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Tax Indexation and Inflationary Finance

by

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ABSTRACT

In this paper we investigate policies for the large and small country that provide complete insulation from foreign real and monetary disturbances. We find that when there exists two channels of transmission, the integrated commodity and capital markets, using only exchange rate policies does not provide complete insulation. However, floating the exchange rate and pursuing a specific interest rate target does. In terms of output variability however, insulating policies may be undesirable.

TAX INDEXATION AND INFLATIONARY FINANCE

Government revenue from inflationary finance has been investigated by Cagan (1956), Bailey (1956), Friedman (1971), Barro (1972), Auernheimer (1974), and others. Primary attention has been directed toward the inflationary implications of revenue-maximizing behavior on the part of government. Typically in that work, government chooses a rate of inflation to maximize the revenue obtained from fiat money creation. The assumption is either no government revenue from tax receipts; or (when taxes are considered), the real level of tax receipts is independent of inflation.

It is questionable whether a government would consider its revenue from money creation without regard to the way inflation affects tax receipts. What happens when taxes and money creation are combined to maximize government revenue? We model the process here in a way that permits direct comparison to pure inflationary finance.

One key feature of this model is a non-indexed tax rate schedule. In pure inflationary finance models, government revenue is maximized by choice of a steady-state rate of inflation; however with a relationship between the price level and tax receipts, the steady state may not be optimal. An interesting and important question is: how does the introduction of a non-indexed tax system affect the optimum inflation rate? Do non-indexed taxes alter the revenue-maximizing time path of the rate of inflation, compared to pure inflationary finance? What are the possible steady states under such a system? In what way does government's time preference intensity affect the optimal time path of inflation?

The first part of this paper is a re-examination of simple inflationary finance, extended to include optimal paths which may not be steady states.

The analysis shows that the optimal path of inflation is constant in time and equal to the rate determined in earlier work, even when the government is not constrained to steady-state behavior.

Part two of the paper analyzes government revenue from tax receipts. The results are (1) with a non-indexed tax system, optimal inflation may initially increase due to the attractiveness of increased tax revenues; but eventually (even before a tax-revenue-maximizing price level is reached) the non-indexed taxes will reduce the optimal inflation rate below the simple inflationary finance rate. Thus, depending on the initial state, less inflation can be expected when the government also relies on a non-indexed tax system for revenue. (2) There are two possible steady states under combined tax and inflationary finance. One is a direct analog of the simple inflationary finance solution. The second involves no inflation; non-indexed taxes have the effect of stablizing the price level. Finally, (3) lessening the government's discount rate increases the likelihood of the steady state solution with no inflation.

I. Inflationary Finance

This section re-examines simple inflationary finance, to include non-steady state solution possibilities. In its structural assumptions the analysis remains as close as possible to earlier literature on inflationary finance.

It is assumed that the rate of inflation is the primary variable governing the demand for real cash balances. The analysis abstracts from population growth, technology, and any real income or real balance effects on the demand for money. The only money is fiat money. The government does not demand money. There are two economic goods--consumable commodities (output of the private sector) and money. There are no interest bearing assets. Violation of these assumptions does not invalidate the analysis; however, additional behavioral relationships are required if they do not all hold.

Individuals' demand for real cash balances is of the form

$$\left(\frac{M_t}{P_t}\right)^d = m(\pi_t), \tag{1}$$

where $m'(\pi_t) < 0$, $m''(\pi_t) > 0$, M represents the nominal stock of cash balances, P denotes the price level, $\pi = \frac{\dot{P}}{P}$ represents the rate of inflation $\left(\dot{P} = \frac{dP}{dt}\right)$, and t is the time index.

The government's flow of revenue (revenue per unit time) from money creation (g_t) is the real value of the increase in the stock of cash balances,

$$g_t = \frac{\dot{M}_t}{P_t}. \tag{2}$$

Government wealth (G) associated with any time path of g_t may be written as

$$G = \int_0^\infty e^{-\delta t} g_t \, dt \tag{3}$$

where $\delta > 0$ represents the subjective rate of time preference of the government.

At each point in time the behavior of the government is constrained by the condition that the demand for money must equal the supply of money. This condition implies a relationship between the rate of growth of the stock of cash balances and the rate of growth of the demand for that stock. Specifically,

$$\dot{M}_t = \dot{P}_t m(\pi_t) + P_t m'(\pi_t)\dot{\pi}_t. \tag{4}$$

Equations (2), (3), and (4) set forth the problem of determining the time path of the rate of inflation which maximizes total government wealth. The typical course at this point is to confine the analysis to an examination of steady states -- time invariant paths of π_t. If we impose the condition that $\dot{\pi}_t = 0$ and allow the government only to make an initial step change in the rate of inflation then the problem is to maximize

$$G = \int_{0+}^{\infty} e^{-\delta t}\, \pi m(\pi)dt + m_{t=0+} - m_{t=0} \tag{5}$$

where $m_{t=0+}$ is the demand for real cash balances in the newly selected steady state and $m_{t=0}$ is the pre-existing demand for real cash balances. The government's choice is limited to a single (time-invariant) value for π. Maximization of G yields the expression

$$\pi = -\delta - \frac{m(\pi)}{m'(\pi)} \tag{6}$$

from which the revenue-maximizing rate of inflation (π^*) may be determined; π^* is the rate of inflation which maximizes the government's revenue from simple inflationary finance under steady state restrictions and corresponds to the value determined by Auernheimer (1974).

The steady state condition is relaxed by dropping the requirement that $\dot{\pi}_t = 0$, and allowing π_t to be any piecewise differentiable function. In particular, jumps in π_t are allowed at any point.

THEOREM 1: The optimum time path for π_t remains $\pi_t = \pi^*$, for all t.

Proof: Substituting for M_t from (4) into (2), and then into (3) gives

$$G[\pi] = \int_{0}^{\infty} e^{-\delta t}\left[\pi_t m(\pi_t) + \dot{\pi}_t m'(\pi_t)\right] dt \tag{7}$$

which may be interpreted as a line integral (useful because of the jumps) and rewritten as

$$G[\pi] = \oint_{\substack{t=0\\ \pi=\pi_0}}^{t=\infty} e^{-\delta t}\left[\pi_t m(\pi_t)dt + m'(\pi_t)d\pi_t\right]. \tag{8}$$

If the path π_t^* is optimal in this more general setting, the value $G[\pi_t^*]$ must be larger than $G[\pi_t]$ for any other admissible path π_t. If the inflation paths are viewed geometrically as curves in the π, t plane, this may be accomplished by appeal to Green's Theorem [Fleming, (1977)].

Figure 1 shows the optimal curve π_t^* and some other arbitrary curve, π_t.

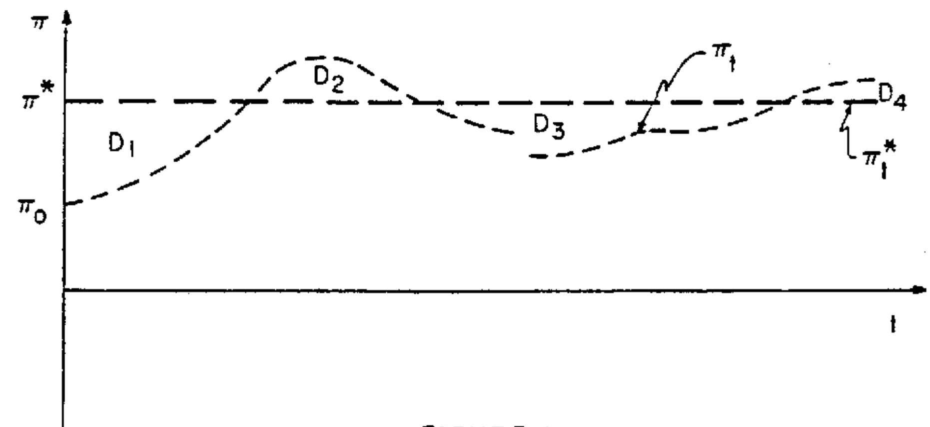

FIGURE 1

THE OPTIMAL PATH π_t^* (__ __) AND AN ARBITRARY COMPETING PATH π_t (----)

The difference $G[\pi_t] - G[\pi_t^*]$, making use of the line integral interpretation, is simply the sum of the integrals (attention must be paid to the orientation) around each of the lobes D_i,

$$\Delta G = G[\pi_t] - G[\pi_t^*] = \int_{\pi_t - \pi_t^*} \pi_t m(\pi_t) e^{-\delta t} dt + m'(\pi_t) e^{-\delta t} d\pi$$

$$= \sum_i \int_{C_i} \pi_t m(\pi_t) e^{-\delta t} dt + m'(\pi_t) e^{-\delta t} d\pi , \qquad (9)$$

where C_i denotes the boundary of the lobe D_i. Next apply Green's Theorem to each of the integrals on the right side of (9) to convert them to integrals over the regions D_i. This results in

$$\Delta G = \sum_i \varepsilon_i \iint_{D_i} e^{-\delta t} f'(\pi_t) dt d\pi , \qquad (10)$$

where

$$f(\pi_t) = (\pi_t + \delta)\, m(\pi_t) \qquad (11)$$

and $\varepsilon_i = \pm 1$ if D_i is above or below π^* respectively due to the opposite orientations of C_i in each case. The function $f(\pi)$ is assumed to have a unique critical point, π^*, (see equation (6)) which is a maximum. (This function recurs throughout and will always be assumed to satisfy this physically reasonable assumption on the demand function, $m(\pi_t)$). Note that this assumption implies $f'(\pi_t) > 0$ for $\pi_t < \pi^*$ and $f'(\pi_t) < 0$ for $\pi_t > \pi^*$. Since $\varepsilon_i f'(\pi) < 0$ in either case, each summand in equation (10) is negative so that $\Delta G < 0$ as required.

II. Combined Tax and Inflationary Finance: Structure

This section is concerned with an extension to include tax finance. We determine the inflation path which maximizes the government's revenue from taxes and inflation, and compare these results to pure inflationary finance.

The tax system is not indexed:

$$\tau_t = \tau(P_t), \tag{12}$$

and a relationship exists between output (y) and the tax rate (τ),

$$y_t = y(\tau_t). \tag{13}$$

By assumption, $\tau'(P_t) > 0$, $0 < \tau_t < \hat{\tau} < 1$, $\lim_{P_t \to \infty} \tau_t = \hat{\tau}$, and $\lim_{P_t \to \infty} \tau'(P_t) = 0$. That is, the tax rate is an increasing function of the price level, up to some maximum tax rate ($\hat{\tau}$) beyond which an increase in the price level can no longer shift individuals into higher tax brackets.

By assumption also, $y'(\tau_t) < 0$ and $y(\tau_t) > 0$ for $\tau_t < \hat{\tau}$, so that tax receipts are positive for all P_t. A number of underlying structures may be invoked, any of which will generate the assumed condition that output is a decreasing function of the tax rate. However, we gain nothing by going behind that assumption; and will not do so.

For simplicity, the analysis will ignore possible further effects (such as wealth effects or inflation effects) on output. All output is assumed to be taxed at an equal marginal and average rate τ_t. The real value of the government's tax receipts per unit time (x_t) may be written as

$$x_t = \tau_t y(\tau_t). \tag{14}$$

The tax rate τ^* which maximizes tax receipts satisfies

$$\frac{\tau^* y'(\tau^*)}{y(\tau^*)} = -1; \tag{15}$$

it is the unit elastic tax rate.

Note from (12) and (14) that total tax revenue is

$$x_t = x(P_t) \tag{16}$$

where $x'(P_t) \gtreqless 0$ when $\tau'(P_t)(\eta + 1) \gtreqless 0$; $\eta = \eta(P_t) = y'(\tau_t)\tau(P_t)/y(\tau_t)$, the elasticity of output with respect to the tax rate. Since $y'(\tau_t) < 0$, $\tau'(P_t) > 0$, and both functions are continuous, there exists a tax-revenue maximizing price level, P^*, which satisfies $\eta(P^*) = -1$. This implies that $x'(P_t) \gtreqless 0$ for $P_t \lesseqgtr P^*$. We further impose two innocuous 'boundedness' conditions $\lim_{P_t \to \infty} x'(P_t) = 0$, and $\lim_{P_t \to \infty} x_t = \hat{\tau} y(\hat{\tau})$. That is, government revenue flow from taxation increases as the price level increases up to P^*, then diminishes for P beyond P^* to a lower bound $\hat{\tau} y(\hat{\tau})$.

The government's discounted real proceeds from combined tax and inflationary finance may now be written as

$$G = \int_0^\infty e^{-\delta t}\left\{\frac{\dot{M}_t}{P_t} + x(P_t)\right\}dt. \tag{17}$$

Substituting $M_t = P_t m(\pi_t)$ as the equilibrium condition in the money market into (4) gives $\frac{\dot{M}_t}{M_t} = \pi_t + \dot{\pi}_t \frac{m'(\pi_t)}{m(\pi_t)}$ which solving for π_t gives $\dot{\pi}_t = \left[\frac{\dot{M}_t}{M_t} - \pi_t\right] \frac{m(\pi_t)}{m'(\pi_t)}$. This implies (since $m(\pi_t)$, $m'(\pi_t) \neq 0$ for every t) that any $\dot{\pi}_t$ can be created by the government by the proper choice of $\dot{M}_t$.

Thus π_t may be regarded as the control variable. Moreover, as before, since jumps in π_t are permissible, π_t as the control variable is selected from the class of piecewise continuously differentiable functions on the interval [0, ∞]. Total government wealth may be expressed, using equation (4) in terms of the control π_t, as

$$G[\pi] = \int_0^\infty e^{-\delta t}\{\pi_t m(\pi_t) + \dot{\pi}_t m'(\pi_t) + x(P_t)\}dt. \qquad (18)$$

where $P_t = P_0 e^{u_t}$, $P_0 = P(t = 0)$, and $u_t = \int_0^t \pi_s ds$. $G[\pi]$ may be rewritten as

$$G[u] = \int_{0+}^\infty e^{-\delta t}\{\dot{u}_t m(\dot{u}_t) + \ddot{u}_t m'(\dot{u}_t) + x(P_0 e^{u_t})\}dt + m_{0+} - m_0 \qquad (19)$$

where $m_0 = m(\pi_{t=0})$ represents the pre-existing equilibrium level of real cash balances and $m_{0+} = m(\pi_{t=0+})$ represents the level of real cash balances immediately following the transition to the optimal path of the rate of inflation. The increment at $t = 0$ is displayed separately to avoid confusion. The expression $\int_{0+}^\infty e^{-\delta t}\,\ddot{u}_t m'(\dot{u}_t)dt$ may be integrated by parts to yield

$$\int_{0+}^\infty e^{-\delta t}\ddot{u}_t m'(\dot{u}_t)dt = e^{-\delta t}m(\dot{u}_t)\Big|_{0+}^\infty + \int_{0+}^\infty e^{-\delta t}\delta m(\dot{u}_t)dt = \int_{0+}^\infty e^{-\delta t}\delta m(\dot{u}_t)dt - m_{0+}.$$

$G[u]$ then becomes

$$G[u] = \int_{0+}^\infty e^{-\delta t}\{(\dot{u}_t + \delta)m(\dot{u}) + x(P_0 e^{u})\}dt - m_0. \qquad (20)$$

Since m_0 is a given constant, maximization is over the integral portion of equation (20) with respect to u_t.

Let $f(\pi_t)$ be as defined in equation (11) and satisfy the same assumptions stated there. The variational equation for stationary points of G is

$$e^{-\delta t}P_0 e^{u_t} x'(P_0 e^{u_t}) = \frac{d}{dt}\{e^{-\delta t} f'(\dot{u}_t)\}. \tag{21}$$

Returning to the primitive variables, the Euler-Lagrange condition may be rewritten in system form

$$\dot{P}_t = \pi_t P_t \tag{22}$$

$$f''(\pi_t)\dot{\pi}_t = \delta f'(\pi_t) + P_t x'(P_t).$$

It is this system that is now analyzed to determine the optimal time path for the rate of inflation. Note that P_0 is specified but π_0 (actually π_{0+}) is free and to be determined. $G[\pi]$ must be maximized over the solutions of equations (22) generated from different choices of π_0.

Figures 2 and 3 show generic plots of the functions $f(\pi)$ and $x(P)$ and their derivatives.

Several results from the calculus of variations theory provide useful information. The Legendre necessary condition [Fleming and Rishel (1975)] states that the second partial of the integrand of G with respect to $\pi\,(=\dot{u})$ must be non-positive along the optimal path, denoted $\hat{\pi}_t$. This condition is easily evaluated and results in $f''(\hat{\pi}_t) \leq 0$. Noting the value $\bar{\pi}$ from Figure 2 and that $\pi_t < -\delta$ is nonphysical, the optimal trajectory for the inflation rate is bounded by

$$-\delta \leq \hat{\pi}_t \leq \bar{\pi}. \tag{23}$$

Continuing, it is next noted that the Weierstrass-Erdmann corner condition [Fleming and Rishel (1975)] implies that since $f'(\pi)$ is monotone in the range delimited by (23) (see Figure 2), then the optimal trajectory, $\hat{\pi}_t$, will have no discontinuities other than the already considered start-up jump at $t = 0$, and therefore, $\hat{\pi}_t$ must in fact be continuously differentiable throughout the time interval $(0,\infty)$.

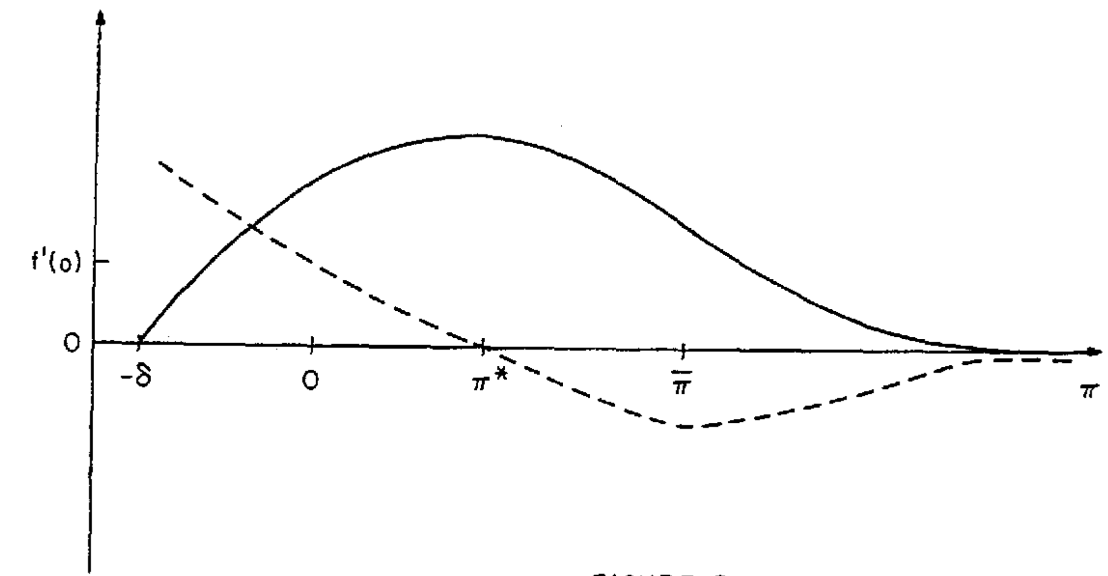

FIGURE 2

THE FUNCTIONS $f(\pi)$ (——) AND $f'(\pi)$ (----)

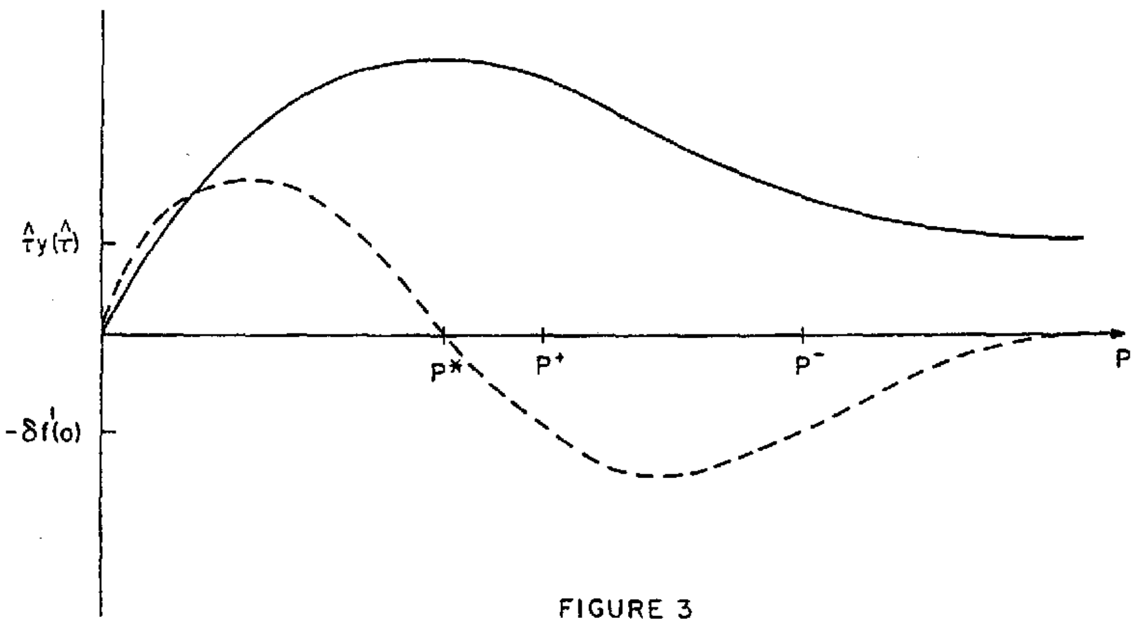

FIGURE 3

THE FUNCTIONS $x(P)$ (——) AND $Px'(P)$ (----)

Finally, the critical points of equations (22) are determined and analyzed. The point $P = 0$, $\pi = \pi^*$ is always a critical point; linearization about this point shows that this is an unstable node. While this is a physically trivial steady-state, it provides useful information for the construction of the phase plane.

Depending on the values of the parameters, there may or may not be additional critical points. Other critical points are given by $\pi = 0$ and P satisfying $Px'(P) = -\delta f'(0)$. If δ is too large there will be no further critical points; if δ is small enough there will be two additional critical points, $\pi = 0$, $P = P^+$ and $\pi = 0$, $P = P^-$ (see Figure 3). (It is possible that $f'(0) < 0$; the story is similar and the analysis is the same.) Linearization about each of the critical points gives the information that $\pi = 0$, $P = P^+$ is always a saddle point and $\pi = 0$, $P = P^-$ is either an unstable node or an unstable spiral, depending on parameter values.

These results and some routine analysis taken together give the phase portraits in the P, π plane shown in Figures 4, 5 and 6. It should be noted that while these plots are generic, different parameters will alter the pictures somewhat. Figure 4 shows the case in which δ is large enough so that there is only one critical point.

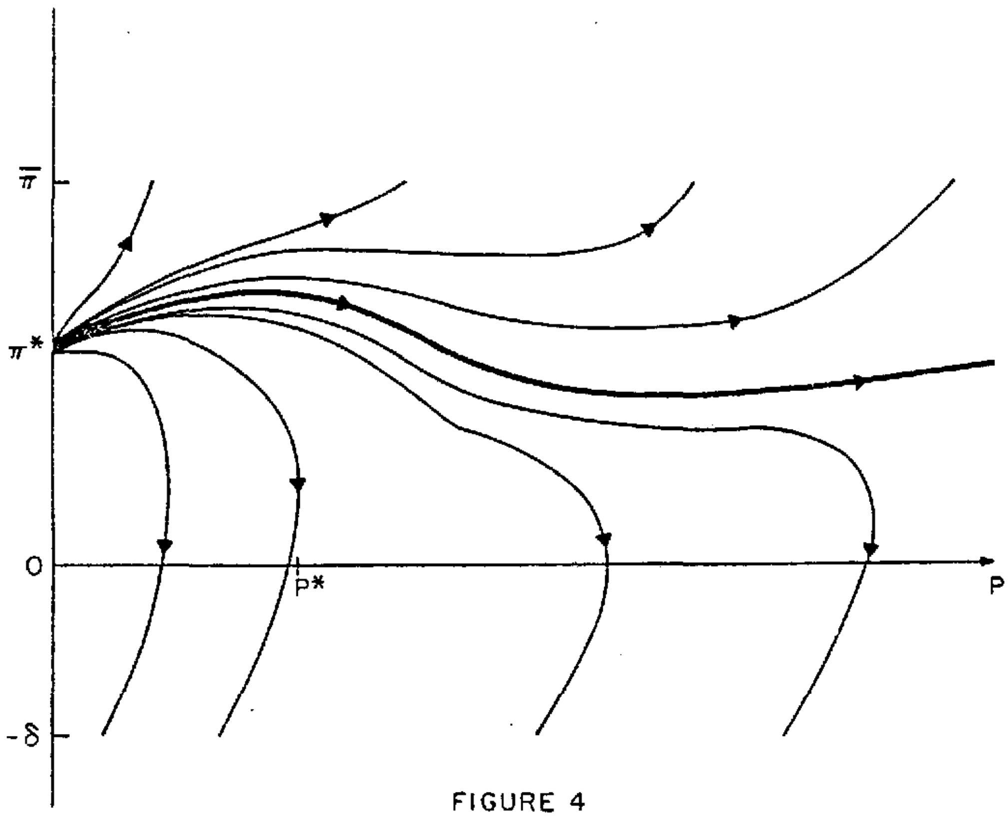

FIGURE 4

PHASE PORTRAIT FOR THE EULER-LANGRANGE EQUATIONS

In this case it is easily seen that the optimal time path, $\hat{\pi}_t$ must be the separatrix (shown with a darker line) which divides the trajectories which eventually reach the line $(P, \pi = -\delta)$ from the trajectories which eventually reach the line $(P, \pi = \bar{\pi})$. This follows from a simple varational argument which shows that trajectories reaching either of these lines have neighboring trajectories which produce larger values in G and therefore cannot be optimal. Notice that the optimal trajectory asymptotically approaches π^*.

The situation where there are additional critical points (δ small enough) is described by Figures 5 and 6. Both possibilities suggested by the phase plane plots may occur depending on the precise shapes of the parameter functions f(•) and x(•). The possible optimal trajectories are again shown with darker lines; the analysis leading to this is the same as in the preceding example.

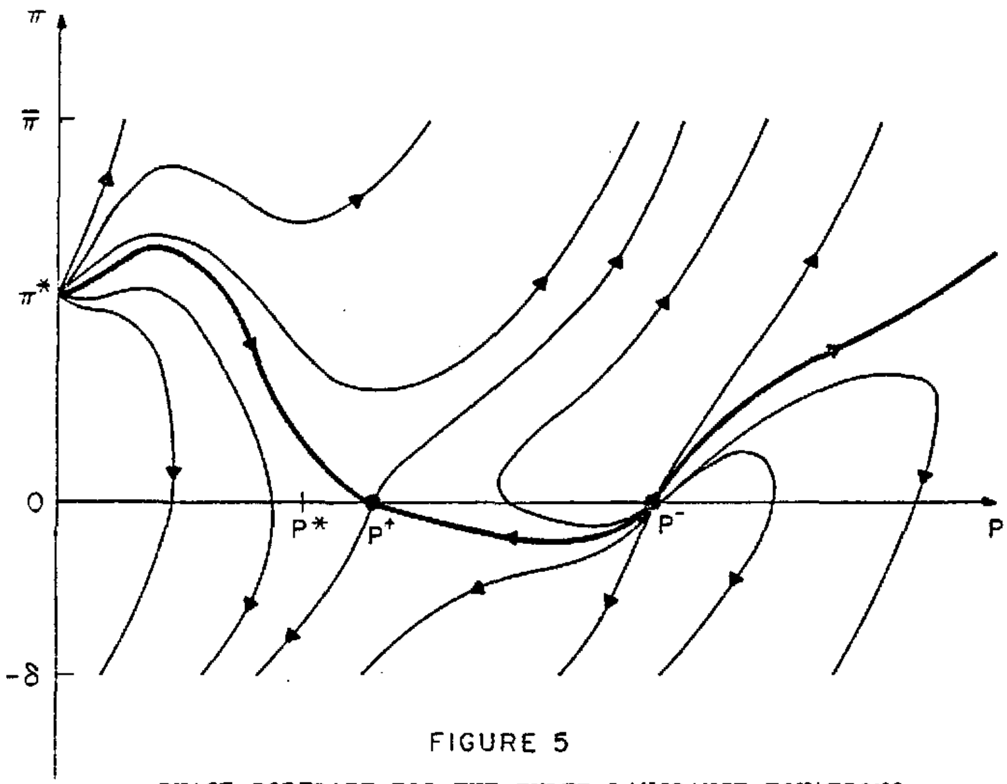

FIGURE 5

PHASE PORTRAIT FOR THE EULER-LANGRANGE EQUATIONS

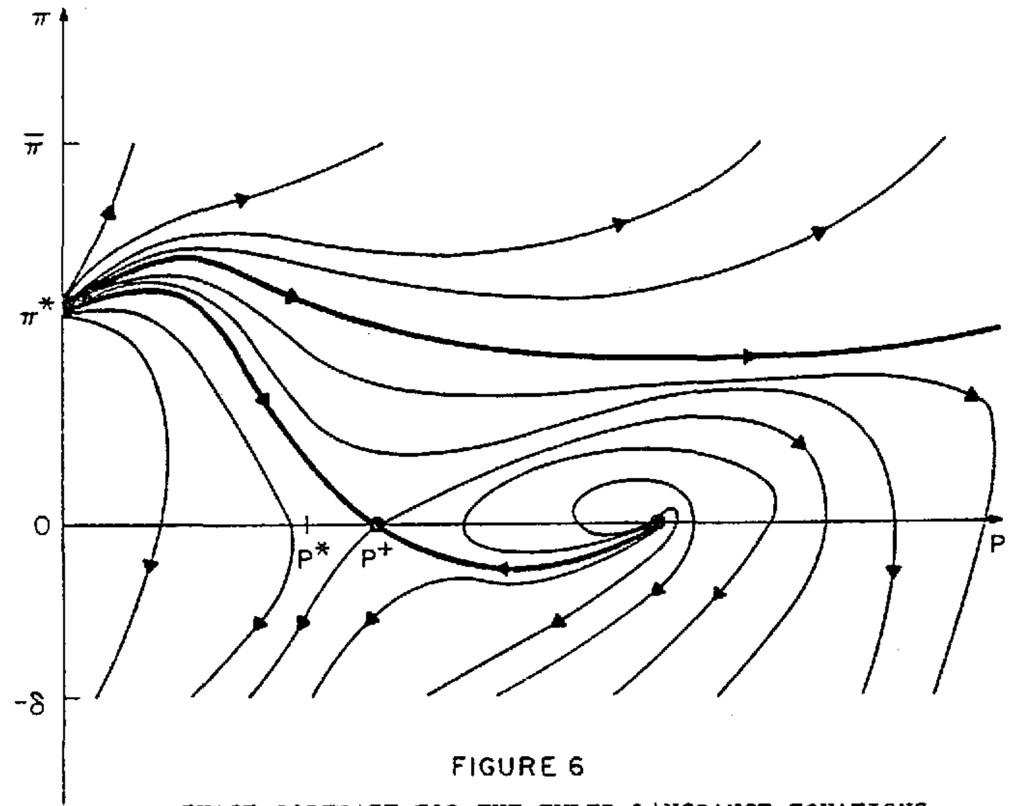

FIGURE 6
PHASE PORTRAIT FOR THE EULER-LANGRANGE EQUATIONS

Should the case delineated by Figure 5 obtain, there is a dichotomy depending on the value of P_0. If $P_0 > P^-$, the behavior is as before: the optimal inflation rate approaches π^* asymptotically. Otherwise, the optimal trajectory is the one which asymptotically approaches the steady-state, $\pi = 0$, $P = P^+$.

The situation described by Figure 6 gives a new twist. For some values of P_0 ($P_0 < P^-$) two choices for optimal trajectories are available. The proper (i.e., maximizing) choice may only be determined by evaluating G along each of the paths and comparing (both are possible depending again on the specific f's and x's).

III. Combined Tax and Inflationary Finance: Optimal Behavior

Some obvious questions are now raised. Does non-indexed taxation provide an incentive for the government to raise or lower the revenue-maximizing time path of the rate of inflation as compared to that of simple inflationary finance? What are the possible steady states under such a system? In what way does the intensity of the government's time preference affect the optimal time path of the rate of inflation?

THEOREM 2: At low price levels, the non-indexed tax system stimulates inflation (because higher tax revenues are obtained at a higher price level). But before the price level reaches the revenue-maximizing level P^*, the government will reduce the rate of inflation, to lengthen the period of time over which it receives high tax revenues.

Proof: From Figure 4, for some initial range of price levels, the revenue-maximizing rate of inflation from combined tax and inflationary finance exceeds that from simple inflationary finance. Note also that beyond some price level (less than P^*), the revenue-maximizing rate of inflation from combined tax and inflationary finance falls below that of pure inflationary finance. As Figures 5 - 6 illustrate, there are two possible steady states under combined tax and inflationary finance. Substituting $\dot{\pi}_t = 0$ into the Euler-Lagrange condition (22) gives

$$\delta f'(\pi) = -P_t x'(P_t) \qquad (24)$$

as the relationship between π and P which must be satisfied in the steady state. This implies that there are two possible steady state paths. As discussed in the previous paragraph, the first (see Figure 4) occurs when $\pi \to \pi^*$ since this implies that $f'(\pi) \to 0$. In this case the steady-state returns to the revenue-maximizing rate of inflation associated with simple inflationary finance. This is because as the price level increases, tax revenues decrease to a lower bound. Further increases in the price level do not result in lower tax revenue and hence do not cause inflationary behavior different from simple inflationary finance.

The second possible steady state is where $\pi_t \to 0$ and $P_t \to P^+$. This case is illustrated in Figure 5. To see how this equilibrium might occur, notice that $P^+ > P^*$ implies that $x'(P^+) < 0$, and notice also that $f'(0) > 0$. Hence there may exist a price level (P^+) where $\delta f'(0) = -P^+x'(P^+)$. If so, then the optimal steady state solution calls for the rate of inflation to fall to zero as the price level approaches P^+. If $\delta f'(0)$ is viewed as a numerical value then, obviously, the conditions which favor a constant price level as a steady state solution are a sharply declining tax revenue function beyond P^*. This is because $P_t x'(P_t)$ (which is continuous in P) will take on a greater range of magnitudes and therefore it is more likely that there exists a value of P such that $Px'(P) = -\delta f'(0)$ is satisfied.

Notice also, as illustrated by Figure 6, that it is possible for the two steady states to coexist. In this case the revenue maximizing choice may only be determined by numerically evaluating and comparing the revenue associated with each path.

THEOREM 3: An increase in the government's rate of time preference δ tends to lower the initial time path of the rate of inflation, but increases the likelihood of inflation (at π^*) in the steady state, whereas a reduction raises the initial time path of the rate of inflation, but tends to induce the steady state with zero inflation.

Proof: First, it is follows from (22) that an increase in δ (an increase in time preference) tends to flatten the revenue-maximizing time path of π_t, so that the rate of inflation diverges less from the simple inflationary solution. More importantly however, an increase in δ lowers the initial optimal time path of the rate of inflation, but also increases the chance of the simple inflationary finance case ($\pi = \pi^*$) as the steady state solution. The first of these responses occurs because (from (24)), π^* is directly reduced to the extent δ is increased. The second response follows because, as δ increases, the steady state condition $\delta f'(0) = -P_t x'(P_t)$ is less likely to have a stable price level solution, $P = P^+$, since such a large value for $-P_t x'(P_t)$ may never be achieved. Notice, on the other hand, that if $\delta = 0$, the initial optimal time path of the rate of inflation is increased but a steady state price level at $P = P^*$ is possible. This is because for $\delta = 0$ the steady-state condition implied by substituting $\delta = \dot{\pi}_t = 0$ into the Euler-Lagrange condition (22) is $P_t x'(P_t) = 0$ which is satisfied at $P = P^*$.

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