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A NOTE ON ENVIRONMENTAL RISK AND THE RATE OF DISCOUNT

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ABSTRACT

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This paper examines the use of risk-adjusted discount rates to evaluate future environmental risks. It is determined that the risk-adjusted discount rate should be lower--not higher--than the risk-free rate if evaluation of future environmental risks is to point toward optimality.

A NOTE ON ENVIRONMENTAL RISK AND THE RATE OF DISCOUNT[1]

INTRODUCTION

In their celebrated paper "Uncertainty and the Evaluation of Public Investment Decisions," Kenneth Arrow and Robert Lind[1] showed that uncertainty in public investments could be pooled across individuals, thus reducing the risk for a single individual to zero. One result was that no risk premium would be utilized in discounting the time stream of benefits and costs. In a later paper, Anthony Fisher[3] narrowed the apparent applicability of the Arrow-Lind Theorem by demonstrating that pure public risk, as evidenced in the pure public bad of fluorocarbon depletion of the ozone layer, could not be pooled across individuals because one individual's assumption of risk does not reduce the risk shouldered by others. It immediately follows from Fisher's analysis that risk premiums should be assigned in the evaluation of particular environmental risks. The present work is an extension of the Fisher paper, and examines the unresolved issue as to what sign should be attached to environmental risk premiums. A surprising conclusion is obtained: environmental risk premiums are negative, and the risk-adjusted discount rate used to evaluate environmental risks is lower than the risk-free rate, if analysis is to point toward optimality.

Our surprising result has important policy implications. If we increase discount rates to adjust for risk (as is the convention) in evaluating future environmental costs, those costs are assigned a lesser weight in a present value cost benefit analysis. If, however, we risk-adjust discount rates downward, future environmental costs attain greater importance in a present value cost benefit analysis.

NEUMANN-MORGENSTERN UTILITY, RISK AVERSION AND NEGATIVE RISKS

The economics literature has a long tradition in the theoretical analysis of risk.[2] It has been clearly established that risk-averse individuals are willing to: 1) accept a sure payment that is less than the mean of a distribution of expected payments, and 2) make a sure payment that is greater than the actuarially fair amount to avoid a distribution of expected payouts. Although the analysis underlying the second proposition is familiar, it is essential to our argument and we recapitulate it here using Neumann-Morgenstern utility.[3]

Consider an individual with an income endowment of Y facing an uncertain loss of income from a bad[4] whose value has a known distribution, B. The resulting expected income is:

$$E(Y') = Y - E(B) \tag{1}$$

where:

$E(Y')$ is the expected value of income, Y', after the loss, B;

Y is the endowment income;

$E(B)$ is the expected value of the loss, B.

For a risk-neutral individual the expected utility of Y', $E(U(Y'))$ equals the utility of the expected income, $U(E(Y'))$. For a risk-loving individual $E(U(Y')) > U(E(Y'))$. For a risk-averse individual $E(U(Y')) < U(E(Y'))$. We continue analysis on the premise that society is comprised of risk-averse individuals.[5]

Given that $U(E(Y')) > E(U(Y'))$ for a risk-averse individual there exists some sure income, $Y^* < E(Y')$ such that $U(Y^*) = E(U(Y'))$. Algebraic manipulation allows to determine that our risk-averse individual is willing to pay up to $Y-Y^*$ to avoid the expected loss, $E(B)$. Algebra further reveals that:

$$Y - Y^* > Y - E(Y') = E(B) \tag{2}$$

Equation (2) is merely an affirmation that a typical household is willing to purchase insurance at a price in excess of actuarial value to avoid facing uncertain losses. If we were examining a regulation or investment to reduce environmental damage, reduction of risk to the representative household is a benefit and should be taken into account. Similarly if we are examining projects accompanied by uncertain--and expectedly detrimental--environmental consequences, increased risk is a cost to the representative household and optimality requires that it be taken into account.

FUTURE ENVIRONMENTAL RISK AND DISCOUNTING

Using the variables of the previous section, $E_t(B_t)$ is the expected cost of future environmental risk and $Y_t - Y_t^*$ is the certain income loss with equivalent utility. Both are valued in each period, t. The discounted present value of future costs is appropriately evaluated as:

$$DPV = \sum_{t=1}^{T} (1 + r)^{-t} \cdot (Y_t - Y_t^*); \tag{3}$$

where: r is the marginal rate of time preference;[6]

T is the time horizon over which the risk is evaluated.

Alternatively, we can choose some risk-adjusted rate of discount such that:

$$DPV = \sum_{t=1}^{T}(1 + i_t)^{-t} \cdot E_t(B_t); \qquad (4)$$

where: i_t is the risk-adjusted discount rate for period t.

Given $Y_t - Y_t^* > E_t(B_t)$ for all t and a positive r value, algebra yields $i_t < r$ for each period t, demonstrating that the discount rate should be decreased to evaluate future environmental risks, if a risk-adjusted discount rate approach is adopted.[7] The alternative to using a risk-adjusted discount rate is found in equation (3) above.[8]

Other risks may warrant attention in a benefit-cost analysis. Most prominent would be the risk inherent in the costs of government action to reduce the environmental damage. If the government intervenes to reduce B_t to zero for all t, income for each period will be $Y_t - C_t$, where C_t is a random variable describing the cost of government intervention. Just as the risk averse individual is willing to pay more than $E_t(B_t)$ to insure against variance of the environmental risk, so will that individual be willing to pay more than $E_t(C_t)$ to insure against variance in the costs of intervention.

The same logic then applies to each of the two courses of action. $E_t(B_t)$ and $E_t(C_t)$ should be discounted at rates below r to make their discounted present values larger, reflecting the willingness to pay to avoid their variances.[9] The two risk-adjusted rates will not generally be the same.

SOCIAL EVALUATION OF ENVIRONMENTAL RISK

To this point, analysis has been on the basis of a single individual facing an environmental risk. Aggregation across individuals with potentially different degrees of risk aversion to find a social risk-adjusted discount rate bears consideration.[10]

Assuming environmental risk is a pure public bad for each time period t, summation of equation (3) over n individuals yields

$$\text{SOCIAL COST} = \sum_{j=1}^{n} DPV_j = \sum_{j=1}^{n} \sum_{t=1}^{T} (1 + r)^{-t} \cdot \Delta Y_{j,t} \qquad (5)$$

where: $\Delta Y_{j,t} \equiv Y_{j,t} - Y_{j,t}^*$ for individual j in period t.

Given $\Delta Y_{j,t} > E_t(B_t)$ for all j and t, algebra is used to show that for each period there is some social risk-adjusted discount rate $\bar{i}_t < r$ such that:[11]

$$\text{SOCIAL COST} = \sum_{j=1}^{n} DPV_j = n \sum_{t=1}^{T} (1 + \bar{i}_t)^{-t} \; E_t(B_t). \qquad (6)$$

Equation (6) is equivalent to:

$$\text{SOCIAL COST} = \sum_{j=1}^{n} DPV_j = \sum_{j=1}^{n} \sum_{t=1}^{T} (1 + i_{j,t})^{-t} \cdot E_t(B_t); \qquad (7)$$

where: $i_{j,t}$ is the risk adjusted discount rate for individual j in period t. However, given all $t \geq 1$, $n > 1$, and $i_{j,t} > 0$ for all j and t it can be shown that:[12]

$$\bar{i}_t \leq 1/n \sum_{j=1}^{n} i_{j,t}, \quad \text{for each } t=1, \ldots, T. \qquad (8)$$

If any two of the $i_{j,t}$'s are unequal for a given t, the inequality in (8) becomes strict. If we are willing to assume that risk increases exponentially with time for each person, the time subscript can be dropped from the risk-adjusted discount rates, i, in (6), (7) and (8).

Given individuals with differing degrees of risk-aversion, (8) reveals that the social risk-adjusted discount rate for evaluating a pure public bad is less than the average of individual rates, reflecting the higher weighting, given to individuals with greater risk-aversion, in the formation of i_t.[13] This suggests that if differing degrees of risk-aversion toward an pure public environmental risk are exhibited, the benefit-cost analyst is advised not to use an arithmetic average risk-adjusted discount rate to represent the social rate. Alternatives are to determine the arithmetic average maximum willingness to pay for use in (5), or to use a number of representative discount rates in (7).

SUMMARY

This note has shown that the risk-adjusted discount rate applied to future environmental risks should be lower than the risk-free rate if benefit-cost analysis is to point toward optimality. Further, for environmental risks that are pure public bads, aggregation across individuals preserves the relationship between the risk-adjusted discount rate and the risk-free rate. However, simple averaging of individual risk-adjusted discount rates may not yield the social risk-adjusted discount rate.

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5. J. v. Neumann and O. Morgenstern (1953), Theory of Games and Economic Behavior, 3rd Ed., Princeton, New Jersey.

6. M. Olson and M. J. Bailey (1981), "Positive Time Preference," Journal of Political Economy, 89, pp. 1-25.

FOOTNOTES

[1]The author is indebted to James C. Owings for helpful comments.

[2]For example see M. Friedman and L. J. Savage[4] or J. v. Neumann and O. Morgenstern[5].

[3]The use of Neumann-Morgenstern utility is purely for expository purposes.

[4]A bad is an object or disservice that confers negative benefits and has positive disposal costs.

[5]This premise is not without foundation. See M. J. Bailey, M. Olson and P. Wonnacott[2].

[6]M. Olson and M. J. Bailey[6] have refined the definition of time preference into pure time preference and marginal time preference. We use the Olson-Bailey terminology.

[7]Lowering the discount rate from r to i_t is equivalent to adding a negative risk premium to r to obtain i_t. In practice a single risk-adjusted discount rate may be utilized for all periods. However, if the use of a single risk-adjusted rate for all periods is to accurately represent the present discounted value of the cost distribution for each period, it is implied that risk, defined as $(Y_t - Y_t^*)/E_t(B_t)$, increases exponentially with time, t.

[8]In utilizing (3) to obtain $i_t < r$, we implicitly assume that the maximum "insurance premium" that the household is willing to pay to avoid the environmental risk is the appropriate measure of cost. If premiums for insurance policies covering purely private expected losses typically afford the purchasing household a consumer surplus, why should we utilize maximum willingness to pay in obtaining $i_t < r$? Because of the possibility of pooling purely private risk across a sufficiently large population, private risk insurance typically sells for actuarial value plus transactions costs. However, as we stated in the opening paragraph Fisher demonstrated that pure public risk cannot be pooled across individuals. Hence, insurance against pure public risk would not be made available by the private sector for a price equal to actuarial value plus transactions costs. Indeed, even if a firm was able to collect premiums equal to the maximum willingness to pay, the possibility of default would remain, given normal--not extreme--risk aversion.

[9]If the risks can be pooled across individuals, a risk-adjusted discount rate would not be appropriate for evaluating intervention costs.

[10]Whether individuals acting in a perfect market can have differing degrees of risk aversion when evaluating a pure public bad may be controversial. The author has chosen not to enter into that argument here.

[11]For an environmental risk that was not a pure public bad, pooling would obtain $\Delta Y_{j,t} = E_t(B_t)$ and we would obtain $\bar{i}_t = r$. Clearly, our social aggregation results are dependent upon Fisher's narrowing of Arrow-Lind.

[12]For convenience let $u_j \equiv (1 + i_{j,t})$. Because the arithmetic mean is nonstrictly greater than the harmonic mean for $u_j > 1$, we may write $1/n\Sigma(u_j) \geq (1/n\Sigma(u_j)^{-1})^{-1}$ (Note that all summation here is over j). We generalize the harmonic mean as: $g = (1/n\Sigma(u_j)^{-t})^{-1/t}$. The generalized harmonic mean decreases nonstrictly as t increases (i.e., $\partial g/\partial t \leq 0$). Thus, $(1/n\Sigma(u_j)^{-t})^{-1/t} \leq 1/n\Sigma(u_j)$ for $t \geq 1$. Obtaining identical values in (6) and (7) for each period, t, requires that $n(1+\bar{i}_t)^{-t} \cdot E_t(B_t) = \Sigma (1+i_{j,t})^{-t} \quad E_t(B_t)$ for each t. Manipulation yields: $1+\bar{i}_t = (1/n\Sigma(1+i_{j,t})^{-t})^{-1/t}$. Thus $1+\bar{i}_t \leq 1/n\Sigma(1+i_{j,t})$, and $\bar{i}_t \leq 1/n\Sigma i_{j,t}$ for each period t. If any two of the $i_{j,t}$'s are unequal for a given period, t, all of the inequalities for that period become strict.

[13]This does not mean that individuals with greater risk-aversion receive a higher weighting in the benefit-cost analysis. The use of i_t in (6) is equivalent to using the arithmetic mean, $\Delta\bar{Y}_t \equiv 1/n\Sigma\Delta Y_{j,t}$, to replace each $\Delta Y_{j,t}$ in (5).