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Research Paper

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ABSTRACT

American yield curves have been characterized by positive slopes when interest rates have been low and by negative slopes when interest rates have been high, with, however, some apparent revisions in the late 1870s and early 1970s of what should be considered "high" and "low". Annual observations on short- and long-term yields between 1862 and 1982 are consistent with both traditional and modern expectations theories under regressive expectations, where "the normal rate" toward which short rates are expected to regress is a function of the monetary standard; specifically, paper or gold. But the model presented here does not allow us to distinguish empirically between the impacts of alternative monetary regimes on the normal rate and term premia.

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MONETARY REGIMES AND THE TERM STRUCTURE OF INTEREST RATES, 1862-1982

1. Introduction

This paper presents some preliminary results of research designed to estimate the impacts of alternative monetary regimes on the term structure of interest rates. The characteristics of yield curves observed since 1862 are described in Sections 2 and 3. These characteristics suggest that a fruitful framework of analysis might be the traditional (or alternatively the modern) expectations theory supplemented by an expectations scheme in which short-term interest rates are expected to regress toward a "normal" level that is itself a function of inflationary expectations and therefore of the monetary standard. The model is given a precise specification in Section 4 and is estimated in Section 5. The theoretical implications, such as they are, of these estimates are discussed in Section 6.

2. American yield curves, 1862-1982

Theories of the term structure of interest rates explain relative yields on default-free securities that are identical in every respect except term-to-maturity. Unfortunately, such securities do not exist -- except, possibly, Treasury bills, which have been issued only since 1929 and are limited to maturities less than one year (less than three months before 1959). However, David Durand and Frederick Macaulay have supplied series on high-grade private short-term and long-term yields dating from 1857, which may allow reasonable approximations of theoretical yield curves.

Durand's yield curves for high-grade corporate bonds[1] from 1900 to 1982 are shown in Chart 1. The mainly flat or falling yield curves of 1900-1930 are indicated by solid lines and the continuously-rising 1931-59 curves are indicated by the dashed lines that dominate the lower portion of the chart. Yield curves for 1960-82 are indicated by the lines with shorter dashes. Curves since 1966 have been identified by year of occurrence.

A striking feature of the yield curves in Chart 1, at least until 1970, is their tendency to rise when yields are low and to fall when yields are high. Later sections of this paper are concerned with the determination of how high is "high" and how low is "low". Until then, let

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[1]Durand's original curves (for 1900-1942) were reported in Durand (1942), updated by Durand and Winn (1947), Durand (1958), and, since 1959, by Scudder, Stevens and Clark, to whom we are grateful for making their estimates available. Selected Durand and Scudder, Stevens and Clark data are available through 1970 in Historical Statistics of the United States (1975, ii, p. 1004) and, more recently, in the annually published Statistical Abstract of the United States. Durand's curves pertain to the first quarter of the year and each yield upon which the estimated curves are based was obtained from an average of six price quotations -- the high and low prices in each of the first three months of the year.

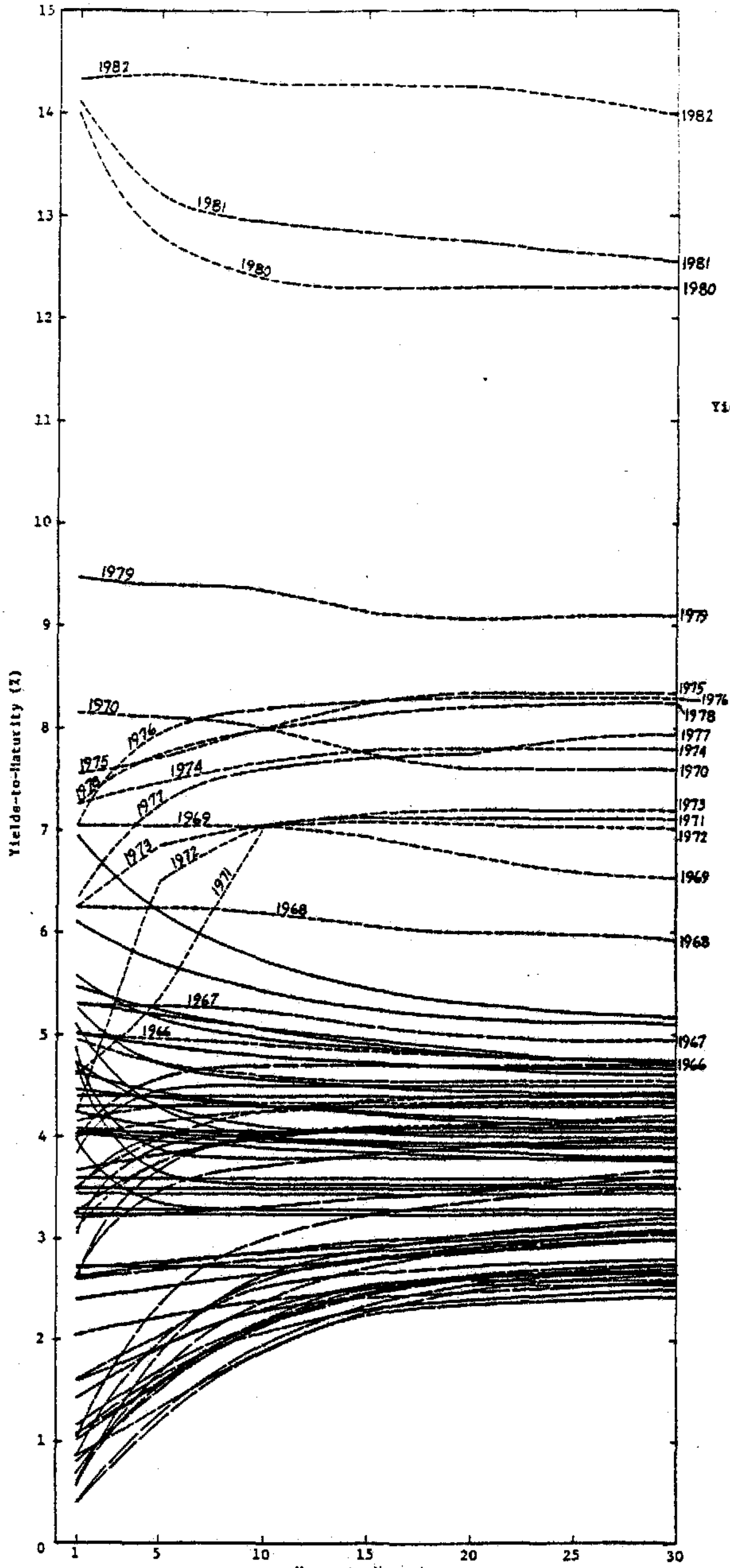
us suppose for the sake of argument that between 1900 and 1970 yields on one-year bonds were considered high when they exceeded 4.40 percent and were thought to be low when they fell short of 3.25 percent. The upper portion of Table 1 shows that, when "high" and "low" are distinguished in this manner, all 1900-1970 yield curves had negative slopes when short-term yields were high and all 1900-1970 yield curves had positive slopes when short-term yields were low. Friedrich Lutz (1940) described these empirical tendencies as consistent with the joint hypotheses that (i) long-term yields are averages of current and expected short-term yields as implied by the traditional expectations (TE) theory and (ii) expectations are regressive in the sense that future yields are expected to evolve toward some "normal" level.

But Lutz's interpretation is strained by the high and rising yield curves of 1971-78, which suggest that either (i) the explanation that is so effective for 1900-1970 has failed in recent years because investors no longer behave according to the tenets of the TE theory and/or they no longer form expectations regressively, or (ii) they have revised their estimate of the normal rate. The extrapolative-expectations version of the TE theory appears broadly consistent with the generally rising yields and positively-sloped yield curves of 1971-78. But this approach does not look as promising in light of the yield curves of 1979-82, which had negative slopes during a period of rapidly rising yields. Extrapolative expectations are rendered additionally suspect by the failure of yield curves to be negatively sloped during the 1930s.[2] A variety of

[2]This does not mean, as we shall see later, that extrapolative expectations may not play <u>some</u> role in the explanation of observed yield curves.

Chart 1

Yield Curves for High-Grade Corporate Bonds, 1900-197



Sources: Durand (1942, 1958), Durand and Winn (1947), and Scudder, Stevens and Clark. (1981 and 1982 estimated from U.S. yield curves.)

Table 1

Frequencies of Rising, Flat, and Falling Yield Curves, 1900-1982

One-year corporate bond yield (percent per annum)	Slope of yield curve: Positive	Flat	Negative
		1900 - 1970	
Above 4.40	0	0	20
3.25 - 4.40	10	10	5
Below 3.25	26	0	0
		1971 - 1982	
Above 8.00	0	0	4
Below 8.00	8	0	0

Sources: Durand, Durand and Winn, and Scudder, Stevens and Clark.

Table 2

Frequencies of Rising and Falling Yield Curves, 1862-1929

Commercial paper yield (percent per annum)	Slope of yield curve: Positive	Negative
	1862 - 1878	
Above 7.57	0	5
Below 7.56	12	0
	1879 - 1929	
Above 5.40	0	16
4.21 - 5.40	4	17
Below 4.21	14	0

Source: Macaulay, Table 10.

alternative explanations of the events of 1971-82 might be worth pursuing, but let us see how far we can go with the explanation emphasized thus far -- the TE theory[3] with regressive expectations. This approach requires an additional hypothesis, one that supplies a rule by which investors revise their estimates of the normal rate. However, such a rule, whatever it is, may not be susceptible to a test on the basis of the data considered so far because we have observed only one unambiguous revision of the normal rate since 1900. For other possible revisions, we must go to the 19th century.

No complete yield curves such as those in Chart 1 are available for the 19th century. However, we may infer the slopes of yield curves from data on the prime commercial paper rate (the short-term yield) and Frederick Macaulay's (1938) unadjusted index of railroad bond yields (the long-term yield).[4] Annual averages of commercial paper and railroad bond

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[3]We will also consider the modern expectations (ME) theory.

[4]"Choice" and "prime" commercial paper rates, which are reported on a discount basis, have been converted to bond equivalent yields. Macaulay tried to construct yield curves for railroad bonds like those later reported by Durand, but he found the correlation between yield and maturity (as well as duration) too small. However, the use of Macaulay's data in Table 2 is consistent with the use of Durand's yield curves in Table 1 because Macaulay found that "when short term rates such as those for ... commercial paper were high, the bonds with shorter durations tended to show the higher yields, and vice versa, when short term rates were low, the bonds with shorter durations tended to show the lower yields ..." (p. 80).

Macaulay's unadjusted index is an unweighted arithmetic average of yields quoted fairly regularly on the New York Stock Exchange, excluding exceptionally high yields as presumably too risky for inclusion in an index of high-grade yields. We use the unadjusted index in Table 2 because it tells our story more dramatically than Macaulay's adjusted index, which was the result of an attempt to present railroad bond yields of "the highest possible grade" (p. 117). Use of the adjusted index produces almost exclusively falling yield curves between 1862 and 1929 (51 of 53 for 1862-1914 and 11 of 15 for 1915-1929). But the adjusted index still gives results consistent with regressive expectations; i.e., the difference between long- and short-term yields tends to vary inversely with the level of the short-term yield.

yields for 1862-1929 are shown in Chart 2. This chart tells, in a different way, essentially the same stories as Chart 1: first, that yield curves tended to be positively-sloped when yields were low and negatively-sloped when yields were high, and, second, that there was apparently a revision of the notions of "high" and "low."[5] However, instead of an upward revision, as in the early 1970s, Chart 2 suggests a downward adjustment of the normal rate in the late 1870s. Notice, for example, that the seven short-term yields between 5.58 percent and 7.55 percent during 1866-75 were all associated with rising yield curves, while after those years all short-term yields above 5.40 percent were associated with falling yield curves.

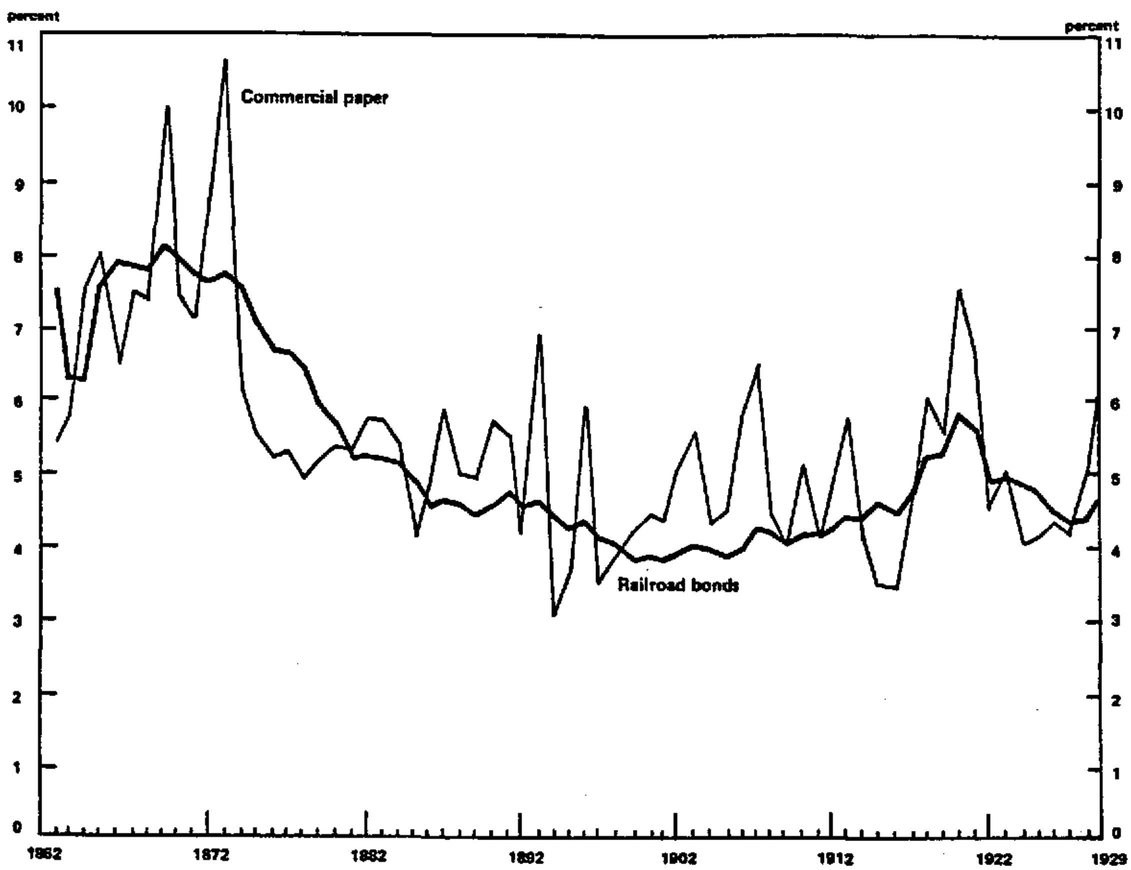
No precise dating of the normal rate's revision, which may have occurred slowly over several years, is immediately obvious from the data. (This is also true of the shift in the 1970s, or perhaps the late 1960s; the reason for our choice of 1971 and 1879 will be made clear below). But suppose, for simplicity of exposition, that most of the adjustment took place early in 1879. Using this date to divide 1862-1929 into two periods, Table 2 suggests that the normal rate may have been in the vicinity of 7.50 percent during 1862-78 and somewhere between 4 and 5.50 percent during 1879-1929.

The values in Table 2 are not directly comparable with those in Table 1 because the yields in the two tables apply to different securities.

[5]During 1900-1929, when Charts 1 and 2 overlap, the yield curves implied by the latter chart have the same sign as those in the former on three-quarters of the occasions on which Durand's yield curves are not flat. Furthermore, the slopes implied by Chart 2 tend to be smaller in absolute value when Durand's curves are flat than when they have non-zero slopes.

Chart 2

Long and Short Rates, 1862-1929



SOURCE: Macaulay, 1938, Table 10.

Nevertheless, these tables and the charts upon which they are based operate as a unit to tell a single story--that American yield curves since 1862 are at least roughly consistent with the traditional expectations theory supplemented by regressive expectations where the normal rate is a function of the monetary standard. This is the hypothesized rule for revising the normal rate that is required for a complete explanation of observed yield curves.

3. The monetary standard and the yield curve

The American monetary standard has undergone the following changes since early in the Civil War: The gold standard was abandoned when banks suspended specie payments on December 30, 1861.[6] In February 1862, Congress authorized the first of several issues of legal tender currency (the famous greenbacks). After a period of monetary expansion accompanied by depreciation of the dollar, followed by prolonged monetary controversy, a bill for the resumption of the gold standard at the prewar exchange rate was passed in January 1875. Resumption was achieved on the target date of January 1, 1879, although success was not assured until late in 1878.[7] The monetary standard remained unchanged until banks were legally prohibited from paying out gold in March 1933. The international gold standard was resumed in January 1934,[8] although the gold value of the dollar was reduced to 59 percent of that prevailing between 1879 and 1933. Finally, in August 1971, the United States suspended the international convertibility of the dollar and embarked on a paper standard identical in all important respects to the greenback era of 1862-78.

The following line of reasoning suggests that the monetary standard should be expected to be an important, perhaps the dominant, influence on the normal rate. First, let the normal rate pertaining to securities of a particular risk class be the yield expected by *investors* to apply to those securities in long-run equilibrium. Second, the available evidence strongly suggests that interest rates are to a considerable extent

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[6]The official standard was bimetallic, but silver did not circulate because it had been undervalued by the official gold-silver exchange rate.

[7]See, for example, Dewey (1936) and Friedman and Schwartz (1963) for histories of American monetary standards.

[8]The domestic circulation of gold was ended by the Gold Reserve Act.

determined by inflationary expectations, which in turn depend on actual inflation.[9] Finally, inflation has for centuries been highly correlated, and widely believed to be highly correlated, with the choice of monetary standard.[10]

These arguments are supported by the data in Charts 1 and 2 and Tables 1 and 2, which are consistent with a downward revision in (or about) 1879 and an upward revision in (or about) 1971 of investor estimates of normal rates. It is not clear from the data whether another revision occurred in the 1930s because the steeply-rising yield curves of that decade (and of the 1940s and 1950s) were, in view of the record-low yields prevailing at the time, consistent with normal rates based on experience of both gold and paper standards.[11] Regression estimates of the impacts of alternative monetary standards on normal rates are presented in later sections, after the specification of our theoretical framework.

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[9]Most observers, including Fisher (1930) and Fama (1975), would agree with this statement. See Wood (1981) for a review of empirical work on the connections between interest rates and inflation.

[10]See Attwood (1819), Lester (1939), Dewey (1936), Friedman and Schwartz (1963), Barro (1980), and Bordo (1981) for discussions of evidence and attitudes regarding inflation under gold and paper standards.

[11]In annual averages, American commercial paper yields have not, except during 1935-46, been less than 1 percent, and they have not, except during 1931-58, been less than 3 percent. They were continuously less than 1 percent during 1935-46 and continuously less than 3 percent during 1931-55. These statements are based on data available from 1819 in Homer (1977).

4. Traditional and modern expectations theories[12]

The traditional expectations (TE) theory asserts that the continuously-compounded yield-to-maturity at time t on a pure discount bond that matures at time T is

$$(1) \quad y(t,T) = \frac{1}{T-t} \int_t^T E[r(s)]ds,$$

where r(s) is the instantaneous spot rate. Thus, according to the TE theory, the yield on a bond is an average of instantaneous spot rates expected to occur over the bond's life. The yield y(t,T) becomes the instantaneous spot rate r(t) as t approaches T.

Suppose expectations of future rates are formed regressively such that

$$(2) \quad E[r(s)] = r(t) + [r^* - r(t)][1 - e^{-b(s-t)}] = r^* - [r^* - r(t)]e^{-b(s-t)},$$

$$s \geq t, \ b \geq 0,$$

where b is the speed at which the short rate is expected to move from its present value r(t) to its long-term normal value r*. For example, $E[r(s)] = r(t)$ for s=t or b=0, and $E[r(s)] \longrightarrow r^*$ for $s \longrightarrow \infty$ or $b \longrightarrow \infty$.[13]

Substituting (2) into (1) and defining T-t = n gives

$$(3) \quad y(t,T) \equiv y_{nt} = r^* - [r^* - r(t)]B_n,$$

where $B_n = (1-e^{-bn})/bn$ and y_{nt} is the yield at time t on a discount bond that matures in n = T-t periods. The spread between the yields on bonds maturing in n and k periods is

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[12]See Appendix A for derivations of the principal implications of the traditional and modern expectations theories.

[13]Further illustrating (2) by means of a numerical example, suppose $r^* = .10$, $r(t) = .05$, and $b = \log 2 \doteq .693$. Then $E[r(1)] = .075$, $E[r(2)] = .0875$, and $E[r(3)] = .09375$.

(4) $y_{nt} - y_{kt} = (1-B)(r^* - y_{kt})$,

where $B = B_n/B_k \leq 1$ as $0<k\leq n$, $b>0$.

The modern expectations (ME) theory implies that long-term yields are biased downward from expected future rates by a negative term premium. Specifically, the ME theory implies that (1) and (3) should be rewritten as[14]

(5) $$y_{nt} = \frac{1}{n}\int_t^{t+n} E[r(s)]ds + \pi(\theta,n) = r^* - [r^* - r(t)]B_n + \pi(\theta,n), \qquad \pi<0,$$

where θ represents the randomness of short rates and the absolute value of π is positively related to term-to-maturity, n.

Equation (4) must also be revised in light of the ME theory:

(6) $y_{nt} - y_{kt} = (1-B)(r^* - y_{kt}) + \pi(\theta,n,k)$,

where $\pi(\theta,n,k) = \pi(\theta,n) - B\pi(\theta,k) < 0$ for $n>k$ and $b>0$ because, under these conditions,

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[14]See Nelson (1972) and Cox, Ingersoll, and Ross (1981) for demonstrations that the TE theory is not certainty-equivalent, as is incorrectly asserted by (1). Equation (5) is identical to Vasicek's (1977) statement of the ME theory when short rates are governed by an elastic random walk

$$dr = b[r^* - r(t)]dt + \sigma dz,$$

where dz is a Wiener process so that the variance of r(s) is proportional to σ^2 and

$$\pi(\theta,n) = \frac{\sigma^2}{2b^2}\left[B_n + \frac{bn}{2}B_n^2 - 1\right].$$

This statement of Vasicek's equation (29) omits the "market price of risk," which represents "the increase in expected instantaneous rate of return on a bond per an additional unit of risk" (p. 181) implied by traditional liquidity preference theories of the term structure.

$\pi(\theta,n) < \pi(\theta,k) < 0$ and $0<B<1$.

Notice that in a steady state, in which $E[r^* - r(t)] = 0$,

$$E(y_{nt} - y_{kt}) = (1-B)[r^* - (r^* + \pi_k)] + \pi_n - B\pi_k = \pi_n - \pi_k,$$

where, for example, $\pi_n \equiv \pi(\theta, n)$.

5. Curve fitting

The initial empirical form. Equation (6) may be written

(7) $y_{nt} - y_{kt} = [(1-B)r^* + \pi] - (1-B)y_{kt} + u_t = \alpha + \beta y_{kt} + u_t,$

where $\pi = \pi(\theta,n,k)$ and the error term u_t may be interpreted as $(1-B)v_t$ when v_t is a random component, with mean zero, in the market's evaluation of the normal rate; r^* is then interpreted as the market's expectation of the normal rate. The regression estimate of β provides an estimate of $\hat{B} = (1+\hat{\beta})$, from which, given n and k, we may compute the speed-of-adjustment, b. Unfortunately, neither r^* nor π can be inferred from $\hat{\alpha}$ and $\hat{\beta}$. Inferences about r^* based upon the TE theory, in which π is ignored, are biased downward because term premia exert a negative influence on the yield curve's slope such that, in contradiction to the TE theory, yields fall below averages of expected spot rates.

The data. There are no continuous series available for short-term or long-term yields for the entire period in which we are interested, 1862-1982. However, Macaulay's Adjusted Index of Railroad Bond Yields, which begins in 1857 and ends in 1936, was virtually identical to Moody's Corporate Aaa yield in 1936. So our long-term yield, y_{nt}, is drawn from Macaulay until 1936 and from Moody's since then.[15] For similar reasons, we have used the "choice 60-90 day two name" commercial paper rates reported in Macaulay for our short-term yield, y_{kt}, until 1919 and the 4-6 month prime commercial paper rates reported by the Federal Reserve between 1920 and 1978. The latter series ended in 1979 and has been succeeded by

[15]Our theory explains yield curves on pure discount bonds but our data on long-term yields are for coupon bonds. The resulting "coupon bias" causes underestimation of the slopes of yield curves and therefore of the speed of adjustment of expected rates toward the normal rate. However, it is shown in Appendix B that coupon bias is small when expectations are regressive.

3-month and 6-month prime commercial paper rates. We have used the unweighted average of these two rates since 1979. Commercial paper discount rates have been converted to bond-equivalent yields.

The estimates. Our estimates of α and β are reported in Table 3. Regression I gives:

$$\hat{B} = (1 + \hat{\beta}) = (1 - .410) = .590$$

and

$$-\hat{\beta}r^* + \pi = \hat{\alpha} \qquad \text{or} \qquad r^* - \frac{\pi}{\hat{\beta}} = -\frac{\hat{\alpha}}{\hat{\beta}} \qquad \text{or} \qquad r^* + 2.439\pi = 4.378.$$

These estimates suggest a normal rate of 4.378 percent under the TE theory and something more than 4.378, depending on the size of π, under the ME theory. However, the low Durbin-Watson statistic casts doubt on these estimates. The remaining regressions represent attempts (i) to capture the influence of alternative monetary systems on the constant term, i.e., on our combined estimate of r^* and π, and/or (ii) to reduce autocorrelation in the residuals.

Regression II introduces a single dummy variable, D_p, that is unity for both paper standard episodes, 1862-78 and 1971-82, and is zero otherwise. Regression III introduces separate dummy variables for the first (D_{p1}) and second (D_{p2}) episodes. These results are consistent with the discussion in Section 2, in which it was argued that a shift from a gold to a paper standard ought to induce an upward revision of the market's estimate of normal rate. Furthermore, we should expect the revision to be greater in the present period, when the abandonment of the gold standard is probably expected to be permanent, than in the earlier period, when a return to the gold standard was widely (though not universally) expected in the not-too-distant future. These implications for r^* are not unambiguous, however, because there is no reason to suppose the market's estimate of the variability of interest rates, and therefore of π, to be independent of the monetary standard.

The monetary standard is not, of course, imposed exogenously upon a society. A shift from a fixed-rate to a flexible-rate system may be viewed as merely one of several reflections of a decision by one or more countries to abandon long-run price stability as a goal. But the interpretation of Regressions II and III remains essentially the same as in the preceding paragraph; i.e., a shift from a gold to a paper standard is likely to be associated with an upward revision of inflationary expectations and therefore of normal interest rates.

Inspection of the residuals of Regression III revealed a strong tendency to over-predict the yield curve's slope (and therefore, perhaps, inflationary expectations) during the early portion of the sample and to under-predict the slope during the later portion, with a marked break occurring about 1914. Regression IV adds a dummy variable, D_{14}, for 1914-82. Continuing our interpretation in terms of monetary standards, at least two classes of events were initiated in 1914 that may have induced or at least have been associated with an upward revision of long-run inflationary expectations. First was the formation of a central bank that was designed to provide an "elastic currency" and was given substantial powers to enlarge the monetary base and to enable the monetary system to economize on gold. Second, World War I began a series of events that led to substantial reductions in the abilities or willingness of countries to achieve or even to pursue the goals of long-run price stability and the maintenance of the gold standard.

Table 3

Regression Estimates of the Determinants of the Difference Between Long- and Short-Term Yields, $y_{nt} - y_{kt}$ (Annual Data, 1862-1982; in Percentages)

	Constant	y_{kt}	D_{pt}	D_{p1t}	D_{p2t}	D_{14t}	$y_{k,t-1}$	$(y_n-y_k)_{t-1}$	SE	$\bar{R}^2$	DW	h	$\hat{\rho}$
				ORDINARY-LEAST-SQUARES									
(I)	1.795	-.410							1.083	.534	.432		
	(.198)	(.035)											
(II)	2.144	-.558	1.569						.948	.643	.376		
	(.183)	(.039)	(.257)										
(III)	2.422	-.628		.709	3.483				.685	.813	.735		
	(.135)	(.029)		(.203)	(.261)								
(IV)	1.561	-.563		1.105	2.765	.976			.562	.874	1.143		
	(.158)	(.025)		(.175)	(.234)	(.128)							
(V)	.768	-.654		.449	1.433	.590	.370	.420	.508	.897	1.321	8.479	
	(.197)	(.033)		(.186)	(.310)	(.139)	(.060)	(.082)					
				COCHRANE-ORCUTT									
(VI)	.330	-.764		.316	.887	.577	.590	.580	.369	.946	1.740	2.155	.165
	(.171)	(.027)		(.158)	(.261)	(.116)	(.054)	(.069)					

y_n = The long-term yield.
- 1861-1936: Annual averages of Macaulay's monthly Adjusted Index of Railroad Bond Yields (based on monthly high and low prices);
- 1937-1982: Annual averages of Moody's daily corporate Aaa bond yields.

y_k = The short-term yield. (Discount rates have been converted to bond-equivalent yields.)
- 1861-1919: Annual averages of "Choice 60-90 day two name paper" rates from Macaulay, whose main source was the weekly highs and lows reported in the Commercial and Financial Chronicle and the Financial Review.
- 1920-1978: Annual averages of daily 4-6 month prime commercial paper rates as reported in the Federal Reserve Board's Banking and Monetary Statistics (1914-41 and 1941-70) and Annual Statistical Digests.
- 1979-1982: The unweighted average of daily 3- and 6-month prime commercial paper rates as reported in Annual Statistical Digests and Federal Reserve Bulletins.

D_p = The overall paper dummy $\begin{cases} =1 \text{ for 1862-78, 1971-82} \\ =0 \text{ otherwise} \end{cases}$

D_{p1} = The first (greenback) paper dummy $\begin{cases} =1 \text{ for 1862-78} \\ =0 \text{ otherwise} \end{cases}$

D_{p2} = The second (post-1970) paper dummy $\begin{cases} =1 \text{ for 1971-82} \\ =0 \text{ otherwise} \end{cases}$

D_{14} = The "1914" dummy $\begin{cases} =1 \text{ for 1914-82} \\ =0 \text{ otherwise} \end{cases}$

SE = Standard error of the regression.

$\bar{R}^2$ = Coefficient of determination adjusted for degrees of freedom.

DW = Durbin-Watson statistic.

h = Durbin's test statistic (a standard normal deviate) for first-order autocorrelated errors.

$\hat{\rho}$ = Cochrane-Orcutt's estimate of the first-order autocorrelation of the errors.

Standard errors of regression coefficients are in parentheses.

Finally, Regression V (and its Cochrane-Orcutt version, Regression VI) take account of the possibility that the estimate of r* may in part be extrapolative. For example, the market's expectations of inflation and interest rates may depend not only upon the freedom allowed a central bank by the existing monetary standard but also upon how the central bank is seen to use that freedom. Suppose that, in addition to the expectations imbedded in the constant terms (including the coefficients of the dummy variables), expectations of y_{kt} depend upon past y_{kt} according to a geometric distributed lag so that our initial empirical form (7) is rewritten

$$(8)\quad (y_n - y_k)_t = \alpha + \beta y_{kt} + \gamma\sum_{i=1}^{\infty} \delta^i y_{k,t-i} + u_t$$

$$= \alpha(1-\delta) + \beta y_{kt} + \delta(\gamma-\beta)y_{k,t-1} + \delta(y_n - y_k)_{t-1} + (u_t - \delta u_{t-1}).$$

Regression VI implies the following values for the parameters:

$\hat{\beta} = -.764$, $\hat{\delta} = .580$, $\hat{\alpha} = .330/(1-.580) = .786$,

$\hat{\gamma} = (.590/.580) - .764 = .253.$

The sum of the geometrically declining weights is:

$$\hat{C} = \hat{\gamma}\sum_{i=1}^{\infty}\hat{\delta}^i = \frac{\hat{\gamma}}{1-\hat{\delta}} = .602$$

and we know from (7) that $\hat{B} = (1 + \hat{\beta}) = .236$.

The constant terms for different monetary regimes are:

$\hat{\alpha}_0 + \hat{\alpha}_1$	$= .330 + .316 = .646$	: 1862-1878, the Greenback era.
$\hat{\alpha}_0$	$= .330$	: 1879-1913, the gold standard before 1914.
$\hat{\alpha}_0 + \hat{\alpha}_{14}$	$= .330 + .577 = .907$	: 1914-1970, the gold standard since 1914.
$\hat{\alpha}_0 + \hat{\alpha}_{14} + \hat{\alpha}_2$	$= .907 + .887 = 1.794$	: 1971-1982, the current paper standard.

The constant terms in (8) require a different interpretation than in (7). In a steady state,

$$E(y_{nt} - y_{kt}) = \pi_n - \pi_k = \alpha - (1-B)(r^* + \pi_k) + C(r^* + \pi_k),$$

where $C = \gamma/(1-\delta)$, so that

$$\alpha = (1-B)r^* + \pi - C(r^* + \pi_k) = (1-B-C)r^* + \pi_n - (B+C)\pi_k.$$

The distributed lag on y_{kt} suggests extrapolative revisions of (i) term premia as well as (ii) the normal rate. This is acceptable if the time variance of interest rates is closely related to their level. Ideally, however, expectations of (i) and (ii) should be formed separately from observations on both the level and the variance of the short rate r(t).

6. Implications of the estimates and directions of future research

The relations between term premia and normal rates under different monetary standards that are implied by Regression VI may be written as follows:

$$r^* + \frac{\pi_n - (\hat{B}+\hat{C})\pi_k}{1-\hat{B}-\hat{C}} = \frac{\hat{\alpha}}{1-\hat{B}-\hat{C}}$$

or

$$r^* + \frac{\pi_n - .838\pi_k}{.162} = \frac{.646}{.162} = 3.988 \qquad \text{during 1862-1878}$$

$$= \frac{.330}{.162} = 2.037 \qquad \text{during 1879-1913}$$

$$= \frac{.907}{.162} = 5.599 \qquad \text{during 1914-1970}$$

$$= \frac{1.794}{.162} = 11.074 \qquad \text{during 1971-1982,}$$

where $\pi_n - .838\pi_k < 0$ so that the normal rates implied by Regression VI exceed 3.988, ..., 11.074. These estimates do not imply that, for example, the normal rate rose 11.074 - 5.599 = 5.475 percent between the 1914-70 and the post-1970 periods because there is no reason to suppose constancy of term premia between monetary regimes. Implications for r^* must await evidence on the variability of short rates, perhaps from daily observations on call loan rates and, more recently, federal funds rates. We can say, however, within our framework, that a shift from a gold to a paper standard is associated with an upward revision of the normal rate and/or a reduction in the absolute values of term premia.

Our estimate of $\hat{B}$ = .236 permits an estimate of the speed-of-adjustment s given the maturities n and k because

$$B = \frac{k(1 - e^{-bn})}{n(1 - e^{-bk})}.$$

Unfortunately, however, n and k are not fixed in our sample, with k varying between about 0.2 (60-90 days from 1862 to 1918) and about 0.4 (4-6 months or 3-6 months since 1918), and n varying over a much wider range. For example, the median maturity of the bonds used by Macaulay to compute his unweighted index of yields was about 17 years in 1870, 47 years in 1900, and 57 years in 1930. The implied value of b is relatively insensitive to variations in k between 0.2 and 0.4 but is quite sensitive to large changes in n, being, for example, b = .220, .144, .085 for n = 20, 30, 50. Future empirical work will attempt to surmount this problem by using constant maturities drawn from Durand's high-grade corporate yield curves available from 1900 and U.S. Treasury yield curves available from 1953. It is not, however, possible to construct reliable yield curves from Macaulay's data, which are concentrated in the nearly flat long-term end of the yield curve.[16]

[16] This problem may be illustrated by an examination of Durand's yield curves (neglecting his flat curves), for which the median proportion of the difference $y_{30} - y_1$ accounted for by $y_{20} - y_1$ is .94.

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APPENDIX A

Derivation of the Modern Expectations Theory's Negative Term Premium

The yield curve under certainty

The continuously-compounded yield-to-maturity at time t on a discount bond that promises $1 at time T is denoted y(t,T) and is defined such that

$$P(t,T) = \exp\left[-y(t,T)(T-t)\right], \tag{A.1}$$

where P(t,T) is the bond's price at time t. This definition holds under both certainty and uncertainty.

A spot rate is the rate-of-return on a bond between the present time t and its maturity date T. The rate-of-return (yield-to-maturity) y(t,T) becomes the instantaneous spot rate r(t) as t approaches T.

In a certain world with perfect markets (including no transaction costs) and continuous trading, all securities must have the same instantaneous rate-of-return to avoid arbitrage opportunities. That is,

$$\frac{dP(t,T)}{P(t,T)} = r(t)dt. \tag{A.2}$$

A solution to (A.2) subject to the terminal condition P(T,T) = 1 is

$$P(t,T) = \exp\left[-\int_t^T r(s)ds\right]. \tag{A.3}$$

Under certainty, the price at time t of a discount bond maturing at time T is the promised terminal payment discounted back to t using the instantaneous rates prevailing at each moment between t and T.

Comparing (A.1) and (A.3) shows that

$$\text{(A.4)} \quad y(t,T) = \frac{1}{T-t} \int_t^T r(s)ds.$$

Under certainty, the observed yield is an average of future instantaneous spot rates over the bond's life.

<u>The (certainty-equivalent) traditional expectations theory</u>

The traditional expectations (TE) theory treats uncertainty by replacing the future spot rates in (A.4) by their expectations, E[r(s)], where all expectations are held as of time t. That is, all of the certainty results described above are carried over to uncertainty, with expected future rates being substituted for known future rates. As a result, the TE theory implies that observed yields-to-maturity are averages of <u>expected</u> future spot rates. We will now show that the modern expectations (ME) theory[1] contradicts this traditional result.

<u>The modern expectations theory</u>

The ME theory asserts that investors behave under uncertainty such that in equilibrium all instantaneous <u>expected</u> rates-of-return are equal. That is,

$$\text{(A.5)} \quad \frac{E[dP(\Theta,t,T)]}{P(\Theta,t,T)} = r(t)dt.$$

and

---

[1]As developed by Cox, Ingersoll, and Ross.

$$(A.6) \quad P(\Theta,t,T) = E\{\exp[-\int_t^T r(s)ds]\},$$

where Θ represents a vector of random variables affecting bond prices and yields.

We will now show that (A.1) and (A.6) imply that

$$(A.7) \quad y(\Theta,t,T) = E\{\frac{1}{T-t}\int_t^T r(s)ds\} + \Pi(\Theta,t,T),$$

where $\Pi \leq 0$ is a negative term premium reducing the yield-to-maturity below the average of instantaneous spot rates expected to prevail over the bond's life and is zero only under certainty.

<u>Proof that $\Pi \leq 0$</u>

Denote the exponential function $\{\exp[-\int_t^T r(s)ds]\} = g(r)$. Then $P(\Theta,t,T) = E[g(r)]$, while the TE theory implies $P(t,T) = g[E(r)]$. Since g is convex, by Jensen's inequality,

$$(A.8) \quad P(\Theta,t,T) = E[g(r)] \geq g[E(r)] = P(t,T)$$

so that

$$(A.9) \quad y(\Theta,t,T) \leq y(t,T) = E\{\frac{1}{T-t}\int_t^T r(s)ds\},$$

with the equality holding only under certainty.

<u>A numerical example</u>

Consider a simple <u>certainty case</u> (or an uncertainty case in which, as in the TE theory, expected future rates are treated as certain) in which $r(s) = r(t) = .10$ for $t \leq s \leq T$. Then, from (A.3),

$$\text{(A.10)} \quad P(t,T) = \exp\left[-\int_t^T r(s)ds\right] = \exp\left[-(T-t)r(t)\right] = \exp\left[-10(.10)\right] = \exp\left[-1\right] = .368,$$

and, from (A.1),

$$\text{(A.11)} \quad y(t,T) = -\frac{\log P(t,T)}{(T-t)} = -\frac{\log\{\exp[-1]\}}{10} = .10.$$

This result could also have been obtained directly from (A.4).

Now consider an <u>uncertainty case</u> in which

$$\text{(A.12)} \quad r(s) = r(t) + \tilde{a}(s - t) \qquad t \leq s \leq T$$

That is, spot rates are expected to vary linearly from their present (time t) value at the rate $\tilde{a}$, which is a random variable with mean $E[\tilde{a}] = 0$.

The average of spot rates between t and T is therefore

$$\text{(A.13)} \quad \frac{1}{T-t}\int_t^T r(s)ds = r(t) + \frac{\tilde{a}}{2}(T-t).$$

Notice that $E[\frac{1}{T-t}\int_t^T r(s)ds] = r(t)$, which was also the average of future spot rates in the certainty case.

Assume as before that $r(t) = .10$ and $T-t = 10$. Furthermore, assume that $\tilde{a}$ may take the values $(-.01,.01)$, each with probability .5. Therefore, since

$$-\int_t^T r(s)ds = -(T-t)[r(t) + \frac{\tilde{a}}{2}(T-t)] = \begin{Bmatrix} -.5 & \text{for} & \tilde{a} = -.01 \\ 1.5 & \text{for} & \tilde{a} = .01 \end{Bmatrix} \quad r(t) = .10,\ T-t = 10,$$

we see from (A.6) that

$$\text{(A.14)} \quad P(\Theta,t,T) = .5\{\exp[-.5]\} + .5\{\exp[-1.5]\} = .5[.6065 + .2231] = .415,$$

so that

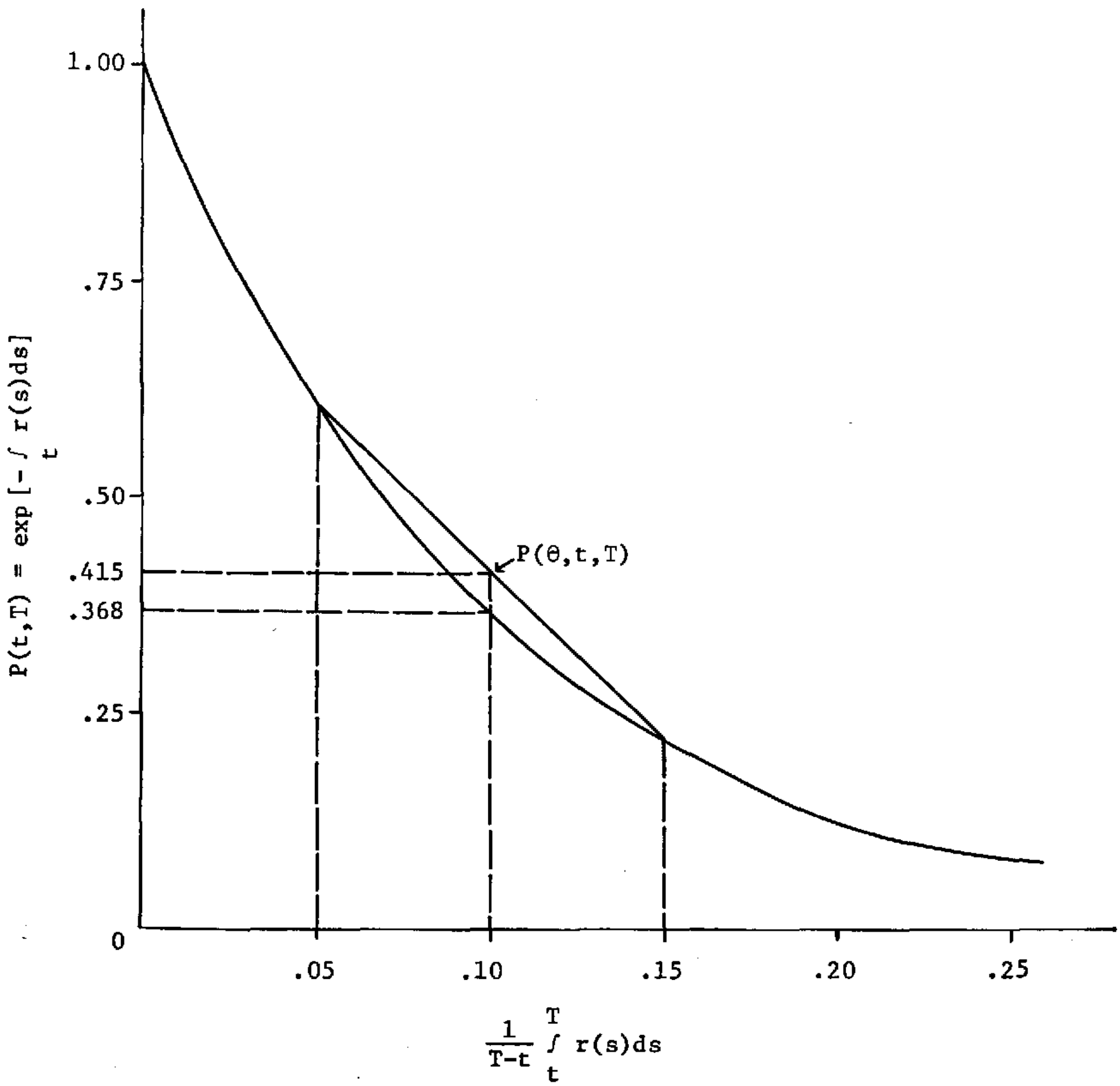
$$\text{(A.15)} \quad y(\Theta,t,T) = \frac{-\log P(\Theta,t,T)}{(T-t)} = .0880 < .10 = y(t,T).$$

These certainty and uncertainty results are illustrated in Figure A.1. The exponential curve shows the certainty price P(t,T) as a function of average future rates. The bond's price is .368 when the average of future rates is .10. Introducing uncertainty, investors are willing to pay .415 > .368 when there is a 50-50 chance that the average of future rates is .05 or .15 because, due to the convexity of the price/average-future-rate function, a decrease (from .10 to .05) in the average of future rates raises the present value of the bond's pay-off more than an increase (from .10 to .15) reduces that present value. Potential capital gains exceed potential capital losses when future short rates are stochastic, causing the bond to be valued higher under uncertainty than when the short rate is expected with certainty to be constant.

The term premium implied by the ME theory in this example is .0880 - .10 = -1.20 percent. The absolute value of this premium is a positive function of both term-to-maturity and risk. For example, $y(\Theta, t, T)$ rises

Figure A.1

Prices of a Discount Bond Under Certainty and Uncertainty



(i) from .0880 to .0984 as term-to-maturity falls[1] from 10 to 5 and (ii) from .0880 to .0969 as the variance of $\tilde{a}$ is reduced by changing the equal-probability outcomes of $\tilde{a}$ from (-.01,.01) to (-.005,.005).

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[1] This yield curve resembles Dothan's Figure 4, which depicts a case in which the instantaneous spot rate follows a geometric Wiener process.

APPENDIX B

Coupon Bias Under Regressive Expectations

1. Introduction

Most theoretical work on the term structure of interest rates has dealt with pure discount bonds while nearly all observed yield curves apply mainly to coupon bonds (exclusively to coupon bonds for maturities exceeding one year). The difference between observed coupon yields and implied discount yields is called coupon bias. After first considering coupon bias on annuities, this appendix presents examples of coupon bias on coupon bonds within the regressive expectations framework presented above.

2. Annuities

Let Y_n^a and Y_n^d be the yields-to-maturity at time t on annuities and pure discount bonds, respectively, that mature at time $t+n$. Then, since an annuity may be regarded as a collection of discount bonds, the following relation must hold in equilibrium.

$$(B.1) \quad C\sum_{i=1}^{n} e^{-iY_n^a} = C\sum_{i=1}^{n} e^{-iY_i^d} ,$$

where C is the payment per period promised by the annuity and the term on the left-hand side of (B.1) is the annuity's price at time t. The yield on an n-period annuity is an average of yields on discount securities maturing after 1, 2, ..., n periods; i.e., Y_n^a is an average of Y_1^d, Y_2^d, ..., Y_n^d.

A 2-period example. Letting $n = 2$, $Y_1^d = .09$, and $Y_2^d = .11$, (B.1) gives $Y_2^a = .1028$ so that

$$(B.2) \quad \text{Coupon bias} = Y_2^a - Y_2^d = .1028 - .11 = -.0072 ,$$

as shown in the top-left portion of Table B.1. Coupon bias increases with the slope of the discount yield curve, as indicated in Table B.1 for the case in which $Y_1^d = .05$, $Y_2^d = .15$, $Y_2^a = .1125$, and coupon bias $= .1125 - .15 = -.0375$.

<u>A linear approximation</u>. Still letting $n = 2$, (B.1) may be written

$$\frac{1}{e^{Y_2^a}} + \frac{1}{e^{2Y_2^a}} = \frac{1}{e^{Y_1^d}} + \frac{1}{e^{2Y_2^d}} \quad \text{or} \quad \frac{1 + e^{-Y_2^a}}{e^{Y_2}} = \frac{1 + e^{2Y_2^d - Y_1^d}}{e^{2Y_2^d}} ,$$

or, letting $\log(1 - x) \doteq -x$ so that $e^{-x} \doteq (1 - x)$ and $(1 + e^{-x}) \doteq (2 - x)$, and

$$e^{-Y_2^a}(2 - Y_2^a) = 2e^{-Y_2^a}(1 - \tfrac{1}{2}Y_2^a) \doteq e^{-2Y_2^d}(2 + 2Y_2^d - Y_1^d) = 2e^{-2Y_2^d}(1 + Y_2^d - \tfrac{1}{2}Y_1^d) .$$

Taking logs and solving for Y_2^a gives

$$\text{(B.3)} \quad Y_2^a \doteq \frac{Y_1^d + 2Y_2^d}{3} .$$

In general,

$$\text{(B.4)} \quad Y_n^a \doteq \frac{Y_1^d + 2Y_2^d + \cdots + nY_n^d}{1 + 2 + \cdots + n} .$$

This approximation is reasonably good when the Y_i^d are close to each other but deteriorates as the Y_i^d become more divergent -- as may be seen by comparing the exact and approximate values of Y_2^a in Table B.1, first for $(Y_1^d, Y_2^d) = (.09, .11)$ and then for $(.05, .15)$.

Table B.1

Yields and Coupon Bias (•) for 2-Period Annuities and Coupon Bonds

Assumed Discount Yields	Exact Values		Linear Approximations	
	Y_2^a (•)	Y_2^c (•)	Y_2^a (•)	Y_2^c (•)
$Y_1^d = .09$, $Y_2^d = .11$	.1028 (-.0072)	.1089 (-.0011)	.1033 (-.0067)	.1100 (0)
$Y_1^d = .05$, $Y_2^d = .15$	.1125 (-.0375)	.1425 (-.0075)	.1167 (-.0333)	.1500 (0)

Exact values are determined according to equations (B.1) and (B.6) and linear approximations according to equations (B.3) and (B.7).

3. Coupon bonds

For simplicity, we will consider only coupon bonds selling at par, i.e., for which $e^{Y_n^c} - 1 = c$, where Y_n^c is the yield-to-maturity at time t on a coupon bond that matures at time t + n and c is its coupon rate. Then the following relation must hold in equilibrium, where the face value of the coupon bond is \$1:

$$\text{(B.5)} \quad \sum_{i=1}^{n} ce^{-iY_n^c} + e^{-nY_n^c} = 1 = \sum_{i=1}^{n} ce^{-iY_i^d} + e^{-nY_n^d}$$

Using the right-hand equation in (B.5), substituting $e^{Y_n^c} - 1$ for c, and rearranging gives

$$\text{(B.6)} \quad e^{Y_n^c} = \frac{1 - e^{-nY_n^d}}{\sum_{i=1}^{n} e^{-iY_i^d}} + 1 \ .$$

Coupon bias tends to be smaller for coupon bonds than for annuities (i.e., Y_n^c tends to be closer than Y_n^a to Y_n^d) because of the greater value taken by the final payment on coupon bonds. This may be seen by comparing Y_2^c, Y_2^a, and Y_2^d in Table B.1.

A linear approximation. Again letting $\log(1-x) \doteq -x$, the following approximate relation between coupon and discount bond yields may be obtained. Consider the case in which $n = 2$.

$$e^{Y_2^c} = \frac{1 + e^{-Y_1^d}}{e^{-Y_1^d}\left(1 + e^{-2Y_2^d + Y_1}\right)} \doteq \frac{2 - Y_1^d}{e^{-Y_1^d}\left(2 - 2Y_2^d + Y_1^d\right)} = \frac{e^{Y_1^d}(1 - \frac{1}{2}Y_1^d)}{\left(1 - Y_2^d + \frac{1}{2}Y_1^d\right)}$$

Taking logs gives

$$\text{(B.7)} \quad Y_2^c \doteq Y_2^d .$$

Exact and approximate values of Y_2^c may be compared in Table B.1 for a gently and a steeply rising yield curve.

4. Regressive expectations

Livingston and Jain (1982) have shown that coupon bond yield curves become flat at long maturities even when discount bond yield curves tend to infinity. Coupon bias and the error of the approximation (B.7) tend to infinity in such cases. In the case of regressive expectations, however, for which discount yield curves become flat at long maturities, coupon bias is limited. Examples of this bias may be seen in Figure B.1, which shows coupon and discount yield curves for three values of the speed-of-adjustment parameter b in the relation that governs market expectations of the path by which the spot rate $r(s)$ will move from its current value $r(o)$ to its "normal" value r^*:[1]

(B.8) $$E[r(s)] = r^* - [r^* - r(o)]e^{-bs}$$

Substituting (B.8) into the equilibrium term structure equation (under certainty) gives

(B.9) $$Y_n^d = \frac{1}{n}\int_o^n E[r(s)]ds = \frac{1}{n}\int_o^n \{r^* - [r^* - r(o)]e^{-bs}\}ds = r^* - [r^* - r(o)]\left[\frac{1 - e^{-bn}}{bn}\right].$$

Discount and coupon yield curves are shown in Figure B.1 for maturities $n = 1, 2, \ldots, 50$ and are listed in the table below the figure for selected maturities from 1 to 100.[2] The middle coupon yield curve is typical of upward-sloping Durand curves, for which the median value of $(Y_{20}^c - Y_1^c)/(Y_{30}^c - Y_1^c)$ is .94, compared to .95 for b = .25.

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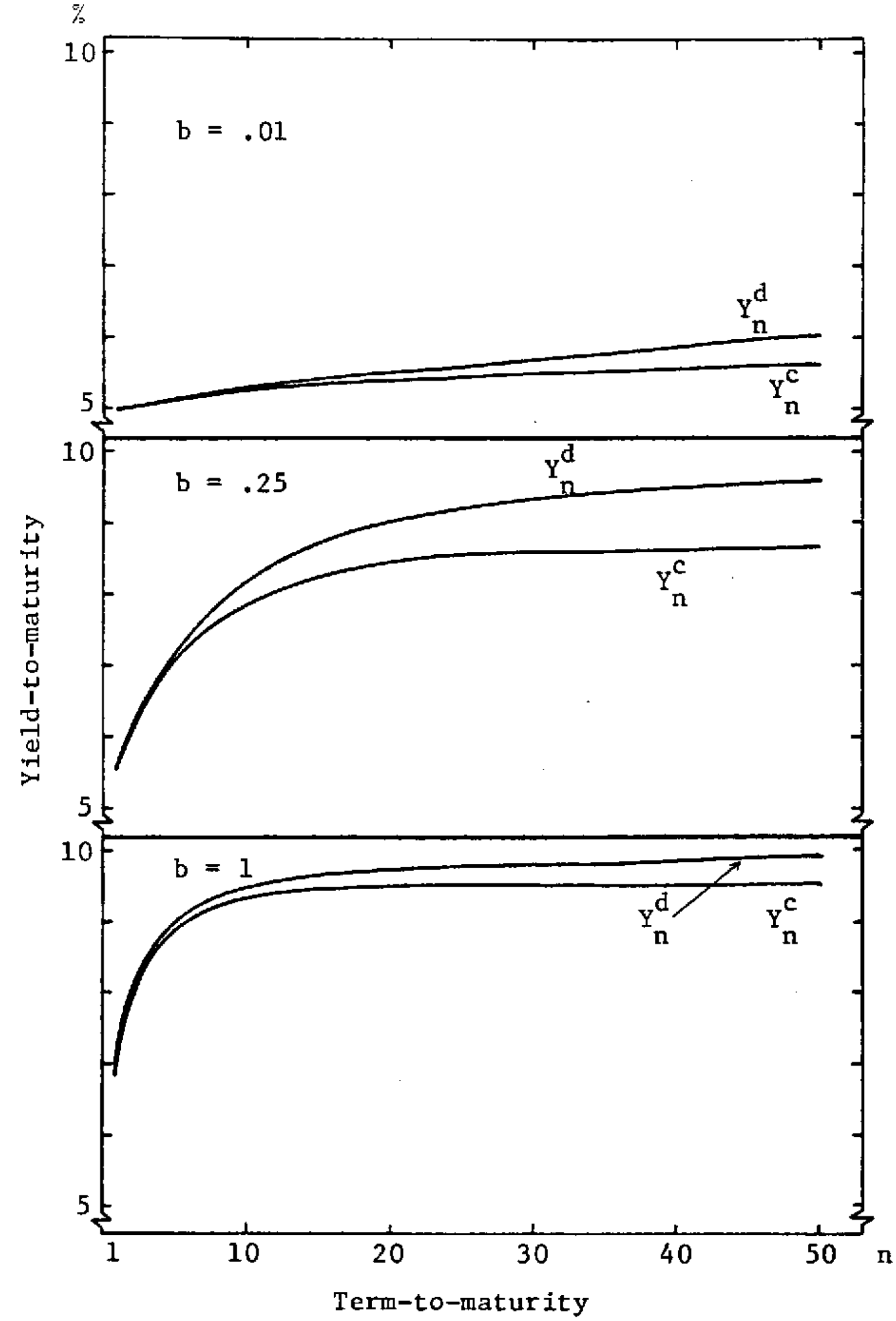
[1] Equations (B.8) and (B.9) are equivalent to (2) and (3) for $T = n$, $t = 0$, $y(t, T) = Y_{nt} \equiv Y_n^d$. Equation (B.9) and Figure B.1 apply precisely only under the Traditional Expectations Theory. If the Modern Expectations Theory applies, the true yield curves are somewhat lower than those in Figure B.1 due to negative term premia. However, the relation between Y_n^d and Y_n^c is governed by (B.6) for both certainty and uncertainty. Neglecting uncertainty causes the slopes of rising yield curves and, therefore, coupon bias to be overstated, while the slopes of falling yield curves and coupon bias are understated.

[2] See Garbade (1982, pp. 293-99) for further examples of coupon bias in a situation consistent with regressive expectations.

Figure B.1

Discount and Coupon Yield Curves Under Regressive Expectations



n	b = .01 Y_n^d	Y_n^c	Coupon bias	b = .25 Y_n^d	Y_n^c	Coupon bias	b = 1 Y_n^d	Y_n^c	Coupon bias
1	5.025	5.025	0	5.576	5.576	0	6.839	6.839	0
5	5.123	5.118	.005	7.146	7.047	.099	9.007	8.874	.133
10	5.242	5.221	.021	8.164	7.882	.282	9.500	9.300	.200
20	5.468	5.388	.080	9.007	8.434	.573	9.750	9.489	.261
30	5.680	5.506	.174	9.334	8.582	.752	9.833	9.535	.298
50	6.065	5.634	.431	9.600	8.647	.953	9.900	9.556	.344
75	6.482	5.685	.797	9.733	8.656	1.077	9.933	9.558	.375
100	6.839	5.695	1.144	9.800	8.657	1.143	9.950	9.559	.391