output when the spending shock is unanticipated i.e. begins in period 0, Model(1) the output when the shock begins in period 1, i.e. is announced one period in advance, etc. As expected, the impact effects on consumption and effort decline the further in advance the war is anticipated. The effects on output increase because of the availability of extra capital during the war. We also see increased disinvestment associated with the period of temporarily high purchases. The effect of the war on interest rates is smaller the further in advance it is anticipated. Finally, note that the path of consumption during the war becomes a lot smoother (despite the uneven pattern of shocks) when households are allowed some scope

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established after the cessation of hostilities. See various issues of the *Survey of Current Business* for 1946. For example in a survey of the postwar adjustment of the U.S. economy in the February 1946 issue there is repeated reference to "restoration of the 40-hour week".

[17] See Hall(1971), and Barro and King(1984).

for capital accumulation beforehand.

The last column of Table 3, Model(U), gives the predictions of the model when households are assumed to be extremely myopic. The high government spending during period 0 is expected to last only one period so households act to buffer it by disinvesting, not realising that there will also be high government purchases in period 1. Thus they begin period 1 with a run down capital stock and are "surprised" with another shock to government spending. They disinvest further, not expecting the war to continue into period 2, and are again surprised when it does. The effect of this succession of surprises is to cause consumption to decline by increasing amounts over the course of the war, output to rise by very little and effort to increase by increasing amounts. All of this is attributable to the manner in which the capital stock behaves.

Focusing on variations in average weekly hours of work as the empirical counterpart of the effort variable in the model may be unnecessarily restrictive. In particular it ignores the big changes in the participation rate that occurred during WWII. Table 4 shows what happens when the model is simulated using an alternative concept of effort. In particular effort is now identified with a composite of weekly hours of work and a measure of the participation rate. I define the participation rate as the ratio of the number of Full Time Equivalent Employees in Private Industry and Government and Government Enterprises to the Total Population. The average value of this ratio over 1929-82 is 0.324, so $N$ is set equal to (0.333)(0.324). A lower value of $N$ effectively means a higher elasticity of labor supply. Bearing this in mind it is straightforward that we get smaller declines in consumption, and greater increases in output, effort and investment in response to the same shocks. Interest rates increase by more,which we can rationalise either in terms of the greater growth in consumption that occurs over the course of the war or in terms of greater effort enhancing the marginal productivity of capital.

## 7. Conclusion

In this paper I attempt to evaluate the ability of the basic neoclassical growth model to explain the effects of temporary government purchases on the macroeconomy. I focus on one particular episode of temporary government purchases, namely World War II, and find that the model achieves some measure of success in explaining movements in quantities over the course of the conflict. The model performs least satisfactorily in explaining movements in interest rates.

A number of points should be noted in conclusion. The analysis in this paper treats all of the variation in government purchases over the course of World War II as changes in purchases of final output. Yet we know that the government also "purchased" significant quantities of inputs, most notably labor by means of the draft. It is common in macroeconomic analysis to abstract from the compositional effects of changes in government purchases, and to the extent that the technology used by the government to produce its final output using inputs of capital and labor is the same as that available to the private sector such abstraction is innocuous. In Wynne(1989) I extend the basic model to allow for government purchases of inputs and find that this extension has some ability to explain the behaviour of interest rates.

A common concern when looking at wartime economies is that there may be some unobservable shift in agents preferneces for the duration of the conflict that induces them to work harder or save more than they normally would at prevailing real wages and interest rates. One would expect such a change in tastes to be reflected in productivity increases over and above those that could be explained by other factors. There were large increases in the productivity of both labor and capital in the United States over the course of the war. But most of the increase can probably be attributed to technological breakthroughs and improvements in the organization of production in response to the high level of demand rather than greater effort associated with a desire to win the war. In support of this argument Milward(1987) notes that there were almost no improvements in productivity in the coal mining industry over the course of the war, despite the fact that coal was still the most important source of energy for industrial production.

I model the increase in government purchases associated with World War II as a purely temporary phenomenon, although it is clear from Figure 2 that there was a permanent increase in the share of GNP absorbed by the government at about the same time. It may be possible to extend the model to allow for this, although determining the timing of the permanent increase would be rather difficult. One alternative would be to look at the experience over the course of World War I, although here we would have to worry about the problem of poor quality data. Two additional advantages to looking at the experience during World War I are that it wasn't preceded by a major depression and that the use of price controls was less extensive than during World War II.

| Table 1.A<br>Sample Autocorrelations | | | | | | |
|---|---|---|---|---|---|---|
| Series | Nobs | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ |
| GNP | 98 | 0.96 | 0.92 | 0.89 | 0.85 | 0.82 |
| Consumption | 98 | 0.96 | 0.93 | 0.89 | 0.86 | 0.82 |
| Investment | 97 | 0.89 | 0.75 | 0.63 | 0.57 | 0.53 |
| Govt Purchases | 98 | 0.95 | 0.88 | 0.83 | 0.78 | 0.75 |

**Notes to Table 1.A.** All series are in natural logs and per capita. For the period 1929-1986, data are from *National Income and Product Accounts of the United States, 1929-1982: Statistical Tables* and recent issues of *Survey of Current Business*. For the period 1889-1928 data are from Table A-IIa of Kendrick *Productivity Trends in the United States*. Data in 1929 dollars were converted to 1982 dollars using 1929 as the year of overlap.

| Table 1.B<br>Sample Autocorrelations: First Differences | | | | | | |
|---|---|---|---|---|---|---|
| Series | Nobs | $r_1$ | $r_2$ | $r_3$ | $r_4$ | $r_5$ |
| GNP | 97 | 0.25 | 0.04 | -0.11 | -0.24 | -0.24 |
| Consumption | 97 | -0.06 | -0.08 | -0.08 | 0.03 | -0.09 |
| Investment | 96 | 0.21 | -0.10 | -0.29 | -0.14 | -0.00 |
| Govt Purchases | 97 | 0.29 | -0.20 | -0.14 | -0.24 | -0.31 |

**Notes to Table 1.B.** All series are first differences of per capita natural logs.

**Table 2**

**Test for unit roots**

Model: $z_t = \mu + \gamma t + \rho_1 z_{t-1} + \ldots + \rho_k(z_{t-k+1} - z_{t-k})$

| Series | Nobs | $k$ | $\hat{\gamma}$ | $t(\hat{\gamma})$ | $\hat{\rho}_1$ | $\tau(\hat{\rho}_1)$ | se |
|---|---|---|---|---|---|---|---|
| GNP | 96 | 2 | 0.004 | 3.693 | 0.801 | −3.755* | 0.056 |
| Consumption | 97 | 1 | 0.002 | 2.330 | 0.887 | -2.306 | 0.0004 |
| Investment | 95 | 2 | 0.003 | 2.539 | 0.816 | −3.538* | 0.215 |
| Govt Purchases | 96 | 2 | 0.007 | 4.214 | 0.724 | −4.678** | 0.178 |

**Notes to Table 2.** $z_t$ is the natural log of the series. $t(\hat{\gamma})$ is the usual t-statistic for $\hat{\gamma}$. $\tau(\hat{\rho}_1)$ is the ratio of $\hat{\rho}_1 - 1$ to its standard error. * denotes reject $H_0 : \rho_1 = 1$ at the 5% level; ** denotes rejection at the 1% level.

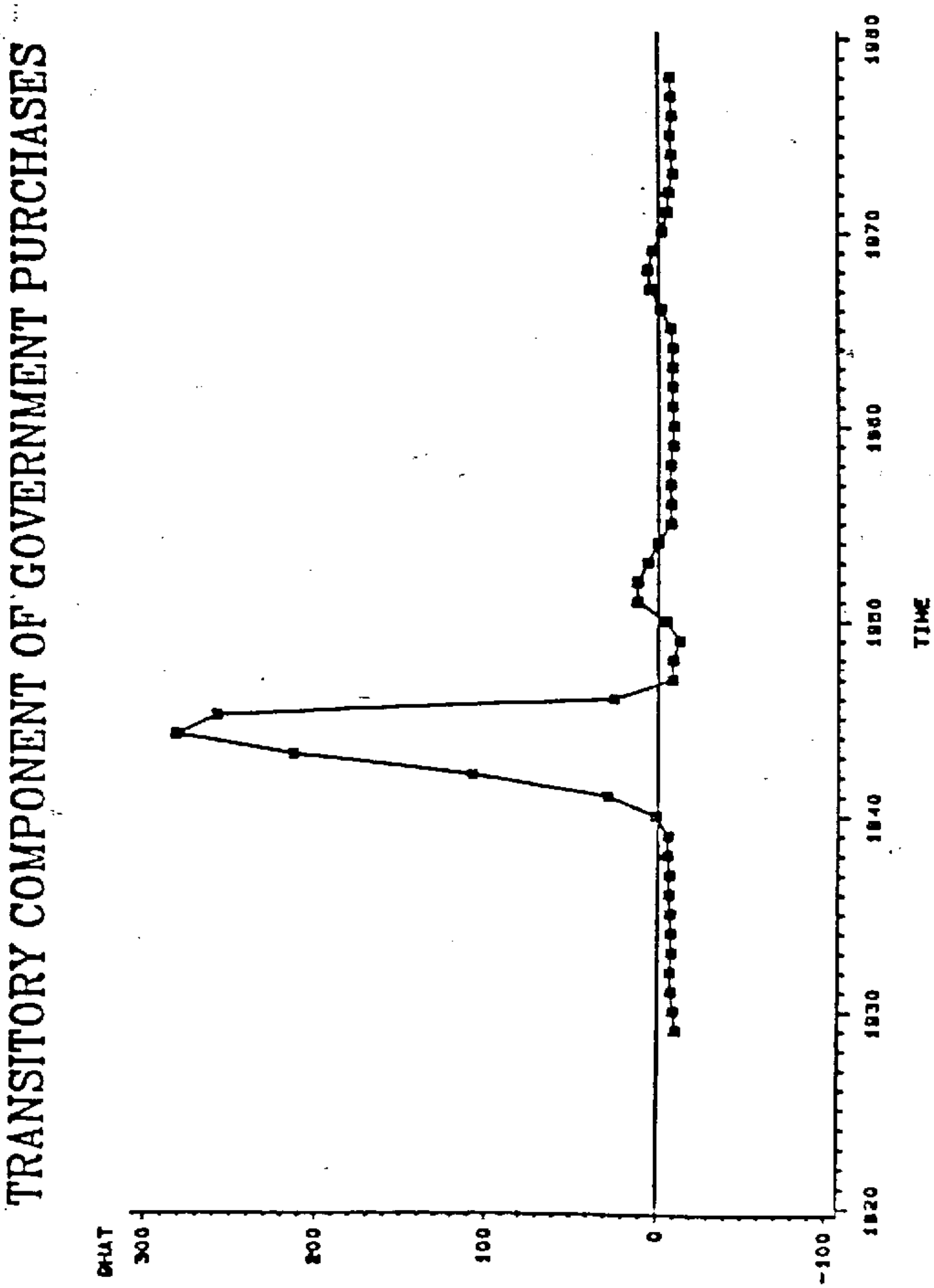
TRANSITORY COMPONENT OF GOVERNMENT PURCHASES
GHAT
300
200
100
0
-100
1920
1930
1940
1950
1960
1970
1980
TIME

**Figure 1**

| Table 3 | | | | | | |
|---|---|---|---|---|---|---|
| **Baseline simulations** | | | | | | |
| **Consumption** | | | | | | |
| Year | Actual | Model(0) | Model(1) | Model(2) | Model(3) | Model(U) |
| 1 | -10.2 | -26.2 | -19.0 | -14.4 | -11.5 | -4.9 |
| 2 | -11.0 | -23.8 | -17.9 | -14.2 | -11.8 | -13.9 |
| 3 | -11.0 | -21.3 | -16.5 | -13.5 | -11.5 | -24.2 |
| 4 | -8.0 | -18.4 | -14.5 | -12.0 | -10.4 | -31.5 |
| **Output** | | | | | | |
| 1 | 14.5 | 21.0 | 23.0 | 24.3 | 25.1 | 4.0 |
| 2 | 31.0 | 22.7 | 24.3 | 25.4 | 26.1 | 6.7 |
| 3 | 37.4 | 18.0 | 19.3 | 20.2 | 20.7 | 7.2 |
| 4 | 31.0 | 7.6 | 8.7 | 9.4 | 9.9 | 3.4 |
| **Effort** | | | | | | |
| 1 | 7.8 | 31.5 | 28.1 | 25.9 | 24.5 | 6.0 |
| 2 | 12.5 | 31.0 | 28.2 | 26.4 | 25.2 | 13.7 |
| 3 | 13.0 | 26.2 | 23.9 | 22.5 | 21.5 | 21.0 |
| 4 | 8.8 | 17.4 | 15.5 | 14.3 | 13.5 | 23.7 |
| **Investment** | | | | | | |
| 1 | -52.1 | 52.7 | 44.9 | 39.9 | 36.7 | -63.1 |
| 2 | -69.5 | -33.1 | -39.4 | -43.5 | -46.1 | -119.5 |
| 3 | -66.9 | -113.9 | -119.1 | -122.4 | -124.5 | -151.1 |
| 4 | -56.4 | -142.3 | -146.5 | -149.2 | -150.9 | -128.0 |
| **Interest Rates** | | | | | | |
| 1 | -3.6 | 2.5 | 1.2 | 0.3 | -0.3 | 0.9 |
| 2 | -2.4 | 2.5 | 1.4 | 0.7 | 0.2 | 2.6 |
| 3 | -3.8 | 2.9 | 2.0 | 1.5 | 1.1 | 4.5 |
| 4 | -23.9 | 3.4 | 2.7 | 2.2 | 1.9 | 5.9 |

**Notes to Table 3.** (1) Percentage deviations from steady state of key aggregates. (2)Parameter values: $\beta = 0.9615, \sigma_{cc} = \sigma_{ll} = -1, \sigma_{cl} = \sigma_{lc} = 0, N = 0.333, \theta_n = 0.667, \delta = 0.1, \sigma_{KN} = 1, \theta_g = 0.2051, \xi_x = 1.018$. (3)Model(0) denotes predictions of the model when the war begins in period 0, Model(1) the predictions when the war begins in period 1, etc.

Model(U) denotes the predictions of the model when the higher purchases are unanticipated each period. (4)Year 1 denotes the first year of the war, Year 2 the second, etc. For the actual data, 1942 is taken to be the first year of WWII, and 1945 the fourth. (5)The data for consumption, output and investment under Actual were obtained by fitting a common deterministic trend to Personal Consumption Expenditures, Gross Private Investment, Government Purchases of Goods and Services, and GNP, all in terms of billions of 1982 dollars and expressed in per capita terms, over the period 1889-1982. The reported numbers are the residuals from this system expressed as a fraction of the predicted values. The data for effort under Actual are the percentage deviations of Average Weekly Hours in Manufacturing during the war years from the average value over 1931-40. The data under Actual for the interest rate are the deviations of the ex post real rate in each of the war years from its sample mean over 1889-1982.

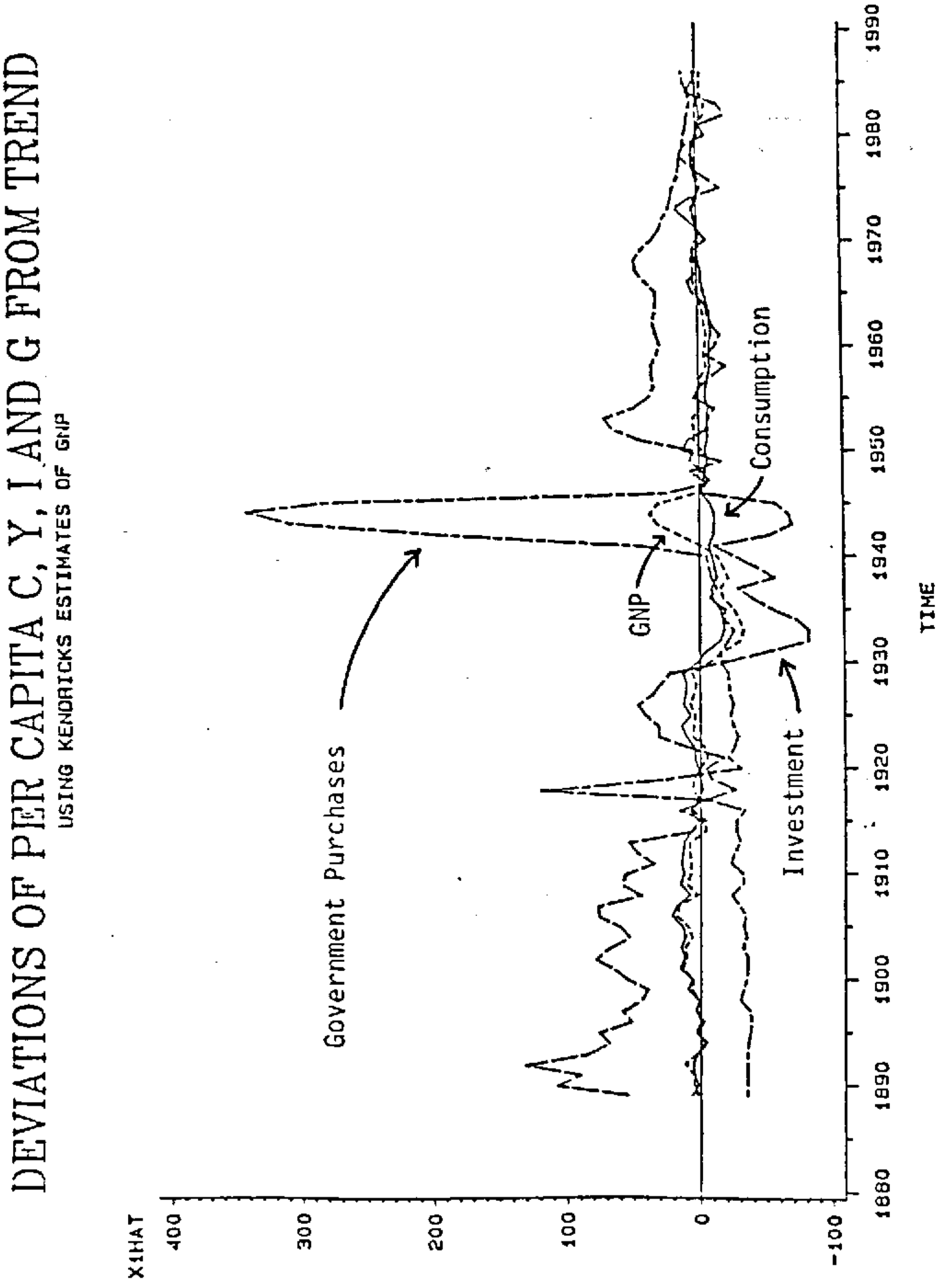
DEVIATIONS OF PER CAPITA C, Y, I AND G FROM TREND
USING KENDRICKS ESTIMATES OF GNP
X1HAT
400
300
200
100
0
-100
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
TIME
Government Purchases
GNP
Consumption
Investment

**Figure 2**

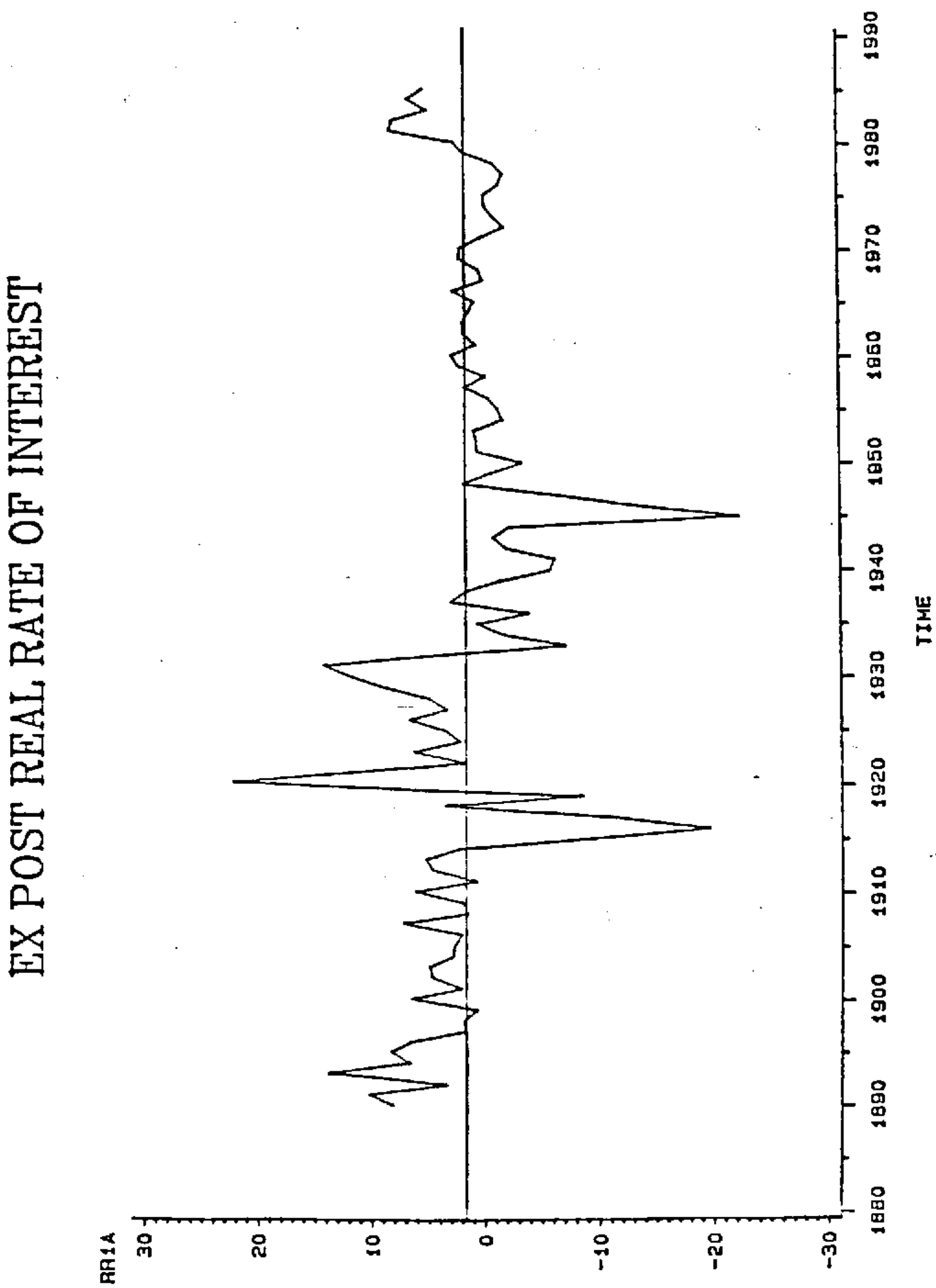
EX POST REAL RATE OF INTEREST
RR1A
30
20
10
0
-10
-20
-30
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
TIME

**Figure 3**

| Table 4 | | | | | | |
|---|---|---|---|---|---|---|
| **Baseline simulations: Alternative definition of effort** | | | | | | |
| **Consumption** | | | | | | |
| Year | Actual | Model(0) | Model(1) | Model(2) | Model(3) | Model(U) |
| 1 | -10.2 | -21.0 | -13.4 | -8.8 | -6.1 | -4.2 |
| 2 | -11.0 | -17.4 | -11.4 | -7.8 | -5.7 | -11.6 |
| 3 | -11.0 | -14.5 | -9.7 | -6.9 | -5.2 | -20.0 |
| 4 | -8.0 | -11.8 | -8.1 | -5.8 | -4.5 | -25.7 |
| **Output** | | | | | | |
| 1 | 14.5 | 30.9 | 32.0 | 32.6 | 33.0 | 6.2 |
| 2 | 31.0 | 33.4 | 34.2 | 34.7 | 35.0 | 11.7 |
| 3 | 37.4 | 27.7 | 24.3 | 28.7 | 29.0 | 14.8 |
| 4 | 31.0 | 14.4 | 15.0 | 15.3 | 15.4 | 12.5 |
| **Effort** | | | | | | |
| 1 | 46.4 | 46.3 | 40.4 | 36.9 | 34.8 | 9.3 |
| 2 | 66.9 | 45.3 | 40.7 | 37.9 | 36.3 | 20.8 |
| 3 | 69.3 | 37.6 | 34.0 | 31.8 | 30.5 | 31.1 |
| 4 | 58.8 | 23.4 | 20.5 | 18.8 | 17.8 | 34.1 |
| **Investment** | | | | | | |
| 1 | -52.1 | 81.0 | 68.3 | 60.8 | 56.3 | -55.9 |
| 2 | -69.5 | -4.0 | -14.0 | -20.0 | -23.5 | -104.3 |
| 3 | -66.9 | -89.8 | -97.7 | -102.3 | -105.1 | -129.6 |
| 4 | -56.4 | -129.4 | -135.6 | -139.3 | -141.5 | -106.3 |
| **Interest Rates** | | | | | | |
| 1 | -3.6 | 3.6 | 2.0 | 1.0 | 0.4 | 0.9 |
| 2 | -2.4 | 3.0 | 1.7 | 0.9 | 0.5 | 2.5 |
| 3 | -3.8 | 2.7 | 1.7 | 1.1 | 0.7 | 4.3 |
| 4 | -23.9 | 2.5 | 1.7 | 1.3 | 1.0 | 5.5 |

**Notes to Table 4.** (1)Percentage deviations from steady state of key aggregates. (2)Parameter values: $\beta = 0.9615, \sigma_{cc} = \sigma_{ll} = -1, \sigma_{cl} = \sigma_{lc} = 0, N = (0.333)(0.324), \theta_N = 0.667, \delta = 0.1, \sigma_{KN} = 1, \theta_g = 0.2051, \xi_x = 1.018$. (3)Model(0) denotes predictions of the model when the war begins in period 0, Model(1) the predictions when the war begins in period 1, etc. Model(U) denotes the predictions of the model when the higher purchases are unanticipated each period. (4)Year 1 denotes the first year of the war, Year 2 the second, etc. For the actual data, 1942 is taken to be the first year of WWII, and 1945 the fourth. (5) The data under Actual for effort are the sum of the changes in Average Weekly Hours in Manufacturing and the participation rate as defined in the text. For definition of the rest of the data under Actual see notes to Table 3.

## Appendix

The equilibrium path of the economy is given by the system of equations

$$D_1U(c_t, 1 - N_t) = \lambda_t$$

$$D_2U(c_t, 1 - N_t) = \lambda_t D_2F(k_t, N_t)$$

$$\beta^* \lambda_{t+1}[(1 - \delta) + D_1F(k_{t+1}, N_{t+1})] = \xi_x \lambda_t$$

$$F(k_t, N_t) + (1 - \delta)k_t = c_t + \xi_x k_{t+1} + g_t.$$

Taking a linear approximation of this system around the steady state yields the following:

$$\sigma_{cc}\hat{c}_t - \sigma_{cl}\frac{N}{1 - N}\hat{N}_t = \hat{\lambda}_t$$

$$\sigma_{lc}\hat{c}_t - \sigma_{ll}\frac{N}{1 - N}\hat{N}_t = \hat{\lambda}_t + \gamma_{Nk}\hat{k}_t + \gamma_{NN}\hat{N}_t$$

$$\hat{\lambda}_{t+1} + \{\frac{(\xi_x - \beta^*(1 - \delta))}{\xi_x}\}(\gamma_{kk}\hat{k}_{t+1} + \gamma_{kN}\hat{N}_{t+1}) = \hat{\lambda}_t$$

$$\theta_k\hat{k}_t + \theta_N\hat{N}_t + (1 - \delta)\frac{k}{y}\hat{k}_t = \theta_c\hat{c}_t + \frac{k}{y}\xi_x\hat{k}_{t+1} + \theta_g\hat{g}_t$$

where $\sigma_{ij}$ = the elasticity of the marginal utility of $i$ (consumption or leisure) with respect to $j$ (leisure or consumption), $\gamma_{ij}$ = the elasticity of the marginal product of $j$ with respect to $i$, $\theta_k$ = the share of output accruing to capital, $\theta_N$ = the share of output accruing to labour, $\theta_c$ = the share of consumption expenditures in steady state output, and $\theta_g$ = the share of government purchases in steady state output. Note that constant returns to scale implies $\gamma_{kk} + \gamma_{kN} = 0$ and $\gamma_{Nk} + \gamma_{NN} = 0$. Also $\gamma_{kN} = \theta_N/\sigma_{KN}$ and $\gamma_{Nk} = \theta_k/\sigma_{KN}$, where $\sigma_{KN}$ is the elasticity of substitution between capital and labour. Thus $\theta_N$ and $\sigma_{KN}$ fully characterize the technology. All elasticities and shares are evaluated at their stationary state values. The hats "^" denote percentage deviations from steady state levels, i.e. $\hat{x}_t = (x_t - x)/x$.

Solution of the linearised model proceeds in two stages. First we solve the pair of simultaneous difference equations in $\hat{k}$ and $\hat{\lambda}$ that produce the dynamics. Then we solve for $\hat{c}$, $\hat{n}$, $\hat{y}$ , $\hat{i}$ and $\hat{r}$ at each point in time as functions of the state variables $\hat{k}$ and $\hat{\lambda}$, and the forcing variable $\hat{g}$. From the first two equations in the linearized system we obtain

$$\begin{pmatrix} \hat{c}_t \\ \hat{n}_t \end{pmatrix} = \begin{pmatrix} \sigma_{cc} & -\sigma_{cl}\frac{n}{1-n} \\ \sigma_{cl} & -\sigma_{ll}\frac{n}{1-n} - \gamma_{nn} \end{pmatrix}^{-1} \begin{pmatrix} 0 & 1 \\ \gamma_{nk} & 1 \end{pmatrix} \begin{pmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{pmatrix}.$$

Using this to eliminate consumption and effort from the second pair, we obtain

$$\begin{pmatrix} \hat{k}_{t+1} \\ \hat{\lambda}_{t+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{pmatrix} + \mathbf{Q}\hat{g}_t$$

where the elements of **A** and **Q** are complicated functions of the parameters of tastes and technology, and the various shares and elasticities appearing in equations (16)-(19). This is the standard linear difference model. The solution of models of this class has been analysed by Blanchard and Kahn(1980) and is discussed at some length in the technical appendix of King, Plosser and Rebelo(1988).

From the structure of the optimization problem that underlies this difference equation system, we know that the solution will be saddle point stable (see Levhari and Liviatan(1972)). One of the eigenvalues of **A** will lie outside the unit circle and one inside. More specifically, the stable root is that associated with $\hat{k}$ and the unstable root that associated with $\hat{\lambda}$. I will not go into the particulars of the solution here, but simply assert that it is

$$\begin{aligned} \begin{pmatrix} \hat{k}_{t+1} \\ \hat{\lambda}_{t+1} \end{pmatrix} &= \mathbf{P} \begin{pmatrix} \mu_1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{P}^{-1} \begin{pmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{pmatrix} \\ &+ \mathbf{P} \begin{pmatrix} p^*_{11}Q_{kg} + p^*_{12}Q_{\lambda g} \\ 0 \end{pmatrix} \hat{g}_t \\ -\mathbf{P} \begin{pmatrix} 0 \\ I \end{pmatrix} &\{\sum_{j=0}^{\infty} \mu_2^{-1-j}(p^*_{21}Q_{kg} + p^*_{22}Q_{\lambda g})\hat{g}_{t+1+j}\}, \end{aligned} \tag{15}$$

where

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} \mathbf{P}^{-1}$$

$$\mathbf{P} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}$$

$$\mathbf{P}^{-1} = \begin{pmatrix} p^*_{11} & p^*_{12} \\ p^*_{21} & p^*_{22} \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} Q_{kg} \\ Q_{\lambda g} \end{pmatrix}$$

and $\mu_1$ is the stable root, and $\mu_2$ the unstable. Note the dependance of $\hat{k}$ and $\hat{\lambda}$ on the entire future path of government spending shocks. This is the outcome of forward looking behaviour and will generate all of the results in response to anticipated spending shocks. For further details on the solution procedure, see the technical appendix to King, Plosser and Rebelo(1988).

Having solved for $\hat{k}$ and $\hat{\lambda}$ it is straightforward to find the values of the other variables of interest. The deviations of consumption and effort from their steady state values are found from

$$\begin{pmatrix} \hat{c}_t \\ \hat{n}_t \end{pmatrix} = \begin{pmatrix} \sigma_{cc} & -\sigma_{cl}\frac{n}{1-n} \\ \sigma_{cl} & -\sigma_{ll}\frac{n}{1-n} - \gamma_{nn} \end{pmatrix}^{-1} \begin{pmatrix} 0 & 1 \\ \gamma_{nk} & 1 \end{pmatrix} \begin{pmatrix} \hat{k}_t \\ \hat{\lambda}_t \end{pmatrix}$$

Output and gross investment are given by the equations

$$\hat{y}_t = \theta_k \hat{k}_t + \theta_n \hat{n}_t,$$

$$\hat{i}_t = \theta_i^{-1}(\hat{y}_t - \theta_c \hat{c}_t - \theta_g \hat{g}_t).$$

The first of these comes from log differentiation of the production function, the second from log differentiation of the aggregate resource constraint. Net investment is defined as

$$\hat{i}_t^n = \hat{i}_t - \delta(\frac{k}{i})\hat{k}_t.$$

Finally we recover (relative) prices. In this model these are the interest rate and the real wage. The interest rate is obtained from the condition for intertemporal efficiency in consumption, which yields

$$r_t - \bar{r} = \sigma_{cc}(\hat{c}_t - \hat{c}_{t+1}) - \frac{N}{1-N}\sigma_{cl}(\hat{N}_t - \hat{N}_{t+1})$$

where $\bar{r}$ is the steady state level of the interest rate[18]. The real wage is simply the marginal product of labour:

$$\hat{w}_t = \gamma_{Nk}\hat{k}_t + \gamma_{NN}\hat{N}_t.$$

[18] $\bar{r} = (\xi_x/\beta^*) - 1$.

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