No. 7807

INTERPERSONAL COMPARISONS

by

Dale K. Osborne

October 1978

Research Paper

FEDERAL RESERVE BANK OF DALLAS

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Interpersonal Comparisons

D. K. Osborne*

1. Introduction

The long controversy over this subject is dying without issue. The annals of the subject having no declared winner to show for all the tedium and confusion, affirmers, deniers, and agnostics mainly go their separate ways convinced that the others are blind. Yet the very persistence of the affirmers is giving them their way. The rest are so tired of denying or demurring to no effect that they surrender in practice if not in theory. The following ritual occurs daily in seminar rooms throughout the English-speaking world (at least): Jones presents an analysis, of some social question or other, that depends in an essential way on interpersonal comparisons. Smith, no affirmer, mildly points out the dependence. Jones, knowing that the new ground rules protect these comparisons once they are acknowledged, cooly admits them. Jones and Smith, having thus duly observed good form, proceed to discuss the remaining assumptions, the method of analysis, the accuracy of the data, and the results as if they had left no great question dangling.

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*Department of Research, Federal Reserve Bank of Dallas. I am grateful for the criticisms given to a previous draft of this paper at the 1977 meeting of the Public Choice Society.

But it is clear that the body of analysis and putative principles that lately constitute our normative theory of economic policy--in other words, modern political economy--lives or dies with interpersonal comparisons. This is the political economy of broad utilitarianism, that is, of social utility together with the stipulation that social utility increases in some manner with individual utility. This political economy aims, therefore, to maximize some increasing function (not necessarily the sum) of individual utilities. Such a function can satisfy Arrow-like conditions, thus imparting some minimal content to the concept of social utility, only if the utilities of different persons are comparable.* The same necessity underlies all the familiar devices, such as compensation schemes, cost/benefit ratios, or consumers' surplus**, that purport to reveal the social utility or its changes, and all such familiar arguments as unemployment, income distribution, or gross national product that often stand in for individual utilities. All calculations with such things ultimately rest on interpersonal comparisons (the rare Pareto-dominant case aside).

These necessities are admitted, at least implicitly, by those who stubbornly affirm interpersonal comparisons in order to "save political economy," as if the modern system were the only one to value individual

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*See Osborne (1976, esp. pp. 1010, 1011).

**A single consumer's surplus might in some cases vary monotonically with his utility. This possibility, beyond our present concern, is not sufficient for interpersonal comparisons via consumers' surplus (with the apostrophe after the s).

concerns. But a political economy can value individual <u>freedom</u> instead of individual <u>preference</u>. Classical liberalism, for instance, aims (in Kant's words) at "a constitution that achieves the greatest possible freedom of human individuals by framing the laws in such a way that the freedom of each can co-exist with that of all others." (<u>Critique of Pure Reason</u>, p. 373.) And it is clear that questions of freedom need not determine a person's preference; he might be freer in state x than state y but prefer y to x on grounds of security. Classical liberalism, then, does not even reach the question of interpersonal comparisons, thus showing that a healthy fear of the collectivist systems, where "society" or the state is supreme, by no means commits us interpersonal comparisons. No more does a distaste for the status quo so commit us. Many, believing that all social change inevitably violates somebody's preferences, go on to declare that the comparisons are needed to justify the change. We may grant the belief, but the declaration presupposes a political economy concerned only with preferences. I can see nothing more than habit and the power of suggestion in this presupposition. The status quo of our time is probably no closer to Kant's aim than the status quo of his, at least in the United States.

The long debate between classical liberalism and broad utilitarianism does not, of course, turn only on interpersonal comparisons.* If these comparisons were possible, thus making possible a respectable utilitarian political economy, they would not for that reason imply its superiority.

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*See Osborne (1978).

We would still have to choose between freedom and utility as the ultimate criterion for economic policy. But if the comparisons are impossible--all putative instances of them being meaningless expressions or confusions of thought--then modern political economy has a vacuum where its principles should be. Politics abhors a vacuum equally with nature, and must fill it with its own principles. The principles of coalition and power explain our actual economic policy, which talks in theory of the public interest while in practice facilitating the mutual coercion of factions. This situation, which few of us like, might not demonstrate our political ignorance or stupidity or ill will so much as our complete and unconscious allegiance to a logically impossible doctrine. Power and coercion easily dress up as the public interest where the formulas claiming to measure it are empty.

These considerations justify a careful account. A complete account being out of the question because of length, I will aim only at the worst and commonest confusions in the subject. These I take to concern compensation schemes, the ordinal/cardinal question, the question of common units or origins of utility scales, and the notion that we compare peoples' utilities in everyday life.* Some simple formal machinery will help all but the last, which I therefore take up prior to the machinery.

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*For the fallacy in the just-noticeable difference approach to interpersonal comparisons, as advanced by Goodman and Markowitz (1952), see Luch and Raiffa (1957, p. 347). This and a few other exceptions aside, I see no purpose in citing the sources or perpetuators of the confusions.

The controversy owes something to differences in usage. As we will use the term, an interpersonal comparison is an expression of the form OC or IC:

> <u>OC (ordinal comparison)</u>: Individual 1 likes x more than individual 2 likes y (or alternatively, 1 is better off at x than 2 is at y).
>
> <u>IC (interval comparison)</u>: Individual 1's preference for w over x exceeds individual 2's preference for y over z (or, alternatively, 1 gains more from the movement from x to w than 2 does from a movement from z to y).

Here, w, x, y, and z belong to the set of social states over which the individuals have preference orderings. In OC, alternatives x and y may or may not be distinct. In IC, w and x are distinct and y and z are distinct but no further distinctions need obtain. Thus IC can be interpreted, if appropriate, as "the move from w to x benefits 1 more than it harms 2."

By this usage, an interpersonal comparison expresses a putative fact, not a value, and must be judged accordingly. It is the most common usage but not the only one to be found in the literature, where interpersonal comparisons are often regarded as value judgments. Graaff (1957, p. 167), for instance, uses "interpersonal comparisons" in the sense illustrated by "x is better than y because it benefits the worthier 1."

This is indeed a value judgment but it is an interpersonal evaluation: it compares the merits of two persons, not the utilities of states to them. And Robbins (1949, pp. 138-141), though using the term in the sense of expression OC, regards it as a value judgment on the ground that OC is not testable--thus implicitly equating the class of value judgments with the class of untestable expressions. In our terms, neither OC nor IC is a value judgment.

2. "We Do It Every Day"

Probably nothing more frequently appears in defense of theoretical appeals to interpersonal comparisons than the assertion, that for all the difficulties these comparisons raise in principle they are nonetheless possible in practice, as shown by our frequent recourse to them in daily living. It is said, for example, that when we give Jones a book and Smith a recording we implicitly compare their utilities: Jones likes the book more than Smith does, Smith likes the recording more than Jones does. Again, when we have a spare ballet ticket, for instance, and give it to Jones instead of Smith, we imply that Jones likes the ballet more than Smith does. For after all, if we want to give to the person who will derive the most pleasure, our very decision entails an interpersonal comparison.

The implication is clear: Since our practice shows (it is said) that the difficulties are strictly theoretical, we can leave theoretical niceties to the future while we get on with the more urgent matters. It is as if the affirmers were the Wright brothers and the deniers those orthodox physicists who continued to prove the impossibility of flying machines right up to 1905. A dollar for every denier silenced by this reasoning would make us rich.*

The reasoning, however, which descends from Little (1957) through Spence (1973, 1977) and others, turns on a mere figure of speech. When we say that Jones likes the book more and the recording less than Smith, we appear to utter two interpersonal comparisons. They are strictly figurative. Behind them stand two real _intra_personal comparisons: Jones prefers the book to the recording, Smith the reverse. That is all we need to know in order to give wisely.

It is the same with respect to the ballet ticket. A figurative comparison, setting Jones's taste for the ballet against Smith's, masks the real comparisons that we must make concerning each person's taste for the ballet relative to other things. Jones greatly enjoys the ballet and Smith does not; i.e., relative to other things that we might give, the ballet stands high in Jones's preference ordering but not so high in Smith's. Over a period of time we try, by a roughly balanced sequence of gifts to both men, to reach the highest feasible place in the preference ordering of each one. For this we need to know something about each man's preferences and nothing else. We might well describe our action as "giving to the person who enjoys it most," but only as a common manner of speaking.

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*Agnostics, on the other hand, often seem to view an interpersonal comparison less as something impossible than as something to be avoided, as if it were fattening.

Similar expressions occur in other contexts. We might say of a man, that he is more intelligent than virtuous. But we understand this to mean that he ranks higher among his fellows in intelligence than in virtue. We realize, when we think about it, that the comparison between intelligence and virtue is only a figurative stand-in for two real comparisons of men. No one would consider it a proof that intelligence and virtue, despite what the philosophers say, are really comparable. In familiar contexts, we can still distinguish between arguments and figures of speech. For our confusions in political economy we have to blame its completely theoretical nature, its complete disjunction from our daily lives.

3. Machinery

Let X be the set of alternatives, N the set of individuals, and S the cartesian product N×X. S contains pairs such as (i,x), the first element of which is a person and the second is an alternative. Let $\geq$ be a binary relation on S with at least some of the properties of a weak ordering--e.g., transitivity--and $>$ be its antisymmetric part. The formal counterpart of expression OC is then $\overline{OC}$,

$$\overline{OC}: \quad (1,x) > (2,y).$$

Now form the cartesian product of X with itself, eliminate the diagonal, and premultiply the result by N to obtain the set T of triples (i,x,y) such that $x \neq y$. Let $\boxed{\geq}$ be a binary relation on T, with antisymmetric part $\boxed{>}$, such that it also has at least some of the properties of an ordering. The formal counterpart of expression IC is $\overline{IC}$,

$\overline{\text{IC}}$: $(1,w,x) \boxed{>} (2,y,z)$

In these terms, the question is whether the assumed relations are well-defined, have observable consequences, and capture the intuitive notions expressed by OC and IC.

Two axioms seem essential.

Ax. 1: $\geq$ and $\boxed{\geq}$ are preorders, i.e., whether surrounded by a circle or a square, $\geq$ is transitive and reflexive while $>$ is transitive, irreflexive, and antisymmetric.

This axiom imposes a connection between the comparisons concerning different pairs of individuals, different pairs or quadruples of states, or both. Thus if $(i,u,v) \boxed{\geq} (j,w,x)$ and $(j,w,x) \boxed{\geq} (k,y,z)$, then $(i,u,v) \boxed{\geq} (k,y,z)$.

For the second axiom, let $\geq_i$ be individual i's preference ordering ($x \geq_i y$ iff i weakly prefers x to y, $x >_i y$ iff he prefers x to y, $x =_i y$ iff he is indifferent between them) and $\boxed{\geq}_i$ be the individual's <u>intensive</u> preference ordering (e.g., $(w,x) \boxed{\geq}_i (y,z)$ iff i prefers w to x more than he prefers y to z).

Ax. 2: For all $i \varepsilon N$ and all $w,x,y,z \varepsilon X$ such that $w \neq x$ and $y \neq z$:

a. $(i,x) \geq (i,y)$ iff $x \geq_i y$;

b. $(i,w,x) \boxed{\geq} (i,y,z)$ iff $(w,x) \boxed{\geq}_i (y,z)$.

This axiom imposes a connection between interpersonal comparisons and individual preferences.*

Besides these axioms, we must also require $\geq$ and $\boxed{\geq}$ to be well defined. This requirement inheres in the very notion of a meaningful relation.**

4. Compensation Schemes

Actually, the only surviving compensation scheme is Scitovsky's (1941). Scitovsky advanced this scheme, based on the "double bribery" condition, as a way of comparing the social welfare of two states. But

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*Paternalists can interpret $\geq_i$ and $\boxed{\geq}_i$ in terms of individual i's welfare as understood by the paternalist rather than the individual. Then Axiom 2 connects interpersonal comparisons with individual welfare as understood by paternalistic observers. On either interpretation, a third essential axiom would impose a continuity on the ordering so that Axiom 2 could not be realized by a trivial definition. This, however, does not figure in our discussion.

**Let a relation R on set $\{a,b,c,\ldots\}$ be determined by another relation Q on set $\{\alpha,\beta,\gamma\ldots\}$, that is, aRb iff $\alpha Q\beta$, and let $\alpha,\beta,\gamma\ldots$ belong to equivalence classes $[\alpha]$, $[\beta]$, $[\gamma]$,... as determined by some equivalence relation. Then R is well-defined whenever it is preserved by substitutions within equivalence classes: aRb iff $\alpha' Q\beta'$ for all $\alpha'\varepsilon[\alpha]$, $\beta'\varepsilon[\beta]$.

it is also advanced, in informal discussions at least, as a way of comparing utilities, and that is how we will treat it. The results apply to both of its putative uses.

Consider the particular comparison between (1,w,x) and (2,x,w), where 1 prefers w to x and 2 prefers x to w. For any alternative $z \in X$ and persons $h,k \in N$, let zt_{hk} denote the state derived from z by transferring some designated commodity from h to k. Then the Scitovsky criterion would imply $(1,w,x) \boxed{>} (2,x,w)$ if whenever there is a transfer from 1 to 2 such that

$$\text{(i)} \quad wt_{12} \succsim_i x \qquad (i = 1,2),$$

there is <u>no</u> transfer from 2 to 1 such that

$$\text{(ii)} \quad xt_{21} \succsim_i w \qquad (i = 1,2),$$

where strict inequality holds at least once in each of (i) and (ii). In other words, 1 prefers w to x more strongly than 2 prefers x to w if he can bribe 2 to accept w but 2 cannot bribe him to accept x. The second part is essential to the antisymmetry of $\boxed{>}$, for, as Scitovsky showed, in some cases 1 and 2 could bribe each other for their preferred states. Without the second part of this criterion we could have both $(1,w,x) \boxed{>} (2,x,w)$ and the reverse. The second part is designed to prevent this, though at the cost of leaving unrelated the elements of T for which reciprocal bribery is possible. Thus the Scitovsky criterion only partially orders T, but that is all right if it orders transitively.

The Scitovsky criterion in fact violates transitivity. To see this, suppose

(a) $(i,w,x) \boxed{>} (j,x,w)$ and $(j,x,w) \boxed{>} (k,w,x)$,

i.e., i can bribe j and j cannot bribe i, but j can bribe k while k cannot bribe j. Clearly, (a) is consistent with the further supposition that

(b) $(k,w,x) \boxed{>} (h,x,w)$ and $(h,x,w) \boxed{>} (i,w,x)$,

for the bribing powers of i,j, and k imply nothing about those of h. In other words, there is no reason to doubt that (a) and (b) can simultaneously meet the Scitovsky criterion. However, (a) and (b) have inconsistent implications. By transitivity, (a) implies $(i,w,x) \boxed{>} (k,w,x)$ while (b) implies the reverse. Hence Scitovsky's scheme--the only surviving compensation scheme--violates Axiom 1.

5. The Connection With Utility Measurement

In view of Axiom 2, expression $\overline{IC}$ is meaningless unless the individuals concerned have intensive preferences, i.e., unless they can say whether their preference for one alternative over another exceeds their preference for some other alternative over yet another one. The existence of such intensity in the preferences does not imply their intensive ("cardinal") measurement; for this, the intensive orderings $\boxed{\geq}_i$ must satisfy several restrictive conditions, as given, for instance, by Krantz, Luces, Suppes, and Tversky (1971). However, the intensity in the preferences is necessary for their intensive measurement.

In short, intensive preferences are necessary for the meaningfulness of expression IC and for the existence of cardinal utility functions. This is the entire connection between utility measurement and interval interpersonal comparisons. Such measurement and such comparisons imply intensive preferences. That is all. Neither implies the other. The next three sections aim to demonstrate this statement.

6. Comparison Normally Precedes Measurement

To believe that utility measurement somehow permits or induces interpersonal comparisons is to put the cart before the horse. Measurement is the assignment of numbers to empirical objects or events in such a way that to every empirical relation describing the property being measured there corresponds a formally equivalent relation between the numbers. The empirical relations come first, the numbers and numerical relations afterward.

Measurement, of any kind, takes as given a set of empirical objects or events together with certain empirical relations between them and, in some cases, one or more empirical operations that can be performed on them. This set, together with the relations and operations, constitutes an empirical relational structure (e.r.s.). The measurement theory of some property begins by imposing conditions on this structure that make the property measureable. It continues with the selection of a set of numbers together with mathematical relations and operations that will mimic the e.r.s.—i.e., it selects a <u>numerical relational structure (n.r.s.)</u>. It then constructs a homomorphism between the e.r.s. and the n.r.s. This homomorphism is a <u>scale</u>.

For a simple example, let Y be a set of steel rods and $\boxed{\geq}$ the ordering of rods by length (e.g., if when placed side by side from the same point rod y does not extend past rod x, then x $\boxed{\geq}$ y). Let $\boxed{+}$ be the operation of laying two rods end to end, so that

$$x \boxed{+} y \boxed{=} z$$

means that x and y laid end to end are equivalent, in the ordering $\boxed{\geq}$, to a rod z. The triple $(Y, \boxed{\geq}, \boxed{+})$ is an e.r.s. An obvious n.r.s. is $(R^1_+, \geq, +)$ where R^1_+ is the set of positive reals, $\geq$ is their usual ordering, and + is their usual addition. (But this is not the only possible n.r.s. for measuring the e.r.s. Numerical multiplication, for instance, rather than addition could model the empirical operation $\boxed{+}$.) A length scale is then a function ϕ from Y to R^1 such that, for all, $x,y \varepsilon Y$,

(i) $\phi(x) \varepsilon R^1_+$

(ii) $\phi(x) \geq \phi(y)$ iff $x \boxed{\geq} y$

(iii) $\phi(x \boxed{+} y) = \phi(x) + \phi(y)$.

The practice of measurement ends with the construction of a suitable scale. The theory of measurement continues by considering the uniqueness of the suitable scales. Measurement scales are rarely unique. In the first place, there normally exist many homomorphisms from the e.r.s. to a given n.r.s. Thus in the preceding example, every function ϕ' such that $\phi'(x) = \alpha\phi(x)$ for some positive real α, is also a homomorphism. The choice of one of these homomorphisms, or scales, is arbitrary, i.e., it is not determined by the e.r.s. In the second place, there normally exist many suitable n.r.s's. Thus, if $\cdot$ represents numerical multiplication then $(R^1_+, \geq, \cdot)$ is an alternative n.r.s. for measuring length. Since the choice of a n.r.s. is arbitrary, the theory of measurement must determine how the numbers assigned to empirical elements change with a different arbitrary choice of n.r.s. as well as with a different arbitrary choice of a homomorphism into a given n.r.s.* These strictly technical matters, though far from easy, are less interesting than the discovery of the axioms that characterize the e.r.s. sufficiently to permit its measurement in the first place, but they are essential to the use of the numbers. In particular, any relations between the numbers, or between the numbers that result from them by further calculation, are meaningful (well defined) only if they are invariant to all arbitrary choices. In the case of such invariance, the numerical relations simply model the underlying empirical relations; in any other case, where the numerical relations are not

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*The preceding description, based on Krantz, et. al. (1971), applies to what is called fundamental or primary measurement. For a theory of derived or secondary measurement see Osborne (1976a).

invariant, they have no empirical significance: they are properties of numbers as numbers not as measurements. It is evident that strictly numerical relations cannot create empirical relations. Measurement models but does not create empirical relations.

If we measured the lengths and weights of steel rods we'd get two sets of numbers related in many ways. But a relation such as

$$\phi(x) > \Psi(y),$$

where $\phi(x)$ is a length measure and $\Psi(y)$ is a weight measure, obviously depends on our choice of length and weight scales and could be reversed by a different choice: it is strictly numerical. No one would suppose that this relation between measurements implied a comparison of length with weight. And if someone did believe the length of rod x to be greater than the weight of rod y, he would not justify it with the measurements but with the rods themselves. He could justify it with the measurements only if it were invariant to all the arbitrary choices that produced them.

A comparison between measurements is meaningful only as a convenient substitute for a comparison between the empirical elements. The possibility of this empirical comparison must exist, at least in principle, before the measurements are made. Measurement reproduces but does not produce meaningful comparisons. This is trivially obvious in the familiar contexts of length and weight measurement but is no less true in the more mysterious contexts of utility measurement.

It is true that we often compare measurements when the underlying empirical comparisons are physically impossible. We can't rearrange Bentham's birth or death to coincide with Sidgwick's; yet, we are sure that Bentham lived a longer life. We can't place Dublin and Odessa on the same line segment from Paris; yet measurements show us that Paris is closer to Dublin than to Odessa.* But though we cannot make these empirical comparisons directly, we could, if we cared to, make them indirectly via some third element of the set in question by using the transitivity of the associated empirical ordering. Indeed, if we did not believe this to be so we would not regard the time intervals corresponding to the ages, or the lengths corresponding to the distances, as belonging to the same e.r.s.; in this case we would not regard the ages or the distances as being comparable.**

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*Actual measurements aren't necessary. Every Texan knows that Paris is in the eastern part of his state while Dublin is near the center and Odessa is in the far west.

**We use quite a bit of theory in imagining these indirect empirical comparisons: for example, that a steel rod does not change when we move it about while ascertaining how many copies of it separate Paris and Dublin. Indirect empirical comparisons--and therefore measurements--presuppose theory. Hence operationalism, the doctrine that all scientific knowledge ultimately rests on the operations that yield numerical measurements, is untenable. See Popper (1962, pp. 59-65) for a fine discussion.

As applied to our question, the above considerations show that no method of interpersonal comparisons can rest essentially on utility measurement. Every comparison made with the aid of utility measurement must be possible without that aid or it is artificial. I will try to amplify this statement by detailed consideration of two confusions. Though for brevity I speak only in terms of preference, or a person's utility as understood by himself, my argument meets the case where utility is something to be understood by a paternalistic observer. The cases differ only in the source of the orderings $\geq_i$ and $\boxed{\geq}_i$: individual i or the paternalist.

7. The Ordinal/Cardinal Confusion

The empirical relational structure (e.r.s.) for individual i's ordinal utility is $(X, \geq_i)$ and the natural numerical relational structure (n.r.s.) for measuring his utility is $(R^1, \geq)$. An ordinal utility scale for i is any function f_i from X to R^1 such that, for all $x,y \varepsilon X$,

$$f_i(x) \geq f_i(y) \text{ iff } x \geq_i y.$$

All such functions constitute a class F_i, the members of which are increasing transformations of each other.* To every possible preference ordering $\geq_i$ (which we assume to be complete and transitive) there corresponds a class F_i of ordinal utility scales. Since "increasing transformation of" is an equivalence relation on the set of maps from X to R^1, the class F_i is an equivalence class.

Individuals 1 and 2 have identical preferences iff their orderings $\geq_1$ and $\geq_2$ agree everywhere on X; in that case $F_1 = F_2$; in every

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*We assume the conditions sufficient for the existence of f_i. See Krantz, et. al. (1971).

other case F_1 and F_2 are disjoint. In other words, the preferences of 1 and 2 can be measured on the same ordinal utility scale if and only if they are identical.

Now suppose we defined the relation (ordinal interpersonal comparison) $\geq$ by:

$$(1,x) > (2,y) \text{ iff } f_1(x) > f_2(y).$$

Then if the preferences differ, so that $F_1 \neq F_2$, the relation is ill defined: the preceding expression is not invariant to substitutions in equivalence classes, for F_1 contains an f_1' such that $f_1'(x) < f_2(y)$ even though $f_1(x)$ exceeds $f_2(y)$.

If on the other hand the preferences agree, so that $F_1 = F_2$, an ordinal interpersonal comparison still does not follow. Reverting to the steel rods of section 6, suppose their diameters and density to be constant. Then weight is proportional to length and every length scale is also a weight scale, i.e., both types of scale belong to the same equivalence class. (One scale is a positive linear transformation of the other, and this is an equivalence relation in the class of real-valued functions on the set of rods). But from this supposition we cannot conclude that length and weight are comparable, and indeed we know better. The mere equality of F_1 and F_2 does not, therefore, justify an ordinal interpersonal comparison. Anyway (not to waste words on it), the equality would remove all motives for the comparison: There would be no interpersonal conflicts to resolve by appeal to the social utility.

We thus reach the widely accepted conclusion that ordinal utility measurement offers no help with interpersonal comparisons. The conclusion is the same, though not so widely accepted*, in the case of cardinal utility.

The e.r.s. for intensive preferences is very rich, containing, besides the ordering $\geq_i$, the set X_i^2 of pairs of states such that $(x,y) \varepsilon X_i^2$ iff $x \geq_i y$ and the ordering $\boxed{\geq}_i$ on X_i^2. Under certain conditions (for which see Krantz, et. al., 1971) there is a map g_i from X to R^1 such that, for all $w,x,y,z \varepsilon X$,

$$g_i(w) \geq g_i(y) \text{ iff } w \geq_i y$$

$$g_i(w) - g_i(x) \geq g_i(y) - g_i(z) \text{ iff } (w,x) \boxed{\geq}_i (y,z).$$

The e.r.s. determines this map up to a positive affine transformation $\alpha g_i(x) + \beta$, $\alpha>0$. The map g_i is an <u>interval ("cardinal") utility scale</u> for i and the class $G_i = \{\alpha g_i(x) + \beta | \alpha>0, x \varepsilon X\}$ is the equivalence class of such scales.**

Individuals 1 and 2 have <u>identical intensive preferences</u> whenever $\geq_1 = \geq_2$ and $\boxed{\geq}_1 = \boxed{\geq}_2$. In that case $G_1 = G_2$; in every other case, G_1 and G_2 are disjoint.

The frequent references to cardinal utility in connection with interpersonal comparisons must reflect the belief that cardinal utility is (a) sufficient, or (b) necessary for the comparisons, as if the richer

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*Indeed, many people justify their denial of interpersonal comparisons by citing their disbelief in cardinal utility or vice versa, as if they had to swallow the one with the other.

**The relation, "positive affine transformation of," is an equivalence relation in the set of real-valued functions on X.

individual e.r.s.'s (a) undoubtedly supplied, or (b) could alone supply, the material needed for the comparisons.

(a) It is obvious, however, that the mere individual orderings $\boxed{\geq}_i$ on X_i^2 imply nothing about the interpersonal orderings, $\geq$ on S or $\boxed{\geq}$ on T, beyond the requirements of Axiom 2. Suppose, then, that $\boxed{\geq}$, for instance, is defined in terms of <u>utility numbers</u>:

$$(1,w,x)\boxed{>}(2,y,z) \text{ iff } g_1(w) - g_1(x) > g_2(y) - g_2(z).$$

But clearly, this relation is ill defined if individuals 1 and 2 have different intensive preferences, for substitutions within the disjoint equivalence classes G_1 and G_2 can convert $>$ to $\leq$. And if the intensive preferences are identical, then no interpersonal comparisons are needed. (This case of identical intensive preferences is so trivial and, with respect to any real problem of social choice, so vacuous, that we will henceforth ignore it.)

(b) Now necessity, if it were true, would imply the sufficiency of interpersonal comparisons for cardinal utility. But supposing the existence of $\geq$ or $\boxed{\geq}$, we obtain at most the e.r.s.'s $(X, \geq_i, X_i^2, \boxed{\geq}_i)$, and this only because of Axiom 2; we do not obtain the conditions on these structures that make them measurable on an interval scale. It is of course possible that such additional axioms as are required to obtain $\boxed{\geq}$ would condition the individual e.r.s.'s sufficiently for their interval measurement. But this possibility, which has never been explored, is not an implication. At the moment, therefore, we have no reason to believe interpersonal comparisons sufficient for interval utility nor, <u>a fortiori</u>, interval utility necessary for interpersonal comparisons.

It thus appears that neither ordinal nor interval utility bear any relation to interpersonal comparisons. With respect to these comparisons, there is no difference between the two types of utility.

8. The Units/Origins Confusion

It is often said that an interpersonal comparison is a matter of common utility units, as if the existence of a common unit for the utilities of two persons were sufficient, or necessary, for the comparison. But this is a simple misconception--and not just because measurement, and hence in particular the choice of units, normally <u>follows</u> empirical comparisons.

Confining attention to interval measurement, the usual context of the discussion, suppose the intensive preferences are thus measurable. To construct an interval utility scale for person i is to assign numbers to the states in a manner that preserves his intensive preferences. One such number is completely arbitrary; we have to choose some state x and then choose some number $g_i(x)$ for it. Then we have to choose some other state y that is not indifferent to x and assign some number $g_i(y)$ to it, subject only to the requirement that $g_i(y) > g_i(x)$ iff $x \, > _i \, y$. These choices determine the numbers $g_i(w)$ to be assigned to all other states w.

Now in the first place, $g_i(x)$ and $g_i(y)$ need not be 1 and 0; they are any two numbers that preserve i's preference between x and y. Furthermore, they need not imply the assignment of 0 or 1 to any state; if x is supremal in X under $\geq_i$, $g_i(x)$ might be -30. Suppose, however, that we choose x as unit and y as origin for person 1:

$$g_1(x) = 1, \qquad g_1(y) = 0.$$

In the second place, the numbers arbitrarily chosen for person 1, which determine his entire utility scale, determine nothing about person 2's scale. The choice of 1's numbers could restrict the choice of 2's only if the two e.r.s.'s were parts of a larger e.r.s., the relations of which our measurements are to preserve. But then such relations would precede the measurement, not follow from it. Failing such relations, that is, given individual utility measurement alone, we can neither justify nor object to the assignment of 1's unit and origin to 2. If 2 prefers x to y we can put $g_2(x) = 1$ and $g_2(y) = 0$ if we want to. Nothing in the problem forbids it. Equally, nothing requires it.

Moreover, except in the trivial and vacuous case of identical intensive preferences*, the assignment of common units or common origins, or both, implies no further relations between the scales. Obviously, such an assignment does not entail a common scale for 1 and 2 when $G_1 \neq G_2$; for in that case no common scale exists.

That persons 1 and 2 have the same utility unit, or different utility units, means only that we have chosen that way. The position would be very peculiar, if an interpersonal comparison were meaningful if (or only

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*Which does not, of course, imply an interval interpersonal comparison any more than identical preferences imply an ordinal interpersonal comparison.

if) we chose a common unit but meaningless if we chose differently, when the choice rides on nothing but our whim. Exactly the same is true of common, or as it may be, different, utility origins.*

Sen (1970) has brought further confusion to the subject with his notion, unfortunately taken up by D'Aspremont and Gevers (1977) among others, that between the case of common units and common origins, assertedly permitting _full_ interpersonal comparisons, and the case of different units and different origins, assertedly permitting _no_ such compariesons, lie intermediate cases of common units but different origins, or vice versa, which according to him permit _partial_ interpersonal comparisons. But the distinction between these four cases is utterly inconsequential. We can give persons 1 and 2 common units but different origins, or the reverse, or neither, at will; and as long as their preferences agree on some pair of states we can give them common units and common origins if we want.**

In short, units and origins have no bearing on the question of interpersonal comparisons. They have seemed important only because of a simple confusion.

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*And, of course, we can always restrict each scale to the same range, say [0,1]. Then, if both persons regard the same state x as minimal and the same state y as maximal, their scales will agree at the extremes; but no interpersonal comparison follows.

**Sen and D'Aspremont and Gevers write as if the cases, (i) $g_2(x) = \alpha g_1(x)$, $\forall x$ $(\alpha>0)$, and (ii) $g_2(x) = g_1(x) + \beta$, $\forall x$, were different. Clearly they are not. In both cases g_2 is a positive affine transformation of g_1, i.e., $G_2 = G_1$, whence individuals 1 and 2 have identical intensive preferences.

9. Proper and Improper Analogies

Our confusions rest, I believe, on the words in which we talk about utility, words that suggest implicit analogies to other fields of measurement. We talk about the "utility of Jones" in the same way we talk about his age or his temperature, and this leads us to think of his utility scale as measuring a property of himself, just as the time and temperature scales do. We are then apt to consider a pair (g_1, g_2) of utility scales as analogous, for instance, to Fahrenheidt and Celsius scales. These temperature scales are certainly different but we know their readings are comparable because they measure the same thing. They assign different temperature numbers to a person but they belong to the same equivalence class. Though we do not describe the matter in terms of equivalence classes, our experience with the word "temperature" leads us to conclude, correctly, that the Fahrenheidt and Celcius scales measure the same thing--that Jones's and Smith's temperatures are comparable even if measured on different scales. By implicit analogy, we falsely conclude from the word "utility" applied to g_1 and g_2 that they, too, measure the same thing.

Or we might think g_1 and g_2 analogous to a pair of thermometers that only have to be calibrated at two points in order to measure all temperatures on the same scale, thus falsely concluding that g_1 and g_2 need (or only need) a similar calibration (say at 1 and 0) to make "the utility of Jones" comparable to Smith's.

Our usage, "the utility of Jones" and "the utility of Smith," leads us unconsciously to think of the men as members of the set being measured, as they are when we speak of their temperature. But in utility measurement, the men do not belong to this set; they measure the set. Properly speaking, we deal not with the utilities of the men but with the utilities of states to them. A person's utility scale measures not a property of himself but a property of the set of states. It is true that any difference in intensive preferences rests ultimately on differences in the persons, which we might call a difference in their properties. But utility scales do not measure this difference, as a temperature scale measures a difference in their temperature; they merely signal its existence.

If we need an analogy, a better one is between (g_1, g_2) and, for instance, the pair (ϕ, Ψ) of temperature and intelligence scales (the latter assumed for the sake of argument to be an interval scale). The source of the analogy is that each pair happens to be defined on a single set, (g_1, g_2) on X and (ϕ, Ψ) on a set of people. Since we would never think the assignment of zero intelligence and temperature to Jones and unit intelligence and temperature to Smith gives us a comparison of intelligence and temperature, we might more easily avoid the interpersonal confusions.

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