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AUGMENTED INFORMATION IN A THEORY OF AMBIGUITY, CREDIBILITY AND INFLATION

Nathan Balke*
Professor of Economics
Southern Methodist University
and
Joseph H. Haslag*
Economist
Research Department
Federal Reserve Bank of Dallas

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"AUGMENTED INFORMATION IN A THEORY
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The paper examines the phenomenon of "Fed Watching" within the context of a macroeconomic policy game. Unlike previous policy game models, individuals are allowed to acquire information about monetary growth in addition to the historical data. Agent's decisions are based on the opportunity costs of resources expended to augment their information set. Incorporated into the Cuikerman and Meltzer model of asymmetric information, the public's optimizing behavior makes the agent's information set a strategic variable. In this setting, it is shown that individuals strategic behavior can influence the monetary authority's strategy with respect to monetary growth control. Further, the policymaker strategically chooses the control variance of money growth to influence agent's information-seeking behavior.

Keywords: endogenous information, asymmetric information, time inconsistency, credibility, policy games.

Section 1: Introduction

The time consistency problem involves the public forming expectations of money growth prior to the monetary authority's implementation of policy. Even though agents form expectations rationally, i.e. use all available information, the lack of pre-commitment on the part of the policymaker allows surprise money growth to be possible. Kydland and Prescott (1977) first described the nature of this policy game and numerous authors have extended their analysis. The primary difference in these extensions of Kydland and Prescott is the treatment of agents' information sets. In Barro and Gordon (1983), agents know the preferences of the policymaker, while in Rogoff (1985), agents know policymaker preferences, but do not observe the productivity shocks until after expectations have been set. Several authors have examined models of asymmetric information. In Canzoneri (1985), agents do not know the monetary authority's forecast of money demand shocks. In Backus and Driffill (1985), Barro (1986), and Cukierman and Meltzer (1986), agents do not know the preferences of the monetary authority, so they must use past observations of money growth to infer policymaker preferences. In all of these papers, the public's rational forecast is based on a predetermined, fixed information set.

This paper considers an extension to the typical macroeconomic policy game by allowing agents to "augment" their information set. That is, allow the information set to be a choice variable. Darby (1976) analyzed the optimal choice of costly information in a general context. Here we allow agents to expend resources to acquire information which improve their forecasts of money growth. We are attempting to capture the phenomenon often described as "Fed Watching" where agents expend resources to gain insight into the actions of the monetary authority. The model of Cukierman and Meltzer (hereafter C-M) provides a framework which is easily modified to analyze the effects of Fed watching.

The paper is organized as follows. Section 2 introduces the notion of augmented information sets into a simple static framework. The static version is characterized by preference shocks that are serially uncorrelated. Representing the policy game in this manner highlights the agent's allocation problem. Section 3 considers the agent's choice of the optimal information set in the Cukierman and Meltzer asymmetric information model. With serially correlated policymaker preferences and positive monetary control variance, agents must decide how to weight past observations and current information purchases when forming expectations. Section 4 examines how agents' optimal choice of a forecast error variance reacts to changes in monetary control variance; and how the monetary authority's optimal control variance reacts to changes in the public's forecast error variance.

Section 2: Serially Uncorrelated Preferences

In formulating policy, the monetary authority is viewed as being subject to various political and social forces. The impact of these forces is manifested as changes in policymaker "preferences." "Preferences" refer to the weight policymakers place on stimulating economic activity versus controlling inflation. The monetary authority possesses exclusive information about the true value of the weighting parameter. The presence of asymmetric information increases the incentive of the policymaker to avoid pre-commitment to a rate of monetary growth.

Before describing the objective functions of the monetary authority and (a representative agent of) the public, we need to outline the structure of the policy game and make the sequence of events explicit. First, a policymaker preference shock occurs. The exact nature of this shock is the private information of the monetary authority. The public forecasts this preference shock; the "quality" of this forecast depends on the level of resources devoted to "monitoring" the monetary authority.[2] Based on an information set "augmented" by Fed watching, the public then forms expectations of the preference shock and future money growth. Finally, the policymaker chooses the rate of money growth taking agent's expectations as given.

The policymaker's objective is to

$$(1)\quad \max \sum_{i=0}^{\infty} \beta^{i} \{[m_i - E(m_i|I_i)]\, x_i - (m_i)^2/2\},$$

where m_i denotes money growth, $E(m_i|I_i)$ is the public's forecast of m_i, given the information set I_i, and β denotes the rate of discount. The variable x_i reflects the policymaker's preferences for stimulating economic activity through surprise money growth relative to controlling inflation. As x_i increases, the policymaker is willing to bear a higher rate inflation in order to further stimulate economic activity. Because we assume that preferences are not serially correlated, the monetary authority takes the public's current and future forecasts of money growth as given. Since future expectations are independent of the monetary authority's current action, the monetary authority's maximization problem can be reduced to the one period maximization problem described by

$$(1a)\quad \max\ [m_i - E(m_i|I_i)]x_i - (m_i)^2/2$$

Equations (1) and (1a) imply that non-zero money growth has a negative impact on the monetary authority's utility, but that surprise money growth has a positive impact.

It is assumed that x_i is random and is described by

$$(2) \quad x_i = A + v_i.$$

Policymaker preferences are on average A, and the variable v_i is independently and identically distributed with mean zero and variance σ_v^2.[3]

Maximizing (1a) with respect to m_i yields

$$(3) \quad m_i = x_i = A + v_i.$$

According to equation (3), the selection of the rate of money growth depends directly upon the relative importance of stimulating economic activity versus creating additional inflation. Note that in the absence of any public information about v_i the expected money growth rate is equal to A.

In a standard time consistency model, the public sets expectations given an information set that includes a conjecture of policymaker preferences. However, we allow agents to augment their information set by expending resources. This permits the public to get an idea of what the policymaker's contemporaneous preference shock is. The public chooses the level of resources to expend in an attempt to minimize unanticipated or "surprise" money creation. For the sake of simplicity, we assume that the public consists of a set of identical individuals. Consequently, the public's problem may be treated from the perspective of a representative agent.[4]

Formally, the representative agent's behavior is characterized as

(4) $$\max_{\sigma_e^2} - E[(m_i - E[m_i | I_i(\sigma_e^2)])]^2 - C(\sigma_e^2).$$

$I_i(\sigma_e^2)$ represents the information set from which agents form their expectations.[5] This information set consists of knowledge of the general motivation of the monetary authority (i.e. knowledge of the functional form of the monetary authority's maximization problem and that $m_i = x_i$), knowledge of the distribution of monetary authority's preferences (i.e. $x_i = A + v_i$, $E(v_i) = 0$, and $Var(v_i) = \sigma_v^2$), and an estimate of the contemporaneous policymaker preference shock, v_i^e. This estimate has the following properties:

$$E[v_i - v_i^e] = 0,\ E[(v_i - v_i^e)^2] = \sigma_e^2,$$
$$E[v_i^e(v_i - v_i^e)] = 0,\ E[v_i(v_i - v_i^e)] = \sigma_e^2, \text{ and}$$
$$E[v_j(v_i - v_i^e)] = 0 \text{ for } j \neq i.$$

Thus, the public's estimate of the preference shock is unbiased and has a variance of σ_e^2.

The public can improve its estimate of the preference shock by expending resources on information acquisition. $C(\sigma_e^2)$ is the amount of resources expended to get a forecast of the policymaker's preference shock of quality σ_e^2. The properties of σ_e^2 are: $C' < 0$, $C'' > 0$, $C(\sigma_v^2) = 0$, and $C(0) = \infty$. The more resources devoted to uncovering the monetary authority's preferences the better is the public's estimate of the preference shock, i.e., lower σ_e^2. If no resources are expended then agents will have no information about the contemporaneous preference shock aside from knowledge of its distribution, i.e., $\sigma_e^2 = \sigma_v^2$. This is the standard assumption of all of the other asymmetric information macropolicy games. However, in our model agents can improve their estimate of the preference shock by increasing the amount resources devoted to monitoring the monetary authority, and if an infinite amount of resources are expended the public will know the policymaker's true preferences with certainty.

Given an estimate of the policymaker's preference shock of quality σ_e^2 and based on the other readily available information, expected money growth is

$$(5) \quad E[m_i | I(\sigma_e^2)] = E[x_i | I(\sigma_e^2)] = A + v_i^e$$

Substituting (3) and (5) into (4), we find that the public's problem is to

$$(6) \quad \max_{\sigma_e^2} \{ - E[(v_i - v_i^e)^2] - C(\sigma_e^2) \},$$

which is equivalent to

$$(6a) \quad \max_{\sigma_e^2} \{ - \sigma_e^2 - C(\sigma_e^2) \}.$$

The first order condition for an interior solution implies that the optimal degree of forecast accuracy, σ_e^2*, will satisfy

$$(7) \quad C'(\sigma_e^2*) = - 1.$$

From equation (7) and the second order conditions, it is obvious that anything that increases the marginal cost of monitoring the monetary authority (i.e., make $C'(\sigma_e^2)$ more negative or $\partial C'(\sigma_e^2)/\partial\mu < 0$, where μ denotes an arbitrary cost-increasing parameter) causes the public to acquire forecasts that are less accurate (σ_e^2 higher).[6]

We can evaluate how the public's ability to forecast the policymaker's preference shock changes the policymaker's expected utility. From equations

(1a), (3), and (5) we find that expected utility of the monetary authority is

$$(8)\quad E[(A + v_i - A - v_i^e)(A + v_i)^2/2] = \sigma_e^2 - \sigma_v^2/2 - A^2/2.$$

Equation (8) suggests that a better forecast of policymaker preferences results in lower expected utility for the monetary authority. If the public has a clearer picture of the objectives of the monetary authority, then it is much more difficult for the monetary authority to generate surprise money growth. Consequently, the monetary authority may have incentive to raise the public's costs of obtaining forecasts and as a result increase σ_e^2. If, as in the analysis of Barro and Gordon (1983), agents have perfect information about the preferences of the monetary authority, i.e., $\sigma_e^2 = 0$, expected utility is $- \sigma_v^2 - A^2/2$. If, as in the C-M analysis, agents have no knowledge about the contemporaneous preference shock, i.e., $\sigma_e^2 = \sigma_v^2$, expected utility is $\sigma_v^2/2 - A^2/2$. If policymakers followed the money growth rule, $m_i = 0$, then expected utility is zero. Thus, when the public is less informed, policymakers are more likely to prefer discretionary policy than to be bound by a rule.

Section 3: Serially Correlated Preferences and Imperfect Control

We now introduce the concept of augmented information into the C-M model. Their model is similar to the one above except that the shocks to the monetary authority's preferences are serially correlated. Because agents cannot observe these preferences directly, they use realizations of past money growth to form expectations of the monetary authority's current preferences.

Policymakers have the same objective as equation (1) above, except current money growth rates will now affect future expectations. However, the monetary authority has imperfect control over money growth. Actual money growth is given by

$$(9) \quad m_i = m_i^p + \Psi_i.$$

m_i^p is planned money growth and Ψ_i is a control error. Ψ_i is i.i.d. with mean zero and variance σ_Ψ^2. Policymaker preferences are given by

$$(10) \quad x_i = A + p_i, \; A > 0,$$

and

$$(11) \quad p_i = \rho p_{i-1} + v_i, \; 0 < \rho < 1.$$

Like the analysis above, v_i is i.i.d. with mean zero and variance σ_v^2.

In C-M, the information set of the public consists of all past realizations of actual money growth, knowledge of the parameter A, knowledge of the distributions of v_i and Ψ_i, and a consistent conjecture of the monetary authority's planned money growth. This conjecture is given by

$$(12) \quad m_i^p = B_0 A + Bp_i.$$

Agents know B_0, A, and B, but do not know p_i.[7] Here, we allow the public to augment this information set by allocating resources in order to uncover the monetary authority's preference shock. As a result, agents get an estimate, v_i^e, of the contemporaneous preference shock v_i. The properties of v_i^e and v_i are given above. The public can improve this estimate, i.e., lower σ_e^2, by committing more resources to monitoring the monetary authority. Thus, the public when forming their expectations of current money growth knows all past realizations of actual money growth, and has estimates of current and past preference shocks along with knowledge of the properties of these estimates. Notice that agents are uncertain about the policymaker's present and past preferences, p_{i-j}; they must try to infer the preference shocks using past realizations of money growth and current and past estimates of the preference shocks.

Following C-M's analysis, we derive the public's conditional expectation of money growth. From equations (9) and (12), the public's conjecture of actual money growth is

$$(13) \quad m_i = B_0 A + Bp_i + \Psi_i.$$

Given the public's information set, expected money growth is

$$(14) \quad E[m_i | I_i(\sigma^2_{e,i})] = B_0 A + B\, E[p_i | m_{i-1}, \ldots, v^e_i, v^e_{i-1}, v^e_{i-2}, \ldots].$$

where

$$(15) \quad E[p_i | m_{i-1}, m_{i-2}, \ldots, v^e_i, v^e_{i-1}, v^e_{i-2}, \ldots] =$$

$$-\frac{(\rho-\lambda)B_0 A}{B(1-\lambda)} + \frac{(\rho-\lambda)}{B} \sum_{j=0}^{\infty} \lambda^j m_{i-1-j} + \sum_{j=0}^{\infty} \lambda^j v^e_{i-j},$$

$$(16) \quad \lambda = \left(\frac{1+r}{\rho} + \rho\right)/2 - \left(1/4 \left(\frac{1+r}{\rho} + \rho\right)^2 - 1\right)^{1/2},$$

and

$$(17) \quad r = B^2(\sigma^2_e/\sigma^2_\Psi).$$

(The derivation of equations (15), (16), and (17) is contained in the Appendix). Equation (15) is virtually identical to that in C-M; however, here the public's estimates of the contemporaneous preference shocks are present in the forecast of money growth, and the term λ depends upon the variance of the public's preference forecast error, σ^2_e, not upon the variance of the preference shock σ_v^2. Implicit in the analysis above is the assumption that the properties of the current and past forecasts of the preference shocks are constant, i.e., $\sigma_e^2 = \sigma_{e,i-j}^2$ $(j \geq 1)$. This assumption turns out to be consistent with maximizing behavior on the part of the public and is shown below.

Notice in equation (15), that as the public's "guess" of the preference shock becomes better (i.e., a smaller σ_e^2) more weight is placed on the estimates of these shocks. In fact, as $\sigma_e^2 \to 0$, $\lambda \to \rho$ which causes agents to place all weight on v_{i-j}^e, $j = 0, \ldots, \infty$, and no weight on past realizations of money growth. On the other hand, as $\sigma_\psi^2 \to 0$, $\lambda \to 0$, which causes agents to place all the weight on the more recent observation of past money growth and on the estimate of the current preference shock and no weight on past estimates of the preference shock.

Proposition 1. The public's conjecture of the monetary authority's behavior (given by equation (13)) and the public's expectations of money growth (described by equation (15)) are consistent with a monetary authority whose objective is to maximize equation (1). The parameters of the agent's conjecture are

$$(18) \quad B_0 = (1-\beta\rho)/(1-\beta\lambda),$$

$$(19) \quad B = (1-\beta\rho^2)/(1-\beta\rho\lambda).$$

(See appendix for proof.)

Thus, it is rational for agents to have expectations of money growth given by equation (15). Equations (18) and (19) imply that the public's expectation of money growth in our model depends on estimates of the policymaker's preference shocks. Like C-M, actions of the monetary authority can only affect agents' expectations through past money growth rates. Because the public's current and future estimates of contemporaneous preference shocks (i.e., v^e_{i+j}, j = 0, 1, ..., n) are independent of actual money growth, the monetary authority's choice of money growth is essentially the same in our model as in C-M. Notice that while individual preference shock estimates (v^e_i) do not affect the monetary authority's behavior, the properties of these estimates (σ^2_e) do affect equations (18) and (19) through the parameter λ.

The degree to which the monetary authority "invests" in credibility is reflected by the terms B_0 and B. Recall from section 2 that if the monetary authority does not take into account future expectations then it would set planned money growth equal to its tradeoff parameter, i.e. $m^p_i = x_i$. When the policymaker takes into account future expectations, we know that the monetary authority's planned money growth is less than its tradeoff parameter since $B_0 \leq 1$ and $B \leq 1$. The monetary authority restrains itself in order to preserve more favorable money growth expectations in the future. As σ^2_e (σ^2_ψ) falls the terms B_0 and B rise (fall) (see Appendix). As monetary control improves, the monetary authority's conduct of policy moves towards a lower planned money growth and away from its own tradeoff parameter. However, as the public's estimates of policymaker's preference shocks improve, planned money growth tends to increase towards the monetary authority's tradeoff parameter. When σ^2_e falls, the public pays more attention to its own estimates of the preference shocks and less attention to past money growth when forming expectations of the monetary authority's preferences. Therefore, the monetary authority has less to gain from investing in credibility because the public is not paying as much attention to past money growth.

Given the expectations mechanism (equation (15)) and actual money growth equation (equation 13)), we can find the variance of the public's money growth forecast. This is given by

$$(20)\quad E\left[(m_i - E[m_i|I_i(\sigma_e^2)])^2\right] = \frac{B^2\ \sigma_e^2}{(1-\lambda^2)} + \frac{(\rho-\lambda)^2\ \sigma_\Psi^2}{(1-\lambda^2)} + \sigma_\Psi^2.$$

The more accurate the public's estimate of the preference shock (lower σ_e^2) and the better the monetary authority's control (lower σ_Ψ^2) the better is the public's overall forecast of money growth.

In general, the public's expected utility is given by

$$(21)\quad -\sum_{i=0}^{\infty} \beta^i\ E\left[(m_i - E[m_i|I_i(\sigma_{e,i}^2, \sigma_{e,i-1}^2, \ldots)])^2\right] + C(\sigma_{e,i}^2)\ .$$

When $\sigma_{e,i}^2$ is the same for all time periods, we can use equation (21) and the variance of the public's expected money growth given by equation (20) to find the public's expected discounted welfare for a forecast of preference shocks of quality σ_e^2. This is

$$(22)\quad -\frac{1}{1-\beta}\quad \frac{B^2\ \sigma_e^2}{(1-\lambda^2)} + \frac{(\rho-\lambda)^2\ \sigma_\Psi^2}{(1-\lambda^2)} + \sigma_\Psi^2 + C(\sigma_e^2)\ .$$

The constant σ_e^2 that maximizes equation (22) will satisfy

$$(23)\quad \frac{-B}{1-\lambda^2} - \frac{2B(dB/d\sigma_e^2)\sigma_e^2}{(1-\lambda^2)} - C'(\sigma_e^2) = 0.$$

Since $B^2 + 2B(dB/d\sigma_e^2)\ \sigma_e^2 > 0$, an interior solution always exist if $C'(\sigma_V^2) = 0$ and $C'(0) = -\infty$.[8]

The resulting choice of σ_e^2 is constant across time given equations (15), (18), and (19). The monetary authority's behavior summarized by equations (7), (18), and (19), the public's expectations mechanism given by equation (16), and a constant optimal σ_e^2 described by equation (23) are all consistent with each other. As long as the structural parameters λ, β, σ_Ψ^2 remain unchanged, the problem facing the public is the same in each period and, consequently, the choice of σ_e^2 is constant over time.[9]

Section 4: Comparative Statics

Again, it is obvious from equation (23) that anything which increases the marginal cost of obtaining preference shock estimates (makes $C'(\sigma_e^2)$ more negative) will cause the public to choose estimates with a higher σ_e^2. The effect of the control variance, σ_Ψ^2, on the choice of σ_e^2 is also of interest.

Proposition 2. Let σ_e^{2*} solve equation (23). A sufficient condition for $\frac{d\sigma_e^{2*}}{d\sigma_\Psi^2} < 0$ is $B^2 \frac{\sigma_e^{2*}}{\sigma_\Psi^2} < 1-\beta\lambda\rho$.

Proposition 2 indicates that when the ratio of preference estimate variance to control variance is small (recall that $0 < \beta < 1$ and $0 < B < 1$), the public will desire better estimates of the preference shock for higher levels of control variance. This implies that as the monetary authority's control gets worse the public is willing to expend more resources to improve the quality of their preference shock forecast. This allows the public to partially offset the increase in the variance of their money growth estimates caused by the increase in the control variance. The condition given in Proposition 2 is a sufficient condition; it is quite possible for the condition not to hold and $d\sigma_e^{2*}/d\sigma_\Psi^2$ still be negative.

Like the example in section 2 where preference shocks are not serially correlated, the ability of the public to form estimates of the monetary authority's preference shocks affects the monetary authority's expected utility. Recall that the expected utility of the monetary authority is given by

$$(24)\quad E\left[\sum_{i=0}^{\infty} \beta^{i} \quad (m_i - E[m_i | I_i(\sigma_e^2)])x_i - (m_i)^2/2\right].$$

Substituting equations (13) and (16) into (25) and taking expectations yields

$$(25)\quad \left[\frac{1}{1-\beta} \quad \frac{B\sigma_e^2}{1-\lambda\rho} - \frac{B_0^2 A^2}{2} - \frac{B^2\sigma_v^2}{2(1-\rho^2)} - \frac{\sigma_\Psi^2}{2}\right].$$

The better the public's preference forecasts are the lower is the monetary authority's expected utility.[10] Comparing equation (25) to the measure of policymaker utility found in C-M, we see (i) σ_e^2 is in the first term instead of σ_v^2; and (ii) σ_Ψ^2 enters explicitly in the policymaker's utility. Once again, the ability of the public to augment their information sets with information about the monetary authority's preference shocks makes it more difficult for discretionary policy to yield the monetary authority greater utility than the fixed money growth rule $m_i^p = 0$.[11]

C-M show that it may be optimal for the monetary authority to have positive control variance. In our model, it is less likely that the monetary authority will choose a positive control variance. If the monetary authority chooses σ_Ψ^2 to maximize (25), taking σ_e^2 as given, σ_Ψ^2 will satisfy

$$(26)\quad \frac{\frac{dB}{d\sigma_\Psi^2}}{1-\lambda\rho}\,\sigma_e^2 + \frac{\rho B\sigma_e^2 \frac{dB}{d\sigma_\Psi^2}}{(1-\lambda\rho)^2} - B_0 \frac{dB_0}{d\sigma_\Psi^2} A^2 - \frac{\frac{dB}{d\sigma_\Psi^2} B\sigma_\Psi^2}{(1-\rho)^2} - 1/2 = 0.$$

Two elements present in our analysis work against positive control variance.[12] First, because $\sigma_e^2 < \sigma_v^2$ the first two terms in equation (26) are smaller in our analysis than in C-M. Second, the presence of the fifth term in equation (26), which is not present in C-M, represents the direct negative effect of the control variance. The element that favors the choice of a positive control variance is that with a higher control variance it is more difficult for the public to ascertain the policymaker's true preferences by observing past money growth rates. However, this effect is weakened by the fact that the public now has an alternative information source on which to base its expectation of policymaker preferences. Notice that if σ_e^2 is not taken as given, then according to Proposition 3 it will be even more unlikely that the optimal control variance is positive.

<u>Proposition 3</u>. Let $\sigma_\Psi^{2}*$ satisfy equation (26), then $d\sigma_\Psi^{2}*/d\sigma_e^2 > 0$.

This is an interesting result. It is similar to Proposition 6 in C-M, but it has a somewhat different intuitive interpretation. Proposition 4 implies that as the public has a better forecast of the monetary authority's preference shocks, the monetary authority will want to improve monetary control. This seems counterintuitive at first glance, because one would expect that as the public's forecasts improve the monetary authority would want to confuse the public by having worse monetary control. However, it seems that in our analysis as the public gets better forecasts of the policymaker's preference shocks they place more weight on their preference shock estimates and less on realizations of past money growth. Therefore, the monetary authority cannot effectively offset the public's improved forecasts by increasing the control variance; the public pays less attention to money growth. So in order to continue to influence the public's expectations, the monetary authority has incentive to improve monetary control and prevent the weights placed on realizations of money growth from falling.

While the monetary authority is less likely to use positive control variance to offset an increase in the quality of the public's preference shock estimate, the monetary authority may use other means to obscure its preferences. As we indicated above, anything that increases the public's marginal cost of obtaining a preference shock estimate causes σ_e^2 to increase. Therefore, the monetary authority can improve its welfare by making it costly for the public to discern the monetary authority's true preferences. The Federal Reserve's penchant for secrecy could be motivated by a desire to raise the costs to the public of discovering the Fed's true preferences. Furthermore, Proposition 3 implies that secrecy may result in the choice of institutions that result in higher control variance.

Section 4: Conclusion

This paper extends the asymmetric information model of Cukierman and Meltzer (1986) by allowing agents to "augment" their information set. By devoting resources to "Fed watching" agents form a rational expectation of the monetary authority's preferences based not only upon past observations of money growth but also upon current and past estimates of the contemporaneous policymaker preference shocks. The quality of the preference shock estimate depends upon the amount of resources expended.

Several results stem from the introduction of augmented information into the C-M model.

(1) The distribution of money growth forecast errors is endogenous -- it is chosen by the public when they choose the amount of resources to devote to monitoring the Fed's preference shocks. Agents weigh the costs of an improved preference shock estimate against improvement in their money growth estimate.

(2) Improvements in the public's estimates of the preference shocks lowers the expected utility of the monetary authority. Furthermore, fixed rules such as $m_i^p = 0$ become more attractive to the monetary authority.

(3) The public's augmented information set makes it less likely that the monetary authority would choose institutions that lead to positive control variance. In C-M the difference between actual and planned money growth masks the preference shock from the public. However, when agents have their own independent estimates of the policymaker's preference shocks, the ability of the control error to obscure policymaker preferences is diminished.

In C-M, credibility depends upon how quickly the public learns. This depends in turn upon the information content of money supply observations. The monetary authority can lessen the information content of money supply observations by having poor monetary control. In our model, the speed of learning depends upon the public's acquisition of information. Agents by investing in information diminish the policymaker's incentive for noisy monetary control. Policymakers can lower monitoring activity by the public by increasing the cost of acquiring information. Perhaps the Fed's desire for secrecy, see Goodfriend [1986], can be rationalized as an attempt to increase the cost of acquiring information.

Several extensions of the above model could be considered. We did not take into account informational differences between agents. How does the model change when only a subset of agents has superior information? Furthermore, if there is a mechanism (i.e. the market, see Grossman and Stiglitz [1980]) that results in private information being dispersed among the entire public, then there may be severe free rider effects with regards to expending resources to uncover policymaker preferences. Examining the role of differential information among the public and possible free rider problems could be enlightening.

Finally, in C-M the monetary authority's preferences and the shocks to these preferences are, presumably, being driven by social and economic forces that are not explicitly modelled. In our model, the public's understanding of these forces are assumed to be summarized by their current and past estimates of the monetary authority's preference shocks. Further work in trying to identify and model the interaction of these socio/economic forces and including agents who have an imperfect understanding of these forces is needed.

Southern Methodist University

and

The Federal Reserve Bank of Dallas

FOOTNOTES

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2. Here the term quality refers to the variance of the current period forecast error.

3. Implicitly we are assuming that the 2 parameter family of distributions describes v_i, or that the other moments of the distribution are unchanged.

4. This assumption maintains the information asymmetry between the monetary authority and the public, not between individuals. Furthermore, the uniform information set across agents implies there is no "free rider" problem whereby "used" information becomes a public good in the acquisition of information.

5. The underlying assumption is that the variance of the forecast error is a monotonically decreasing function with respect to agent's resource expenditure. Or, $\sigma_e^2 = f(R)$, with $f' < 0$, where R is the agent's resource expenditure. Clearly, R is the true choice variable. In order to minimize notation, we have chosen to substitute σ_e^2 as the choice variable in the agent's objective function, i.e. $R = f^{-1}(\sigma_e^2) = c(\sigma_e^2)$.

6. Notice that the second order conditions hold, i.e., $-C''(\sigma_e^2) < 0$.

7. The parameters B_0 and B are taken as given.

8. The public's "optimal" choice of a constant σ_e^2, which satisfies equation (23), is not time consistent. According to the decision rule prescribed by the first-order condition represented by equation (23), agents are committed to choosing the constant forecast error variance for periods $i + j$ $(j \geq 1)$. Time inconsistency arises because the current value of σ_e^2 affects future money growth forecast error. This dependence is obvious from equation (A6) in the Appendix. In addition, changes in current period σ_e^2 affect future money growth forecast errors through changes in the weights placed on current preference shock estimates (i.e., the a_j's and b_j's in equations (A7) and (A8)). If agents take the reaction function of the monetary authority as constant (i.e., B_0 and B are constant) then the time consistent choice of $\sigma^2_{e,i+j}$ $(j \geq 0)$ will satisfy

$$- B^2/(1-\beta\lambda^2) - C'(\sigma^2_{e,i}) = 0 \quad i = 0, 1, \ldots, \infty.$$

(For an analytical derivation of the optimal, time consistent σ_e^2, see section 5 of the appendix.)

9. There may be other equilibria where $\sigma^2_{e,i}$ is not constant.

10. Expected utility of the monetary authority in Cukierman and Meltzer is:

$$\frac{1}{1-\beta}\left[\frac{B\sigma_v^2}{1-\lambda\rho} - \frac{B_0^2A^2}{2} - \frac{B^2\sigma_v^2}{2(1-\rho^2)}\right]$$

11. Expected utility for the monetary authority under the fixed rule is
$-\frac{\sigma_e^2}{2}$.

12. The first order conditions in Cukierman and Meltzer's model are:

$$\frac{1}{1-\beta}\left[\frac{dB}{d\lambda}\frac{\sigma_v^2}{(1-\lambda\rho)}+\frac{\rho B\sigma_v^2}{(1-\lambda\rho^2)}-B_0\frac{dB_0}{d\lambda}A^2-\frac{B\frac{dB}{d\lambda}}{(1-\rho^2)}\sigma_v^2\right]\frac{d\lambda}{d\sigma_\psi^2}=0.$$

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Appendix for

Augmented Information in a Theory of Ambiguity, Credibility, and Inflation

Appendices

1. Derivation of $E[p_i | m_{i-1}, m_{i-2}, \cdots, v^e_i, v^e_{i-1}, v^e_{i-2}, \cdots]$

Let $y_i = m_i - B_0 A$.

The information set consists of observations of $y_{i-1}, y_{i-2}, \cdots$ and $v^e_i, v^e_{i-1}, v^e_{i-2}, \cdots$, and knowledge of σ^2_Ψ, B_0, B, A, σ^2_v, and the properties of the preference shock estimates. These properties are:

$$E[(v_{i-j} - v^e_{i-j})] = 0, \quad E[(v_{i-j} - v^e_{i-j})^2] = \sigma^2_{e,i-j},$$
$$E[v_{i-j}(v_{i-j} - v^e_{i-j})] = \sigma^2_{e,i-j}, \quad E[v^e_{i-j}(v_{i-j} - v^e_{i-j})] = 0,$$

and $E[v_{i-j}(v_{i-k} - v^e_{i-k})] = 0$ for $j \neq k$. Agents will use this information set efficiently to minimize $E[(p_i - E(p_i | I_i))^2]$.

Let

$$\text{(A1)} \quad E[p_i | I_i] = \sum_{j=1}^{\infty} a_j \frac{y_{i-j}}{B} + \sum_{j=0}^{\infty} b_j v^e_{i-j}$$

The trick is to choose the parameters a_j and b_j to minimize $E[(p_i - E(p|I_i))^2]$.

Set

$$\text{(A2)} \quad E[(p_i - E(p_i|I_i))^2] = E[p_i - \sum_{j=1}^{\infty} a_j(p_{i-j} + \frac{\Psi_{i-j}}{B}) - \sum_{j=0}^{\infty} b_j v^e_{i-j}]^2$$

$$\text{(A3)} \quad E[\sum_{j=0}^{\infty} \rho^j v_{i-j} - \sum_{j=1}^{\infty} a_j(\sum_{k=0}^{\infty} \rho^k v_{i-j-k} + \frac{\Psi_{i-j}}{B}) - \sum_{j=0}^{\infty} b_j v^e_{i-j}]^2.$$

Rewriting (A3), we get

$$\text{(A4)} \quad E[(v_i - b_0 v^e_i) + [(\rho - a_1) v_{i-1} - b_1 v^e_{i-1}] + [(\rho^2 - \rho a_1 - a_2) v_{i-2} - b_2 v^e_{i-2}]$$
$$+ [(\rho^3 - \rho^2 a_1 - \rho a_2 - a_3) v_{i-3} - b_3 v^e_{i-3}] + \cdots$$
$$- \sum_{j=1}^{\infty} a_j \frac{\Psi_{i-j}}{B}]^2.$$

Further rewriting yields

$$\text{(A5)} \quad E[(1-b_0)v_i + b_0(v_i - v^e_i) + (\rho - a_1 - b_1)v_{i-1} + b_1(v_{i-1} - v^e_{i-1})$$

$$+ (\rho^2 - a_1\rho - a_2 - b_2)v_{i-2} + b_2(v_{i-2} - v^e_{i-2})$$

$$+ (\rho^3 - a_1\rho^2 - a_2\rho - a_3 - b_3)v_{i-3} + b_3(v_{i-3} - v^e_{i-3}) + \cdots$$

$$- \sum_{j=1}^{\infty} a_j \frac{\Psi_{i-j}}{B}]^2.$$

By taking expectations of (A5) and using the properties of v^e_{i-j}, we get

$$\text{(A6)}\quad E[(p_i - E[p_i|I_i])^2] = (1-b_0)^2\,\sigma_v^2 + b_0^2\sigma^2_{e,i} + 2(1-b_0)b_0\sigma^2_{e,i}$$

$$+ (\rho-a_1-b_1)^2\sigma_v^2 + b_1^2\sigma^2_{e,i-1} + 2(\rho-a_1-b_1)b_1\sigma^2_{e,i-1}$$

$$+ (\rho^2 - \rho a_1 - a_2 - b_2)^2\sigma_v^2 + b_2^2\sigma^2_{e,i-2} + 2(\rho^2 - \rho a_2 - b_2)b_2\sigma^2_{e,i-2} + \cdots$$

$$+ \sum_{j=1}^{\infty} a_j^2\,\frac{\sigma_\Psi^2}{B^2}.$$

Choose a_j, b_j to minimize (A6). The first order conditions for the b_j's are

$$\text{(A7a)}\quad -2(1-b_0)\sigma_v^2 + 2(1-b_0)\sigma^2_{e,i} = 0$$

$$\text{(A7b)}\quad -2(\rho-a_1-b_1)\,\sigma_v^2 + 2(\rho-a_1-b_1)\sigma^2_{e,i-1} = 0$$

$$\text{(A7c)}\quad -2(\rho^2-\rho a_1-a_2-b_2)\,\sigma_v^2 + 2(\rho^2-\rho a_1-a_2-b_2)\sigma^2_{e,i-2} = 0$$

$$\vdots \qquad\qquad \vdots$$

Since $\sigma^2_{e,k} \leq \sigma^2_v$ $\forall k$, then the FOC imply that $b_0 = 1$, $b_1 = \rho - a_1$, $b_2 = \rho^2 - \rho a_1 - a_2$, $b_3 = \rho^3 - \rho^2 a_1 - \rho a_2 - a_3$, etc. The first order conditions with respect to the a_j's are:

(A8a) $2[\rho-a_1-b_1]\sigma_v^2 - 2b_1\sigma_{e,i-1}^2 - 2\rho[\rho^2-\rho a_1-a_2-b_2]\ \sigma_v^2$

$-2\rho b_2\sigma_{e,i-2}^2 - 2\rho^2[\rho^3-\rho^2 a_1-\rho a_2-a_3-b_3]\sigma_v^2 - 2\rho^2 b_3\sigma_{e,i-3}^2$

$- \quad \cdots\cdots$

$$+2a_1 \frac{\sigma_\psi^2}{B^2} = 0$$

(A8b) $-2[\rho^2-\rho a_1-a_2-b_2]\sigma_v^2 - 2b_2\sigma_{e,i-2}^2 - 2\rho[\rho^3-\rho^2 a_1-\rho a_2-a_3-b_3]\sigma_v^2 - 2\rho b_{e,i-3}$

$$- \cdot \cdot \cdot \qquad + 2a_2 \frac{\sigma_\psi^2}{B^2} = 0$$

(A8c) $-2[\rho^3-\rho^2 a_1-\rho a_2-a_3-b_3]\sigma_v^2 - 2b_3\sigma_{e,i-3}^2 - 2\rho[\rho^4 - \rho^3 a_1 - \rho^2 a_2-\rho a_3-a_4-b_4]\sigma_v^2$

$$-2\rho b_4\sigma_{e,i-4}^2 \qquad - \cdot \cdot \cdot \qquad + 2a_3 \frac{\sigma_\psi^2}{B^2} = 0$$

Using the FOC's for the b_j's, we can rewrite (A8a), (A8b),$\cdots$ as

(A9a) $-2[\rho-a_1]\sigma_{e,i-1}^2 - 2\rho[\rho^2-\rho a_1-a_2]\sigma_{e,i-2}^2 - 2\rho^2[\rho^3-\rho^2 a_1-\rho a_2-a_3]\sigma_{e,i-3}^2$

$$- \cdot \cdot \cdot \quad + 2a_2 \frac{\sigma_\psi^2}{B} = 0$$

(A9b) $-2[\rho^2-\rho a_1-a_2]\sigma_{e,i-2}^2 - 2\rho[\rho^3- \rho^2 a_1-\rho a_2 - a_3]\sigma_{e,i-3}^2 - 2\rho^2[\rho^4-\rho^3 a_1-\rho^2 a_2-\rho a_3-a_4]\sigma_{e,i-4}^2$

$$- \cdot \cdot \cdot + \quad 2a\frac{\sigma_\psi^2}{B} = 0.$$

If $\sigma_{e,k}^2 = \sigma_{e-k}^2$, then equations (A9a), (A9b),$\cdots$ are exactly the same as equation (A4) in C-M except that σ_v^2 is replaced by σ_e^2. Therefore, we can use their solution for the a_j's. C-M find that

(A10) $a_j = (\rho-\lambda)\lambda^{j-1}$

where $\lambda = \frac{1}{2}(\frac{1+r}{\rho} + \rho) - \left(\frac{1}{4}(\frac{1+r}{\rho} + \rho)^2-1\right)^{\frac{1}{2}}$

and $r = \frac{B^2\sigma_e^2}{\sigma_\Psi^2}$.

Using (A10) and the first order conditions for the b_j's, we find that

(A11) $b_j = \lambda^j$.

Therefore,

(A12) $E[p_i|I_i] = \frac{(\rho-\lambda)}{B} \sum_{j=0}^{\infty} \lambda^j y_{i-1-j} + \sum_{j=0}^{\infty} \lambda^j v^e_{i-j}$.

Using (A12), and the fact that $y_i = m_i - B_0 A$, we find that

(15) $E[p_i|I_i] = - \frac{(\rho-\lambda)B_0 A}{B(1-\lambda)} + \frac{(\rho-\lambda)}{B} \sum_{j=0}^{\infty} \lambda^j m_{i-1-j} + \sum_{j=0}^{\infty} \lambda^j v^e_{i-j}$.

2. Proof of Proposition 1

The policymakers problem is to

(A14) $\max_{m^p_{i,i=0},\cdots} E\left[\sum_{i=0}^{\infty} \beta^i\left([m_i - E(m_i|I_i)]x_i - \frac{(m_i)^2}{2}\right)\right]$.

Rewriting (A14)

(A15) $\max_{m^p_{i,i=0},\cdots} E\left[\sum_{i=0}^{\infty} \beta^i\left(m^p_i + \Psi_i - \frac{(1-p)}{(1-\lambda)} B_0 A - (\rho-\lambda) \sum_{j=0}^{\infty} \lambda^j (m^p_{i-j-1} + \right.\right.$

$\left.\left.\Psi_{i-j-1})x_i - B \sum_{i=0}^{\infty} \lambda^j v^e_{i-j}]x_i - \frac{(m^p_i + \Psi_i)^2}{2}\right)\right]$.

The first order conditions imply

(A16) $x_i - (\rho-\lambda)[\beta E(x_{i+2}) + \beta \frac{\partial E(v^e_{i+1} x_{i+1})}{\partial m^p_i} + \beta^2 \lambda E(x_{i+2}) + \beta^2 \frac{\partial E(v^e_{i+2} x_{i+2})}{\partial m^p_i}$

$+ \beta^2 \lambda \frac{\partial E(v^e_{i+1} x_{i+1})}{\partial m^p_i} + \beta^3 \lambda^2 E(x_{i+3}) + \beta^3 \frac{\partial E(v^e_{i+3} x_{i+3})}{\partial m^p_i} + \beta^3 \lambda \frac{\partial E(v^e_{i+2} x_{i+2})}{\partial m^p_i}$

$+ \beta^3 \lambda^2 + \frac{\partial E(v^e_{i+1} x_{i+1})}{\partial m^p_i} + \cdots] = m^p_i + E(\Psi_i)$

If σ_e^2 is constant over time then $\frac{\partial E(v_j^e\ x_j)}{\partial m_i^p} = 0$. Also $E(\Psi_i) = 0$. Therefore (A16) can be written as

(A17) $x_i - (\rho-\lambda)[\beta E(x_{i+1}) + \beta^2\lambda\ E(x_{i+2}) + \beta^3\lambda^2 E(x_{i+3}) + \cdot\cdot\cdot] = m_i^p.$

This is the same first order condition as in C-M; therefore, we can use their results to obtain

(A18) $m_i^p = B_0A + Bp_i$, where

(A19) $B_0 = \frac{1-\beta\rho}{1-\beta\lambda},$

(A20) $B = \frac{1-\beta\rho^2}{1-\beta\rho\lambda}.$

3. Derivation of Equation (21).

Recall that

(A21) $E[m_i|I_i] = B_0A + BE[p_i|I_i].$

Using (A21) and equation (13)

(A22) $E[(m_i - E[m_i|I_i])^2] = E[B^2(p_i - E[p_i|I_i])^2] + \sigma_\Psi^2.$

Substituting (A12) in for $E[p_i|I_i]$ in equation (A22), we get

(A23) $B^2E[p_i - (\rho-\lambda)\sum_{j=0}^{\infty}\lambda^j \frac{y_{i-j-1}}{B} - \sum_{j=0}^{\infty}\lambda^j v_{i-j}^e]^2 + \sigma_\Psi^2.$

This in turn can be written as

(A24) $B^2E[p_i - (\rho-\lambda)\sum_{j=0}^{\infty}\lambda^j[p_{i-j-1} + \frac{\Psi_{i-j-1}}{B}] - \sum_{j=0}^{\infty}\lambda^j v_{i-j}^e]^2 + \sigma_\Psi^2.$

Further substition yields,

(A25) $B^2E[\sum_{j=0}^{\infty}\rho^j v_{i-j} - (\rho-\lambda)\sum_{j=0}^{\infty}\lambda^j[\sum_{k=0}^{\infty}(\rho^k v_{i-1-j-k}) + \frac{\Psi_{i-j-1}}{B}] - \sum_{j=0}^{\infty}\lambda^j v_{i-j}^e]^2 + \sigma_\Psi^2.$

Rewriting (A25), we get after some algebra

(A26) $B^2E[\sum_{j=0}^{\infty}\lambda^j(v_{i-j} - v_{i-j}^e) - (\rho-\lambda)\sum_{j=0}^{\infty}\lambda^j \frac{\Psi_{i-j-1}}{B}]^2 + \sigma_\Psi^2.$

Taking expectations,

$$\text{(A27)}\quad B^2\left[\sum_{j=0}^{\infty}\lambda^{2j}\sigma_e^2 + (\rho-\lambda)^2\sum_{j=0}^{\infty}\lambda^{2j}\frac{\sigma_\Psi^2}{B^2}\right] + \sigma_\Psi^2.$$

This implies that

$$\text{(A28)}\quad E[(m_i - E(m_i|I_i])^2] = \frac{B^2\sigma_e^2}{1-\lambda^2} + \frac{(\rho-\lambda)^2}{1-\lambda^2}\sigma_\Psi^2 + \sigma_\Psi^2.$$

4. The optimal constant σ_e^2. (Derivation of Equation 23).

The public's problem is to

$$\text{(A29)}\quad \max_{\sigma_e^2} - \frac{1}{(1-\beta)}\left\{B^2\frac{\sigma_e^2}{1-\lambda^2} + \frac{(\rho-\lambda)^2}{1-\lambda^2}\sigma_\Psi^2 + \sigma_\Psi^2 + c(\sigma_e^2)\right\}$$

Taking derivative of (A29) with respect to σ_e^2 yields.

$$\text{(A30)}\quad \frac{-1}{(1-\beta)}\left\{\frac{2B\frac{dB}{d\sigma_e^2}\sigma_e^2}{1-\lambda^2} + \frac{B^2}{1-\lambda^2} + \left[\frac{2\lambda B^2\sigma_e^2 2\lambda}{(1-\lambda^2)^2} + \frac{2\lambda(\rho-\lambda)^2 2\lambda}{(1-\lambda^2)^2}\sigma_\Psi^2 - \frac{2(\rho-\lambda)}{(1-\lambda^2)}\sigma_\Psi^2\right]\frac{d\lambda}{d\sigma_e^2}\right.$$

$$\left. + c'(\sigma_e^2)\right\} = 0.$$

Algebra yields

$$\text{(A31)}\quad \frac{-1}{(1-\beta)}\left\{\frac{2B\frac{dB}{d\sigma_e^2}\sigma_e^2}{1-\lambda^2} + \frac{B^2}{1-\lambda^2} + \left[\lambda\sigma_e^2 - (\rho-\lambda)(1-\lambda\rho)\frac{\sigma_\Psi^2}{B^2}\right]\left(\frac{d\lambda}{d\sigma_e^2}\right)\frac{2B^2}{(1-\lambda^2)^2}\right.$$

$$\left. + c'(\sigma_e^2)\right\} = 0.$$

From the equation (A9a) and the form of the parameters a_j, we find that

$\frac{-2\lambda}{1-p\lambda}\sigma_e^2 + 2(p-\lambda)\frac{\sigma_\Psi^2}{B^2} = 0.$ This implies that the first order condition for

maximization problem (A29) is

$$\text{(A32)}\quad -\left\{\frac{2B\,\frac{dB}{d\sigma_e^2}\,\sigma_e^2}{1-\lambda^2} + \frac{B^2}{1-\lambda^2} + c'(\sigma_e^2)\right\} = 0.$$

5. Derivation of the time consistent optimal σ_e^2.

The public's problem is to

$$\text{(A33)}\quad \max_{\sigma^2_{e,i}} -\left\{\sum_{i=0}^{\infty} \beta^i \left\{E\left[\left(m_i - E[m_i|I_i(\sigma^2_{e,i}, \sigma^2_{e,i-1},\dots)]\right)^2 + c(\sigma_e^2, i)\right\}\right\}.$$

Rewriting (A33), we get

$$\text{(A34)}\quad \max_{\sigma^2_{e,i}} -\left\{\sum_{i=0}^{\infty} \beta^i \left\{E\left[B^2(p_i - E[p_i|I_i])^2\right] + \sigma_\Psi^2 + c(\sigma^2_{e,i})\right\}\right\}.$$

Using equation (A6) and remembering that the parameters $a_{j,i}$ and $b_{j,i}$ are chosen to minimize $E\left[\left(m_i - E(m_i|I_i)\right)^2\right]$ as in appendix 1, we get

$$\text{(A35)}\quad \max_{\sigma^2_{e,i}} -\Bigg\{\sum_{i=0}^{\infty} \beta^i \Bigg\{B^2\Bigg[(1-b_{0,i})^2\sigma_v^2 + b^2_{0,i}\sigma^2_{e,i} + 2(1-b_{0,i})b_{0,i}\sigma^2_{e,i}$$

$$+(\rho-a_{1,i}-b_{1,i})^2\sigma_v^2 + b^2_{1-i}\sigma^2_{e,i-1} + 2(\rho-a_{1,i}-b_{1,i})b_{1,i}\sigma^2_{e,i-1}$$

$$+(\rho^2-\rho a_{1,i}-a_{2,i}-b_{2},i)^2\sigma_v^2 + b^2_{2,i}\sigma^2_{e,i-2} + 2(\rho^2-\rho a_{1,i}-a_{2,i}-b_{2,i})b_{2,i}\,\sigma^2_{e,i-2}$$

$$+\cdots \qquad + \sum_{j=1}^{\infty} a_{j,i}\frac{\sigma_\Psi^2}{B^2}\Bigg] + \sigma_\Psi^2 + c(\sigma^2_{e,i})\Bigg\}\Bigg\}.$$

Taking the derivative of (A35) with respect to $\sigma^2_{e,i}$ remembering that B_0 and B^2 are taken as given, we get

$$\text{(A36)}\quad -\quad \beta^i B^2\left[b^2_{0,i} + \beta b^2_{1,i+1} + \beta^2 b^2_{2,i+2} + \beta^3 b^2_{3,i+3} + \cdots\right]$$

$$+B^2\sum_{k=1}^{\infty}\beta^k\sum_{j=1}^{\infty}\frac{\partial E[p_k - E(p_k|I_k)]^2}{\partial a_{j,k}}\,\frac{\partial a_{j,k}}{\partial\sigma^2_{e,i}} + B^2\sum_{k=i}^{\infty}\beta^k\sum_{j=0}^{\infty}\frac{\partial E[p_k - E(p_k|I_k)]^2}{\partial b_{j,k}}\,\frac{\partial b_{j,k}}{\partial\sigma^2_{e,i}}$$

$$+ \beta^{i} c'(\sigma^2_{e,i}) \quad = 0.$$

Recall that for the case where $\sigma^2_{e,i} = \sigma^2_e$, for all i, $b_{j,i}$ and $a_{j,i}$ are given by equations (A10) and (A11). Also, since $a_{j,i}$ and $b_{j,i}$ are chosen optimally (see Appendix 1), $\frac{\partial E[p_k - E(p_k | I_k)]^2}{\partial a_{j,k}} = 0$ and $\frac{\partial E[p_k - E(p_k | J_k)]^2}{\partial b_{j,k}} = 0.$

Therefore, we can rewrite (A36) as

$$\text{(A37)} \quad - \beta^{i} \left\{ B^2[1+\beta\lambda^2 + \beta^2\lambda^4 + \beta^3\lambda^6 + \cdot \cdot \cdot] + c'(\sigma^2_e) \right\} = 0.$$

Thus, the time consistent optimal σ^2_e will satisfy

$$\text{(A38)} \quad - \frac{B^2}{1-\beta\lambda^2} + c'(\sigma^2_e) = 0.$$

6. Proof of Proposition 2.

Let FOC be first order condition given by equation (A31). Let SOC be the second order conditions for the maximization problem (A29) (SOC < 0).

$$\text{(A39)} \quad \frac{d\sigma^{2*}_e}{d\sigma^2_\Psi} = - \frac{\frac{\partial(\text{FOC})}{\partial\sigma^2_\Psi}}{\text{SOC}}$$

$$\text{(A40)} \quad \frac{\partial(\text{FOC})}{\partial\sigma^2_\Psi} = \frac{-1}{(1-\beta)} \left\{ \frac{2\left(\frac{\partial B}{d\sigma^2_\Psi}\right)\left(\frac{\partial B}{\partial\sigma^2_e}\right)\sigma^2_e}{(1-\lambda^2)} + \frac{2B\frac{\partial^2 B}{\partial\sigma^2_e\partial\sigma^2_\Psi}\sigma^2_e}{1-\lambda^2} + \frac{2B}{1-\lambda^2}\frac{\partial B}{\partial\sigma^2_\Psi} \right.$$
$$+ \frac{2B\left(\frac{\partial B}{\partial\sigma^2_e}\right)\sigma^2_e + B^2}{(1-\lambda^2)^2} 2\lambda\frac{\partial\lambda}{\partial\sigma^2_\Psi}$$
$$+ \frac{2B^2}{(1-\lambda^2)^2}\left[\sigma^2_e \frac{\partial\lambda}{\partial\sigma^2_\Psi} + [p(p-\lambda) + (1-\lambda p)]\frac{\sigma^2_\Psi}{B}\frac{\partial\lambda}{\partial\sigma^2_\Psi} + \frac{2(\rho-\lambda)(1-\lambda\rho)\sigma^2_\Psi}{B^3}\frac{\partial B}{\partial\sigma^2_\Psi}\right.$$
$$\left.\left. - \frac{(\rho-\lambda(1-\lambda\rho)}{B^2}\right]\frac{\partial\lambda}{\partial\sigma^2_e}\right\}.$$

Using the relation between σ^2_e and σ^2_Ψ in terms B and λ, we know that

$$\frac{\partial B}{\partial \sigma_\psi^2} = \frac{-\partial B}{\partial \sigma_e^2}\,\frac{\sigma_e^2}{\sigma_e^2} \quad \text{and} \quad \frac{\partial \lambda}{\partial \sigma_\psi^2} = \frac{-\partial \lambda}{\partial \sigma_e^2}\,\frac{\sigma_e^2}{\sigma_\psi^2}\,.$$

Also from equation (A9a), we know that

$$2\lambda\sigma_e^2 = 2(1-\rho\lambda)(\rho-\lambda)\,\frac{\sigma_\psi^2}{B}\,.$$

Rearranging (A40), we get

$$\text{(A41)} \quad \left(\frac{-1}{1-\beta}\right)\left\{\frac{\frac{\partial B}{\partial \sigma_\psi^2}}{1-\lambda^2}\left[2B + 2\,\frac{\partial B}{\partial \sigma_e^2}\,\sigma_e^2\right] + \frac{4B\lambda}{(1-\lambda^2)^2}\,\frac{\partial \lambda}{\partial \sigma_\psi^2}\left[2\sigma_e^2\,\frac{\partial B}{\partial \sigma_e^2} + B\right]\right.$$

$$\left.\frac{2B^2}{(1-\lambda^2)^2}\,\frac{\partial \lambda}{\partial \sigma_\psi^2}\left(\frac{1-\lambda\rho}{\lambda}\right)\frac{\sigma_\psi^2}{B^2} + 2B\,\frac{\partial^2 B}{\partial \sigma_e^2\,\partial \sigma_\psi^2}\,\frac{\sigma_e^2}{(1-\lambda^2)}\right\}.$$

Recall that

$$B = \frac{1-\beta\rho^2}{1-\beta\rho\lambda} \quad \text{and} \quad \lambda(r) = \frac{\frac{1+r}{\rho} + \rho}{2} - \left(\frac{1}{4}\left(\frac{1+r}{\rho} + \rho\right)^2 - 1\right)^{\frac{1}{2}}$$

where $r = B^2\,\dfrac{\sigma_e^2}{\sigma_\psi^2}$.

$$\text{(A42)} \quad \lambda'(r) = -\,\frac{\lambda(r)\left(\frac{1}{4}\left(\frac{1+r}{\rho} + \rho\right)^2 - 1\right)^{-\frac{1}{2}}}{2\rho} < 0$$

To find $\dfrac{\partial B}{\partial \sigma_\psi^2}$ we use the equation:

$$B(1-\beta\rho\lambda) = 1-\beta\rho^2$$

$$\left(1-\beta\rho\lambda - \beta\rho\lambda' 2B^2\,\frac{\sigma_e^2}{\sigma_\psi^2}\right)dB = -\,\beta\rho\lambda' B^3\,\frac{\sigma_e^2}{(\sigma_\psi^2)^2}\,d\sigma_\psi^2$$

Therefore,

$$\text{(A43)} \quad \frac{\partial B}{\partial \sigma_\psi^2} = \frac{-\beta\rho\lambda'\,\frac{B^3\sigma_e^2}{(\sigma_\psi^2)^2}}{1-\beta\rho\lambda - \beta\rho\lambda' 2B^2\,\frac{\sigma_e^2}{\sigma_\psi^2}} > 0.$$

Furthermore,

(A44) $$\frac{\partial\lambda}{\partial\sigma_\Psi^2} = \lambda'\left(\frac{-B^2\sigma_e^2}{\sigma_\Psi^2} + \frac{2B\sigma_e^2}{\sigma_\Psi^2}\frac{\partial B}{\partial\sigma_\Psi^2}\right) = \frac{\lambda'(-1+\beta\rho\lambda)\dfrac{B^2\sigma_e^2}{(\sigma_\Psi^2)^2}}{1-\beta\rho\lambda - \beta\rho\lambda' 2B^2\dfrac{\sigma_e^2}{\sigma_\Psi^2}} > 0.$$

Finally,

(A45) $$\frac{\partial^2 B}{\partial\sigma_\Psi^2\,\partial\sigma_e^2} = \frac{\partial B}{\partial\sigma_e^2}\frac{\partial B}{\sigma_\Psi^2}\frac{1}{B}\left[4 + \partial\frac{\sigma_e^2}{B}\frac{\partial B}{\partial\sigma_e^2}\right] - \frac{\partial B}{\partial\sigma_e^2}\frac{1}{\sigma_\Psi^2} + \frac{\partial\lambda'}{\partial\sigma_\Psi^2}\frac{1}{\lambda'}\frac{\partial B}{\partial\sigma_e^2}\left[1 + \frac{2}{B}\frac{\partial B}{\partial\sigma_e^2}\right]$$

Substituting equation (A45) into (A41)

(A46) $$-\frac{1}{(1-\beta)}\left\{\frac{\frac{\partial B}{\sigma_\Psi^2}}{(1-\lambda^2)}\left[4B + 10\frac{\partial B}{\sigma_e^2}\sigma_e^2 + 12\frac{(\sigma_e^2)^2}{B}\left(\frac{\partial B}{\partial\sigma_e^2}\right)^2\right] + \frac{4B\lambda}{(1-\lambda^2)}\frac{\partial\lambda}{\partial\sigma_\Psi^2}\left[2\sigma_e^2\frac{\partial B}{\partial\sigma_e^2} + B\right]\right.$$

$$\left. + \frac{2(1-\lambda\rho)}{(1-\lambda^2)^2\lambda}\sigma_\Psi^2\frac{d\lambda}{\partial\sigma_\Psi^2} + \frac{2\sigma_e^2}{1-\lambda^2}\frac{\partial B}{\partial\sigma_e^2}\left[B + 2\frac{\partial B}{\partial\sigma_e^2}\sigma_e^2\right]\frac{\partial\lambda'}{\partial\sigma_\Psi^2}\frac{1}{\lambda'}\right\}.$$

Further rewriting yields,

(A47) $$-\frac{1}{(1-\beta)}\left\{\frac{\frac{\partial B}{\sigma_\Psi^2}}{(1-\lambda^2)}\left[4B + 10\frac{\partial B}{\partial\sigma_e^2}\partial_e^2 + 12\frac{(\sigma_e^2)^2}{B}\left(\frac{\partial B}{\partial\sigma_e^2}\right)^2\right.\right.$$

$$+ \frac{\partial\lambda}{\partial\sigma_\Psi^2}\frac{1}{(1-\lambda^2)}\left[\frac{4B\lambda}{(1-\lambda^2)}[2\sigma_e^2\frac{\partial B}{\sigma_e^2} + B] + \frac{2(1-\lambda\rho)}{(1-\lambda^2)}\frac{\sigma_\Psi^2}{\lambda} + 2\sigma_e^2\frac{\partial B}{\partial\sigma_e^2}\left[B + 2\sigma_e^2\frac{\partial B}{\partial\sigma_e^2}\right] x\right.$$

$$\left.\left.\left(\frac{1}{\lambda} + (\frac{1+r}{\rho} + \rho)(\frac{1}{4}(\frac{1+r}{\rho} + \rho)^2 - 1)^{\frac{1}{2}}\right)\right]\right\}.$$

If one writes out the term $\left[4B + 10\dfrac{\partial B}{\partial\sigma_e^2}\sigma_e^2 + 12\dfrac{(\sigma_e^2)^2}{B}\left(\dfrac{\partial B}{\partial\sigma_e^2}\right)^2\right]$, one finds that this term is always positive. Thus, to sign equation (A47) we need to look at the second term in (A47). The second term can be written as

$$\text{(A48)}\quad \frac{\partial\lambda}{\partial\sigma_\Psi^2}\frac{1}{(1-\lambda^2)}\left[\left[2\sigma_e^2\frac{\partial B}{\partial\sigma_e^2}+B\right]\left[\frac{4B\lambda}{1-\lambda^2}+\left(\frac{1}{\lambda}+\frac{(\frac{1+r}{\rho}\,p)\,(\frac{1}{4}\,(\frac{1+r}{\rho}+\rho)^2-1)^{-1/2}}{2\lambda}\right)2\sigma_e^2\frac{\partial B}{\partial\sigma_e^2}\right]\right.$$

$$\left.+\frac{2(1-\lambda p)}{(1-\lambda^2)}\frac{\sigma_\Psi^2}{\lambda}\right].$$

If we look at the term

$$\frac{4B\lambda}{1-\lambda^2}+\left(\frac{1}{\lambda}+\frac{(\frac{1+r}{\rho}+p)\,(\frac{1}{4}\,(\frac{1+r}{\rho}\,p)^2-1)^{-1/2}}{2\lambda}\right)2\sigma_e^2\,\frac{\partial B}{\partial\sigma_e^2},$$

we can write this term as

$$\text{(A49)}\quad \frac{4B\lambda}{(1-\lambda^2)}+\left(\frac{1}{\lambda}+\frac{\left(\frac{1+\lambda^2}{\lambda}\right)_2\left(\frac{\lambda}{1-\lambda^2}\right)}{2\lambda}\right)2\sigma_e^2\,\frac{\partial B}{\partial\sigma_e^2}.$$

This equals

$$\text{(A50)}\quad \frac{4}{(1-\lambda^2)\lambda}\left(B\lambda^2+\sigma_e^2\frac{\partial B}{\partial\sigma_e^2}\right).$$

Using (A43), we can write (A50) as

$$\text{(A51)}\quad \frac{4B\lambda^2}{(1-\lambda^2)\lambda}\left[\left(1-\beta\lambda p-B^2\beta\,\frac{\sigma_e^2}{\sigma_\Psi^2}\right)(1-\lambda^2)+\lambda^2B^2\beta\,\frac{\sigma_e^2}{\sigma_\Psi^2}\right].$$

This term will always be positive if $1-\beta\lambda\rho > \beta B^2\,\frac{\sigma_e^2}{\sigma_\Psi^2}$.

Therefore, if

$$\text{(A52)}\quad 1-\beta\lambda\rho > B\beta^2\,\frac{\sigma_e^2}{\sigma_\Psi^2}$$

we know that the term (A48) is positive which in turn implies that (A47) is negative. Consequently, $\frac{\partial[FOC]}{\partial\sigma_\Psi^2} < 0$ which implies that $\frac{\partial\sigma_e^{2*}}{\partial\sigma_\Psi^2} < 0$.

It is possible for (A52) not hold and equation (A51) still be positive. Furthermore, it may be possible for $\frac{\partial \sigma_e^2}{\partial \sigma_\Psi^2}$ to be negative even if equation (A51) is not positive since euqation (A48) may still be positive. Finally, even if equation (A48) is negative it still may be possible for (A47) to be negative and, consequently, $\frac{\partial \sigma_e^{2*}}{\partial \sigma_\Psi^2} < 0$. Thus, condition (A52) represents a sufficient condition.

7. Derivation of Monetary authority expected utility.

The monetary expected utility is

$$\text{(A53)} \quad \sum_{i=0}^{\infty} \beta^i E\left[\left(m_i - E[m_i|I_i]\right. x_i - \frac{(m_i)^2}{2}\right]$$

$$\text{(A54)} \quad = \sum_{i=0}^{\infty} \beta^i E\left[B(p_i - E[p_i|I_i]\ (A+p_i) - \frac{(B_0 A + B\rho_i + \Psi_i)^2}{2}\right]$$

$$\text{(A55)} \quad = \sum_{i=0}^{\infty} \beta^i E\left[B\left(\sum_{i=0}^{\infty} \rho^i v_{i-j} - (\rho-\lambda)\sum_{j=0}^{\infty} \lambda^i\left[\sum_{k=0}^{\infty} \rho^k v_{i-j-k-1} + \frac{\Psi_{i-j-1}}{B}\right]\right.\right.$$

$$\left. - \sum_{j=0}^{\infty} \lambda^i v^e_{i-j}\right)\left(\sum_{j=0}^{\infty} \rho^i v_{i-j}\right) - \frac{(B_0 A + B\sum_{j=0}^{\infty} \rho^j v_{i-j} + \Psi_i)^2}{2}\Bigg]$$

$$\text{(A56)} \quad = \sum_{i=0}^{\infty} \beta^i \left\{E\left[B\left(\sum_{j=0}^{\infty} \lambda^j(v_{i-j} - v^e_{i-j}) - (\rho-\lambda)\sum_{j=0}^{\infty} \lambda^i \frac{\Psi_{i-j-1}}{B}\right)\left(\sum_{j=0}^{\infty} \rho^j v_{i-j}\right)\right]\right.$$

$$\left. - \frac{B_0 A}{2} - \frac{B^2\sigma_v^2}{2(1-\rho^2)} - \frac{\sigma_\Psi^2}{2}\right\}$$

$$\text{(A57)} \quad = \sum_{i=0}^{\infty} \beta^i \left\{B\sum_{j=0}^{\infty} \lambda^j \rho^j\ E[(v_{i-j} - v^e_{i-j})v_{i-j}] - \frac{B_0 A}{2} - \frac{B^2\sigma_v^2}{2(1-\rho^2)} - \frac{\sigma_\Psi^2}{2}\right\}$$

$$\text{(A58)} \quad = \sum_{i=0}^{\infty} \beta^i \left\{ B \sum_{j=0}^{\infty} \lambda^j \rho^j \sigma_e^2 - \frac{B_0 A}{2} - \frac{B^2 \sigma_v^2}{2(1-\rho^2)} - \frac{\sigma_\Psi^2}{2} \right\}$$

$$\text{(A59)} \quad = \sum_{i=0}^{\infty} \beta^i \left\{ \frac{B \sigma_e^2}{(1-\lambda\rho)} - \frac{B_0 A}{2} - \frac{B^2 \sigma_v^2}{2(1-\rho^2)} - \frac{\sigma_\Psi^2}{2} \right\}$$

$$\text{(A60)} \quad = \frac{1}{(1-\beta)} \left\{ \frac{B \sigma_e^2}{(1-\lambda\rho)} - \frac{B_0 A}{2} - \frac{B^2 \sigma_v^2}{2(1-\rho^2)} - \frac{\sigma_\Psi^2}{2} \right\}$$

8. Proof of proposition 3.

$$\text{(A61)} \qquad \frac{\partial \sigma_\Psi^2}{\partial \sigma_e^2} = - \frac{\dfrac{\partial(\text{FOC})}{\partial \sigma_e^2}}{\text{SOC}}$$

Recall that the FOC are given by equation (26) and that SOC < 0.

$$\text{(A62)} \quad \frac{\partial(\text{FOC})}{\partial \sigma_e^2} = \frac{1}{1-\beta} \Bigg[\frac{\dfrac{\partial B}{\partial \sigma_e^2 \partial \sigma_\Psi^2}}{(1-\lambda\rho)} \sigma_e^2 + \frac{\rho \sigma_e^2 \dfrac{\partial \lambda}{\partial \sigma_\Psi^2} \dfrac{\partial B}{\partial \sigma_e^2}}{(1-\lambda\rho)^2} + \frac{\rho B \sigma_e^2 \dfrac{\partial^2 \lambda}{\partial \sigma_\Psi^2 \partial \sigma_e^2}}{(1-\lambda\rho)^2}$$

$$+ \frac{\rho \dfrac{\partial B}{\partial \sigma_\Psi^2} \sigma_e^2}{(1-\lambda\rho)^2} \frac{\partial \lambda}{\partial \sigma_e^2} + \frac{2\rho^2 \sigma_e^2 \dfrac{\partial \lambda}{\partial \sigma_\Psi^2}}{(1-\lambda\rho)^3} \frac{\partial \lambda}{\partial \sigma_e^2} - \frac{\partial B_0}{\partial \sigma_e^2} \frac{\partial B_0}{\partial \sigma_\Psi^2} A^2 - B_0 \frac{\partial^2 B_0}{\partial \sigma_\Psi^2 \partial \sigma_e^2} A^2$$

$$- \frac{\dfrac{\partial B}{\partial \sigma_e^2} \dfrac{\partial B}{\partial \sigma_\Psi^2}}{(1-\rho^2)} \sigma_v^2 - B \frac{\dfrac{\partial^2 B}{\partial \sigma_\Psi^2 \partial \sigma_e^2}}{(1-\lambda\rho)} \sigma_v^2 + \frac{\dfrac{\partial B}{\partial \sigma_\Psi^2}}{1-\lambda\rho} + \frac{\rho B \dfrac{\partial \lambda}{\partial \sigma_\Psi^2}}{(1-\lambda\rho)^2} \Bigg].$$

Notice that $\dfrac{\partial B}{\partial \sigma_e^2} = - \dfrac{\partial B}{\partial \sigma_\Psi^2} \dfrac{\sigma_\Psi^2}{\sigma_e^2}$, $\dfrac{\partial^2 B}{\partial \sigma_e^2 \partial \sigma_\Psi^2} = - \dfrac{\partial^2 B}{\partial \sigma_\Psi^{2\,2}} \dfrac{\sigma_\Psi^2}{\sigma_e^2}$,

$$\frac{\partial \lambda}{\partial \sigma_e^2} = - \frac{\partial \lambda}{\partial \sigma_\Psi^2} \frac{\sigma_\Psi^2}{\sigma_e^2}, \quad \frac{\partial^2 \lambda}{\partial \sigma_\Psi^2 \partial \sigma_e^2} = - \frac{\partial^2 \lambda}{\partial \sigma_\Psi^{2\,2}} \frac{\sigma_\Psi^2}{\sigma_e^2}, \quad \frac{\partial B_0}{\partial \sigma_e^2} = - \frac{\partial B_0}{\partial \sigma_\Psi^2} \frac{\sigma_\Psi^2}{\sigma_e^2},$$

and $$\frac{\partial^2 B_0}{\partial\sigma_\Psi^2\,\partial\sigma_e^2} = -\frac{\partial^2 B_0}{(\partial\sigma_\Psi^2)^2}\frac{\sigma_\Psi^2}{\sigma_e^2}.$$

Using these facts, we know that

$$\text{(A63)}\quad \frac{\partial(\text{FOC})}{\partial\sigma_e^2} = -\,\text{SOC}\,\frac{\sigma_\Psi^2}{\sigma_e^2} + \frac{1}{1-\beta}\left[\frac{\frac{\partial B}{\partial\sigma_\Psi^2}}{1-\lambda\rho} + \frac{pB\partial\sigma_\Psi^2}{(1-\lambda\rho)^2}\right]$$

Therefore, $\frac{\partial \text{FOC}}{\partial\sigma_e^2} > 0$ and, consequently, $\frac{\partial\sigma_\Psi^{2*}}{\partial\sigma_e^2} > 0$.