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INVESTMENT AND THE NOMINAL INTEREST RATE THE VARIABLE VELOCITY CASE

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Investment and the Nominal Interest Rate: The Variable Velocity Case

Models in which money enters as an argument of the utility function and models in which money enters as an argument of the production function are special cases of a more general model in which both households and firms hold money as a buffer between receipts and expenditures, in an effort to avoid brokerage fees. A high nominal interest rate discourages those purchases which must be financed out of previously accumulated cash balances while, at the same time, increasing the real resources devoted to intermediation. Depending on the relative strengths of these two effects, investment may be stimulated or depressed.

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I. INTRODUCTION

In an earlier paper (Koenig [1987b]), I examined the relationship between investment and the nominal interest rate in an economy where the cash balances of households and firms were rigidly linked to their respective expenditures. In such an economy the nominal interest rate acts like a tax on agents' purchases. As a result, households are inclined to save, rather than spend, when the nominal interest rate is relatively high. Provided that, at the margin, firms are able to finance at least some of their capital spending out of contemporaneous earnings, this increase in desired saving is only partially offset by a decrease in desired investment. *Realized* investment is thus greatest when the nominal interest rate is high in comparison to its own moving average—a "short-run Tobin effect."

This paper extends my earlier analysis to the case in which households and firms, at a cost in terms of real resources, are able to increase the velocity of money. In this context it is shown that the short-run Tobin effect may be reversed. Intuitively, when the nominal interest rate is high, consumption of financial services by households and firms may rise to such an extent that any decline in the consumption of *non*-financial goods and services is overwhelmed, reducing the resources available for investment.

As in my earlier paper, I allow for the possibility that some fraction, $1-\mu$, of firms' investment expenditures can be financed from contemporaneous earnings. Unless μ equals zero, there is a "Stockman effect": A tendency for investment to be depressed whenever the nominal interest rate is expected to be higher, on average, in the future than it has been in the past (Stockman [1981]). When μ equals unity, so all investment must be financed out of accumulated assets, the money balances of the representative firm can be interpreted as an argument of its production function. Regardless of the value of μ, the money balances of households may be thought of as an argument—along with gross household spending—of an indirect utility function. Thus both of the most frequently used methods of modeling the role of money—as an argument of the household utility function, and as an argument of the production function—are special cases of model developed here.

This is a substantially less restrictive framework than that employed by Fischer [1979], Sweeney

[1984], Cohen [1985], and Obstfeld [1985], each of whom has also examined the effects of anticipated policy in an economy where the velocity of money is variable.[1]

Section II discusses the utility and profit maximization problems facing households and firms, respectively, and shows how the behavior of economic agents is affected by the presence of real costs of financial management. A dynamic analysis is undertaken in Section III, and it is shown that the short–run Tobin effect may operate in reverse. A summary concludes the paper. Throughout, agents are assumed to possess perfect foresight.

II. THE MODEL: PRELIMINARY ANALYSIS

Intuition

Imagine a world in which each household receives wage and dividend income at discrete intervals, but desires to purchase and consume output continuously through time. As in Baumol [1952] or Tobin [1956], if it is costly to move into or out of interest–bearing securities, the household will choose to hold money as a buffer between its receipts and its expenditures. The more money held, on average, between wage and dividend distributions, the lower the brokerage fees paid by the household, but, also, the lower the household's interest income.

Imagine, similarly, that each firm receives a continuous flow of revenue, but makes wage and dividend distributions, and some investment expenditures, at discrete intervals.[2] Then firms, like households, will use money as a buffer between receipts and disbursements, in an effort to hold down brokerage costs.

If disbursements are staggered through time across a large number of firms, their *aggregate* flow will appear continuous, yet both firms and households will maintain positive money balances. This suggests that the economy described above be approximated by a continuous–time model of an "average" household and "average" firm, each of which carries money in order to avoid brokerage fees.[3] It is just such a model which I develop below.

The Representative Household

Explicit Transaction Costs. The representative household chooses time–paths for its real balances (m) and the rate of change of its total real wealth ($\dot{a}$) so as to maximize

$$\int_0^\infty u(c(t))exp\ (-\beta t)dt \tag{1}$$

subject to

$$\int_0^\infty [c(t) + m(t)i(t) + T(c(t), m(t))]exp\ [-\int_0^t r(s)ds]dt \leq a(0) \tag{2}$$

$$\dot{a}(t) = a(t)r(t) - [c(t) + m(t)i(t) + T(c(t), m(t))], \tag{3}$$

and $a(t) \geq 0$, where i is the nominal rate of interest, r the real interest rate, and c consumption net of "transactions costs"—i.e., net of the cost of those financial services which the household purchases in an effort to economize on its money balances. The household is assumed to regard its initial wealth and the real and nominal interest rates as exogenous. The wealth variable is meant to include not only the asset–holdings of the household, but also the present discounted value of future wages and net (lump–sum) governmental transfers. Transaction costs, $T(c, m)$, are a function of net consumption and the real balances held by the household. In the model of the demand for money due to Baumol and Tobin, for example, transaction costs are proportional to the velocity of money: $T(c, m) = \tau c/m$, where τ represents the real cost of each money–bond or bond–money exchange. In a cash–in–advance model, on the other hand, transaction costs are zero for c less than m, and explode to infinity if households attempt to increase c beyond m.

The utility function, $u(\cdot)$, is assumed to be increasing and strictly concave. Further, $1 + T_c$ is assumed to be positive, so that an increase in net consumption, real balances held constant, must imply an increase in gross consumption expenditures, $x \equiv c + T$. With $u'/(1 + T_c)$ greater than zero, equality will hold in constraint (2). It is also reasonable to require that $T_{cc}T_{mm} - T_{cm}^2$ be negative: without increasing returns in the transactions technology, using money as a buffer between receipts and expenditures would make little sense.

The necessary conditions for an optimum include:

$$0 = H_{\dot{a}} = \lambda - u'/(1 + T_c), \tag{4}$$

$$0 = H_m = -(i + T_m)u'/(1 + T_c), \tag{5}$$

$$\dot{\lambda} = \beta\lambda - H_a = \beta\lambda - ru'/(1 + T_c), \tag{6}$$

where $H \equiv u(c) + \lambda\dot{a}$ is the Hamiltonian function. It is also required that (4) and (5) correspond to a maximum of the Hamiltonian. For this it is sufficient that the Hamiltonian function be strictly concave in the control variables. This means that the utility function must be sufficiently concave to overcome increasing returns in the transactions technology. Strict concavity of H also implies that $T_{mm} > 0$, so that transaction costs decrease at a decreasing rate as real balances expand. (For details, see the Appendix.) From (5), households add to their real balances until the resultant marginal decrease in the costs of cash–management equals the nominal interest rate.

Marginal transaction costs, T_c, act just like a sales tax levied on household purchases: one unit of gross expenditure will purchase only $1/(1+T_c)$ units of output for actual consumption. Accordingly, households prefer to make their purchases when marginal transaction costs are relatively low. Formally, combine equations (4) and (6) and rearrange terms to obtain

$$\dot{u}'/u' = \beta - r + \dot{T}_c/(1 + T_c). \tag{7}$$

The corresponding condition would be $\dot{u}'/u' = \beta - r$ in an economy in which transactions costs were identically equal to zero.

It seems reasonable to suppose that, given the nominal interest rate, the more purchases an individual expects to make the more cash he will plan to carry. Since, from equation (5),

$$dm = -(T_{cm}/T_{mm})dc - (1/T_{mm})di, \tag{8}$$

this means that T_{cm} must be negative. It also seems reasonable to suppose that for a given nominal interest rate, by spending more (increasing x), an individual ought to be able to purchase more output (increase c). Using (8) to eliminate dm from the equation $dx = (1 + T_c)dc + T_m dm$, then solving for dc,

$$dc = [dx + (T_m/T_{mm})di]/[(1+T_c) - T_{cm}(T_m/T_{mm})]. \tag{9}$$

Net consumption will be increasing in gross consumption expenditure, given the nominal interest rate, if and only if $1 + T_c$ is greater than $T_{cm}(T_m/T_{mm})$.

From equation (8), given net consumption, the higher is the nominal interest rate, the lower are real money balances. But T_{cm} is less than zero—i.e., the marginal cost of purchasing output is high when real balances are low. It follows that for any given real interest rate, net consumption will be lower the greater is the nominal interest rate and, consequently, net consumption will fall through time if the nominal interest rate is rising through time. The latter result can be formalized by expanding equation (7), using (8) to eliminate $\dot{m}$:

$$\dot{c} = \lambda^2[T_{mm}(r-\beta) + (T_{cm}/(1+T_c))\dot{i}]/[(H_{\dot{a}\dot{a}}H_{mm} - H_{\dot{a}m}^2)(1+T_c)]. \tag{10}$$

By strict concavity of H, the denominator of the expression on the right–hand–side of this equation is positive. Thus $\dot{c}$ is increasing in the real rate of interest and decreasing in $\dot{i}$. The smaller is u''/u' in magnitude, the smaller is $H_{\dot{a}\dot{a}}H_{mm} - H_{\dot{a}m}^2$, and so the more sensitive is net consumption to changes in interest rates. Intuitively, the less curvature there is in the utility function, the easier it is for households to shift consumption through time.

Total transaction costs respond ambiguously to changes in the nominal interest rate. An increase in i reduces real balances, placing *upward* pressure on transaction costs, but also decreases net consumption, which may place *downward* pressure on transaction costs. Formally, $\dot{T} = \dot{c}T_c + \dot{m}T_m$, or, using equations (8) and (10) to eliminate $\dot{m}$ and $\dot{c}$,

$$\begin{aligned}\dot{T} = \lambda^2\{&(T_cT_{mm} - T_mT_{cm})(r-\beta) \\ &+ [(T_cT_{cm} - T_mT_{cc})/(1+T_c) + T_m u''/u']\dot{i}\}/[(H_{\dot{a}\dot{a}}H_{mm} - H_{\dot{a}m}^2)(1+T_c)]\end{aligned} \tag{11}$$

The coefficients of r and $\dot{i}$ cannot, in general, be signed.

By adding equations (10) and (11), one can obtain a formula for the rate of change of gross consumption expenditure:

$$\dot{x} = A(r-\beta) - B\dot{i}. \tag{12}$$

Here

$$A \equiv \lambda^2[(1+T_c)T_{mm} - T_m T_{cm}]/[(H_{\dot{a}\dot{a}}H_{mm} - H_{\dot{a}m}^2)(1+T_c)]$$

$$B \equiv \lambda^2[(T_m T_{cc}/(1+T_c) - T_{cm}) - T_m u''/u']/[(H_{\dot{a}\dot{a}}H_{mm} - H_{\dot{a}m}^2)(1+T_c)].$$

From strict concavity of H and the discussion accompanying equation (9), A is positive. The sign of B, on the other hand, is ambiguous. If the utility function has little curvature ($|u''/u'|$ is small), so that net consumption is sensitive to changes in the nominal interest rate, gross spending and net consumption will both fall when the nominal interest rate is high (B will be positive), and the amount of output available for investment will tend to rise. If the utility function is sufficiently concave however, B will be negative, and the decline in net consumption which occurs as the nominal interest rate goes up will be more than offset by increased expenditure on financial services. Total household expenditure will increase, tending to reduce the amount of output available for investment.

Real Balances as an Argument of the Utility Function.[4] The analysis of the representative household can easily be translated into the more streamlined notation of Sidrauski [1967]. Define a new instantaneous utility function, $U(\cdot,\cdot)$, by $U(c+T(c,m),m) \equiv u(c)$.[5] One then has $U_x = \lambda = u'/(1+T_c) > 0$, and $U_m = -\lambda T_m = -T_m u'/(1+T_c)$, from which it follows that the necessary conditions for an optimum can be rewritten as

$$U_m/U_x = i \tag{5'}$$

$$\dot{U}_x/U_x = \beta - r. \tag{6'}$$

Strict concavity of the function H is equivalent to strict concavity of $U(\cdot,\cdot)$. Further, the conditions $T_{cm} < 0$ and $(1+T_c) > T_{cm}(T_m/T_{mm})$ are satisfied if and only if both gross consumption and real balances are normal goods. (See the Appendix.)

In the new notation, equation (12) becomes

$$\dot{x} = -[\Delta_2 U_x^2/\Delta](r-\beta) - [U_{xm}U_x/\Delta]\dot{i}, \tag{12'}$$

where $\Delta \equiv U_{xx}U_{mm} - U_{xm}^2 > 0$ and, by normality, $\Delta_2 \equiv (U_{mm}U_x - U_mU_{xm})/U_x^2 < 0$. In the discussion following equation (12) it was pointed out that gross expenditures need not decline with net consumption in response to an increase in the nominal interest rate. Equation (12′) says that this will occur precisely when the cross–partial derivative of the Sidrauski utility function is negative.[6]

Though the Sidrauski formulation of the household maximization problem is notationally convenient, it is not very intuitive, and so will not be employed in the analysis which follows.

The Representative Firm

Explicit Transaction Costs. The representative firm has real gross revenue $Nf(K/N)$, where K and N denote, respectively, the capital and labor available to the firm. The production function, $f(\cdot)$, is required to be strictly concave, with $f'(0) = \infty$ and $f'(\infty) < \beta + \delta$, where δ is the rate of depreciation. For analytic simplicity, each worker is assumed to provide one unit of labor, and the number of workers per firm is exogenously fixed.[7]

There are two different types of expenditures: those financed out of contemporaneous earnings, and those financed out of accumulated assets. The former are meant to correspond to expenditures which are, in the real world, for any particular firm, "smooth" with respect to time: routine, daily purchases which are made using same–day receipts. The latter are meant to correspond to expenditures which are, in contrast, "lumpy": wage and dividend distributions for example, made once per month or once per quarter. Assuming that sales revenue is smooth, it is only lumpy expenditures which present a financial management problem. If it was interested solely in minimizing lost interest, a firm would immediately put any revenues, net of smooth expenditures, into bonds, selling off the bonds whenever a lumpy payment fell due. In fact though, the firm will want to keep the number of money–bond transactions—and, hence, brokerage fees—in check. It can do so by allowing net receipts to accumulate as cash for a time between each purchase of bonds. The higher its average cash balances relative to its flow of net revenue, the lower the brokerage fees the firm incurs.

Here $S = S(E, M)$ will be used to denote the brokerage costs which the representative firm must pay, as a function of its net revenues, E, and real money balances, M. The most reasonable assumption is that firms and households have access to the same transactions technology, so that $S(E, M) = T(E, M)$ for all E and M. It follows that $1 + S_E > 0, S_{MM} > 0, S_{EM} < 0, 1 + S_E > S_{EM}(S_M/S_{MM})$, and $S_{EE}S_{MM} - S_{EM}^2 < 0$.

Both transaction costs and a fixed fraction, $1 - \mu$, of investment expenditures will be assumed to be financed out of contemporaneous earnings.[8] Thus

$$\begin{aligned} E(t) = {} & Nf(K(t)/N) - (1-\mu)[\dot{K}(t) + \delta K(t) + K(t)\phi(\dot{K}(t)/K(t))] \\ & - S(E(t), M(t)), \end{aligned} \tag{13}$$

where $K\phi(\dot{K}/K)$ is the cost of installing new equipment. Following Hayashi [1982], ϕ is assumed strictly convex, with $\phi(0) = \phi'(0) = 0$.

The firm chooses time–paths for $\dot{K}$ and $\dot{M}$ which maximize the present discounted value of its distributions to households:[9]

$$\int_0^{\infty} D(t) exp\, [-\int_0^t r(s)ds]dt, \tag{14}$$

subject to

$$\begin{aligned} D(t) = {} & E(t) - \mu[\dot{K}(t) + \delta K(t) + K(t)\phi(\dot{K}(t)/K(t))] - \dot{M}(t) \\ & - \pi(t)M(t) + G(t), \end{aligned} \tag{15}$$

where π is the inflation rate and G represents lump–sum transfers received by the firm from the monetary authority. Both π and G are exogenous to the firm, as is the real interest rate, r.

For optimality it is necessary that

$$0 = J_{\dot{K}} = q - (1 + \phi')[\mu + (1-\mu)/(1 + S_E)] \tag{16}$$

$$0 = J_{\dot{M}} = n - 1 \tag{17}$$

$$\dot{q} = rq - J_K = rq - \{f'/(1 + S_E) - (\delta + \phi - \phi'\dot{K}/K)[\mu + (1-\mu)/(1 + S_E)]\} \tag{18}$$

$$\dot{n} = rn - J_M = rn + \pi + S_M/(1 + S_E), \tag{19}$$

where $J \equiv D + q\dot{K} + n\dot{M}$ is the Hamiltonian function.

Just as marginal transactions costs, T_c, act like a sales tax on household purchases, here S_E acts like a tax on the firm's net revenue (E). Thus, according to (18), it is the "after tax" marginal product of capital which measures the payoff to be derived in the future from an additional unit of capital. The marginal cost of investment is also distorted by marginal transactions costs (c.f. eq. (16)), but only insofar as investment reduces net revenue—i.e., only insofar as μ is less than unity.

The implications of these distortions are most easily seen when one ignores installation costs. Using equation (16) to eliminate the shadow price, q, from (18), one then has:

$$f' = [\mu(1 + S_E) + (1 - \mu)](r + \delta) + (1 - \mu)\dot{S}_E/(1 + S_E). \tag{20}$$

When all investment expenditures are financed from contemporaneous earnings ($\mu = 0$), only *changes* in marginal transaction costs distort the desired capital stock: firms want a large capital stock when they expect marginal transaction costs to be lower in the future than they are currently. Investment will thus tend to be greatest when S_E is high relative to its past and future values. When μ equals unity, only the *level* of marginal transaction costs matters: the higher is S_E, the lower the capital stock desired. Hence investment will tend to be greatest when S_E is expected to be lower in the future than it has been in the past.

Combining (17) and (19), one has

$$-S_M/(1 + S_E) = i, \tag{21}$$

which says firms add to their real money balances until the resultant marginal decrease in transaction costs, adjusted for the "tax" on net revenue, is equal to the nominal interest rate. Differentiate (21) to obtain

$$dM = [(S_M S_{EE} - (1 + S_E)S_{EM})dE - (1 + S_E)^2 di]/[(1 + S_E)S_{MM} - S_M S_{EM}]. \tag{22}$$

Thus the firm's demand for real balances is decreasing in the nominal interest rate. It will be increasing in the flow of net revenue provided that $S_M S_{EE} - (1 + S_E)S_{EM}$ is positive, which I will assume to be the case.[10]

Second–order conditions are satisfied if the Hamiltonian is strictly concave in $\dot{K}, K$, and M (Mangasarian [1966]). This means that f'' and ϕ'' must be large enough in magnitude to offset increasing returns to scale in the transactions technology. (See the Appendix.)

Real Balances as a Factor of Production. According to equation (13), when μ equals unity, net revenue is a function of capital, labor, and real balances alone. Thus one can think of real balances as an argument of a net production function, $E(K, N, M)$,à la Levhari and Patinkin [1968]. Differentiation of (13) establishes that $E_K = f'/(1 + S_E)$ and $E_M = -S_M/(1 + S_E)$. Further differentiation establishes that to assume $S_M S_{EE} - (1 + S_E)S_{EM} > 0$ in equation (22) is equivalent to requiring that E_{KM} be positive—i.e., to requiring that capital and money be complements in production.

Identities and Equilibrium Conditions

The wealth of the representative household consists of its real money balances, the present discounted value of the distributions which it receives from firms, and the present discounted value of the lump–sum transfers it receives from the government:

$$a(t) = m(t) + \int_t^{\infty} [D(s)/N + g(s)] exp\,[-\int_t^s r(v)dv]ds. \tag{23}$$

Of course,

$$\pi(t) = i(t) - r(t). \tag{24}$$

If the government distributes money to households and to firms at equal percentage rates, one will also have

$$\theta(t) = \dot{m}(t)/m(t) + \pi(t) \tag{25a}$$

$$\theta(t) = \dot{M}(t)/M(t) + \pi(t), \tag{25b}$$

where $\theta(t)$ is the common growth rate of nominal balances. While households and firms treat governmental transfers as exogenous, in market equilibrium $g = \theta m$ and $G = \theta M$.

Take the time–derivative of the right–hand–side of equation (23) and equate it to the right–hand–side of equation (3). Using (15), (24), and (25) one obtains

$$f = c + (\dot{k} + \delta k + k\phi) + (T + S/N), \tag{26}$$

where $k(t) \equiv K(t)/N$ is the capital/labor ratio. Thus output per worker is split between consumption, investment, and transaction costs per worker.

Steady State Comparative Statics

Equation (7) implies that in steady state the real rate of interest will equal the rate of time preference. If μ equals zero, so that all investment is financed from contemporaneous earnings, equation (18) implies that the amount of capital per household in steady state is independent of monetary policy. It then follows that gross output and gross consumption spending per household are, in steady state, also independent of monetary policy. In general though, with $\mu > 0$, the higher is the steady–state nominal interest rate—or, equivalently, the higher is the steady–state inflation rate—the lower are the capital stock, output, and household spending. Regardless of the value of μ, a high nominal interest rate leads to reductions in net consumption and in the real balances of households and firms. Since utility is a function of net consumption alone, if the government wants to maximize the steady–state welfare of the representative household, it ought to drive the nominal interest rate to zero.[11]

III. ANALYSIS OF THE MODEL: DYNAMICS

Intuition

The intuition underlying the dynamic analysis is simple. Marginal transaction costs, T_c, act like a sales tax on households' purchases of output, the proceeds of which are thrown away. A constant sales tax does not affect the timing of desired consumption. Anticipated *changes* in the tax rate *do* have an impact however: households want to concentrate their consumption in periods during which the tax rate is relatively low. Whether households' gross spending moves with net consumption or against it depends on how responsive net consumption is to changes in the tax rate: if the elasticity of demand for net consumption is high, gross spending and net consumption will move together (the change in the latter overwhelming any opposing change in total tax payments); if the elasticity of demand for net consumption is low, gross spending and net consumption will move inversely. If gross spending and net purchases move together, the supply of savings will be greatest when the marginal tax rate on household purchases is relatively high. If gross spending and net purchases move in opposition to one another, the supply of savings will be greatest when the tax rate is relatively *low*.

In the present context, the marginal "tax" rate on household purchases, T_c, is higher the higher is the nominal interest rate. Accordingly, net consumption tends to be lowest when the nominal interest rate is high relative to its own moving average (c.f. eq. (10)). Gross household spending moves *with* net consumption if the utility function has little curvature (u''/u' is small in magnitude), for then it is easy for households to substitute between consumption at different dates, with the result that net consumption is very responsive to changes in the nominal interest rate. Similarly, gross household spending moves *opposite* net consumption if u''/u' is large in magnitude. In the first case, when $|u''/u'|$ is small, there tends to be a positive short–run Tobin effect: a relatively high nominal interest rate reduces household spending, freeing output for capital investment. (In eq. (12), B is positive.) If $|u''/u'|$ is sufficiently large, on the other hand (so that in eq. (12), B is negative), the short–run Tobin effect may operate in reverse.

Similarly, S_E acts like a tax on firms' net earnings, the proceeds of which are, again, thrown away. The parameter μ measures the fraction of investment expenditures which are subject to taxation: in calculating taxable earnings the firm is able to deduct $1 - \mu$ times its investment expenditures from its gross revenue. If μ equals zero, the firm is able to fully expense its investment spending. In this case it is well known that a invariant tax rate has no distortionary effect on the timing of investment. Investment demand *is* distorted, however, by anticipated *changes* in the marginal tax rate: firms will want to concentrate investment in periods in which the tax rate is relatively high, for the revenues earned in the future from a machine installed today will then be taxed at a low rate, while the current cost of the machine will be fully deductible now, while the tax rate is elevated.[12]

If μ is greater than zero, investment expenditures are only partially deductible, while the future earnings from an addition to the capital stock are fully taxed. Not surprisingly, in this situation the higher the tax rate, the lower the desired capital stock. Investment will tend to be greatest when the tax rate is expected to be lower in the future than it has been in the past.

Regardless of the value of μ, the higher in total are the tax liabilities of firms, the fewer are the resources available to the private sector for investment.

In the present context the marginal tax rate on net earnings, S_E, is higher the higher is the nominal interest rate. Accordingly, insofar as μ is less than unity there is a tendency for investment to be greatest when the nominal interest rate is thought to be high relative to its own moving average: a positive short-run Tobin effect. Insofar as μ is greater than zero there is also a Stockman effect: a tendency for investment to be greatest when the nominal interest rate is expected to be lower in the future than it has been in the past. Regardless of the value of the parameter μ, when the nominal interest rate is high, firms devote more resources to financial management, reducing the resources available for investment and tending to reverse the short-run Tobin effect.

In summary, a nominal interest rate which is high relative to its own moving average tends to increase savings by reducing household demand for non-financial goods and services. It also tends, insofar as they can be financed from contemporaneous earnings, to increase firms' desire to undertake investment projects. On the other hand, both firms and households may increase spending on financial services when the nominal interest rate is high, tending to reduce the resources available for investment. The net effect is unclear: it depends, among other things, upon how easy it is to substitute between consumption at different dates, and upon the fraction of investment expenditures which can be financed from contemporaneous earnings.

Formal Analysis

The model developed in Section II can be linearized about its steady state, and reduced to a second-order differential equation in k, the capital stock per worker:

$$\ddot{k}(t) - A_1\dot{k}(t) - A_0(k(t) - k^*) = \alpha_0(i(t) - i^*) + \alpha_1\dot{i}(t). \tag{27}$$

Here A_0, A_1, and A_2 are positive constants, and

$$\alpha_0 \equiv [\mu(\beta+\delta)(S_M S_{EE}/(1+S_E) - S_{EM})]^*/A_2$$

$$\alpha_1 \equiv \{(1-\mu)[S_M S_{EE}/(1+S_E) - S_{EM}]/(1+S_E)$$
$$+ (1/A)(f'/(\beta+\delta))[S_M/(N(1+S_E)) - BJ_{MM}]\}^*/A_2.$$

The parameters A and B are defined as in equation (12), while J, recall, is the Hamiltonian function from the representative firm's dynamic optimization problem. An asterisk indicates that an expression is evaluated at steady state. From the second–order conditions, $J_{MM} < 0$. It has already been assumed that $1 + S_E$ and $S_M S_{EE}/(1+S_E) - S_{EM}$ are positive. (See the discussion preceding eq. (13), and following eq. (22).) The coefficient α_0 is thus non–negative. The coefficient α_1 is ambiguous in sign.

In its homogeneous form (with $i(t) - i^* = \dot{i}(t) = 0$), equation (27) has two roots, z_1 and z_2, both real, with $z_1 < 0 < z_2$. The equation has, accordingly, but one convergent solution:

$$k(t) - k^* = \int_{-\infty}^{t} [\alpha_1(i(s) - \bar{i}(s)) - (\alpha_0/(z_2 - z_1))(\bar{i}^+(s) - \bar{i}^-(s))]ds \tag{28}$$

where

$$\bar{i}^+(s) \equiv z_2 \int_s^\infty i(v) exp\ [(s-v)z_2]dv$$

$$\bar{i}^-(s) \equiv -z_1 \int_{-\infty}^{s} i(v) exp\ [(s-v)z_1]dv$$

$$\bar{i}(s) \equiv [z_2/(z_2 - z_1)]\bar{i}^+(s) + [-z_1/(z_2 - z_1)]\bar{i}^-(s).$$

It follows immediately that investment is governed by

$$\dot{k}(t) = \alpha_1(i(t) - \bar{i}(t)) - (\alpha_0/(z_2 - z_1))(\bar{i}^+(t) - \bar{i}^-(t)). \tag{29}$$

Thus the rate of investment depends, in general, upon both the deviation of the nominal interest rate from its own moving average—the short–run Tobin effect—and upon the difference between weighted averages of future and past nominal interest rates—the Stockman effect. The Stockman effect operates only insofar as μ, the fraction of investment which cannot be financed out of contemporaneous earnings, is greater than zero. (If $\mu = 0, \alpha_0 = 0$.) The effect is negative in the sense that a rising nominal interest rate tends to depress investment. The short–run Tobin effect is ambiguous in sign: when α_1 is positive, a nominal interest rate which is high relative to its own moving average will tend to stimulate investment; when α_1 is negative, a relatively high nominal interest rate will depress investment.

Household behavior influences the sign of α_1—and, hence, the sign of the short–run Tobin effect—through B. The sign of B in turn depends upon the magnitude of u''/u', which measures how easy it is for households to substitute between consumption at different dates. The larger is $|u''/u'|$, the less willing are households to shift net consumption, and the smaller will be the fall in net consumption caused by an increase in the nominal interest rate. If $|u''/u'|$ is so large as to make B negative, any fall in net consumption is overwhelmed by increased spending on financial services, so that gross consumption spending actually rises, tending to reduce the resources available for investment.

The behavior of firms influences the sign of α_1 in two conflicting directions. Insofar as μ is less than unity, so that investment expenditures can be financed from contemporaneous earnings, firms have an incentive to concentrate investment in periods during which the nominal interest rate is relatively high—for in so doing, they can shift net earnings into periods during which marginal transactions costs are relatively *low*. This effect is captured by the term $[(1-\mu)(S_M S_{EE}/(1+S_E) - S_{EM})/(1+S_E)]^*/A_2$, which is always non–negative, and which is larger the smaller is μ. On the other hand, when the nominal interest rate is high, firms devote more resources to financial management, reducing the resources available for investment. This is captured by the term $[(f'/(\beta+\delta))S_M/(NA(1+S_E))]^*/A_2$, which, because S_M is less than zero, is always negative.

Special Cases

Money as a Factor of Production. Real money balances are often included as an argument of the representative firm's production function. When this is done the money balances of households are usually ignored. As noted in Section II, treating money as a factor of production is justified as long as μ equals unity, so that all investment must be financed out of previously accumulated assets. To ignore the money balances of households amounts to assuming that households face a transactions cost function which is identically equal to zero. What are the implications of these assumptions for the relationship between investment and the nominal interest rate?

With $T(c,m) \equiv 0, x$ is identically equal to c, and equation (7) becomes $\dot{u}'/u' = \beta - r$. It follows that the coefficients A and B in equation (12) take on the values $-u'/u''$ and zero, respectively. The coefficient α_1, which determines the magnitude and direction of the short–run Tobin effect, becomes

$$\alpha_1 = -S_M^*/[(1+S_E)(u'/u'')(J_{\dot{K}\dot{K}}J_{MM} - J_{\dot{K}M}^2)N^2]^*.$$

Since S_M and u'' are both negative, while $J_{\dot{K}\dot{K}}J_{MM} - J_{\dot{K}M}^2$ is positive, the short–run Tobin effect unambiguously negative—i.e., investment is concentrated in periods during which the nominal interest rate is *low* relative to its own moving average.

With μ equal to unity, α_0 is greater than zero, so there is also a Stockman effect.

Money as a Consumer Good. In Section II it was argued that one can legitimately include the representative household's real money balances along with its gross consumption spending in an indirect utility function, à la Sidraudski [1967]. When this is done the real money balances held by firms are usually ignored, which amounts to assuming $S(E,M)$ is identically zero.

With $S(E,M) \equiv 0$, desired investment is governed by the conventional Tobin's q model, as formalized by Hayashi [1982]. Household saving continues to be distorted by changes in the nominal interest rate however. Equations (27) and (29) become

$$\ddot{k}(t) - \beta\dot{k}(t) + [Af''/(1+A\phi''/k)]^*(k(t)-k^*) = [B/(1+A\phi''/k)]^*\dot{i} \tag{27'}$$

$$\dot{k}(t) = [B/(1+A\phi''/k)]^*(i(t) - \bar{i}(T)), \tag{29'}$$

respectively.[13] Thus there is no Stockman effect, and the sign of the short–run Tobin effect is the same as that of B, which, in turn, is the same as that of the cross–partial derivative of the Sidrauski utility function.

Baumol–Tobin Technology with Large Firms. In the two special cases examined thus far, either the transaction costs facing households or those facing firms have been arbitrarily set equal to zero. Are there situations in which households and firms have access to the same transactions technology, and yet the real balances of one or the other can be ignored? The answer appears to be "yes": for certain transactions technologies the money balances held by firms are negligible, provided each firm has large number of employees and can finance all of its investment from contemporaneous earnings.[14]

Suppose, for example, that the transaction costs borne by households and firms obey the Baumol–Tobin formulas $T(c,m) = \tau c/m$ and $S(E,M) = \tau E/M$, respectively, where τ is a fixed parameter. It is well–known that with a Baumol–Tobin transactions technology, agents' demand for money rises with the square root of the volume of transactions, given the nominal interest rate. In the present context, M varies with the square root of E. Now it is easy to show that the representative firm's steady–state net earnings, E^*, vary directly in proportion to the number of workers employed by the firm, N, as N rises. (Formally, E^*/N has a positive, finite limit as $N \to \infty$.) It follows that as firms get larger and larger, the steady–state money balances which they maintain *per employee* (M^*/N), shrink to zero, even though *total* money holdings (M^*) increase. More importantly, both marginal transaction costs, S_E^*, and many of the distortions to firm behavior due to *changes* in marginal transaction costs, disappear in the limit as N goes to infinity.[15] In particular, equations (27) and (29) simplify to

$$\ddot{k}(t) - [(A\beta\phi''/k + (\beta + \mu\delta))/R]^* \dot{k}(t) + (Af''/R)^*(k(t) - k^*) = (B/R)^* \dot{i} \qquad (27'')$$

$$\dot{k}(t) = (B/R)^*(i(t) - \bar{i}(t)), \qquad (29'')$$

where $R \equiv A\phi''/k + (1-\mu)$ is always greater than zero. When firms are able to finance all of their investment from contemporaneous earnings—so μ equals zero—these equations are identical to those obtained from a model in which money is purely a consumer's good. (Compare equations (27″) and (29″) to (27′) and (29′).)

Regardless of whether or not μ equals zero, according to equation (29″) there is no Stockman effect. The short–run Tobin effect has the same sign as B^*. When the transactions technology is

Baumol–Tobin, $B = m(m+r)[1-(-cu''/u')]/[2(-cu''/u')-r]$. Thus the short-run Tobin effect is positive if and only if the steady-state elasticity of intertemporal substitution, $1/(-cu''/u')^*$, is greater than unity.[16]

IV. SUMMARY

Households purchase financial services because the flow of wage and dividend distributions which they receive from firms is uneven relative to the flow of net consumption which they desire. Firms purchase financial services because the flow of their wage and dividend distributions, and, perhaps, a portion of their investment expenditures, is uneven relative to the flow of their sales receipts. Households and firms can reduce the real costs of financial management by holding money as a buffer between receipts and expenditures.

When the nominal interest rate is high, households and firms reduce their money balances. With lower money balances, households find it more difficult, at the margin, to purchase output. Consumption of non-financial goods and services accordingly tends to fall. Consumption of financial services by households and firms, on the other hand, may well rise. If *total* consumption spending falls, the resources available for investment increase. If total consumption spending rises, investment is depressed.

APPENDIX: SECOND-ORDER CONDITIONS

Households

As noted in Section II, the second-order conditions associated with the utility maximization problem of the representative household will be satisfied if the Hamiltonian function is strictly concave in the control variables, $\dot{a}$ and m:

$$0 > H_{\dot{a}\dot{a}} = \lambda[u''/u' - T_{cc}/(1+T_c)]/(1+T_c) \tag{A.1}$$

$$0 < H_{\dot{a}\dot{a}}H_{mm} - H_{\dot{a}m}^2 = -\lambda^2[T_{mm}(u''/u') - (T_{cc}T_{mm} - T_{cm}^2)/(1+T_c)]/(1+T_c). \tag{A.2}$$

Since $1+T_c>0, u'>0$, and $T_{cc}T_{mm}-T_{cm}^2<0$, equation (A.2) is satisfied only if u'' and T_{mm} are opposite in sign. Since $u(\cdot)$ is strictly concave, T_{mm} must be positive.

To impose inequalities (A.1) and (A.2) is equivalent to requiring that the Sidrauski utility function be strictly concave in gross consumption expenditures, $x=c+T(c,m)$, and real balances, m. To see this, differentiate the formulas for U_x and U_m contained in Section II to obtain:

$$U_{xx}=(u''-\lambda T_{cc})/(1+T_c)^2, \tag{A.3}$$

$$U_{xm}=-[(u''-\lambda T_{cc})T_m/(1+T_c)+\lambda T_{cm}]/(1+T_c), \tag{A.4}$$

$$U_{mm}=(u''-\lambda T_{cc})[T_m/(1+T_c)]^2-\lambda T_{mm}+2\lambda T_{cm}T_m/(1+T_c). \tag{A.5}$$

Equations (A.3) – (A.5) also imply that the conditions $T_{cm}<0$ and $(1+T_c)>T_{cm}(T_m/T_{mm})$ are satisfied if and only if both gross consumption and real balances are normal goods.

Firms

As noted in Section II, in order to guarantee that the time–paths of capital and real balances which satisfy the first–order conditions of the firm are maximizing, it is sufficient that the Hamiltonian function be strictly concave in $\dot{K}, K$, and M. When evaluated at steady state, the concavity conditions take the form:[17]

$$0>(J_{\dot{K}\dot{K}})^*=-\{(1-\mu)^2S_{EE}/(1+S_E)^3+[\mu+(1-\mu)/(1+S_E)]\phi''/K\}^* \tag{A.6}$$

$$0>(J_{KK})^*=-\{[f'-(1-\mu)\delta]^2S_{EE}/(1+S_E)^3-f''/[N(1+S_E)]\}^* \tag{A.7}$$

$$0>(J_{MM})^*=-[1/(1+S_E)]^*\{[S_{MM}+iS_{EM}]+i[S_{EE}i+S_{EM}]\}^* \tag{A.8}$$

$$\begin{aligned}0<(J_{\dot{K}\dot{K}}J_{MM}-J_{\dot{K}M}^2)^* &= \{(1-\mu)^2(S_{EE}S_{MM}-S_{EM}^2)/(1+S_E)^4\\ &\quad -J_{MM}[\mu+(1-\mu)/(1+S_E)]\phi''/K\}^*\end{aligned} \tag{A.9}$$

$$0 < (J_{KK}J_{MM} - J_{KM}^2)^* = \{[f' - (1-\mu)\delta]^2(S_{EE}S_{MM} - S_{EM}^2)/(1+S_E)^4 + J_{MM}f''/[N(1+S_E)]\}^* \tag{A.10}$$

$$0 < (J_{\dot{K}\dot{K}}J_{KK} - J_{\dot{K}K}^2)^* = [(1-\mu)/(1+S_E)^2]^*\{S_{EE}[(f' - (1-\mu)\delta)/(1+S_E)]^2\phi''/K - f''\phi''/(NK) - (1-\mu)S_{EE}f''/[N(1+S_E)^2]\}^*. \tag{A.11}$$

In the special case in which $\mu = 1$, requiring that (A.10) hold is equivalent to requiring that the firm's net production function, $E(K, N, M)$, exhibit decreasing returns to scale in capital and money when evaluated at steady state—i.e., to requiring $(E_{KK}E_{MM} - E_{KM}^2)^* > 0$. To require that (A.7) and (A.8) hold is equivalent to insisting that there be diminishing marginal returns to K and M—i.e., to requiring $(E_{KK})^* < 0$ and $(E_{MM})^* < 0$.

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FOOTNOTES

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1 In each of the articles cited, the money balances of firms are ignored. In addition, Fischer and Cohen impose severe restrictions on the indirect utility function of the representative household.

2 Administrative economies of scale could easily make such occasional, discrete distributions profit–maximizing.

3 The "representative agent" approach is analytically convenient, but has the disadvantage that the role of money as a buffer between receipts and disbursements is not explicit. In adopting this approach I am also forced to abstract from the impact which monetary policy might have on the economy through distributional channels. See, for example, Rotemberg [1984].

4 The analysis presented here differs from that of Feenstra [1986] principally in that Feenstra finds conditions on transaction costs which are equivalent to *quasi*–concavity of Sidrauski's utility function, while the analysis presented here finds conditions on transaction costs and $U(\cdot)$ which are equivalent to *full* concavity. In a dynamic model, quasi–concavity is *not* sufficient to guarantee that the first–order conditions correspond to a maximum.

5 The new function will be well–defined for all positive x and m provided that transaction costs are zero when no output is purchased ($T(0,m) = 0$ for all $m > 0$), gross expenditures are strictly increasing in net consumption ($1 + T_c > 0$ for all $m > 0$), and gross expenditures increase without bound as net consumption goes to infinity $\lim_{c\to\infty}[c + T(c,m)] = \infty$ for all $m > 0$). The second assumption, which guarantees that $U(\cdot,\cdot)$ is single–valued, has already been discussed. The first

rules out utility functions which are additively separable in gross expenditures and real balances, except those for which $U(0, m)$ equals $-\infty$ for all $m > 0$.

6 My own empirical work suggests strongly, however, that U_{xm} is positive. See Koenig [1987a].

7 Perhaps each firm is located in a separate town. People are immobile between towns, but output can be transported costlessly.

8 This notation is consistent with that used in my earlier paper (Koenig [1987b]). Empirical results obtained by Mankiw and Summers [1986] suggest that μ is close to zero.

9 Since both the supply of labor per household and the number of households per firm are exogenously fixed, the split of D between dividends and wages is irrelevant.

10 The reader may readily verify that this condition is satisfied by the Baumol–Tobin transactions technology, $S(E, M) = \tau E/M$.

11 This presupposes that the government can use lump–sum taxation to remove money from circulation—a dubious assumption at best.

12 Basically, firms want to concentrate net earnings in periods during which the tax on net earnings is relatively low.

13 The latter equations are derived in an earlier version of this paper (Koenig [1986]), for the case in which $\phi''^{*} \equiv 0$.

14 A complete characterization of the transactions technologies which have this property is beyond the scope of this paper. My impression is that any such characterization would have to impose restrictions on the third–order partial derivatives of the transactions cost function, $S(\cdot, \cdot)$.

15 Firms will *want* to be as large as possible, in order to take advantage of economies of scale in the transactions technology.

16 The second–order conditions from the household maximization problem require that the term $-cu''/u'$ be greater than $\tau/2$.

[17] Only the steady–state versions of the second–order conditions are relevant to the linearized dynamics of the model.