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FISHER TO FAMA TO FISHER: INFLATION AND INTEREST RATES, 1890-1981

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Research Paper

Federal Reserve Bank of Dallas

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At least since 1930, empirical researchers have grappled with the issue of whether investors immediately assimilate anticipated inflation into required returns so that variations in nominal interest rates are the best available predictors of variations in expected inflation. Irving Fisher and others demonstrated that pre-World War II interest rates responded slowly and incompletely to changes in inflation. But Shiller and Fama derived strikingly contrasting results for postwar data (particularly 1953-71): nominal interest rates (on average) reflected anticipated inflation so that real interest rates were independent of price changes. Fama argued that the contrast resulted from an improved Consumer Price Index (CPI) in 1953 and that the "Gibson paradox" observed in prewar data arose from spurious autocorrelations induced by a "noisy" price index.

The present study demonstrates that the differences between Fisher's and Fama's results stem solely from Fama's selection of a sample period--January 1953 to July 1971--of unprecedented price stability in U.S. history. The famous theoretical "Fisher effect" -- wherein interest rates correctly anticipate inflation -- holds during Fama's period, and the result is robust with respect to price index used, yield source, or type of short-term instrument studied. However, in the period since April 1974 the economy has reverted to characteristically higher price volatility, and empirical Fisherian results are again apparent. Once again, evidence resoundingly rejects the hypothesis that variations in nominal interest rates are appropriate measures of variations in expected inflation.

Consequently, it appears that in general the theoretical "Fisher effect" cannot be interpreted as a reduced-form equation for the nominal interest rate. Evidence suggests that the interest rate is determined simultaneously by interaction of the commodity and financial sectors, so an appropriate reduced-form equation must emanate from a complete macroeconomic system. Furthermore, the reduced form must be consistent with findings which corroborate a complete reflection of anticipated inflation in nominal interest rates during periods of relative price stability while generating the Gibson paradox during periods of normal price volatility.

The results presented here are not necessarily inconsistent with rational expectations (efficient markets). However, they reject rational expectations in conjunction with a variety of maintained hypotheses specified below.

I. The Fisher equation as a reduced form

Under certainty, i.e., a deterministic macroeconomic system, an investor holding claims denominated in money terms will demand a nominal holding-period return

$$1)\quad (1 + R_t) = (1 + r_t)(1 + \Pi_t)$$

where R_t, r_t, and Π_t are the nominal rate of return, real rate of return and inflation rate over [t, t+1]. The relationship between inflation and nominal interest rates described by (1) is the famous Fisher effect.[1]

An "expectations" version[2] of the Fisher equation under uncertainty is

$$2)\quad (1 + R_t^e) = (1 + r_t^e)\ (1 + \Pi_t^e) + \mathrm{Cov}(r_t, \Pi_t)$$

with its consequent linear approximation

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[1]The equilibrium relationship was known prior to Fisher (see his historical survey (1907, pp. 356-358)). However, (1) is commonly ascribed to Fisher since he was the first to carry out extensive statistical studies of the relationship.

[2]Ignoring risk-aversion is perhaps attributable to the belief that the short holding periods (six months or less) in subsequent statistical tests obviated the need to hedge against shifting investment opportunities and that money market instruments generating tested returns are (nearly) free of default risk.

2a) $R_t^e \cong r_t^e + \Pi_t^e + \text{Cov}(r_t, \Pi_t)$.

Assume inflation is initiated by exogenous shifts in the supply or demand for money. Further assume -- as in most "classical" macroeconomic models -- that money is "neutral": shocks to the monetary system leave real variables unaffected. Classical neutrality results require that <u>relative</u> prices be unchanged by the price level or inflation. A key relative price is the expected <u>real</u> holding-period return which is the opportunity cost of current goods, $(1 + R_t)P_t$, relative to expected costs of future goods, P_{t+1}^e, or

$$3) \quad (1 + r_t^e) = \frac{(1+R_t^e)P_t}{P_t^e} = (1 + R_t^e)(1 + \Pi_t^e)^{-1}$$

which is merely a rearrangement of (2) with $\text{Cov}(r_t, \Pi_t) = 0$ since other real variables like r_t are also independent of inflation. If the security has no default risk, (3) and its linear approximation

3a) $R_t \cong r_t^e + \Pi_t^e$

become the reduced form equation[3] for the nominal interest rate.

Finally, introduce two maintained hypotheses. First, if technology and tastes are stable through time, r_t^e can be replaced by a constant, (arbitrarily called) β_0. Second, if investors have "rational expectations", they forecast inflation (or any other variable) utilizing the minimum mean-square error estimator called the projection, $\hat{\Pi}_t \equiv P(\Pi_t | \Theta_t)$, (or regression) of Π_t onto the information set Θ_t. In linear systems, the projection is the conditional expectation, calculated assuming investors know the probability distribution of the exogenous variables and lagged endogenous variables comprising Θ_t.[4] The rational expectations assumption is called the "efficient markets hypothesis" in the finance literature and is interpreted to mean that "investors utilize all available information" in forecasting, where "available information" then includes the "true" probability distributions of variables in Θ_t.

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[3]Sargent (1976) has noted that in some classical models, fiscal variables affect both R_t and Π_t, so (3) is not a true reduced form equation. However, inclusion ofa disturbance term δ_t to capture such omitted variables ($R_t = r_t^e + \Pi_t^e + \delta_T$) would also be inappropriate since δ_t would not then be orthogonal to Π_t^e.

[4]For further details of linear least squares projections, see Sargent (1979), Ch. 10.

Given our maintained hypotheses, investors forecast inflation as

$$4) \quad \Pi_t^e = \hat{\Pi}_t = E[\Pi_t | \Theta_t] = \sum_{i=1}^{N} \beta_i \theta_{it}$$

where $\{\theta_{it}\}_{i=1}^{N}$ may be any subset of Θ_t, and the β_i are parameters.

Then the forecast error $\varepsilon_t \equiv \Pi_t - \hat{\Pi}_t$ has the properties that $E(\varepsilon_t | \Theta_t) = 0$ and $E(\varepsilon_t \varepsilon_{t-i} | \Theta_t) = 0$ for $\forall i > 0$.[5] Consequently, under our maintained hypotheses, (3a) may be written

$$5) \quad \Pi_t = -\beta_0 + R_t + \varepsilon_t.$$

Then the projection (regression) of inflation on any subset of Θ_t may be described with zero coefficients on everything except R_t and β_0. Equation (5) represents the regression most statistical researchers (including Fisher) have studied: project inflation onto a subset $\{\theta_{it}\} = \{1, R_t\}$. Its interpretation is the classic "Fisher effect": a rise in anticipated inflation is fully reflected in the nominal interest

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[5]This follows in the specific case (3a) since $E[\varepsilon_t \theta_{it}] = 0$ for all $\theta_{it} \in \Theta_t$ and R_{t-i}, $\Pi_{t-i} \in \Theta_t$ for $i > 0$.

rate, leaving the real rate unchanged. Hence (short-term) financial instruments' returns would provide the best available information about expected inflation if empirical studies substantiate the theory.

Many macroeconomists remain skeptical of results derived from "classical" precepts. However, Sargent (1973, 1976) has demonstrated that the "Fisher effect" carries over to a (Keynesian) IS-LM framework under a modification of our prior maintained hypotheses: the real rate of interest will still be statistically independent of systematic shifts in the money supply so that foreseen changes in monetary policy will affect R_t only to the extent they alter Π_t^e. This Fisherian result requires that investors forecast according to the rational expectations mechanism previously posited and that the Friedman-Phelps "natural rate" hypothesis of employment holds so the aggregate supply function is not <u>permanently</u> affected by systematic monetary or fiscal policies.

Consider the following system of structural equations (aggregate supply, aggregate demand (IS), and portfolio balance (LM), respectively):

6) $y_t = y_t^n + a(p_t - p_{t-1}^e) + v_{1t}\ , \quad a \geq 0$

7) $y_t = y_t^n + b[R_t - (p_t^e - p_t)] + cZ_t + v_{2t},\ b < 0$

8) $m_t = p_t + y_t + dR_t + v_{3t},\ d \leq 0$

where all variables are logarithms except the nominal interest rate R_t. Variables y_t and p_t represent endogenous real income and the price leve respectively. Z_t, y_t^n, p_t^e, and m_t are exogenous: they represent fiscal variables, a measure of "natural output", price forecasts, and the money supply, respectively. The v_{it} are mutually uncorrelated (but possibly serially correlated) random variables. Since prices are logs, $(p_t^e - p_t) = \Pi_t^e$ is an inflation forecast.

Note that the aggregate supply function is a (Lucas-type) version of a Phillips curve allowing short-run deviation in output from the natural rate due to errors in suppliers' price forecasts.

The system is completed by specifying the disturbances as autoregressive processes mixed with a zero mean white noise ($v_{it} = \sum_j w_{ij} v_{i,t-j} + \xi_{it}$, $i=1,2,3$, $j= 1,2,\ldots$, with ξ_{it} mutually uncorrelated) and assuming the money supply is governed by a linear feedback rule defined as a mixture of autoregressive processes on the exogenous variables m_t, y_t^n, Z_t and the disturbances v_i, and an additional white noise.

For this system the reduced-form equation for the nominal interest rate is

$$9)\quad R_t = q[(1+a)v_{2t} - v_{1t}] - qa[m_t - E_{t-1}p_t - y_t^n - v_{3t}]$$

$$+ q(1+a)cZ_t - q(1+a)b\Pi_t$$

where $q \equiv [-1/(b+ab+ad)] > 0$.[6] The reduced-equation (9) is equivalent to the regression (5) (the standard form of the Fisher equation) if the coefficient of Π_t equals one and the remaining terms can (on average) be viewed as constants. Conversely, if empirical evidence suggests that (5) holds, we can draw implications about parameters of the macro-economic system required for the equivalence of (9) and (5).

For the coefficient of Π_t to be (close to) one, either parameters "a" or "d" (or both) must be (close to) zero. Thus the Phillips curve must be vertical even in the short-run or the interest rate must be determined solely in the commodity market. A vertical Phillips curve might be observed empirically when price volatility is so low that realizations rarely deviate from forecasts; a resultant empirical estimate of (6) might imply a=0.

To interpret the terms other than inflation as (approximately) constant in (9) requires that i) the disturbances' autoregressive components be constant functions; ii) fiscal policy be stable; and iii) investors maintain a constant relationship between the nominal interest rate and the deviation of forecast real balances from the "natural" level of output.

Substituting (6) into (8) and taking conditional expectations at t-1 reveals that

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[6]In (9) and henceforth, the short-hand notation $E_t x_t$ stands for $E(x_t|\Theta_t)$ for any stochastic process x_t.

$$10)\quad E_{t-1}[m_t - p_t - y_t^n - v_{3t}] = dE_{t-1}R_t + E_{t-1}v_{1t}.$$

Equation (10) implies that any forecast change in the money supply is reflected solely in changed expectations for the price level and the nominal interest rate (plus altered expectations for an exogenous variable and disturbance); there is no systematic effect on the real interest rate. Furthermore, the anticipated relationship between real balances and "natural" output is indeed reflected by a constant proportional adjustment to the expected nominal rate (given the disturbances' autoregressive components are constant).

Alternatively, taking conditional expectations of (9) at t-1 and substituting from (10) shows that when suppliers make price forecasts an appropriate regression for the nominal interest rate is

$$11)\quad E_{t-1}R_t = (1-qad)^{-1}q(1+a)[E_{t-1}(v_{2t}-v_{1t}) + cE_{t-1}Z_t - bE_{t-1}\Pi_t].$$

Hence under requirements (i) and (ii) above, investors' forecasts of nominal rates will indeed be a linear combination of a constant and their forecasts of inflation as (5) suggests.

Recapitulating, under rational expectations and either a "classical" or IS-LM-Phillips curve (neo-Keynesian) economy, a standard interpretation of the "Fisher effect" results: anticipated changes in the money supply affect nominal interest rates only to the extent they alter forecasts of inflation. Empirical substantiation of the standard regression tested by most statistical researchers,

$$5') \quad \Pi_t = \beta_0 + \beta_1 R_t + \varepsilon_t$$

$$\text{S.T. } \beta_1 = 1$$

(with β_0 the implied constant real rate) would lend credence to rational expectations forecasts in conjunction with either a "classical" economy and constant technology or a neo-Keynesian economy characterized by a vertical long-run Phillips curve, stable fiscal policy, and a variety of restrictions on the evolution of disturbances. Conversely, empirical repudiation of (5') could be interpreted as absence of a stable technology in a "classical" economy or lack of a stable fiscal policy or non-constant autoregressive disturbance functions in a neo-Keynesian economy; it would not merit summary rejection of rational expectations. However, it would clearly imply that variations in nominal interest rates would not be accurate indicators of variations in expected inflation. Furthermore, the failure of (5') would cast doubt on the premise that "nominal interest rates summarize all the information about future inflation rates that is in time-series of past inflation rates".[7] Verification of that hypothesis would require identification of a reduced-form equation adequate to describe interaction of commodity and monetary sectors to jointly determine R_t.

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[7]Fama (1975), p. 269.

II. Conflicting empirical results: Fisher versus Fama

A. Fisher's empirical "contradiction" of the theory

Fisher (1930) was the first to subject (3a) and (5) to rigorous statistical testing. Analyzing data on short-and long-term interest rates in Britain and the United States, Fisher found no corroboration for the hypothesis that expected inflation is immediately and perfectly reflected in variations of nominal interest rates. Fisher's results may be characterized as follows:

1) Over long periods, R and Π moved together; however short-term contemporanous correlations between R and Π were not high (some were even negative). For example, decennial averages of R and Π were highly correlated but quarterly or annual averages were not.

2) Variations in R tended to be less than those in Π. Quoting Fisher,

> when prices are rising, the rate of interest tends to be high but not so high as it should be to compensate for the rise; and when prices are falling, the rate of interest tends to be low but not so low as it should be to compensate for the fall.[8]

3) The influence of inflation seemed to be "distributed over time". Fisher found that correlations between R and distributed lags of

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[8] Fisher (1930, p. 43).

past inflation rates rose consistently with the number of lags. He interpreted the finding to indicate "that interest rates follow price changes closely in degree, though rather distantly in time".[9]

Subsequent researchers have hypothesized that investors forecast inflation by "adaptive expectations". Then (3a) becomes

$$12)\quad R_t = \beta_o + \sum_{i=1}^{\infty} \beta_i \Pi_{t-i} + \varepsilon_t.$$

Investigators like Sargent (1969), Yohe and Karnosky, and Gibson found in tests of (12) that coefficients on past rates of inflation were statistically significiant for very long lags; furthermore, the weights summed to less than one. If prices are in logs ($p_t \equiv \log P_t$) so inflation can be defined as $\Pi_t \equiv p_{t+1} - p_t$, (12) can be rewritten as

$$(12')\quad R_t = \beta_o + \sum_{i=1}^{\infty} \beta_i^* p_{t-i} + \varepsilon_t.$$

This is a statement of the Gibson paradox: the tendency of nominal interest rates to be highly correlated with a time series of the price level rather then the expected inflation rate. Sargent (1973) has suggested how the Gibson paradox could arise in a stochastic macroeconomic system even if expectations are "rational". Consequently, empirical evidence supporting (12') is insufficient to disprove rational expectations (efficient markets). Conversely, any reduced form equation

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[9]Ibid., p. 451.

for the nominal interest rate must be rich enough to explain the empirical evidence for the Gibson paradox.

B. Fama and efficient markets

Using data from a different time period--January 1953 to July 1971--Fama (1975) claimed to find a very different set of facts. He concluded that

> ...one ...cannot reject the hypothesis that all variation through time in one- to six-month nominal rates of interest mirrors variation in correctly assessed one- to six-month expected rates of change in purchasing power... The nominal interest rate R_t ... is the best possible predictor of inflation from t-1 to t.

Fama chose his sample period for the following reasons: (i) the Federal Reserve's bond support program during and after World War II prevented interest rates from reflecting expected inflation; (ii) the Consumer

Price Index (CPI) was "improved" in 1953;[10] and (iii) price controls from August 1971 to mid-1974 meant that "observed values of the CPI" differed from the "true costs of goods to consumers" (Fama, 1975, p. 275).

Rather than inflation (Π_t), Fama defined the variable "purchasing power change" (Δ_t) such that

$$13) \quad (1+\Pi_t) = (1+\Delta_t)^{-1}.$$

Then appropriate restatements of (3a) and (5) are

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[10]According to Fama, the "substantial upgrading of the CPI "in 1953 was because the "number of items in the Index increased substantially, and monthly sampling of major items become the general rule. For tests of market efficiency based on monthly data, monthly sampling of major items in the CPI is critical. Sampling items less frequently than monthly... creates spurious autocorrelation in the Index," making tests of "market efficiency on pre-1953 data difficult to interpret." (Fama, 1975, p. 274).

While it is true that the numbers of items and sources sampled were increased in 1953 (See BLS Bulletins 1948, p. 69, and 1711, p. 75), we have been able to find no reference to an increase in the frequency of sampling at that time. In fact, the frequency of sampling since 1953, and indeed since 1948, has probably been considerably less than between 1940 and 1948. "A reduction of one-third in appropriations for fiscal year 1948 necessitated changes in the frequency of pricing for nonfood items and in the number of items currently priced" (BLS, Bulletin 1517, p. 4; also see Bulletin 965, pp. 35-36). Since that time, most nonfood items have been sampled every month only in the five largest urban areas and once every three months in other areas. Several items are sampled only semiannually or annually.

This does not mean that Fama was wrong to begin with 1953, for the Federal Reserve's bond support program during and after World War II prevented interest rates from reflecting expected inflation. But it does suggest that he may have exaggerated the superiority of the post-1952 years as the appropriate sample for tests of market efficiency. And his emphasis on the importance of sampling frequency lends support to use of the WPI, where most items really are sampled monthly.

14) $R_t = r_t - \Delta_t$;

15) $\Delta_t = \beta_o - R_t + \varepsilon_t$

where ε_t is orthogonal to the constant real rate β_o and nominal rate R_t. Equation (15) may be tested by the OLS regression

16) $\Delta_t = \alpha_o + \alpha_1 R_t + \varepsilon_t^*$

where we expect $\alpha_1 = -1$. From the properties of the forecast error for the projection developed earlier, ε_t (and hence ε_t^*) should contain no serial correlation. Furthermore, combining (14) and (15) shows

17) $r_t - \beta_o = \varepsilon_t$

so autocorrelations of the time series of realized real rates have the properties of the forecast error; hence autocorrelations of r_t should be zero for all lags. Finally, if $E_t\Delta_t = \beta_o - E_tR_t$ is the projection of Δ_t onto Θ_t, then the addition of $\Delta_{t-1} \varepsilon \Theta_t$ to the right hand side of (15) should result in $\alpha_2 = 0$ when testing the OLS regression

18) $\Delta_t = \alpha_o + \alpha_1 R_t + \alpha_2 \Delta_{t-1} + \varepsilon_t^*$.

Using price data from Salomon Brothers' daily release for one- to six-month Treasury bills, selecting bills maturing closest to month's end, and employing the revised CPI, Fama calculated time series for

Δ_t, R_t, and r_t to test the OLS regressions (16) and (18) and autocorrelations of the time series r_t.[11] When maturity exceeded one month, non-overlapping data were selected to avoid introducing spurious autocorrelation.

For his sample period, Fama discovered that autocorrelations of the the real rate were indeed zero for all lags, α_1 was close to -1 as predicted by theory, and α_2 (the coefficient of purchasing power change lagged one period) was insignificantly different from zero.[12] He concluded that variations in nominal interest rates were the best available predictors of inflation: current market values appeared to incorporate all information available in time series of lagged purchasing power change (inflation).

Fama explained the contrast of his results with his predecessors' findings by noting that[13]

> ...earlier studies... are based primarily on pre-1953 data, and the negative results on market efficiency may to a large extent just reflect poor commodity price data. By the same token, the success of the tests reported here is probably to a non-negligible extent a consequence of the availability of good data beginning in 1953.

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[11]Although theory has been developed in terms of linear approximations like (3a), the time series r_t was calculated from the precise relationship $r_t = (1+R_t)(1+\Delta_t) - 1.$

[12]Numerical highlights of Fama's findings appear in the next section for contrast.

[13]Shiller obtained results similar to Fama's on postwar data.

He further hypothesized that the Gibson paradox prevalent in data prior to 1953 resulted from poor price data (a "noisy" price index) obscuring the Fisher effect by spurious autocorrelation while picking up long-run price movements.[14]

III. A return to Fishter's world: evidence conflicting with theory

Sufficient time has elapsed since the end of the price controls in April 1974 to test Fama's hypotheses outside his sample period.[15] Unfortunately, it appears that postwar years (particulary 1953-71) were not earmarked by improved data collection or a public more highly attuned to the costs of inflation. Rather, Fama's results stem from the selection of a sample period of price placidity unparalleled during the twentieth century. Perusal of Charts 1 and 2 indicates that 1953-71 was a period of such price tranquility that even the most ardent foes of rational expectations should not react incredulously to assertions that agents acted as though they <u>knew</u> the underlying

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[14]Sargent (1976) agreed with Fama that a break in data behavior apparently occurred near World War II: postwar tests seemed to support a rational expectations hypothesis. However, Sargent suggested that any conclusions would be speculative in the absence of a verifiable theory for the Gibson paradox in prewar data.

[15]Sector-by-sector decontrol began October 25, 1973, so when the Economic Stabilization Act expired in April 1974 less than one-fourth of the economy was subject to controls. Thus we would not expect any sudden rise in prices in May 1974 due to a "build-up" effect. For further details, see Department of the Treasury, <u>Historical Working Paper on the Economic Stabilization Program: 8/15/71-4/30/74</u>, Office of Economic Stabilization. In any case, what matters to the theory is prediction of price movements, whatever the cause, whether due to the ending of controls or not.

Chart 1. Inflation and real and nominal interest rates

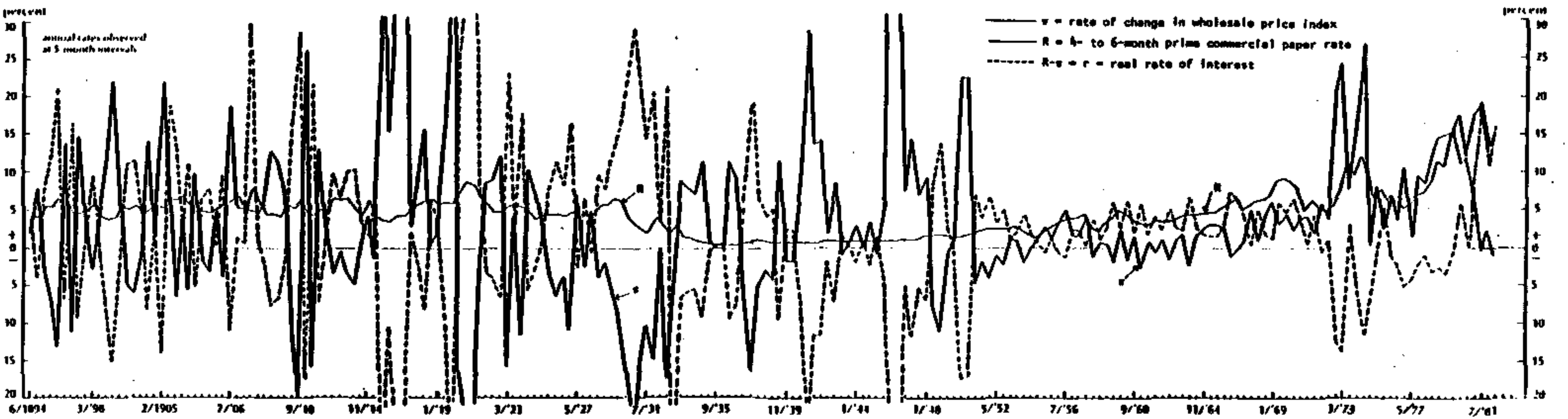
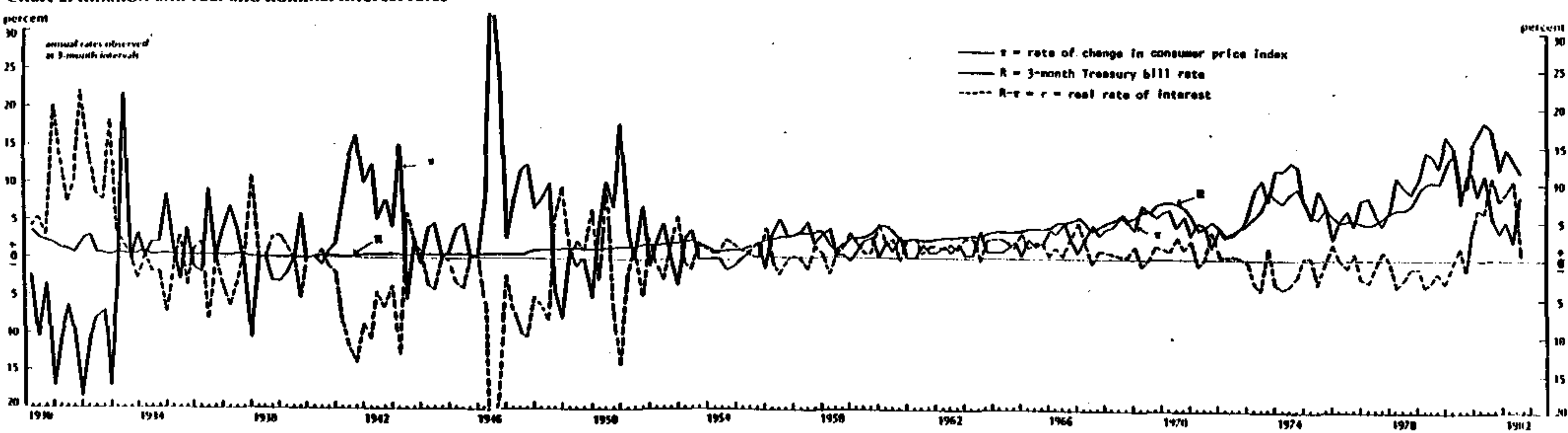


Chart 2. Inflation and real and nominal interest rates



probability distribution of the price level during the era. Chart 1 shows 5-month observations on the 4-6 month prime commercial paper rate (R, the fairly steady solid line), the rate of change in the Wholesale Price Index (Π, the volatile solid line), and R-Π (the dashed line) between 1894 and 1981. Chart 2 displays 3-month observations on the 3-month Treasury bill rate and the CPI from 1929 to 1982.[16]

During the 1953-71 period of low price volatility, the rational expectations "Fisher effect" model is easily corroborable. The result is highly robust and apparently independent of i) the source of yield data; ii) the price index used (CPI or WPI); and iii) the type of short-term instrument (Treasury bill or commercial paper) generating the nominal rate series. However, in periods characterized by higher price volatility (viz. prior to 1953 or subsequent to the recent removal of price controls, i.e., 4/74-present), regression tests of (16) and (18) and autocorrelation tests on the realized real rate series soundly reject rational expectations in conjunction with the maintained hypotheses described earlier.

Table 1 presents a comparison of sample autocorrelations for three-month real rates during Fama's sample period. The time-series for r_t generating the autocorrelations were calculated using the CPI and three data sources: i) Fama's data supplied from Salomon Brothers' daily quote sheets for the bill maturing closest to month's end; ii) data

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[16]Data for the 4-6 month prime commercial paper rate and the 3-month Treasury bill rate first became available in 1894 and 1929, respectively.

Table 1

Comparison of sample autocorrelations on three-month real rates (1/53-4/71) calculated from the CPI and three data sources: 1) Fama's data supplied from Salomon Bros. quote sheets (last day of the month); 2) Salomon Bros. Yield Book*; 3) Federal Reserve Board averages (non-overlapping data).

Statistic/ Source	(1) Fama	(2) SBYB	(3) FRB
$\hat{\rho}_1$	.00	.04	.03
$\hat{\rho}_2$	.02	-.14	-.17
$\hat{\rho}_3$	.08	.16	.17
$\hat{\rho}_4$	.26	.23	.23
$\hat{\rho}_5$	.16	.09	.09
$\hat{\rho}_6$	-.09	-.08	-.08
$\hat{\rho}_7$	.06	.01	-.00
$\hat{\rho}_8$	-.01	.18	.21
$\hat{\rho}_9$	.08	-.14	-.15
$\hat{\rho}_{10}$	-.32	-.23	-.21
$\hat{\rho}_{11}$	.11	.06	.09
$\hat{\rho}_{12}$	.19	.19	.19
$\sigma(\hat{\rho}_1)$	.12	.12	.12
$\overline{r}$**	.00306	.00313	.00307
s(r)	.00371	.00400	.00400
T-1	73	73	73

*The Yield Book reported a quote on the mid-day of the month prior to 1964 and on the first day of the month subsequently. Tests indicate that real rate series calculated before and after the reporting change were indistinguishable.

**On a three-month (rather than annual) basis.

Table 2

Sample Autocorrelations on Five-Month Real Rates
4-6 Month Commercial Paper and the WPI (Non-Overlapping Data

Statistic/ Period	8/94-3/29	8/29-12/52	2/53-6/71	5/53-9/8
$\hat{\rho}_1$	.43**	.60**	.10	.49**
$\hat{\rho}_2$	.02	.35	.25	.30
$\hat{\rho}_3$	-.14	.18	.01	.20
$\hat{\rho}_4$	-.11	.05	.08	.05
$\hat{\rho}_5$	-.02	-.08	.08	-.08
$\hat{\rho}_6$	.09	-.14	.01	-.03
$\hat{\rho}_7$	.11	-.12	.10	-.04
$\hat{\rho}_8$	-.08	-.10	.21	-.09
$\hat{\rho}_9$	-.29	-.10	.10	-.06
$\hat{\rho}_{10}$	-.24	-.02	-.16	.06
$\hat{\rho}_{11}$	-.14	-.00	-.12	-.01
$\hat{\rho}_{12}$	-.12	.05	-.23	.11
$\sigma(\hat{\rho}_1)$	.11	.13	.15	.12
$\overline{r}$***	.01501	-.00261	.01154	.0065
s(r)	.07051	.05010	.00966	.0193
T-1	83	56	44	68

*Due to use of non-overlapping data, there are 5 sets of data for ever time period, each starting in a different month. This set was chosen random and is representative. The highest estimator $\hat{\rho}_1$ for the 1953-7 period in alternative sets was 0.27 which is still statistically insignificant at the 0.95 confidence level.

**Statistically different from zero at the 0.95 confidence level.

***On a five-month (rather than annual) basis.

from Salomon Brothers' Yield Book which reported a quote on the mid-day of the month prior to 1964 and the first day of the month subsequently; and iii) Federal Reserve Board data, reported as monthly averages of daily quotes. Non-overlapping data were utilized throughout the study to avoid introducing spurious autocorrelations. Note that the sample autocorrelation series for twelve lags are nearly indistinguishable in magnitude and sign pattern regardless of data source.

Table 2 displays sample autocorrelations on five-month real rates generated from 4-6 month prime commercial paper and the WPI. Hence the instrument used is not totally riskless (in the sense of default risk) and the price index is more volatile than the CPI. Note that in each historical period except Fama's 1953-71 subperiod, first-order autocorrelation coefficients are significantly different from zero, violating the theory.

Table 3 extends the evidence that Fama's conclusions result from the sample period selected rather than removal of noise from the price index. It depicts sample autocorrelations on three-month real rates generated from Federal Reserve Board average yields and, alternatively, the CPI or WPI, over three sample periods: pre-Fama, Fama, and post-Fama. As anticipated, autocorrelations are close to zero for all lags during the Fama period regardless of the price index used to generate the real rate time series. In both pre- and post-Fama periods, first-order autocorrelation estimates are significantly different from zero at the 0.95 confidence level.

Table 3

COMPARING SAMPLE AUTOCORRELATIONS ON THREE-MONTH REAL RATES OVER THREE PERIODS
3-MONTH T-BILLS (FEDERAL RESERVE AVERAGES) WITH THE CPI OR WPI (NON-OVERLAPPING DATA)*

Statistic/ Data Source#	1/34 - 10/52 FRB(CPI)	1/34 - 10/52 FRB(WPI)	1/53 - 4/71** FRB(CPI)	1/53 - 4/71** FRB(WPI)	4/74 - 10/81 FRB(CPI)	4/74 - 10/81 FRB(WPI)
$\hat{\rho}_1$	.52***	.60***	.03	.01	.40***	.48***
$\hat{\rho}_2$	.34	.30	-.17	.31	.21	.02
$\hat{\rho}_3$	.23	.08	.17	-.00	.32	.09
$\hat{\rho}_4$	.08	-.06	.23	.18	.12	.14
$\hat{\rho}_5$	.06	-.03	.09	-.07	.02	-.08
$\hat{\rho}_6$	-.08	-.09	-.08	.09	-.02	-.04
$\hat{\rho}_7$	-.17	-.19	-.00	.05	-.11	-.04
$\hat{\rho}_8$	-.17	-.16	.21	.14	-.10	-.18
$\hat{\rho}_9$	-.26	-.16	-.15	-.04	-.15	-.22
$\hat{\rho}_{10}$	-.24	-.14	-.21	.07	-.11	.00
$\hat{\rho}_{11}$	-.22	-.17	.09	.09	-.06	-.02
$\hat{\rho}_{12}$	-.19	-.17	.19	.15	.02	-.18
$\sigma(\hat{\rho}_1)$	.12	.12	.12	.12	.18	.18
$\bar{r}$##	-.00807	-.00943	.00307	.00510	-.00032	-.00007
s(r)	.01501	.02703	.00400	.00631	.00939	.01593
T-1	75	75	73	73	30	30

*Due to use of non-overlapping data, there are three data sets for every time period, each starting in a different month. This set was chosen at random and is representative. The highest estimators $\hat{\rho}_1$ for the 1953-71 period in alternative sets were 0.18 and 0.11 for the CPI and WPI respectively; neither is significantly different from zero at the 0.95 confidence level.

**Note the 4/71 observation uses the 7/71 price level.

***Statistically different from zero at the 0.95 confidence level.

#The data legends stand for "Federal Reserve Board" averages with the Consumer Price Index (FRB(CPI)) and with the Wholesale Price Index (FRB(WPI)). Federal Reserve Board data are monthly averages of daily quotes.

On a three-month (rather than annual) basis.

These results are duplicated in Table 4 for one-month real rates generated from Salomon Brothers' Yield Book quotes and either the CPI or WPI in Fama and post-Fama settings. Fama's sample autocorrelations are included for comparison. Once again, the data source or price index used is immaterial between 1953-71; the "efficient markets" hypothesis is upheld even if the price index is the WPI. However, in the 4/74-9/81 period, the same hypothesis is rejected. Combining the results of Tables 1-4 supports our previous conclusions about the robustness of the "efficient markets" hypothesis during the 1953-71 sample period and its complete failure elsewhere.

For those unconvinced by autocorrelation tests, the results of regressions (16) and (18) are reported in Table 5 for one-month Treasury bills (quotes from Salomon Brothers' Yield Book) and the CPI during Fama's period, the period subsequent to removal of price controls, and the two periods jointly. Table 6 displays corresponding outcomes for the WPI. Fama's regression results are reproduced for contrast since his data source for bond yields is slightly different from ours.

Once again, our results duplicate Fama's during the 1953-71 sample period. However, for the period 4/74-9/81, corroboration of the "efficient markets" hypothesis fails: the coefficient on the nominal interest rate is not close to -1.0 and the first three sample autocorrelation coefficients for the residuals are significantly different from zero at the 0.95 confidence level. Seriously autocorrelated residuals appear in the combined sample period 1/53-9/81 (excluding price controls), although the coefficient of R_t is again close to -1.0.

TABLE 4

Comparing Sample Autocorrelations on One-Month Real Rates Over Two Periods
1-Month T-Bills (Salomon Bros. Yield Book) with the CPI or WPI

Statistic Source	1/53 - 6/71 FAMA(CPI)*	SBYB(CPI)	SBYB(WPI)	4/74 -9/81 SBYB(CPI)	SBYB(WPI)
$\hat{\rho}_1$	.09	.10	.01	.40**	.35**
$\hat{\rho}_2$	.13	.13	.09	.30	.10
$\hat{\rho}_3$	-.02	-.03	-.11	.24	.23
$\hat{\rho}_4$	-.01	-.01	.10	.14	-.11
$\hat{\rho}_5$	-.02	-.03	-.05	.07	-.05
$\hat{\rho}_6$	-.02	-.03	.18	.05	.02
$\hat{\rho}_7$	-.07	-.07	.03	.05	-.18
$\hat{\rho}_8$	.04	.03	.13	.13	-.04
$\hat{\rho}_9$	.11	.11	-.08	.13	.15
$\hat{\rho}_{10}$	.10	.09	.06	.09	.01
$\hat{\rho}_{11}$	.03	.03	-.01	.13	.10
$\hat{\rho}_{12}$	.19	.18	.13	.08	.18
$\sigma(\hat{\rho}_1)$	.07	.07	.07	.11	.11
$\bar{r}$***	.00074	.00086	.00154	-.00074	-.00069
$s(r)$	.00197	.00194	.00353	.00301	.00732
T-1	222	221	221	89	89

*Fama's results, which include 7/71, are reproduced here for comparison. We have omitted the 7/71 observation since it requires the price index in 8/71 when price controls had been instituted.

**Significantly different from zero at the 0.95 confidence level.

***On a one-month (rather than annual) basis.

TABLE 5

REGRESSION TESTS ON ONE-MONTH BILLS
(The Price Index for Calculating Δ_t Is the CPI.)

Simple Regression

$$\Delta_t = a_0 + a_1 R_t + e_t$$

Period	a_0	t-value	a_1	t-value	R^2	s(e)	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$	obs.
1/53-7/71	.0007	2.39	-.942	-9.76	.301	.0019	.094	.125	-.031	223
4/74-9/81	-.0036	-4.66	-.557	-5.12	.230	.0028	.382*	.273*	.212*	90
1/53-9/81 (excluding 8/71-3/74)	.0006	2.33	-1.040	-19.50	.551	.0024	.304*	.274*	.187*	313
Fama's result# (1/53-7/71)	.0007	2.33	-.98	-9.8	.29	.0020	.09	.13	-.02	223

Multiple Regression

$$\Delta_t = a_0 + a_1 R_t + a_2 \Delta_{t-1} + e_t$$

Period	a_0	t-value	a_1	t-value	a_2	t-value	R^2	s(e)	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$	obs.
1/53-7/71	.0006	1.98	-.843	-7.60	.092	1.43	.309	.0019	.005	.128	-.034	212
4/74-9/81	-.0023	-2.90	-.247	-2.03	.468	4.42	.371	.0025	-.043	.070	.100	90
1/53-9/81 (excluding 8/71-3/74)	.0003	1.20	-.647	-8.45	.360	6.59	.608	.0022	-.075	.158*	.069	312
Fama's result# (1/53-7/71)	.0006	1.97	-.87	-7.25	.110	1.57	.30	.0020	-.05	.13	-.04	212

*Note that $\sigma(\hat{\rho}_t) \approx \sqrt{1/(T-t)}$ under the hypothesis that the true autocorrelation coefficient is zero. See Nelson (pp. 71-74) for a more precise approximation using Bartlett's method. Under either method with a 95% confidence statistic, we reject the hypothesis that residuals are not autocorrelated in the simple regression. Addition of a lagged dependent variable adds significant explanatory power and eliminates autocorrelation in the residuals.

#Fama's results are included for comparison since his data source is different. His R^2 is adjusted for degrees of freedom. Note our regression results are nearly coincident with Fama's over the 1953-71 period as expected.

TABLE 6

REGRESSION TESTS ON ONE-MONTH BILLS
(The Price Index for Calculating Δ_t Is the WPI.)

Simple Regression
$$\Delta_t = a_0 + a_1 R_t + e_t$$

Period	a_0	t-value	a_1	t-value	R^2	s(e)	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$	obs.
1/53-7/71	.0003	.63	-.565	-3.25	.046	.0035	-.018	.075	-.133	223
4/74-9/81	-.0054	-2.71	-.288	-1.04	.012	.0070	.316*	.104	.282*	90
1/53-9/81 (excluding 8/71-3/74)	.0007	1.44	-.955	-8.62	.193	.0050	.250*	.138*	.145*	313

Multiple Regression
$$\Delta_t = a_0 + a_1 R_t + a_2 \Delta_{t-1} + e_t$$

Period	a_0	t-value	a_1	t-value	a_2	t-value	R^2	s(e)	$\hat{\rho}_1(e)$	$\hat{\rho}_2(e)$	$\hat{\rho}_3(e)$	obs.
1/53-7/71	.0003	.58	-.570	-3.17	-.0194	-.29	.045	.0035	.006	.066	-.126	222
4/74-9/81	-.0041	-2.05	-.196	-.73	.2696	2.71	.089	.0068	.042	-.006	.360*	90
1/53-9/81 (excluding 8/71-3/74)	.0005	1.03	-.725	-6.02	.2346	4.29	.237	.0049	-.003	.078	.151*	312

*Note that $\sigma(\hat{\rho}_t) \approx \sqrt{1/(T-t)}$ under the hypothesis that the true autocorrelation coefficient is zero. See Nelson (pp. 71-74) for a more precise approximation using Bartlett's method. Under either method with a 95% confidence statistic, we reject the hypothesis that residuals are not autocorrelated in the simple regression. Addition of a lagged dependent variable adds significant explanatory power and eliminates autocorrelation in the residuals.

The addition of a single lagged purchasing-power explanatory variable, Δ_{t-1}, transforms the simple regression (16) to the multiple regression (18). During Fama's period, the coefficient of Δ_{t-1} is not significantly different from zero, further corroborating the "efficient markets" hypothesis.[17] However, during the post-Fama period and the joint period 1/53-9/81 (excluding price controls), the coefficient of Δ_{t-1} is relatively large, significantly different from zero, and adds a large amount of explanatory power to the regressions. Furthermore, the residual sample autocorrelations are close to zero.[18] Consequently, variations in nominal interest rates are clearly acceptable as "best" predictors of forecast inflation during the period 1/1953-7/1971 but fail from 4/1974-9/1981 (and from 1/53-9/81 with the price control subset excluded).[19] Table 6 shows analogous results when the price

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[17]However, Nelson and Schwert found that a Box-Jenkins distributed lag on inflation (in place of Fama's Δ_{t-1}) improved the R^2 slightly (from .29 to .31) and was statistically significant.

[18]This should not be surprising since the residuals in the sample regression (16) are autocorrelated during post-Fama and joint periods. Assuming an AR(1) process for the errors, multiple regression (18) is highly analogous to a generalized (first) difference regression of (16). The lack of autocorrelation is important in the presence of a lagged dependent variable because we can then claim our parameter estimates are unbiased and consistent.

[19]For statistical purists, an F-test can be constructed to test whether parameters in the multiple regression (18) come from the same population during the 1/53-7/71 and 4/74-9/81 periods. The null hypothesis is

$$H_0 : \beta_F = \beta_{PF}$$

where β_F and β_{PF} are the parameter vectors in the Fama and post-Fama period regressions respectively. It is rejected if the F-statistic from the regression (F*) exceeds the critical value $F(1-\alpha;\, R,\, N-K)$ where $(1-\alpha)$ is the confidence level, R the number of restrictions, and N-K the degrees of freedom. For the multiple regression reported in Table 5, $F^* = 14.86 \geq F_{cv}(.99, 3, 306) \cong 3.8$ so we can safely reject the hypothesis that the two are similar.

index is the WPI. The major difference between the WPI and CPI regressions is the lower explanatory power of the WPI.

IV. Conclusions

It is evident that in general, variations in nominal interest rates are not appropriate measures of variations in expected inflation. The classical "Fisher effect" equation is inadequate to describe the intricate interactions between financial and commodity sectors in determining the interest rate. The Gibson paradox observed in pre-World War II data does not stem from "noisy" commodity price data; it reappears with the resurgence of sustained higher levels of price volatility as suggested by Charts 1 and 2. Inflation forecasts can evidently only be inferred from a non-classical reduced-form equation.

It is relatively easy to argue that (9) or an equation implied by the linear least-squares projection (11) might fit the data for both periods 1/53-7/71 and 4/74-9/81. During the earlier period, fiscal policy was relatively stable; furthermore, price volatility was sufficiently low that one might argue price forecasts differed from realizations too little to generate even short-run systematic deviations from "natural" output. With the resurrection of greater price volatility and more erratic fiscal policy in the 4/74-9/81 period, several maintained hypotheses could be violated, resulting in the failure of (9) (or its analogue implied by (11)) to collapse to the classical Fisher equation; nevertheless, expectations could still be formed rationally. Thus a system with a non-constant real rate (where we interpret the "real rate" as the sum of all terms in a reduced form equation like (9) other than the inflation term) is broadly consistent with rational expectations (efficient markets).

Much recent empirical work on the relationship between interest rates and inflation has focused on this implied non-constancy of the real rate. For example, Tanzi, Cargill and Meyer, Fama and Gibbons, Mishkin, and Startz have estimated equations of the Fisher-Fama type using a variety of distributed lag schemes (intended to capture inflationary expectations) and real variables (presumed to influence the real rate of interest) and most have concluded that the real rate is not constant. Essentially the same result has been obtained within multi-equation, macroeconomic frameworks by Elliott and Shiller and Siegel. All of these studies except the last assume, with Fisher and Fama, that money is both exogenous and the primary source of the inflationary process, i.e., that inflations are precipitated by exogenous monetary disturbances. Thus, these studies are consistent with the assumed exogeneity of money in the macroeconomic system (6)-(8) which led to the reduced form (9).

Clearly, however, other macro systems, price forecasting mechanisms, and monetary endogeneity, would lead to alternative feasible reduced-form equations. Another explanation of the data reported above might emanate from a model completed by i) endogenizing the behavior of the central bank and therefore of the money suppply and ii) making explicit the causal connections between the real rate of interest and other real variables. Such a model is best described as Wicksellian, in which i) disturbances emanate from the real side of the economy through forces affecting supplies and demands for goods, the most important of which may be categorized under the headings "savings" and "investment"; ii) these disturbances affect the equilibrium real rate of interest (Wicksell's natural rate); and iii) a monetary authority fond of stable interest rates resists the adjustment of the market rate to the natural rate, iv) thereby accentuating the price changes induced by the initiating real disturbances.[20] This approach, which must take disequilibria into account, could explain the strong, long-term tendencies of money, inflation, nominal interest rates, and real investment and/or government expenditures (as the most volatile real variables in the system) to vary together. However, since nominal interest rates vary less than inflation, partly due to the central bank's resistance, real interest rates move oppositely to these variables. The high autocorrelations observed in expost real rates are thus broadly consistent with Wicksell's analysis.

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[20]Shiller and Siegel rejected their version of Wicksell's model but their tests treated high-powered money as exogenous. However, they admitted the possibility "that central bank behavior, in attempting to attenuate interest rate changes, will result in a correlation between high-powered money and interest rates and hence give rise to the Gibson Paradox".

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