No. 8002

OFFICIAL RESERVE ASSET CHOICE AND SUBSTITUTION ACCOUNT PROPOSALS

by

Leroy O. Laney

April 1980

Research Paper

FEDERAL RESERVE BANK OF DALLAS

No. 8002

OFFICIAL RESERVE ASSET CHOICE AND SUBSTITUTION ACCOUNT PROPOSALS

by

Leroy O. Laney

April 1980

Weakness in the foreign exchange value of the U.S. dollar in recent experience focused greater attention on the dollar's future as a reserve currency.[1] Reports of permanent diversification into other reserve currencies, even by official entities, increased markedly during 1978 and 1979. Most frequently mentioned alternative currencies were the Deutschemark and the Swiss franc, and some reference to the Japanese yen and other currencies also has been made.[2] There has even been some reported sentiment, primarily from European quarters, for reviving the official monetary role of gold--not necessarily by reinstituting any form

---

1. Although economists long have recognized the potential for greater reserve diversification under a system of flexible exchange rates, some uncertainty seemed to exist as to whether it would actually occur in practice. Some analyses about the time of the advent of managed floating rates were more optimistic, if guardedly so at points, about the dollar's continued role at the center of the world financial system. See for example Cooper [1972 and 1973] and Whitman [1974]. Not surprisingly more concern about such diversification seems to arise when the dollar is under downward pressure in the market. This was true to an extent in 1973 as the dollar's value continued to fall after the second discrete devaluation of the decade. See Salant [1973]. More recent analyses, in addition to generally somewhat more alarmist versions in the popular press (see, for example, Business Week [1979] and U.S. News and World Report [1979]), include Euromoney [1978], Laney [1978], Triffin [1979], and Rose [1979].

2. On the yen see Gregory [1979]. A weaker yen more recently has suppressed some reference to this currency, and it is unclear in any case whether it might achieve a truly international role or be the center of some sort of "Asian currency area."

Among other currencies, it has even been suggested that with the removal of exchange controls the role of the British pound might be revived. See Brown [1979].

For an analysis of the rising role of the primary alternative currency to the dollar, the German mark, see Monthly Report of the Deutsche Bundesbank [1979]. The Bundesbank estimates that in mid-1979 the mark accounted for 11.3 percent of total official foreign exchange reserves excluding West Germany's own, compared to 7.7 percent in 1975.

of the gold standard, but simply by recognizing its potential as a free-asset alternative to currencies in additions to central bank portfolios.[3]

Official reserve asset diversification has reportedly occurred in both national currency markets and the euromarkets, and has been recognized formally as a likely future trend by national and international agencies.[4] For its part, the United States, while drawing attention to such factors as greater depth of U.S. capital markets and other aspects that tend other things equal to underpin a continued major role for the dollar, has stated that it will not attempt to artificially perpetuate the international status of its currency. And inherent in November 1978 U.S. dollar support initiatives was at least some willingness to institute greater symmetry among currencies: by issuing bonds denominated in foreign currencies to finance balance-of-payments deficits, the United States demonstrated a minor inclination to borrow and amass its own foreign currency reserves, rather than rely strictly on short-term central bank swap lines and continued traditional financing of the deficit by increasing dollar-denominated liabilities to foreigners. Spokesmen for potential new reserve currency countries have acquiesced to the point that they recognize at least some enlarged role for them is quite likely, even though generally they remain extremely reluctant to assume significant reserve center

---

3. See, for example, The Wall Street Journal [1980].

4. See, for example, Bank for International Settlements [1979] and Economic Report of the President [1980].

responsibilities because of the effects that it would have and the constraints that it would put on their relatively more open economies.[5]

Against this backdrop, more serious and urgent discussion has been revived for instituting a larger role for the International Monetary Fund's Special Drawing Right (SDR) as the primary store of international liquidity.[6] In the 1960's, the original impetus toward making the SDR the world monetary system's principal reserve asset came from concern about adequate international liquidity in a gold exchange standard world. The floating of exchange rates generally in the early 1970's made this concern rather obsolete, and resulted in the redefinition of the SDR in 1974 as a basket of sixteen currencies rather than continuing its fixed link via the U.S. dollar to gold.[7] This method of defining the SDR's value means that

---

5. In general, Switzerland and Japan have been less adamant in their opposition than West Germany, but this may be largely because of the mark's apparent position as the front-running alternative currency to the dollar. For one statement of the Swiss attitude see Leutwiler [1980]. Hayami [1979] and Bundesbank [1979, op. cit.] provide some insight into Japanese and German attitudes, respectively.

6. For discussions of the evolution of the SDR and measures taken to enhance its role, see the IMF Survey, various issues. More negative recent assessments of the SDR's role are found in Chrystal [1978], Haberler [1979], Murphy [1979], and Sacchetti [1979].

7. Originally one SDR was defined as equal to one U.S. dollar when the U.S. unit was still linked to gold at $35 per ounce. The SDR/gold link was maintained after the dollar price of gold changed in 1971, but the variation in the value of the SDR vis-a-vis individual currencies in a floating rate world ultimately forced its redefinition as a weighted basket of national currency units. Weights in the basket were originally determined by countries' shares in world exports in the 1968-1972 period, but the U.S. dollar's share was modified somewhat to account for nontrade aspects of the currency's importance. One redefinition occurred in July 1978 when the Saudi Arabian riyal and the Iranian rial replaced the Danish krone and the South African rand in the basket. The basket, under existing IMF plans, is subject to further revision at five-year intervals. The next such review of weights in the basket is scheduled for 1983. At that time it is currently intended that modification of weights to reflect a currency's financial importance will be extended to nondollar currencies as well, based on the amount of a country's currency held in other members' reserves. See Polak (1979) for discussion.

it is likely to be more stable in terms of a chosen national currency unit than some other single national currency is vis-a-vis that unit, since exchange rate risk is diversified among the various currencies in the basket.[8] If official monetary agencies were to hold a major portion of their international reserves in SDR's, it is argued, valuation risk on these reserves would tend to be less than if a single currency such as the dollar were held instead.

Even though exchange rate risk might be reduced by SDR consolidation of the "dollar overhang," however, it would still not be minimized in a portfolio context. This paper will demonstrate that, if participation in the IMF's proposed substitution account is not compulsory, some incentives for reserve asset diversification may still exist. Moreover, the SDR proportions in optimally diversified portfolios might be smaller than many currently envision. It is not argued here that an evolving multiple currency reserve asset system is desirable necessarily, even though its drawbacks ultimately may be less than some have contended. But it is argued that even if other obstacles to institution of the substitution account are overcome, it still may not accomplish, as long as the SDR is defined as it is presently, the intended goal of eliminating potentially destablizing portfolio shifts among various reserve assets.

Substitution Account Proposals

It is currently proposed that the SDR substitution account would take in as assets the existing U.S. dollar foreign exchange reserves of

---

8. As pointed out by Chrystal [op. cit., p. 21], however, the SDR's value need not necessarily be more stable than all currencies included, or not included, in the basket.

participating central banks, issuing in their place SDR-denominated claims on the account. The account is intended to affect only the composition of total international reserves, not their level. The incentive for central bank participation in the account, which would be voluntary, is presumed to derive largely from the above mentioned lower volatility of the SDR, so that overall the risk on international reserves inherent in a managed floating exchange rate world would be less.

Table 1 includes the units of each individual currency in the SDR's present composition, along with percentage weights prevailing on one given recent date. Although the SDR floats separately from all national currencies because of this definition, it would not be strictly correct to say that it floats independently from them. The more important weight for the U.S. dollar in the definition, for example, means that there will be some tendency for the SDR to fluctuate with the dollar vis-a-vis any third unit, even though in terms of that third unit the fluctuations of the SDR will tend to be less volatile because of other currencies in the basket.

While risk may be reduced, however, there is a negative aspect with respect to the SDR's yield: currently it is less than that attainable on investments in currencies. Table 1 also indicated weights and interest rates from which this yield is determined. Only some fraction of the computed weighted market rate is paid on existing SDR balances.[9] Figure 1 compares the weighted market rate with that actually paid on existing SDR

---

9. This fraction was 60 percent from mid-1974 through 1978. At the beginning of 1979 it was changed to 72 percent on creditor SDR positions and 80 percent on debtor positions.

Table 1

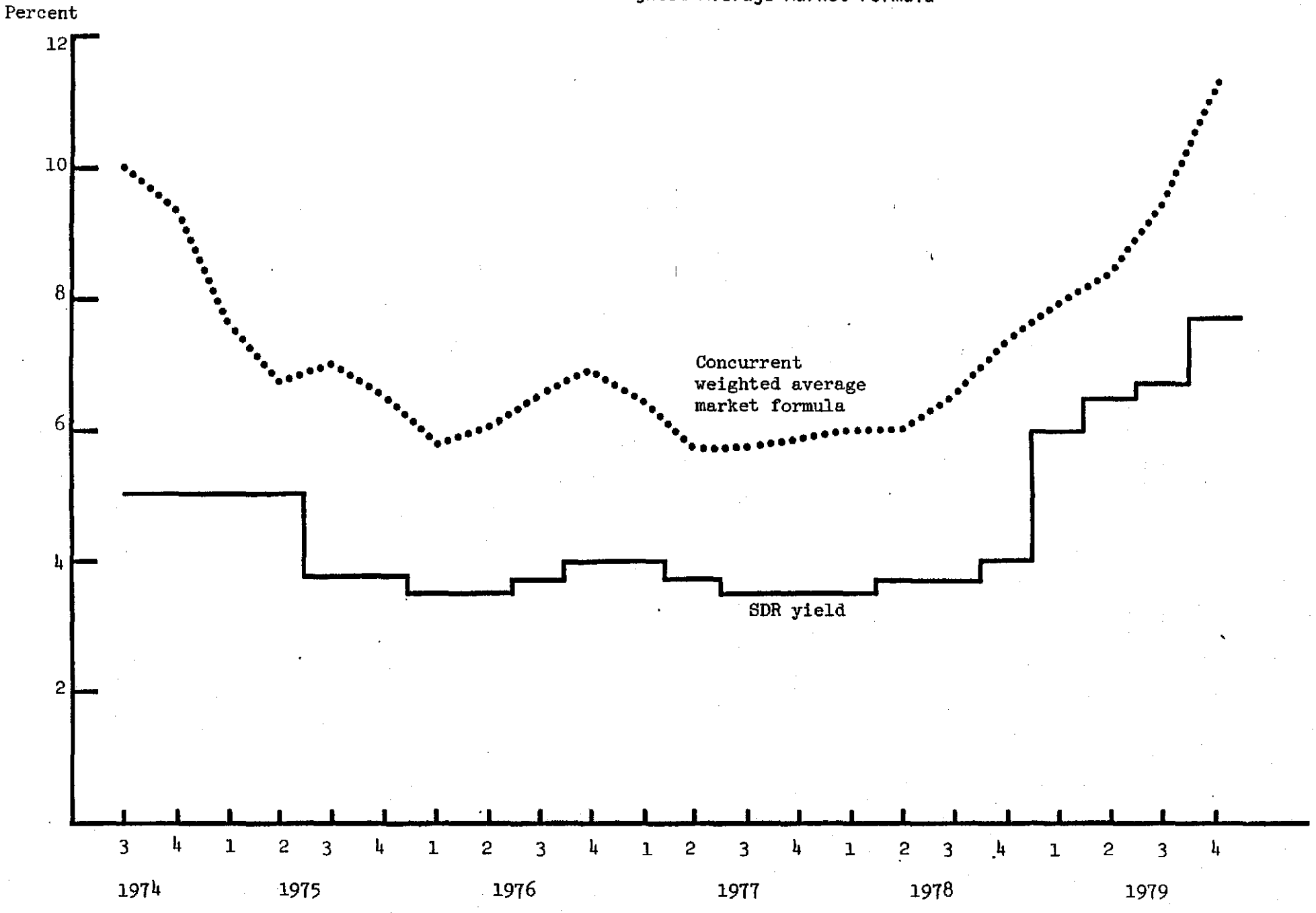
Special Drawing Right Capital Valuation Weights
and Yield Formula Weights

	Currency units	Percentage Weights at 1/31/80 exchange rates	Interest rate percentage weights 1/
U.S. dollar	.400	30.4	49
German mark	.320	14.0	18
British pound	.050	8.6	11
French franc	.420	7.9	11
Japanese yen	21.000	6.7	11
Canadian dollar	.070	4.6	
Dutch guilder	.140	5.5	
Italian lira	52.000	4.9	
Belgian franc	1.600	4.3	
Saudi Arabian riyal	.130	2.9	
Iranian rial	1.700	1.8	
Swedish krona	.110	2.0	
Australian dollar	.017	1.4	
Austrian schilling	.280	1.7	
Norwegian krone	.100	1.6	
Spanish peseta	1.500	1.7	
		100.0	100

1/ Interest rate weights are applied to the 3-month Treasury bill rate in the U.S. and the U.K., 3-month interbank deposit rates in West Germany and France, and the unconditional call money rate in Japan.

Figure 1

SDR Yield and Concurrent Weighted Average Market Formula



holdings. Substitution account claims may not be exactly equivalent to the presently existing SDR assets, and the interest yield paid on them would not necessarily be the same. But one overall goal is to enhance the role of SDR in the monetary system, and this would not be achieved by having the existing balances and substitution account claims considered as two separate assets, so that similarity of yield and basic definitional characteristics is likely.

If the substitution account proposal is to be successful, several hurdles must be overcome. The basic issues are those concerning the liquidity of the SDR, the solvency of the substitution account itself, and the fundamental desirability of the SDR as a reserve asset in a portfolio context.[10]

If the SDR is to compete with national currencies, especially one such as the U.S. dollar in which money markets are highly developed, then its liquidity obviously is important. But since the SDR is not held and traded in private markets, it cannot be used as an intervention medium by central banks. Some currency balances for intervention would be required then in addition to the SDR and this automatically precludes the existence of a single asset reserve system. With at least one and perhaps several reserve currencies held also, the tendency toward a multiple currency system might not be completely eliminated. Ultimately, extensive private use of the SDR might be possible, but the record to date does not cause one to be optimistic on this. And private markets are quite capable of

---

10. For discussion of these issues see Sobol [1979] and Morgan Guaranty Bank [1979a].

inventing their own currency baskets when the demand for them arises, tailored to specific needs.[11]

The solvency issue arises from the substitution account's balance sheet. If the account is sponsored by the IMF but is not backed by any of the Fund's own resources, then dollar assets must be balanced against SDR-denominated liabilities. If the interest yield on U.S. Treasury obligations in which the account's assets are held is less than the yield the account commits itself to pay on its liabilities, or if the dollar's own exchange value depreciates relative to the SDR, then solvency of the account could be questioned. It may be politically unrealistic to expect the United States to consent to guaranteeing the account's solvency. This could entail the controversial payment of higher interest rates by the U.S. government to the account than to other holders of government debt, or alternatively it might simply dictate higher U.S. interest rates overall. But the latter has elements of allowing the international constraint to dictate U.S. monetary policy, and while this constraint has in fact been an important one in recent experience, part of the U.S. support for the account may derive from the hope that the constraint can be relaxed. Direct U.S. exchange rate guarantees on the account's dollar assets are also controversial, since ultimately they entail having the U.S. taxpayer assume the exchange rate risk of foreign central banks. Basically, if the U.S were going to take such steps to ensure the yield and exchange value of

---

11. Several such private sector currency baskets that have been used since the advent of managed floating are discussed in see Aschheim and Park [1976]. But interestingly none of these, however imaginative, has really caught on either.

its foreign dollar liabilities, it could do so without invoking IMF substitution account auspicies at all.

One recently proposed attempt to overcome the solvency problem involves backing the account's SDR-denominated liabilities with some portion of the IMF's gold stock. Since this gold is owned ultimately by IMF members, their approval of this plan is required, however, and there may be some opposition. Support is more likely from countries that do not wish to see their currencies used as reserves than from some less-developed countries that previously have benefited from the sale of IMF gold. If the gold backing plan is approved, proponents envision that the problem of exchange rate guarantees will be overcome, and it may enable the payment of a more competitive yield on substitution account claims.

The issue of SDR desirability relative to other reserve assets is partially linked to the solvency issue by the determination of the yield to be paid on the SDR-denominated claims. But in addition to relative yield aspects of SDR desirability and the previously noted aspect of the volatility or risk of the SDR relative to other reserve asset choices, the typical relationship of all potential reserve assets to each also can be important.

The following analysis does not address the liquidity or solvency problems directly, even though they both relate to the desirability

question.[12] It is the purpose of this paper to approach the issue of SDR appeal to central bank investors from the standpoint of rudimentary portfolio analysis, assuming voluntary substitution account participation and that the current definition of the SDR as a basket of currencies remains intact for substitution account claims.

Reserve Asset Choice in a Portfolio Context

It seems quite appropriate to analyse the problem of official reserve asset choice in a world of generalized floating exchange rates using a standard portfolio framework.[13,14] The essential aspect of asset diversification to minimize total portfolio risk for any given expected portfolio return involves the correlation among individual assets. In this context the standard two-asset portfolio problem is to minimize portfolio variance,

---

12. In addition to the relationship of desirability to the solvency question noted above, liquidity also is obviously related to desirability via the imputed risk associated with holding various assets.

13. Earlier literature on the official reserve asset choice decision under the gold exchange standard focused more on why countries held their reserves in gold versus foreign exchange generally. For analyses of reserve choice under the gold exchange standard, see Kenen [1963], Greene [1968], Hagemann [1969], Officer and Willett [1969], and Makin [1971 and 1972a]. See also Makin [1972b] on coexistence of SDRs and a reserve currency under fixed parities, and Dreyer [1977] for discussion of an SDR exchange standard. An empirical investigation of central bank reserve currency preferences under managed floating is Heller and Knight [1978].

This portfolio approach under managed floating also may be seen to derive some theoretical underpinning from the "asset market" approach to exchange rate determination that has gained acceptance in recent years. See, for example, Frenkel [1976] and Bilson [1979] among many others.

14. Typical textbook portfolio expositions are Sharpe [1970] and Fama [1976].

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + (2x_1x_2\sigma_1\sigma_2)\rho_{12} \ , \qquad (1)$$

subject to

$$\mu_p = x_1\mu_1 + x_2\ \mu_2, \qquad (2)$$

$$x_1 + x_2 = 1, \text{ and} \qquad (3)$$

$$x_1, x_2 \geq 0, \qquad (4)$$

where:

σ_p^2 = total portfolio variance

μ_p = total portfolio return

σ_1 = standard deviation of asset 1

σ_2 = standard deviation of asset 2

μ_1 = return on asset 1

μ_2 = return on asset 2

x_1 = proportion of portfolio held in asset 1

x_2 = proportion of portfolio held in asset 2

ρ_{12} = correlation between assets 1 and 2; $(-1 \leq \rho_{12} + 1)$.

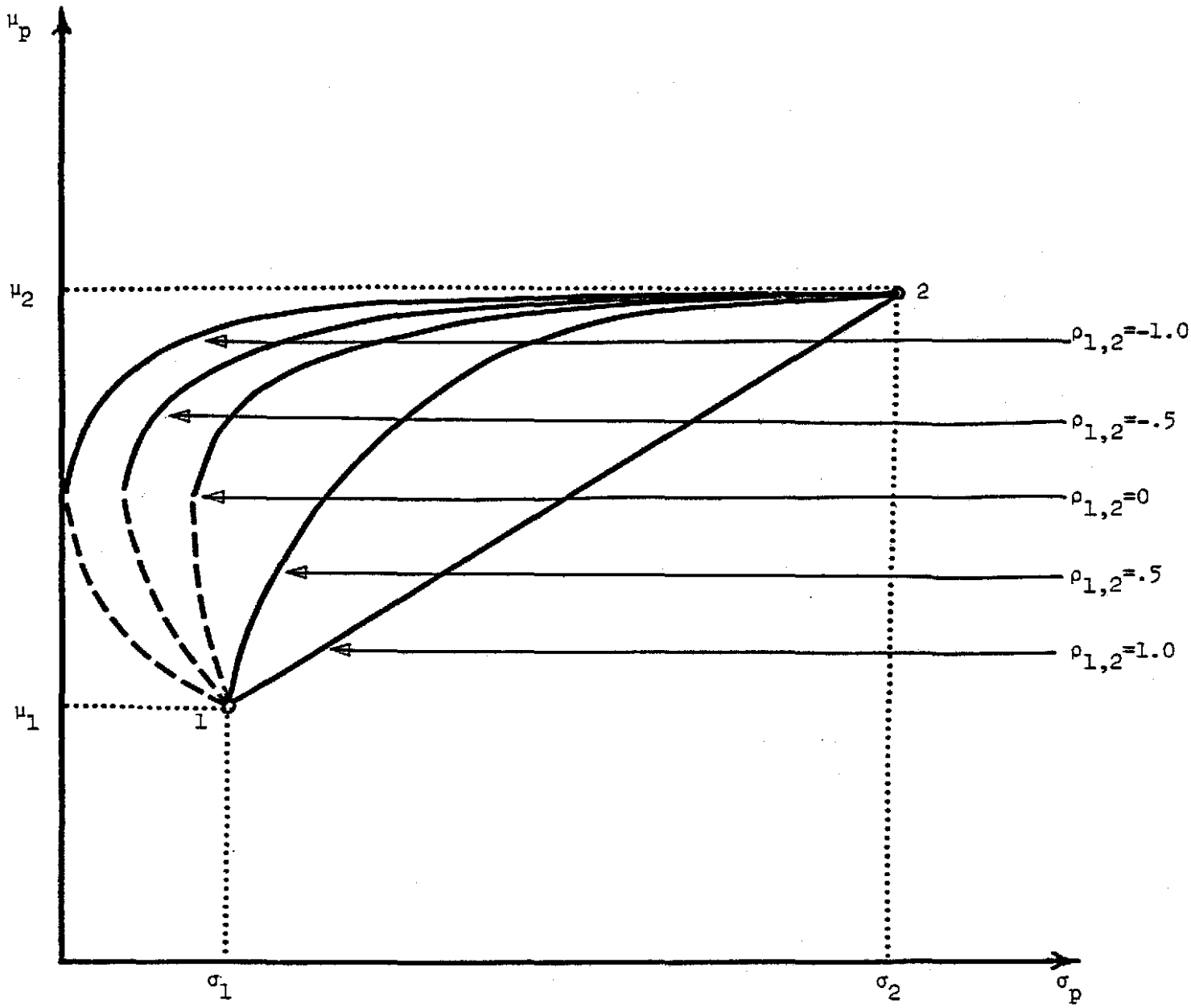
Constraint (2) specifies some level of desired portfolio return, (3) insures a fully invested portfolio, and (4) limits optimal x's to assets, or long positions only.

Given other parameters, σ_p^2 will be reduced as ρ_{12} approaches minus one. Figure 2 illustrates four possibilities for ρ_{12} in μ_p - σ_p space. If $\rho_{12} = 1.0$, equation (1) reduces to

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_1\sigma_2 = (x_1\sigma_1 + x_2\sigma_2)^2 \ , \qquad (5)$$

Figure 2

Reserve Choice Between Two Assets



or $$\sigma_p = x_1\sigma_1 + x_2\sigma_2. \qquad (6)$$

Expression (6) is linear, and assets 1 and 2 behave as one asset; $\sigma_p = \sigma_1$ if only asset 1 is held and $\sigma_p = \sigma_2$ if only asset 2 is held. There is no gain from diversification.

At the other extreme, if $\rho_{12} = -1.0$, then equation (1) reduces to

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 - 2x_1x_2\sigma_1\sigma_2 = (x_1\sigma_1 - x_2\sigma_2)^2 \qquad (7)$$

or $$\sigma_p = x_1\sigma_1 - x_2\sigma_2. \qquad (8)$$

For some combination of assets 1 and 2 in expression (8) $\sigma_p = 0$, and portfolio risk is completely eliminated. Only the upper portion of the curve in the figure comprises the "efficient" border, however, since along the lower portion a higher portfolio return is possible for the same risk. The efficient portfolio frontier is also illustrated in the figure for other values of ρ_{12}.

If assets 1 and 2 in this simple example were alternative international reserve assets, then the correlation between them obviously would be important in reducing overall risk on a country's total international reserves. And it is reasonable to assume at least two such assets if a secondary market in SDR claims is not developed and at least some currency balances must be held for intervention purposes. It is also not unrealistic to say that the SDR, as presently constituted, is roughly equivalent to asset 1 as a low-risk, low-return asset, and that the dollar

may be represented by asset 2 as a higher-risk, higher-return asset.[15] If correlation between variations in the SDR's value and that of the dollar were low, then the more curvilinear efficient frontiers in Figure 2 would be relevant. But the dollar might tend to dominate efficient portfolios in this case since that higher return portion of the frontier closer to the dollar is the efficient portion. If the SDR basket continues to give a major weight to the U.S. currency in its composition, however, then correlation between their movements is likely to be higher. The SDR would dominate efficient portfolios if lower portfolio returns were acceptable, but less would be gained in terms of risk reduction by diversification between the two.

If the correlation between movements in the SDR and the dollar is in fact closer to +1, there is some incentive to add a third asset to the portfolio, or to substitute that asset for either the dollar or the SDR, if risk reduction on the total reserve portfolio is an important goal. Figure 3 introduces such an asset. Again seeking real world analogies, an even higher-risk, higher-return asset 3 might be representative of an alternative reserve currency such as the German mark.[16] If correlations among these three were roughly as depicted in 3(a), then the SDR and the mark would dominate efficient portfolios in the range of relevant returns; the dollar would be inferior and excluded from the optimal asset mix. But the dollar would be included along the overall efficient frontier if either

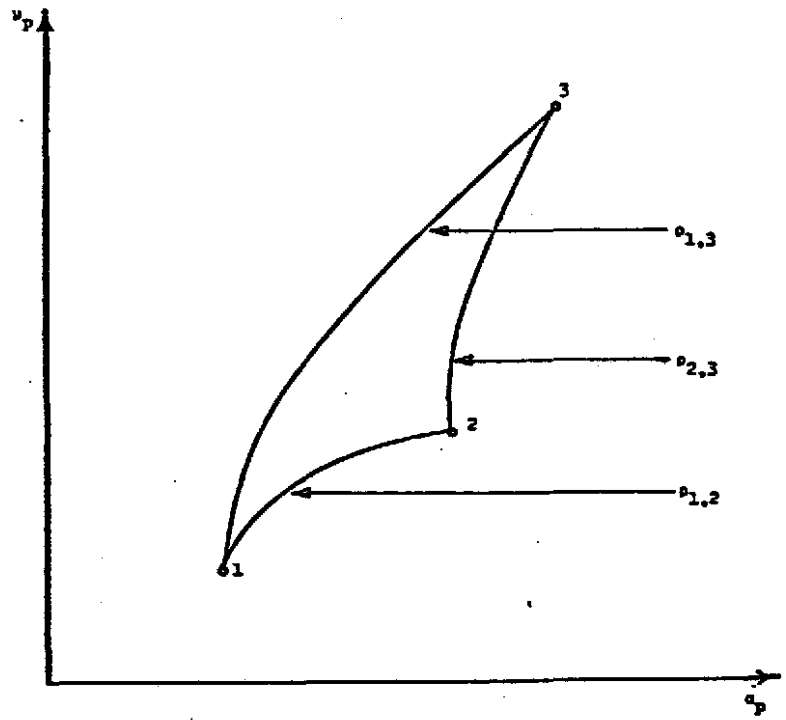
---

15. This depends upon the unit by which risk and return are measured, of course, but the following empirical section lends some support to these assumptions.

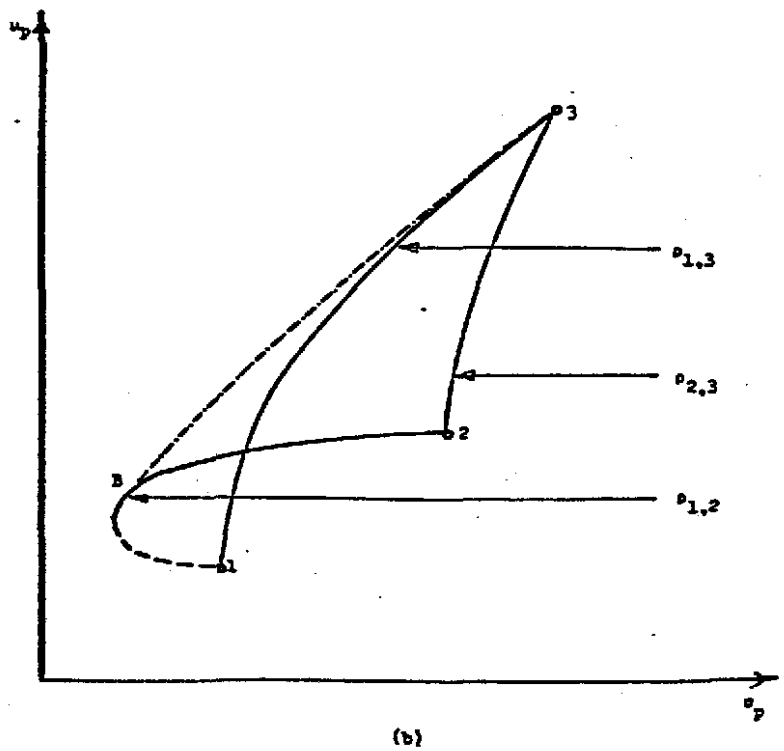
16. Note (15) applies here also.

Figure 3

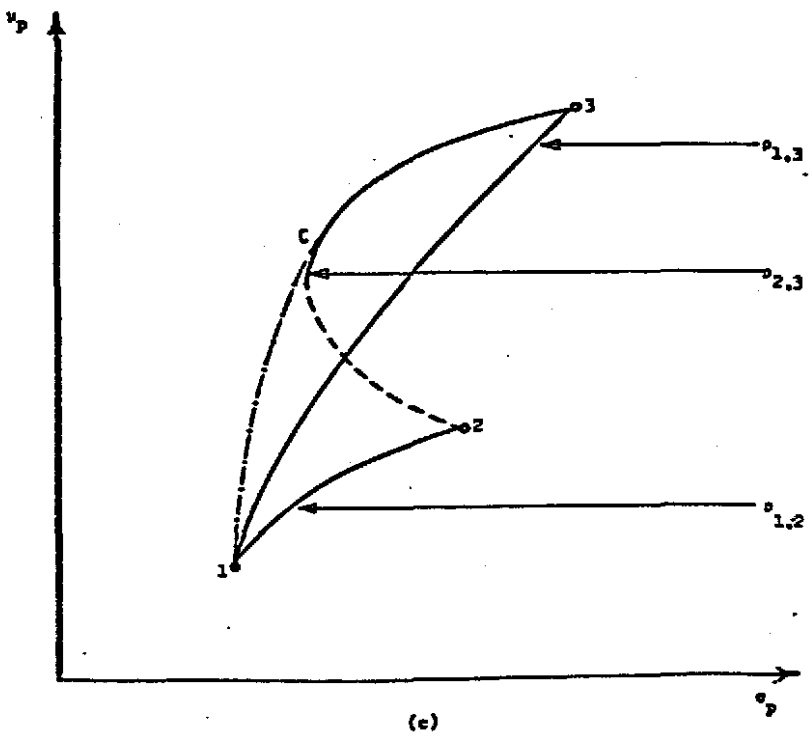
Reserve Choice Among Three Assets



(a)



(b)



(c)

3(b) or 3(c) were descriptive of asset 2 correlations with assets 1 or 3. In 3(b), some fixed combination of assets 1 and 2 that applies at point B would combine in varying proportions with asset 3 along the curve from B to 3; from B to 1 assets 1 and 2 only would dominate. In 3(c), a fixed combination of assets 2 and 3 at point C would combine in varying proportions with asset 1 along the curve from C to 1; from C to 3 only assets 2 and 3 would be included in efficient portfolios.

If assets 1, 2, and 3 were in fact representative respectively of the SDR, the U.S. dollar, and the German mark as potential reserve asset choices here, it is likely empirically that 3(a) would be most relevant. Situation 3(b) is not likely because of the dollar's weight in the SDR basket of currencies, and 3(c) is similarly unlikely because if the SDR and the dollar are highly correlated bilaterally, then their respective movements vis-a-vis any third asset are also likely to be similar.

This analysis might be developed further, perhaps including other potential reserve assets, but it is clear how the optimal mix of assets depends upon returns and risks associated with each, their correlations with each other, and the overall return required on the total reserve portfolio. Since different countries logically have different units in which these parameters are measured, this aspect also can be crucial. It is obvious, however, that there is nothing that would necessarily dictate that the SDR-denominated asset would dominate the optimal asset mix, as is sometimes assumed in pro-substitution account arguments.

The optimal asset mix is at bottom an empirical question that must be answered by each individual country. The problem is a difficult one in practice because the measurement of expected return and risk on

various assets is ultimately subjective. And the choice of a yardstick against which to gauge them is also ambiguous. The value of domestic currency might be chosen as the relevant unit, but perhaps the price of some exported or imported commodity, or some price index of exports, imports, or world inflation generally would be judged more appropriate.

The following section is an empirical demonstration of these aspects of reserve asset portfolio choice. Several cavaets are required before turning to the results, however.

First, the portfolio approach taken here is chosen only to illustrate a particular aspect of the official reserve asset mix problem. There are a number of reasons why a given monetary agency might not be able to, or might not wish to, achieve the asset mix that is indicated as optimal in the following results, even if measures of relevant parameters were accepted. Central banks do not manage their reserve portfolios in a fashion that the private sector might find optimal. Their basic exchange market intervention function, for example, can dictate the acquisition of a currency even if it is not a desirable addition to their portfolio. Also, constraints on portfolio proportions can be imposed by national controls or binding international agreements.[17] And a number of factors that can influence their international reserve portfolio are not independent of their own actions, as would be the case for many private entities.

Second, in addition to the fact that the measurement unit used here for risk and return might not be that chosen by an individual country,

---

17. Participants in the European joint float, for example, traditionally have restricted their holdings of other members' currencies to negligible amounts.

the relevant parameters are calculated only from past data. The asset mix that is found to be optimal here says little in a prescriptive sense about what the optimal mix should be in the future. It is only a static portfolio analysis of what the optimal mix would be if all assumptions were valid and if calculated parameters were truly descriptive. In spite of these cavaets, however, the empirical results do illustrate intended points.

An Empirical Illustration of Official Reserve Asset Choice

Using a standard mean-variance portfolio selection technique, several countries are analysed in this section for a hypothetical optimal reserve asset mix. The countries examined, chosen on the basis of the absolute size of their foreign exchange reserves, were West Germany, Japan, Switzerland, France, Italy, the United Kingdom, and Saudi Arabia. These were the countries that had foreign exchange reserves valued in excess of 10 billion SDRs at the end of 1979. All except Saudi Arabia are industrialized nations, and the currencies of some of them are targets of greater reserve asset diversification. As such, these countries especially may not be able to diversify along the lines indicated as optimal in the following outcomes, since by intervening to stem the rise in their currencies brought about by the diversification of others, they are forced to acquire unwanted U.S. dollars. But it should be underlined that these countries are analysed here not as potential reserve centers, with the constraints on their reserve composition that go with this, but simply as holders of large quantities of reserves. The question to be answered here is more simply: What would these countries do to minimize risk on their reserves if no such constraints applied? From a practical point of view,

Saudi Arabia may be more interesting, since it is typical of a country with the incentive and the relative ability to actually accomplish the desired diversification over time.

For the six industrialized countries, the domestic monetary unit is assumed to be the riskless numeraire against which foreign asset return and risk are measured. Quarterly annualized percentage rates of change, in domestic currency per unit of each potential international reserve asset, were taken from 1975:1, shortly after the SDR's redefinition as a basket of currencies, through 1979:4. Possible international reserve assets considered were SDR substitution account claims, assuming that they would be defined in terms of the standard basket as existing SDR's are, the U.S. dollar, the German mark, the Swiss franc, the Japanese yen, and gold. In the cases of Germany, Switzerland, and Japan, their own currencies naturally were excluded from the international reserve portfolio. The currencies include the current and historically most important reserve unit, the dollar, as well as all of those that have been mentioned seriously as major candidates for diversification in recent experience, in spite of the fact that obstacles to these currencies achieving an important reserve role may exist. To complete the list, gold is also included, because the metal continues to have monetary status in the eyes of some in spite of efforts during the past decade to demonetize it. But it is not included here as any international standard of value, nor as the unit to which any major currency reserve unit is fixed as the dollar was under the Bretton Woods system, but instead simply as an alternative official reserve asset valued at market prices.

To the percent changes in exchange rates were added interest yields on each of the currencies and on the SDR-denominated unit. Three-month interest rates in national markets were taken as an approximation of yield for the currencies, and the SDR's yield was approximated in alternative calculations by taking first 80 percent and then 100 percent of the market formula.[18] No such income yield was included for gold.

For each country j and asset i, then, with t = 1975:1 to 1979:4, returns were computed as

$$R^j_{i,t} = \left[\left(\frac{p^j_{i,t}}{p^j_{i,t-1}} \right)^4 - 1 \right] \cdot 100 + r^j_{i,t} \qquad (9)$$

where p^i_j is the domestic currency price of the international reserve asset i and r^j_i is the approximated interest on that asset. Means and standard deviations of the resulting series were then taken as representative of

---

18. Since credit balances in existing SDR's at this writing pay 72 percent of the weighted average of market rates computed as in Table 1, while debit balances are charged 80 percent of the weighted average rate, by assuming an 80 percent rate to be paid on substitution account credit claims the outcomes here may be biased somewhat toward favoring the SDR. But one goal of substitution account proponents has been to raise the yield on the SDR in order to make it more attractive to potential holders, even to the point of paying 100 percent of the market formula. Alternative outcomes are accessed for this reason which do assume even the 100 percent payments. As discussed earlier, the actual payment of such higher yields depends crucially on successful resolution of the account's solvency problem.

return and risk associated with each asset,[19] and covariations $\rho_{ij}\sigma_i\sigma_j$ measure interrelationships among assets.

While the remaining country analysed here, Saudi Arabia, may be more relevant for its comparative ability to diversify efficiently in a portfolio context over the long run, it is less clear what standard to choose in measuring portfolio risk and return. For a country with developed capital markets in its currency, and whose central monetary authorities have a significant domestic component to their own portfolio, the choice of the domestic currency as a measuring unit is reasonable. But since many less-developed countries that have a large amount of diversifiable international reserves, for which Saudi Arabia stands here as an illustration, do not have such domestic markets nor a currency that is traded on any significant basis internationally, some other numeraire is desirable. In this example, the export price of crude petroleum was chosen. In no sense does this mean by analogy that oil is equivalent to the Saudi Arabian currency, but it is reasonable to believe that the country would wish to minimize risk and/or maximize return on investments it has acquired as a result of selling oil to the world in terms of that commodity. If the country's return on its reserves in any given period is less than the value of the commodity oil, then it would have been more

---

19. Use of standard deviation or variance to gauge foreign exchange risk accurately is dangerous, since return distributions tend like those of some other assets to be highly leptokurtic, but it may serve in this illustrative example. Conclusions similar to those here would likely emerge using other measures of risk as long as variability rankings remain the same. Westerfield [1977] finds standard deviation to be a misleading measure of exchange rate variability, but does find rankings among currencies using standard deviations to be the same as those using two other measures of variability under flexible rates of the 1970's.

rational economic behavior to leave the oil in the ground. Other choices suggest that themselves, such as some index of the country's imports, or perhaps some measure of world inflation, might give similar results to those reported here, but they were not investigated in computations to follow. All such calculations in this section, it may be recalled, can only serve as an example of how diversification might work once these and other assumptions are settled in the minds of those making the portfolio decision. Except for the choice of the numeraire unit, all procedures were the same for Saudi Arabia as for the other countries.

Means and standard deviations for each asset, across countries analysed, are given in Table 2, as well as correlation matrices for returns over the chosen time frame. The mean return column in the table highlights how choice of the risk-return measuring unit can be important. Average returns in all assets for countries with weaker domestic currencies over the period, for example Italy or the United Kingdom, are higher than those with a numeraire that itself was rising generally against included international assets, for example West Germany, Switzerland and Saudi Arabia. When the SDR yield is calculated at 80 percent of the market formula it is usually the lowest return asset, but its return is always fairly close to that for the U.S. dollar. When SDR yield is calculated at 100 percent of the formula, its return is slightly higher than the dollar in every case. (While mean returns for the SDR asset are changed somewhat by alternative assumptions along these lines, its standard deviations and correlations with other assets are not changed very much at all.) Low return assets are sometimes the less risky ones as measured by standard deviations, but ranking assets by return and risk for each country always

Table 2

Means, Standard Deviations, and Correlation Matrices of Percentage Returns for Five Potential Reserve Assets Across Seven Countries (1975:1-1979:4)*

	Mean μ	Std. dev. σ	Correlation Matrix					
			SDR	$	DM	SF	Yen	Gold
I. West Germany								
(a) SDR	.46(1.95)	7.91(7.90)	1.00		-			
(b) $	1.43	13.90	.93(.93)	1.00	-			
(c) SF	6.46	15.48	.09(.07)	-.06	-	1.00		
(d) Yen	2.91	21.17	.49(.46)	.30	-	.59	1.00	
(e) Gold	17.91	46.59	-.10(-.03)	-.02	-	-.14	-.35	1.00
II. Japan								
(a) SDR	4.09(5.58)	18.23(18.64)	1.00				-	
(b) $	4.81	20.90	.95(.95)	1.00			-	
(c) DM	9.58	22.94	.93(.93)	.79	1.00		-	
(d) SF	9.69	18.91	.65(.65)	.49	.77	1.00	-	
(e) Gold	29.10	79.84	.77(.79)	.71	.79	.60	-	1.00
III. Switzerland								
(a) SDR	-2.06(-.57)	13.45(13.58)	1.00			-		
(b) $	-.92	17.51	.94(.95)	1.00		-		
(c) DM	2.24	13.42	.82(.83)	.66	1.00	-		
(d) Yen	-1.12	16.74	.41(.39)	.38	.08	-	1.00	
(e) Gold	16.35	53.94	.37(.41)	.34	.57	-	-.28	1.00
IV. France								
(a) SDR	5.24(6.73)	8.60(8.73)	1.00					
(b) $	6.00	13.09	.85(.86)	1.00				
(c) DM	10.43	12.49	.71(.72)	.35	1.00			
(d) SF	12.18	19.14	.39(.38)	.08	.51	1.00		
(e) Yen	7.60	20.17	.42(.38)	.23	.15	.59	1.00	
(f) Gold	24.41	50.11	.02(.09)	.01	.29	-.02	-.36	1.00
V. Italy								
(a) SDR	12.56(14.05)	15.97(15.81)	1.00					
(b) $	13.33	19.76	.93(.93)	1.00				
(c) DM	18.43	20.51	.90(.90)	.72	1.00			
(d) SF	20.72	27.58	.74(.74)	.55	.75	1.00		
(e) Yen	15.45	26.38	.68(.66)	.56	.52	.75	1.00	
(f) Gold	32.90	53.37	.10(.14)	.09	.26	.05	-.22	1.00
VI. United Kingdom								
(a) SDR	9.88(11.37)	15.95(15.92)	1.00					
(b) $	10.85	20.15	.93(.94)	1.00				
(c) DM	15.35	19.51	.87(.87)	.69	1.00			
(d) SF	16.93	22.33	.65(.64)	.48	.70	1.00		
(e) Yen	12.60	25.56	.66(.64)	.56	.45	.68	1.00	
(f) Gold	29.44	59.10	.21(.25)	.19	.40	.08	-.20	1.00
VII. Saudi Arabia								
(a) SDR	-4.34(-2.85)	19.71(19.29)	1.00					
(b) $	-4.37	16.34	.94(.94)	1.00				
(c) DM	.80	24.34	.95(.95)	.83	1.00			
(d) SF	3.33	33.81	.87(.87)	.74	.86	1.00		
(e) Yen	-.05	32.72	.88(.89)	.85	.75	.86	1.00	
(f) Gold	7.76	35.85	.29(.31)	.16	.43	.40	.29	1.00

* Parameters calculated using an SDR yield equal to 80 percent of the market formula. Figures in parentheses are those using 100 percent of the market formula.

presents at least somewhat different orderings. Gold valued at market prices is unsurprisingly always the highest return asset even without any income yield, but it is also the most risky. Gold does demonstrate one other desirable portfolio characteristic in that its correlations with other assets are lower, and correlations between the SDR-denominated asset and the U.S. dollar are almost always higher than any other cross-correlation in the presented matrices. In the case of Saudi Arabia the cross-correlation between the German mark and the SDR asset is very slightly higher than that between the dollar and the SDR, and the mark-SDR correlation is always second highest in other cases. But this is also not surprising since the weight of the German mark in the SDR basket is second to the U.S. dollar.

When risk is measured in terms of domestic currency, the SDR asset is always the most stable. As mentioned previously it is this quality that those in favor of SDR consolidation frequently emphasize in recommending claims denominated in the asset as the primary store of international liquidity. In the case of Saudi Arabia the variation of the dollar is marginally less than that of the SDR in terms of the oil price,[20] but both assets are noticeably more stable than any other potential assets in the portfolio. It becomes obvious from inspection of these parameters, however, that focusing solely on the SDR's stability

---

20. This may derive unintentionally from the fact that oil is priced in U.S. dollars, so that the oil price in terms of any other unit that fluctuates vis-a-vis the dollar will give a series demonstrating at least slightly more variation. But this may be realistic to the extent that oil is a relevant unit of measurement here. The fact that Saudi Arabia and other OPEC nations continue to price and receive payment for their oil in U.S. dollars, in spite of periodic mention that they might switch to the SDR or some other currency basket, lends some support.

characteristic without also considering its return and correlation with other possible reserve assets, is insufficient to determine its desirability.

Using parameters in Table 2, a standard portfolio selection approach such as that developed in the previous section can determine a dominant asset mix for various levels of total portfolio return and risk. The algorithm utilized here[21] minimizes the objective function

$$-\lambda\mu x + x'\Phi x \tag{10}$$

subject to the linear constraints

$$\Psi x \leq z_1 \tag{11}$$

$$\Omega x \geq z_2 \tag{12}$$

$$\Theta x = z_3 \tag{13}$$

$$x \geq 0 \tag{14}$$

where in matrix notation x is the vector of optimal asset proportions, μ is the vector of asset returns, and Φ is the variance-covariance matrix of returns. Constraints (11) and (12) can be specified, by choice of some appropriate matrices Ψ and Ω and vectors z_1 and z_2, to confine various assets to less than or greater than some given proportion. Although these constraints were not imposed for the following empirical outcomes, they could easily be used to characterize various institutional constraints on reserve asset composition. In examples reported here Θ was chosen to be

---

21. This algorithm is an adaptation of program QPF4 available from the Rand Corporation, Santa Monica, California.

simply a unit vector and z_3 was set equal to one, imposing the fully invested portfolio constraint, but this general equality constraint also might be used for certain institutional requirements on the portfolio. Since constraint (14) requires optimal portfolio proportions to be positive, the following outcomes do not consider the extent to which borrowed reserves, or negative portfolio proportions, might be used to reduce overall risk. This is equivalent to focusing only upon the asset side of a central bank balance sheet, but this constraint might be relaxed for some applications. By iteratively choosing various values of λ, equivalent to the slope of the efficient portfolio frontier at some maximum return and minimum risk, the locus of efficient points in μ-σ space can be traced and optimal asset proportions associated with these points can be determined.

Figures 4 through 10 depict the efficient portfolio frontier for the seven countries analysed here, using parameters calculated assuming an SDR yield of 80 percent of the market formula. Various points on the locus are identified as well as points representing return and risk for each reserve asset individually. Presented in the inset to the figures are the dominant asset proportions associated with efficient points identified. In each figure the point a represents the minimum portfolio risk point over the entire range of returns, and points b, c, and d represent risks at higher return levels. The figures also include, where applicable, a point R on the portfolio border, with portfolio return constrained to be equal to the average interest rate on three-month domestic deposits in the relevant country over the chosen time period. This is roughly equivalent to choosing a minimum risk international reserve asset mix that would yield

Figure 4

Efficient Portfolios and Individual Potential Reserve Assets

WEST GERMANY

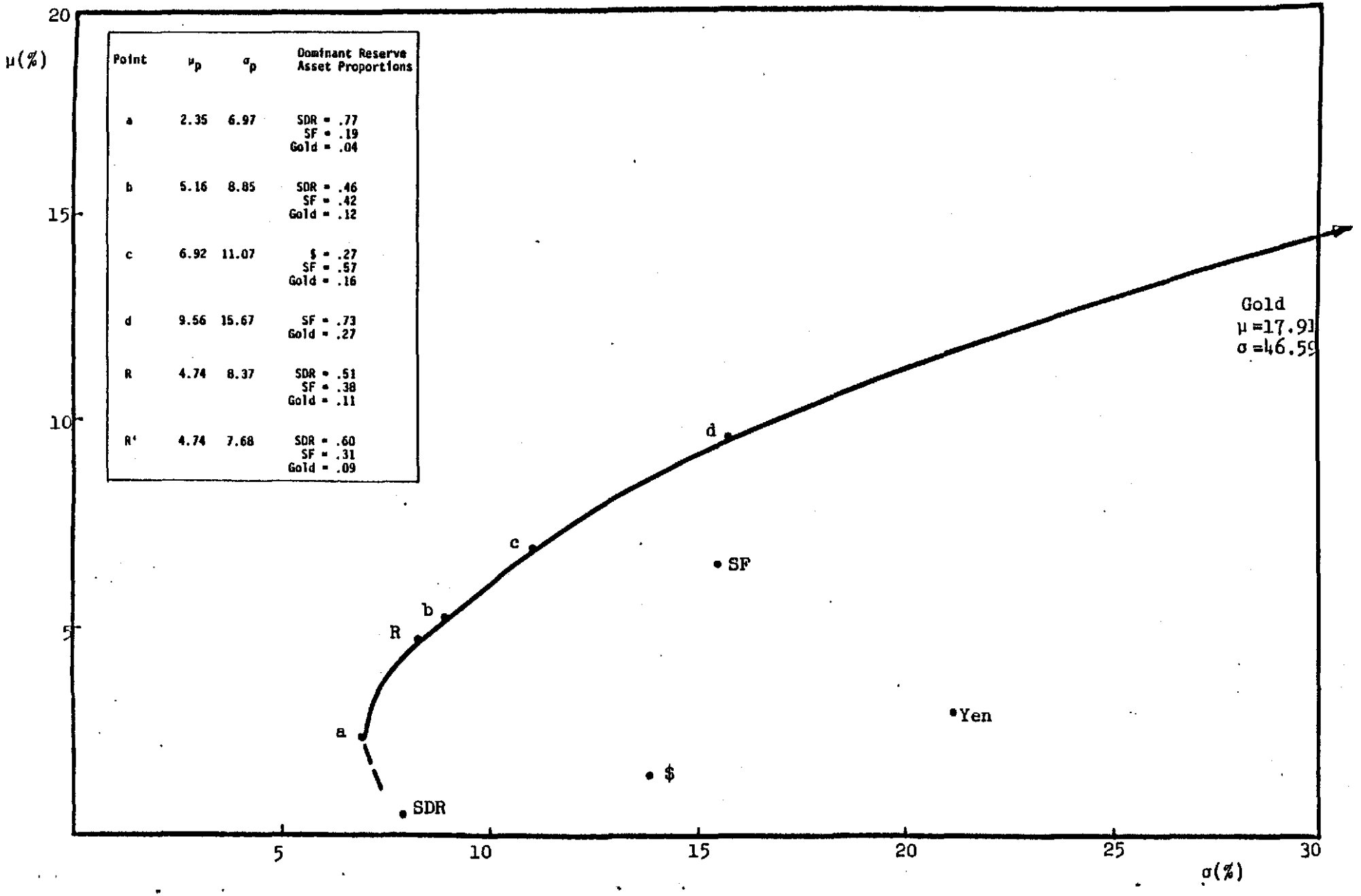


Figure 5

Efficient Portfolios and Individual Potential Reserve Assets

JAPAN

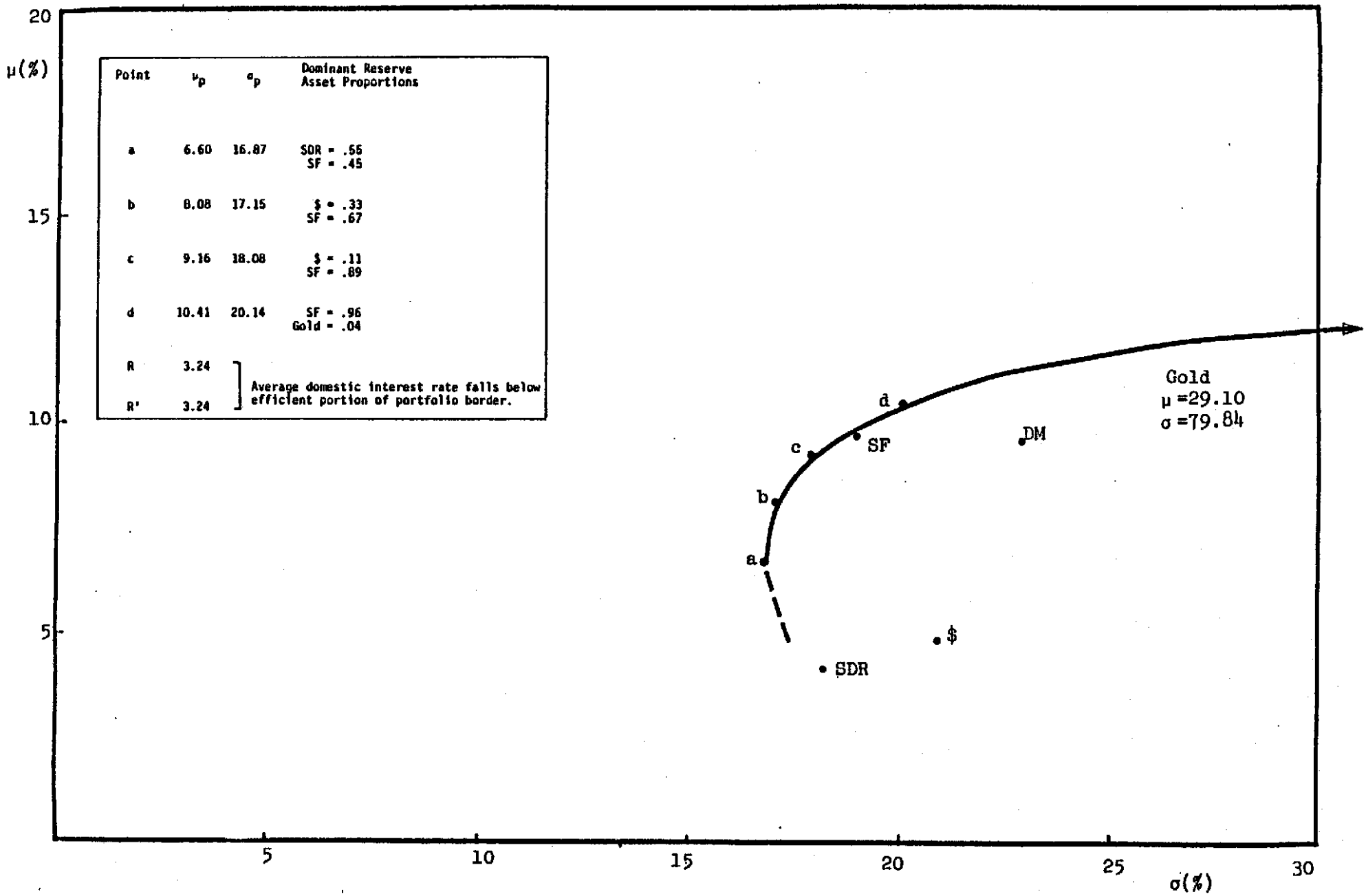


Figure 6

Efficient Portfolios and Individual Potential Reserve Assets

SWITZERLAND

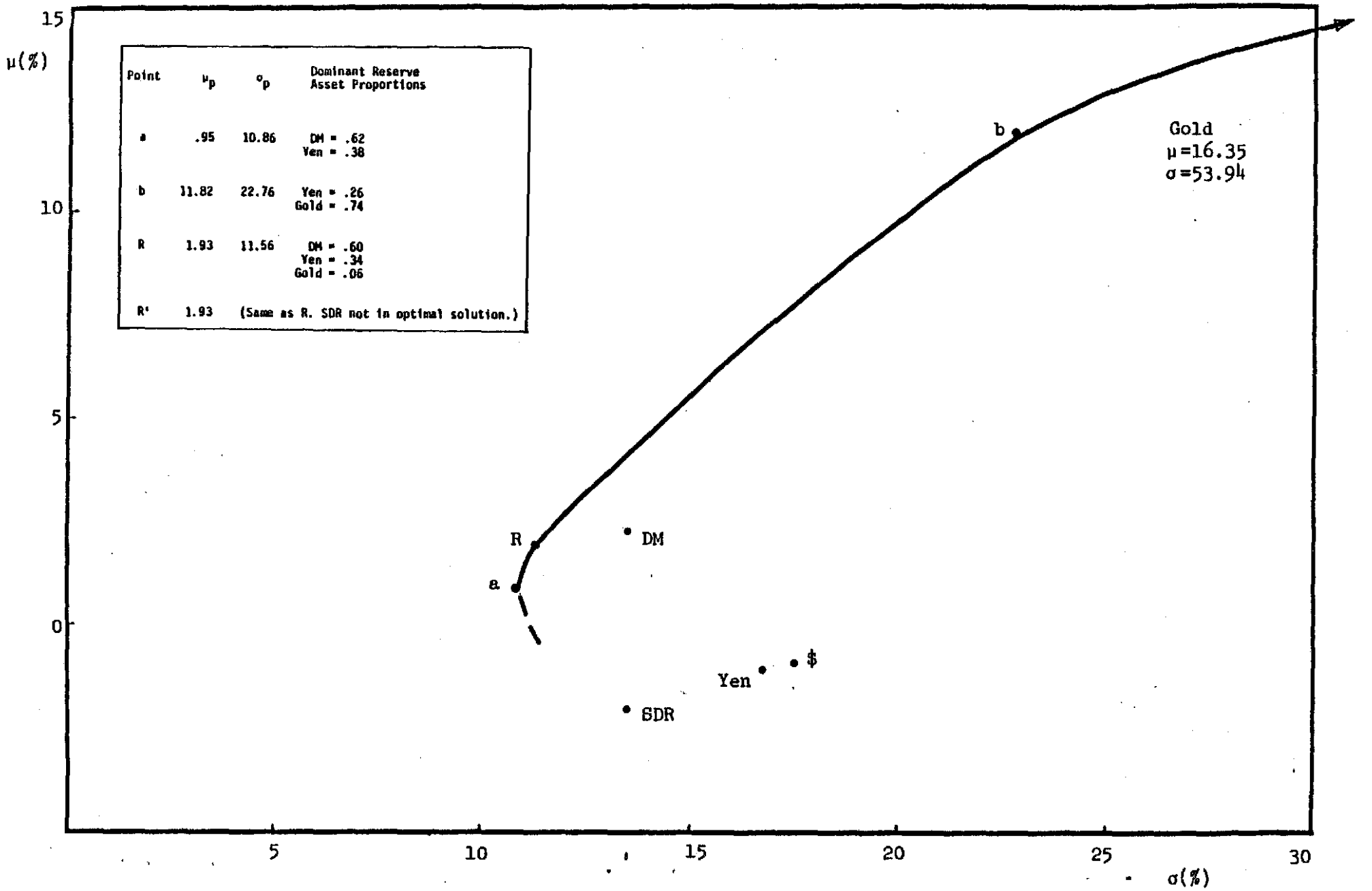
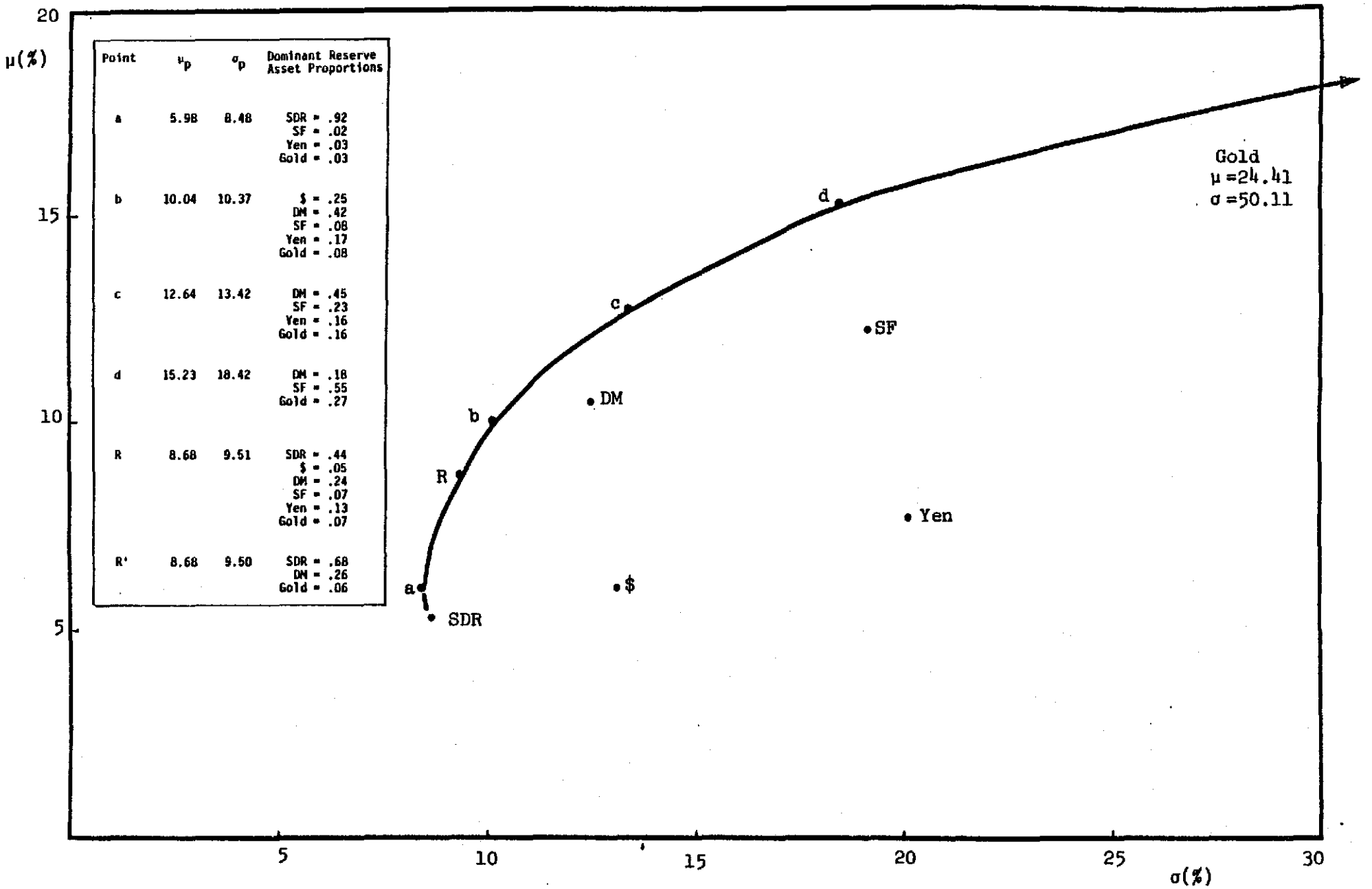


Figure 7

Efficient Portfolios and Individual Potential Reserve Assets

FRANCE



Point	μ_p	σ_p	Dominant Reserve Asset Proportions
a	5.98	8.48	SDR = .92 SF = .02 Yen = .03 Gold = .03
b	10.04	10.37	$ = .25 DM = .42 SF = .08 Yen = .17 Gold = .08
c	12.64	13.42	DM = .45 SF = .23 Yen = .16 Gold = .16
d	15.23	18.42	DM = .18 SF = .55 Gold = .27
R	8.68	9.51	SDR = .44 $ = .05 DM = .24 SF = .07 Yen = .13 Gold = .07
R'	8.68	9.50	SDR = .68 DM = .26 Gold = .06

Figure 8

Efficient Portfolios and Individual Potential Reserve Assets

ITALY

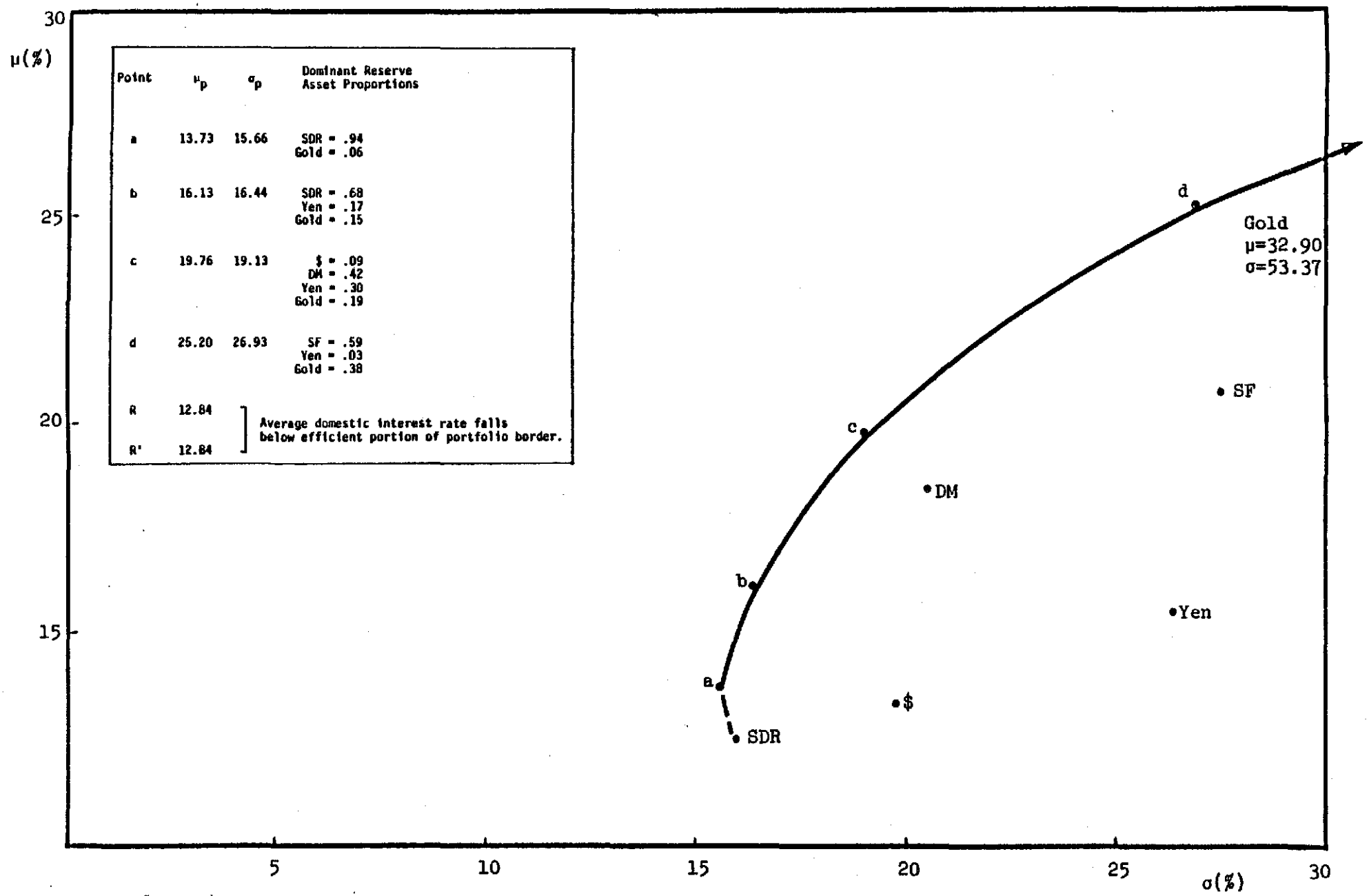
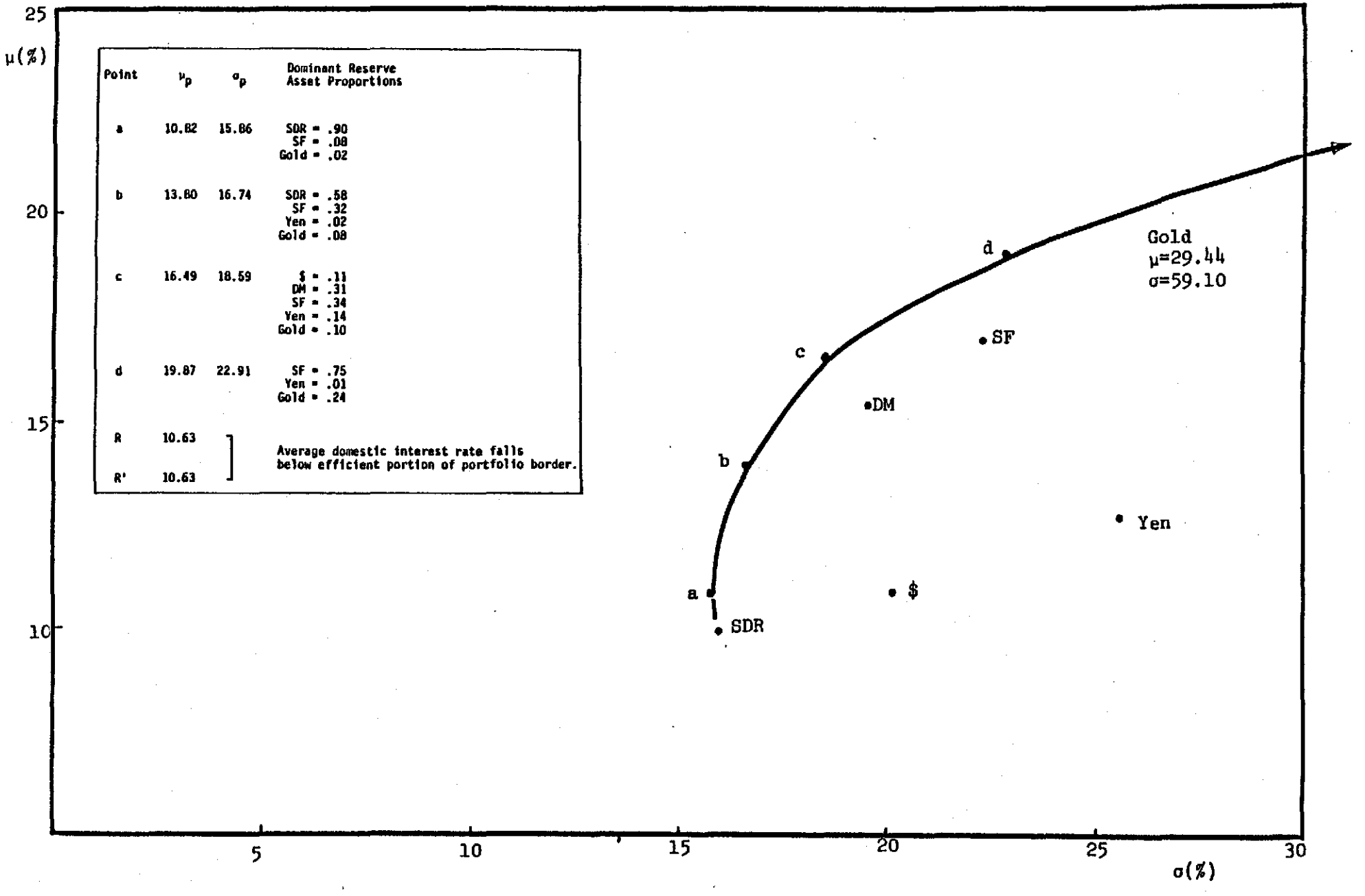


Figure 9

Efficient Portfolios and Individual Potential Reserve Assets

UNITED KINGDOM

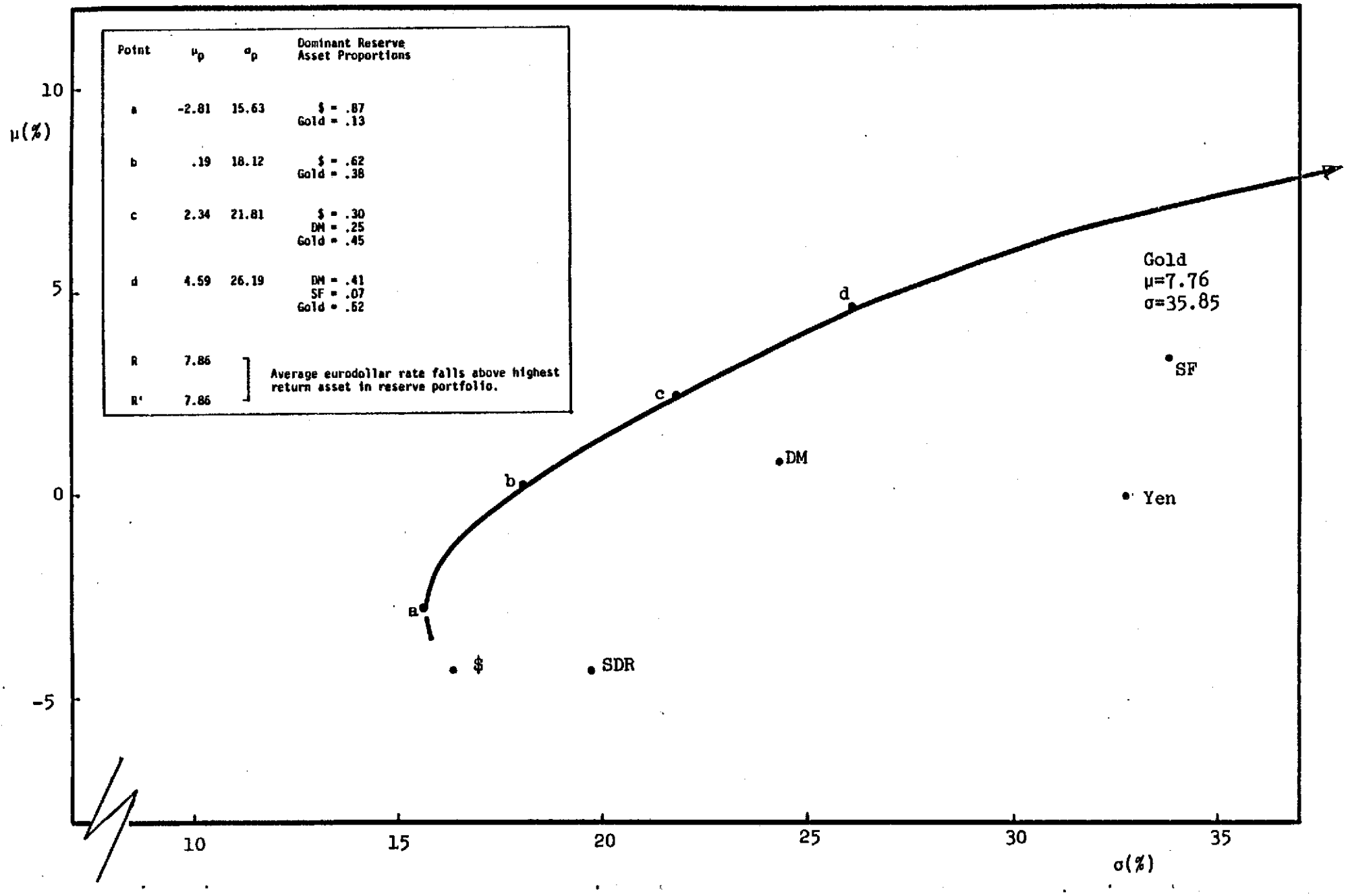


Point	μ_p	σ_p	Dominant Reserve Asset Proportions
a	10.82	15.86	SDR = .90 SF = .08 Gold = .02
b	13.80	16.74	SDR = .58 SF = .32 Yen = .02 Gold = .08
c	16.49	18.59	$ = .11 DM = .31 SF = .34 Yen = .14 Gold = .10
d	19.87	22.91	SF = .75 Yen = .01 Gold = .24
R	10.63		Average domestic interest rate falls below efficient portion of portfolio border.
R'	10.63		

Figure 10

Efficient Portfolios and Individual Potential Reserve Assets

SAUDI ARABIA



the same return as that which might accrue to the domestic component of the central bank's portfolio, if that component were invested in securities yielding a return similar to the chosen average interest rate.[22] In some cases, this computed return R falls below efficient portion of the computed portfolio border, so that a higher return than R could be obtained at lower risk. (See Japan, Italy, and the United Kingdom.) In one case, Saudi Arabia, the entire computed efficient frontier falls below several proxies for R. (Again, since the domestic component of the monetary authorities' portfolio is irrelevant in this case, some other benchmark must be chosen. If R were proxied by the period's average return on three-month eurodollar deposits, for example, this would be 7.86 > 7.76 percent, the portfolio's return if it were totally invested in the highest return asset here, gold.)

Another efficient portfolio frontier (not depicted) was computed using the SDR asset yield estimated at 100 percent of the market formula. Often the shape of the frontier and the optimal asset mix did not vary appreciably at some portfolio return levels from those reported using the SDR yield calculated at 80 percent of the market formula. But especially at lower portfolio return levels the SDR asset more frequently eclipsed the dollar in the optimal asset mix, since, as indicated in Table 2, the SDR under this assumption is both a higher return and a lower risk asset than the dollar. For comparison, one other risk-return point R' is reported in the insets to the figures where applicable, that corresponding to the return R on this alternative portfolio frontier.

---

22. As in computing the returns on reserve assets themselves, the nearest equivalent of a three-month interbank loan rate was used for this calculation. To the extent that either international reserve assets in foreign markets or domestic assets are invested in instruments for which these interest rates are not representative, outcomes may not be accurate, but this assumption may suffice for approximations in these examples.

It is evident from inspection of the dominant asset proportions that the mix can vary substantially depending upon the specified level of portfolio return. At lower portfolio return levels the SDR frequently has a relatively large share of the optimal asset mix. (See West Germany, Japan, France, Italy, and the United Kingdom.) Even when the SDR income yield is calculated at 80 percent of the market formula, then, its low risk characteristics can make it a desirable asset if return demanded on the overall reserve asset portfolio is low. Its share diminishes as higher levels of portfolio return are specified, however, and it frequently loses ground very quickly to other assets in the mix as specified portfolio return is increased. Using the returns R as some gauge of desired return the SDR would constitute about half of the German portfolio and slightly less of the French portfolio. Since R falls below the efficient part of the portfolio frontier for Japan, Italy, and the United Kingdom, and since the SDR occupies a large share of those portfolios at the minimum risk point a, especially for the latter two, it is reasonable to assume a large SDR share for these portfolios also. But the SDR does not appear in the Swiss or Saudi Arabian portfolios as computed here at all.

Interestingly the dollar does not make a very strong showing in most portfolios. The U.S. currency is dominant at low return levels in Saudi Arabian portfolios, owing largely to the fact that measurement of risk and return for this country in terms of the exported commodity oil makes the dollar a lower risk asset than the SDR. In German, Japanese, and French portfolios at some levels of return, and to a lesser extent in some Italian and British portfolios, the dollar does appear in the optimal mix of assets. But its proportion in all of these is much less than its actual current share in official reserves.

Frequently, especially in higher return portfolios, the German mark, Swiss franc, and Japanese yen occupy important positions. And as portfolio returns become very high, gold naturally dominates, since in the limit the highest possible return portfolio for all countries is composed entirely of gold. Gold's high risk, however, generally prevents the metal from having a very large share, except in a case such as Saudi Arabia in which alternative asset choices offer such a relatively low return.

By calculating the efficient portfolio border using the SDR with 100 percent of market formula yield, then measuring minimum risk at the average domestic interest rate by point R', some information is gleaned. Among the cases in which the domestic interest rate falls within the range of the computed efficient border--Germany, Switzerland, and France--only Germany shows a reduction in portfolio risk at the chosen return. Portfolio proportions do not vary much for Germany between R and R'. The SDR share rises to 60 percent from 51 percent. For Switzerland there is no difference between R and R' since the SDR does not appear in the dominant asset mix. And for France, although the minimum risk point changes hardly at all, the SDR's share rises to 68 percent from 44 percent, driving out the smaller currency shares for the dollar, the Swiss franc, and the yen that are included in the 80 percent SDR yield portfolio.

Conclusions

The previously mentioned limitations on interpretation of these results bear reiteration. The choice of portfolio numeraire units, the estimation of portfolio parameters from historical data, the choice of assets that are considered candidates for optimal portfolios, and the portfolio return at which an optimal asset mix is evaluated all involve

rather heroic assumptions if one interprets the outcomes to be normative in any sense. In addition, correspondence of the portfolio model to the real world is damaged by assuming the measured parameters to encompass all those factors that enter into the formation of central monetary authority attitudes toward chosen assets. For example, there is no consideration of the extent to which depth of markets in which the assets are held can influence such attitudes,[23] and no incorporation of the fact that central banks might not be able to acquire assets in indicated proportions even if they did wish to do so. Finally, existing or potential future institutional constraints on reserve asset composition are ignored.

Even so, the results illustrate how it is at least possible for the indicated combinations of reserve assets to be optimal if one concentrates solely on the portfolio aspects isolated here. The fact that the SDR-denominated asset does dominate several portfolios from this standpoint suggests that the substitution account might be subject to substantial voluntary use if other problems surrounding its institution can be overcome. But the presence of significant amounts of the other assets in some portfolios also suggests that, when viewed from a global perspective, the SDR-denominated substitution account claims might not occupy the central and most dominant position among reserve assets. The underlying goal of making the SDR the world's primary international reserve asset might, therefore, continue to be frustrated.

---

23. It might not be difficult to incorporate some measure of relative liquidity of alternative reserve assets, based perhaps on some measure of the depth of various capital markets, into the portfolio framework utilized here, but such a measure would likely be rather subjective. If this were done, it would quite probably enhance the U.S. dollar's share in portfolios, but it would almost certainly not help the SDR since it is the least liquid among assets considered here.

Particularly instructive may be the case of Saudi Arabia, since it represents here the kind of country that would be most able to diversify more over time. This is the only country investigated here for which a noticeable portion of its computed efficient portfolio border falls at netagive rates of return, and even along the higher-return, higher-risk portion of it the achievable returns are not very great. This would be the case, moreover, regardless of the measurement unit chosen as long as that unit moved approximately in line with world inflation. The SDR never appears in optimal portfolios here, and even if, by some alternative choice of the numeraire unit, it did appear at lower returns, it would like the dollar in the example drop out at higher levels in favor of the higher return currencies and gold. If this single illustration characterizes the possibly large number of outer countries that are likely to be most active in diversifying, then alternative reserve currencies might be even more dominant on a global scale than a more superficial inspection of the results here would indicate.

International monetary reformers are therefore left to contemplate, barring alternative means of SDR valuation and perhaps even then, the possibility of a multiple reserve asset system even if substitution account plans go more smoothly than they are likely to go. If the multiple reserve system is inevitable the question of whether it is desirable may be moot, but there will still likely be some disagreement on just how disruptive such a system will be. To an extent some surface opposition to it may come from a confusion of stocks and flows. Official portfolio shifts in the transition to some desired currency mix may be disruptive, but once that mix is achieved approximately central banks may

not be nearly as active in reacting to possibly transient changes in exchange rate expectations as private entities are.[24] During that transition, the use of off-market diversification facilities has been suggested to ease the strain,[25] and there has been some recent indication that such channels may actually be used.[26]

Some followers of recent developments no doubt conclude that the only hopes for a workable and achievable system still lie with the U.S. dollar, and therefore the U.S. monetary policies that are so important in underpinning this currency's international role. Since October 1979, and especially since the beginning of the year 1980, strength in the dollar has reportedly halted substantially the diversification out of the currency. But strictly in the portfolio context that has been a vehicle for this paper, even if the expected return on any single asset is quite attractive

---

24. Those less upset by the prospect of a multiple currency system also may argue that any change is de facto one of degree rather than kind. Other reserve currencies have existed along with the U.S. dollar to a minor extent since the beginning of managed floating, and at no point in modern experience has any single asset dominated entirely. The British pound coexisted with the U.S. dollar for years under the Bretton Woods system, and even under the pre-World War I classical gold standard the German mark and the French franc, along with the pound sterling, comprised significant portions of the world's official foreign exchange balances. A multiple currency system under the gold standard or Bretton Woods is markedly different than one under managed floating, of course, since no form of exchange rate guarantee exists under the current system. But it is interesting to note (see Lindert [1969]) that liquid claims on the major reserve centers in the pre-1914 world did exceed their own total reserves, the same kind of situation that later in this century led to the discontinuation of the U.S. dollar's gold convertibility.

25. See Morgan Guaranty Bank [1979b].

26. See The Banker [1980].

the incentives for diversification still exist. One is left to conclude tentatively that reserve asset diversification is likely to continue to at least some degree over the longer run even if (1) U.S. policies are relatively disciplined, and (2) the substitution account is instituted. While some countries may avail themselves of the substitution account, others may not. One may argue for the institution of such an account for the benefit of the former, but the latter will likely insure a multiple reserve system on a global basis in any case. This paper is not intended to argue for the relative viability of a multiple reserve system, however. It only suggests that it may be the most likely course.

REFERENCES

Aschheim, Joseph, and Y.S. Park. Artificial Currency Units: The Foundation of Functional Currency Areas. Princeton Essays in International Finance No. 114 (Princeton, New Jersey: International Finance Section, Department of Economics, Princeton University, 1976).

Bilson, John F.O. "Recent Developments in Monetary Models of Exchange Rate Determination," International Monetary Fund Staff Papers, June 1979.

Brown, Brendan. "The Internationalisation of the Petro-pound," The Banker, December 1979.

Chrystal, K. Alec. International Money and the Future of the SDR. Princeton Essays in International Finance No. 128 (Princeton, New Jersey: International Finance Section, Department of Economics, Princeton University, 1978).

Cooper, Richard N. "Eurodollars, Reserve Dollars, and Asymmetries in the International Monetary System," Journal of International Economics 2, 1972.

_____. "The Future of the Dollar," with comments by Peter Oppenheimer, Pascal Salin, and Motoo Kaji, Foreign Policy, Summer 1973.

Dreyer, Jacob S. Composite Reserve Assets in the International Monetary System (Greenwich, Conn.: JAI Press, 1977).

Fama, Eugene F. Foundations of Finance (New York: Basic Books, 1976).

Frenkel, Jacob. "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence," Scandinavian Journal of Economics, vol. 78, no. 2, 1976.

Greene, Margaret L. "Reserve Asset Preferences Revisited," in The Open Economy: Essays on International Trade and Finance, eds. P.B. Kenen and R. Lawrence (New York: Columbia University Press, 1968).

Gregory, Gene. "No Yen for Regional Monetary Schemes," The Banker, March 1979.

Haberler, Gottfried. The State of the World Economy and the International Monetary System, reprint no. 92 (Washington: American Enterprise Institute, 1979).

Hagemann, Helmut. "Reserve Policies for Central Banks and Their Implications for U.S. Balance of Payments Policy," American Economic Review, March 1979.

Hayami, Masaru. "Japan's Role in the International Financial Community," The Banker, March 1979.

Heller, H. Robert and Malcolm Knight. Reserve Currency Preferences of Central Banks, Princeton Essay in International Finance No. 131, (Princeton, New Jersey: International Finance Section, Department of Economics, Princeton University, 1978).

Kenen, Peter B. "International Liquidity and the Balance of Payments of a Reserve Currency Country" Quarterly Journal of Economics, November 1960.

Laney, Leroy O. "A Diminished Role for the Dollar as a Reserve Currency?" Voice of the Federal Reserve Bank of Dallas, December 1978.

Leutwiler, Fritz. "An Interview with the President of the Swiss National Bank," The Banker, February 1979.

Lindert, Peter H. Key Currencies and Gold, 1900-1913. Princeton Essay in International Finance no. 24 (Princeton, New Jersey: International Finance Section, Department of Economics, Princeton University, 1969).

Makin, John. "The Composition of International Reserve Holdings: A Problem of Choice Involving Risk," American Economic Review, December 1971.

_____. "On the Success of the Reserve Currency System in the Crisis Zone," Journal of International Economics, February 1972.

_____. "The Problem of Coexistence of SDRs and a Reserve Currency," Journal of Money, Credit, and Banking, August 1972.

Murphy, J. Carter. "SDR's, Dollars, and Other International Reserves," paper presented at the Meetings of the Southwest Social Science Association, Fort Worth, Texas, March 1979.

Officer, L. H. and Thomas D. Willett. "Reserve Asset Preferences and the Confidence Problem in the Crisis Zone," Quarterly Journal of Economics, November 1969.

Polak, J. J. "The SDR as a Basket of Currencies," International Monetary Fund Staff Papers, December 1979.

Rose, Harold. "The Future of the Dollar as a Reserve Currency," Barclay's Review, February 1979.

Sacchetti, Ugo. "The SDR: Ten Years of Experience," Banca Nazionale del Lavoro Quarterly Review, December 1979.

Salant, Walter. "The Post-Devaluation Weakness of the Dollar," Brookings Papers on Economic Activity: 2, 1973.

Sharpe, William F. Portfolio Theory and Capital Markets (New York: McGraw-Hill, 1970).

Sobol, Dorothy Meadow. "A Substitution Account: Precedents and Issues," Federal Reserve Bank of New York Quarterly Review, Summer 1979.

Triffin, Robert. "The International Role and Fate of the Dollar," Foreign Affairs, Winter 1978/79.

Westerfield, Janice Moulton. "An Examination of Foreign Exchange Risk Under Fixed and Floating Rate Regimes," Journal of International Economics, May 1977.

Whitman, Marina v. N. "The Current and Future Role of the Dollar: How Much Symmetry?" Brookings Papers on Economic Activity: 3, 1974.

Bank for International Settlements, Forty-Ninth Annual Report, June 1979.

Council of Economic Advisers, Economic Report of the President, 1980.

Deutsche Bundesbank, "The Deutsche Mark as an International Investment Currency," Monthly Report, November 1979.

International Monetary Fund, IMF Survey, various issues.

Morgan Guaranty Trust Company of New York, "Reserve Diversification and the IMF Substitution Account," World Financial Markets, September 1979.

_____. "In Quest of International Monetary Stability," World Financial Markets, October 1979.

"How Central Banks are Ditching the Dollar," Euromoney, October 1978.

"The Crumbling Cornerstone of the Alliance," Business Week, March 12, 1979.

"Is the Dollar Washed Up as a World Currency?" U.S. News and World Report, October 15, 1979.

"Europeans Think Gold Will Expand Its Role in Monetary Dealings," Wall Street Journal, January 31, 1980.

"Off-market Reserves," The Banker, February 1980.