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Revisiting the Great Ratios Hypothesis*

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Abstract

Kaldor called the constancy of certain ratios stylized facts, Klein and Kosobud called them great ratios. While they often appear in theoretical models, the empirical literature finds little evidence for them, perhaps because the procedures used cannot deal with lack of cointegration, two-way causality, and cross-country error dependence. We propose a new system pooled mean group estimator that can deal with these features. Monte Carlo results show it performs well compared with other estimators, and using it on a dataset over 150 years and 17 countries, we find support for five of the seven ratios considered.

Keywords: Great ratios, heterogeneous panels, cointegration, two-way long-run causality, error cross-sectional dependence.

JEL Classification: B4, C18, C33, C5

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1 Introduction

The idea that certain economic ratios are roughly constant in the long run is an old one. Kaldor (1957, 1961) described them as *stylized facts* and Klein and Kosobud (1961) labelled them *great ratios*. Kaldor (1957, p591) wrote: "A satisfactory model concerning the nature of the growth process in a capitalist economy must also account for the remarkable historical constancies revealed by recent empirical investigations." Other ratios thought to be constant appear in finance including the dividend-price ratio, discussed by Campbell and Shiller (1988).

Although there was some skepticism about these stylized facts,[1] they have been widely adopted in theoretical models in economics and finance, and are often implied by economic theories about the conditions required for balanced growth, arbitrage, or debt solvency. Jones and Romer (2010, p225) in a paper called "The New Kaldor facts," take the old ones for granted. They say: "Redoing this exercise nearly 50 years later shows just how much progress we have made. Kaldor's first five facts have moved from research papers to textbooks. There is no longer any interesting debate about the features that a model must contain to explain them."

The great ratios hypothesis has also prompted a large empirical literature, including recent contributions by Müller and Watson (2018), Kapetanios et al. (2020), and Harding (2020). This literature is less supportive of the hypothesis. Harding provides a survey of the literature and notes: "econometric tests reject the great ratios hypothesis but economic growth theorists and quantitative macroeconomic model builders continue to embed that hypothesis in their work." Most, but not all, of these studies focus on individual countries and consider two variables, say y_t and x_t, $t = 1, 2, ..., T$, the logarithms of the numerator and denominator for instance, and investigate if their difference, $z_t = y_t - x_t$, is stationary over a reasonably long period. This might be done by estimating a long run coefficient θ in $y_t = \theta x_t + u_t$ and testing whether $\theta = 1$ or by testing whether the difference $y_t - x_t$ is stationary,

[1] For example, Robert Solow in his classic book, Growth Theory, commented that there "is no doubt that they are stylized, though it is possible to question whether they are facts," Solow (1970, p2).

having neither stochastic nor deterministic trends.[2] Stationarity of $y_t - x_t$ can be tested either using Dickey and Fuller (1979) type tests or the KPSS test proposed by Kwiatkowski et al. (1992). Both approaches are known to have important limitations. Testing for unit roots tends to lack power, particularly against highly persistent yet stationary alternatives. The KPSS test involves estimating the long run variance of the partial sum series, $s_t = \sum_{\tau=1}^{t} z_\tau$, which requires quite long time series if the size of the test is to be controlled, particularly when z_t is stationary but highly persistent. A number of other studies treat y_t and x_t as unit root processes, or more generally as first order integrated processes, $I(1)$, and test whether they are cointegrated with a unit long run coefficient, so that z_t becomes stationary, or an $I(0)$ process.[3] The cointegration approach allows separation of the possibility of cointegration between y_t and x_t from the requirement of a unit long run coefficient. For example, it is often found that logs of real income and consumption cointegrate but their long run coefficient is not unity. Cointegration tests are more informative but suffer from similar limitations to unit root testing. Müller and Watson (2018) find that inference on long run covariability of y_t and x_t is complicated and critically depends on the exact form of their long run persistence. Harding (2020) argues that unit root and cointegration tests cannot produce valid inference on the great ratios hypothesis, because of the nature of the stationary distribution of the ratio.

This paper suggests that a possible reason for the lack of evidence for the great ratios hypothesis is the inability of the econometric procedures used to deal with lack of cointegration, two-way causality between the variables in the ratios, and cross-country error dependence; all likely features of the data. Lack of cointegration could result from failures of the error correction mechanisms that bring y_t and x_t back to their long run equilibrium relationship. Such failures of the error correction mechanisms, which may be associated with major shocks such as wars, depressions, natural disasters, or important policy failures, make it difficult

[2] If there are deterministic trends in x_t and y_t they must be co-trending such that the trend cancels out for the difference not to have a trend.

[3] There can be a long-run relationship whether the variables are both $I(0)$ or both $I(1)$ but it will only be a cointegrating relationship in the $I(1)$ case. When the variables are $I(0)$ they must be co-trending, the trends cancel out, for the great ratios hypothesis to hold.

to estimate the long run coefficients with any precision from the relatively short-span time series that are typically used for a single country. A panel data approach that considers the great ratios hypothesis across many countries may be more effective. There are a number of relevant panel estimators. The pooled mean group (PMG) estimator of Pesaran, Shin, and Smith (1999) has heterogeneous short run coefficients and homogeneous long run coefficients. Heterogeneous short run coefficients allow for failure of cointegration, since the country-specific error correction coefficients, that govern adjustments towards the long run equilibrium, can be zero in some countries. Homogeneous long run coefficients are appropriate for the great ratios, since they imply a homogeneous long run coefficient of unity across all countries. The PMG is a single equation estimator and so cannot handle two-way long run causality. Breitung (2005) modified PMG to handle two-way long run causality, but unfortunately his two-step estimator breaks down when there are non-cointegrating episodes in the panel under consideration. Similar limitations apply to other panel estimators that we consider later, including the panel dynamic OLS (PDOLS) estimator proposed by Mark and Sul (2003).

To deal with the three problems, lack of cointegration, two-way causality and error cross-section dependence, we propose a new system PMG estimator, or SPMG for short.[4] This has a vector error correction form for the two equations for y_t and x_t with a long-run coefficient θ which is homogeneous across countries, and which should take the value one if the great ratio hypothesis is true. However, the estimation method does allow for heterogeneous short run coefficients, namely the error correction coefficients and the coefficients of lagged changes across the country-specific equations. In the absence of cointegration both error-correction coefficients will be zero. SPMG is robust to lack of cointegration for some units because the error-correction coefficients effectively act as weights, and therefore the contribution of the units without cointegration to the SPMG estimate of the long-run coefficient will

[4] The SPMG estimator can also be viewed as a panel version of Johansen (1991) maximum likelihood approach where the cointegrating vectors, $\{\boldsymbol{\beta}_i, i = 1, 2, ..., n\}$ are assumed to be the same across all i ($\boldsymbol{\beta}_i = \boldsymbol{\beta}$). In this set up Johansen's reduced rank computational algorithm is no longer applicable and the estimates of the common long run coefficients, $\boldsymbol{\beta}$, and the n unit-specific error correction coefficients must be computed iteratively.

be negligible. Under regularity conditions typically assumed in the literature, the SPMG estimator of θ can be viewed as a quasi-ML (QML) estimator which is asymptotically normal, super consistent in T, the number of time periods, and converges to its true value at the rate of $T\sqrt{(1-\pi)n}$, where n is the number of countries and π is the constant fraction ($0 < \pi < 1$) of countries that do not cointegrate. No restrictions on the relative rate of $n(1-\pi)$ and T will be needed for consistency of $\hat{\theta}$ as $n(1-\pi)$ and $T \to \infty$. However, for valid inference, $(1-\pi)n/T \to 0$ is required. This condition is likely to be met in most cross-country applications, including the empirical application considered in this paper where $n = 17$ and $T = 140$. It is well known that T needs to be relatively large for estimation of long run effects. Pooling with respect to θ helps with the power of testing for $\theta = 1$, while allowing for heterogeneity in short-run dynamics.

Error cross-sectional dependence is likely in cross-country panel studies and ignoring it could lead to artificially low standard errors. To achieve reliable small sample inference in the presence of cross-sectional error dependence, we adopt a bootstrap procedure.[5] Extensive Monte Carlo (MC) experiments show that the SPMG estimator with bootstrapped confidence intervals is the only estimator that meets all three robustness criteria (two-way long run causality, lack of cointegration, and error cross-section dependence) and at the same time has satisfactory small sample properties. The MC results also show that the performance of other panel estimators considered are not satisfactory. Firstly they all suffer from large size distortions in the case of sample sizes that are typically available, even if their assumptions are met. Second, and more importantly, the assumptions they require are too restrictive for the analysis of great ratios, as they either require cointegration conditions to hold in all countries and/or the direction of long run causality between y_t and x_t to be known.

In the empirical application we use the long-span data covering the period 1870-2016 for a panel of 17 countries that has been made available in the Jordà-Schularick-Taylor (JST) macrohistory database.[6] We focus on the SPMG estimator, but for comparison we also

[5] Cross-sectional dependence is often modelled by an unobserved factor, but having another factor shift the long run relationship is not compatible with the great ratio hypothesis. Thus correcting the standard errors is appropriate.

[6] See Jordà, Schularick, and Taylor (2017) and Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019),

report estimates obtained using PMG, PDOLS, Breitung's estimator, and the mean group estimator based on individual Müller and Watson (2018) estimates. We present estimates of the mean long run coefficient for seven theoretical relationships. For consumption on GDP, the SPMG estimator of the long run coefficient is 0.907 (0.884-0.930), with the bootstrapped 95% confidence interval in brackets; for investment on GDP the long run coefficient is 1.044 (1.029-1.059); for imports on exports the long run coefficient is 0.967 (0.961-0.973); for government debt on GDP we have 1.051 (0.993-1.108); for short on long interest rates we obtain 1.010 (0.912-1.108); for inflation on long interest rates 0.653 (0.419-0.888); and finally for inflation on money growth 1.227 (1.153-1.300). From a statistical perspective, with the exception of two ratios (debt-GDP and long-short rates), all the remaining long run estimates are significantly different from one at the 5% level. But given that the long run estimates are based on a large number of observations (147 years pooled across 17 countries), perhaps it is not surprising that some of the point estimates that are close to unity are still found to be statistically different from unity. From an economic perspective, all but two (those involving inflation) are quite close to unity, with their point estimates falling in the narrow range of 0.9 to 1.1. Of the seven long run relations considered we can confidently conclude that our empirical results do not support the hypothesis of a unit long run relationship between inflation and the long-term interest rate, and inflation and money supply growth. The remaining five long run relations tend to support the great ratio hypothesis, and their inclusion in macroeconomic model seems to us to be justified.

The rest of the paper is set out as follows: Section 2 reviews the literature on the theory and econometrics of the great ratios hypothesis. Section 3 introduces the system pooled mean group estimator. Section 4 summarizes Monte Carlo evidence on the performance of individual estimators. Section 5 provides empirical evidence for the presence of unit elasticities for seven great ratios using the different estimators, and discusses the findings. Section 6 ends with some concluding remarks. A Data Appendix provides additional information on the data and an online supplement provides details of the individual panel estimators; the boot-

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and the link http://www.macrohistory.net/data.

strapping procedure adopted; a description of the design of the Monte Carlo experiments; and Monte Carlo results.

2 The great ratios hypothesis

Three types of theoretical mechanisms for stationary great ratios have been suggested in the literature. These arise from the requirements for balanced growth, arbitrage or solvency.

The first mechanism relates to the conditions required for the existence of steady states in growth models. In deterministic neoclassical growth models where technical progress happens at a constant rate, balanced growth paths require output, capital, investment and consumption to grow at the same rate, implying constant consumption-output, capital-output and investment-output ratios. In stochastic settings these ratios vary over time but must be stationary.[7] Attfield and Temple (2010) point out that the equilibrium values of the great ratios depend on the structural parameters of the growth model, which may vary over time.

A second mechanism is through arbitrage where profitable opportunities due to misalignments of prices in product or asset markets are exploited, removing the divergence. The central issue is that certain pairs of variables cannot diverge indefinitely, since even small differences in growth rates blow up over the longer term. Examples are real wages and labour productivity, that underlie the labour share; stock prices and dividends as in Campbell and Shiller (1988); or prices of similar goods in different countries denominated in the same currency.

A third mechanism operates through solvency conditions on variables like the balance of payments and government debt as shares of income, which requires that countries cannot accumulate debt indefinitely. However, as argued by Bohn (2007), solvency cannot be inferred from the statistical properties of debt because the inter-temporal budget constraint and transversality condition impose little restriction on the time series properties of the vari-

[7] Deterministic models of growth were considered by Solow (1956), Swan (1956), and Barro and Sala-i-Martin (1995), among others. Stochastic growth models were developed by Merton (1975), Donaldson and Mehra (1983) and Binder and Pesaran (1999) among others.

ables. Chudik et al. (2017) show for a panel of countries that, while log public debt and income are I(1), they do not cointegrate for around half of the countries considered. Even for those that cointegrate, there are statistically significant departures from the unit elasticity.

Empirical studies that attempt to test the great ratios hypotheses face a range of difficulties There are measurement issues. Cette et al. (2019), and Barro (2021) raise these in the case of the share of wages,. There are issues with the techniques. Müller and Watson (2018) argue that problems arise due to lack of sufficiently long samples and the fact that inference depends on the long run persistence properties of the underlying variables. There are also well known problems with testing for unit roots, discussed in the introduction. Stock and Watson (2017) following Elliott (1998) emphasize that evidence for cointegration can be very fragile in the case of departures from exact unit roots. There are further problems with bounded time series, such as ratios that lie between zero and one, Cavaliere and Xu (2014). Testing for deterministic trends can also be problematic as shown recently by Elliott (2020). However empirical studies such as Harvey et al. (2003), who examine four ratios for the G7 countries, Mills (2009) who analyses the Klein-Kosobud data using modern econometric techniques and Tropimov (2017) who considers the stability of the capital output ratio, all tend to question the long run stability of the great ratios.

The possibility that lack of cointegration results from the failure of the error correction mechanisms that bring y_t and x_t back to their long run equilibrium relationship has been considered. Siklos and Granger (1997) proposed the concept of regime-sensitive cointegration whereby the variables fall in and out of an equilibrium relationship and the underlying series need not be cointegrated at all times. Cointegration is switched off when a common stochastic trend is added. Psaradakis et al. (2004) consider Markov switching error-correction models where the speed of adjustment to equilibrium could be different in different regimes.

To examine why the great ratios hypothesis might fail, consider two variables, x_{it} and y_{it}, which might be the logarithms of the numerator or denominator of one of the great ratios, for country $i = 1, 2, ..., n$ over the period $t = 1, 2, ..., T$, with the error correction representation:[8]

$$\begin{aligned} \Delta y_{it} &= a_{yi} - \phi_{yi}\xi_{i,t-1} + \sum_{\ell=1}^{p-1} \boldsymbol{\psi}'_{yi\ell}\Delta \mathbf{w}_{i,t-\ell} + u_{yit}, & (1) \\ \Delta x_{it} &= a_{xi} - \phi_{xi}\xi_{i,t-1} + \sum_{\ell=1}^{p-1} \boldsymbol{\psi}'_{xi\ell}\Delta \mathbf{w}_{i,t-\ell} + u_{xit}, & (2) \end{aligned}$$

where $\Delta y_{it} = y_{it} - y_{i,t-1}$, $\Delta x_{it} = x_{it} - x_{i,t-1}$, $\Delta \mathbf{w}_{it} = (\Delta y_{it}, \Delta x_{it})'$, with the common error correction term defined by

$$\xi_{it}(\theta_i) = y_{it} - \theta_i x_{it} - \mu_{it}. \quad (3)$$

The above system of equations allows for two-way short run as well as long run feedbacks between y_{it} and x_{it}. Also by allowing the error correction coefficients, ϕ_{yi} and ϕ_{xi}, to vary over countries the above specification can deal with failure of error correction in some countries such that $\phi_{xi} = \phi_{yi} = 0$.

In the above set up, the great ratios hypothesis applied to $\mathbf{w}_{it} = (y_{it}, x_{it})'$ may fail because (i) the long run coefficient is not unity, $\theta_i \neq 1$; (ii) there are trends or level shifts in the long run relationship, $\mu_{it} \neq 0$; (iii) there is no adjustment, $\phi_{xi} = \phi_{yi} = 0$. For instance, Kapetanios et al. (2020) assume $\theta_i = 1$, and show that for a number of ratios calculated using recent UK data ξ_{it} is $I(1)$ using a constant μ_{it}, but $I(0)$ using a non-parametric estimate of a slowly varying μ_{it}. We abstract from intercept shifts in the long run relations and set $\mu_{it} = 0$ in (1).

3 System pooled mean group estimator

Since the null of the great ratios hypothesis is a homogeneous long run coefficient, namely $\theta_i = \theta = 1$, for all i, the objective is to estimate θ and test $H_0 : \theta = 1$, allowing for the fact that there may be failure of error correction such that ϕ_{yi} or ϕ_{xi} can be zero in some countries. To do this we need an estimator of θ that allows for (i) two-way long run causality between x_{it} and y_{it} (ii) possible failure of error correction, $\phi_{yi} = \phi_{xi} = 0$ for some countries,

[8] Given our interest in great ratios we focus on bivariate relationships. Applications with more than two variables are subjects of further research.

and (iii) robust inference on testing $\theta = 1$, that allows for the cross-section dependence. We propose a system pooled mean group (SPMG) estimator that has these characteristics.

As noted above, since under the great ratios hypothesis the long run coefficients, θ_i, are the same across countries, then the PMG estimator of Pesaran, Shin, and Smith (1999) is a natural starting choice. But it only applies if direction of long run causality between y_{it} and x_{it} is known, and does not allow for two-way long run causality between y_{it} and x_{it}. The concept of long run causality is discussed in Granger and Lin (1995) and Pesaran, Shin, and Smith (2001), and in the context of (1) and (2) is defined in terms of ϕ_{yi} and ϕ_{xi}. Specifically, x_{it} (y_{it}) is said to long run cause y_{it} (x_{it}) if $\phi_{yi} \neq 0$ and $\phi_{xi} = 0$ ($\phi_{yi} = 0$ and $\phi_{xi} \neq 0$). Two-way long run causality arises when ϕ_{yi} and ϕ_{xi} are both non-zero. It is also worth noting that long run causality does not rule out short-term feedbacks from $\Delta y_{i,t-\ell}$, $\ell = 1, 2, ...p-1$, to Δx_{it} (and *vice versa*). As can be seen from (1) and (2) lagged changes of both variables appear in both equations. In the case of most great ratios, there is no reason to believe that there is a single known direction of causality, whether in the short or long run.

Denoting the 2×1 vector $\boldsymbol{\phi}_i = \left(\phi_{yi}, \phi_{xi}\right)'$, we allow for $\boldsymbol{\phi}_i = \mathbf{0}$ for some countries. The system of equations (1)-(3) can be written as

$$\Delta \mathbf{w}_{it} = -\boldsymbol{\phi}_i \boldsymbol{\beta}' \mathbf{w}_{i,t-1} + \boldsymbol{\Upsilon}_i \mathbf{q}_{it} + \mathbf{u}_{it}, \tag{4}$$

where $\mathbf{w}_{it} = (y_{it}, x_{it})'$, $\boldsymbol{\beta} = (1, -\theta)'$, $\boldsymbol{\Upsilon}_i = (\mathbf{a}_i, \boldsymbol{\Psi}_{i,1}, \boldsymbol{\Psi}_{i,2}, ..., \boldsymbol{\Psi}_{i,p-1})'$, $\mathbf{a}_i = (a_{yi}, a_{xi})'$, $\boldsymbol{\Psi}_{i\ell} = \left(\boldsymbol{\psi}_{yi\ell}, \boldsymbol{\psi}_{xi\ell}\right)'$, for $\ell = 1, 2, ..., p-1$, $\mathbf{q}_{it} = \left(1, \Delta \mathbf{w}'_{i,t-1}, \Delta \mathbf{w}'_{i,t-2}, ..., \Delta \mathbf{w}'_{i,t-p+1}\right)'$, and $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ is a 2×1 error vector with $E(\mathbf{u}_{it}) = \mathbf{0}$, and $E(\mathbf{u}'_{it}\mathbf{u}_{it}) = \boldsymbol{\Sigma}_i$, a positive definite covariance matrix.

To deal with two-way long run causality, Breitung (2005) considers the 2×1 vector $\boldsymbol{\gamma}_i = \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i$ and assumes that $\boldsymbol{\gamma}'_i \boldsymbol{\phi}_i = \boldsymbol{\phi}'_i \boldsymbol{\Sigma}_i^{-1} \boldsymbol{\phi}_i \neq 0$, for all i. Then pre-multiplying both sides of

(4) by $\boldsymbol{\gamma}_i'$ he obtains[9]

$$\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\Delta\mathbf{w}_{it} = -\left(\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right)\boldsymbol{\beta}'\mathbf{w}_{i,t-1} + \boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\Upsilon}_i\mathbf{q}_{it} + \boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{u}_{it},$$

which in turn yields

$$z_{it} = -\boldsymbol{\beta}'\mathbf{w}_{i,t-1} + \boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\Upsilon}_i\mathbf{q}_{it} + \mathbf{v}_{it}, \tag{5}$$

where $z_{it} = \left(\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right)^{-1}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\Delta\mathbf{w}_{it}$, and $\mathbf{v}_{it} = \left(\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right)^{-1}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\mathbf{u}_{it}$. Under the normalization $\boldsymbol{\beta} = (1, -\theta)'$, the above equation can be written equivalently as

$$z_{it}^{+} = \theta x_{i,t-1} + \boldsymbol{\kappa}_i'\mathbf{q}_{it} + v_{it}, \tag{6}$$

where $z_{it}^{+} = \left(\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right)^{-1}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\Delta\mathbf{w}_{it} + y_{i,t-1}$, and $\boldsymbol{\kappa}_i = \boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\Upsilon}_i$.[10] Breitung (2005) proposed two-step estimator where the initial estimates of $\boldsymbol{\phi}_i$ and $\boldsymbol{\Sigma}_i$ are obtained from the first step regressions (4) for each cross section unit not imposing homogeneity of θ (using Johansen or Engle-Granger approach), while pooled θ is estimated from the second stage (6). However, when $\boldsymbol{\phi}_i = \mathbf{0}$ for some i, then (5)-(6) are no longer defined, and the two-step Breitung's estimator is not robust to absence of cointegration in some cross section units, since the variance of $\left(\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}\right)^{-1}$ blows up if $\boldsymbol{\phi}_i = \mathbf{0}$.

The SPMG allows for two-way causality without needing to use the inverse of $\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i$. We follow the same likelihood approach as the PMG but maximize the system log-likelihood function for $\mathbf{w}_{it}$ given by the model (4), allowing for one or both elements of $\boldsymbol{\phi}_i$ to be zero for some i.

Under Gaussian errors, the log-likelihood function for unit i conditional on the initial

[9] Breitung considers a more general set up where there are $k \geq 2$ variables and $0 < r < k$ cointegrating relationships.

[10] Abstracting from higher order lags, and from deterministic terms, equations (5) and (6) correspond to equations (3) and (6) of Breitung (2005) for $k = 2$, and $r = 1$.

observations $\mathbf{w}_{i,1}, \mathbf{w}_{i,2}, ..., \mathbf{w}_{i,p}$ is given by

$$\begin{aligned}\mathcal{L}_{i,T}\left(\theta,\boldsymbol{\phi}_i,\boldsymbol{\Sigma}_i\right) &= -\frac{(T-p)}{2}\ln\left(2\pi\right)+(T-p)\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| \\ &\quad -\frac{1}{2}\sum_{t=p+1}^{T}\mathbf{u}_{it}'\boldsymbol{\Sigma}_i^{-1}\mathbf{u}_{it},\end{aligned}$$

where $\mathbf{u}_{it}\left(\theta,\boldsymbol{\phi}_i\right)$ is defined by (4) which we write more compactly as

$$\mathbf{u}_{it} = \Delta\mathbf{w}_{it} + \boldsymbol{\phi}_i\xi_{i,t-1}\left(\theta\right) - \boldsymbol{\Upsilon}_i\mathbf{q}_{it},$$

with the error correction term, $\xi_{it}\left(\theta\right) = y_{it} - \theta x_{it}$. Concentrating out the effects of short-term dynamics (represented by $\mathbf{q}_{it}$) and pooling the individual log-likelihood functions under error cross-sectional independence, we now obtain the following concentrated system log-likelihood function

$$\begin{aligned}\mathcal{L}_{n,T}\left(\theta,\boldsymbol{\phi},\boldsymbol{\Sigma}\right) &= -\frac{(T-p)\,n}{2}\ln\left(2\pi\right)+(T-p)\sum_{i=1}^{n}\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| \\ &\quad -\frac{1}{2}\sum_{i=1}^{n}\sum_{t=p+1}^{T}\tilde{\mathbf{u}}_{it}\left(\theta,\boldsymbol{\phi}_i\right)'\boldsymbol{\Sigma}_i^{-1}\tilde{\mathbf{u}}_{it}\left(\theta,\boldsymbol{\phi}_i\right),\end{aligned} \tag{7}$$

where $\boldsymbol{\phi} = (\boldsymbol{\phi}_1', \boldsymbol{\phi}_2', ..., \boldsymbol{\phi}_n')'$, $\boldsymbol{\Sigma} = (\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, ..., \boldsymbol{\Sigma}_n)'$, $\tilde{\mathbf{u}}_{it}\left(\theta,\boldsymbol{\phi}_i\right)'$are row-vectors of $\tilde{\mathbf{U}}_i = [\tilde{\mathbf{u}}_{i,p+1}\left(\theta,\boldsymbol{\phi}_i\right),$ $\tilde{\mathbf{u}}_{i,2}\left(\theta,\boldsymbol{\phi}_i\right),\ ...,\ \tilde{\mathbf{u}}_{i,T}\left(\theta,\boldsymbol{\phi}_i\right)]'$ given by

$$\tilde{\mathbf{U}}_i = \tilde{\mathbf{U}}_i\left(\theta,\boldsymbol{\phi}_i\right) = \mathbf{H}_i\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}\left(\theta\right)\boldsymbol{\phi}_i'\right], \tag{8}$$

in which $\boldsymbol{\xi}_{i,-1}\left(\theta\right) = \left[\xi_{ip}\left(\theta\right), \xi_{i,p+1}\left(\theta\right), ..., \xi_{i,T-1}\left(\theta\right)\right]'$, $\Delta\mathbf{W}_i = (\Delta\mathbf{w}_{i,p+1}, \Delta\mathbf{w}_{i,p+2,}, ..., \Delta\mathbf{w}_{iT})'$, $\Delta\mathbf{w}_{it} = (\Delta y_{it}, \Delta x_{it})'$. $\mathbf{H}_i$ is orthogonal projection matrix given by $\mathbf{H}_i = \mathbf{I}_{T-p} - \mathbf{Q}_i(\mathbf{Q}_i'\mathbf{Q}_i)^{-1}\mathbf{Q}_i'$, where $\mathbf{Q}_i$ is matrix of observations on $\mathbf{q}_{it}$, namely $\mathbf{Q}_i = (\mathbf{q}_{i,p+1}, \mathbf{q}_{i,p+2}, ..., \mathbf{q}_{i,T})'$.

Following this approach, the first order conditions for θ, $\boldsymbol{\phi}_i$, and $\boldsymbol{\Sigma}_i$ imply the following implicit solutions for MLE estimators, $\hat{\theta}$, $\hat{\boldsymbol{\phi}}_i$ and $\hat{\boldsymbol{\Sigma}}_i$:

$$\hat{\theta} = -\left[\sum_{i=1}^{n}\left(\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i\right)\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right]^{-1}\sum_{i=1}^{n}\mathbf{x}_{i,-1}'\mathbf{H}_i\left(\Delta\mathbf{W}_i + \mathbf{y}_{i,-1}\hat{\boldsymbol{\phi}}_i'\right)\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i, \tag{9}$$

$$\hat{\boldsymbol{\phi}}_i = -\left[\boldsymbol{\xi}_{i,-1}'(\theta)\,\mathbf{H}_i\boldsymbol{\xi}_{i,-1}(\theta)\right]^{-1}\Delta\mathbf{W}_i'\mathbf{H}_i\boldsymbol{\xi}_{i,-1}'(\theta), \tag{10}$$

and

$$\hat{\boldsymbol{\Sigma}}_i = (T-p)^{-1}\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}(\theta)\,\hat{\boldsymbol{\phi}}_i'\right]'\mathbf{H}_i\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}(\theta)\,\hat{\boldsymbol{\phi}}_i'\right], \tag{11}$$

where $\mathbf{x}_{i,-1} = (x_{ip}, x_{i,p+1}, ..., x_{i,T-1})'$, $\mathbf{y}_{i,-1} = (y_{ip}, y_{i,p+1}, ..., y_{i,T-1})'$. Given an initial estimate of θ, say $\hat{\theta}^{(1)}$, initial estimates $\hat{\boldsymbol{\phi}}_i^{(1)}$ and $\hat{\boldsymbol{\Sigma}}_i^{(1)}$ can be computed using (10) and (11). The estimates $\hat{\boldsymbol{\phi}}_i^{(1)}$ and $\hat{\boldsymbol{\Sigma}}_i^{(1)}$, can then be used to update the estimate of θ, say $\hat{\theta}^{(2)}$, using (9), and so on until convergence.

Note that $\hat{\boldsymbol{\phi}}_i$ (and $\hat{\boldsymbol{\Sigma}}_i^{-1}$) effectively act as weights in (9). Specifically, when $\hat{\boldsymbol{\phi}}_i \rightarrow_p \mathbf{0}$ for some i, then the contribution of this unit to the pooled estimate of $\hat{\theta}$ in (9) will be negligible, and tends to zero as $T \rightarrow \infty$. This is why SPMG continues to be applicable even if y_{it} and x_{it} do not cointegrate for some (but not all) units. In addition, SPMG is invariant to the ordering of the variables, whilst the two-step version of Breitung (2005) is not. More formally, following the literature, assume that $\mathbf{u}_{it}$ are independently distributed over i and t, with mean $\mathbf{0}$ and finite positive definite covariances, $\boldsymbol{\Sigma}_i$ (specifically $0 < c < \lambda_{min}(\boldsymbol{\Sigma}_i) < \lambda_{max}(\boldsymbol{\Sigma}_i) < C < \infty$), and $\mathbf{w}_{it} \sim I(1)$ for all i, but allow a number of error correction vectors, $\boldsymbol{\phi}_i$, to be zero (non-cointegrating). Specifically, without loss of generality, suppose that

$$\boldsymbol{\phi}_i'\boldsymbol{\phi}_i > 0, \text{ for } i = 1, 2, ..., m, \tag{12}$$

$$\boldsymbol{\phi}_i = \mathbf{0}, \text{ for } i = m+1, m+2, ..., n. \tag{13}$$

Using standard results from the literature we have $\hat{\boldsymbol{\phi}}_i = \boldsymbol{\phi}_i + o_p(1)$, and $\hat{\boldsymbol{\Sigma}}_i^{-1} = \boldsymbol{\Sigma}_i^{-1} + o_p(1)$, and[11]

$$\mathbf{H}_i \left(\Delta \mathbf{W}_i + \mathbf{y}_{i,-1}\hat{\boldsymbol{\phi}}_i'\right) = \mathbf{H}_i \left(\mathbf{U}_i - \theta_0 \mathbf{H}_i \mathbf{x}_{i,-1}\boldsymbol{\phi}_i'\right) + o_p(1).$$

where θ_0 is the true value of θ. Using these results in (9) it now follows that

$$n^{1/2}T\left(\hat{\theta} - \theta_0\right) = - \ Q_{nT}^{-1} q_{nT} + o_p(1), \tag{14}$$

where

$$Q_{nT} = n^{-1}\sum_{i=1}^{n}\left(\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right)\left(T^{-2}\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right), \text{ and } q_{nT} = n^{-1/2}\sum_{i=1}^{n}\left(T^{-1}\sum_{t=1}^{T}\tilde{x}_{i,t-1}\mathbf{u}_{it}'\right)\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i. \tag{15}$$

Under cross-sectional independence, and certain regularity conditions concerning the moments of $\mathbf{u}_{it}$, it then follows that $n^{1/2}T\left(\hat{\theta} - \theta\right)$ tends to a Gaussian distribution so long as Q_{nT} tends to a non-zero limit as n and $T \to \infty$.[12] Since $\mathbf{w}_{it}$ is a unit root process, then $T^{-2}\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}$ converges to a stochastically bounded and strictly positive random variable, and it is sufficient that $n^{-1}\sum_{i=1}^{n}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i$ also converges to a non-zero limit. Under (12) and (13) we have (note that by assumption $\lambda_{max}\left(\boldsymbol{\Sigma}_i\right) < C < \infty$)

$$n^{-1}\sum_{i=1}^{n}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i = \left(\frac{m}{n}\right)\left[m^{-1}\sum_{i=1}^{m}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right] > (1/C)\left(\frac{m}{n}\right)\left(m^{-1}\sum_{i=1}^{m}\boldsymbol{\phi}_i'\boldsymbol{\phi}_i\right),$$

and Q_{nT} tends to a non-zero limit as n and $m \to \infty$, if $m/n = 1 - \pi > 0$, where π is the fraction of units that are not cointegrating. Therefore, under regularity conditions typically assumed in the literature, the SPMG estimator of θ can be viewed as a quasi-ML (QML) estimator which is asymptotically normal and converges to its true value at the rate of $T\sqrt{(1-\pi)n}$. The convergence rate of SPMG estimator in terms of n and T is not affected by a non-zero π, so long as π is not too close to unity. However, in practice the effective number of units in the panel is discounted by the proportion of non-cointegrating units.

[11] See, for example, Pesaran, Shin, and Smith (1999) and Breitung (2005), and the references cited therein.

[12] As noted in the introduction, no restrictions on the relative rate of n and T is needed for consistency of $\hat{\theta}$ as n and $T \to \infty$. However, $n/T \to 0$ is required for valid inference, but this is likely to be satisfied in the type of applications considered.

We carry out inference using conventional standard errors (assuming error cross-sectional independence), robust standard errors (allowing for arbitrary error cross-sectional dependence), and bootstrapped confidence intervals, outlined in Sections S.1 and S.2.3 of the online supplement.

There are also a number of time series estimators of unit-specific cointegrating vectors, such as the original Engle-Granger, fully-modified OLS, dynamic OLS, and ARDL as well as system estimators like Johansen that can be used to investigate the empirical validity of the great ratios hypothesis.[13] These estimators can be averaged across countries to yield corresponding mean group (MG) panel data estimators introduced in Pesaran and Smith (1995). This not only reduces country-specific sampling errors, but also allows the calculation of non-parametric standard errors directly based on the individual estimates, which are robust to serial correlation and heteroskedasticity at the individual country level, avoiding complicated inference problems. Furthermore, as shown in Chudik and Pesaran (2019), the MG group procedure is valid if the error cross-sectional dependence is weak. However, the MG estimator requires that all country-specific estimators are consistent, a condition which will not be met when the cointegration condition does not hold for some of the units under consideration. To ensure that all units being considered are in fact cointegrating involves pre-testing and is subject to further complications. For these reasons we focus on PMG and system PMG estimators. To deal with the possibility of strong error cross-sectional dependence we consider robust standard errors (outlined in equation (S.26)) and, in addition, we adopt a bootstrap algorithm, with details provided in the online supplement.[14]

In our application, the SPMG estimator does not allow for time variation in $\boldsymbol{\phi}$. The analysis can be extended to scenarios where $\boldsymbol{\phi} = \mathbf{0}$ for some sub-periods and $\boldsymbol{\phi} \neq \mathbf{0}$ in others, by considering each sub-period as if it relates to a new synthetic country. This extension was not pursued because identification of such sub-periods is not straightforward,

[13] See Engle and Granger (1987), Phillips and Hansen (1990), Johansen (1991), Stock and Watson (1993), Phillips (1995), and Pesaran and Shin (1999).

[14] While the robust standard errors work well for a sufficiently large T, regardless of cross sectional dependence, bootstrap SPMG confidence intervals performed the best in all our Monte Carlo experiments summarized in the next section.

and the time series data available for many countries is rather short and any sample splits can lead to biased estimates due to short sub-samples.

4 Monte Carlo evidence

In this section we investigate the finite-sample performance of alternative panel estimators of long run relationships, and accuracy (confidence interval coverage rate) of tests of the common long run coefficient which is of interest in the analysis of great ratios. As noted already, the focus of our Monte Carlo experiments is on (i) robustness of the estimators to possible failure of cointegration in the case of some units, (ii) the ability of the estimator to perform well regardless of the direction of long run causality, and last but not least (iii) the robustness of inference based on the long run estimates to heteroskedasticity and cross-sectional error dependence. This section provides summary of Monte Carlo findings. A detailed account of the estimators used and the design and results of these experiments is presented in the online supplement.

4.1 Summary of data generating process

In the discussion above and in the empirical application we assume that while $\boldsymbol{\phi}_i$ may equal zero, it is not time varying. For the Monte Carlo we represent the failure of cointegration by allowing the $\boldsymbol{\phi}$ to be time varying and to be zero for a proportion of the time. Accordingly, we generate $w_{it} = (y_{it}, x_{it})'$ using a VAR(2) model, which we write in the error-correcting representation as

$$\Delta y_{it} = a_{yi} - \phi_{yit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{yyi}\Delta y_{i,t-1} + \psi_{yxi}\Delta x_{i,t-1} + u_{yit},$$

$$\Delta x_{it} = a_{xi} - \phi_{xit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{xyi}\Delta y_{i,t-1} + \psi_{xxi}\Delta x_{i,t-1} + u_{xit}.$$

The coefficient of interest is θ, the long run coefficient, which we set equal to one.

When the error-correcting coefficients $\phi_{yit} \neq 0$ and $\phi_{xit} = 0$, there is a long run relationship with long run causality from x to y, which we denote as $x \to y$. When both $\phi_{yit}, \phi_{xit} \neq 0$ the long run causality runs both ways, which we denote as $x \leftrightarrow y$. Initially, we consider data generating processes (DGP) where the direction of long run causality is from x to y ($x \to y$) and set $\phi_{xit} = 0$ for all i, t, and generate ϕ_{yit} to be non-zero except for a number of non-cointegrating episodes with durations that vary from a minimum of 10 periods and a maximum of T periods (namely the full sample). This is achieved by setting

$$\phi_{yit} = \begin{cases} 0, & \text{for } t \in \mathcal{T}_i \\ \phi_{yi}, & \text{for } t \notin \mathcal{T}_i \end{cases},$$

where $\phi_{yi} \sim IIDU\,(0.1, 0.25)$ and $\mathcal{T}_i$ denotes the set of non-cointegrating episodes for unit i, with each episode having duration T_i. With probability $(1 - \pi)$, we set $T_i = 0$ (namely $\mathcal{T}_i = \emptyset$), and with probability π, we draw T_i uniformly from the set of integers $\{10, 11, ..., T\}$. The start of non-cointegrating episodes in $\mathcal{T}_i$ is also generated stochastically. In experiments where $x \leftrightarrow y$, we generate ϕ_{xit} similarly to ϕ_{yit}, namely using the identical index sets $\mathcal{T}_i$, and generate ϕ_{xi} as $\phi_{xi} \sim IIDU\,(-0.15, -0.05)$. The parameter π controls the occurrence of non-cointegrating episodes. We consider $\pi = 0$, the benchmark case with no non-cointegrating episodes, $\pi = 0.05$ with relatively few non-cointegrating episodes, and $\pi = 0.2$ with more frequent non-cointegrating episodes.

Another key aspect of our design is the strength of error cross-sectional dependence which has importance implications for inference, in particular. We consider three options, the independent case, where $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ is independent of $\mathbf{u}_{jt}$ for all $i \neq j$, and two cross sectionally correlated cases with spatial and latent factor dependence. Specifically, we consider a spatial autoregressive model (SAR) and a mixed spatial factor model. We set the spatial autoregressive parameter to 0.6, and allow the factors to be strong. In all cases, $\mathbf{u}_{it}$ are generated allowing for non-zero contemporaneous covariances, $Cov\,(u_{yit}, u_{xit}) \neq 0$, using both Gaussian and non-Gaussian error distributions, to check the robustness of different estimation methods to departures from Gaussianity.

In total we consider 36 different experiments, spanning the choices of (a) π (probability of non-cointegration), (b) the direction of long run causality, (c) error cross section dependence, and (d) error distributions.[15] For each experiment we consider 20 pairs of sample size combinations obtained from $T \in \{50, 100, 150, 200\}$ and $n \in \{17, 30, 50, 100, 200\}$, and use $R_{MC} = 2000$ replications to obtain the results. We include $n = 17$ because that is the number of countries in our empirical application. Among the choices of (n, T), the smaller values are relevant for typical empirical applications in economics, whereas the larger choices of (n, T) are interesting from an econometric perspective as they shed more light on the consistency, validity of asymptotic standard errors, and the relative importance of n and T dimensions. We consider five estimators outlined below.

4.2 Five estimators

We consider the PMG estimator by Pesaran, Shin, and Smith (1999) (with asymptotic inference or with bootstrapped inference outlined in online supplement), and the system PMG estimator outlined above. The third estimator is mean group estimator based on individual Müller and Watson (2018) estimates. In particular, we split the sample period into q non-overlapping sub-samples of (approximately) equal size, where q is treated as fixed as (n, T) changes. We then take simple temporal averages for each of the subsamples, and average individual cross-section specific least squares estimates computed using the sample of q temporally averaged periods. We refer to this estimator as MGMW. Due to temporal averaging, this estimator has the potential to be quite robust. The fourth estimator included in this study is two-step estimator by Breitung (2005), and the fifth estimator is the off-the-shelf popular panel dynamic OLS (PDOLS) estimator by Mark and Sul (2003). We do not make any modifications to PDOLS estimator and its inclusion in this study is for completeness and for the comparison purposes. PDOLS need not be robust to episodes of non-cointegration and/or cross-sectionally correlated panels, but it is an important benchmark in the litera-

[15] In the online supplement we also consider time series heteroskedasticity and show that this does not substantially affect coverage rates.

ture.[16]

4.3 Summary of Monte Carlo findings

We begin with reporting the results for the baseline case in Tables 1 and 3 where the errors are cross-sectionally independent and Gaussian, the long run causality is known to run from x to y, and there are no episodes of non-cointegration. In this case we expect all five estimators to work reasonably well. Focusing on the sample size combination closest to our empirical applications, namely $T = 100$, and $n = 17$, the best RMSE value of 0.0231 is achieved by PMG estimator, followed closely by SPMG with RMSE of 0.0253, two-step Breitung with RMSE of 0.0256, with RMSEs of the remaining estimators quite a bit higher, falling in the range 0.0289 to 0.0528. (See Table 1) From an econometric perspective, a difference in RMSE of over 125% between the worst (MGMW) and the best (PMG) estimators is large. From an economic perspective, however, even the worst RMSE is rather small and all five estimators yield reasonably precise estimates of the long run coefficient in the baseline experiments. A similar ordering of the five estimators is obtained when we consider bias, except for the SPMG which now has the smallest bias followed by PMG. The results also show the importance of the T dimension for the performance of all the five estimators.

The 95 per cent coverage rate for the different estimators of the long run coefficient are reported in Tables 1 and Table 3. For ($T = 100$, $n = 17$) sample size the simulated coverage rates vary from 79.4% (for PDOLS, $p = 1$, in Table 1) to 91.5% - 94.0% (bootstrapped confidence intervals of the PMG and the SPMG reported in Table 3). Bootstrapped inference reported in Table 3 appears to be uniformly better than the conventional alternatives reported in Table 1.[17] While for the values of $T > 100$, coverage rates are reasonably good, this is not the case for $T = 50$, where the conventional confidence intervals are in the range

[16] Additional panel estimators in the literature are the panel Fully modified OLS (FMOLS) estimator by Pedroni (2001) and the recently introduced panel Bewley estimator by Chudik, Pesaran, and Smith (2021).

[17] We have also considered a robust alternative to the conventional estimator of SPMG variance, outlined in equation (S.26) in the online supplement, and its thresholding version (S.27). While the robust standard errors work very well for $T = 200$, regardless of cross sectional dependence (and for all our choices of n, see Table S38), they suffer from the same small sample drawbacks as the conventional standard errors. Bootstrap SPMG confidence intervals performed the best in all Monte Carlo experiments.

of 8.5% to 86.7%.

Consider now the most "demanding" experiment under which we allow for non-Gaussian and cross-sectionally correlated errors with both spatial and factor dependencies (denoted as factor + SAR), two-way long run causality, and $\pi = 0.2$ (a relatively high occurrence of non-cointegrating episodes). The results of this case are summarized in Tables 2 and 3.[18] The SPMG (with bootstrapped confidence intervals in Table 3) emerges as the only reliable estimator and is therefore a clear winner. It is also the only estimator without serious bias. For $T = 100$, and $n = 17$, its bias is only 0.0004 compared with the range of -0.0373 to -0.0098 for the other estimators. Similarly the RMSE value of SPMG estimator at 0.0183 is substantially smaller than the RMSE obtained for the other estimators, which are 0.0210 for PMG, 0.0669 for MGMW, 0.0723 for two-step Breitung estimator, and 0.0634, 0.0660, 0.0781, for PDOLS(1,4,8). The large gap between SPMG and two-step Breitung estimators is primarily due to the fact that 20% of the cross-section units in these experiments are not cointegrating, and this is not allowed under two-step Breitung estimator. The need to bootstrap for accurate inference is again confirmed by the 94.0% coverage of the SPMG bootstrapped confidence interval compared with the conventional coverage of only 67.6%. Coverage rates of the remaining estimators are poor, and fall in the range 58% to 88%.

While no Monte Carlo exercise, regardless of how extensive or carefully designed, can guarantee the reliability of any particular estimator in real datasets, it can illuminate lack of robustness or other problems in a controlled setting containing features thought to be found in real world data. As is well known, confidence intervals designed for cross sectionally independent errors are invalid when errors are in fact cross sectionally dependent, and the seriousness of this problem is clearly documented by the detailed Monte Carlo results in the online supplement. In addition, while it is not guaranteed that confidence intervals that are robust to cross section dependence will perform well in practice, the bootstrapped confidence intervals adopted in this paper had rather good coverage rates.

[18] Summary results for all other experiments (between these two extremes reported in Tables 1-3) are provided in the online supplement. Findings presented in the supplement suggest that non-Gaussianity does not have significant influence on the results. This suggests that the distributional form of the errors is unlikely to be of great importance in practice.

Tests indicate that cross section correlation of residuals is clearly present in the data used in this paper. The other two aspects of our design - two way causality and the existence of non-converging episodes - cannot be as easily validated, but are both plausible a priori. Thus it is reassuring to have estimator that is robust to those features.

We also considered the small sample properties of the MG estimators based on Johansen and ARDL individual country estimates, but found that they did not perform well as compared to the pooled approach, particularly when compared to the system PMG estimator. As is well known, MG estimators requires the underlying individual estimates to have finite moments, and this condition does not hold in general. This was found to be the case when we used country-specific Johansen's estimates.

We also considered a number of other approaches to deal with non-cointegrating episodes. Given the long span of our data and the possibility of no cointegration during particular episodes we tried averaging estimates over sub-periods within each cross-section unit. We also tried pre-testing whereby we first tested for cointegration before including the estimate when computing the MG estimator. After considerable investigation, we found out that splitting sample into subperiods, does not seem to be beneficial, because (as the Monte Carlo experiments show), having a large T dimension is crucial for estimation and inference. There was mixed evidence on the value of pre-testing for the existence of level relationships. Pre-testing is not required for PMG and system PMG estimators since the individual estimates are weighted by the magnitude of error-correcting coefficients, which makes them robust to non-cointegrating units.

5 Empirical evidence

In the empirical application, we estimate pooled long run coefficients for seven bivariate relationships using a panel of 17 countries over the years 1870-2016 from the Jordà-Schularick-Taylor macro-history database. The relations are: (1) the logarithms of real consumption per capita and real GDP per capita; (2) the logarithms of investment and GDP; (3) the logarithms of imports and exports; (4) the logarithms of public debt and GDP; (5) short and long interest rates, (6) inflation and long interest rates; and (7) inflation and money growth.[19] In each case we estimate the long run coefficient and its 95% confidence interval. While the dynamics of adjustments might differ across countries, the great ratios hypothesis implies that the long run coefficient will take a common value of unity. Details of the data and variables are given in the data appendix. Except for the export-import relationship, these pairs overlap with those considered by MW. They use a 68 year post-war US sample and consider data on some other variables including unemployment, total factor productivity, stock returns, dividends and earnings.

We consider the same estimators investigated in the Monte Carlo section. Regarding the PDOLS and its corresponding Monte Carlo evidence on negative consequences of a too short lead/lag order, we only consider the longer lead and lag orders, $p = 4$ and 8. The estimators reported below are (1) PMG, the Pooled Mean Group estimator of Pesaran, Shin, and Smith (1999); (2) SPMG; (3) MGMW, the mean group estimator based on Müller and Watson (2018) country-specific estimates, using temporally aggregated data into $q = 5$ sub-periods; (4) the two-step Breitung (2005) estimator; (5) PDOLS, $p = 4$ is the panel dynamic OLS estimator by Mark and Sul (2003) using 4 leads and lags; and (6) PDOLS, $p = 8$. For the PMG and SPMG two sets of confidence intervals are provided, asymptotic and bootstrap, for the other estimators only asymptotic confidence intervals are given. More details can be found in the online supplement.

We use the largest available balanced panel for each pair of variables, as described in data appendix. The number of countries (n) ranges from 14 (investment) to 17 and the number of time periods (T) from 121 to 143 years.

Other than the SPMG the estimators are not invariant to normalization and like Müller and Watson (2018) we provide estimates under both long run causal ordering, namely $\hat{\theta}_{y.x}$ and $\hat{\theta}_{x.y}$. Only in the case of SPMG is one estimator exactly the reciprocal of the other, namely $\hat{\theta}_{y.x}\hat{\theta}_{x.y} = 1$. This property does not hold for other estimators that depend on

[19] Strictly the last three relationships are not ratios as such, but their long run constancy is often assumed.

whether y_t is regressed on x_t or *vice versa.*

The pooled estimates of the long run coefficients together with their 95% confidence intervals are summarized in Table 4. For each pair of variables (y, x) we report six different estimates of $\hat{\theta}_{y.x}$ and $\hat{\theta}_{x.y}$, namely PMG, SPMG, PMW, Breitung, PDOLS(4), and PDOLS(8). To check for possible error cross-sectional dependence, at the bottom of Table 4 we also report the average pair wise correlation coefficient of residuals from the panel data models and related CD test statistics due to Pesaran (2004, 2015). It is clear that there are significant degrees of error cross-section dependence and for statistical testing it is prudent to focus on bootstrapped confidence intervals reported for PMG and SPMG estimators.

With such a large number of observations and quite small standard errors, in some cases, it is not clear that traditional significance testing is the appropriate criteria for judging closeness to unity. For debt-GDP and long-short interest rates relationships, the estimated long run coefficients are close to unity and not significantly different from it at the 5 per cent level. The SPMG (with bootstrapped 95% confidence interval) gives long run coefficients of $\hat{\theta}_{Debt-GDP} = 1.05$ (0.993-1.108) for debt on GDP and $\hat{\theta}_{Short-Long} = 1.01$ (0.912-1.108) for short on long rates. Similar results are obtained using the other estimators. The bracketed figures refer to 95% confidence intervals.

For investment-GDP and imports-exports, the long run coefficients are also estimated to be close to unity. But due to their high precision the null hypothesis that the long run coefficient is in fact unity gets rejected. The SPMG gives $\hat{\theta}_{INV-GDP} = 1.044$ (1.029-1.059) for investment on GDP and $\hat{\theta}_{IM-EX} = 0.967$ (0.961-0.973) for imports on exports. Again similar results are obtained when other estimators are considered.

The estimates of the long run coefficient for consumption-GDP pair are somewhat away from one. For example, using the SPMG method we obtain $\hat{\theta}_{CON-GDP} = 0.907$ (0.884-0.930), with the other estimates slightly lower ranging from a low of 0.883 when we use PDOLS(4), and 0.900 when we use PMG.

For the remaining two relationships the evidence is more mixed, with different estimators yielding different results. For regressions of inflation on money supply growth the estimates of $\theta_{INF-Money}$ are not significantly different from one in the case of MGMW, Breitung, PDOLS(4), and PDOLS(8) estimators. But an opposite conclusion is reached if the long run coefficient is estimated by running regressions of money supply growth on inflation. This is the case if we consider Breitung, PDOLS(4) and PDOLS(8) estimators. A unit long run relationship between money supply growth and inflation is supported only by MGMW estimator irrespective of which way the regressions are implemented. PMG and SPMG both strongly reject the null of a unit long run relationship between inflation and money supply growth.

Almost all estimates of the long run coefficient of inflation on long term interest rate are significantly below unity - the exception being when the long run coefficient is estimated by PDOLS(8) using the regression of inflation on the long term interest rate. Even in this case the long run coefficient is poorly estimated and an opposite conclusion is reached if the long run estimate is computed from the reverse regression of the long term rate on inflation.

Overall, it is quite encouraging that five out of the seven long run coefficients are quite close to unity, with substantial empirical evidence in support of Debt to GDP and Imports to Exports as being great ratios, and the difference between long and short interest rates being stationary. The evidence on consumption-GDP and investment-GDP as great ratios is less overwhelming. This is particularly problematic for consumption-GDP ratio where the largest estimate obtained for the long run elasticity of consumption to GDP is 0.907 (using SPMG) which is difficult to rationalize. At the level of the cross section of individual households this could correspond to the well established pattern that savings as a proportion of income increase with income, the rich save more. At the level of the time series for a country, this could correspond to the fact that the measured private consumption is only a part of total consumption and with increasing income, government consumption has accounted for a growing part.

6 Concluding comments

By using long span panel data and a robust estimator we provide more evidence for close to unit elasticities for two balanced growth conditions, two solvency conditions and a stable term structure, but found evidence against a unit long run coefficient in the case of the Fisher relationship and the inflation money growth relationship.

We relied on long-span panel data to overcome the drawbacks of the single-country regressions. However, panel analysis of great ratios presents its own challenges - namely possible failures of cointegration, the unknown direction of long run causality, and cross-sectionally correlated observations. To overcome these challenges, we have proposed a new system pooled mean group estimator. Monte Carlo simulations show that SPMG performs better in small samples than alternative estimators even in the presence of two-way long run causality, failure of cointegration, and error cross-sectional dependence.

TABLE 1

MC results for the estimation of long run coefficient $\theta_0 = 1$ in the baseline experiments

	Bias (×100)				**RMSE (×100)**				**Coverage rate (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator											
17	-1.05	-0.22	-0.09	-0.06	5.49	2.31	1.40	1.00	71.1	83.8	89.0	90.5
30	-0.86	-0.19	-0.09	-0.05	4.22	1.64	1.03	0.74	67.7	85.1	88.2	90.3
50	-1.03	-0.26	-0.10	-0.05	3.21	1.32	0.79	0.57	69.9	83.6	88.8	89.5
100	-0.91	-0.22	-0.08	-0.05	2.26	0.89	0.55	0.39	68.2	85.0	88.8	91.5
200	-0.98	-0.21	-0.09	-0.05	1.76	0.64	0.38	0.28	62.0	84.0	89.4	91.2
	SPMG estimator											
17	0.12	0.06	0.04	0.01	6.71	2.53	1.47	1.03	59.5	79.4	86.2	89.1
30	0.24	0.07	0.02	0.01	4.95	1.74	1.06	0.76	59.1	80.5	86.8	88.7
50	-0.01	-0.04	-0.01	0.00	3.59	1.38	0.80	0.59	59.9	79.6	87.0	87.8
100	0.10	0.01	0.01	0.00	2.46	0.93	0.57	0.40	61.2	81.0	86.0	89.6
200	0.00	0.01	0.01	0.00	1.71	0.65	0.39	0.28	61.3	81.4	87.2	88.6
	MGMW estimator, $q = 5$											
17	-5.54	-1.97	-0.98	-0.60	10.53	5.28	3.56	2.62	86.7	92.1	93.9	94.1
30	-5.64	-1.84	-0.87	-0.49	8.92	4.19	2.80	2.04	82.2	90.7	92.5	92.3
50	-5.64	-2.09	-0.99	-0.55	7.63	3.52	2.18	1.56	76.8	87.2	90.3	92.3
100	-5.59	-1.91	-0.96	-0.55	6.76	2.77	1.66	1.15	63.4	82.4	89.0	91.8
200	-5.55	-1.92	-0.93	-0.54	6.14	2.38	1.36	0.90	40.7	71.4	82.2	88.2
	two-step Breitung's estimator											
17	-2.56	-0.66	-0.30	-0.18	5.64	2.56	1.60	1.14	76.2	85.5	90.1	91.2
30	-2.25	-0.57	-0.26	-0.13	4.46	1.90	1.14	0.85	74.6	85.7	90.5	91.1
50	-2.34	-0.65	-0.28	-0.15	3.71	1.54	0.91	0.66	68.9	84.5	89.7	91.8
100	-2.30	-0.62	-0.28	-0.15	3.08	1.14	0.66	0.47	57.2	81.8	88.9	90.7
200	-2.31	-0.62	-0.27	-0.15	2.72	0.92	0.51	0.35	39.2	75.3	85.2	87.9
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.86	-2.39	-1.58	-1.17	6.82	3.44	2.27	1.67	75.8	79.4	80.5	81.8
30	-4.65	-2.30	-1.52	-1.11	5.94	2.93	1.91	1.42	69.2	70.9	71.8	73.1
50	-4.73	-2.38	-1.54	-1.12	5.50	2.76	1.79	1.31	56.0	54.1	55.9	59.4
100	-4.65	-2.31	-1.52	-1.11	5.07	2.51	1.65	1.21	31.7	30.4	31.4	32.8
200	-4.63	-2.30	-1.50	-1.10	4.83	2.40	1.57	1.15	8.5	7.1	7.6	7.8
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.73	-1.20	-0.76	-0.56	6.59	2.89	1.79	1.30	83.9	88.6	90.1	90.7
30	-2.47	-1.09	-0.71	-0.51	5.23	2.23	1.36	1.01	82.9	87.0	89.3	88.8
50	-2.49	-1.16	-0.74	-0.52	4.18	1.88	1.17	0.84	80.8	83.9	84.6	85.8
100	-2.50	-1.14	-0.73	-0.52	3.48	1.55	0.97	0.70	72.3	75.0	77.4	77.2
200	-2.48	-1.14	-0.72	-0.52	3.01	1.35	0.85	0.62	59.7	61.0	61.2	60.8
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.34	-0.60	-0.32	-0.24	10.82	3.27	1.83	1.28	70.4	87.7	90.8	92.4
30	-1.48	-0.46	-0.30	-0.20	8.14	2.41	1.32	0.95	71.1	89.6	92.1	92.2
50	-1.42	-0.52	-0.31	-0.21	5.95	1.83	1.05	0.73	75.2	89.5	90.8	92.4
100	-1.33	-0.49	-0.31	-0.21	4.35	1.34	0.77	0.54	75.1	88.6	90.0	90.9
200	-1.30	-0.52	-0.31	-0.22	3.24	1.02	0.59	0.42	76.1	85.6	87.8	88.2

Notes: Coverage rate is 95% confidence interval coverage rate. This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0$, and no cross section dependence of errors. See Section S.3.1 of the online supplement for full details of the data generating process. Description of the PMG, SPMG, MGMW, and 2-step Breitung estimators, and the description of bootstrapping procedures are provided in Sections S.2.1-S.2.3 of the online Supplement. PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). The number of Monte Carlo replications is $R_{MC} = 2000$.

TABLE 2

MC results for the estimation of long run coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, $x \leftrightarrow y$, $\pi = 0.2$ and CS dependence of errors

	Bias (×100)				**RMSE (×100)**				**Coverage rate (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator											
17	-1.98	-0.98	-0.65	-0.51	4.54	2.10	1.34	1.02	54.9	62.0	65.8	66.2
30	-1.74	-0.87	-0.62	-0.45	3.40	1.72	1.12	0.83	50.4	57.8	58.0	59.7
50	-1.77	-0.85	-0.58	-0.42	3.01	1.43	0.93	0.70	45.2	50.7	52.9	55.0
100	-1.62	-0.85	-0.56	-0.41	2.44	1.23	0.80	0.60	35.9	38.7	39.7	43.4
200	-1.59	-0.82	-0.55	-0.41	2.20	1.11	0.73	0.54	26.9	27.6	27.5	28.8
	SPMG estimator											
17	-0.02	0.04	0.01	0.01	5.92	1.83	1.12	0.85	53.8	67.6	71.6	72.3
30	0.12	0.05	-0.02	0.00	3.23	1.40	0.88	0.64	53.0	64.3	67.5	69.8
50	0.00	0.05	0.03	0.02	2.43	1.05	0.66	0.48	53.0	64.0	68.6	71.4
100	0.10	-0.01	0.00	0.01	1.85	0.74	0.46	0.34	51.2	63.2	69.9	71.4
200	0.05	0.01	0.01	0.01	1.24	0.53	0.33	0.24	50.1	63.0	67.5	70.3
	MGMW estimator, $q = 5$											
17	-2.65	-1.92	-1.39	-1.26	8.97	6.69	5.72	5.45	83.9	87.8	89.2	92.3
30	-2.04	-1.62	-1.23	-1.11	7.06	5.18	4.74	4.40	83.5	87.6	89.3	90.1
50	-2.41	-1.48	-1.21	-0.95	6.02	4.08	3.66	3.36	81.3	84.1	87.6	88.5
100	-1.95	-1.40	-1.21	-0.95	4.59	3.22	2.87	2.55	78.5	82.7	83.8	85.9
200	-2.04	-1.32	-1.09	-1.01	4.08	2.67	2.30	2.17	72.3	78.1	79.6	80.3
	two-step Breitung's estimator											
17	-4.15	-2.82	-2.20	-2.26	10.12	7.23	6.47	7.65	50.8	58.4	61.6	61.4
30	-4.04	-2.69	-2.21	-1.94	8.42	6.28	5.31	4.90	49.0	54.2	58.8	60.9
50	-4.19	-2.53	-2.18	-1.83	7.14	5.13	4.76	4.28	41.0	52.6	55.8	59.5
100	-4.04	-2.47	-2.16	-1.99	6.13	4.56	3.78	4.20	33.2	45.5	48.1	49.9
200	-4.04	-2.51	-2.11	-1.85	5.28	3.69	3.10	2.82	22.9	35.6	38.0	41.6
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.16	-3.73	-2.97	-2.74	8.72	6.34	5.42	5.17	75.4	83.0	82.7	86.1
30	-4.97	-3.61	-2.95	-2.59	7.31	5.37	4.64	4.15	74.5	79.3	81.2	84.6
50	-5.14	-3.39	-2.88	-2.51	6.81	4.64	3.98	3.62	65.2	75.0	74.2	78.7
100	-4.97	-3.33	-2.90	-2.54	6.03	4.10	3.62	3.24	56.4	62.3	62.7	64.4
200	-4.89	-3.35	-2.82	-2.51	5.65	3.85	3.29	2.95	44.8	44.2	45.0	50.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.42	-3.34	-2.69	-2.53	9.70	6.60	5.61	5.33	75.9	84.6	85.5	88.4
30	-4.27	-3.27	-2.69	-2.37	7.91	5.51	4.68	4.16	73.3	83.0	84.8	88.2
50	-4.52	-3.02	-2.60	-2.30	7.04	4.59	3.93	3.57	70.9	81.5	82.0	84.5
100	-4.38	-2.96	-2.65	-2.33	5.99	3.94	3.50	3.14	63.7	74.5	75.1	74.1
200	-4.35	-3.01	-2.57	-2.30	5.48	3.64	3.11	2.79	56.9	62.1	60.7	63.9
	PDOLS estimator, leads and lags order $p = 8$											
17	-3.65	-3.29	-2.64	-2.51	13.62	7.41	6.10	5.69	58.2	83.0	85.1	87.8
30	-3.62	-3.23	-2.65	-2.32	10.63	6.17	5.00	4.38	56.5	81.0	84.2	88.0
50	-3.93	-2.92	-2.54	-2.23	8.74	4.95	4.15	3.69	57.5	80.7	82.7	86.1
100	-3.88	-2.84	-2.62	-2.29	7.12	4.14	3.64	3.22	52.9	75.5	77.2	77.4
200	-4.04	-2.91	-2.53	-2.27	6.24	3.77	3.18	2.83	48.2	66.7	67.1	69.0

Notes: Coverage rate is 95% confidence interval coverage rate. This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table 1.

TABLE 3

Robust bootstrapped 95% confidence interval coverage rates (×100) for PMG and SPMG estimators in MC experiments

$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	Baseline experiments							
	PMG estimator				SPMG estimator			
17	90.1	93.5	94.4	94.4	86.7	91.5	93.6	94.2
30	89.0	94.3	94.0	94.9	86.1	93.8	93.7	94.6
50	89.0	92.7	94.7	94.2	86.2	92.1	94.2	94.2
100	89.9	93.5	94.5	94.8	86.0	93.5	93.9	93.9
200	85.8	93.3	94.9	95.0	87.5	93.1	94.7	95.0
	Experiments with non-Gaussian errors, $x \leftrightarrow y$, $\pi = 0.2$ and CS dependence of errors							
	PMG estimator				SPMG estimator			
17	87.8	91.8	90.6	91.2	88.5	94.0	93.8	95.0
30	87.6	87.9	88.2	88.1	89.4	92.3	93.6	93.2
50	83.9	86.1	86.4	87.3	89.5	92.4	93.5	94.3
100	78.3	78.8	79.7	81.2	87.5	92.4	94.0	94.1
200	71.8	71.5	71.8	71.9	88.7	92.4	93.6	94.7

Notes: Bootstrapped critical values are computed in each of the Monte Carlo replication as described in Sections S.2.1-S.2.3 of the online supplement, based on $R_b = 2000$ bootstrap replications. See notes to Table 1 and 2.

TABLE 4

Direct and reverse estimates of long run coefficient θ and 95% confidence intervals (CI)

y_{it} :	Real consumption per capita		Investment		Imports		Debt	
x_{it} :	Real GDP per capita		GDP		Exports		GDP	
	y on x	x on y	y on x	x on y	y on x	x on y	y on x	x on y
	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$	$\hat{\theta}_{y.x}$	$\hat{\theta}_{x.y}$
PMG	**0.900**	**1.109**	**1.043**	**0.965**	**0.963**	**1.033**	**1.054**	**1.002**
asymptotic CI	(0.884-0.915)	(1.083-1.134)	(1.034-1.052)	(0.955-0.975)	(0.957-0.969)	(1.028-1.039)	(1.026-1.082)	(0.982-1.022)
bootstrap CI	(0.876-0.923)	(1.063-1.154)	(1.027-1.058)	(0.950-0.980)	(0.954-0.972)	(1.026-1.041)	(0.986-1.122)	(0.969-1.035)
SPMG	**0.907**	**1.103**	**1.044**	**0.958**	**0.967**	**1.035**	**1.051**	**0.952**
asymptotic CI	(0.893-0.921)	(1.086-1.120)	(1.035-1.052)	(0.950-0.966)	(0.962-0.971)	(1.029-1.040)	(1.026-1.075)	(0.929-0.974)
bootstrap CI	(0.884-0.930)	(1.074-1.132)	(1.029-1.059)	(0.945-0.971)	(0.961-0.973)	(1.028-1.041)	(0.993-1.108)	(0.905-0.998)
MGMW	**0.885**	**1.128**	**1.070**	**0.934**	**0.977**	**1.025**	**1.046**	**0.956**
asymptotic CI	(0.844-0.926)	(1.080-1.176)	(1.050-1.090)	(0.917-0.952)	(0.956-0.997)	(1.004-1.046)	(0.975-1.117)	(0.893-1.018)
Breitung	**0.898**	**1.107**	**1.035**	**0.966**	**0.983**	**1.017**	**1.012**	**0.980**
asymptotic CI	(0.872-0.923)	(1.076-1.138)	(1.027-1.042)	(0.959-0.973)	(0.979-0.986)	(1.013-1.021)	(0.987-1.037)	(0.956-1.004)
PDOLS, p=4	**0.883**	**1.109**	**1.036**	**0.962**	**0.983**	**1.017**	**1.002**	**0.984**
asymptotic CI	(0.846-0.919)	(1.069-1.150)	(1.021-1.052)	(0.945-0.978)	(0.975-0.991)	(1.008-1.025)	(0.969-1.035)	(0.948-1.020)
PDOLS, p=8	**0.887**	**1.110**	**1.035**	**0.963**	**0.982**	**1.019**	**0.996**	**0.995**
asymptotic CI	(0.848-0.925)	(1.070-1.150)	(1.023-1.048)	(0.950-0.977)	(0.975-0.989)	(1.008-1.029)	(0.966-1.026)	(0.963-1.026)
n	15	15	14	14	17	17	16	16
T	143	143	131	131	131	131	121	121
	Cross section dependence of residuals							
	u_{yt}	u_{xt}	u_{yt}	u_{xt}	u_{yt}	u_{xt}	u_{yt}	u_{xt}
CD test	22.16	21.32	16.21	14.98	37.77	33.39	3.72	16.22
Ave. pair-wise corr.	0.182	0.175	0.150	0.138	0.285	0.252	0.031	0.136

Notes: Description of the PMG, SPMG, MGMW and Breitung estimators is provided in Sections S.2.1-S.2.3 of the online supplement. PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). Bootstrapped confidence intervals are based on $R_b = 2000$ bootstrap replications. The source for all variables is the Jordà-Schularick-Taylor (JST) macro-history database available at http://www.macrohistory.net/data/, see, Jordà, Schularick, and Taylor (2017) and Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019). See Table A1 in the data appendix for variable descriptions. Estimations are conducted using the largest balanced panel as described in the data appendix.

TABLE 4 (CONTINUED)

Direct and reverse estimates of long run coefficient θ and 95% confidence intervals (CI)

y_{it} :	Short IR		Inflation		Inflation	
x_{it} :	Long IR		Long IR		Money	
	y on x $\hat{\theta}_{y.x}$	x on y $\hat{\theta}_{x.y}$	y on x $\hat{\theta}_{y.x}$	x on y $\hat{\theta}_{x.y}$	y on x $\hat{\theta}_{y.x}$	x on y $\hat{\theta}_{x.y}$
PMG	**0.971**	**0.865**	**0.521**	**0.567**	**0.677**	**0.650**
asymptotic CI	(0.920-1.022)	(0.815-0.915)	(0.356-0.685)	(0.455-0.679)	(0.619-0.736)	(0.606-0.695)
bootstrap CI	(0.867-1.075)	(0.755-0.975)	(0.303-0.739)	(0.380-0.754)	(0.601-0.753)	(0.595-0.706)
SPMG	**1.010**	**0.990**	**0.653**	**1.530**	**1.227**	**0.815**
asymptotic CI	(0.963-1.057)	(0.947-1.032)	(0.498-0.809)	(1.418-1.641)	(1.169-1.284)	(0.775-0.854)
bootstrap CI	(0.912-1.108)	(0.899-1.081)	(0.419-0.888)	(1.256-1.803)	(1.153-1.300)	(0.763-0.866)
MGMW	**1.020**	**0.857**	**0.878**	**0.429**	**0.943**	**0.952**
asymptotic CI	(0.929-1.110)	(0.774-0.939)	(0.758-0.999)	(0.273-0.585)	(0.805-1.082)	(0.829-1.074)
Breitung	**1.001**	**0.945**	**0.665**	**0.239**	**1.041**	**0.696**
asymptotic CI	(0.932-1.069)	(0.884-1.005)	(0.380-0.949)	(0.179-0.299)	(0.964-1.118)	(0.649-0.743)
PDOLS, p=4	**0.986**	**0.868**	**0.630**	**0.127**	**1.051**	**0.756**
asymptotic CI	(0.892-1.079)	(0.779-0.956)	(0.309-0.950)	(0.029-0.226)	(0.933-1.169)	(0.675-0.837)
PDOLS, p=8	**0.960**	**0.901**	**0.747**	**0.194**	**1.025**	**0.795**
asymptotic CI	(0.862-1.058)	(0.798-1.004)	(0.369-1.126)	(0.062-0.326)	(0.884-1.166)	(0.685-0.906)
n	16	16	17	17	16	16
T	120	120	137	137	133	133
Cross section dependence of residuals						
	u_{yt}	u_{xt}	u_{yt}	u_{xt}	u_{yt}	u_{xt}
CD test	33.15	40.34	31.58	50.96	19.69	13.47
Ave. pair-wise corr.	0.279	0.339	0.233	0.376	0.157	0.107

See notes on the previous page.

A Data appendix

The data are taken from the Jordà-Schularick-Taylor (JST) Macrohistory Database (available at http://www.macrohistory.net/data/),[20] see Jordà, Schularick, and Taylor (2017). Jordà et al. (2019) provides further discussion of the rate of return data. JST provide data for 17 countries over the period 1870-2016. There are clearly issues with the measurement of economic variables over such a long span. However, JST is a carefully compiled database which has been widely used. The data are assembled from a wide variety of sources with different definitions of variables and countries vary because of boundary changes.[21] For instance, the long interest rate is on government bonds, with a maturity typically around 10 years, but sometimes longer like the British Consols which were perpetuals. From about 1950 the maturity is fairly accurately defined at about 10 years. While the series may be noisy, there is a lot of variation, so the signal-noise ratio may be high.

Table A1 lists the series we use together with their availability. We use the largest balanced panel for estimations. For each country i, we omit gap years (if any) and compute the number of available time periods for each country, denoted as T_i. Then we re-order countries so that $T_1 \geq T_2 \geq ... \geq T_{n_{\max}}$. Note that T_1 is the largest time dimension (and the largest number of observations) if only one country was to be used for estimation, $2T_2$ is the largest number of observations for a balanced panel if two countries were chosen for estimation, and so on. We find $n^* = \max_{1 \leq n \leq n_{\max}} \{nT_n\}$, and the largest balanced panel features n^* countries and T_{n^*} periods.

[20] We downloaded version JSTdatasetR4 (Release 4, May 2019), in particular we have downloaded datafile http://www.macrohistory.net/JST/JSTdatasetR4.xlsx.

[21] There is detailed documentation of the sources at http://www.macrohistory.net/data/.

TABLE A1

*Variable description and data availability**

Name	Gap years**	Description	Variable construction***
Real consumption per capita	88 (3.5%)	log of real consumption per capita index	log(rconpc)
Real GDP per capita	0 (0%)	log of real GDP per capita index	log(rgdppc)
GDP	25 (1%)	log of nominal GDP (local ccy)	log(gdp)
Investment	220 (8.8%)	log of nominal investment (local ccy)	log(iy*gdp)
Imports	41 (1.6%)	log of nominal imports (local ccy)	log(imports)
Exports	41 (1.6%)	log of nominal exports (local ccy)	log(exports)
Public Debt	184 (7.4%)	log of public debt (local ccy)	log(debtgdp*gdp)
Short IR	148 (5.9%)	short nominal interest rate, log(1+r/100), r is in % per year	log(1+stir/100)
Long IR	35 (1.4%)	long nominal interest rate, log(1+r/100), r is in % per year	log(1+ltrate/100)
Inflation	17 (0.7%)	Annual Consumer Price Inflation	log(cpi/cpi(-1))
Money	172 (6.9%)	Annual nominal broad money growth	log(money/money(-1))

Notes: (*) The source for all variables is the Jordà-Schularick-Taylor (JST) macrohistory database available at http://www.macrohistory.net/data/, see, Jordà, Schularick, and Taylor (2017) and Jordà, Knoll, Kuvshinov, Schularick, and Taylor (2019). We have downloaded the latest version available at the beginning of September 2020, which is "JSTdatasetR4" (Release 4, May 2019).

(**) The full sample covers $n_{\max} = 17$ countries and $T = 147$ years, together $n_{\max} T_{\max} = 2499$ country-year datapoints. Countries are: Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, UK, Italy, Japan, Netherlands, Norway, Portugal, Sweden, and USA. Time is 1870-2016. The column 'Gap years' reports the number of country-year data points with missing data. The shares of the gap years in the overall sample are reported in the parentheses.

(***) The column 'Variable construction' shows variable transformations referencing the underlying variable codes in the JST database.

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Online Supplement to "Revisiting the Great Ratios Hypothesis"

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This online supplement consists of three sections. Section S.1 provides the quasi log-likelihood function, first order conditions, and asymptotic variance of the SPMG estimator. Section S.2 describes implementation of the PMG, SPMG, 2-step Breitung, and MGMW estimators, including bootstrapping procedures. Section S.3 provides a full account of Monte Carlo (MC) experiments, including a detailed description of MC design, and the full set of MC findings.

S.1 Likelihood function, first order conditions, and asymptotic variance of SPMG estimator

We assume observations on $\mathbf{w}_{it} = (y_{it}, x_{it})'$ are available for $i = 1, 2, ..., n$ and $t = 1, 2, ..., T$. Our model is given by (1)-(3), which can be conveniently written as (4), namely:

$$\Delta \mathbf{w}_{it} = -\boldsymbol{\phi}_i \boldsymbol{\beta}' \mathbf{w}_{i,t-1} + \boldsymbol{\Upsilon}_i \mathbf{q}_{it} + \mathbf{u}_{it}, \tag{S.1}$$

for $i = 1, 2, ..., n$ and $t = p+1, p+2, ..., T$, where $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ is a reduced-form error vector with $E(\mathbf{u}_{it}) = \mathbf{0}$, and $E(\mathbf{u}_{it}'\mathbf{u}_{it}) = \boldsymbol{\Sigma}_i$, a positive definite covariance matrix. Also, $\boldsymbol{\beta} = (1, -\theta)'$, $\mathbf{q}_{it} = \left(1, \Delta\mathbf{w}_{i,t-1}', \Delta\mathbf{w}_{i,t-2}', ..., \Delta\mathbf{w}_{i,t-p+1}'\right)'$, and $\boldsymbol{\Upsilon} = (\mathbf{a}_i, \boldsymbol{\Psi}_{i,1}, \boldsymbol{\Psi}_{i,2}, ..., \boldsymbol{\Psi}_{i,p-1})'$, with $\mathbf{a}_i = (a_{yi}, a_{xi})'$ and $\boldsymbol{\Psi}_{i\ell} = \left(\boldsymbol{\psi}_{yi\ell}, \boldsymbol{\psi}_{xi\ell}\right)'$ for $\ell = 1, 2, ..., p-1$.

For notational convenience, we also define $\boldsymbol{\phi} = \left(\boldsymbol{\phi}_1', \boldsymbol{\phi}_2', ..., \boldsymbol{\phi}_n'\right)'$, $\boldsymbol{\Upsilon} = (\boldsymbol{\Upsilon}_1', \boldsymbol{\Upsilon}_2', ..., \boldsymbol{\Upsilon}_n')'$, and $\boldsymbol{\Sigma} = (\boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, ..., \boldsymbol{\Sigma}_n)'$. In addition, let

$$\boldsymbol{\beta}'\mathbf{w}_{i,t-1} = y_{it} - \theta x_{it} = \xi_{i,t-1}(\theta).$$

Assuming $\mathbf{u}_{it} \sim IIDN\left(\mathbf{0}, \boldsymbol{\Sigma}_i\right)$, independently distributed over i, the log-likelihood function conditional on the initial observations $\mathbf{w}_{i,1}, \mathbf{w}_{i,2}, ..., \mathbf{w}_{i,p}$, is given by:

$$\begin{aligned}\mathcal{L}_{n,T}\left(\theta, \boldsymbol{\phi}, \boldsymbol{\Upsilon}, \boldsymbol{\Sigma}\right) &= -\frac{(T-p)\,n}{2}\ln\left(2\pi\right) + \frac{(T-p)}{2}\sum_{i=1}^{n}\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| \\ &\quad -\frac{1}{2}\sum_{i=1}^{n}\sum_{t=p+1}^{T}\left(\Delta\mathbf{w}_{it} + \boldsymbol{\phi}_i\xi_{i,t-1}\left(\theta\right) - \boldsymbol{\Upsilon}_i\mathbf{q}_{it}\right)'\boldsymbol{\Sigma}_i^{-1}\left(\Delta\mathbf{w}_{it} + \boldsymbol{\phi}_i\xi_{i,t-1}\left(\theta\right) - \boldsymbol{\Upsilon}_i\mathbf{q}_{it}\right).\end{aligned}$$

Consider the projection matrix $\mathbf{H}_i = \mathbf{I}_{T-p} - \mathbf{Q}_i(\mathbf{Q}_i'\mathbf{Q}_i)^{-1}\mathbf{Q}_i'$, where $\mathbf{Q}_i$ is a matrix of observations on $\mathbf{q}_{it} = \left(1, \Delta\mathbf{w}_{i,t-1}', \Delta\mathbf{w}_{i,t-2}', ..., \Delta\mathbf{w}_{i,t-p+1}'\right)'$, namely $\mathbf{Q}_i = \left(\mathbf{q}_{i,p+1}, \mathbf{q}_{i,p+2}, ..., \mathbf{q}_{i,T}\right)'$. In addition, let $\mathbf{x}_{i,-1} = \left(x_{i,p}, x_{i,p+1}, ..., x_{i,T-1}\right)'$, $\mathbf{y}_{i,-1} = \left(y_{i,p}, y_{i,p+1}, ..., y_{i,T-1}\right)'$, $\boldsymbol{\xi}_{i,-1} = \left(\xi_{ip}, \xi_{i,p+1}, ..., \xi_{i,T-1}\right)'$, $\Delta\mathbf{W}_i = \left(\Delta\mathbf{w}_{i,p+1,}\Delta\mathbf{w}_{i,p+2,}, ..., \Delta\mathbf{w}_{iT}\right)'$, and define

$$\Delta\tilde{\mathbf{W}}_i = \mathbf{H}_i\Delta\mathbf{W}_i,\ \tilde{\boldsymbol{\xi}}_{i,-1}\left(\theta\right) = \mathbf{H}_i\boldsymbol{\xi}_{i,-1}\left(\theta\right),$$

where row vectors of $\Delta\tilde{\mathbf{W}}_i$ and elements of $\tilde{\boldsymbol{\xi}}_{i,-1}\left(\theta\right)$ are denoted as $\Delta\tilde{\mathbf{W}}_i = \left(\Delta\tilde{\mathbf{w}}_{i,p+1,}\Delta\tilde{\mathbf{w}}_{i,p+2,}, ..., \Delta\tilde{\mathbf{w}}_{iT}\right)'$ and $\tilde{\boldsymbol{\xi}}_{i,-1}\left(\theta\right) = \left[\tilde{\xi}_{ip}\left(\theta\right), \tilde{\xi}_{i,p+1}\left(\theta\right), ..., \tilde{\xi}_{i,T-1}\left(\theta\right)\right]'$.

Concentrating $\boldsymbol{\Upsilon}$ out, the concentrated log-likelihood function is given by

$$\begin{aligned}\mathcal{L}_{n,T}\left(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma}\right) &= -\frac{(T-p)\,n}{2}\ln\left(2\pi\right) + (T-p)\sum_{i=1}^{n}\ln\left|\boldsymbol{\Sigma}_i^{-1}\right| \\ &\quad -\frac{1}{2}\sum_{i=1}^{n}\sum_{t=p+1}^{T}\left[\Delta\tilde{\mathbf{w}}_{it} + \boldsymbol{\phi}_i\tilde{\xi}_{i,t-1}\left(\theta\right)\right]'\boldsymbol{\Sigma}_i^{-1}\left[\Delta\tilde{\mathbf{w}}_{it} + \boldsymbol{\phi}_i\tilde{\xi}_{i,t-1}\left(\theta\right)\right],\end{aligned}$$

which corresponds to equation (7) in the paper.

Partial derivatives of $\mathcal{L}_{n,T}\left(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma}\right)$ with respect to θ and $\boldsymbol{\phi}$ are

$$\begin{aligned}\frac{\partial\mathcal{L}_{n,T}\left(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma}\right)}{\partial\theta} &= \sum_{i=1}^{n}\sum_{t=p+1}^{T}\left(\tilde{x}_{i,t-1}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\Delta\tilde{\mathbf{w}}_{it} + \boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\tilde{x}_{i,t-1}\tilde{y}_{i,t-1}\right) \\ &\quad -\sum_{i=1}^{n}\sum_{t=p+1}^{T}\boldsymbol{\phi}_i'\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\tilde{x}_{i,t-1}^2,\end{aligned} \tag{S.2}$$

and

$$\frac{\partial\mathcal{L}_{n,T}\left(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma}\right)}{\partial\boldsymbol{\phi}_i} = \sum_{t=p+1}^{T}\left[\tilde{\xi}_{i,t-1}\left(\theta\right)\boldsymbol{\Sigma}_i^{-1}\Delta\tilde{\mathbf{w}}_{it} + \tilde{\xi}_{i,t-1}^2\left(\theta\right)\boldsymbol{\Sigma}_i^{-1}\boldsymbol{\phi}_i\right], \tag{S.3}$$

respectively. Setting partial derivatives (S.2)-(S.3) equal to zero gives implicit solutions (9)-(10) in the paper. Similarly, setting partial derivatives of $\mathcal{L}_{n,T}\left(\theta, \boldsymbol{\phi}, \boldsymbol{\Sigma}\right)$ with respect to elements of $\boldsymbol{\Sigma}_i$ equal to zero yields implicit solution (11) for $\hat{\boldsymbol{\Sigma}}_i$ in the paper.

Using (14)-(15), asymptotic variance of the SPMG estimator is given by $p\lim_{n,T\to\infty} Q_{nT}^{-1}V_{qnT}Q_{nT}^{-1}$, where $V_{qnT} = E\left(q_{nT}q_{nT}'\right) = E\left(q_{nT}^2\right)$. Under error cross-sectional independence, we have $V_{qnT} =$

Q_{nT}, in which case the asymptotic variance reduces to $p\lim_{n,T\to\infty} Q_{nT}^{-1}$, and it can be consistently estimated by $\hat{Q}_{nT}^{-1}$, where

$$\hat{Q}_{nT} = n^{-1}\sum_{i=1}^{n}\left(\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i\right)\left(T^{-2}\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right), \tag{S.4}$$

with $\hat{\boldsymbol{\phi}}_i$ and $\hat{\boldsymbol{\Sigma}}_i$ denoting the SPMG estimates of $\boldsymbol{\phi}_i$ and $\boldsymbol{\Sigma}_i$, respectively, as given by (10) and (11). Under weak error cross-sectional dependence, it is no longer the case that $V_{qnT} = Q_{nT}$, but it is possible to consistently estimate V_{qnT} as

$$\hat{V}_{qnT} = \frac{1}{nT^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=2}^{T}\hat{\zeta}_{it}\hat{\zeta}_{jt}, \tag{S.5}$$

where $\hat{\zeta}_{it} = \tilde{x}_{i,t-1}\hat{\tilde{\mathbf{u}}}_{it}'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i$, $\hat{\tilde{\mathbf{u}}}_{it}'$, for $t = p{+}1, p{+}2, ..., T$, are the rows of $\hat{\tilde{\mathbf{U}}}_i\left(\hat{\theta}, \hat{\boldsymbol{\phi}}_i\right) = \mathbf{H}_i\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}\left(\hat{\theta}\right)\hat{\boldsymbol{\phi}}_i'\right]$, see (8). A consistent estimator of the asymptotic variance of SPMG estimator $\hat{\theta}$, regardless of error cross-sectional dependence, is given by $\hat{Q}_{nT}^{-1}\hat{V}_{qnT}\hat{Q}_{nT}^{-1}$.

S.2 Implementation of individual estimators and bootstrapping procedures

This section of the online supplement provide detailed description of the implementation of PMG, 2-step Breitung, SPMG, and MGMW estimators. Computation of bootstrapped confidence intervals for PMG and SPMG estimators is outlined as well.

S.2.1 PMG estimator

PMG estimator of the level coefficient θ is based on the (conditional) ARDL specification

$$y_{it} = c_i + \sum_{\ell=1}^{p}\alpha_{i,\ell}y_{i,t-1} + \sum_{\ell=0}^{q}\beta_{i,\ell}x_{i,t-\ell} + \varepsilon_{it}, \tag{S.6}$$

where

$$\theta = \frac{\sum_{\ell=0}^{q}\beta_{i,\ell}}{1-\sum_{\ell=1}^{p}\alpha_{i,\ell}},$$

is homogenous across i. Conditional model (S.6) can be obtained from (1)-(3), assuming that $E\left(u_{xit}u_{yit}\right)/E\left(u_{xit}^2\right)$ is time invariant (see, for instance, Section 5 of Chudik and Pesaran, 2021).

For expositional convenience, we set $p = q = 2$ below. The same choices of lag orders are used in the Monte Carlo section, and in the empirical section of the main paper. For $p = q = 2$, (S.6) can be written as

$$\Delta y_{it} = c_i - \phi_i\xi_{i,t-1}(\theta) + \omega_{yi}\Delta y_{i,t-1} + \omega_{xi,0}\Delta x_{it} + \omega_{xi,1}\Delta x_{i,t-1} + \varepsilon_{it}, \tag{S.7}$$

where

$$\xi_{i,t-1}\left(\theta\right)=y_{i,t-1}-\theta x_{i,t-1}. \tag{S.8}$$

We continue to assume observations on (x_{it},y_{it}) for $t=1,2...,T$ time periods and $i=1,2,...,n$ cross-section units are available for estimation. The PMG estimator is computed by a back-substitution algorithm. Assuming $\varepsilon_{it}\sim IIDN\left(0,\sigma_{\varepsilon i}^2\right)$, and setting the first derivatives of the concentrated log-likelihood function

$$\ell_{nT}(\boldsymbol{\varphi})=-\frac{T-2}{2}\sum_{i=1}^{n}\ln 2\pi\sigma_i^2-\frac{1}{2}\sum_{i=1}^{n}\frac{1}{\sigma_{\varepsilon i}^2}\left[(\Delta\mathbf{y}_i+\phi_i\boldsymbol{\xi}_{i,-1}(\theta)\right]'\mathbf{H}_i^{\diamondsuit}\left[\Delta\mathbf{y}_i+\phi_i\boldsymbol{\xi}_{i,-1}(\theta)\right], \tag{S.9}$$

with respect to $\boldsymbol{\varphi}=\left(\theta,\phi_1,\phi_2,...,\phi_n,\sigma_{\varepsilon i}^2,\sigma_{\varepsilon 2}^2,...,\sigma_{\varepsilon n}^2\right)'$ to $\mathbf{0}$ yields the following implicit expressions for $\hat{\theta}^{PMG}$, $\hat{\phi}_i^{PMG}$, and $\hat{\sigma}_{\varepsilon i}^2$ which we solve iteratively:

$$\hat{\theta}^{PMG}=-\left\{\sum_{i=1}^{n}\frac{\left(\hat{\phi}_i^{PMG}\right)^2}{\hat{\sigma}_{\varepsilon i}^2}\mathbf{x}_{i,-1}'\mathbf{H}_i^{\diamondsuit}\mathbf{x}_{i,-1}\right\}^{-1}\left\{\sum_{i=1}^{n}\frac{\hat{\phi}_i^{PMG}}{\hat{\sigma}_{\varepsilon i}^2}\mathbf{x}_{i,-1}'\mathbf{H}_i^{\diamondsuit}\left(\Delta\mathbf{y}_i+\hat{\phi}_i^{PMG}\mathbf{y}_{i,-1}\right)\right\}, \tag{S.10}$$

$$\hat{\phi}_i^{PMG}=-\left[\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{PMG}\right)\mathbf{H}_i^{\diamondsuit}\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right)\right]^{-1}\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{PMG}\right)\mathbf{H}_i^{\diamondsuit}\Delta\mathbf{y}_i,\; i=1,2,...,n, \tag{S.11}$$

$$\hat{\sigma}_{\varepsilon i}^2=(T-2)^{-1}\left[\Delta\mathbf{y}_i+\hat{\phi}_i^{PMG}\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right)\right]'\mathbf{H}_i^{\diamondsuit}\left[\Delta\mathbf{y}_i+\hat{\phi}_i^{PMG}\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right)\right],\; i=1,2,...,n, \tag{S.12}$$

where $\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{PMG}\right)=\mathbf{y}_{i,-1}-\hat{\theta}^{PMG}\mathbf{x}_{i,-1}$, and $\mathbf{H}_i^{\diamondsuit}=\mathbf{I}_{T-2}-\mathbf{Q}_i^{\diamondsuit}(\mathbf{Q}_i^{\diamondsuit\prime}\mathbf{Q}_i^{\diamondsuit})^{-1}\mathbf{Q}_i^{\diamondsuit\prime}$, $\mathbf{Q}_i^{\diamondsuit}=(\Delta\mathbf{y}_{i,-1},\Delta\mathbf{x}_i,\Delta\mathbf{x}_{i,-1},\boldsymbol{\tau}_{T-2})$, $\Delta\mathbf{y}_{i,-1}=(\Delta y_{i,2},\Delta y_{i,3},...,\Delta y_{i,T-1})'$, $\mathbf{y}_{i,-1}=(y_{i,2},y_{i,3},...,y_{i,T-1})'$, $\mathbf{x}_{i,-1}=(x_{i,2},x_{i,3},...,x_{i,T-1})'$, $\Delta\mathbf{x}_i=(\Delta x_{i,3},\Delta x_{i,4},...,\Delta x_{i,T})'$, $\Delta\mathbf{x}_{i,-1}=(\Delta x_{i,2},\Delta x_{i,3},...,\Delta x_{i,T-1})'$, and $\boldsymbol{\tau}_{T-2}$ is $T-2$ dimensional column vector of ones. Higher order lags of Δy_{it} and Δx_{it} in (S.7) could be accommodated by augmenting $\mathbf{Q}_i^{\diamondsuit}$ by these terms.

Starting with a consistent initial estimate of $\boldsymbol{\theta}$, say $\hat{\boldsymbol{\theta}}_{(0)}^{PMG}$, estimates of ϕ_i and σ_i^2 are computed using (S.11) and (S.12), which can then be substituted in (S.10) to obtain a new estimate of $\boldsymbol{\theta}$, say $\hat{\boldsymbol{\theta}}_{(1)}^{PMG}$, and so on until convergence is achieved. Our criterion for convergence is $\left\|\hat{\boldsymbol{\theta}}_{(j)}^{PMG}-\hat{\boldsymbol{\theta}}_{(j-1)}^{PMG}\right\|<10^{-4}$. Convergence is usually achieved very fast with average number of iterations generally quite small (<10 in most cases).

The initial estimate $\hat{\boldsymbol{\theta}}_{(0)}^{PMG}$ is taken to be the FE estimator of the coefficient $\boldsymbol{\theta}$ in the following panel Engle-Granger regression,

$$y_{it}=\mu_i+\boldsymbol{\theta}x_{it}+e_{it}, \tag{S.13}$$

for $t=1,2,...,T$, $i=1,2,...,n$.

Inference for the PMG estimator $\hat{\boldsymbol{\theta}}^{PMG}$ is conducted using

$$\widehat{Var}\left(\hat{\boldsymbol{\theta}}^{PMG}\right)=\hat{\Omega}_{PMG}=\left(\sum_{i=1}^{n}\frac{\left(\hat{\phi}_i^{PMG}\right)^2}{\hat{\sigma}_{\varepsilon i}^2}\mathbf{x}_{i,-1}'\mathbf{H}_i^{\diamondsuit}\mathbf{x}_{i,-1}\right)^{-1}. \tag{S.14}$$

S.2.1.1 Bootstrapping critical values

In addition to asymptotic critical values, based on (S.14) we also consider bootstrap critical values of the test statistics

$$t_{\hat{\theta}}\left(\theta_0\right) = \frac{\hat{\theta}^{PMG} - \theta_0}{\widehat{s.e.}\left(\hat{\theta}^{PMG}\right)}, \tag{S.15}$$

where

$$\widehat{s.e.}\left(\hat{\theta}^{PMG}\right) = \hat{\Omega}_{PMG}^{1/2}, \tag{S.16}$$

and $\hat{\Omega}_{PMG}$ is given by (S.14). Similarly to the bootstrapping procedure for the SPMG estimator outlined below, we adopt recursive-design residual-based Wild bootstrap, where bootstrap data samples are recursively generated using resampled residuals and coefficients of the estimated model. Wild bootstrap algorithm implemented in this paper is similar to Herwartz and Walle (2018). Specifically, we adopted two bootstrapping procedures: (i) conditional on x_{it} and (ii) unconditional, where model for x_{it} is also utilized. Conditional procedure in Subsection S.2.1.1 is valid in the case of strict exogeneity of x_{it}, whereas the unconditional procedure outlined in Subsection S.2.1.1 is valid also in the case of short run feedbacks from y_{it} to x_{it}. However, in all of our MC experiments, these two procedures performed very similarly, regardless of the presence of the short run feedbacks from y_{it} to x_{it}. In the empirical section, we reported confidence intervals based on the unconditional procedure. Both types of confidence intervals are available in our codes package.

Bootstrapping critical values (conditional on x_{it}) The following procedure is adopted.

1. Using data $y_{i1}, y_{i2}, ..., y_{iT}$ and $x_{i1}, x_{i2}, ..., x_{iT}$ $(i = 1, 2, ..., n)$ estimate all parameters of

$$\Delta y_{it} = c_i - \phi_i(y_{i,t-1} - \theta x_{i,t-1}) + \omega_{yi}\Delta y_{i,t-1} + \omega_{xi,0}\Delta x_{it} + \omega_{xi,1}\Delta x_{i,t-1} + \varepsilon_{it}. \tag{S.17}$$

   Denote the corresponding PMG estimates of the unknown coefficients $\hat{\theta}$, $\{\hat{c}_i\}_{i=1}^n$, $\left\{\hat{\phi}_i\right\}_{i=1}^n$, $\{\hat{\omega}_{yi}\}_{i=1}^n$, $\{\hat{\omega}_{xi,0}\}_{i=1}^n$, $\{\hat{\omega}_{xi,1}\}_{i=1}^n$. We omit superscript "PMG" in this procedure to simplify notations.

2. Obtain residuals, denoted as $\hat{\varepsilon}_{it}$.

3. Repeat for $b = 1, 2, ..., B$:

   (a) Generate $\varepsilon_{it}^{(b)} = \varkappa_t^{(b)}\hat{\varepsilon}_{y,it}$, where $\varkappa_t$ is randomly drawn from Rademacher distribution (Liu, 1988, Davidson and Flachaire, 2008)

$$\varkappa_t^{(b)} = \begin{cases} -1, & \text{with probability } 1/2 \\ 1, & \text{with probability } 1/2 \end{cases}.$$

(b) Generate

$$y_{it}^{(b)} = y_{i,t-1}^{(b)} + \hat{c}_i - \hat{\phi}_i \left(y_{i,t-1}^{(b)} - \hat{\theta} x_{i,t-1} \right) + \hat{\omega}_{y,i} \Delta y_{i,t-1}^{(b)} + \hat{\omega}_{xi,0} \Delta x_{it} + \hat{\omega}_{xi,1} \Delta x_{i,t-1} + \varepsilon_{it}^{(b)}, \tag{S.18}$$

for $t = 3, 4, ..., T$, with initial values $y_{i1}^{(b)} \equiv y_{i1}$ and $y_{i2}^{(b)} \equiv y_{i2}$.

(c) Using the bootstrap sample $\mathbf{y}_i^{(b)} = \left(y_{i1}, y_{i2}, y_{i3}^{(b)}, y_{i4}^{(b)}, ..., y_{iT}^{(b)} \right)'$ and $\mathbf{x}_i = (x_{i1}, x_{i2}, ..., x_{iT})'$ $(i = 1, 2, ..., n)$, compute PMG estimate $\hat{\theta}^{(b)}$, and the associated test statistics

$$t_{\hat{\theta}}^{(b)} = \frac{\hat{\theta}^{(b)} - \hat{\theta}}{\widehat{s.e.}\left(\hat{\theta}^{(b)} \right)}, \tag{S.19}$$

where $\widehat{s.e.}\left(\hat{\theta}^{(b)} \right) = \left(\hat{\Omega}_{PMG}^{(b)} \right)^{1/2}$.

4. Inference (at 5% nominal level) is conducted using the 95 percent quantile of $\left\{ \left| t_{\hat{\theta}}^{(b)} \right|, b = 1, 2, ..., B \right\}$ as critical value.

Bootstrapping critical values (unconditional) This procedure is the same as above, but we allow for feedback from y_{it} to x_{it}. Specifically, we estimate the conditional model

$$\Delta y_{it} = c_i - \phi_i (y_{i,t-1} - \theta x_{i,t-1}) + \omega_{yi} \Delta y_{i,t-1} + \omega_{xi,0} \Delta x_{it} + \omega_{xi,1} \Delta x_{i,t-1} + \varepsilon_{it}.$$

and the marginal model

$$\Delta x_{it} = d_i + \delta_{xi} \Delta x_{i,t-1} + \delta_{yi} \Delta y_{i,t-1} + v_{it}. \tag{S.20}$$

Bootstrap samples are consequently generated as

$$x_{it}^{(b)} = x_{i,t-1}^{(b)} + \hat{d}_i + \hat{\delta}_{xi} \Delta x_{i,t-1}^{(b)} + \hat{\delta}_{yi} \Delta y_{i,t-1}^{(b)} + v_{it}^{(b)},$$

and

$$y_{it}^{(b)} = y_{i,t-1}^{(b)} + \hat{c}_i - \hat{\phi}_i \left(y_{i,t-1}^{(b)} - \hat{\theta} x_{i,t-1}^{(b)} \right) + \hat{\omega}_{y,i} \Delta y_{i,t-1}^{(b)} + \hat{\omega}_{xi,0} \Delta x_{it}^{(b)} + \hat{\omega}_{xi,1} \Delta x_{i,t-1}^{(b)} + \varepsilon_{it}^{(b)},$$

for $t = 3, 4, ..., T$ using the initial values $y_{it}^{(b)} = y_{it}$, for $t = 1, 2$ and $x_{it}^{(b)} = x_{it}$, for $t = 1, 2$, in which $v_{it}^{(b)}$ is generated similarly to $\varepsilon_{it}^{(b)}$.

S.2.2 2-step Breitung's (2005) panel data estimator

Breitung's 2-step panel data estimator requires an initial consistent estimator. We follow the same choices for the implementation of Breitung's estimator as in the original paper Breitung (2005), and in the accompanying Gauss codes we received from Joerg Breitung, which we gratefully acknowledge. For each unit $i = 1, 2, ..., n$, we compute

$$\hat{\theta}_i = \left(\tilde{\mathbf{x}}_i'\tilde{\mathbf{x}}_i\right)^{-1}\tilde{\mathbf{x}}_i'\tilde{\mathbf{y}}_i, \tag{S.21}$$

where $\tilde{\mathbf{x}}_i = \mathbf{M}_\tau \mathbf{x}_i$, $\tilde{\mathbf{y}}_i = \mathbf{M}_\tau \mathbf{y}_i$, $\mathbf{x}_i = (x_{i1}, x_{i1}, ..., x_{iT})'$, $\mathbf{y}_i = (y_{i1}, y_{i1}, ..., y_{iT})'$, $\mathbf{M}_\tau = \mathbf{I}_T - \boldsymbol{\tau}\boldsymbol{\tau}'/T$, $\mathbf{I}_T$ is $T \times T$ identity matrix and $\boldsymbol{\tau}$ is $T \times 1$ vector of ones. Next, we compute

$$\xi_{it}\left(\hat{\theta}_i\right) = y_{it} - \hat{\theta}_i x_{it}.$$

We continue to assume $p = q = 2$. Define the following data vectors

$$\begin{aligned}
\Delta\mathbf{x}_i &= \left(\Delta x_{i,3}, \Delta x_{i,4}, ..., \Delta x_{i,T}\right)' \\
\Delta\mathbf{y}_i &= \left(\Delta y_{i,3}, \Delta y_{i,4}, ..., \Delta y_{i,T}\right)', \\
\mathbf{x}_{i-1} &= \left(x_{i,2}, x_{i,3}, ..., x_{i,T-1}\right)', \\
\mathbf{y}_{i-1} &= \left(y_{i,2}, y_{i,3}, ..., y_{i,T-1}\right)', \\
\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}_i\right) &= \left[\xi_{i,2}\left(\hat{\theta}_i\right), \xi_{i,3}\left(\hat{\theta}_i\right), ..., \xi_{i,T-1}\left(\hat{\theta}_i\right)\right]' \\
\mathbf{q}_{i,t-1} &= \left(1, \Delta x_{i,t-1}, \Delta y_{i,t-1}\right)',
\end{aligned}$$

and the following data matrices

$$\Delta\mathbf{W}_i = \left(\Delta\mathbf{y}_i, \Delta\mathbf{x}_i\right), \ \mathbf{W}_{i,-1} = \left(\mathbf{y}_{i,-1}, \mathbf{x}_{i,-1}\right),$$

$$\mathbf{Q}_i = \left(\mathbf{q}_{i,2}, \mathbf{q}_{i,3}, ..., \mathbf{q}_{i,T-1}\right)'.$$

Let $\mathbf{H}_i = \mathbf{I}_{T-2} - \mathbf{Q}_i\left(\mathbf{Q}_i'\mathbf{Q}_i\right)^{-1}\mathbf{Q}_i$, and

$$\Delta\tilde{\mathbf{W}}_i = \mathbf{H}_i\Delta\mathbf{W}_i, \ \tilde{\mathbf{W}}_{i,-1} = \mathbf{H}_i\mathbf{W}_{i,-1}, \ \tilde{\boldsymbol{\xi}}_{i,-1}\left(\hat{\theta}_i\right) = \mathbf{H}_i\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}_i\right).$$

In addition, let

$$\begin{aligned}
\hat{\boldsymbol{\phi}}_i &= \left[\tilde{\boldsymbol{\xi}}_{i,-1}'\left(\hat{\theta}_i\right)\tilde{\boldsymbol{\xi}}_{i,-1}\left(\hat{\theta}_i\right)\right]^{-1}\tilde{\boldsymbol{\xi}}_{i,-1}'\left(\hat{\theta}_i\right)\Delta\tilde{\mathbf{W}}_i, \\
\hat{\mathbf{U}}_i &= \Delta\tilde{\mathbf{W}}_i + \tilde{\boldsymbol{\xi}}_{i,-1}\left(\hat{\theta}_i\right)\hat{\boldsymbol{\phi}}_i', \\
\hat{\boldsymbol{\Sigma}}_i &= \hat{\mathbf{U}}_i'\hat{\mathbf{U}}_i/\left(T-3\right), \\
\tilde{\mathbf{z}}_i^+ &= \tilde{\mathbf{W}}_{i,-1}\mathbf{s}_1 + \left(\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\phi}}_i\right)^{-1}\Delta\tilde{\mathbf{W}}_i\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i
\end{aligned}$$

$\mathbf{s}_1 = (1, 0)'$, and $\mathbf{s}_2 = (0, 1)'$. Then 2-step Breitung (2005) estimator of θ is computed as

$$\hat{\theta}^{Br} = \left(\sum_{i=1}^{n}\mathbf{s}_2'\tilde{\mathbf{W}}_{i,-1}'\tilde{\mathbf{W}}_{i,-1}\mathbf{s}_2\right)^{-1}\sum_{i=1}^{n}\mathbf{s}_2'\tilde{\mathbf{W}}_{i,-1}'\Delta\tilde{\mathbf{z}}_i^+.$$

Inference is conducted using the "2S-OLS" and "2S-robust" standard errors as described in Breitung (2005).

S.2.3 SPMG estimator

SPMG estimator avoids inversion of $\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\phi}}_i$, since it can be the case that $\hat{\boldsymbol{\phi}}_i = o_p(1)$ for some units that do not have long run relationship (or $\hat{\boldsymbol{\phi}}_i$ can converge to a rank-deficient matrix in probability in a more general case). SPMG estimator of θ is solved iteratively using

$$\hat{\theta}^{\text{SPMG}} = -\left(\sum_{i=1}^{n}\hat{\boldsymbol{\phi}}_i'\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right)^{-1}\sum_{i=1}^{n}\mathbf{x}_{i,-1}'\mathbf{H}_i\left(\Delta\mathbf{W}_i + \mathbf{y}_{i,-1}\hat{\boldsymbol{\phi}}_i'\right)\hat{\boldsymbol{\Sigma}}_i^{-1}\hat{\boldsymbol{\phi}}_i, \tag{S.22}$$

$$\hat{\boldsymbol{\phi}}_i^{\text{SPMG}} = -\left[\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{\text{SPMG}}\right)\mathbf{H}_i\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right)\right]^{-1}\boldsymbol{\xi}_{i,-1}'\left(\hat{\theta}^{\text{SPMG}}\right)\mathbf{H}_i\Delta\mathbf{W}_i, \tag{S.23}$$

$$\hat{\boldsymbol{\Sigma}}_i^{\text{SPMG}} = (T-2)^{-1}\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right)\hat{\boldsymbol{\phi}}_i^{\text{SPMG}\prime}\right]'\mathbf{H}_i\left[\Delta\mathbf{W}_i + \boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right)\hat{\boldsymbol{\phi}}_i^{\text{SPMG}\prime}\right], \tag{S.24}$$

in which $\mathbf{x}_{i,-1} = (x_{i,1}, x_{i,2}, ..., x_{i,T-1})'$, $\mathbf{y}_{i,-1} = (y_{i,1}, y_{i,2}, ..., y_{i,T-1})'$, $\boldsymbol{\xi}_{i,-1}\left(\hat{\theta}^{\text{SPMG}}\right) = \left[\xi_{i1}\left(\hat{\theta}^{\text{SPMG}}\right), \xi_{i2}\left(\hat{\theta}^{\text{SPMG}}\right), ..., \xi_{iT-1}\left(\hat{\theta}^{\text{SPMG}}\right)\right]$, $\boldsymbol{\xi}_{it}\left(\hat{\theta}^{\text{SPMG}}\right) = y_{it} - \hat{\theta}^{\text{SPMG}}x_{it}$, $\Delta\mathbf{W}_i = (\Delta\mathbf{w}_{i2,}\Delta\mathbf{w}_{i3,}, ..., \Delta\mathbf{w}_{iT})'$,and $\Delta\mathbf{w}_{it} = (\Delta y_{it}, \Delta x_{it})'$. $\mathbf{H}_i = \mathbf{I}_{T-2} - \mathbf{Q}_i\left(\mathbf{Q}_i'\mathbf{Q}_i\right)^{-1}\mathbf{Q}_i$, $\mathbf{Q}_i = (\mathbf{q}_{i,2}, \mathbf{q}_{i,3}, ..., \mathbf{q}_{i,T-1})'$ and $\mathbf{q}_{i,t-1} = (1, \Delta x_{i,t-1}, \Delta y_{i,t-1})'$.

The conventional estimator of the variance of $\hat{\theta}^{\text{SPMG}}$ is computed, using (S.4), as

$$\widehat{Var}\left(\hat{\theta}^{\text{SPMG}}\right) = \hat{\Omega}_{\text{SPMG}} = n^{-1}T^{-2}\hat{Q}_{nT} = \left[\sum_{i=1}^{n}\hat{\boldsymbol{\phi}}_i^{\text{SPMG}\prime}\left(\hat{\boldsymbol{\Sigma}}_i^{\text{SPMG}}\right)^{-1}\hat{\boldsymbol{\phi}}_i^{\text{SPMG}}\mathbf{x}_{i,-1}'\mathbf{H}_i\mathbf{x}_{i,-1}\right]^{-1}. \tag{S.25}$$

Estimators of $Var\left(\hat{\theta}^{\text{SPMG}}\right)$ that are robust to error cross-sectional dependence are computed, using (S.4)-(S.5), as

$$\widetilde{Var}\left(\hat{\theta}^{\text{SPMG}}\right) = n^{-1}T^{-2}\hat{Q}_{nT}^{-1}\hat{V}_{qnT}\hat{Q}_{nT}^{-1} = \hat{\Omega}_{\text{SPMG}}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=2}^{T}\hat{\zeta}_{it}\hat{\zeta}_{jt}\right)\hat{\Omega}_{\text{SPMG}}, \tag{S.26}$$

where

$$\hat{\zeta}_{it} = \tilde{x}_{i,t-1}\widehat{\tilde{\mathbf{u}}}_{it}'\left(\hat{\boldsymbol{\Sigma}}_i^{\text{SPMG}}\right)^{-1}\hat{\boldsymbol{\phi}}_i^{\text{SPMG}},$$

and $\widehat{\tilde{\mathbf{u}}}_{it}'$ for $t = 2, 3, ..., T$, are the rows of $\mathbf{H}_i\hat{\mathbf{U}}_i$, in which $\hat{\mathbf{U}}_i = (\hat{\mathbf{u}}_{i,2}, \hat{\mathbf{u}}_{i,3}, ..., \hat{\mathbf{u}}_{i,T})'$ is the matrix of SPMG residuals. We consider inference based on (S.25) using asymptotic critical values, inference based on (S.26) with asymptotic critical values (reported in Table S38), and inference based on (S.25) using bootstrapped critical values outlined below. Our bootstrapping procedure is robust to error cross sectional dependence, and it outperforms the other alternatives in our Monte Carlo experiments in terms of accuracy of the confidence intervals coverage rates. In situations where n

is large relative to T, one could also consider a threshold version of (S.26), given by:

$$\widetilde{Var}\left(\hat{\theta}^{\text{SPMG}}\right) = \hat{\Omega}_{\text{SPMG}}\left[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{t=2}^{T}\hat{\zeta}_{it}\hat{\zeta}_{jt}I\left(\left|\hat{\rho}_{\zeta ij}\right| > \sqrt{T}c_p\left(n\right)\right)\right]\hat{\Omega}_{\text{SPMG}}, \tag{S.27}$$

where $\hat{\rho}_{\zeta ij}$ is the sample correlation of $\hat{\zeta}_{it}$ and $\hat{\zeta}_{jt}$, and $c_p\left(n\right)$ is a suitably chosen thresholding critical value, as considered by Bailey et al. (2019), $c_p\left(n\right) = \Phi^{-1}\left(1 - pn^{-\delta}/2\right)$, with $p = 0.05$ and $\delta = 2$. Our Monte Carlo findings suggest thresholding is not required for the sample sizes we consider.

S.2.3.1 Bootstrapping critical values

We consider bootstrap critical values of the test statistics

$$t_{\hat{\theta}^{\text{SPMG}}}\left(\theta_0\right) = \frac{\hat{\theta}^{\text{SPMG}} - \theta_0}{\widehat{s.e.}\left(\hat{\theta}^{\text{SPMG}}\right)}, \tag{S.28}$$

where

$$\widehat{s.e.}\left(\hat{\theta}^{\text{SPMG}}\right) = \hat{\Omega}_{\text{SPMG}}^{1/2}, \tag{S.29}$$

and $\hat{\Omega}_{\text{SPMG}}$ is given by (S.25).

The bootstrap algorithm adopted in this paper is essentially the same as in Herwartz and Walle (2018). In the context of dynamic panel data models it is natural to generate bootstrap data samples recursively using resampled residuals and the coefficients of the estimated model. As to the choice of residual resampling, we follow the Wild bootstrap literature and premultiply the estimated residuals with a conveniently chosen zero-mean, unit-variance random variable.[1] We choose Rademacher distribution (Liu, 1988).[2] Given that our time dimension is large compared with the cross-sectional dimension, and to preserve cross-sectional correlations of residuals, we follow Herwartz and Walle (2018) and resample columns of cross-sectionally stacked residuals (see also Kapetanios, 2008 for related discussion). Specifically, the following procedure is adopted.

1. Using data $y_{i1}, y_{i2}, ..., y_{iT}$ and $x_{i1}, x_{i2}, ..., x_{iT}$ $(i = 1, 2, ..., n)$ estimate all parameters of

$$\Delta y_{it} = a_{yi} - \phi_{yi}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{yyi}\Delta y_{i,t-1} + \psi_{yxi}\Delta x_{i,t-1} + u_{yit}. \tag{S.30}$$

$$\Delta x_{it} = a_{xi} - \phi_{xi}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{xxi}\Delta x_{i,t-1} + \psi_{xyi}\Delta y_{i,t-1} + u_{xit}. \tag{S.31}$$

[1] Wild bootstrap was originally proposed by Wu (1986) and Liu (1988). There are numerous applications of Wild bootsrap in the literature, including Gonçlaves and Killian (2004) in the context of autoregressive models, and Gonçlaves and Kaffo (2015) in the context of dynamic panel data models, among others.

[2] Other widely adopted distributions considered in the Wild bootstrap literature are $N\left(0, 1\right)$ distribution and the binary distribution suggested by Mammen (1993). Herwartz and Walle (2018) found Rademacher distribution to be most effective in the context of panel unit root inference, while Gonçlaves and Killian (2004) found no indication that the choice of the distribution makes much difference in practice. We did not investigate alternative distribution choices in this paper.

Denote the corresponding SPMG estimates of the unknown coefficients as $\hat{\theta}$, $\{\hat{a}_{yi}\}_{i=1}^{n}$, $\{\hat{a}_{xi}\}_{i=1}^{n}$, $\left\{\hat{\phi}_{yi}\right\}_{i=1}^{n}$, $\left\{\hat{\phi}_{xi}\right\}_{i=1}^{n}$, $\{\psi_{yyi}\}_{i=1}^{n}$, $\{\psi_{yxi}\}_{i=1}^{n}$, $\{\psi_{xyi}\}_{i=1}^{n}$, and $\{\psi_{xxi}\}_{i=1}^{n}$. We omit superscript "SPMG" in this procedure to simplify notations.

2. Obtain residuals, denoted as $\hat{\mathbf{u}}_{it} = (\hat{u}_{yit}, \hat{u}_{xit})$.

3. Repeat for $b = 1, 2, ..., B$:

   (a) Generate $\mathbf{u}_{it}^{(b)} = \left(u_{yit}^{(b)}, u_{xit}^{(b)}\right)' = \varkappa_t^{(b)} \hat{\mathbf{u}}_{it}$, where $\varkappa_t$ is randomly drawn from Rademacher distribution (Liu, 1988, Davidson and Flachaire, 2008)

$$\varkappa_t^{(b)} = \begin{cases} -1, & \text{with probability } 1/2 \\ 1, & \text{with probability } 1/2 \end{cases}.$$

   (b) Generate

$$y_{it}^{(b)} = y_{i,t-1}^{(b)} + \hat{a}_{yi} - \hat{\phi}_{yi}\left(y_{i,t-1}^{(b)} - \hat{\theta} x_{i,t-1}^{(b)}\right) + \hat{\psi}_{yyi}\Delta y_{i,t-1}^{(b)} + \hat{\psi}_{yxi}\Delta x_{i,t-1}^{(b)} + u_{yit}^{(b)}, \tag{S.32}$$

$$x_{it}^{(b)} = x_{i,t-1}^{(b)} + \hat{a}_{xi} - \hat{\phi}_{xi}(y_{i,t-1}^{(b)} - \hat{\theta} x_{i,t-1}^{(b)}) + \hat{\psi}_{xxi}\Delta x_{i,t-1}^{(b)} + \hat{\psi}_{xyi}\Delta y_{i,t-1}^{(b)} + u_{xit}^{(b)}, \tag{S.33}$$

   for $t = 3, 4, ..., T$, with initial values $y_{i1}^{(b)} \equiv y_{i1}$, $y_{i2}^{(b)} \equiv y_{i2}$, $x_{i1}^{(b)} \equiv x_{i1}$, and $x_{i2}^{(b)} \equiv x_{i2}$.

   (c) Using the bootstrap sample $\mathbf{y}_i^{(b)} = \left(y_{i1}, y_{i2}, y_{i3}^{(b)}, y_{i4}^{(b)}, ..., y_{iT}^{(b)}\right)'$ and $\mathbf{x}_i^{(b)} = \left(x_{i1}, x_{i2}, x_{i3}^{(b)}, x_{i4}^{(b)}, ..., x_{iT}^{(b)}\right)'$ $(i = 1, 2, ..., n)$, compute SPMG estimate $\hat{\theta}^{(b)}$, $\hat{\Omega}_{\text{SPMG}}^{(b)}$ (based on (S.25)), and the associated test statistics

$$t_{\hat{\theta}}^{(b)} = \frac{\hat{\theta}^{(b)} - \hat{\theta}}{\widehat{s.e.}\left(\hat{\theta}^{(b)}\right)}, \tag{S.34}$$

   where $\widehat{s.e.}\left(\hat{\theta}^{(b)}\right) = \sqrt{\hat{\Omega}_{\text{SPMG}}^{(b)}}$.

4. Inference (at 5% nominal level) is conducted using the 95 percent quantile of $\left\{\left|t_{\hat{\theta}}^{(b)}\right|, b = 1, 2, ..., B\right\}$ as critical value.

S.2.4 Mean Group MW estimator

Our "MGMW" estimator is a mean group estimator that utilizes cross-section specific estimates by Müller and Watson (2018). We split the T time period into q subsamples of (approximately) equal size. In the paper we set $q = 5$ both in the Monte Carlo and empirical sections. Specifically, let m be the integer part of T/q, denoted as $m = floor\,(T/q)$, and let $r = (T/q - m)\,q$ be the remainder of the division of T by q. Let $\mathcal{H}_s$ be the index set of time periods belonging to the sub-period s, for $s = 1, 2, ..., q$, defined as

$$\mathcal{H}_s = \{H_{s-1} + 1, H_{s-1} + 2, ..., H_s\},$$

where H_s is the last period of the subsample s, and it is defined recursively as

$$\begin{aligned} H_0 &= 0, \\ H_s &= H_{s-1} + T_s, \text{ for } s = 1, 2, ..., q, \end{aligned}$$

in which T_s is the number of time periods in the subsample s, defined as

$$T_s = \begin{cases} m+1 & \text{for } s \leq r \\ m & \text{for } s > r \end{cases}, \ s = 1, 2, ..., q.$$

Note that $H_q = T$. Define the temporal aggregates:

$$\bar{y}_{i,s} = \frac{1}{T_s} \sum_{s \in \mathcal{H}_s} y_{it}, \text{ and similarly } \bar{x}_{i,s} = \frac{1}{T_s} \sum_{s \in \mathcal{H}_s} x_{it},$$

for $i = 1, 2, ..., n$, and $s = 1, 2, ..., q$. MGMW estimator is the average of cross-section specific Least Squares estimates of θ_i in the regression

$$\bar{y}_{is} = \mu_i + \theta_i \bar{x}_{is} + \epsilon_{is}, \tag{S.35}$$

for $i = 1, 2, ..., n$, and $s = 1, 2, ..., q$. Inference is conducted using the conventional nonparametric standard errors, as outlined in Chapter 28.5 of Pesaran (2015).

S.3 Monte Carlo evidence

S.3.1 Design

We generate $\Delta \mathbf{w}_{it} = (\Delta y_{it}, \Delta x_{it})'$ based on the following panel vector error correction model, for $i = 1, 2, ..., n$ and $t = 1, 2, ..., T$,

$$\Delta y_{it} = a_{yi} - \phi_{yit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{yyi} \Delta y_{i,t-1} + \psi_{yxi} \Delta x_{i,t-1} + u_{yit}, \tag{S.36}$$

and

$$\Delta x_{it} = a_{xi} - \phi_{xit}(y_{i,t-1} - \theta x_{i,t-1}) + \psi_{xxi} \Delta x_{i,t-1} + \psi_{xyi} \Delta y_{i,t-1} + u_{xit}, \tag{S.37}$$

which can be equivalently written using a vector notation as

$$\Delta \mathbf{w}_{it} = \mathbf{a}_i - \boldsymbol{\phi}_{it} \boldsymbol{\beta}' \mathbf{w}_{i,t-1} + \boldsymbol{\Psi}_i \Delta \mathbf{w}_{i,t-1} + \mathbf{u}_{it}, \tag{S.38}$$

for $i=1,2,...,n$ and $t=1,2,...,T$ with initial values $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta\mathbf{w}_{i0}$, where $\mathbf{u}_{it}=(u_{yit},u_{xit})'\sim(\mathbf{0},\boldsymbol{\Sigma}_i)$, $\mathbf{a}_i=(a_{yi},a_{xi})'$, $\boldsymbol{\phi}_{it}=(\phi_{yit},\phi_{xit})'$, $\boldsymbol{\beta}=(1,-\theta)'$, we set the long-run coefficient $\theta=1$, and

$$\boldsymbol{\Psi}_i=\begin{pmatrix}\psi_{yyi} & \psi_{yxi}\\ \psi_{xyi} & \psi_{xxi}\end{pmatrix}.$$

We generate $\psi_{yyi},\psi_{xxi}\sim IIDU(0,0.4)$, and $\psi_{yxi},\psi_{xyi}\sim IIDU(-0.1,0.2)$, for $i=1,2,...,n$.

Initial values: To generate the initial values $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta\mathbf{w}_{i0}$ we assume that initially up to date $t=1$, $\boldsymbol{\phi}_{it}=\boldsymbol{\phi}_i$ for all i and all $t<1$. In such a case $\boldsymbol{\beta}'\mathbf{w}_{it}$ and $\Delta\mathbf{w}_{it}$ are covariance stationary for $t<1$, and we can use the the Granger representation theorem (Engle and Granger (1987), Johansen (1991), Hansen (2005)), to obtain a moving average representation for $\boldsymbol{\beta}'\mathbf{w}_{it}$ and $\Delta\mathbf{w}_{it}$. By Corollary 1 of Hansen (2005), we have the following representations

$$\boldsymbol{\beta}'\mathbf{w}_{it}=\boldsymbol{\beta}'\mathbf{C}_i^*(L)(\mathbf{u}_{it}+\mathbf{a}_i)=d_i+\boldsymbol{\beta}'\mathbf{C}_i^*(L)\mathbf{u}_{it},$$

and

$$\Delta\mathbf{w}_{it}=\mathbf{C}_i(L)(\mathbf{u}_{it}+\mathbf{a}_i)=\boldsymbol{\tau}\gamma_i+\mathbf{C}_i(L)\mathbf{u}_{it},$$

for $i=1,2,...,n$ and $t<1$, where $\boldsymbol{\tau}=(1,1)'$, $\mathbf{C}_i(L)=\sum_{\ell=0}^{\infty}\mathbf{C}_{i\ell}$, $\mathbf{C}_i(L)$ can be partitioned as $\mathbf{C}_i(L)=\mathbf{C}_i(1)+(1-L)\mathbf{C}_i^*(L)$, $\mathbf{C}_i^*(L)=\sum_{\ell=0}^{\infty}\mathbf{C}_{i\ell}^*L^{\ell}$, matrices $\mathbf{C}_{i\ell}$ are defined recursively by (S.44)-(S.46), matrices $\mathbf{C}_{i\ell}^*$ are defined by (S.47)-(S.48), $d_i=\boldsymbol{\beta}'\mathbf{C}_i^*(1)\mathbf{a}_i$, and $\mathbf{C}_i(1)\mathbf{a}_i=\boldsymbol{\tau}\gamma_i$.[3] We generate the initial values $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta\mathbf{w}_{i0}$ as

$$\boldsymbol{\beta}'\mathbf{w}_{i0}=d_i+\sum_{\ell=0}^{M}\boldsymbol{\beta}'\mathbf{C}_{i,-\ell}^*\mathbf{u}_{i,-\ell},$$

and

$$\Delta\mathbf{w}_{i0}=\tau\gamma_i+\sum_{\ell=0}^{M}\mathbf{C}_{i,-\ell}\mathbf{u}_{i,-\ell},$$

where we set $M=50$. The slopes of the linear trends in levels, γ_i, and the means $d_i=E(\boldsymbol{\beta}'\mathbf{w}_{i0})$ are generated as $\gamma_i,d_i\sim IIDN(0.02,0.01^2)$. Given (d_i,γ_i) we recover the corresponding values for $\mathbf{a}_i$ in (S.38).[4] Using generated initial values, $\boldsymbol{\beta}'\mathbf{w}_{i0}$ and $\Delta\mathbf{w}_{i0}$, we then generate $\Delta\mathbf{w}_{it}$ for $t=1,2,...,T$ and $i=1,2,...,n$, using (S.38).

Three options are considered for the innovations $\mathbf{u}_{it}=(u_{yit},u_{xit})'$ regarding the cross sectional dependence:

1. Cross sectionally independent errors. $\mathbf{u}_{it}=(u_{yit},u_{xit})'$ is generated as

$$u_{yit}=\kappa u_{xit}+\sigma_{yi}\varepsilon_{yit},\ u_{xit}=\sigma_{xi}\varepsilon_{xit}, \tag{S.39}$$

[3] γ_i is the common slope of the linear trend of x_{it} and y_{it}. Model (S.38) features unrestricted intercepts $\mathbf{a}_i$, and therefore the level variables x_{it} and y_{it} can feature linear trends. The slope of the linear trend of x_{it} and y_{it} is common in this design because $\boldsymbol{\beta}=(1,-1)'$ and linear trends are not allowed in (S.38), which also implies the error-correcting term $\boldsymbol{\beta}'\mathbf{w}_{it}=y_{it}-x_{it}$ does not feature a linear trend.

[4] Specifically, $\mathbf{a}_i=\boldsymbol{\Upsilon}_i^{-1}(d_i,\gamma_i)'$, where $\boldsymbol{\Upsilon}_i=[\boldsymbol{\phi}_i,(\mathbf{I}-\boldsymbol{\Psi}_i)\boldsymbol{\tau}]$.

where $\kappa = 0.5$, σ^2_{yi},$\sigma^2_{xi} \sim 0.1 + 0.1 \cdot IID\chi^2(2)$. We consider Gaussian and non-Gaussian cases to generate ε_{yit} and ε_{xit} (to illustrate robustness from departures from Gaussianity). In the Gaussian case, we generate $\varepsilon_{yit}, \varepsilon_{xit} \sim IIDN(0,1)$. In the non-Gaussian case, we generate

$$\varepsilon_{yit}, \varepsilon_{xit} \sim \frac{IIDN\chi^2(1) - 1}{\sqrt{2}}. \tag{S.40}$$

2. "SAR" errors. We generate $\mathbf{u}_{it} = (u_{yit}, u_{xit})'$ based on (S.39) with (as before) $\kappa = 0.5$, and σ^2_{yi},$\sigma^2_{xi} \sim 0.1 + 0.1 \cdot IID\chi^2(2)$, but we generate ε_{xit} and ε_{yit} based on the following spatial autoregressive model,

$$\varepsilon_{hit} = \delta \sum_{j=1}^{n} d_{ij}\varepsilon_{hjt} + v_{hit}, \text{ for } h = x, y, \tag{S.41}$$

where $\delta = 0.6$. Similarly to the cross sectionally independent case, we consider Gaussian and non-Gaussian cases to generate v_{yit} and v_{xit}. In the Gaussian case, $v_{yit}, v_{xit} \sim IIDN(0,1)$. In the non-Gaussian case, $v_{yit}, v_{xit} \sim 2^{-1/2}\left[IIDN\chi^2(1) - 1\right]$. d_{ij} are the elements of the $n \times n$ spatial weights matrix $\mathbf{D} = (d_{ij})$. We follow Kelejian and Prucha (2007) and assume units are set out on a rectangular grid at locations (s, r), for $r = 1, 2, ..., m_1$ and $s = 1, 2, ..., m_2$ such that $n = m_1 m_2$.[5] $\mathbf{W}$ is a rook type matrix, where two units are neighbors if their Euclidean distance is less than or equal to one. The weights matrix is normalized such that rows sum to one.

3. "Factor + SAR" errors. Errors are generated as

$$u_{yit} = \gamma_{yi} f_t + u^*_{xit}, \text{ and } u_{xit} = \gamma_{xi} f_t + u^*_{xit},$$

where $\gamma_{yi}, \gamma_{xi} \sim IIDN\left(1, 0.25^2\right)$, $f_t \sim IIDN(0,1)$, and u^*_{yit} and u^*_{xit} are generated in the same way as the SAR errors outlined above.

We initially consider the direction of long-run causality from x to y $(x \to y)$ by setting $\phi_{xit} = 0$ in (S.36)-(S.37). We generate ϕ_{yit} based on the two episodes,

$$\phi_{yit} = \begin{cases} \phi_{yi}, & \text{for } t \notin \mathcal{T}_i \\ 0, & \text{for } t \in \mathcal{T}_i \end{cases}, \tag{S.42}$$

where $\phi_{yi} \sim IIDU(0.1, 0.25)$, and $\mathcal{T}_i$ defines the sample index set for the episode with no convergence towards the long-run.

We also consider experiments with the two-way long-run causality, where ϕ_{yit} is generated according to (S.42), and ϕ_{xit} is generated similarly as

$$\phi_{xit} = \begin{cases} \phi_{xi}, & \text{for } t \notin \mathcal{T}_i \\ 0, & \text{for } t \in \mathcal{T}_i \end{cases}, \tag{S.43}$$

[5] We consider $(m_1, m_2) \in \{(6,5), (10,5), (10,10), (20,10), (20,25)\}$, for $n = 30$, 50, 100, 200, and 300, respectively.

where $\phi_{xi} \sim IIDU\left(-0.15, -0.05\right)$.

The episode sample index $\mathcal{T}_i$ defines the non-equilibrating episodes. It is stochastically generated as follows. For each $i = 1, 2, ..., n$, we draw indicator s_i from the Bernoulli distribution with probability parameter π, hence $s_i = 1$ with probability π and $s_i = 0$ with probability $1 - \pi$. We consider three options for π:

1. $\pi = 0$ no non-equilibrating episodes (benchmark case),

2. $\pi = 0.05$ (moderate occurrence of non-equilibrating episodes), and

3. $\pi = 0.2$ (high occurrence of non-equilibrating episodes).

If $s_i = 0$ then we set $\mathcal{T}_i = \emptyset$. If $s_i = 1$, we generate the starting point and the duration of non-equilibrating episode randomly. We first draw the duration d_i from uniform distribution from $\{T_{\min}, T_{\min} + 1, ..., T\}$, with $T_{\min} = 10$, ensuring non-equilibrating episode is at least 10 periods long (the average duration is $T/2 - 5$ periods, and the max duration is $T - 9$). Then we draw a starting period t_i^s from uniform distribution on $\{1, 2, ..., T - d_i\}$. We set $\mathcal{T}_i = \{t_i^s, t_i^s + 1, ..., t_i^s + d_i - 1\}$.

To obtain variable in levels ($\mathbf{w}_{it}$), we generate initial level values $x_{i,0} \sim N(1, 1)$ for $i = 1, 2, ..., n$. We then compute $y_{i,0}$ using $x_{i,0}$ and the initial value $\boldsymbol{\beta}'\mathbf{w}_{i0}$.[6] Given $\mathbf{w}_{i0}$, and $\Delta\mathbf{w}_{it}$ for $t = 1, 2, ..., T$, we compute $\mathbf{w}_{it} = \mathbf{w}_{i0} + \sum_{\ell=1}^{t} \Delta\mathbf{w}_{i\ell}$ for $t = 1, 2, ..., T$. Sample of n cross section units and T time periods, $\{\mathbf{w}_{it},\ i = 1, 2, ..., n,\ t = 1, 2, ..., T\}$ is used for estimation of $\theta = 1$ and the corresponding inference. The following choices for sample size are considered: $T \in \{50, 100, 150, 200\}$, and $n \in \{30, 50, 100, 200\}$. $R_{MC} = 2\ 000$ replications are conducted for each experiment.

S.3.1.1 Definition of $\mathbf{C}_{i\ell}$ and $\mathbf{C}_{i\ell}^*$

The coefficient matrices $\mathbf{C}_{i\ell}$ and $\mathbf{C}_{i\ell}^*$ are given recursively by

$$\mathbf{C}_{i0} = \mathbf{I}_2 \tag{S.44}$$
$$\mathbf{C}_{i1} = \mathbf{C}_{i0}\mathbf{\Phi}_{i1} - \mathbf{I}_2 \tag{S.45}$$
$$\mathbf{C}_{i\ell} = \mathbf{C}_{i,\ell-1}\mathbf{\Phi}_{i1} + \mathbf{C}_{i,\ell-2}\mathbf{\Phi}_{i2},\ \ell = 2, 3, ... \tag{S.46}$$

and

$$\mathbf{C}_{i0}^* = \mathbf{I}_2 - \mathbf{C}_i\left(1\right) \tag{S.47}$$
$$\mathbf{C}_{ij}^* = \mathbf{C}_{i,j-1}^* + \mathbf{C}_{ij},\ \text{for } j = 1, 2, ..., \tag{S.48}$$

where $\mathbf{C}_i\left(1\right) = \sum_{\ell=0}^{\infty} \mathbf{C}_{i\ell}$,

$$\mathbf{\Phi}_{i1} = \mathbf{I}_2 - \boldsymbol{\phi}_i\boldsymbol{\beta}' + \mathbf{\Psi}_i, \tag{S.49}$$
$$\mathbf{\Phi}_{i2} = -\mathbf{\Psi}_i, \tag{S.50}$$

[6]Noting that $\boldsymbol{\beta}'\mathbf{w}_{i0} = y_{i,0} - x_{i0}$, it follows $y_{i,0} = \boldsymbol{\beta}'\mathbf{w}_{i0} + x_{i,0}$.

and $\boldsymbol{\phi}_i = (\phi_{yi}, \phi_{xi})'$.

S.3.2 Summary of Monte Carlo experiments

We consider 36 experiments in total, based on the individual choices for:

1. distribution of errors (Gaussian and non-Gaussian),
2. direction of long run causality ($x \rightarrow y$, or $x \leftrightarrow y$),
3. the choice of the non-cointegration probability parameter π ($0, 0.05$ or 0.2), and
4. the choice of cross-sectional dependence of errors (none, SAR, or factor+SAR).

The following summary table provides summary of all experiments:

Table S1: Summary of Monte Carlo experiments

Experiment No.	Results reported in	Error distribution	LR causality	π	CS dependence of errors
1	Table S2	Gaussian	$x \rightarrow y$	0	none
2	Table S3	Gaussian	$x \rightarrow y$	0	SAR
3	Table S4	Gaussian	$x \rightarrow y$	0	Factor + SAR
4	Table S5	Gaussian	$x \rightarrow y$	0.05	none
5	Table S6	Gaussian	$x \rightarrow y$	0.05	SAR
6	Table S7	Gaussian	$x \rightarrow y$	0.05	Factor + SAR
7	Table S8	Gaussian	$x \rightarrow y$	0.2	none
8	Table S9	Gaussian	$x \rightarrow y$	0.2	SAR
9	Table S10	Gaussian	$x \rightarrow y$	0.2	Factor + SAR
10	Table S11	Gaussian	$x \leftrightarrow y$	0	none
11	Table S12	Gaussian	$x \leftrightarrow y$	0	SAR
12	Table S13	Gaussian	$x \leftrightarrow y$	0	Factor + SAR
13	Table S14	Gaussian	$x \leftrightarrow y$	0.05	none
14	Table S15	Gaussian	$x \leftrightarrow y$	0.05	SAR
15	Table S16	Gaussian	$x \leftrightarrow y$	0.05	Factor + SAR
16	Table S17	Gaussian	$x \leftrightarrow y$	0.2	none
17	Table S18	Gaussian	$x \leftrightarrow y$	0.2	SAR
18	Table S19	Gaussian	$x \leftrightarrow y$	0.2	Factor + SAR
19	Table S20	Non-Gaussian	$x \rightarrow y$	0	none
20	Table S21	Non-Gaussian	$x \rightarrow y$	0	SAR
21	Table S22	Non-Gaussian	$x \rightarrow y$	0	Factor + SAR
22	Table S23	Non-Gaussian	$x \rightarrow y$	0.05	none
23	Table S24	Non-Gaussian	$x \rightarrow y$	0.05	SAR
24	Table S25	Non-Gaussian	$x \rightarrow y$	0.05	Factor + SAR
25	Table S26	Non-Gaussian	$x \rightarrow y$	0.2	none
26	Table S27	Non-Gaussian	$x \rightarrow y$	0.2	SAR
27	Table S28	Non-Gaussian	$x \rightarrow y$	0.2	Factor + SAR
28	Table S29	Non-Gaussian	$x \leftrightarrow y$	0	none
29	Table S30	Non-Gaussian	$x \leftrightarrow y$	0	SAR
30	Table S31	Non-Gaussian	$x \leftrightarrow y$	0	Factor + SAR
31	Table S32	Non-Gaussian	$x \leftrightarrow y$	0.05	none
32	Table S33	Non-Gaussian	$x \leftrightarrow y$	0.05	SAR
33	Table S34	Non-Gaussian	$x \leftrightarrow y$	0.05	Factor + SAR
34	Table S35	Non-Gaussian	$x \leftrightarrow y$	0.2	none
35	Table S36	Non-Gaussian	$x \leftrightarrow y$	0.2	SAR
36	Table S37	Non-Gaussian	$x \leftrightarrow y$	0.2	Factor + SAR

Notes: See Section S.3.1 for details of the Monte Carlo design.

S.3.3 Monte Carlo results

This section presents Monte Carlo results, as outlined in Section S.3.2.

Table S2: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0$ and no CS dependence of errors.

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.05	-0.22	-0.09	-0.06	5.49	2.31	1.40	1.00	71.1	83.8	89.0	90.5	90.1	93.5	94.4	94.4
30	-0.86	-0.19	-0.09	-0.05	4.22	1.64	1.03	0.74	67.7	85.1	88.2	90.3	89.0	94.3	94.0	94.9
50	-1.03	-0.26	-0.10	-0.05	3.21	1.32	0.79	0.57	69.9	83.6	88.8	89.5	89.0	92.7	94.7	94.2
100	-0.91	-0.22	-0.08	-0.05	2.26	0.89	0.55	0.39	68.2	85.0	88.8	91.5	89.9	93.5	94.5	94.8
200	-0.98	-0.21	-0.09	-0.05	1.76	0.64	0.38	0.28	62.0	84.0	89.4	91.2	85.8	93.3	94.9	95.0
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.56	-0.66	-0.30	-0.18	5.64	2.56	1.60	1.14	76.2	85.5	90.1	91.2	74.7	84.9	89.5	91.2
30	-2.25	-0.57	-0.26	-0.13	4.46	1.90	1.14	0.85	74.6	85.7	90.5	91.1	72.2	84.4	89.5	91.1
50	-2.34	-0.65	-0.28	-0.15	3.71	1.54	0.91	0.66	68.9	84.5	89.7	91.8	66.5	83.4	89.0	91.5
100	-2.30	-0.62	-0.28	-0.15	3.08	1.14	0.66	0.47	57.2	81.8	88.9	90.7	56.0	80.5	88.8	90.3
200	-2.31	-0.62	-0.27	-0.15	2.72	0.92	0.51	0.35	39.2	75.3	85.2	87.9	38.0	74.5	84.4	86.9
	SPMG estimator								Standard				Robust bootstrapped			
17	0.12	0.06	0.04	0.01	6.71	2.53	1.47	1.03	59.5	79.4	86.2	89.1	86.7	91.5	93.6	94.2
30	0.24	0.07	0.02	0.01	4.95	1.74	1.06	0.76	59.1	80.5	86.8	88.7	86.1	93.8	93.7	94.6
50	-0.01	-0.04	-0.01	0.00	3.59	1.38	0.80	0.59	59.9	79.6	87.0	87.8	86.2	92.1	94.2	94.2
100	0.10	0.01	0.01	0.00	2.46	0.93	0.57	0.40	61.2	81.0	86.0	89.6	86.0	93.5	93.9	93.9
200	0.00	0.01	0.01	0.00	1.71	0.65	0.39	0.28	61.3	81.4	87.2	88.6	87.5	93.1	94.7	95.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.86	-2.39	-1.58	-1.17	6.82	3.44	2.27	1.67	75.8	79.4	80.5	81.8
30	-4.65	-2.30	-1.52	-1.11	5.94	2.93	1.91	1.42	69.2	70.9	71.8	73.1
50	-4.73	-2.38	-1.54	-1.12	5.50	2.76	1.79	1.31	56.0	54.1	55.9	59.4
100	-4.65	-2.31	-1.52	-1.11	5.07	2.51	1.65	1.21	31.7	30.4	31.4	32.8
200	-4.63	-2.30	-1.50	-1.10	4.83	2.40	1.57	1.15	8.5	7.1	7.6	7.8
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.73	-1.20	-0.76	-0.56	6.59	2.89	1.79	1.30	83.9	88.6	90.1	90.7
30	-2.47	-1.09	-0.71	-0.51	5.23	2.23	1.36	1.01	82.9	87.0	89.3	88.8
50	-2.49	-1.16	-0.74	-0.52	4.18	1.88	1.17	0.84	80.8	83.9	84.6	85.8
100	-2.50	-1.14	-0.73	-0.52	3.48	1.55	0.97	0.70	72.3	75.0	77.4	77.2
200	-2.48	-1.14	-0.72	-0.52	3.01	1.35	0.85	0.62	59.7	61.0	61.2	60.8
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.34	-0.60	-0.32	-0.24	10.82	3.27	1.83	1.28	70.4	87.7	90.8	92.4
30	-1.48	-0.46	-0.30	-0.20	8.14	2.41	1.32	0.95	71.1	89.6	92.1	92.2
50	-1.42	-0.52	-0.31	-0.21	5.95	1.83	1.05	0.73	75.2	89.5	90.8	92.4
100	-1.33	-0.49	-0.31	-0.21	4.35	1.34	0.77	0.54	75.1	88.6	90.0	90.9
200	-1.30	-0.52	-0.31	-0.22	3.24	1.02	0.59	0.42	76.1	85.6	87.8	88.2
	MGMW estimator, $q = 5$											
17	-5.54	-1.97	-0.98	-0.60	10.53	5.28	3.56	2.62	86.7	92.1	93.9	94.1
30	-5.64	-1.84	-0.87	-0.49	8.92	4.19	2.80	2.04	82.2	90.7	92.5	92.3
50	-5.64	-2.09	-0.99	-0.55	7.63	3.52	2.18	1.56	76.8	87.2	90.3	92.3
100	-5.59	-1.91	-0.96	-0.55	6.76	2.77	1.66	1.15	63.4	82.4	89.0	91.8
200	-5.55	-1.92	-0.93	-0.54	6.14	2.38	1.36	0.90	40.7	71.4	82.2	88.2

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0$, and no cross section dependence of errors. See Section S.3.1 for details of the data generating process. Description of the PMG, 2-step Breitung, SPMG, and MGMW estimators, and the description of bootstrapping procedures are provided in Sections S.2.1-S.2.3 of the online supplement. PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). The number of Monte Carlo replications is $R_{MC} = 2000$. Bootstrapped critical values are computed in each of the Monte Carlo replication as described in Sections S.2.1-S.2.3 of the online supplement, based on $R_b = 2000$ bootstrap replications.

Table S3: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.13	-0.24	-0.09	-0.06	6.87	3.03	1.88	1.37	62.3	75.0	78.5	81.6	90.5	93.4	93.9	94.5
30	-0.95	-0.23	-0.10	-0.07	4.90	2.12	1.34	0.98	62.7	75.7	79.4	80.1	89.2	93.8	94.4	94.2
50	-1.07	-0.24	-0.09	-0.06	3.77	1.62	1.00	0.77	61.8	74.4	79.5	79.1	88.5	92.3	94.4	93.5
100	-0.97	-0.21	-0.09	-0.06	2.71	1.10	0.69	0.50	60.1	75.7	79.9	82.3	87.2	93.1	94.8	95.7
200	-0.98	-0.20	-0.08	-0.05	2.01	0.80	0.50	0.36	56.0	73.6	79.3	81.7	86.8	93.0	94.1	94.4
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.87	-0.75	-0.31	-0.19	7.26	3.34	2.16	1.61	66.1	76.8	79.5	80.7	74.4	86.5	89.3	90.4
30	-2.36	-0.63	-0.29	-0.16	5.33	2.48	1.55	1.15	66.6	76.6	79.7	81.3	73.2	85.7	88.9	89.9
50	-2.50	-0.65	-0.28	-0.16	4.43	1.90	1.19	0.90	60.2	75.8	79.7	79.4	67.2	83.9	88.4	89.9
100	-2.39	-0.64	-0.30	-0.17	3.47	1.39	0.84	0.60	52.8	72.6	80.0	82.9	60.7	82.2	88.0	91.6
200	-2.32	-0.61	-0.28	-0.15	2.94	1.07	0.63	0.45	40.4	67.4	77.8	78.9	47.7	77.9	86.7	88.9
	SPMG estimator								Standard				Robust bootstrapped			
17	0.17	0.09	0.05	0.01	8.41	3.28	1.96	1.41	53.1	70.5	75.6	79.5	86.2	92.3	93.5	94.0
30	0.15	0.05	0.02	0.00	5.68	2.26	1.39	1.01	53.9	70.3	76.5	78.9	86.3	92.6	93.7	93.8
50	-0.03	0.00	0.00	-0.01	4.36	1.69	1.03	0.79	52.5	70.4	75.5	76.7	84.4	91.7	94.3	93.2
100	0.04	0.02	0.00	-0.01	2.97	1.14	0.70	0.51	53.0	71.9	77.9	80.3	85.8	93.0	94.2	95.2
200	0.05	0.04	0.02	0.01	2.03	0.81	0.51	0.37	54.0	72.3	76.8	79.1	86.5	92.2	93.7	93.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.19	-2.55	-1.69	-1.26	8.14	4.12	2.74	2.06	68.8	72.3	73.2	72.7
30	-4.73	-2.40	-1.62	-1.20	6.58	3.40	2.25	1.69	64.5	65.1	65.0	66.0
50	-4.92	-2.43	-1.59	-1.19	6.03	3.01	1.99	1.50	52.1	52.4	54.1	54.8
100	-4.76	-2.36	-1.58	-1.17	5.35	2.68	1.78	1.32	33.0	33.4	33.0	34.2
200	-4.67	-2.34	-1.54	-1.14	4.98	2.50	1.65	1.22	14.5	12.8	12.3	12.6
	PDOLS estimator, leads and lags order $p = 4$											
17	-3.03	-1.29	-0.80	-0.58	8.37	3.72	2.34	1.73	74.2	79.3	80.1	80.9
30	-2.46	-1.15	-0.76	-0.56	6.21	2.82	1.76	1.30	74.3	79.0	78.1	79.6
50	-2.68	-1.18	-0.76	-0.56	4.95	2.20	1.42	1.05	73.4	76.8	76.2	75.7
100	-2.55	-1.16	-0.76	-0.56	3.92	1.77	1.12	0.82	67.6	69.3	69.9	71.8
200	-2.47	-1.15	-0.74	-0.54	3.28	1.48	0.94	0.69	56.8	57.4	58.1	57.2
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.88	-0.64	-0.32	-0.20	13.34	4.28	2.47	1.75	61.0	79.1	80.8	82.4
30	-1.35	-0.49	-0.32	-0.24	9.37	3.10	1.77	1.27	65.6	81.3	82.5	83.3
50	-1.66	-0.49	-0.30	-0.22	7.05	2.27	1.35	0.98	68.8	81.8	82.0	81.2
100	-1.28	-0.48	-0.32	-0.23	5.11	1.64	0.96	0.67	66.8	80.7	81.8	83.7
200	-1.29	-0.53	-0.31	-0.22	3.70	1.23	0.72	0.51	68.4	78.0	79.6	81.4
	MGMW estimator, $q = 5$											
17	-5.69	-1.95	-0.96	-0.57	11.73	6.05	4.03	3.08	81.3	85.5	86.5	86.2
30	-5.76	-1.94	-0.96	-0.57	9.58	4.82	3.21	2.35	79.1	84.3	85.1	85.9
50	-5.78	-2.10	-1.00	-0.56	8.27	3.80	2.42	1.80	72.7	83.4	86.6	87.1
100	-5.73	-1.92	-0.98	-0.61	7.11	2.97	1.86	1.33	59.9	78.6	84.3	87.0
200	-5.58	-1.94	-0.97	-0.55	6.37	2.57	1.50	1.00	42.5	67.0	78.9	83.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0$, and SAR cross section dependence of errors. See notes to Table S1.

Table S4: MC findings for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-0.48	-0.16	-0.03	-0.02	5.12	2.27	1.42	1.07	55.2	64.3	70.0	71.0	89.2	93.3	93.7	94.2
30	-0.32	-0.08	0.00	-0.04	3.69	1.69	1.05	0.79	53.8	63.8	68.7	67.8	89.8	92.6	94.5	93.9
50	-0.36	-0.07	-0.03	0.02	2.97	1.29	0.83	0.62	50.9	62.9	64.9	67.4	89.8	94.3	94.1	94.2
100	-0.34	-0.02	-0.01	-0.01	2.14	0.99	0.61	0.44	48.3	58.6	63.9	65.2	88.8	93.7	95.0	94.7
200	-0.22	-0.04	-0.03	-0.01	1.59	0.74	0.45	0.34	47.5	54.6	60.0	60.5	90.4	94.4	95.6	95.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-1.50	-0.35	-0.12	-0.09	5.36	2.56	1.66	1.23	60.4	66.3	69.8	70.7	78.9	87.9	90.6	91.7
30	-1.17	-0.32	-0.10	-0.09	4.16	1.95	1.22	0.92	58.5	66.2	69.5	69.3	79.0	87.5	91.6	92.1
50	-1.16	-0.33	-0.16	-0.05	3.24	1.52	1.00	0.72	55.6	64.5	64.9	67.8	79.4	90.3	91.8	93.6
100	-1.21	-0.26	-0.13	-0.08	2.60	1.14	0.74	0.53	49.0	60.0	62.4	65.7	79.0	91.3	94.2	94.6
200	-1.04	-0.28	-0.14	-0.07	2.06	0.92	0.56	0.41	45.0	54.3	60.4	60.5	84.1	93.6	95.9	96.0
	SPMG estimator								Standard				Robust bootstrapped			
17	0.18	-0.05	0.03	0.02	7.09	2.50	1.52	1.10	45.2	58.9	66.3	67.8	85.1	92.2	92.9	93.8
30	0.05	0.02	0.06	0.00	5.17	1.84	1.10	0.81	44.4	58.8	64.0	65.2	83.9	90.8	93.4	93.7
50	-0.07	0.02	0.01	0.04	4.30	1.38	0.88	0.65	43.8	58.8	61.5	64.6	84.4	92.6	92.8	93.4
100	-0.11	0.08	0.03	0.01	4.42	1.08	0.63	0.46	40.9	53.2	60.0	62.8	83.3	91.3	94.1	94.1
200	0.16	0.05	0.01	0.01	2.34	0.84	0.48	0.34	40.6	51.1	58.6	60.4	83.8	91.6	94.3	94.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-2.89	-1.46	-0.95	-0.74	5.65	2.91	1.96	1.48	69.4	70.0	70.2	70.2
30	-2.53	-1.41	-0.96	-0.73	4.55	2.43	1.63	1.22	65.2	64.3	64.4	64.5
50	-2.70	-1.43	-1.00	-0.73	3.94	2.10	1.47	1.09	56.7	56.1	55.8	56.3
100	-2.67	-1.38	-0.99	-0.74	3.45	1.80	1.28	0.96	44.3	45.1	42.0	40.5
200	-2.51	-1.40	-0.94	-0.71	3.04	1.69	1.12	0.86	29.4	25.3	23.9	25.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.01	-0.91	-0.52	-0.41	6.42	2.85	1.81	1.31	68.6	71.1	72.9	73.2
30	-1.74	-0.88	-0.55	-0.42	4.98	2.29	1.38	1.03	66.9	68.4	69.8	69.6
50	-1.85	-0.86	-0.59	-0.41	4.11	1.79	1.18	0.85	63.2	66.1	64.8	65.5
100	-1.79	-0.81	-0.57	-0.42	3.31	1.42	0.95	0.70	56.6	59.7	59.5	58.4
200	-1.61	-0.85	-0.56	-0.41	2.69	1.25	0.79	0.58	49.1	47.6	46.7	46.2
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.37	-0.52	-0.26	-0.19	10.84	3.37	1.97	1.37	54.3	70.5	72.0	73.1
30	-1.22	-0.51	-0.28	-0.23	8.30	2.64	1.43	1.06	53.9	66.7	72.3	69.7
50	-1.23	-0.47	-0.31	-0.19	6.63	1.97	1.17	0.81	50.8	67.3	67.3	70.2
100	-1.07	-0.43	-0.31	-0.22	5.08	1.52	0.90	0.63	47.6	62.4	65.6	64.6
200	-0.89	-0.47	-0.31	-0.21	4.30	1.26	0.71	0.50	42.4	55.6	58.7	59.5
	MGMW estimator, $q = 5$											
17	-1.45	-0.56	-0.17	-0.13	7.25	3.99	2.70	1.97	78.8	79.0	78.4	78.9
30	-0.82	-0.40	-0.15	-0.13	5.38	3.06	2.01	1.56	80.1	79.1	78.9	78.3
50	-0.99	-0.51	-0.26	-0.13	4.34	2.32	1.60	1.21	79.0	80.9	78.3	79.1
100	-0.96	-0.33	-0.22	-0.12	3.32	1.66	1.17	0.87	76.5	79.1	79.8	78.0
200	-0.84	-0.39	-0.24	-0.15	2.58	1.28	0.85	0.62	73.1	77.3	78.0	78.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0$, and factor+SAR cross section dependence of errors. See notes to Table S1.

Table S5: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.05$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.15	-0.22	-0.10	-0.06	5.65	2.38	1.45	1.03	71.2	84.4	88.9	90.4	89.9	93.8	94.5	94.2
30	-0.95	-0.20	-0.10	-0.06	4.36	1.71	1.06	0.75	67.7	84.6	87.6	90.2	88.7	94.2	94.0	94.5
50	-1.11	-0.27	-0.10	-0.05	3.34	1.36	0.81	0.59	68.6	83.3	88.9	89.6	89.4	92.6	93.9	94.9
100	-0.96	-0.22	-0.08	-0.05	2.33	0.91	0.57	0.40	67.1	84.8	88.1	91.2	89.3	93.5	94.1	94.5
200	-1.06	-0.22	-0.09	-0.05	1.83	0.66	0.39	0.29	61.8	84.1	89.2	91.2	85.1	93.6	95.1	94.8
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-3.63	-1.25	-0.77	-0.48	8.04	5.50	4.74	4.50	70.4	78.2	83.8	84.6	68.7	77.5	82.9	84.4
30	-3.54	-1.27	-0.86	-0.48	9.29	3.92	4.57	3.11	66.4	78.6	80.5	80.3	64.6	77.1	79.8	79.2
50	-3.45	-1.34	-0.80	-0.65	5.27	3.17	2.81	2.68	60.3	74.5	76.8	76.9	58.2	73.8	76.4	77.1
100	-3.44	-1.34	-0.80	-0.57	4.53	2.75	1.97	2.01	45.2	66.6	72.6	72.9	44.2	65.6	72.1	72.4
200	-3.48	-1.25	-0.77	-0.58	4.02	2.03	1.96	1.37	26.8	58.6	66.5	68.1	25.5	57.4	65.7	68.0
	SPMG estimator								Standard				Robust bootstrapped			
17	0.13	0.08	0.04	0.01	6.82	2.60	1.52	1.06	60.1	79.5	85.8	89.3	86.8	92.0	94.0	94.1
30	0.23	0.08	0.02	0.01	5.10	1.81	1.09	0.77	59.1	80.1	86.4	88.3	86.0	93.2	93.9	94.3
50	0.00	-0.04	0.00	0.00	3.72	1.43	0.83	0.60	60.4	78.9	87.4	88.1	86.0	92.0	94.0	94.3
100	0.13	0.02	0.02	0.01	2.53	0.95	0.59	0.41	60.1	81.2	86.0	89.6	86.2	93.3	93.4	93.7
200	-0.01	0.02	0.01	0.00	1.76	0.67	0.40	0.29	61.9	81.0	87.3	88.6	87.0	92.5	94.2	94.9

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals.

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.19	-3.38	-2.40	-1.93	8.79	5.26	3.98	3.58	75.5	80.8	82.8	85.3
30	-6.01	-3.26	-2.29	-1.79	7.67	4.45	3.30	2.76	67.7	74.0	77.7	80.3
50	-6.01	-3.30	-2.34	-1.89	7.02	4.02	3.05	2.64	57.1	63.3	67.8	73.2
100	-5.98	-3.28	-2.33	-1.82	6.52	3.67	2.69	2.20	33.9	41.8	49.7	56.5
200	-5.97	-3.23	-2.29	-1.81	6.23	3.44	2.48	2.02	16.9	21.1	28.1	36.9
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.22	-2.29	-1.64	-1.35	8.76	5.05	3.74	3.46	82.1	89.3	90.7	91.9
30	-4.03	-2.15	-1.54	-1.23	7.13	3.99	2.96	2.52	80.4	88.5	89.1	90.0
50	-3.95	-2.17	-1.60	-1.34	5.89	3.30	2.62	2.37	78.5	84.7	85.6	87.7
100	-4.01	-2.20	-1.59	-1.28	5.08	2.86	2.16	1.82	67.6	74.3	76.7	80.6
200	-4.02	-2.16	-1.56	-1.27	4.58	2.52	1.86	1.58	49.9	60.4	65.1	71.0
17	PDOLS estimator, leads and lags order $p = 8$											
	-2.93	-1.78	-1.26	-1.07	12.79	5.67	3.93	3.61	69.7	88.0	91.9	93.3
30	-3.19	-1.62	-1.19	-0.95	9.92	4.31	3.05	2.54	69.4	90.3	91.6	92.6
50	-2.99	-1.58	-1.22	-1.07	7.39	3.27	2.60	2.38	73.3	88.7	90.2	92.0
100	-3.01	-1.65	-1.23	-1.00	5.75	2.71	2.03	1.72	68.9	84.7	86.9	89.8
200	-2.95	-1.62	-1.20	-1.01	4.57	2.21	1.64	1.42	65.5	79.2	83.0	85.0
	MGMW estimator, $q = 5$											
17	-6.87	-2.91	-1.87	-1.29	12.70	7.40	5.97	5.50	85.6	92.0	94.4	94.7
30	-6.90	-2.87	-1.68	-1.15	10.58	5.95	4.55	4.04	81.0	89.8	92.0	93.5
50	-6.82	-3.00	-1.86	-1.45	9.00	4.95	3.75	3.11	74.7	85.9	90.6	92.9
100	-6.90	-2.96	-1.80	-1.29	8.16	4.13	2.91	2.39	60.0	77.7	86.5	89.4
200	-6.86	-2.93	-1.76	-1.28	7.49	3.50	2.41	1.93	34.2	63.6	76.4	83.1

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S6: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.22	-0.23	-0.09	-0.07	7.10	3.12	1.93	1.40	62.2	74.8	78.2	81.8	90.3	93.2	94.2	94.6
30	-1.04	-0.24	-0.11	-0.07	5.04	2.16	1.36	1.00	63.0	75.1	79.3	80.3	88.8	93.9	94.4	94.1
50	-1.14	-0.26	-0.10	-0.06	3.88	1.65	1.02	0.78	62.1	74.3	79.3	79.5	88.3	92.6	94.5	93.6
100	-1.03	-0.22	-0.09	-0.06	2.78	1.13	0.70	0.51	59.3	75.6	80.6	82.0	87.5	93.1	95.1	95.5
200	-1.05	-0.21	-0.08	-0.04	2.08	0.82	0.51	0.37	54.9	73.9	79.1	80.9	86.5	92.8	95.1	94.7
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-3.93	-1.40	-0.77	-0.67	9.74	6.97	4.77	4.85	62.6	72.5	75.8	75.8	69.2	81.2	83.7	84.2
30	-3.42	-1.28	-0.78	-0.51	6.95	4.41	3.66	3.65	60.4	71.4	73.0	74.7	66.8	78.6	81.7	81.3
50	-3.65	-1.36	-0.86	-0.77	5.83	3.31	3.14	3.45	53.9	67.6	72.1	70.9	60.2	75.2	79.0	78.0
100	-3.63	-1.33	-0.83	-0.63	5.06	2.76	2.10	2.22	43.3	61.9	66.9	70.1	49.9	69.8	73.7	76.1
200	-3.46	-1.32	-0.82	-0.58	4.27	2.09	1.91	1.92	29.5	56.2	63.6	65.7	34.2	61.9	69.3	72.2
	SPMG estimator								Standard				Robust bootstrapped			
17	0.17	0.12	0.06	0.01	8.64	3.37	2.01	1.44	52.4	70.1	75.5	79.7	85.9	92.2	93.5	94.2
30	0.13	0.07	0.02	0.00	5.81	2.31	1.42	1.03	54.9	70.4	76.3	78.5	85.7	93.0	93.3	94.2
50	0.00	-0.01	0.00	-0.01	4.48	1.73	1.05	0.81	53.7	70.1	76.1	77.1	84.4	92.0	93.7	93.2
100	0.06	0.03	0.01	0.00	3.02	1.17	0.72	0.52	53.4	71.0	78.4	80.5	85.5	93.0	94.4	95.5
200	0.04	0.05	0.03	0.02	2.08	0.83	0.52	0.38	53.5	72.0	76.9	79.2	86.5	91.8	94.0	93.8

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.47	-3.54	-2.60	-2.12	9.94	5.95	4.57	4.18	68.6	75.4	77.3	77.9
30	-6.08	-3.36	-2.41	-1.92	8.16	4.81	3.57	3.02	64.5	69.8	72.5	75.4
50	-6.24	-3.40	-2.43	-2.00	7.52	4.27	3.24	2.82	53.5	60.4	65.2	68.2
100	-6.11	-3.34	-2.42	-1.94	6.80	3.83	2.85	2.36	34.2	41.5	47.0	55.1
200	-6.02	-3.29	-2.35	-1.88	6.38	3.54	2.57	2.11	18.4	23.5	30.1	37.5
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.50	-2.39	-1.79	-1.48	10.37	5.89	4.37	4.07	73.9	81.3	83.3	83.4
30	-4.00	-2.20	-1.61	-1.32	7.87	4.40	3.23	2.77	74.3	82.1	82.2	85.1
50	-4.19	-2.23	-1.66	-1.41	6.60	3.57	2.83	2.55	72.0	78.5	80.0	82.5
100	-4.10	-2.23	-1.66	-1.37	5.47	3.03	2.30	1.97	64.7	70.5	72.8	77.7
200	-4.01	-2.19	-1.60	-1.32	4.78	2.63	1.93	1.65	50.2	58.5	63.8	67.7
	PDOLS estimator, leads and lags order $p = 8$											
17	-3.55	-1.84	-1.37	-1.15	15.42	6.66	4.65	4.26	60.0	80.4	84.3	85.0
30	-3.07	-1.63	-1.23	-1.02	11.02	4.80	3.35	2.80	66.2	82.9	85.5	87.5
50	-3.36	-1.61	-1.26	-1.12	8.67	3.57	2.82	2.57	67.1	83.4	86.0	86.7
100	-2.99	-1.65	-1.28	-1.08	6.38	2.90	2.17	1.87	64.6	80.6	82.9	86.3
200	-2.94	-1.65	-1.23	-1.04	4.90	2.37	1.72	1.49	62.9	75.8	78.2	82.5
	MGMW estimator, $q = 5$											
17	-7.01	-3.01	-1.81	-1.33	13.51	8.13	6.52	6.38	80.3	87.0	89.1	88.8
30	-6.95	-2.89	-1.79	-1.36	11.11	6.55	4.82	4.28	77.8	85.6	87.1	88.8
50	-7.00	-3.11	-1.85	-1.43	9.66	5.32	4.03	3.38	71.0	81.9	87.8	89.5
100	-7.11	-2.99	-1.90	-1.45	8.53	4.29	3.11	2.59	57.6	76.8	82.7	86.0
200	-6.88	-2.97	-1.80	-1.35	7.69	3.69	2.47	2.02	38.0	62.9	75.4	80.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S7: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-0.53	-0.15	-0.04	-0.02	5.22	2.27	1.44	1.08	55.1	66.3	71.0	71.5	89.6	93.8	93.8	94.3
30	-0.36	-0.08	0.00	-0.04	3.77	1.71	1.07	0.80	53.6	64.3	69.2	68.4	89.7	92.9	94.2	93.9
50	-0.38	-0.07	-0.03	0.02	3.03	1.31	0.84	0.63	51.7	63.6	65.4	68.3	89.6	94.4	94.2	94.2
100	-0.37	-0.02	-0.01	-0.01	2.19	1.00	0.62	0.45	48.7	58.8	64.6	66.2	89.2	94.0	95.1	94.7
200	-0.25	-0.05	-0.03	-0.01	1.63	0.75	0.46	0.34	47.8	55.1	61.1	60.8	90.7	94.7	95.9	95.5
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.00	-0.66	-0.35	-0.34	7.03	4.42	3.79	3.60	57.4	65.6	66.8	68.4	75.1	84.0	84.7	85.3
30	-1.68	-0.63	-0.39	-0.30	5.07	3.60	2.69	2.25	57.3	63.8	67.1	64.6	74.8	82.5	84.9	82.6
50	-1.71	-0.79	-0.56	-0.38	4.04	2.67	2.14	1.96	53.0	62.5	63.5	63.9	75.5	82.1	83.2	82.6
100	-1.77	-0.65	-0.44	-0.37	3.33	2.05	1.57	1.53	45.4	58.2	60.6	63.7	71.7	84.6	84.1	82.9
200	-1.59	-0.65	-0.40	-0.34	2.68	1.48	1.15	1.13	40.3	51.6	58.7	61.0	73.8	84.3	86.0	84.3
	SPMG estimator								Standard				Robust bootstrapped			
17	0.15	-0.02	0.03	0.02	7.41	2.52	1.55	1.11	45.3	60.1	66.4	68.8	85.0	92.6	93.1	93.9
30	0.00	0.03	0.06	0.00	5.34	1.86	1.12	0.82	45.3	59.3	65.6	66.2	83.9	90.9	93.0	93.4
50	-0.10	0.02	0.01	0.04	4.67	1.41	0.89	0.66	44.0	59.1	61.4	64.8	84.3	92.4	93.2	93.7
100	-0.14	0.09	0.03	0.01	5.01	1.09	0.64	0.46	41.0	54.4	60.8	63.5	83.7	91.5	94.1	93.8
200	0.14	0.05	0.02	0.01	2.47	0.86	0.48	0.35	41.2	51.6	58.9	61.2	83.5	91.6	94.5	94.5

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-3.47	-1.91	-1.35	-1.14	6.52	3.81	2.99	2.51	71.3	74.9	76.1	76.8
30	-3.09	-1.79	-1.36	-1.07	5.31	3.10	2.35	1.96	68.6	71.8	73.0	75.5
50	-3.32	-1.86	-1.42	-1.12	4.64	2.73	2.17	1.75	62.0	67.3	67.2	72.7
100	-3.24	-1.82	-1.40	-1.11	4.08	2.33	1.85	1.50	50.3	56.7	59.0	65.4
200	-3.12	-1.85	-1.32	-1.07	3.69	2.21	1.57	1.31	36.9	40.1	43.3	51.4
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.72	-1.42	-0.96	-0.84	7.44	3.95	3.00	2.49	71.1	76.3	78.5	80.6
30	-2.39	-1.30	-0.99	-0.78	5.78	3.09	2.22	1.86	70.0	75.6	78.9	79.3
50	-2.60	-1.34	-1.05	-0.83	4.89	2.53	1.99	1.60	66.0	75.9	76.2	80.9
100	-2.48	-1.31	-1.03	-0.83	3.97	2.00	1.58	1.30	60.6	71.7	73.3	76.8
200	-2.35	-1.36	-0.97	-0.79	3.38	1.83	1.28	1.07	53.5	61.6	65.4	71.2
	PDOLS estimator, leads and lags order $p = 8$											
17	-2.12	-1.09	-0.75	-0.66	11.65	4.60	3.29	2.68	54.4	75.6	77.2	80.8
30	-1.97	-0.97	-0.77	-0.62	8.96	3.55	2.37	1.93	55.0	73.8	80.3	79.2
50	-2.07	-1.02	-0.81	-0.64	7.27	2.75	2.03	1.62	52.3	75.8	79.6	83.7
100	-1.85	-0.99	-0.81	-0.65	5.58	2.08	1.56	1.26	50.7	75.1	79.0	83.4
200	-1.70	-1.05	-0.76	-0.62	4.71	1.82	1.19	0.99	43.8	69.2	73.7	79.0
	MGMW estimator, $q = 5$											
17	-1.69	-0.66	-0.27	-0.22	8.09	4.93	4.01	3.45	81.0	82.3	82.5	83.5
30	-0.94	-0.44	-0.30	-0.24	5.90	3.84	2.79	2.39	82.4	83.1	84.4	85.2
50	-1.19	-0.61	-0.45	-0.26	4.88	2.98	2.30	1.93	81.6	84.4	84.2	86.9
100	-1.10	-0.48	-0.36	-0.25	3.70	2.10	1.62	1.53	79.7	84.0	86.2	87.0
200	-1.00	-0.52	-0.34	-0.24	2.93	1.67	1.20	1.03	76.8	81.3	84.9	86.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S8: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.56	-0.28	-0.12	-0.07	6.53	2.64	1.59	1.15	69.3	83.3	89.0	90.8	88.9	94.0	94.7	94.1
30	-1.28	-0.25	-0.12	-0.07	4.92	1.86	1.16	0.83	67.2	84.9	88.1	90.0	88.0	94.3	94.5	94.1
50	-1.38	-0.31	-0.12	-0.07	3.76	1.48	0.89	0.64	66.0	82.6	88.2	90.1	88.0	92.4	93.7	94.2
100	-1.24	-0.28	-0.10	-0.05	2.64	1.01	0.62	0.43	63.9	83.8	88.2	91.0	88.4	93.3	94.3	95.0
200	-1.29	-0.26	-0.10	-0.06	2.08	0.73	0.44	0.32	57.7	82.9	88.1	89.9	84.5	93.4	94.7	94.5
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-7.14	-3.40	-2.58	-1.97	13.70	10.30	9.68	8.53	58.2	68.4	72.0	71.3	57.5	67.2	71.0	70.9
30	-6.92	-3.03	-2.26	-1.52	13.29	8.25	7.10	6.43	50.9	66.8	69.2	70.7	49.2	66.5	68.4	70.8
50	-7.05	-3.30	-2.35	-1.94	10.15	6.46	5.69	5.65	40.9	60.2	63.4	66.1	39.5	59.3	62.7	65.6
100	-7.00	-3.33	-2.09	-1.90	8.61	5.73	4.79	4.81	26.2	51.9	60.9	63.9	25.9	51.2	60.4	63.5
200	-7.12	-3.22	-2.22	-1.96	8.06	4.39	3.84	7.77	12.3	38.8	50.0	57.4	11.8	38.8	50.0	57.4
	SPMG estimator								Standard				Robust bootstrapped			
17	0.08	0.12	0.07	0.03	8.06	2.92	1.67	1.19	59.2	79.0	85.6	88.9	85.9	92.2	93.8	94.2
30	0.27	0.11	0.04	0.02	5.80	1.99	1.20	0.85	57.5	80.7	86.5	87.8	85.3	93.6	93.9	94.5
50	0.05	0.00	0.01	0.00	4.18	1.55	0.92	0.66	58.8	79.8	86.7	89.0	85.2	92.0	93.4	93.9
100	0.13	0.03	0.03	0.02	2.80	1.05	0.64	0.44	58.6	81.0	87.0	89.6	86.7	92.9	93.5	95.0
200	0.03	0.05	0.03	0.02	1.91	0.73	0.45	0.32	60.1	80.4	86.3	88.6	88.4	92.7	93.9	94.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-10.27	-6.31	-4.98	-4.14	13.62	9.18	7.73	6.89	72.7	81.8	85.5	87.8
30	-9.92	-5.89	-4.53	-3.79	12.06	7.61	6.18	5.45	64.3	77.1	81.5	85.6
50	-10.19	-6.17	-4.84	-4.14	11.49	7.27	5.94	5.30	53.8	66.3	72.7	75.7
100	-10.10	-6.14	-4.69	-3.93	10.84	6.77	5.28	4.51	35.4	46.8	54.4	61.6
200	-10.10	-6.10	-4.67	-3.95	10.48	6.43	4.97	4.27	24.8	26.3	32.0	39.6
	PDOLS estimator, leads and lags order $p = 4$											
17	-8.90	-5.50	-4.39	-3.70	14.19	9.29	7.72	6.93	76.3	86.6	89.3	90.6
30	-8.45	-5.03	-3.93	-3.33	11.88	7.40	6.02	5.33	72.5	85.3	86.3	88.4
50	-8.70	-5.29	-4.27	-3.71	10.85	6.80	5.66	5.11	67.6	80.3	80.1	82.0
100	-8.71	-5.34	-4.12	-3.50	9.92	6.21	4.88	4.22	52.9	66.4	68.9	73.0
200	-8.72	-5.31	-4.11	-3.52	9.35	5.77	4.50	3.92	35.7	49.9	52.0	57.8
	PDOLS estimator, leads and lags order $p = 8$											
17	-8.06	-5.21	-4.14	-3.55	17.85	10.28	8.10	7.28	64.7	85.0	88.8	90.8
30	-7.91	-4.74	-3.72	-3.16	14.59	7.94	6.29	5.49	63.7	85.2	88.4	89.8
50	-8.26	-4.93	-4.06	-3.55	12.51	6.96	5.77	5.20	61.2	82.1	83.7	85.9
100	-8.18	-5.05	-3.92	-3.34	10.64	6.26	4.87	4.20	50.5	71.3	76.9	79.6
200	-8.15	-5.02	-3.91	-3.36	9.51	5.65	4.41	3.85	37.7	57.9	64.7	66.8
	MGMW estimator, $q = 5$											
17	-10.54	-5.50	-4.63	-3.57	17.46	12.32	10.48	9.56	84.2	90.2	93.5	93.6
30	-10.59	-5.62	-4.21	-3.38	15.01	9.85	8.12	7.58	76.3	85.9	89.5	91.4
50	-10.73	-5.97	-4.44	-3.92	13.27	8.54	7.06	6.30	66.4	79.4	83.8	87.2
100	-10.76	-5.81	-4.20	-3.48	12.19	7.35	5.76	5.00	48.3	68.0	75.7	79.1
200	-10.74	-5.89	-4.28	-3.52	11.47	6.65	5.06	4.33	21.9	45.8	58.7	66.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S9: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.65	-0.27	-0.10	-0.07	7.87	3.33	2.05	1.49	61.2	75.2	79.8	81.9	89.7	94.1	94.9	94.7
30	-1.34	-0.30	-0.13	-0.09	5.48	2.32	1.46	1.07	61.9	76.2	78.9	81.0	89.3	94.5	94.6	94.5
50	-1.41	-0.29	-0.12	-0.08	4.24	1.78	1.10	0.84	62.2	75.1	80.9	79.8	88.7	92.7	94.2	93.8
100	-1.32	-0.27	-0.11	-0.06	3.06	1.22	0.75	0.54	58.5	76.2	81.4	83.5	87.4	93.7	95.0	95.3
200	-1.29	-0.25	-0.10	-0.05	2.32	0.88	0.55	0.40	53.7	74.4	78.9	82.7	85.9	93.0	94.7	93.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-7.32	-3.68	-2.25	-2.10	14.38	13.14	12.06	9.58	54.7	65.0	66.6	67.3	60.1	71.8	72.7	72.8
30	-6.85	-3.18	-2.35	-1.77	11.67	9.88	7.15	8.68	48.7	61.9	66.0	68.0	53.5	67.0	71.1	71.8
50	-7.21	-3.45	-2.61	-2.15	10.42	6.44	6.43	6.27	40.2	57.9	62.8	63.0	44.4	63.1	66.5	67.2
100	-7.22	-3.33	-2.33	-1.94	9.18	5.48	4.63	4.42	26.7	51.2	58.5	60.3	29.7	54.4	62.4	63.8
200	-7.00	-3.36	-2.35	-1.87	8.28	4.53	3.86	3.44	14.3	38.6	49.8	54.5	16.3	42.1	53.5	57.3
	SPMG estimator								Standard				Robust bootstrapped			
17	0.21	0.20	0.09	0.03	9.74	3.62	2.14	1.55	52.2	72.5	76.6	81.4	86.3	92.3	94.4	94.4
30	0.21	0.10	0.04	0.01	6.37	2.48	1.52	1.10	52.9	70.8	76.9	79.3	85.8	93.3	93.6	94.3
50	0.06	0.05	0.02	0.00	4.85	1.86	1.14	0.86	53.4	71.8	78.1	77.8	85.4	92.4	93.6	93.2
100	0.03	0.04	0.02	0.01	3.22	1.25	0.77	0.56	53.3	72.8	79.7	81.3	86.4	93.3	94.6	95.2
200	0.07	0.07	0.04	0.03	2.23	0.90	0.56	0.41	53.7	71.2	76.5	79.4	85.9	92.0	93.5	93.8

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-10.53	-6.53	-5.19	-4.43	14.55	9.70	8.17	7.41	69.2	78.1	82.2	85.2
30	-10.13	-6.12	-4.80	-4.10	12.72	8.01	6.60	5.85	63.2	73.2	78.0	81.8
50	-10.51	-6.39	-5.07	-4.37	12.07	7.66	6.35	5.69	51.6	61.9	68.5	74.1
100	-10.33	-6.29	-4.91	-4.19	11.19	6.99	5.55	4.82	35.6	45.5	52.5	57.9
200	-10.13	-6.22	-4.83	-4.14	10.57	6.57	5.17	4.48	25.5	26.6	33.0	38.5
	PDOLS estimator, leads and lags order $p = 4$											
17	-9.16	-5.66	-4.55	-3.92	15.42	9.86	8.19	7.46	72.6	82.4	85.5	87.7
30	-8.57	-5.22	-4.17	-3.61	12.67	7.81	6.45	5.72	70.3	82.2	83.9	86.4
50	-9.06	-5.50	-4.48	-3.91	11.58	7.22	6.10	5.51	63.8	75.2	76.9	80.1
100	-8.91	-5.46	-4.32	-3.73	10.31	6.43	5.14	4.52	51.1	63.7	66.8	69.6
200	-8.70	-5.41	-4.26	-3.69	9.44	5.91	4.69	4.13	37.4	48.6	50.2	54.4
	PDOLS estimator, leads and lags order $p = 8$											
17	-8.46	-5.35	-4.27	-3.72	20.21	10.89	8.68	7.86	59.5	81.1	85.2	87.8
30	-8.07	-4.89	-3.95	-3.43	15.66	8.41	6.76	5.90	61.3	81.6	85.2	87.8
50	-8.71	-5.12	-4.26	-3.73	13.73	7.43	6.23	5.61	58.6	79.0	82.4	83.1
100	-8.24	-5.15	-4.11	-3.57	11.07	6.48	5.14	4.51	50.0	69.6	74.7	76.5
200	-8.16	-5.13	-4.06	-3.53	9.68	5.82	4.61	4.06	37.9	57.2	62.3	64.7
	MGMW estimator, $q = 5$											
17	-10.73	-5.98	-4.47	-3.65	18.69	12.85	11.38	10.58	80.4	88.2	90.1	91.0
30	-10.90	-6.05	-4.47	-3.98	15.81	10.70	8.56	8.14	72.6	83.0	87.3	88.6
50	-11.08	-6.32	-4.62	-3.96	14.11	9.21	7.65	6.73	63.3	76.9	81.2	84.0
100	-10.96	-5.88	-4.40	-3.75	12.59	7.55	6.01	5.35	46.3	65.7	73.5	77.8
200	-10.82	-5.99	-4.40	-3.71	11.69	6.82	5.25	4.61	24.0	46.9	57.8	63.2

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S10: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-0.62	-0.15	-0.05	-0.03	5.61	2.38	1.54	1.14	55.3	67.6	72.7	73.7	89.7	93.8	94.1	93.9
30	-0.47	-0.09	0.00	-0.04	4.13	1.80	1.12	0.84	54.9	65.9	71.4	70.2	90.1	93.4	94.8	94.2
50	-0.45	-0.09	-0.03	0.01	3.20	1.37	0.89	0.66	52.7	65.7	68.0	69.6	89.8	94.5	94.3	94.0
100	-0.46	-0.04	-0.01	-0.01	2.33	1.05	0.64	0.47	48.2	61.3	66.2	68.7	89.5	94.3	94.9	95.0
200	-0.32	-0.06	-0.03	-0.01	1.75	0.79	0.48	0.36	48.7	56.8	63.6	62.3	90.1	94.6	95.9	95.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-3.58	-2.07	-1.34	-0.86	11.44	12.44	7.08	7.33	53.6	60.2	62.8	64.5	66.7	73.9	75.3	75.9
30	-3.23	-1.65	-1.16	-1.20	7.84	6.85	5.40	5.93	50.2	59.4	61.7	61.4	65.0	74.0	73.9	73.3
50	-3.28	-1.78	-1.53	-1.19	8.53	4.72	4.29	3.81	45.3	56.3	59.8	61.8	62.5	71.2	72.0	72.8
100	-3.50	-1.64	-1.30	-1.13	5.55	3.58	3.45	2.97	37.6	54.3	57.1	59.5	54.0	71.1	71.3	72.0
200	-3.25	-1.68	-1.25	-1.16	4.68	2.91	2.75	2.44	31.2	46.7	52.4	54.5	50.4	67.2	72.3	70.5
	SPMG estimator								Standard				Robust bootstrapped			
17	0.15	0.01	0.04	0.03	10.91	2.72	1.64	1.17	46.5	62.2	68.2	71.2	84.9	92.7	93.1	94.0
30	0.10	0.05	0.06	0.00	6.79	1.97	1.18	0.86	44.2	59.8	67.1	67.8	84.4	91.6	94.3	93.6
50	-0.20	0.04	0.02	0.03	7.15	1.48	0.94	0.69	44.5	60.9	64.2	65.8	84.2	92.5	93.5	93.2
100	-0.15	0.09	0.04	0.02	5.59	1.15	0.67	0.48	41.6	56.0	62.4	65.3	84.1	92.1	94.8	94.2
200	0.14	0.06	0.02	0.02	3.37	0.92	0.50	0.36	40.7	53.6	62.1	61.5	84.2	91.9	94.0	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.15	-3.32	-2.54	-2.10	8.85	5.94	4.95	4.36	75.9	82.0	84.6	86.2
30	-4.71	-3.05	-2.46	-2.07	7.36	4.79	4.04	3.47	72.7	81.5	82.0	85.6
50	-5.11	-3.16	-2.60	-2.22	6.69	4.33	3.72	3.20	68.3	75.3	77.0	82.5
100	-5.04	-3.15	-2.60	-2.16	6.06	3.88	3.26	2.76	56.9	63.8	66.9	72.1
200	-4.83	-3.16	-2.44	-2.15	5.52	3.63	2.85	2.55	45.2	48.2	52.2	57.8
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.70	-3.02	-2.25	-1.86	10.06	6.36	5.11	4.49	74.2	83.8	85.8	88.4
30	-4.33	-2.72	-2.19	-1.85	8.12	4.95	4.08	3.47	73.4	84.0	85.7	88.6
50	-4.74	-2.80	-2.34	-2.00	7.22	4.31	3.67	3.14	69.7	82.9	82.9	86.8
100	-4.64	-2.81	-2.34	-1.95	6.22	3.73	3.12	2.63	62.3	74.7	76.3	79.9
200	-4.37	-2.85	-2.19	-1.94	5.47	3.44	2.67	2.39	54.7	65.4	66.9	70.1
	PDOLS estimator, leads and lags order $p = 8$											
17	-4.33	-2.89	-2.15	-1.76	14.35	7.34	5.56	4.84	57.6	81.6	84.4	87.4
30	-4.13	-2.59	-2.09	-1.77	11.15	5.54	4.39	3.65	56.0	82.1	86.7	88.6
50	-4.45	-2.65	-2.23	-1.90	9.49	4.71	3.84	3.25	54.5	82.2	83.6	88.7
100	-4.26	-2.67	-2.25	-1.85	7.62	3.92	3.23	2.65	50.3	77.8	80.6	84.1
200	-3.92	-2.75	-2.09	-1.86	6.53	3.59	2.68	2.38	42.7	69.1	74.3	76.5
	MGMW estimator, $q = 5$											
17	-2.12	-1.26	-0.77	-0.53	10.28	7.39	6.25	5.76	83.9	87.8	89.5	90.3
30	-1.25	-0.83	-0.69	-0.61	7.62	5.62	4.78	4.31	85.0	87.4	87.4	91.2
50	-1.69	-0.92	-0.89	-0.58	6.40	4.49	3.83	3.44	83.5	87.3	88.1	90.5
100	-1.63	-0.88	-0.69	-0.52	4.99	3.29	2.81	2.61	80.3	86.2	88.3	89.2
200	-1.43	-0.90	-0.65	-0.55	4.12	2.74	2.17	2.02	76.6	81.6	84.9	85.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S11: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-6.29	-2.95	-1.84	-1.32	8.08	3.65	2.26	1.64	41.2	50.7	56.9	59.1	68.4	67.0	67.6	67.4
30	-5.99	-2.81	-1.77	-1.27	7.00	3.22	2.04	1.45	27.4	33.7	38.4	41.7	52.7	49.8	51.8	50.9
50	-5.84	-2.65	-1.70	-1.22	6.50	2.92	1.86	1.34	14.6	19.9	21.5	23.7	34.0	32.8	31.8	31.7
100	-5.68	-2.67	-1.69	-1.21	6.03	2.79	1.77	1.27	2.5	2.9	3.6	4.9	10.0	7.1	6.4	7.8
200	-5.59	-2.63	-1.66	-1.19	5.75	2.70	1.70	1.22	0.0	0.0	0.0	0.1	0.8	0.3	0.1	0.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.77	-0.80	-0.36	-0.20	4.95	2.09	1.26	0.91	74.6	87.1	91.4	92.1	72.3	86.1	90.8	91.8
30	-2.76	-0.81	-0.36	-0.21	4.15	1.65	1.00	0.70	67.2	85.2	88.5	91.4	65.6	83.9	88.4	90.9
50	-2.61	-0.68	-0.30	-0.17	3.50	1.29	0.76	0.54	62.6	84.4	89.7	91.8	59.9	83.3	89.1	91.2
100	-2.59	-0.73	-0.33	-0.18	3.08	1.06	0.59	0.40	44.7	75.8	86.5	90.0	43.3	74.5	85.5	89.5
200	-2.59	-0.72	-0.31	-0.17	2.83	0.91	0.47	0.31	22.3	63.8	80.5	86.5	21.0	61.6	79.6	85.6
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.04	-0.05	-0.03	-0.03	4.84	1.91	1.16	0.84	67.5	84.2	88.5	90.6	88.6	93.3	94.3	94.3
30	-0.03	-0.04	-0.01	-0.01	3.62	1.43	0.89	0.62	66.9	83.6	88.2	90.1	88.1	92.6	93.8	94.7
50	0.00	0.04	0.01	0.01	2.77	1.08	0.64	0.48	67.2	83.6	89.6	90.5	88.2	93.0	95.1	94.4
100	0.01	-0.03	-0.01	0.00	1.91	0.77	0.47	0.32	66.5	83.5	88.3	91.6	90.3	93.1	93.9	95.6
200	-0.01	-0.01	0.00	0.00	1.35	0.55	0.33	0.23	68.1	82.4	88.6	90.3	88.5	92.6	94.0	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.75	-2.36	-1.49	-1.08	6.17	3.07	1.96	1.43	72.0	74.0	74.2	74.9
30	-4.73	-2.32	-1.47	-1.07	5.64	2.74	1.77	1.28	57.5	59.1	61.2	61.7
50	-4.57	-2.17	-1.40	-1.02	5.11	2.44	1.58	1.15	43.2	45.7	47.8	48.4
100	-4.51	-2.20	-1.41	-1.02	4.81	2.34	1.50	1.09	17.1	17.5	17.4	18.3
200	-4.53	-2.18	-1.40	-1.01	4.67	2.26	1.44	1.04	1.6	2.2	2.3	2.8
	PDOLS estimator, leads and lags order $p = 4$											
17	-1.59	-0.75	-0.46	-0.33	5.26	2.21	1.36	0.98	87.3	91.4	92.2	91.9
30	-1.69	-0.76	-0.46	-0.33	4.21	1.73	1.09	0.78	84.6	90.0	90.0	90.9
50	-1.56	-0.63	-0.40	-0.29	3.29	1.35	0.84	0.61	86.1	89.4	89.2	90.4
100	-1.50	-0.69	-0.43	-0.31	2.55	1.09	0.68	0.48	82.0	84.4	85.9	86.1
200	-1.59	-0.68	-0.42	-0.30	2.13	0.91	0.56	0.40	72.9	76.1	76.5	78.3
	PDOLS estimator, leads and lags order $p = 8$											
17	-0.53	-0.23	-0.15	-0.11	9.37	2.59	1.46	1.00	71.7	90.0	91.8	93.0
30	-0.51	-0.26	-0.14	-0.10	6.98	1.93	1.10	0.77	73.8	89.9	91.9	93.5
50	-0.60	-0.16	-0.10	-0.07	5.23	1.46	0.84	0.59	75.9	91.2	93.1	93.0
100	-0.52	-0.21	-0.13	-0.08	3.75	1.06	0.61	0.41	77.2	90.3	91.9	92.4
200	-0.71	-0.20	-0.11	-0.08	2.67	0.76	0.43	0.30	76.7	90.9	92.5	92.5
	MGMW estimator, $q = 5$											
17	-5.01	-1.58	-0.70	-0.42	8.47	3.87	2.48	1.86	84.9	91.9	92.4	94.0
30	-5.18	-1.66	-0.76	-0.43	7.24	3.16	1.97	1.43	79.7	88.7	92.1	93.3
50	-4.82	-1.45	-0.66	-0.38	6.17	2.50	1.53	1.14	74.4	89.9	92.0	93.7
100	-5.03	-1.50	-0.68	-0.39	5.75	2.12	1.22	0.84	53.8	81.0	89.5	90.9
200	-4.90	-1.51	-0.67	-0.37	5.30	1.84	0.98	0.64	30.7	68.9	83.7	88.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and no CS dependence of errors. See notes to Table S1.

Table S12: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-6.46	-3.14	-2.02	-1.50	8.97	4.19	2.67	2.00	41.6	49.1	51.7	53.5	74.7	75.9	77.2	77.5
30	-6.14	-2.94	-1.91	-1.37	7.56	3.55	2.32	1.67	28.9	36.9	38.3	41.0	61.6	63.7	64.6	64.8
50	-5.95	-2.75	-1.79	-1.29	6.91	3.18	2.06	1.48	18.7	24.6	25.8	27.0	46.6	49.6	49.1	51.0
100	-5.78	-2.73	-1.76	-1.27	6.27	2.93	1.89	1.37	5.4	6.0	8.0	8.6	21.6	20.6	20.8	22.5
200	-5.65	-2.71	-1.73	-1.25	5.88	2.82	1.79	1.30	0.4	0.5	0.9	0.7	3.2	3.3	2.9	3.6
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.93	-0.88	-0.43	-0.27	6.08	2.76	1.72	1.27	66.4	76.0	80.1	82.3	74.1	86.7	90.5	92.0
30	-2.88	-0.84	-0.40	-0.22	4.85	2.06	1.31	0.94	62.2	75.1	79.6	81.1	70.2	85.0	88.4	91.0
50	-2.59	-0.69	-0.31	-0.16	3.95	1.61	1.00	0.71	58.5	76.0	79.6	82.0	64.9	84.2	88.2	91.1
100	-2.66	-0.75	-0.34	-0.18	3.38	1.25	0.73	0.51	44.4	68.1	77.8	81.3	52.8	78.1	87.2	90.0
200	-2.64	-0.76	-0.34	-0.18	3.01	1.05	0.58	0.39	25.1	57.9	72.0	77.9	31.6	68.8	82.1	88.1
	SPMG estimator								Standard				Robust bootstrapped			
17	0.00	-0.05	-0.05	-0.07	6.03	2.56	1.56	1.14	59.0	72.2	78.5	80.3	88.8	92.4	94.5	94.5
30	-0.06	0.00	-0.01	-0.01	4.45	1.88	1.17	0.84	55.8	73.4	77.5	78.7	88.5	92.5	93.6	94.1
50	0.13	0.06	0.02	0.02	3.39	1.41	0.89	0.65	58.8	72.9	77.0	79.0	87.6	92.4	93.9	93.8
100	0.02	0.00	0.00	0.01	2.31	0.98	0.62	0.45	58.9	71.9	77.1	79.1	88.6	92.1	93.3	94.4
200	-0.03	-0.02	0.00	0.00	1.62	0.71	0.44	0.31	60.1	72.5	77.9	79.4	87.1	91.5	93.4	93.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.94	-2.51	-1.66	-1.23	7.17	3.64	2.39	1.80	65.5	67.4	66.4	66.8
30	-4.88	-2.42	-1.59	-1.16	6.23	3.08	2.05	1.50	55.8	56.4	56.8	56.7
50	-4.62	-2.23	-1.46	-1.07	5.45	2.68	1.76	1.29	45.0	47.1	46.4	47.8
100	-4.63	-2.26	-1.47	-1.07	5.08	2.49	1.62	1.18	21.2	22.6	22.7	24.3
200	-4.59	-2.25	-1.46	-1.06	4.80	2.36	1.53	1.12	4.3	5.0	5.4	5.9
	PDOLS estimator, leads and lags order $p = 4$											
17	-1.59	-0.79	-0.54	-0.41	6.62	2.97	1.85	1.36	76.5	80.4	81.9	82.7
30	-1.71	-0.79	-0.50	-0.36	5.15	2.18	1.41	1.01	75.8	80.7	80.2	82.5
50	-1.50	-0.64	-0.41	-0.30	3.98	1.69	1.09	0.78	77.9	81.4	80.6	81.9
100	-1.56	-0.70	-0.44	-0.31	3.01	1.29	0.82	0.59	73.4	76.6	77.0	78.0
200	-1.62	-0.71	-0.45	-0.31	2.40	1.05	0.66	0.47	66.6	68.3	69.9	70.5
	PDOLS estimator, leads and lags order $p = 8$											
17	-0.23	-0.24	-0.21	-0.16	11.43	3.47	1.97	1.40	63.2	79.7	82.2	83.6
30	-0.43	-0.27	-0.16	-0.11	8.27	2.48	1.48	1.03	66.7	82.1	82.6	83.9
50	-0.59	-0.15	-0.10	-0.06	6.32	1.90	1.13	0.78	67.5	82.4	81.7	83.5
100	-0.55	-0.20	-0.13	-0.07	4.41	1.31	0.77	0.54	69.1	82.1	84.1	84.9
200	-0.72	-0.21	-0.12	-0.08	3.07	0.95	0.55	0.39	71.9	82.1	82.7	84.2
	MGMW estimator, $q = 5$											
17	-5.22	-1.48	-0.83	-0.47	9.65	4.52	2.99	2.20	78.7	84.7	85.7	85.7
30	-5.26	-1.59	-0.77	-0.45	7.84	3.58	2.32	1.70	75.4	83.7	86.0	86.3
50	-4.71	-1.43	-0.71	-0.42	6.46	2.81	1.76	1.31	72.6	84.2	87.3	88.3
100	-5.03	-1.52	-0.71	-0.39	5.96	2.32	1.35	0.96	54.1	76.9	84.1	86.8
200	-4.95	-1.52	-0.70	-0.39	5.42	1.96	1.07	0.73	32.5	66.5	79.8	84.4

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and SAR CS dependence of errors. See notes to Table S1.

Table S13: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-2.04	-1.00	-0.67	-0.52	4.35	2.06	1.33	1.01	52.7	59.7	60.9	63.7	87.1	88.8	88.2	89.7
30	-1.95	-0.94	-0.61	-0.45	3.54	1.65	1.08	0.78	47.2	53.6	54.2	57.0	83.2	85.9	89.2	88.8
50	-1.84	-0.86	-0.59	-0.42	3.08	1.41	0.93	0.67	40.7	46.3	48.7	49.8	79.6	83.3	83.6	86.2
100	-1.75	-0.85	-0.57	-0.41	2.54	1.21	0.80	0.58	30.5	35.1	36.8	38.2	74.8	77.2	77.6	79.5
200	-1.68	-0.84	-0.56	-0.41	2.28	1.11	0.73	0.54	22.1	24.2	24.0	25.1	67.1	68.4	68.3	68.6
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-1.22	-0.38	-0.18	-0.11	4.07	1.89	1.22	0.87	60.4	70.3	72.3	74.6	79.4	87.1	89.1	92.4
30	-1.32	-0.39	-0.14	-0.11	3.30	1.46	0.91	0.67	57.5	66.7	71.0	71.1	77.2	86.6	90.9	91.3
50	-1.18	-0.31	-0.16	-0.08	2.70	1.13	0.71	0.51	53.5	65.3	68.5	70.2	75.0	87.0	90.5	91.3
100	-1.14	-0.29	-0.15	-0.07	2.06	0.84	0.51	0.37	49.5	62.6	67.4	69.9	73.1	86.5	90.3	92.2
200	-1.11	-0.31	-0.15	-0.08	1.77	0.68	0.39	0.28	43.5	57.9	64.0	65.8	66.3	83.4	89.4	90.9
	SPMG estimator								Standard				Robust bootstrapped			
17	0.11	0.01	-0.02	-0.01	4.97	1.77	1.11	0.80	51.3	65.2	67.8	72.6	88.5	91.7	92.7	94.1
30	-0.05	-0.04	0.01	-0.01	3.25	1.32	0.82	0.59	49.1	60.9	66.8	69.4	86.5	92.3	93.7	94.6
50	-0.01	0.02	-0.01	0.01	2.60	1.00	0.63	0.46	46.6	61.5	64.4	68.1	86.4	92.0	94.2	93.8
100	0.03	0.02	-0.01	0.01	1.77	0.70	0.44	0.33	49.5	61.4	65.0	67.3	87.8	92.1	93.7	93.9
200	0.04	0.00	0.00	0.00	1.42	0.52	0.31	0.23	46.2	59.9	64.6	67.0	87.4	91.4	93.6	93.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-2.26	-1.16	-0.76	-0.58	4.43	2.25	1.48	1.11	65.7	68.1	68.0	68.6
30	-2.24	-1.14	-0.73	-0.56	3.75	1.88	1.22	0.92	60.1	61.3	60.2	59.6
50	-2.21	-1.05	-0.72	-0.53	3.30	1.58	1.05	0.79	52.2	54.6	54.1	54.4
100	-2.08	-1.02	-0.71	-0.51	2.73	1.35	0.93	0.68	41.4	41.5	40.0	43.3
200	-2.07	-1.04	-0.69	-0.52	2.54	1.26	0.85	0.64	25.0	24.1	23.2	24.6
	PDOLS estimator, leads and lags order $p = 4$											
17	-0.89	-0.49	-0.32	-0.25	5.02	2.10	1.30	0.94	68.3	72.8	73.0	75.5
30	-1.20	-0.52	-0.29	-0.25	3.94	1.64	1.00	0.74	65.0	70.2	71.1	70.3
50	-1.16	-0.44	-0.30	-0.21	3.21	1.25	0.80	0.58	62.8	68.6	69.5	68.1
100	-1.05	-0.43	-0.30	-0.20	2.38	0.96	0.61	0.44	59.3	65.0	66.2	66.3
200	-1.05	-0.47	-0.29	-0.21	1.97	0.81	0.49	0.36	54.3	58.0	58.9	57.3
	PDOLS estimator, leads and lags order $p = 8$											
17	0.15	-0.22	-0.16	-0.11	8.83	2.52	1.44	1.00	56.7	72.4	73.0	75.8
30	-0.60	-0.23	-0.10	-0.11	6.74	1.91	1.10	0.76	53.7	69.3	72.6	71.6
50	-0.52	-0.18	-0.11	-0.07	5.36	1.45	0.85	0.59	52.1	69.2	70.1	71.6
100	-0.46	-0.17	-0.12	-0.06	3.95	1.07	0.59	0.41	53.3	66.8	71.2	70.3
200	-0.64	-0.20	-0.11	-0.08	2.99	0.81	0.44	0.31	51.1	66.4	67.2	70.0
	MGMW estimator, $q = 5$											
17	-1.38	-0.44	-0.30	-0.12	5.29	2.80	1.86	1.40	76.2	76.8	78.2	77.2
30	-1.51	-0.49	-0.24	-0.17	4.22	2.08	1.44	1.03	75.5	76.2	77.1	78.3
50	-1.47	-0.43	-0.22	-0.13	3.51	1.63	1.08	0.79	73.1	77.0	79.6	78.9
100	-1.35	-0.44	-0.23	-0.11	2.63	1.23	0.79	0.57	70.0	75.3	78.7	78.8
200	-1.22	-0.43	-0.22	-0.13	2.16	0.94	0.59	0.44	65.6	72.9	76.3	77.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S14: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-6.41	-3.01	-1.87	-1.35	8.33	3.74	2.30	1.67	42.2	51.0	57.8	59.0	68.7	67.5	68.1	68.1
30	-6.11	-2.87	-1.79	-1.28	7.16	3.30	2.08	1.48	27.7	34.8	39.3	42.1	53.8	50.1	52.5	51.8
50	-5.91	-2.67	-1.72	-1.23	6.62	2.96	1.89	1.35	15.7	20.4	23.3	25.3	36.8	34.7	33.8	33.4
100	-5.77	-2.69	-1.70	-1.22	6.14	2.82	1.79	1.28	2.6	3.1	3.8	5.2	11.2	8.1	7.1	8.4
200	-5.66	-2.66	-1.67	-1.20	5.84	2.73	1.72	1.23	0.0	0.0	0.0	0.1	0.8	0.2	0.3	0.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-4.66	-2.03	-1.37	-1.11	11.06	5.99	5.25	4.42	66.4	76.6	80.0	81.6	64.8	75.5	80.0	81.4
30	-4.44	-1.88	-1.28	-1.12	6.97	6.08	3.62	3.27	58.7	70.8	74.0	74.9	57.2	70.1	73.7	74.9
50	-4.29	-1.87	-1.44	-1.10	6.01	3.58	3.64	3.01	48.0	66.7	69.9	72.1	46.4	66.0	69.9	71.5
100	-4.15	-1.79	-1.28	-1.04	5.31	3.85	2.22	2.08	31.2	53.7	63.0	63.4	30.2	53.7	62.8	62.6
200	-4.30	-1.99	-1.39	-1.07	4.82	2.57	2.12	1.70	12.4	36.0	47.5	53.0	11.8	36.1	47.9	53.4
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.04	-0.07	-0.04	-0.04	5.01	1.99	1.19	0.86	67.5	83.5	88.8	90.3	89.1	93.3	94.4	94.3
30	-0.04	-0.06	-0.01	-0.01	3.72	1.47	0.91	0.64	67.5	83.8	88.4	90.4	88.6	92.9	94.0	95.1
50	0.02	0.04	0.01	0.01	2.85	1.11	0.66	0.49	66.9	83.2	89.9	90.8	88.4	93.2	95.5	94.1
100	0.00	-0.03	-0.01	0.00	1.97	0.79	0.48	0.33	67.3	83.1	88.3	91.2	90.2	93.1	94.1	95.5
200	-0.01	0.00	0.00	0.00	1.38	0.56	0.34	0.24	67.7	83.0	89.0	90.5	89.2	93.0	94.4	95.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.57	-3.88	-2.77	-2.24	9.08	5.87	4.75	4.26	71.2	75.5	78.1	80.5
30	-6.48	-3.73	-2.68	-2.19	8.09	5.05	3.96	3.39	59.2	65.7	70.1	74.1
50	-6.31	-3.54	-2.62	-2.11	7.33	4.39	3.45	2.92	45.1	56.8	61.4	67.2
100	-6.19	-3.49	-2.55	-2.06	6.71	3.94	3.00	2.50	25.9	34.8	39.7	49.7
200	-6.30	-3.59	-2.62	-2.11	6.58	3.84	2.87	2.36	12.8	16.0	22.9	29.9
	PDOLS estimator, leads and lags order $p = 4$											
17	-3.53	-2.38	-1.79	-1.52	8.25	5.39	4.51	4.09	83.7	89.5	91.3	91.6
30	-3.54	-2.25	-1.71	-1.48	6.84	4.36	3.53	3.09	81.4	87.8	89.8	91.8
50	-3.46	-2.09	-1.67	-1.41	5.63	3.54	2.92	2.56	79.2	86.2	87.0	89.2
100	-3.31	-2.05	-1.61	-1.37	4.55	2.85	2.34	2.03	72.8	76.9	81.4	83.2
200	-3.51	-2.17	-1.69	-1.43	4.18	2.63	2.10	1.81	55.3	63.0	65.3	70.5
	PDOLS estimator, leads and lags order $p = 8$											
17	-2.55	-1.92	-1.50	-1.30	12.32	5.89	4.76	4.21	70.4	87.8	91.0	92.0
30	-2.33	-1.75	-1.39	-1.25	9.33	4.53	3.63	3.16	71.4	88.5	90.8	93.0
50	-2.51	-1.63	-1.38	-1.20	7.10	3.62	2.96	2.58	73.6	88.5	90.7	92.5
100	-2.26	-1.58	-1.32	-1.16	5.14	2.72	2.26	1.97	73.7	84.6	87.4	88.1
200	-2.57	-1.70	-1.39	-1.22	4.25	2.39	1.93	1.69	66.8	77.7	80.0	80.9
	MGMW estimator, $q = 5$											
17	-6.37	-3.00	-1.77	-1.27	10.44	6.76	5.63	4.90	84.7	91.5	92.9	95.0
30	-6.60	-2.92	-1.78	-1.51	9.46	5.27	4.80	3.73	78.0	88.2	92.9	93.8
50	-6.24	-2.66	-1.81	-1.41	8.00	4.24	3.28	2.93	70.6	87.5	90.9	94.4
100	-6.45	-2.72	-1.68	-1.33	7.36	3.66	2.60	2.30	49.7	76.2	85.0	88.6
200	-6.36	-2.78	-1.80	-1.35	6.86	3.28	2.33	1.91	25.1	56.4	71.2	79.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S15: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-6.54	-3.20	-2.05	-1.53	9.13	4.29	2.71	2.04	41.8	49.9	52.4	53.8	74.9	76.3	77.5	77.5
30	-6.25	-2.99	-1.93	-1.39	7.72	3.63	2.35	1.69	29.3	36.4	39.4	42.1	62.3	64.6	64.9	65.8
50	-6.04	-2.78	-1.81	-1.30	7.03	3.22	2.08	1.50	19.2	25.3	26.4	28.1	47.8	49.5	49.7	51.9
100	-5.86	-2.75	-1.77	-1.28	6.37	2.95	1.90	1.38	6.0	6.7	7.6	9.7	22.3	22.2	21.2	23.3
200	-5.73	-2.74	-1.74	-1.26	5.96	2.85	1.81	1.31	0.4	0.5	1.0	0.9	3.5	3.6	2.8	3.8
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-4.68	-2.28	-1.64	-1.24	9.30	6.92	5.10	4.47	59.7	69.6	72.3	74.2	66.0	77.5	79.8	82.9
30	-4.68	-2.21	-1.49	-1.27	7.67	4.74	4.29	3.62	54.4	65.4	68.7	70.2	60.3	72.9	76.1	77.0
50	-4.31	-1.90	-1.44	-1.15	6.39	3.71	3.21	3.01	48.2	63.5	66.6	68.5	53.0	69.3	73.2	74.2
100	-4.25	-1.99	-1.41	-1.11	5.42	3.04	2.49	2.16	32.0	52.0	59.0	60.9	38.8	58.0	65.7	67.3
200	-4.37	-2.09	-1.43	-1.12	5.08	2.78	2.45	2.04	13.5	36.3	45.1	52.3	16.5	41.7	50.9	56.7
	SPMG estimator								Standard				Robust bootstrapped			
17	0.04	-0.06	-0.06	-0.08	6.20	2.62	1.58	1.17	59.3	72.7	79.1	80.9	88.7	92.1	94.7	94.7
30	-0.04	-0.01	-0.02	-0.01	4.54	1.92	1.19	0.85	57.2	73.7	77.5	80.1	88.5	92.9	94.0	94.5
50	0.15	0.06	0.02	0.03	3.52	1.44	0.90	0.66	57.6	72.9	76.7	79.4	86.9	92.2	93.4	94.0
100	0.02	0.00	0.00	0.01	2.36	1.00	0.63	0.46	59.7	73.0	77.3	79.8	88.5	92.4	93.6	94.4
200	-0.02	-0.02	-0.01	0.00	1.65	0.72	0.44	0.31	59.4	72.7	77.9	79.8	87.8	91.8	93.8	93.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.77	-4.04	-2.98	-2.44	10.01	6.17	4.92	4.33	65.6	71.2	72.5	74.1
30	-6.68	-3.94	-2.93	-2.39	8.57	5.42	4.34	3.71	57.6	62.6	66.6	71.1
50	-6.40	-3.60	-2.71	-2.22	7.63	4.54	3.61	3.12	46.6	55.7	59.0	64.0
100	-6.35	-3.61	-2.68	-2.17	7.01	4.13	3.18	2.67	26.8	36.6	41.5	48.7
200	-6.39	-3.69	-2.72	-2.21	6.73	3.99	2.99	2.48	13.7	16.3	21.9	30.3
	PDOLS estimator, leads and lags order $p = 4$											
17	-3.57	-2.43	-1.91	-1.66	9.63	5.70	4.63	4.08	75.7	81.3	85.0	85.7
30	-3.65	-2.40	-1.90	-1.63	7.53	4.73	3.89	3.39	76.1	82.8	84.2	87.4
50	-3.43	-2.08	-1.71	-1.48	6.09	3.65	3.05	2.72	75.0	81.7	82.5	85.5
100	-3.40	-2.12	-1.70	-1.45	4.90	3.06	2.49	2.18	69.4	74.0	77.6	80.8
200	-3.58	-2.25	-1.76	-1.50	4.37	2.78	2.20	1.91	55.1	61.7	63.2	67.8
	PDOLS estimator, leads and lags order $p = 8$											
17	-2.22	-1.92	-1.60	-1.42	14.01	6.31	4.94	4.22	62.0	81.0	84.6	86.6
30	-2.38	-1.90	-1.57	-1.39	10.49	4.97	4.01	3.46	67.2	82.2	85.2	88.6
50	-2.50	-1.59	-1.41	-1.26	7.97	3.73	3.08	2.74	66.1	82.6	85.7	87.9
100	-2.34	-1.63	-1.40	-1.23	5.72	2.95	2.42	2.12	66.8	80.8	83.5	86.0
200	-2.65	-1.77	-1.44	-1.28	4.53	2.56	2.02	1.78	63.6	73.7	76.5	78.6
	MGMW estimator, $q = 5$											
17	-6.55	-2.87	-2.06	-1.53	11.67	7.31	5.78	5.05	79.4	86.1	87.7	89.3
30	-6.72	-2.96	-1.93	-1.57	9.88	5.67	4.44	3.90	74.7	85.0	88.4	90.3
50	-6.14	-2.63	-1.89	-1.50	8.15	4.41	3.52	3.19	71.3	84.1	89.0	90.6
100	-6.46	-2.80	-1.76	-1.38	7.52	3.82	2.80	2.38	49.6	73.7	82.1	85.9
200	-6.43	-2.84	-1.86	-1.41	7.00	3.41	2.41	2.00	27.9	55.8	70.0	78.1

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S16: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-2.04	-1.02	-0.67	-0.53	4.39	2.10	1.35	1.02	53.8	60.7	62.1	64.1	87.5	88.5	88.9	89.8
30	-1.97	-0.95	-0.62	-0.46	3.59	1.67	1.09	0.80	48.1	53.7	55.0	57.2	83.1	86.6	89.2	88.4
50	-1.85	-0.87	-0.60	-0.42	3.11	1.42	0.94	0.67	41.1	47.1	48.7	50.7	81.1	83.3	83.5	85.7
100	-1.77	-0.85	-0.58	-0.41	2.57	1.23	0.81	0.59	30.7	35.9	37.3	39.8	74.9	77.2	78.0	79.5
200	-1.69	-0.85	-0.57	-0.42	2.30	1.12	0.73	0.55	22.2	24.5	24.8	25.9	67.6	68.3	68.3	68.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.09	-0.95	-0.71	-0.57	5.91	3.55	3.04	3.36	57.7	66.2	68.8	70.1	73.6	80.2	81.5	83.3
30	-2.01	-0.91	-0.60	-0.62	4.56	3.10	2.40	2.28	54.5	61.8	66.0	66.0	71.9	78.0	80.3	79.0
50	-1.87	-0.86	-0.68	-0.60	3.89	2.33	2.09	1.92	50.3	61.8	63.0	63.7	67.6	77.4	77.4	76.4
100	-1.83	-0.84	-0.66	-0.53	3.03	1.76	1.51	1.60	43.2	55.7	59.4	60.8	61.1	71.6	74.2	73.1
200	-1.85	-0.88	-0.63	-0.54	2.70	1.51	1.29	1.15	35.3	49.2	55.4	57.6	53.4	65.3	68.5	68.9
	SPMG estimator								Standard				Robust bootstrapped			
17	0.11	0.01	-0.02	-0.02	5.23	1.79	1.12	0.81	51.4	65.7	68.2	73.0	88.7	91.8	92.7	94.3
30	-0.06	-0.04	0.01	-0.01	3.29	1.34	0.83	0.60	49.6	61.4	68.6	70.0	87.1	92.2	93.8	94.1
50	0.01	0.03	-0.01	0.01	2.65	1.02	0.64	0.46	47.6	62.2	65.5	68.5	86.8	91.7	93.8	94.0
100	0.03	0.02	-0.01	0.01	1.86	0.72	0.45	0.33	50.7	61.4	66.8	67.2	87.5	92.0	93.8	94.0
200	0.03	0.00	0.00	0.00	1.87	0.52	0.31	0.24	47.2	61.5	64.8	67.2	87.5	91.4	93.4	93.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-3.02	-1.82	-1.33	-1.14	5.76	3.43	2.79	2.51	69.3	74.1	75.1	77.9
30	-2.95	-1.76	-1.26	-1.07	4.77	2.93	2.35	2.01	65.5	70.6	74.7	75.8
50	-2.91	-1.62	-1.26	-1.05	4.19	2.41	1.94	1.72	59.4	69.1	70.9	74.9
100	-2.78	-1.59	-1.24	-1.00	3.57	2.13	1.68	1.43	49.4	60.2	62.3	69.0
200	-2.81	-1.64	-1.23	-1.02	3.37	1.99	1.52	1.30	37.4	44.3	48.0	54.8
	PDOLS estimator, leads and lags order $p = 4$											
17	-1.76	-1.20	-0.93	-0.82	6.46	3.41	2.76	2.46	71.0	78.9	81.0	83.0
30	-1.99	-1.20	-0.86	-0.78	5.00	2.81	2.27	1.93	70.1	78.3	80.7	83.4
50	-1.97	-1.06	-0.87	-0.76	4.14	2.16	1.77	1.60	69.8	79.5	81.8	83.4
100	-1.85	-1.06	-0.87	-0.72	3.26	1.82	1.44	1.26	65.7	77.9	79.8	83.8
200	-1.90	-1.14	-0.86	-0.75	2.83	1.60	1.23	1.08	61.6	70.3	73.4	76.9
	PDOLS estimator, leads and lags order $p = 8$											
17	-0.67	-0.98	-0.81	-0.70	10.31	3.95	3.01	2.59	57.6	77.0	80.3	83.0
30	-1.49	-0.95	-0.69	-0.66	7.68	3.15	2.45	2.01	57.3	77.5	82.1	83.4
50	-1.38	-0.85	-0.72	-0.65	6.20	2.40	1.88	1.66	55.5	79.6	82.7	85.3
100	-1.28	-0.84	-0.72	-0.60	4.65	1.94	1.46	1.28	56.4	80.4	83.8	87.3
200	-1.52	-0.93	-0.72	-0.64	3.64	1.60	1.20	1.05	53.1	76.8	80.9	84.1
	MGMW estimator, $q = 5$											
17	-1.68	-0.84	-0.56	-0.42	6.34	4.09	3.60	3.04	79.2	81.6	84.0	84.1
30	-1.76	-0.77	-0.44	-0.42	5.07	2.94	2.46	2.17	79.3	81.8	84.8	87.1
50	-1.65	-0.64	-0.46	-0.41	4.11	2.40	1.92	1.69	78.2	83.0	86.3	88.0
100	-1.56	-0.70	-0.44	-0.30	3.13	1.86	1.41	1.24	76.0	82.1	87.6	88.3
200	-1.43	-0.68	-0.45	-0.35	2.61	1.43	1.11	0.96	73.2	79.6	83.2	85.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S17: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-6.97	-3.17	-1.96	-1.40	9.20	4.02	2.47		42.9	54.4	60.2	62.1	69.5	71.1	70.5	71.1
30	-6.47	-2.99	-1.86	-1.32	7.75	3.49	2.18	1.56	30.6	37.3	44.0	47.2	56.8	54.1	55.9	56.5
50	-6.24	-2.78	-1.79	-1.27	7.06	3.11	1.99	1.42	17.4	23.1	27.6	29.2	39.8	39.2	38.0	38.5
100	-6.06	-2.78	-1.75	-1.25	6.48	2.94	1.86	1.32	3.6	4.8	6.0	6.9	13.8	11.6	12.0	11.0
200	-5.92	-2.76	-1.72	-1.23	6.12	2.84	1.78	1.27	0.1	0.1	0.3	0.3	1.3	0.8	0.6	1.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-9.45	-5.35	-4.59	-3.81	16.78	11.30	13.29		50.0	60.3	63.3	65.5	49.1	59.9	63.3	65.4
30	-9.50	-5.27	-4.20	-3.94	13.76	10.15	8.37	7.99	41.3	54.1	57.5	58.1	39.5	53.1	57.6	58.1
50	-9.52	-5.49	-4.41	-3.69	12.26	8.07	7.26	6.34	27.4	45.7	48.4	53.5	27.4	45.7	48.6	53.4
100	-9.18	-5.55	-4.43	-3.85	11.00	8.06	5.95	5.43	14.2	30.0	36.8	40.9	14.7	30.0	36.6	40.5
200	-9.19	-5.56	-4.41	-3.71	9.91	6.29	5.27	4.57	4.4	14.2	22.2	26.7	4.1	14.5	22.4	27.5
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.18	-0.09	-0.06	-0.04	5.60	2.19	1.31		68.0	83.2	88.1	89.8	88.9	93.5	95.1	94.3
30	-0.01	-0.06	-0.01	-0.01	4.13	1.63	0.99	0.70	65.6	83.5	87.7	89.7	88.5	93.4	94.2	94.6
50	0.04	0.05	0.01	0.02	3.18	1.21	0.74	0.54	65.3	84.5	88.7	90.1	88.1	93.5	94.6	94.1
100	0.01	-0.02	-0.01	0.00	2.18	0.87	0.53	0.36	66.3	82.1	87.4	90.8	89.2	93.4	93.9	95.5
200	0.00	-0.01	0.00	0.01	1.50	0.63	0.37	0.26	66.3	81.4	88.1	90.2	88.6	92.0	94.2	94.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-11.32	-7.66	-6.15	-5.33	14.81	10.72	9.18		67.2	76.9	78.9	81.9
30	-11.57	-7.67	-6.14	-5.39	13.84	9.60	7.98	7.22	58.1	66.6	71.7	75.1
50	-11.50	-7.61	-6.12	-5.26	13.00	8.99	7.40	6.49	45.5	55.2	59.7	66.0
100	-11.29	-7.66	-6.25	-5.43	12.06	8.34	6.89	6.09	30.9	33.4	36.7	43.4
200	-11.41	-7.61	-6.14	-5.35	11.82	7.98	6.50	5.70	20.5	16.2	19.6	22.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-8.70	-6.36	-5.29	-4.68	14.32	10.44	9.04		76.9	84.6	86.0	87.2
30	-9.13	-6.39	-5.30	-4.76	12.94	9.04	7.61	6.94	70.4	77.9	80.8	83.6
50	-9.09	-6.41	-5.32	-4.67	11.58	8.31	6.94	6.15	64.2	73.3	73.1	76.6
100	-8.85	-6.49	-5.47	-4.84	10.18	7.43	6.30	5.65	50.7	54.3	55.5	59.7
200	-9.08	-6.43	-5.35	-4.76	9.78	6.96	5.80	5.19	29.9	35.3	36.6	37.5
	PDOLS estimator, leads and lags order $p = 8$											
17	-7.68	-5.89	-5.00	-4.46	18.46	11.06	9.43		66.0	83.0	85.3	86.9
30	-7.74	-5.89	-4.99	-4.54	15.04	9.33	7.75	7.02	62.7	78.1	82.8	84.4
50	-8.03	-6.01	-5.05	-4.48	12.61	8.41	6.99	6.20	58.5	73.6	75.3	79.7
100	-7.81	-6.10	-5.24	-4.67	10.40	7.33	6.24	5.60	52.6	59.5	60.9	65.3
200	-8.02	-5.99	-5.08	-4.58	9.47	6.69	5.63	5.09	35.5	42.1	45.4	45.7
	MGMW estimator, $q = 5$											
17	-10.74	-6.26	-4.79	-4.10	16.22	11.50	10.28		81.3	89.7	92.5	94.3
30	-11.14	-6.76	-5.07	-4.40	14.66	10.10	8.74	7.69	71.4	82.6	87.0	90.5
50	-10.58	-6.26	-4.99	-4.32	12.84	8.39	7.09	6.42	61.6	77.5	82.0	86.0
100	-10.85	-6.51	-5.07	-4.37	12.09	7.69	6.21	5.57	39.1	56.8	64.9	70.2
200	-10.56	-6.45	-5.02	-4.31	11.16	7.03	5.66	4.97	15.7	32.9	42.8	49.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S18: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-7.11	-3.35	-2.14	-1.58	10.06	4.58	2.86	2.12	42.4	51.8	54.7	56.1	75.8	77.8	79.3	79.3
30	-6.64	-3.14	-2.01	-1.43	8.28	3.84	2.47	1.77	31.1	39.0	42.4	44.9	63.4	66.7	67.0	68.0
50	-6.37	-2.89	-1.88	-1.33	7.50	3.37	2.17	1.55	20.6	27.3	28.7	30.1	49.3	52.0	52.3	55.0
100	-6.15	-2.85	-1.83	-1.32	6.71	3.08	1.98	1.43	6.3	8.2	9.6	10.8	24.1	25.4	24.3	25.9
200	-5.99	-2.84	-1.79	-1.30	6.26	2.96	1.87	1.35	0.5	0.4	1.3	1.8	4.5	4.5	4.6	5.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-9.58	-5.96	-4.90	-4.24	18.27	12.36	11.11	9.76	48.4	57.4	60.3	63.7	54.1	62.9	65.4	69.0
30	-9.84	-5.64	-4.68	-4.37	14.03	10.00	8.77	9.33	38.6	52.2	55.3	55.9	42.5	57.2	59.7	59.9
50	-9.57	-5.62	-4.57	-3.92	12.41	8.24	7.48	6.79	30.0	45.3	48.6	50.4	33.5	49.7	52.2	53.8
100	-9.26	-5.76	-4.72	-4.18	10.96	7.24	6.45	5.85	16.5	29.9	35.6	40.1	18.8	32.2	38.1	42.6
200	-9.44	-5.84	-4.61	-3.97	10.32	6.64	5.57	4.95	4.7	13.9	22.0	26.9	5.4	16.1	24.4	29.3
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.10	-0.08	-0.06	-0.07	6.87	2.83	1.67	1.23	59.2	73.8	79.9	81.7	89.6	92.7	95.4	94.8
30	-0.02	-0.01	-0.01	0.00	4.94	2.05	1.27	0.91	57.2	74.1	78.5	81.3	89.3	93.2	94.2	94.1
50	0.16	0.08	0.03	0.03	3.86	1.53	0.95	0.70	58.4	75.2	78.2	80.7	86.3	92.9	93.9	93.8
100	0.05	0.01	0.00	0.01	2.55	1.07	0.67	0.48	59.5	73.0	77.5	81.7	88.9	92.5	93.9	94.9
200	-0.01	-0.01	0.00	0.00	1.78	0.77	0.47	0.33	58.6	72.1	76.9	82.0	88.1	91.6	93.0	93.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-11.72	-7.97	-6.55	-5.82	15.94	11.20	9.73	9.01	64.3	73.3	75.5	78.2
30	-11.83	-8.01	-6.66	-5.89	14.43	10.14	8.69	7.85	55.9	63.5	68.3	72.8
50	-11.73	-7.82	-6.40	-5.59	13.43	9.32	7.81	6.95	44.7	55.2	56.1	62.3
100	-11.62	-7.95	-6.57	-5.76	12.51	8.73	7.30	6.49	28.9	32.9	36.9	42.6
200	-11.60	-7.87	-6.43	-5.65	12.05	8.28	6.80	6.02	19.7	17.3	19.1	21.4
	PDOLS estimator, leads and lags order $p = 4$											
17	-9.08	-6.59	-5.62	-5.11	15.99	10.92	9.57	8.84	72.0	80.5	82.4	84.3
30	-9.34	-6.68	-5.76	-5.22	13.62	9.59	8.32	7.56	68.2	75.3	79.2	81.1
50	-9.23	-6.56	-5.54	-4.97	12.07	8.62	7.33	6.60	63.6	70.7	70.1	74.2
100	-9.14	-6.74	-5.76	-5.16	10.66	7.83	6.70	6.04	49.5	52.9	54.4	56.2
200	-9.24	-6.67	-5.61	-5.04	10.04	7.25	6.09	5.50	30.9	34.6	35.1	36.0
	PDOLS estimator, leads and lags order $p = 8$											
17	-7.81	-6.11	-5.31	-4.88	20.52	11.68	10.02	9.10	60.7	79.8	82.7	84.4
30	-8.10	-6.18	-5.46	-4.99	16.17	9.91	8.53	7.66	59.2	76.0	80.2	82.4
50	-8.25	-6.15	-5.28	-4.79	13.40	8.73	7.39	6.66	56.3	73.1	72.4	75.8
100	-8.13	-6.34	-5.53	-4.98	11.03	7.76	6.66	6.00	49.4	58.2	60.8	62.4
200	-8.23	-6.25	-5.34	-4.86	9.80	7.01	5.93	5.39	35.3	40.7	44.4	43.4
	MGMW estimator, $q = 5$											
17	-11.01	-6.35	-5.27	-4.47	17.44	12.31	10.90	9.84	76.7	86.4	88.3	92.3
30	-11.21	-6.89	-5.35	-4.81	15.17	10.47	9.06	8.06	70.0	81.1	85.9	87.4
50	-10.57	-6.36	-5.13	-4.55	13.07	8.76	7.49	6.78	61.0	75.0	80.2	82.0
100	-11.02	-6.74	-5.26	-4.57	12.38	7.96	6.47	5.79	38.3	53.4	62.7	67.4
200	-10.75	-6.65	-5.23	-4.52	11.42	7.27	5.88	5.22	16.2	30.6	39.8	50.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S19: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-2.20	-1.07	-0.72	-0.54	4.69	2.20	1.42	1.05	56.3	62.8	63.7	67.0	87.5	89.5	89.9	90.9
30	-2.05	-0.99	-0.64	-0.47	3.78	1.75	1.14	0.83	49.5	55.9	57.6	59.9	84.0	86.9	89.4	88.1
50	-1.93	-0.89	-0.62	-0.43	3.27	1.48	0.97	0.69	42.4	50.2	51.0	53.5	81.4	84.7	84.6	87.0
100	-1.85	-0.87	-0.59	-0.42	2.70	1.27	0.84	0.60	33.3	37.7	38.7	41.8	75.3	78.9	78.5	80.4
200	-1.76	-0.88	-0.58	-0.43	2.40	1.16	0.75	0.57	23.3	26.6	26.3	27.3	69.4	69.3	69.0	69.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-4.37	-2.61	-2.01	-1.73	9.78	7.37	6.39	10.30	52.1	58.2	62.6	61.8	63.4	69.1	72.1	71.0
30	-4.38	-2.29	-2.04	-1.97	8.18	6.09	5.26	5.21	47.5	57.9	60.3	61.4	59.4	67.6	70.1	69.9
50	-4.10	-2.58	-2.08	-1.94	6.96	4.86	4.14	4.10	41.1	52.7	56.2	55.9	52.4	62.4	65.9	64.6
100	-4.02	-2.47	-2.21	-1.91	5.73	3.97	3.60	3.35	33.2	44.6	46.9	50.6	42.8	53.7	54.8	58.5
200	-4.08	-2.50	-2.06	-1.90	5.52	3.58	3.02	2.79	21.4	34.8	40.3	41.9	30.4	44.5	48.4	50.3
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.16	0.01	-0.04	-0.01	7.34	1.89	1.18	0.84	53.7	66.9	71.6	74.6	89.1	92.2	93.0	94.8
30	-0.05	-0.05	0.01	-0.01	3.63	1.40	0.87	0.63	49.7	63.8	71.0	71.6	87.5	92.4	93.9	94.0
50	0.00	0.04	-0.01	0.02	2.83	1.07	0.66	0.48	49.6	63.4	68.2	71.2	86.7	91.8	94.4	94.3
100	0.05	0.02	-0.01	0.01	2.32	0.75	0.47	0.34	49.8	63.9	67.1	69.8	87.9	92.8	93.9	94.4
200	-0.01	0.00	0.00	0.00	3.43	0.55	0.33	0.25	49.2	63.1	67.8	69.2	88.0	91.9	93.8	94.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.34	-3.50	-2.87	-2.57	8.85	6.03	5.22	4.79	73.4	82.7	83.9	86.4
30	-5.17	-3.47	-2.82	-2.52	7.44	5.19	4.53	4.01	71.1	78.8	82.8	84.4
50	-5.18	-3.41	-2.85	-2.50	6.83	4.66	3.95	3.56	64.7	74.4	75.8	78.8
100	-4.97	-3.37	-2.89	-2.54	5.99	4.18	3.58	3.19	55.9	63.3	62.1	67.4
200	-4.99	-3.36	-2.79	-2.52	5.76	3.89	3.25	2.97	42.2	45.3	46.3	50.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.45	-3.01	-2.57	-2.32	9.89	6.25	5.35	4.87	73.8	84.2	86.9	88.6
30	-4.56	-3.06	-2.52	-2.30	8.07	5.28	4.57	4.03	72.5	83.1	86.4	88.0
50	-4.59	-3.03	-2.57	-2.29	7.10	4.63	3.89	3.51	70.2	80.6	82.9	85.3
100	-4.34	-3.03	-2.63	-2.34	5.92	4.06	3.45	3.10	63.6	74.4	74.0	76.5
200	-4.43	-3.03	-2.54	-2.33	5.59	3.68	3.06	2.82	54.6	60.2	63.2	62.1
	PDOLS estimator, leads and lags order $p = 8$											
17	-3.46	-2.89	-2.54	-2.24	13.73	7.06	5.84	5.17	58.4	81.5	85.3	88.8
30	-4.15	-2.91	-2.44	-2.24	10.81	5.78	4.91	4.24	58.0	81.6	86.0	88.0
50	-4.21	-2.96	-2.49	-2.23	9.08	5.07	4.09	3.65	55.6	79.0	84.0	85.8
100	-3.87	-2.95	-2.59	-2.30	7.10	4.36	3.57	3.18	54.1	74.6	77.2	79.4
200	-4.17	-2.95	-2.48	-2.29	6.41	3.82	3.11	2.86	45.5	65.2	69.5	69.3
	MGMW estimator, $q = 5$											
17	-2.75	-1.70	-1.35	-1.19	8.85	6.63	5.92	5.52	83.8	87.9	89.8	91.8
30	-2.64	-1.46	-1.00	-1.10	7.07	5.09	4.48	4.10	82.1	87.1	89.4	90.9
50	-2.47	-1.46	-1.12	-0.95	5.96	4.05	3.72	3.38	80.7	85.1	86.9	89.8
100	-2.25	-1.46	-1.19	-1.01	4.70	3.38	2.84	2.61	76.7	81.8	85.3	86.5
200	-2.11	-1.38	-1.12	-1.04	4.08	2.69	2.33	2.21	71.8	76.2	79.0	79.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S20: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-0.94	-0.29	-0.10	-0.06	5.21	2.21	1.38	1.02	70.9	85.4	89.1	90.3	90.3	93.4	94.7	94.5
30	-0.71	-0.12	-0.08	-0.06	3.74	1.60	1.02	0.73	71.5	85.2	87.4	90.3	91.4	94.4	93.9	94.2
50	-0.55	-0.17	-0.09	-0.04	2.84	1.23	0.74	0.54	71.7	85.8	89.8	91.7	91.4	93.8	95.2	95.5
100	-0.61	-0.14	-0.06	-0.04	2.09	0.87	0.55	0.39	68.0	84.9	88.2	90.4	89.8	93.4	93.9	94.6
200	-0.68	-0.16	-0.07	-0.04	1.52	0.61	0.37	0.27	67.7	84.2	89.6	90.6	89.3	93.8	94.7	95.7
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.40	-0.70	-0.30	-0.18	5.69	2.58	1.60	1.18	79.0	87.0	91.1	91.7	75.7	85.6	90.3	91.2
30	-2.28	-0.51	-0.24	-0.15	4.42	1.88	1.20	0.87	75.6	87.4	90.4	91.2	72.7	86.5	90.0	90.4
50	-2.02	-0.61	-0.28	-0.14	3.56	1.53	0.93	0.65	70.9	84.8	89.2	92.1	69.4	83.5	88.7	91.4
100	-2.02	-0.53	-0.24	-0.13	2.89	1.12	0.69	0.49	63.5	83.5	87.5	90.1	61.3	82.5	86.9	89.9
200	-2.06	-0.56	-0.24	-0.14	2.51	0.89	0.49	0.34	47.3	76.9	86.6	89.3	46.0	75.1	85.7	88.9
	SPMG estimator								Standard				Robust bootstrapped			
17	0.09	-0.04	0.00	0.00	6.24	2.39	1.44	1.05	62.2	80.7	86.8	88.3	86.6	92.6	94.8	94.5
30	0.29	0.12	0.02	-0.01	4.41	1.72	1.07	0.75	63.0	80.3	85.8	88.7	88.0	92.8	93.1	94.4
50	0.43	0.04	0.00	0.01	3.31	1.31	0.77	0.55	61.9	81.8	87.8	90.4	86.9	93.7	94.5	95.5
100	0.28	0.08	0.03	0.02	2.35	0.90	0.56	0.40	59.0	81.4	87.0	88.0	86.6	93.1	94.3	94.3
200	0.19	0.05	0.02	0.01	1.60	0.63	0.38	0.27	62.8	81.2	88.4	90.3	87.3	93.9	95.1	95.0

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.92	-2.43	-1.58	-1.19	7.00	3.48	2.27	1.71	75.9	78.5	80.5	80.2
30	-4.85	-2.27	-1.51	-1.13	6.01	2.91	1.95	1.44	68.4	72.8	70.9	72.3
50	-4.68	-2.39	-1.55	-1.12	5.48	2.79	1.81	1.31	55.6	55.7	56.8	58.6
100	-4.63	-2.29	-1.51	-1.11	5.06	2.51	1.66	1.22	34.8	32.8	34.4	34.5
200	-4.67	-2.32	-1.50	-1.10	4.87	2.42	1.57	1.15	11.2	9.2	8.6	9.2
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.69	-1.23	-0.77	-0.58	6.77	2.92	1.82	1.34	83.4	88.9	89.3	89.9
30	-2.61	-1.07	-0.70	-0.53	5.09	2.22	1.42	1.03	83.3	87.7	87.9	87.5
50	-2.45	-1.19	-0.76	-0.54	4.28	1.91	1.19	0.85	78.8	82.8	83.4	84.9
100	-2.49	-1.12	-0.72	-0.52	3.48	1.54	0.98	0.71	72.8	76.2	76.1	77.6
200	-2.56	-1.16	-0.72	-0.52	3.06	1.37	0.85	0.61	59.3	58.1	60.5	61.7
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.55	-0.60	-0.36	-0.26	11.19	3.23	1.87	1.32	70.6	90.1	91.3	92.4
30	-1.37	-0.40	-0.28	-0.22	7.78	2.38	1.40	0.97	73.5	90.3	90.8	91.8
50	-1.25	-0.55	-0.34	-0.23	6.16	1.85	1.05	0.74	73.8	88.6	90.9	92.6
100	-1.34	-0.49	-0.31	-0.21	4.42	1.35	0.80	0.56	73.1	88.6	88.2	90.3
200	-1.38	-0.52	-0.30	-0.21	3.20	1.01	0.58	0.41	74.7	85.6	87.4	87.8
	MGMW estimator, $q = 5$											
17	-5.72	-1.90	-1.01	-0.56	11.66	5.43	3.49	2.64	88.4	91.8	92.7	93.4
30	-5.52	-1.74	-0.81	-0.58	9.06	4.15	2.80	2.01	84.4	92.2	94.1	93.8
50	-5.33	-1.96	-0.93	-0.58	7.76	3.63	2.22	1.59	79.9	88.5	92.4	93.2
100	-5.47	-1.77	-0.92	-0.56	6.77	2.83	1.75	1.24	67.9	85.0	89.1	91.0
200	-5.50	-1.89	-0.93	-0.54	6.16	2.41	1.38	0.94	46.5	74.0	83.8	88.4

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$, and no CS dependence of errors. See notes to Table S1.

Table S21: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.24	-0.36	-0.14	-0.09	6.87	3.00	1.87	1.36	61.0	73.8	78.7	80.3	90.1	92.8	93.9	95.1
30	-0.83	-0.20	-0.12	-0.09	4.68	2.06	1.33	0.98	64.0	74.9	78.2	81.5	89.6	93.3	94.5	94.0
50	-0.73	-0.20	-0.09	-0.05	3.54	1.56	0.98	0.74	62.5	74.7	79.1	80.7	90.2	94.0	95.2	94.9
100	-0.70	-0.17	-0.06	-0.04	2.54	1.11	0.70	0.51	61.4	73.4	79.6	80.5	88.2	93.2	93.7	94.4
200	-0.77	-0.18	-0.08	-0.04	1.86	0.77	0.48	0.35	58.7	75.9	79.4	82.3	88.1	93.0	94.7	94.7
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.63	-0.82	-0.34	-0.22	7.20	3.42	2.12	1.60	67.5	75.6	79.6	81.1	75.8	86.8	89.0	90.9
30	-2.39	-0.60	-0.31	-0.19	5.41	2.40	1.55	1.15	68.2	77.9	79.1	81.3	73.8	85.8	89.5	90.6
50	-2.19	-0.67	-0.31	-0.17	4.28	1.92	1.21	0.88	65.1	76.1	79.3	81.7	70.7	85.0	89.2	91.5
100	-2.13	-0.58	-0.25	-0.14	3.36	1.40	0.86	0.62	57.2	73.4	78.9	80.9	63.5	81.9	88.2	89.8
200	-2.19	-0.60	-0.26	-0.15	2.85	1.05	0.62	0.44	43.4	68.5	76.7	80.6	50.8	77.6	86.9	89.7
	SPMG estimator								Standard				Robust bootstrapped			
17	0.12	-0.07	-0.02	-0.02	8.71	3.20	1.95	1.41	52.4	68.5	75.1	77.9	85.7	92.2	93.6	94.6
30	0.24	0.05	-0.02	-0.03	5.50	2.17	1.37	1.00	56.0	71.5	74.4	78.6	86.7	92.5	93.8	93.4
50	0.29	0.02	0.00	0.00	4.05	1.65	1.01	0.75	53.6	69.5	77.0	78.3	87.3	92.7	94.8	94.4
100	0.24	0.06	0.05	0.02	2.87	1.15	0.72	0.53	52.3	70.1	75.5	79.0	85.0	92.0	92.8	93.2
200	0.14	0.03	0.01	0.01	1.95	0.79	0.49	0.35	54.5	72.0	76.8	81.0	86.0	93.0	94.3	94.2

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.15	-2.59	-1.73	-1.33	8.24	4.18	2.76	2.11	70.0	71.9	73.0	71.8
30	-4.95	-2.38	-1.61	-1.22	6.76	3.36	2.26	1.71	63.5	65.7	65.0	65.5
50	-4.79	-2.48	-1.63	-1.19	5.96	3.08	2.03	1.50	53.7	54.1	53.5	55.6
100	-4.65	-2.35	-1.55	-1.15	5.29	2.68	1.77	1.32	36.5	35.7	37.4	36.8
200	-4.76	-2.37	-1.56	-1.14	5.06	2.53	1.66	1.22	14.4	11.6	12.7	12.9
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.92	-1.31	-0.85	-0.65	8.56	3.77	2.35	1.77	73.6	79.1	80.3	80.4
30	-2.66	-1.12	-0.76	-0.58	6.28	2.74	1.77	1.31	75.3	79.7	78.1	79.5
50	-2.49	-1.23	-0.80	-0.57	5.05	2.29	1.44	1.06	73.5	74.7	75.5	76.6
100	-2.47	-1.15	-0.73	-0.54	3.94	1.79	1.13	0.83	67.8	69.2	69.7	70.6
200	-2.61	-1.18	-0.75	-0.54	3.35	1.49	0.95	0.68	56.2	56.8	56.0	57.7
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.74	-0.64	-0.41	-0.31	13.44	4.30	2.48	1.80	60.7	78.9	82.4	81.6
30	-1.38	-0.42	-0.30	-0.24	9.27	3.00	1.80	1.28	65.6	80.8	81.5	82.2
50	-1.12	-0.58	-0.35	-0.25	7.17	2.34	1.35	0.98	65.2	80.5	82.2	81.9
100	-1.31	-0.52	-0.31	-0.21	5.07	1.69	1.00	0.70	68.6	79.6	80.6	82.5
200	-1.44	-0.52	-0.32	-0.22	3.67	1.20	0.72	0.50	68.4	78.6	78.6	80.6
	MGMW estimator, $q = 5$											
17	-5.72	-1.90	-0.99	-0.62	12.31	6.26	4.11	3.07	80.8	84.8	85.9	86.5
30	-5.84	-1.89	-0.96	-0.64	9.87	4.72	3.18	2.36	79.5	85.7	87.0	86.9
50	-5.63	-1.99	-0.98	-0.62	8.47	3.92	2.48	1.88	75.2	82.9	86.8	87.4
100	-5.40	-1.83	-0.93	-0.55	6.92	3.08	1.91	1.36	65.4	79.8	85.4	86.8
200	-5.63	-1.97	-1.00	-0.58	6.41	2.57	1.51	1.06	44.0	70.7	78.9	84.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0$, and SAR CS dependence of errors. See notes to Table S1.

Table S22: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-0.51	-0.16	-0.01	-0.02	4.87	2.28	1.43	1.08	57.5	63.3	67.7	71.2	90.0	93.1	93.4	94.1
30	-0.22	-0.02	0.00	-0.01	3.69	1.65	1.05	0.80	50.8	61.6	68.0	68.2	90.1	93.8	94.6	93.7
50	-0.35	-0.07	-0.01	-0.01	2.78	1.28	0.78	0.60	51.3	62.1	67.8	68.9	90.4	93.4	95.0	95.2
100	-0.25	-0.07	-0.01	-0.01	2.01	0.96	0.60	0.45	50.2	59.1	64.9	64.4	90.8	94.2	94.8	95.1
200	-0.19	-0.03	-0.03	-0.01	1.55	0.70	0.45	0.33	45.0	57.4	59.6	61.7	92.0	95.0	95.2	96.2
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-1.45	-0.39	-0.14	-0.09	5.37	2.58	1.63	1.24	60.4	67.2	71.7	72.4	77.7	87.3	90.3	91.7
30	-1.03	-0.30	-0.10	-0.06	3.99	1.94	1.25	0.94	58.5	64.0	67.4	68.9	80.4	89.0	92.2	91.8
50	-1.22	-0.32	-0.10	-0.08	3.26	1.52	0.95	0.70	54.2	63.4	68.4	67.9	79.0	88.8	93.6	94.3
100	-1.09	-0.30	-0.13	-0.06	2.54	1.17	0.72	0.54	50.2	61.6	65.2	65.3	80.6	90.8	94.3	94.5
200	-1.03	-0.27	-0.14	-0.07	2.08	0.90	0.56	0.41	44.7	53.9	59.0	62.1	81.8	93.6	96.0	96.4
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.14	-0.07	0.02	0.01	8.29	2.49	1.51	1.12	47.2	59.9	64.8	69.3	85.2	91.9	92.7	93.0
30	0.10	0.06	0.05	0.01	5.53	1.79	1.10	0.83	43.6	57.7	64.7	65.5	84.8	92.3	94.0	93.7
50	-0.08	0.03	0.03	0.01	3.48	1.40	0.82	0.62	42.7	59.1	64.6	67.0	85.6	90.5	94.5	94.4
100	-0.05	0.03	0.04	0.02	5.27	1.03	0.62	0.46	41.1	55.9	61.2	63.8	85.3	92.2	94.0	94.0
200	0.03	0.05	0.01	0.01	3.56	0.78	0.46	0.34	38.6	53.9	57.5	59.5	84.9	92.2	93.8	94.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-2.89	-1.46	-0.96	-0.74	5.66	2.93	1.94	1.50	69.8	69.4	70.9	71.5
30	-2.50	-1.38	-0.94	-0.72	4.45	2.39	1.62	1.23	64.5	62.9	63.2	64.0
50	-2.76	-1.45	-0.96	-0.75	4.02	2.10	1.42	1.08	55.5	55.9	57.4	55.8
100	-2.58	-1.42	-0.98	-0.73	3.40	1.86	1.27	0.95	45.2	43.1	42.9	41.2
200	-2.56	-1.40	-0.95	-0.71	3.08	1.68	1.13	0.86	29.6	26.3	22.5	24.7
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.06	-0.91	-0.56	-0.43	6.64	2.84	1.77	1.34	66.4	71.8	73.2	74.4
30	-1.61	-0.82	-0.51	-0.40	4.95	2.22	1.39	1.05	64.5	68.3	69.4	68.8
50	-1.91	-0.87	-0.54	-0.43	4.13	1.80	1.13	0.84	62.7	65.1	67.6	66.3
100	-1.74	-0.87	-0.56	-0.41	3.26	1.48	0.94	0.68	57.0	59.2	59.5	58.1
200	-1.70	-0.85	-0.57	-0.41	2.77	1.24	0.79	0.58	47.4	47.9	46.5	47.5
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.25	-0.59	-0.29	-0.24	11.23	3.34	1.93	1.38	51.9	70.8	71.7	74.4
30	-0.91	-0.41	-0.24	-0.19	8.19	2.52	1.46	1.06	51.5	68.2	69.2	70.9
50	-1.29	-0.49	-0.27	-0.21	6.42	2.03	1.15	0.80	52.7	65.7	70.0	71.3
100	-1.16	-0.51	-0.31	-0.19	5.20	1.56	0.93	0.62	48.4	62.6	64.7	65.4
200	-1.04	-0.47	-0.30	-0.21	4.28	1.26	0.71	0.49	41.6	55.5	59.0	60.5
	MGMW estimator, $q = 5$											
17	-1.43	-0.46	-0.19	-0.13	7.03	4.01	2.76	2.01	78.7	77.3	79.0	80.7
30	-0.97	-0.45	-0.16	-0.08	5.67	2.95	2.02	1.54	79.6	78.8	79.5	79.0
50	-1.16	-0.41	-0.19	-0.15	4.36	2.30	1.54	1.18	77.8	78.4	81.2	80.6
100	-0.81	-0.38	-0.22	-0.14	3.17	1.71	1.16	0.85	78.0	79.2	80.6	79.6
200	-0.78	-0.38	-0.20	-0.12	2.45	1.27	0.82	0.63	75.6	77.6	79.4	78.8

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S23: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.03	-0.29	-0.11	-0.06	5.52	2.26	1.41	1.05	70.9	85.2	88.9	89.7	89.6	93.7	94.3	94.3
30	-0.76	-0.12	-0.08	-0.06	3.90	1.65	1.06	0.75	70.7	84.9	87.7	89.5	90.6	94.2	93.8	94.2
50	-0.60	-0.18	-0.09	-0.05	2.92	1.27	0.76	0.55	71.7	85.4	89.3	91.8	90.6	93.1	95.0	95.7
100	-0.69	-0.15	-0.07	-0.04	2.16	0.89	0.56	0.40	68.3	84.3	88.5	90.2	89.3	93.8	94.1	94.7
200	-0.74	-0.16	-0.07	-0.04	1.58	0.63	0.38	0.27	66.8	84.3	89.3	91.1	89.1	93.7	94.4	95.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-3.58	-1.44	-0.81	-0.59	8.27	5.95	4.49	5.01	72.4	80.4	83.7	83.2	70.5	79.3	83.2	83.0
30	-3.50	-1.19	-0.75	-0.79	6.47	4.30	3.61	8.74	67.5	78.6	80.8	80.9	65.0	78.0	80.8	80.9
50	-3.16	-1.29	-0.79	-0.53	5.21	3.54	2.93	2.24	62.2	73.9	77.2	79.2	61.1	73.2	77.0	78.9
100	-3.15	-1.19	-0.69	-0.51	5.36	2.70	3.60	2.01	49.3	69.4	72.3	73.5	48.8	68.9	71.7	73.5
200	-3.28	-1.22	-0.73	-0.58	4.13	2.00	1.68	1.44	31.4	58.8	67.7	70.9	30.6	58.4	67.4	70.6
	SPMG estimator								Standard				Robust bootstrapped			
17	0.11	-0.02	0.01	0.01	6.61	2.45	1.48	1.08	61.0	81.2	87.0	88.3	86.3	92.9	94.6	94.2
30	0.29	0.14	0.03	0.00	4.60	1.79	1.11	0.77	62.6	80.3	85.9	88.2	88.2	92.8	93.2	94.0
50	0.45	0.06	0.00	0.01	3.41	1.35	0.79	0.57	61.7	80.6	87.9	90.9	86.6	93.5	94.3	95.3
100	0.26	0.08	0.04	0.02	2.42	0.93	0.57	0.41	59.0	81.0	86.5	87.9	86.6	92.9	94.5	94.3
200	0.20	0.06	0.02	0.01	1.65	0.65	0.39	0.28	62.5	81.2	87.9	90.3	86.9	94.1	94.8	95.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.31	-3.47	-2.40	-1.96	9.05	5.38	4.04	3.55	75.8	79.6	82.5	83.6
30	-6.22	-3.26	-2.33	-1.88	7.82	4.41	3.37	2.83	67.5	74.4	76.3	78.6
50	-5.98	-3.36	-2.36	-1.83	7.05	4.17	3.05	2.46	57.8	61.4	66.0	70.6
100	-6.02	-3.25	-2.33	-1.83	6.65	3.68	2.74	2.24	36.9	44.9	50.0	57.0
200	-6.03	-3.27	-2.32	-1.82	6.33	3.49	2.52	2.02	20.1	21.3	26.9	35.4
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.32	-2.39	-1.65	-1.38	9.16	5.12	3.83	3.41	82.1	88.4	89.8	90.5
30	-4.14	-2.15	-1.58	-1.32	7.10	3.91	3.03	2.59	81.3	87.5	87.5	89.7
50	-3.98	-2.25	-1.63	-1.29	6.10	3.52	2.63	2.15	76.4	84.0	82.9	87.2
100	-4.09	-2.16	-1.59	-1.28	5.30	2.88	2.21	1.87	68.1	74.4	77.9	80.8
200	-4.11	-2.20	-1.59	-1.28	4.70	2.57	1.91	1.58	51.6	57.3	62.9	68.5
	PDOLS estimator, leads and lags order $p = 8$											
17	-3.38	-1.87	-1.30	-1.11	13.45	5.54	4.04	3.54	70.2	88.0	91.4	92.0
30	-3.00	-1.58	-1.21	-1.06	9.45	4.06	3.09	2.62	71.7	89.7	90.8	92.7
50	-2.91	-1.68	-1.25	-1.02	7.81	3.54	2.59	2.13	72.4	87.9	89.8	91.8
100	-3.13	-1.62	-1.22	-1.01	6.02	2.72	2.10	1.79	68.5	85.0	87.4	89.4
200	-3.06	-1.64	-1.23	-1.01	4.67	2.26	1.69	1.43	66.2	78.8	81.6	84.6
	MGMW estimator, $q = 5$											
17	-6.84	-3.01	-1.85	-1.24	13.35	7.51	5.86	5.01	87.9	92.0	93.5	94.9
30	-6.71	-2.78	-1.65	-1.34	10.90	5.80	4.82	3.99	84.0	91.6	93.9	94.5
50	-6.58	-3.03	-1.85	-1.41	9.32	5.08	3.72	3.18	78.2	86.4	91.4	93.7
100	-6.79	-2.79	-1.74	-1.30	8.26	4.16	2.99	2.54	63.3	82.8	86.3	88.4
200	-6.85	-2.88	-1.78	-1.27	7.56	3.52	2.44	1.97	40.0	66.0	78.7	83.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S24: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
n\T	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.35	-0.37	-0.15	-0.10	7.15	3.04	1.90	1.39	61.2	74.1	78.7	80.1	89.4	93.1	93.8	94.9
30	-0.89	-0.21	-0.12	-0.08	4.78	2.09	1.35	0.99	63.9	75.4	78.8	81.2	90.5	93.4	94.7	94.3
50	-0.77	-0.21	-0.10	-0.05	3.60	1.59	1.00	0.75	62.5	75.3	79.5	80.9	89.9	93.6	95.3	94.5
100	-0.78	-0.19	-0.06	-0.04	2.59	1.13	0.70	0.52	61.8	74.1	80.7	81.2	87.9	93.2	93.8	94.1
200	-0.84	-0.19	-0.08	-0.05	1.92	0.78	0.49	0.35	58.1	75.6	80.0	82.9	87.6	93.3	94.2	94.4
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-3.80	-1.56	-1.14	-0.72	9.23	5.86	7.64	4.59	63.4	71.2	75.8	76.7	70.8	80.9	83.9	84.9
30	-3.63	-1.31	-0.64	-0.66	7.58	4.56	9.49	3.31	60.5	71.6	74.1	74.5	66.6	79.2	82.6	82.4
50	-3.37	-1.41	-0.90	-0.61	5.94	3.39	3.06	2.67	56.9	67.8	71.0	72.2	62.9	75.7	79.1	80.1
100	-3.34	-1.25	-0.86	-0.58	4.85	3.03	2.18	1.92	46.5	63.5	67.2	68.9	52.6	71.2	73.8	75.6
200	-3.27	-1.28	-0.77	-0.61	7.15	2.13	1.66	1.56	29.9	56.0	64.3	67.5	36.2	62.8	70.3	72.6
	SPMG estimator								Standard				Robust bootstrapped			
17	0.17	-0.05	-0.01	-0.02	9.11	3.24	1.98	1.44	52.0	69.1	75.6	77.7	85.7	92.2	93.8	94.2
30	0.27	0.06	-0.01	-0.02	5.68	2.22	1.40	1.02	55.4	71.6	74.9	78.9	86.8	92.2	93.7	93.5
50	0.32	0.03	0.00	0.00	4.14	1.69	1.03	0.76	53.5	71.0	77.8	79.3	86.9	93.2	94.8	94.5
100	0.23	0.06	0.06	0.03	2.91	1.17	0.73	0.53	53.3	70.2	76.3	80.6	85.3	92.1	93.4	93.4
200	0.15	0.04	0.02	0.01	2.00	0.80	0.50	0.36	54.0	72.1	77.7	81.7	85.2	92.3	94.5	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
n\T	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.47	-3.62	-2.60	-2.14	9.95	5.84	4.52	3.90	70.1	74.8	77.5	77.0
30	-6.37	-3.39	-2.45	-1.99	8.45	4.71	3.57	3.06	64.7	69.1	71.7	73.3
50	-6.12	-3.49	-2.48	-1.95	7.49	4.44	3.25	2.62	54.3	59.1	61.9	66.7
100	-6.03	-3.34	-2.41	-1.93	6.81	3.87	2.89	2.37	38.3	44.9	49.4	55.7
200	-6.14	-3.35	-2.41	-1.90	6.50	3.61	2.64	2.14	19.8	21.5	27.6	34.8
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.46	-2.46	-1.79	-1.52	10.48	5.65	4.33	3.73	73.3	80.5	82.8	84.3
30	-4.24	-2.23	-1.65	-1.40	8.01	4.19	3.20	2.80	74.1	81.5	81.4	83.8
50	-4.03	-2.34	-1.71	-1.37	6.69	3.79	2.81	2.29	71.2	76.8	77.9	81.6
100	-4.05	-2.23	-1.65	-1.36	5.56	3.08	2.36	1.99	65.2	71.6	73.9	76.9
200	-4.18	-2.25	-1.66	-1.34	4.92	2.68	2.02	1.69	50.4	56.7	59.5	66.3
	PDOLS estimator, leads and lags order $p = 8$											
17	-3.47	-1.90	-1.42	-1.22	15.22	6.28	4.63	3.87	61.2	79.7	85.2	85.9
30	-3.14	-1.61	-1.25	-1.10	10.92	4.40	3.27	2.83	64.6	83.1	84.6	87.0
50	-2.80	-1.77	-1.32	-1.09	8.53	3.88	2.78	2.25	66.2	81.9	84.1	85.8
100	-3.04	-1.69	-1.28	-1.08	6.50	2.96	2.26	1.91	64.7	79.2	82.1	85.1
200	-3.14	-1.68	-1.29	-1.07	5.06	2.40	1.82	1.53	61.1	74.7	78.2	81.0
	MGMW estimator, $q = 5$											
17	-7.01	-2.85	-1.91	-1.37	14.12	8.00	6.81	5.16	81.0	86.5	88.3	89.6
30	-7.15	-2.98	-1.96	-1.56	11.45	6.51	4.81	4.47	79.1	86.8	88.8	90.1
50	-7.00	-3.09	-1.94	-1.49	10.18	5.45	4.03	3.47	73.2	82.9	86.9	89.8
100	-6.70	-2.83	-1.82	-1.37	8.34	4.29	3.16	2.65	62.5	76.6	83.2	86.0
200	-6.98	-2.96	-1.87	-1.39	7.82	3.65	2.57	2.14	38.0	63.9	74.6	81.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S25: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.05$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-0.54	-0.17	-0.02	-0.02	4.98	2.31	1.45	1.09	57.4	63.7	69.1	71.9	89.6	93.0	93.4	94.1
30	-0.25	-0.02	-0.01	-0.01	3.75	1.67	1.06	0.81	51.9	62.3	68.2	69.3	90.1	94.1	94.6	93.6
50	-0.38	-0.08	-0.01	-0.01	2.83	1.30	0.79	0.61	52.5	62.7	67.4	69.8	90.3	93.4	95.3	95.4
100	-0.28	-0.07	-0.01	-0.01	2.05	0.97	0.60	0.46	49.5	60.4	65.5	65.3	91.0	94.1	95.0	94.9
200	-0.21	-0.04	-0.04	-0.01	1.58	0.71	0.45	0.33	45.2	58.4	60.1	62.2	91.9	94.8	95.4	96.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.05	-0.71	-0.41	-0.15	6.47	3.86	3.43	7.96	57.2	64.9	69.1	70.6	74.6	82.5	85.0	85.4
30	-1.65	-0.59	-0.30	-0.30	5.01	3.54	2.47	2.26	55.5	63.6	66.3	65.9	75.0	82.9	85.0	83.1
50	-1.69	-0.72	-0.47	-0.31	4.08	2.42	1.95	1.79	53.5	60.8	64.2	64.3	74.1	82.1	84.6	83.2
100	-1.68	-0.61	-0.47	-0.30	3.29	2.22	2.05	1.50	47.6	57.2	60.3	62.6	72.2	82.5	83.0	82.4
200	-1.60	-0.60	-0.35	-0.32	2.82	1.44	2.62	0.98	40.2	54.6	56.4	62.2	73.6	85.2	85.1	85.4
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.15	-0.07	0.02	0.01	7.79	2.52	1.54	1.14	46.9	60.7	65.7	69.9	85.3	92.0	92.7	93.0
30	0.06	0.07	0.04	0.01	5.93	1.81	1.11	0.84	43.6	58.1	65.6	65.9	85.3	92.4	94.1	93.9
50	-0.09	0.04	0.03	0.01	3.64	1.43	0.83	0.63	42.2	59.5	65.0	67.9	86.1	90.9	94.8	94.2
100	0.03	0.04	0.04	0.02	3.10	1.04	0.63	0.47	41.2	56.5	62.0	64.1	85.5	92.4	94.2	94.1
200	0.01	0.05	0.01	0.01	4.06	0.79	0.47	0.35	37.9	54.2	57.8	60.6	85.0	92.0	94.0	95.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-3.50	-1.92	-1.36	-1.15	6.48	3.76	2.92	2.41	71.6	74.1	77.6	78.2
30	-3.13	-1.81	-1.31	-1.07	5.23	3.10	2.27	2.02	67.6	70.5	73.1	75.7
50	-3.31	-1.91	-1.37	-1.12	4.64	2.75	2.08	1.68	60.6	66.0	68.9	71.0
100	-3.16	-1.87	-1.38	-1.09	4.06	2.41	1.82	1.46	51.7	56.0	59.6	64.3
200	-3.16	-1.85	-1.35	-1.07	3.71	2.18	1.61	1.31	36.7	39.6	42.7	50.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.76	-1.43	-1.00	-0.86	7.49	3.80	2.92	2.37	68.7	76.7	79.4	80.6
30	-2.37	-1.31	-0.92	-0.78	5.76	3.02	2.11	1.93	68.4	76.6	78.1	79.8
50	-2.56	-1.38	-1.00	-0.83	4.81	2.54	1.88	1.50	66.8	75.8	77.3	79.8
100	-2.43	-1.37	-1.00	-0.80	3.97	2.10	1.56	1.26	61.6	69.1	73.7	77.2
200	-2.43	-1.35	-1.00	-0.79	3.44	1.79	1.31	1.07	52.5	60.7	62.9	70.4
	PDOLS estimator, leads and lags order $p = 8$											
17	-2.08	-1.20	-0.77	-0.70	12.08	4.45	3.20	2.52	53.8	74.6	77.8	80.1
30	-1.80	-0.98	-0.68	-0.59	8.93	3.42	2.23	1.98	53.4	75.4	79.0	81.2
50	-1.99	-1.05	-0.76	-0.64	6.98	2.78	1.93	1.48	53.8	74.9	79.6	83.7
100	-1.96	-1.08	-0.79	-0.61	5.71	2.19	1.55	1.22	51.9	73.1	78.7	82.1
200	-1.86	-1.03	-0.78	-0.62	4.76	1.79	1.22	0.99	44.1	68.7	73.6	79.6
	MGMW estimator, $q = 5$											
17	-1.63	-0.61	-0.35	-0.35	7.75	4.78	3.72	3.71	80.7	80.4	84.2	84.9
30	-1.21	-0.57	-0.28	-0.20	6.18	3.59	2.85	2.60	82.1	83.2	85.0	85.5
50	-1.29	-0.57	-0.38	-0.33	4.89	3.00	2.14	1.92	81.6	82.4	85.6	86.9
100	-0.92	-0.54	-0.33	-0.24	3.55	2.19	1.71	1.37	80.4	82.2	86.6	86.9
200	-0.92	-0.52	-0.31	-0.18	2.77	1.62	1.22	1.02	78.4	81.7	83.7	88.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S26: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.48	-0.38	-0.13	-0.08	6.31	2.59	1.55	1.17	70.3	84.6	88.8	89.6	89.7	94.0	94.8	94.5
30	-0.99	-0.16	-0.10	-0.07	4.39	1.83	1.16	0.83	70.8	84.5	88.3	90.2	90.5	94.4	93.6	94.2
50	-0.79	-0.24	-0.11	-0.05	3.24	1.39	0.83	0.60	71.4	84.8	89.9	91.6	90.3	92.9	95.6	95.9
100	-0.90	-0.18	-0.08	-0.05	2.42	0.97	0.60	0.44	65.5	83.9	89.1	90.0	89.0	93.4	94.3	94.3
200	-0.94	-0.20	-0.09	-0.05	1.80	0.69	0.41	0.30	63.1	83.4	89.7	91.0	88.2	93.6	94.9	95.0
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-7.46	-3.18	-2.46	-2.04	14.35	13.92	10.05	13.71	57.9	67.7	71.5	72.7	56.1	66.9	71.2	73.2
30	-7.28	-3.16	-2.23	-2.01	11.94	8.51	6.89	10.95	51.3	64.5	68.0	70.2	50.1	64.4	67.5	70.4
50	-6.89	-3.24	-2.30	-1.75	10.02	6.21	5.85	5.00	44.3	61.8	65.5	69.0	43.7	62.2	65.1	69.1
100	-6.66	-3.07	-2.26	-1.84	8.90	5.50	5.43	4.85	29.1	52.5	58.8	63.7	29.5	53.3	58.9	63.9
200	-6.85	-3.17	-2.18	-1.93	8.70	4.35	3.76	4.02	13.8	42.1	52.8	58.3	14.2	42.0	53.3	59.0
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.03	-0.01	0.04	0.01	7.53	2.83	1.62	1.20	60.4	79.5	86.5	88.1	86.6	93.1	94.9	94.0
30	0.42	0.19	0.05	0.01	5.22	1.98	1.22	0.85	59.4	79.3	85.9	88.2	87.7	93.4	93.1	93.8
50	0.55	0.07	0.02	0.02	3.82	1.47	0.86	0.62	59.1	80.0	88.0	90.7	87.1	93.1	95.4	95.5
100	0.32	0.13	0.06	0.03	2.70	1.03	0.62	0.45	57.2	79.8	87.3	88.3	86.0	92.6	94.5	94.8
200	0.24	0.09	0.03	0.02	1.80	0.70	0.42	0.30	61.7	81.2	88.1	89.8	87.1	92.7	94.8	94.9

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-10.53	-6.33	-4.88	-4.11	14.03	9.13	7.61	6.71	72.8	81.5	84.5	87.6
30	-10.38	-6.26	-4.79	-3.93	12.58	8.17	6.56	5.65	65.9	74.4	78.4	83.9
50	-10.29	-6.32	-4.76	-4.01	11.71	7.48	5.89	5.10	54.7	64.2	70.6	75.7
100	-10.08	-6.15	-4.76	-4.05	10.86	6.79	5.43	4.70	38.1	45.3	55.1	61.8
200	-10.21	-6.14	-4.73	-4.00	10.59	6.47	5.05	4.33	27.5	26.2	31.1	38.4
	PDOLS estimator, leads and lags order $p = 4$											
17	-9.22	-5.55	-4.30	-3.65	14.73	9.25	7.67	6.72	78.1	86.3	87.9	90.4
30	-8.87	-5.44	-4.20	-3.48	12.39	8.02	6.40	5.53	72.5	83.4	84.5	87.7
50	-8.91	-5.50	-4.18	-3.57	11.32	7.09	5.61	4.89	67.7	78.0	80.2	82.7
100	-8.73	-5.36	-4.19	-3.63	10.02	6.25	5.05	4.43	57.1	67.3	69.1	73.9
200	-8.88	-5.34	-4.17	-3.57	9.51	5.81	4.59	3.99	40.8	50.1	52.3	56.4
	PDOLS estimator, leads and lags order $p = 8$											
17	-8.65	-5.29	-4.10	-3.48	18.91	10.04	8.13	7.04	64.9	86.2	88.4	90.6
30	-8.07	-5.15	-3.97	-3.30	14.73	8.53	6.63	5.68	63.7	84.2	85.5	89.7
50	-8.30	-5.23	-3.95	-3.41	13.05	7.36	5.73	4.97	61.9	80.7	84.2	85.7
100	-8.11	-5.09	-3.99	-3.48	10.69	6.32	5.08	4.45	53.7	72.1	76.5	80.6
200	-8.23	-5.05	-3.96	-3.42	9.59	5.71	4.50	3.93	38.9	60.2	64.3	67.2
	MGMW estimator, $q = 5$											
17	-10.86	-5.96	-4.23	-3.33	18.55	12.98	10.57	9.55	84.1	90.1	91.7	93.9
30	-10.81	-5.75	-4.20	-3.51	15.78	10.00	9.05	7.84	77.0	87.3	90.5	92.5
50	-10.66	-5.96	-4.35	-3.56	13.83	8.92	7.19	6.15	69.7	79.9	86.4	88.6
100	-10.70	-5.87	-4.28	-3.67	12.28	7.48	5.94	5.27	51.4	70.2	74.3	78.6
200	-10.88	-5.90	-4.24	-3.55	11.68	6.73	5.14	4.47	25.7	49.7	62.3	66.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S27: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.73	-0.43	-0.17	-0.11	7.91	3.28	2.05	1.50	61.2	74.7	80.3	80.3	89.4	93.7	94.3	95.3
30	-1.17	-0.26	-0.14	-0.10	5.30	2.27	1.46	1.06	63.8	76.0	79.6	81.6	90.3	93.3	94.1	94.2
50	-1.00	-0.29	-0.12	-0.06	3.97	1.70	1.06	0.79	61.6	76.3	81.2	82.1	90.6	93.9	95.1	94.9
100	-1.03	-0.21	-0.07	-0.04	2.85	1.19	0.74	0.55	60.2	75.9	80.8	81.9	88.0	94.1	94.3	94.1
200	-1.06	-0.23	-0.10	-0.06	2.13	0.84	0.52	0.38	55.9	76.1	80.9	82.8	87.0	93.4	94.0	94.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-7.59	-3.66	-2.76	-2.28	15.75	11.53	14.22	9.03	54.1	63.7	66.8	69.9	60.4	69.8	73.5	75.8
30	-6.18	-3.36	-2.33	-1.84	50.67	8.29	11.62	6.90	48.6	62.0	64.7	66.4	53.9	68.2	70.8	71.2
50	-7.11	-3.58	-2.70	-2.24	10.34	6.47	6.36	7.40	41.8	56.9	62.7	65.3	45.8	62.5	68.0	69.4
100	-6.95	-3.32	-2.40	-1.87	9.25	5.62	4.66	6.12	30.1	50.2	56.3	60.9	34.1	55.7	60.7	64.5
200	-7.04	-3.30	-2.29	-2.04	10.11	4.58	3.75	3.73	14.0	38.6	50.4	55.9	16.1	42.9	54.7	59.0
	SPMG estimator								Standard				Robust bootstrapped			
17	0.24	0.00	0.03	-0.01	10.75	3.52	2.16	1.54	51.1	69.9	76.0	78.4	85.7	92.2	94.2	94.5
30	0.35	0.10	0.01	-0.01	6.27	2.40	1.52	1.10	55.4	72.2	76.5	79.2	86.8	92.7	93.2	93.7
50	0.39	0.03	0.01	0.01	4.60	1.82	1.10	0.81	54.3	71.9	78.6	80.3	86.7	93.6	94.8	94.7
100	0.24	0.10	0.08	0.04	3.16	1.25	0.76	0.57	53.1	71.6	77.9	80.6	85.0	92.7	94.4	93.9
200	0.19	0.07	0.03	0.02	2.15	0.86	0.53	0.38	53.5	73.8	79.0	81.8	87.1	93.1	94.3	94.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-10.67	-6.52	-5.28	-4.53	14.73	9.70	8.38	7.35	70.8	78.7	81.1	84.6
30	-10.56	-6.45	-5.00	-4.21	13.05	8.52	6.90	6.10	62.6	72.3	76.1	80.7
50	-10.48	-6.57	-5.07	-4.29	12.10	7.86	6.27	5.43	53.6	61.7	67.7	73.8
100	-10.17	-6.31	-4.95	-4.29	11.06	7.04	5.66	4.97	37.7	46.8	53.2	59.8
200	-10.40	-6.29	-4.92	-4.19	10.84	6.65	5.27	4.55	26.6	25.5	31.3	37.3
	PDOLS estimator, leads and lags order $p = 4$											
17	-9.37	-5.65	-4.66	-4.04	15.72	9.87	8.51	7.36	72.3	83.3	84.4	87.2
30	-9.03	-5.59	-4.39	-3.74	13.01	8.36	6.74	5.97	70.2	79.7	81.9	85.3
50	-9.05	-5.72	-4.46	-3.83	11.76	7.47	5.99	5.21	63.8	73.4	77.7	79.8
100	-8.80	-5.50	-4.37	-3.85	10.28	6.51	5.28	4.69	56.2	65.2	67.1	70.6
200	-9.05	-5.47	-4.34	-3.75	9.78	5.97	4.80	4.20	39.7	47.7	50.1	54.3
	PDOLS estimator, leads and lags order $p = 8$											
17	-8.97	-5.36	-4.47	-3.88	20.56	10.88	9.12	7.73	60.3	80.4	85.1	87.4
30	-8.33	-5.27	-4.14	-3.56	15.81	8.89	7.00	6.15	59.7	80.5	83.4	86.5
50	-8.26	-5.43	-4.23	-3.66	13.63	7.76	6.13	5.29	59.0	78.3	80.9	83.3
100	-8.23	-5.24	-4.18	-3.71	11.18	6.63	5.32	4.72	51.6	70.2	74.4	76.6
200	-8.44	-5.17	-4.14	-3.60	9.95	5.87	4.71	4.14	38.3	57.3	61.3	64.5
	MGMW estimator, $q = 5$											
17	-11.17	-5.59	-4.62	-3.85	19.25	12.76	11.51	9.76	78.6	86.7	88.9	91.0
30	-11.19	-6.16	-4.71	-3.95	16.04	10.77	9.03	8.39	75.7	83.8	86.7	88.9
50	-10.97	-6.20	-4.47	-3.90	14.58	9.16	7.41	6.67	66.6	78.5	83.1	84.9
100	-10.65	-5.99	-4.48	-3.78	12.48	7.64	6.16	5.46	50.6	67.5	72.9	76.3
200	-10.98	-6.06	-4.53	-3.79	11.92	6.90	5.41	4.69	25.7	45.7	57.5	63.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S28: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.2$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-0.71	-0.18	-0.02	-0.02	5.37	2.46	1.54	1.15	57.2	65.9	70.0	73.5	89.9	93.3	93.8	94.4
30	-0.35	-0.02	-0.03	-0.01	3.98	1.76	1.11	0.85	54.0	64.5	70.6	70.5	90.2	93.7	94.8	94.1
50	-0.47	-0.11	-0.02	-0.02	3.02	1.36	0.81	0.63	53.4	64.5	69.7	72.1	90.8	94.0	95.6	95.8
100	-0.36	-0.07	-0.01	-0.01	2.18	1.01	0.63	0.48	51.2	62.7	68.0	66.8	91.5	94.5	95.1	95.1
200	-0.28	-0.05	-0.04	-0.02	1.67	0.74	0.48	0.35	46.0	58.8	61.7	63.2	92.2	95.1	95.2	95.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-3.68	-1.66	-1.32	-1.13	9.39	7.06	6.98	9.58	52.8	62.8	62.5	64.6	65.8	75.4	74.1	73.7
30	-3.36	-1.65	-1.07	-1.03	7.59	5.94	4.81	4.87	51.7	61.4	62.7	63.4	66.1	75.9	75.9	74.6
50	-3.43	-1.85	-1.27	-1.10	7.55	4.72	3.79	3.54	43.4	56.4	60.1	60.9	59.6	72.2	73.4	73.4
100	-3.39	-1.68	-1.41	-1.12	5.26	3.77	3.48	3.08	38.7	52.0	56.8	60.6	55.8	69.4	69.9	72.4
200	-3.29	-1.66	-1.20	-1.14	4.78	3.01	3.49	2.25	29.9	45.7	50.4	54.0	50.2	68.0	68.7	69.7
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.31	-0.06	0.04	0.02	9.25	2.71	1.64	1.19	46.7	61.0	66.5	71.3	84.9	91.6	93.2	93.4
30	0.02	0.10	0.04	0.02	7.15	1.91	1.16	0.89	44.2	61.4	67.7	67.6	85.0	92.5	94.7	93.8
50	-0.12	0.03	0.04	0.02	4.45	1.49	0.87	0.65	43.1	60.7	67.4	70.2	85.6	91.1	95.2	95.0
100	-0.12	0.07	0.05	0.03	7.04	1.10	0.66	0.49	41.4	58.7	64.7	65.6	85.2	92.6	94.0	94.4
200	-0.08	0.07	0.02	0.01	5.90	0.83	0.50	0.36	39.0	56.0	59.5	61.9	85.0	92.3	93.8	95.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.25	-3.18	-2.49	-2.27	8.94	5.84	4.90	4.49	75.5	82.5	85.2	88.1
30	-4.92	-3.12	-2.47	-2.03	7.38	4.91	3.98	3.57	73.1	79.3	81.1	86.9
50	-5.19	-3.25	-2.53	-2.22	6.79	4.41	3.59	3.16	65.3	74.9	77.9	80.8
100	-4.98	-3.18	-2.58	-2.20	6.01	3.90	3.25	2.80	56.0	65.4	66.0	71.5
200	-4.94	-3.17	-2.52	-2.15	5.60	3.62	2.92	2.55	45.5	46.2	50.7	58.2
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.87	-2.86	-2.21	-2.07	10.33	6.24	5.09	4.61	73.1	84.8	86.4	88.7
30	-4.52	-2.79	-2.19	-1.81	8.21	5.06	3.99	3.60	73.3	83.5	85.2	89.0
50	-4.81	-2.91	-2.26	-2.00	7.20	4.41	3.51	3.08	69.3	81.5	84.4	86.7
100	-4.63	-2.87	-2.32	-1.99	6.18	3.77	3.11	2.67	62.9	76.4	77.1	80.2
200	-4.55	-2.85	-2.27	-1.94	5.62	3.42	2.74	2.39	52.4	64.3	66.0	70.4
	PDOLS estimator, leads and lags order $p = 8$											
17	-4.49	-2.80	-2.08	-2.01	14.87	7.25	5.59	4.95	55.1	81.3	85.1	88.7
30	-4.22	-2.65	-2.06	-1.70	11.34	5.67	4.25	3.81	56.5	81.3	86.6	89.7
50	-4.49	-2.77	-2.15	-1.90	9.38	4.83	3.70	3.16	53.2	80.5	86.1	88.1
100	-4.42	-2.79	-2.23	-1.90	7.76	4.03	3.20	2.71	50.4	76.5	80.6	84.9
200	-4.28	-2.73	-2.16	-1.85	6.71	3.54	2.74	2.38	41.2	67.3	72.6	77.7
	MGMW estimator, $q = 5$											
17	-2.15	-1.12	-0.87	-0.78	10.16	7.02	6.24	6.15	82.8	86.1	88.9	90.3
30	-1.78	-1.01	-0.75	-0.43	8.03	5.41	4.70	4.41	85.0	87.3	89.6	91.8
50	-1.92	-0.96	-0.75	-0.62	6.48	4.47	3.77	3.46	83.2	86.4	89.2	90.8
100	-1.38	-0.90	-0.72	-0.58	4.76	3.34	2.95	2.59	80.2	85.3	86.7	89.0
200	-1.26	-0.93	-0.60	-0.46	3.81	2.70	2.21	2.02	78.7	80.9	84.0	86.6

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \to y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S29: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-5.44	-2.78	-1.78	-1.28	7.37	3.50	2.23	1.60	46.2	53.2	59.6	60.7	72.0	68.2	70.3	69.3
30	-5.24	-2.67	-1.71	-1.24	6.37	3.14	1.99	1.43	32.7	36.3	41.0	43.0	57.4	51.8	54.2	53.0
50	-5.01	-2.48	-1.62	-1.18	5.67	2.74	1.79	1.30	19.1	22.1	24.0	27.1	41.6	37.5	34.9	35.5
100	-4.89	-2.50	-1.63	-1.18	5.26	2.63	1.71	1.24	5.0	3.5	4.3	5.2	16.5	8.6	8.0	8.6
200	-4.77	-2.46	-1.60	-1.17	4.96	2.53	1.64	1.20	0.3	0.0	0.1	0.0	2.3	0.6	0.4	0.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.35	-0.74	-0.33	-0.18	4.73	2.10	1.34	0.94	78.8	86.5	89.9	92.2	76.4	86.0	89.9	92.6
30	-2.49	-0.74	-0.34	-0.19	3.94	1.64	1.02	0.72	71.2	84.2	89.0	90.4	70.3	83.5	89.5	90.5
50	-2.50	-0.64	-0.28	-0.15	3.46	1.29	0.77	0.55	63.6	84.6	90.3	91.6	61.7	84.4	89.7	91.5
100	-2.41	-0.69	-0.30	-0.17	2.94	1.05	0.61	0.42	50.6	77.0	85.6	89.2	50.1	77.3	85.2	89.4
200	-2.36	-0.68	-0.30	-0.17	2.64	0.87	0.47	0.31	30.4	68.1	80.9	86.2	29.8	66.9	80.2	85.9
	SPMG estimator								Standard				Robust bootstrapped			
17	0.40	-0.01	0.01	0.01	4.58	1.88	1.19	0.83	67.6	83.9	87.8	90.8	90.4	93.4	93.6	95.0
30	0.08	-0.03	-0.01	0.00	3.40	1.42	0.88	0.63	69.0	83.1	87.7	90.2	89.1	92.1	94.2	93.6
50	0.13	0.07	0.04	0.03	2.58	1.06	0.66	0.48	67.7	84.5	88.4	89.9	90.2	94.6	94.0	94.5
100	0.12	0.00	0.00	0.00	1.78	0.73	0.46	0.33	68.7	85.4	87.7	90.2	89.1	94.2	93.7	94.6
200	0.16	0.02	0.01	0.00	1.28	0.53	0.32	0.23	68.0	83.7	89.3	90.5	89.8	93.0	95.2	95.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.51	-2.33	-1.51	-1.08	6.02	3.08	2.01	1.44	75.0	74.6	74.6	75.2
30	-4.65	-2.29	-1.47	-1.06	5.53	2.74	1.77	1.28	59.3	62.6	62.5	63.5
50	-4.67	-2.17	-1.39	-1.02	5.24	2.45	1.58	1.16	43.5	48.1	47.0	50.6
100	-4.54	-2.20	-1.41	-1.02	4.85	2.35	1.51	1.09	20.0	19.7	20.1	22.6
200	-4.47	-2.19	-1.40	-1.02	4.62	2.26	1.45	1.05	4.7	2.5	2.6	2.6
	PDOLS estimator, leads and lags order $p = 4$											
17	-1.42	-0.75	-0.47	-0.32	5.35	2.24	1.44	1.01	86.6	90.8	90.4	91.2
30	-1.67	-0.74	-0.47	-0.32	4.18	1.77	1.11	0.79	84.5	89.1	89.6	89.8
50	-1.64	-0.65	-0.40	-0.29	3.38	1.38	0.85	0.62	83.7	90.0	90.4	90.1
100	-1.56	-0.68	-0.42	-0.30	2.63	1.11	0.70	0.50	80.6	84.3	83.8	85.5
200	-1.53	-0.68	-0.42	-0.30	2.12	0.91	0.56	0.41	75.0	76.2	77.4	76.1
	PDOLS estimator, leads and lags order $p = 8$											
17	-0.53	-0.24	-0.16	-0.09	9.77	2.60	1.52	1.05	71.9	90.9	92.2	92.8
30	-0.46	-0.26	-0.16	-0.10	7.11	1.99	1.13	0.79	73.0	90.6	92.0	91.8
50	-0.58	-0.17	-0.09	-0.07	5.44	1.54	0.86	0.60	74.8	90.4	92.8	94.1
100	-0.45	-0.20	-0.12	-0.08	3.85	1.09	0.64	0.44	76.3	90.3	90.9	92.1
200	-0.59	-0.21	-0.12	-0.09	2.75	0.78	0.44	0.31	77.7	90.4	91.2	91.9
	MGMW estimator, $q = 5$											
17	-4.79	-1.54	-0.72	-0.33	8.49	4.13	2.64	1.90	85.7	91.2	92.8	93.8
30	-5.11	-1.51	-0.67	-0.39	7.48	3.18	2.00	1.45	81.3	89.0	92.2	92.9
50	-5.00	-1.42	-0.64	-0.36	6.57	2.56	1.58	1.14	73.8	89.6	93.1	93.1
100	-4.92	-1.50	-0.69	-0.38	5.73	2.12	1.23	0.86	58.8	82.3	89.9	91.9
200	-4.87	-1.49	-0.71	-0.39	5.30	1.83	1.00	0.66	33.5	70.6	82.6	89.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and no CS dependence of errors. See notes to Table S1.

Table S30: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-5.79	-3.03	-1.99	-1.46	8.36	4.10	2.68	1.97	46.2	50.0	51.7	54.1	77.9	77.2	77.6	77.6
30	-5.57	-2.83	-1.84	-1.34	7.14	3.51	2.26	1.66	34.0	37.9	40.6	41.8	66.0	64.0	64.5	64.9
50	-5.36	-2.64	-1.74	-1.29	6.31	3.06	1.99	1.48	21.4	25.7	26.9	28.5	52.1	50.6	50.9	52.0
100	-5.14	-2.62	-1.72	-1.26	5.66	2.83	1.86	1.36	7.2	7.2	7.7	9.2	27.0	22.2	21.4	21.6
200	-5.09	-2.55	-1.67	-1.23	5.34	2.66	1.74	1.28	1.1	0.6	0.8	0.5	6.8	4.3	3.8	3.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.60	-0.85	-0.38	-0.22	6.06	2.78	1.78	1.30	68.2	76.5	80.1	81.5	76.9	86.9	90.5	90.8
30	-2.64	-0.79	-0.37	-0.20	4.73	2.08	1.34	0.98	65.0	75.7	78.7	80.1	72.4	85.7	89.5	91.0
50	-2.62	-0.67	-0.31	-0.18	4.01	1.64	1.01	0.74	57.0	74.6	80.1	81.5	65.2	84.9	90.0	90.8
100	-2.49	-0.75	-0.34	-0.19	3.27	1.28	0.77	0.55	48.8	68.8	76.2	79.1	56.8	79.4	86.1	88.8
200	-2.46	-0.72	-0.32	-0.19	2.87	1.01	0.57	0.39	30.9	62.3	72.7	76.3	38.2	72.7	83.8	87.9
	SPMG estimator								Standard				Robust bootstrapped			
17	0.34	-0.02	0.00	0.00	5.93	2.55	1.60	1.17	58.4	71.3	77.6	79.2	90.3	91.9	94.1	93.9
30	0.10	0.00	0.00	0.01	4.24	1.80	1.18	0.87	58.7	72.8	76.3	78.1	88.7	93.3	93.7	92.9
50	0.18	0.09	0.04	0.02	3.28	1.39	0.86	0.64	57.9	72.3	78.1	80.1	89.3	93.2	94.4	95.0
100	0.13	-0.01	-0.01	-0.01	2.22	0.96	0.61	0.45	58.6	73.2	77.2	78.8	88.5	92.6	93.2	94.2
200	0.12	0.02	0.01	0.00	1.57	0.67	0.42	0.31	58.9	73.5	76.9	79.7	88.5	92.6	94.1	94.2

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.75	-2.49	-1.65	-1.21	7.04	3.67	2.42	1.79	66.7	67.4	66.4	67.5
30	-4.79	-2.39	-1.58	-1.14	6.18	3.11	2.06	1.51	56.7	58.1	56.6	57.8
50	-4.81	-2.26	-1.49	-1.10	5.68	2.73	1.79	1.33	41.5	46.8	45.7	46.7
100	-4.59	-2.28	-1.48	-1.08	5.06	2.52	1.64	1.21	24.2	23.9	23.9	24.5
200	-4.55	-2.25	-1.45	-1.07	4.78	2.37	1.53	1.13	6.8	4.8	5.8	6.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-1.52	-0.81	-0.53	-0.37	6.79	2.96	1.89	1.38	77.6	80.9	81.3	81.7
30	-1.70	-0.75	-0.50	-0.34	5.13	2.22	1.44	1.05	76.3	80.9	80.4	81.1
50	-1.64	-0.66	-0.43	-0.32	4.04	1.74	1.10	0.81	77.7	80.1	81.1	79.7
100	-1.54	-0.71	-0.45	-0.33	3.08	1.34	0.86	0.63	73.5	75.3	75.0	76.2
200	-1.55	-0.71	-0.43	-0.32	2.38	1.05	0.66	0.48	70.4	69.4	70.0	69.7
	PDOLS estimator, leads and lags order $p = 8$											
17	-0.73	-0.32	-0.19	-0.11	11.78	3.48	2.04	1.44	63.3	80.4	82.1	82.6
30	-0.63	-0.25	-0.17	-0.09	8.23	2.53	1.50	1.07	67.1	81.7	81.5	82.7
50	-0.57	-0.15	-0.09	-0.08	6.47	1.98	1.14	0.80	68.8	81.7	83.9	82.7
100	-0.47	-0.20	-0.13	-0.09	4.55	1.39	0.83	0.59	69.1	81.1	80.9	82.1
200	-0.57	-0.23	-0.12	-0.09	3.16	0.98	0.58	0.40	71.2	81.2	82.4	82.2
	MGMW estimator, $q = 5$											
17	-4.76	-1.63	-0.73	-0.39	9.32	4.59	3.09	2.30	80.2	84.5	86.2	85.9
30	-5.01	-1.64	-0.71	-0.37	7.88	3.71	2.38	1.69	75.7	84.7	86.1	87.6
50	-5.12	-1.42	-0.64	-0.39	6.93	2.82	1.83	1.35	70.1	84.0	87.1	87.8
100	-4.92	-1.54	-0.71	-0.40	5.94	2.35	1.39	0.99	57.2	76.4	84.2	86.4
200	-4.84	-1.51	-0.71	-0.42	5.36	1.95	1.08	0.75	35.2	68.1	79.4	83.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and SAR CS dependence of errors. See notes to Table S1.

Table S31: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.84	-0.95	-0.62	-0.48	4.17	1.97	1.26	0.97	52.6	59.4	63.0	62.8	87.7	91.3	89.8	90.5
30	-1.67	-0.83	-0.60	-0.44	3.24	1.62	1.07	0.80	48.8	54.6	55.2	55.9	85.4	87.8	88.0	87.7
50	-1.67	-0.82	-0.55	-0.41	2.83	1.36	0.89	0.67	42.9	48.1	49.8	51.7	83.3	85.2	85.8	86.0
100	-1.53	-0.83	-0.54	-0.40	2.30	1.18	0.77	0.58	33.8	35.2	36.8	39.9	78.0	78.5	79.3	80.0
200	-1.52	-0.80	-0.53	-0.40	2.08	1.07	0.70	0.52	24.8	25.9	25.7	26.0	70.9	70.5	71.0	70.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-1.27	-0.33	-0.16	-0.08	4.04	1.83	1.16	0.88	62.7	69.8	72.3	73.2	79.6	88.4	91.1	92.3
30	-1.08	-0.29	-0.17	-0.09	3.05	1.45	0.94	0.68	60.0	65.7	67.3	70.5	79.2	87.4	89.9	91.2
50	-1.20	-0.28	-0.12	-0.06	2.63	1.11	0.70	0.52	57.3	65.8	70.4	71.6	76.0	87.5	90.8	91.9
100	-1.07	-0.33	-0.15	-0.08	2.05	0.85	0.52	0.38	51.5	62.9	66.1	69.1	74.2	86.3	90.0	91.5
200	-1.07	-0.31	-0.13	-0.08	1.74	0.66	0.39	0.27	46.0	58.0	65.0	68.6	68.4	84.6	90.1	92.2
	SPMG estimator								Standard				Robust bootstrapped			
17	0.07	0.03	0.00	0.02	4.12	1.68	1.05	0.80	50.8	65.3	69.5	69.0	89.5	93.8	93.7	94.2
30	0.09	0.05	-0.02	-0.01	3.04	1.33	0.83	0.61	50.2	60.9	64.1	66.3	88.3	92.4	93.4	92.7
50	0.01	0.03	0.02	0.02	2.28	0.99	0.63	0.46	50.0	61.5	67.2	67.7	89.1	91.9	93.2	93.7
100	0.09	-0.02	0.00	0.00	1.66	0.69	0.44	0.32	49.2	59.9	64.8	66.3	87.0	92.5	93.5	94.4
200	0.04	0.01	0.01	0.01	1.37	0.49	0.31	0.23	48.4	61.4	65.6	66.7	89.1	92.3	93.2	94.3

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-2.29	-1.11	-0.74	-0.55	4.41	2.19	1.42	1.08	67.9	69.0	68.3	69.3
30	-2.10	-1.06	-0.75	-0.54	3.55	1.84	1.26	0.92	62.9	62.0	61.6	60.4
50	-2.24	-1.03	-0.69	-0.52	3.28	1.57	1.03	0.79	54.0	55.6	55.9	56.4
100	-2.06	-1.06	-0.70	-0.52	2.74	1.39	0.92	0.69	43.1	40.5	40.9	40.7
200	-2.05	-1.04	-0.68	-0.52	2.51	1.25	0.84	0.63	27.2	23.8	25.3	23.6
	PDOLS estimator, leads and lags order $p = 4$											
17	-1.17	-0.46	-0.30	-0.21	4.93	1.99	1.26	0.94	67.6	74.3	72.8	75.0
30	-0.98	-0.45	-0.33	-0.22	3.74	1.61	1.04	0.74	66.6	68.4	68.6	70.8
50	-1.17	-0.42	-0.27	-0.20	3.14	1.26	0.79	0.58	64.3	69.5	68.6	70.7
100	-1.01	-0.48	-0.30	-0.21	2.36	1.00	0.62	0.45	61.4	65.0	64.4	66.2
200	-1.06	-0.47	-0.28	-0.21	1.92	0.79	0.49	0.35	55.9	56.2	59.2	58.7
	PDOLS estimator, leads and lags order $p = 8$											
17	-0.35	-0.17	-0.13	-0.07	8.96	2.39	1.39	1.02	55.9	73.7	72.7	74.6
30	-0.24	-0.17	-0.16	-0.09	6.65	1.91	1.12	0.77	53.6	70.3	67.8	71.5
50	-0.49	-0.14	-0.07	-0.06	5.17	1.45	0.84	0.58	54.0	70.8	72.0	73.0
100	-0.36	-0.19	-0.14	-0.07	3.83	1.06	0.62	0.43	53.0	67.5	68.2	71.1
200	-0.57	-0.19	-0.10	-0.09	2.94	0.77	0.44	0.31	51.2	67.0	68.5	69.5
	MGMW estimator, $q = 5$											
17	-1.56	-0.51	-0.25	-0.16	5.25	2.67	1.84	1.43	76.9	78.3	77.0	77.4
30	-1.26	-0.36	-0.26	-0.17	4.01	2.09	1.47	1.06	76.8	78.3	74.9	76.5
50	-1.44	-0.40	-0.19	-0.09	3.43	1.65	1.08	0.80	73.8	77.0	78.3	78.5
100	-1.21	-0.46	-0.22	-0.11	2.54	1.22	0.80	0.57	72.6	74.9	76.6	78.6
200	-1.23	-0.45	-0.20	-0.14	2.13	0.96	0.58	0.43	66.9	72.0	78.3	78.1

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S32: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-5.54	-2.80	-1.80	-1.30	7.58	3.54	2.26	1.64	47.1	54.7	60.1	61.5	72.9	69.9	70.6	69.8
30	-5.34	-2.72	-1.74	-1.26	6.54	3.20	2.03	1.46	33.2	37.8	42.6	44.4	59.3	52.5	53.9	53.6
50	-5.09	-2.51	-1.64	-1.19	5.78	2.79	1.81	1.32	20.2	23.2	24.1	28.5	43.0	38.9	36.2	37.2
100	-4.97	-2.52	-1.64	-1.19	5.37	2.66	1.73	1.25	5.3	3.8	4.5	6.2	17.8	9.2	8.8	9.4
200	-4.84	-2.48	-1.62	-1.17	5.03	2.56	1.66	1.21	0.4	0.1	0.1	0.2	2.7	0.7	0.4	0.4
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-4.07	-1.93	-1.30	-1.10	9.10	4.94	4.24	3.99	70.3	77.4	80.1	80.6	68.7	76.8	79.8	81.1
30	-4.17	-1.99	-1.35	-1.11	6.82	4.61	4.50	3.20	61.2	69.7	74.8	76.8	60.5	69.0	74.7	77.0
50	-4.25	-1.80	-1.35	-1.06	6.30	3.54	3.30	2.76	49.2	65.5	69.6	70.9	48.5	65.6	69.7	70.5
100	-4.09	-1.86	-1.27	-1.01	5.14	2.96	2.31	2.14	36.4	54.9	62.3	64.3	36.0	55.1	63.1	64.7
200	-4.03	-1.94	-1.31	-1.05	4.65	2.59	2.15	1.86	17.6	37.5	49.8	53.8	17.7	37.4	49.0	54.4
	SPMG estimator								Standard				Robust bootstrapped			
17	0.39	0.00	0.02	0.01	4.75	1.95	1.23	0.87	67.7	83.6	87.6	90.4	90.1	93.2	93.4	94.4
30	0.10	-0.04	-0.02	-0.01	3.56	1.46	0.91	0.64	68.0	83.5	88.2	89.7	89.0	92.5	94.2	94.0
50	0.12	0.07	0.04	0.03	2.65	1.09	0.68	0.49	67.5	83.5	89.1	89.8	90.5	94.3	94.6	94.7
100	0.13	0.00	0.00	0.00	1.84	0.75	0.47	0.34	68.2	84.0	87.8	89.3	89.1	94.6	93.3	93.8
200	0.17	0.02	0.01	0.00	1.30	0.54	0.33	0.24	68.7	84.0	89.1	90.3	89.8	93.3	94.8	94.9

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.22	-3.63	-2.68	-2.18	8.89	5.46	4.47	4.02	74.4	77.4	77.8	80.2
30	-6.40	-3.74	-2.67	-2.19	8.06	5.07	3.86	3.42	61.1	67.6	70.9	73.0
50	-6.43	-3.51	-2.58	-2.12	7.50	4.38	3.38	2.99	46.3	56.3	60.9	66.2
100	-6.27	-3.52	-2.58	-2.11	6.87	3.99	3.08	2.60	26.7	34.5	41.3	49.9
200	-6.22	-3.57	-2.59	-2.10	6.54	3.83	2.84	2.33	15.7	16.7	23.6	30.6
	PDOLS estimator, leads and lags order $p = 4$											
17	-3.30	-2.13	-1.69	-1.46	8.48	4.94	4.18	3.87	84.3	90.1	90.6	91.1
30	-3.50	-2.29	-1.72	-1.48	6.93	4.46	3.42	3.12	81.5	86.5	89.2	90.7
50	-3.54	-2.06	-1.62	-1.42	5.83	3.54	2.85	2.63	78.5	86.0	88.9	89.3
100	-3.46	-2.09	-1.64	-1.42	4.83	2.92	2.42	2.14	73.5	78.8	79.2	84.3
200	-3.40	-2.15	-1.66	-1.41	4.16	2.63	2.07	1.78	61.1	63.1	66.4	68.5
	PDOLS estimator, leads and lags order $p = 8$											
17	-2.42	-1.66	-1.39	-1.24	12.54	5.38	4.37	4.04	70.3	90.0	91.4	92.3
30	-2.24	-1.83	-1.41	-1.25	9.45	4.80	3.50	3.15	72.1	88.2	91.1	91.6
50	-2.52	-1.59	-1.33	-1.20	7.48	3.66	2.88	2.65	72.5	87.4	90.9	91.6
100	-2.38	-1.63	-1.35	-1.22	5.54	2.86	2.34	2.09	74.0	86.2	84.9	88.7
200	-2.39	-1.70	-1.38	-1.20	4.30	2.42	1.92	1.66	69.4	77.9	79.2	80.7
	MGMW estimator, $q = 5$											
17	-6.21	-2.77	-1.77	-1.28	11.07	7.16	5.41	4.94	85.3	91.2	93.1	94.7
30	-6.58	-2.80	-1.75	-1.41	9.77	5.46	4.15	3.62	80.3	89.5	93.2	94.8
50	-6.50	-2.66	-1.78	-1.37	8.64	4.53	3.52	3.10	70.7	86.5	91.9	94.2
100	-6.42	-2.81	-1.80	-1.39	7.47	3.82	2.79	2.31	52.6	77.4	85.4	90.0
200	-6.32	-2.80	-1.79	-1.39	6.91	3.34	2.33	1.98	29.4	57.9	71.2	80.5

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and no CS dependence of errors. See notes to Table S1.

Table S33: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-5.91	-3.05	-2.01	-1.48	8.61	4.15	2.71	2.00	46.6	50.6	52.3	54.9	77.0	77.4	78.0	77.7
30	-5.71	-2.88	-1.87	-1.36	7.35	3.57	2.30	1.69	34.4	37.5	41.1	42.3	66.0	64.9	64.8	65.2
50	-5.45	-2.67	-1.76	-1.29	6.43	3.10	2.02	1.49	22.2	26.4	27.7	29.1	52.4	52.7	51.9	52.3
100	-5.22	-2.64	-1.74	-1.27	5.75	2.86	1.87	1.37	7.6	8.1	8.4	10.0	27.9	23.0	21.5	22.7
200	-5.16	-2.58	-1.69	-1.24	5.42	2.69	1.76	1.29	1.1	0.8	0.9	0.5	7.0	4.5	4.0	4.3
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-4.45	-2.21	-1.49	-1.29	9.19	6.61	4.75	4.62	61.4	70.4	72.1	73.4	68.4	79.1	81.1	81.3
30	-4.33	-2.07	-1.40	-1.12	7.36	5.41	4.97	4.81	57.9	63.9	68.9	71.6	63.9	71.9	76.3	78.5
50	-4.36	-1.88	-1.46	-1.20	6.42	3.72	3.47	3.09	49.0	62.5	64.6	67.4	53.9	70.6	71.0	73.6
100	-4.17	-1.94	-1.39	-1.14	5.46	3.08	2.53	2.32	36.4	53.1	58.4	60.5	42.5	60.1	65.2	66.3
200	-4.13	-1.97	-1.37	-1.12	5.03	2.72	2.04	1.73	20.0	40.1	48.3	52.9	24.8	45.6	53.8	57.6
	SPMG estimator								Standard				Robust bootstrapped			
17	0.37	0.01	0.01	0.00	6.17	2.61	1.63	1.19	57.5	71.7	77.7	78.9	89.8	92.2	93.7	94.2
30	0.11	-0.01	-0.01	0.01	4.38	1.83	1.20	0.89	58.4	73.1	76.7	79.0	89.1	93.1	93.5	93.2
50	0.18	0.09	0.03	0.02	3.35	1.42	0.87	0.65	58.8	72.3	78.8	79.8	89.3	93.3	94.6	94.7
100	0.14	-0.01	-0.01	0.00	2.27	0.98	0.62	0.46	58.9	72.8	77.5	79.4	88.3	93.1	93.3	93.6
200	0.13	0.02	0.01	0.00	1.60	0.68	0.43	0.32	59.6	74.4	77.7	79.7	87.9	92.7	94.4	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-6.63	-3.95	-2.98	-2.50	9.85	6.10	4.94	4.48	67.6	71.3	71.4	73.3
30	-6.59	-3.89	-2.85	-2.37	8.59	5.41	4.15	3.70	58.5	63.8	65.8	68.5
50	-6.62	-3.66	-2.76	-2.31	7.91	4.65	3.66	3.25	45.6	56.1	58.2	64.5
100	-6.32	-3.63	-2.69	-2.24	7.02	4.15	3.22	2.78	29.4	35.9	41.6	47.8
200	-6.36	-3.68	-2.71	-2.21	6.75	3.97	2.98	2.46	16.5	17.2	24.0	31.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-3.59	-2.36	-1.91	-1.71	9.65	5.58	4.59	4.26	76.7	83.4	83.7	85.2
30	-3.61	-2.35	-1.82	-1.59	7.64	4.78	3.68	3.37	75.8	82.3	83.6	86.4
50	-3.60	-2.13	-1.75	-1.56	6.35	3.78	3.10	2.86	73.7	81.2	83.5	85.3
100	-3.44	-2.14	-1.71	-1.52	5.09	3.07	2.54	2.29	70.0	75.3	76.1	78.9
200	-3.51	-2.22	-1.74	-1.49	4.40	2.76	2.18	1.88	58.5	62.3	64.9	68.0
	PDOLS estimator, leads and lags order $p = 8$											
17	-2.79	-1.90	-1.60	-1.47	14.31	6.18	4.81	4.39	62.4	81.7	83.7	85.6
30	-2.54	-1.86	-1.51	-1.35	10.59	5.14	3.78	3.42	66.2	82.8	85.6	87.2
50	-2.60	-1.63	-1.44	-1.34	8.37	3.90	3.15	2.88	67.3	82.6	86.1	87.3
100	-2.39	-1.66	-1.40	-1.30	6.05	3.02	2.46	2.23	68.2	81.0	82.8	84.7
200	-2.49	-1.75	-1.44	-1.27	4.63	2.57	2.03	1.76	65.9	75.0	75.9	78.1
	MGMW estimator, $q = 5$											
17	-6.28	-3.02	-1.88	-1.44	11.95	7.04	5.49	5.23	80.3	86.2	88.7	89.7
30	-6.43	-2.95	-1.83	-1.50	9.83	5.68	4.57	3.83	75.5	85.5	88.7	91.8
50	-6.65	-2.70	-1.84	-1.51	8.82	4.62	3.82	3.25	68.0	83.2	88.8	90.4
100	-6.40	-2.81	-1.83	-1.41	7.58	3.86	2.85	2.36	52.7	74.6	82.2	86.3
200	-6.32	-2.77	-1.82	-1.45	6.92	3.39	2.42	2.02	30.3	59.6	71.2	76.8

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and SAR CS dependence of errors. See notes to Table S1.

Table S34: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.86	-0.94	-0.63	-0.49	4.26	1.98	1.27	0.98	53.3	60.4	64.2	63.2	87.7	91.2	90.0	90.6
30	-1.69	-0.84	-0.60	-0.45	3.28	1.64	1.08	0.80	50.1	56.1	55.4	56.6	86.0	88.3	88.1	87.8
50	-1.70	-0.83	-0.56	-0.41	2.87	1.37	0.90	0.67	43.2	49.3	50.7	52.5	83.3	85.0	85.2	86.3
100	-1.55	-0.83	-0.54	-0.41	2.32	1.19	0.78	0.58	34.6	36.7	38.2	40.6	77.9	78.6	79.0	80.3
200	-1.53	-0.80	-0.53	-0.40	2.10	1.08	0.71	0.52	25.5	26.3	25.9	26.2	71.2	70.8	71.0	70.0
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.02	-0.99	-0.67	-0.74	6.37	3.54	3.39	4.57	59.3	66.6	68.2	67.9	73.6	82.0	83.4	82.0
30	-1.82	-0.88	-0.74	-0.54	4.42	2.78	2.44	2.31	56.0	62.5	63.9	64.9	72.1	78.9	79.2	80.3
50	-1.96	-0.84	-0.65	-0.53	3.97	2.23	2.02	1.95	50.8	61.1	61.5	65.3	67.7	77.4	76.5	76.7
100	-1.80	-0.86	-0.64	-0.53	3.01	1.78	1.48	1.42	45.2	57.0	57.9	60.7	62.3	72.9	72.1	72.8
200	-1.83	-0.84	-0.61	-0.54	2.67	1.65	1.28	1.07	36.2	50.4	55.7	57.2	54.1	65.9	69.2	69.3
	SPMG estimator								Standard				Robust bootstrapped			
17	0.09	0.04	0.00	0.01	4.27	1.70	1.06	0.81	51.2	65.8	69.9	69.2	89.1	93.8	93.5	94.2
30	0.09	0.05	-0.02	-0.01	3.08	1.34	0.83	0.61	51.7	61.7	66.0	67.5	88.9	92.1	93.5	92.9
50	0.00	0.03	0.03	0.02	2.31	1.01	0.64	0.46	50.3	62.5	67.2	69.2	89.4	92.1	93.6	93.9
100	0.09	-0.02	0.00	0.00	1.68	0.70	0.44	0.32	49.4	61.6	66.4	67.8	87.7	92.2	93.6	94.1
200	0.03	0.01	0.01	0.01	1.45	0.50	0.31	0.23	48.4	62.0	66.3	67.1	89.2	92.1	93.3	94.2

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-3.01	-1.81	-1.28	-1.17	5.78	3.49	2.73	2.67	71.8	75.1	74.9	77.6
30	-2.80	-1.65	-1.33	-1.05	4.55	2.88	2.32	2.01	68.5	72.4	73.8	75.3
50	-2.99	-1.58	-1.23	-1.03	4.28	2.43	1.98	1.72	60.6	69.6	71.7	76.0
100	-2.77	-1.63	-1.24	-1.02	3.60	2.16	1.69	1.47	52.3	59.3	61.0	67.4
200	-2.78	-1.64	-1.22	-1.03	3.34	1.97	1.51	1.30	37.6	43.0	49.6	55.2
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.01	-1.24	-0.87	-0.87	6.44	3.43	2.70	2.67	71.1	80.4	79.3	82.6
30	-1.77	-1.11	-0.94	-0.76	4.87	2.79	2.19	1.93	70.7	79.0	79.7	83.6
50	-2.05	-1.03	-0.84	-0.74	4.27	2.23	1.85	1.60	70.7	79.6	82.0	85.1
100	-1.84	-1.10	-0.88	-0.73	3.28	1.85	1.45	1.29	68.5	76.5	78.6	83.4
200	-1.90	-1.14	-0.86	-0.75	2.82	1.57	1.22	1.07	61.4	68.9	73.3	77.0
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.19	-1.01	-0.73	-0.77	10.51	3.89	2.94	2.83	56.7	78.7	79.2	82.6
30	-1.08	-0.88	-0.80	-0.65	7.75	3.17	2.31	2.04	55.8	77.2	79.7	83.0
50	-1.40	-0.79	-0.68	-0.61	6.15	2.44	1.97	1.63	57.3	79.4	82.5	86.9
100	-1.20	-0.86	-0.75	-0.62	4.55	1.93	1.47	1.31	53.8	78.4	83.4	86.5
200	-1.46	-0.91	-0.72	-0.65	3.63	1.56	1.18	1.04	51.5	77.8	80.8	83.8
	MGMW estimator, $q = 5$											
17	-1.80	-0.91	-0.49	-0.48	6.32	3.82	3.21	3.19	79.3	82.6	82.9	84.5
30	-1.42	-0.61	-0.52	-0.40	4.79	2.99	2.52	2.45	80.8	84.0	82.8	85.9
50	-1.63	-0.64	-0.40	-0.31	4.12	2.33	1.98	1.75	78.5	84.3	85.2	88.5
100	-1.32	-0.70	-0.47	-0.30	3.04	1.88	1.41	1.26	77.8	82.0	85.1	88.1
200	-1.42	-0.67	-0.43	-0.36	2.59	1.42	1.10	0.99	71.5	79.9	83.8	85.4

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.05$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S35: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and no CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				RMSE (×100)				95% CI coverage rate (x100)							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-6.04	-2.95	-1.88	-1.36	8.47	3.82	2.43	1.76	47.3	57.2	61.8	63.3	73.3	73.6	73.2	71.5
30	-5.57	-2.82	-1.80	-1.29	6.98	3.37	2.14	1.54	35.2	40.9	45.7	48.0	60.7	57.3	57.8	57.4
50	-5.32	-2.60	-1.70	-1.23	6.13	2.93	1.89	1.38	21.6	27.3	28.4	32.6	47.3	42.4	39.3	41.6
100	-5.23	-2.62	-1.70	-1.22	5.69	2.78	1.80	1.29	6.7	5.6	6.7	8.2	20.9	13.2	11.7	11.8
200	-5.07	-2.56	-1.66	-1.21	5.30	2.65	1.72	1.25	0.7	0.4	0.3	0.3	4.1	1.3	0.9	1.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-8.65	-5.74	-4.46	-4.06	15.81	10.83	12.50	10.48	52.5	61.5	64.9	65.2	52.2	61.4	64.9	65.6
30	-9.22	-5.68	-4.45	-3.88	13.82	9.64	9.21	7.47	41.5	52.5	57.3	60.3	40.7	52.9	57.3	60.3
50	-9.15	-5.24	-4.27	-3.72	11.99	7.62	7.09	6.97	31.0	47.3	50.8	55.3	30.9	46.7	51.2	55.7
100	-9.17	-5.36	-4.20	-3.75	10.74	7.42	5.77	5.37	16.6	34.1	43.0	44.1	17.2	34.2	42.7	44.9
200	-8.90	-5.50	-4.24	-3.72	9.67	6.38	6.33	4.65	5.8	16.1	24.1	28.3	6.4	16.8	25.2	28.8
	SPMG estimator								Standard				Robust bootstrapped			
17	0.43	0.01	0.03	0.01	5.40	2.14	1.34	0.95	66.4	83.7	87.9	90.4	88.9	93.3	93.9	94.5
30	0.16	-0.02	-0.01	0.00	4.02	1.59	1.00	0.71	66.7	83.3	87.8	89.7	89.2	93.0	94.1	94.1
50	0.18	0.09	0.05	0.03	2.89	1.20	0.74	0.53	67.2	83.6	88.8	89.7	90.9	94.4	95.3	94.5
100	0.13	0.00	-0.01	0.00	2.04	0.83	0.52	0.37	67.3	83.5	87.4	89.7	89.1	93.4	93.3	94.4
200	0.19	0.02	0.01	0.01	1.45	0.59	0.36	0.26	66.8	83.6	89.3	90.1	89.3	93.3	94.6	94.5

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				RMSE (×100)				95% CI cov.r. (x100)			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-11.25	-7.66	-6.34	-5.45	14.85	10.58	9.47	8.49	69.9	78.2	79.6	83.3
30	-11.49	-7.81	-6.22	-5.53	13.86	9.83	8.10	7.40	59.4	68.3	71.9	77.0
50	-11.36	-7.52	-6.13	-5.34	12.85	8.76	7.30	6.51	46.2	55.9	59.3	65.7
100	-11.45	-7.58	-6.17	-5.40	12.28	8.34	6.93	6.12	29.6	36.2	39.1	45.2
200	-11.27	-7.59	-6.16	-5.33	11.70	7.97	6.53	5.70	20.7	17.3	21.4	24.4
	PDOLS estimator, leads and lags order $p = 4$											
17	-8.83	-6.38	-5.50	-4.82	14.65	10.24	9.32	8.42	77.3	85.1	86.5	88.9
30	-9.07	-6.60	-5.40	-4.92	13.02	9.35	7.75	7.15	72.3	78.1	81.4	84.7
50	-8.95	-6.30	-5.32	-4.73	11.48	8.03	6.81	6.15	64.8	73.4	74.7	78.1
100	-9.12	-6.37	-5.38	-4.82	10.54	7.44	6.34	5.70	51.8	58.8	58.2	61.1
200	-8.90	-6.39	-5.38	-4.74	9.66	6.93	5.86	5.21	38.9	36.2	38.8	39.3
	PDOLS estimator, leads and lags order $p = 8$											
17	-8.15	-5.96	-5.25	-4.62	18.82	10.86	9.70	8.72	64.5	84.0	86.9	89.0
30	-7.80	-6.23	-5.13	-4.69	15.12	9.80	7.96	7.24	64.8	77.2	82.3	84.4
50	-7.99	-5.85	-5.07	-4.53	12.90	8.13	6.86	6.16	60.3	74.7	75.9	79.4
100	-7.96	-5.94	-5.11	-4.64	10.72	7.36	6.27	5.66	54.1	64.2	63.2	67.0
200	-7.83	-5.95	-5.14	-4.56	9.40	6.68	5.72	5.10	41.4	47.2	46.9	48.7
	MGMW estimator, $q = 5$											
17	-10.78	-6.53	-5.31	-4.40	17.27	12.28	10.33	9.89	82.0	89.4	92.6	94.2
30	-10.93	-6.53	-5.21	-4.51	15.06	10.32	9.14	7.86	73.9	84.3	87.0	89.9
50	-10.80	-6.52	-5.13	-4.38	13.50	9.00	7.51	6.64	63.1	76.9	82.8	85.2
100	-10.94	-6.62	-5.05	-4.35	12.24	7.88	6.30	5.64	41.9	57.9	65.8	70.4
200	-10.73	-6.58	-5.11	-4.42	11.44	7.25	5.75	5.14	19.2	34.2	44.4	51.7

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and no CS dependence of errors. See notes to Table S1.

Table S36: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-6.49	-3.21	-2.10	-1.55	9.66	4.45	2.87	2.13	46.2	53.0	55.9	57.4	77.6	78.9	79.6	78.6
30	-6.05	-2.99	-1.93	-1.40	7.88	3.76	2.41	1.76	35.3	39.7	43.5	45.3	67.2	66.2	67.3	66.3
50	-5.69	-2.76	-1.82	-1.34	6.77	3.23	2.12	1.56	23.3	28.9	30.6	32.0	55.3	55.3	53.8	54.9
100	-5.49	-2.73	-1.80	-1.31	6.09	2.99	1.95	1.42	8.2	9.4	10.1	10.6	31.1	26.0	25.6	25.5
200	-5.40	-2.66	-1.74	-1.27	5.70	2.79	1.81	1.33	1.3	1.1	1.1	1.2	9.1	5.8	5.2	6.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-9.47	-6.32	-4.88	-4.49	16.15	13.09	10.92	9.94	48.7	58.8	61.7	61.6	54.4	63.7	66.7	67.2
30	-9.61	-6.04	-4.81	-4.10	13.54	10.71	10.02	8.56	39.8	49.8	54.5	57.4	43.8	54.2	58.7	60.7
50	-9.28	-5.34	-4.36	-4.09	11.99	8.96	11.95	6.77	31.1	45.1	48.9	51.3	35.5	49.5	52.2	55.1
100	-9.21	-5.54	-4.51	-4.07	10.88	7.45	6.19	5.68	17.2	32.1	38.9	40.4	21.1	36.0	41.9	43.8
200	-9.16	-5.68	-4.51	-4.02	10.23	6.81	5.40	4.99	7.1	17.0	23.7	27.8	7.8	19.0	25.6	30.0
	SPMG estimator								Standard				Robust bootstrapped			
17	0.42	0.01	0.02	0.00	6.90	2.79	1.74	1.28	56.9	72.9	78.3	80.5	90.0	92.3	93.8	94.7
30	0.18	0.02	0.00	0.02	4.78	1.96	1.28	0.94	60.4	75.1	78.0	79.8	89.1	93.3	93.6	93.3
50	0.28	0.12	0.04	0.02	3.64	1.51	0.94	0.68	58.4	74.6	79.4	81.6	89.3	93.7	95.0	95.0
100	0.16	0.00	-0.01	0.00	2.46	1.04	0.66	0.48	60.2	74.3	78.5	80.5	88.4	93.3	94.0	94.0
200	0.14	0.03	0.01	0.00	1.74	0.73	0.45	0.33	60.3	75.6	78.7	81.4	88.0	92.7	94.4	94.4

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-11.77	-8.24	-6.93	-6.11	16.05	11.57	10.29	9.38	65.2	74.3	76.4	79.6
30	-11.90	-8.24	-6.70	-6.07	14.57	10.51	8.77	8.14	56.3	64.7	69.0	72.0
50	-11.69	-7.85	-6.51	-5.75	13.38	9.24	7.83	7.08	45.4	55.5	56.7	61.6
100	-11.55	-7.84	-6.48	-5.77	12.51	8.69	7.33	6.59	31.6	35.0	36.1	40.5
200	-11.54	-7.85	-6.44	-5.63	12.04	8.27	6.84	6.03	21.6	17.4	19.7	23.8
	PDOLS estimator, leads and lags order $p = 4$											
17	-9.22	-6.91	-6.05	-5.43	15.95	11.24	10.17	9.29	72.8	80.8	83.1	86.0
30	-9.44	-6.96	-5.83	-5.42	13.78	10.02	8.40	7.87	67.7	75.0	79.1	81.7
50	-9.14	-6.58	-5.66	-5.11	12.06	8.49	7.31	6.70	64.4	71.7	70.6	75.7
100	-9.17	-6.60	-5.66	-5.16	10.83	7.78	6.73	6.14	52.2	56.4	53.7	56.3
200	-9.14	-6.63	-5.63	-5.02	10.01	7.23	6.15	5.52	37.0	37.2	36.4	38.9
	PDOLS estimator, leads and lags order $p = 8$											
17	-8.44	-6.57	-5.82	-5.24	20.44	11.96	10.63	9.58	59.8	80.0	82.8	86.8
30	-8.35	-6.55	-5.58	-5.22	16.08	10.55	8.62	8.00	60.8	74.9	79.6	82.8
50	-8.24	-6.15	-5.40	-4.91	13.71	8.63	7.38	6.74	57.9	72.3	73.5	77.7
100	-8.15	-6.15	-5.39	-4.99	11.31	7.72	6.67	6.11	52.0	61.9	60.4	62.6
200	-8.05	-6.21	-5.38	-4.83	9.78	7.01	6.01	5.42	40.7	46.8	45.0	47.1
	MGMW estimator, $q = 5$											
17	-10.49	-6.81	-5.50	-4.80	17.94	12.45	11.33	10.25	78.8	85.7	89.7	91.4
30	-11.14	-7.01	-5.44	-4.87	15.23	10.84	9.28	8.36	71.9	81.4	85.4	87.8
50	-11.09	-6.57	-5.39	-4.64	13.85	9.08	7.97	7.05	60.2	74.3	80.3	82.9
100	-10.97	-6.67	-5.31	-4.52	12.35	8.01	6.66	5.88	42.4	56.2	63.7	66.7
200	-10.66	-6.63	-5.18	-4.59	11.43	7.31	5.89	5.30	19.1	34.6	44.8	47.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and SAR CS dependence of errors. See notes to Table S1.

Table S37: MC results for the estimation of LR coefficient $\theta_0 = 1$ in experiments with Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and factor+SAR CS dependence of errors

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.98	-0.98	-0.65	-0.51	4.54	2.10	1.34	1.02	54.9	62.0	65.8	66.2	87.8	91.8	90.6	91.2
30	-1.74	-0.87	-0.62	-0.45	3.40	1.72	1.12	0.83	50.4	57.8	58.0	59.7	87.6	87.9	88.2	88.1
50	-1.77	-0.85	-0.58	-0.42	3.01	1.43	0.93	0.70	45.2	50.7	52.9	55.0	83.9	86.1	86.4	87.3
100	-1.62	-0.85	-0.56	-0.41	2.44	1.23	0.80	0.60	35.9	38.7	39.7	43.4	78.3	78.8	79.7	81.2
200	-1.59	-0.82	-0.55	-0.41	2.20	1.11	0.73	0.54	26.9	27.6	27.5	28.8	71.8	71.5	71.8	71.9
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-4.15	-2.82	-2.20	-2.26	10.12	7.23	6.47	7.65	50.8	58.4	61.6	61.4	63.1	69.0	71.1	70.9
30	-4.04	-2.69	-2.21	-1.94	8.42	6.28	5.31	4.90	49.0	54.2	58.8	60.9	60.5	65.9	67.6	69.6
50	-4.19	-2.53	-2.18	-1.83	7.14	5.13	4.76	4.28	41.0	52.6	55.8	59.5	52.0	62.8	63.6	66.3
100	-4.04	-2.47	-2.16	-1.99	6.13	4.56	3.78	4.20	33.2	45.5	48.1	49.9	43.1	55.2	55.7	57.7
200	-4.04	-2.51	-2.11	-1.85	5.28	3.69	3.10	2.82	22.9	35.6	38.0	41.6	32.5	45.2	47.5	49.4
	SPMG estimator								Standard				Robust bootstrapped			
17	-0.02	0.04	0.01	0.01	5.92	1.83	1.12	0.85	53.8	67.6	71.6	72.3	88.5	94.0	93.8	95.0
30	0.12	0.05	-0.02	0.00	3.23	1.40	0.88	0.64	53.0	64.3	67.5	69.8	89.4	92.3	93.6	93.2
50	0.00	0.05	0.03	0.02	2.43	1.05	0.66	0.48	53.0	64.0	68.6	71.4	89.5	92.4	93.5	94.3
100	0.10	-0.01	0.00	0.01	1.85	0.74	0.46	0.34	51.2	63.2	69.9	71.4	87.5	92.4	94.0	94.1
200	0.05	0.01	0.01	0.01	1.24	0.53	0.33	0.24	50.1	63.0	67.5	70.3	88.7	92.4	93.6	94.7

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.16	-3.73	-2.97	-2.74	8.72	6.34	5.42	5.17	75.4	83.0	82.7	86.1
30	-4.97	-3.61	-2.95	-2.59	7.31	5.37	4.64	4.15	74.5	79.3	81.2	84.6
50	-5.14	-3.39	-2.88	-2.51	6.81	4.64	3.98	3.62	65.2	75.0	74.2	78.7
100	-4.97	-3.33	-2.90	-2.54	6.03	4.10	3.62	3.24	56.4	62.3	62.7	64.4
200	-4.89	-3.35	-2.82	-2.51	5.65	3.85	3.29	2.95	44.8	44.2	45.0	50.0
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.42	-3.34	-2.69	-2.53	9.70	6.60	5.61	5.33	75.9	84.6	85.5	88.4
30	-4.27	-3.27	-2.69	-2.37	7.91	5.51	4.68	4.16	73.3	83.0	84.8	88.2
50	-4.52	-3.02	-2.60	-2.30	7.04	4.59	3.93	3.57	70.9	81.5	82.0	84.5
100	-4.38	-2.96	-2.65	-2.33	5.99	3.94	3.50	3.14	63.7	74.5	75.1	74.1
200	-4.35	-3.01	-2.57	-2.30	5.48	3.64	3.11	2.79	56.9	62.1	60.7	63.9
	PDOLS estimator, leads and lags order $p = 8$											
17	-3.65	-3.29	-2.64	-2.51	13.62	7.41	6.10	5.69	58.2	83.0	85.1	87.8
30	-3.62	-3.23	-2.65	-2.32	10.63	6.17	5.00	4.38	56.5	81.0	84.2	88.0
50	-3.93	-2.92	-2.54	-2.23	8.74	4.95	4.15	3.69	57.5	80.7	82.7	86.1
100	-3.88	-2.84	-2.62	-2.29	7.12	4.14	3.64	3.22	52.9	75.5	77.2	77.4
200	-4.04	-2.91	-2.53	-2.27	6.24	3.77	3.18	2.83	48.2	66.7	67.1	69.0
	MGMW estimator, $q = 5$											
17	-2.65	-1.92	-1.39	-1.26	8.97	6.69	5.72	5.45	83.9	87.8	89.2	92.3
30	-2.04	-1.62	-1.23	-1.11	7.06	5.18	4.74	4.40	83.5	87.6	89.3	90.1
50	-2.41	-1.48	-1.21	-0.95	6.02	4.08	3.66	3.36	81.3	84.1	87.6	88.5
100	-1.95	-1.40	-1.21	-0.95	4.59	3.22	2.87	2.55	78.5	82.7	83.8	85.9
200	-2.04	-1.32	-1.09	-1.01	4.08	2.67	2.30	2.17	72.3	78.1	79.6	80.3

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$, and factor+SAR CS dependence of errors. See notes to Table S1.

Table S38: Coverage rate of robust asymptotic 95 percent confidence intervals of SPMG estimator of LR coefficient θ

A: Experiments 1-18 (summarized in Table S1)

$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	Experiment 1				**Experiment 2**				**Experiment 3**			
17	58.30	78.35	85.60	88.85	61.95	80.85	86.60	89.25	61.95	81.40	87.15	90.40
30	57.80	80.00	86.05	88.25	61.20	80.80	86.20	88.80	63.15	81.15	88.95	90.90
50	58.10	78.80	86.35	87.45	59.10	80.35	87.55	87.85	63.50	84.90	88.45	90.80
100	59.80	80.15	86.20	89.30	60.35	81.70	87.55	90.40	64.70	84.50	91.15	92.50
200	59.90	80.25	87.00	88.50	60.90	81.60	87.25	89.25	68.15	86.20	92.75	94.35
	Experiment 4				**Experiment 5**				**Experiment 6**			
17	58.90	78.80	84.70	88.95	61.00	81.50	86.20	89.25	62.60	81.40	87.30	89.90
30	57.75	80.00	86.00	88.10	61.10	80.20	86.00	88.55	62.90	81.00	88.55	90.45
50	58.35	78.35	87.10	87.40	59.55	80.55	87.80	88.30	63.25	84.70	88.50	90.80
100	58.75	80.55	85.55	89.50	60.20	82.40	87.60	90.00	63.15	84.60	90.70	92.35
200	60.20	79.70	87.05	88.55	61.80	81.10	87.10	88.95	68.20	85.65	92.75	94.25
	Experiment 7				**Experiment 8**				**Experiment 9**			
17	57.50	77.85	84.85	88.55	59.50	81.85	86.35	89.50	61.50	82.00	87.30	90.10
30	55.80	79.80	86.35	87.10	60.25	80.05	86.45	88.60	60.90	81.75	88.70	90.80
50	57.00	78.65	86.00	88.40	59.40	79.90	86.85	88.00	63.50	83.75	88.30	90.50
100	57.65	80.05	86.45	89.45	60.00	80.85	87.90	90.15	63.25	84.10	90.75	92.05
200	59.15	80.00	86.35	88.35	60.85	80.55	85.70	87.95	66.50	85.90	92.05	93.90
	Experiment 10				**Experiment 11**				**Experiment 12**			
17	66.80	83.45	88.10	89.55	68.40	83.35	89.25	90.45	69.75	84.10	89.10	90.95
30	65.70	83.40	87.60	89.65	65.15	83.40	88.50	90.15	68.90	84.65	89.15	91.30
50	65.65	82.15	88.95	90.15	66.60	81.65	87.15	90.00	68.15	83.50	89.45	90.55
100	64.95	82.85	88.10	91.60	66.90	82.35	87.80	90.90	70.10	84.60	89.25	90.35
200	66.05	81.55	87.85	90.05	66.80	82.55	87.65	89.55	68.40	83.65	89.70	90.35
	Experiment 13				**Experiment 14**				**Experiment 15**			
17	65.90	83.25	87.90	89.20	68.45	83.30	89.50	90.45	69.20	84.40	89.25	91.50
30	65.65	83.35	87.75	90.25	66.20	83.55	88.50	90.60	69.70	84.75	90.05	91.00
50	64.70	82.15	89.10	90.15	65.65	81.85	87.40	90.00	68.10	84.00	89.30	90.85
100	66.25	82.30	87.95	90.80	67.55	82.60	88.05	91.10	70.25	84.45	89.25	90.45
200	65.90	82.35	88.60	90.20	66.95	82.45	88.00	89.90	68.90	84.05	90.00	90.75
	Experiment 16				**Experiment 17**				**Experiment 18**			
17	67.40	82.40	87.45	89.15	66.45	82.65	89.70	90.90	69.35	84.55	89.40	91.65
30	64.45	82.85	87.25	89.30	65.45	83.25	88.50	90.65	70.05	84.80	89.70	91.05
50	63.95	82.75	88.00	89.35	65.10	82.80	88.05	90.10	67.30	83.95	89.45	91.40
100	64.95	81.65	87.20	91.00	65.90	81.35	88.25	91.20	69.20	84.60	89.65	90.80
200	64.85	80.05	87.45	89.65	65.25	82.00	86.65	89.45	70.15	84.30	89.40	91.40

Notes: This table reports coverage rate MC findings for 95 percent confidence intervals computed based on the robust variance matrix estimator (S.26), using asymptotic critical values. See Table S1 fof the list of individual experiments.

Table S38 (Continued): Coverage rate of robust asymptotic 95 percent confidence intervals of SPMG estimator of LR coefficient θ

B: Experiments 19-36 (summarized in Table S1)

$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	Experiment 19				**Experiment 20**				**Experiment 21**			
17	61.35	79.80	86.45	88.00	61.05	80.10	86.85	89.35	63.00	80.70	86.90	89.60
30	61.80	79.50	85.25	88.65	64.15	81.45	86.65	88.85	64.00	83.50	89.20	90.70
50	60.55	80.70	86.90	90.00	61.00	80.90	87.30	89.95	64.30	83.20	90.45	92.10
100	57.90	80.45	85.85	87.90	59.60	79.45	86.80	89.40	65.85	86.50	90.65	92.60
200	61.10	80.50	87.60	89.20	62.75	81.50	88.00	89.70	68.60	87.65	92.05	94.65
	Experiment 22				**Experiment 23**				**Experiment 24**			
17	59.85	79.85	86.60	87.75	61.20	80.60	87.25	89.35	62.20	80.95	86.80	89.50
30	60.90	79.35	85.55	88.25	62.90	81.10	86.55	89.20	64.55	83.65	89.25	90.55
50	60.65	79.35	87.35	90.30	60.65	80.55	87.35	89.80	64.45	82.95	90.45	91.75
100	57.85	80.15	86.20	87.60	60.70	80.25	87.05	89.10	65.10	86.10	90.50	92.35
200	60.50	80.10	87.25	89.60	61.80	81.90	87.45	89.60	67.65	87.25	92.20	94.75
	Experiment 25				**Experiment 26**				**Experiment 27**			
17	60.15	78.60	85.95	87.55	59.10	79.40	86.65	88.65	60.90	80.60	86.75	89.65
30	58.40	78.90	85.10	88.30	62.60	81.70	86.55	88.55	63.85	82.85	89.25	90.10
50	58.00	79.15	87.15	90.55	60.80	80.20	87.65	89.75	62.70	83.30	90.30	92.20
100	56.60	78.90	86.75	87.70	59.90	80.30	86.05	88.75	63.55	85.15	90.65	92.55
200	60.75	80.05	87.35	89.30	60.40	81.45	87.40	89.80	65.85	86.65	91.85	94.25
	Experiment 28				**Experiment 29**				**Experiment 30**			
17	66.75	83.00	88.00	90.70	69.25	83.05	88.65	90.05	70.60	86.30	89.65	90.75
30	67.30	81.85	87.35	89.50	67.20	84.60	87.70	89.05	70.75	84.05	88.95	89.15
50	66.35	83.40	87.90	89.25	67.35	83.30	88.90	90.35	70.45	84.30	88.45	90.45
100	67.35	84.30	87.25	89.35	66.60	83.90	87.60	90.30	70.05	85.05	90.00	91.35
200	67.00	83.05	88.65	90.10	67.15	83.60	88.70	90.65	71.80	84.60	89.05	91.65
	Experiment 31				**Experiment 32**				**Experiment 33**			
17	66.55	82.85	87.40	90.25	67.95	82.90	88.60	90.00	69.80	86.05	89.85	91.40
30	66.45	82.55	87.85	89.30	66.95	83.95	87.10	89.35	71.05	84.20	89.15	89.25
50	65.95	83.35	88.15	89.40	67.40	83.90	88.40	90.45	70.70	83.90	88.85	90.75
100	67.40	83.15	87.05	89.10	66.45	84.20	87.60	89.65	69.35	85.00	89.70	91.35
200	67.75	83.25	89.10	90.10	67.45	83.65	88.85	90.00	72.30	85.00	89.35	91.95
	Experiment 34				**Experiment 35**				**Experiment 36**			
17	65.75	83.80	87.25	90.50	66.50	83.15	87.65	90.00	69.45	85.80	89.55	91.25
30	66.20	82.50	87.10	89.05	67.45	83.50	87.40	89.30	71.20	83.90	88.60	89.35
50	66.45	83.75	88.40	89.10	66.85	84.40	88.60	91.10	70.50	84.45	88.80	90.85
100	66.10	82.25	87.00	88.95	66.50	83.85	88.45	89.75	69.05	84.95	89.70	91.50
200	65.55	82.70	88.55	89.65	66.65	83.90	87.95	90.30	71.65	84.45	89.20	92.00

Notes: This table reports coverage rate MC findings for 95 percent confidence intervals computed based on the robust variance matrix estimator (S.26), using asymptotic critical values. See Table S1 fof the list of individual experiments.

S.3.4 Additional MC Experiments featuring time series heteroskedasticity

Monte Carlo experiments in the previous section allowed for cross sectional heteroskedasticity of errors, but ruled out time series heteroskedasticity. This section presents additional MC findings that allows for error heteroskedasticity both over time and across cross-sectional units. Specifically, we use GARCH(1,1) to generate the errors. For the sake of brevity, we focus on the following two experiments (out of 36): (a) the baseline experiment with Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0$, no CS dependence of errors, and (b) the most demanding experiment featuring non-Gaussian errors, LR causality $x \leftrightarrow y$, $\pi = 0.2$ and factor+SAR CS error dependence. In the baseline experiment (a) we generate $\varepsilon_{yit}, \varepsilon_{xit}$, in (S.39) as $\varepsilon_{yit} \sim IIDN\left(0, \sigma^2_{\varepsilon yt}\right)$, $\varepsilon_{xit} \sim IIDN\left(0, \sigma^2_{\varepsilon xt}\right)$, where

$$\sigma^2_{\varepsilon yt} = (1 - \alpha_{1\varepsilon y} - \alpha_{2\varepsilon y}) + \alpha_{1\varepsilon y}\varepsilon^2_{yi,t-1} + \alpha_{2\varepsilon y}\sigma^2_{\varepsilon y,t-1} \tag{S.51}$$

$$\sigma^2_{\varepsilon xt} = (1 - \alpha_{1\varepsilon x} - \alpha_{2\varepsilon x}) + \alpha_{1\varepsilon x}\varepsilon^2_{xi,t-1} + \alpha_{2\varepsilon x}\sigma^2_{\varepsilon x,t-1}, \tag{S.52}$$

and we set $\alpha_{1\varepsilon x} = \alpha_{1\varepsilon y} = 0.2$ and $\alpha_{2\varepsilon x} = \alpha_{2\varepsilon y} = 0.75$. In the experiment (b), we generate $\varepsilon_{yit}, \varepsilon_{xit}$ according to the spatial model (S.41) with $v_{yit} \sim \sigma_{vyt}2^{-1/2}\left[IIDN\chi^2(1) - 1\right]$, and $v_{xit} \sim \sigma_{vxt}2^{-1/2}\left[IIDN\chi^2(1) - 1\right]$, where σ^2_{vyt} and σ^2_{vxt} are generated similarly to (S.51)-(S.52), namely

$$\begin{aligned}\sigma^2_{vyt} &= (1 - \alpha_{1vy} - \alpha_{2vy}) + \alpha_{1vy}v^2_{yi,t-1} + \alpha_{2vy}\sigma^2_{vy,t-1}\\ \sigma^2_{vxt} &= (1 - \alpha_{1vx} - \alpha_{2vx}) + \alpha_{1vx}v^2_{xi,t-1} + \alpha_{2vx}\sigma^2_{vx,t-1},\end{aligned}$$

with $\alpha_{1vx} = \alpha_{1vy} = 0.2$ and $\alpha_{2vx} = \alpha_{2vy} = 0.75$. In addition, the common factor, f_t, is also generated as GARCH(1,1), $f_t \sim IIDN\left(0, \sigma^2_{ft}\right)$ with

$$\sigma^2_{ft} = (1 - \alpha_{1f} - \alpha_{2f}) + \alpha_{1f}f^2_{t-1} + \alpha_{2f}\sigma^2_{f,t-1},$$

$\alpha_{1f} = 0.2$ and $\alpha_{2f} = 0.75$.

Table S39 presents MC findings for the baseline experiment (a) with GARCH effects, and Table S40 presents results for the more demanding experiment (b). Comparing these tables with Tables S2 and S37, we see that introduction of GARCH effects in the error processes do not substantially affect the coverage rates.

Table S39: MC results for the estimation of LR coefficient $\theta_0 = 1$ in the baseline experiment (a) with GARCH effects
(Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0$ and no CS dependence of errors)

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-1.28	-0.34	-0.14	-0.09	5.56	2.25	1.40	1.00	70.5	84.8	88.6	90.7	90.2	93.4	94.1	94.9
30	-1.23	-0.35	-0.13	-0.08	4.06	1.68	1.04	0.74	69.9	84.5	88.0	90.2	89.4	93.2	93.9	94.0
50	-1.02	-0.20	-0.08	-0.05	3.19	1.25	0.75	0.55	69.0	83.9	89.4	90.6	89.3	94.1	94.7	94.1
100	-0.94	-0.25	-0.12	-0.06	2.30	0.91	0.56	0.40	66.9	82.5	87.6	90.6	87.9	92.9	93.8	94.9
200	-0.96	-0.24	-0.10	-0.06	1.74	0.66	0.39	0.28	62.4	81.6	88.4	89.8	87.2	92.7	94.8	94.5
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-2.40	-0.71	-0.33	-0.17	5.82	2.58	1.62	1.16	77.3	85.6	89.1	91.1	74.9	85.1	88.1	90.9
30	-2.37	-0.69	-0.30	-0.16	4.69	1.92	1.20	0.84	72.5	86.0	89.2	92.0	69.6	85.2	88.9	91.2
50	-2.32	-0.59	-0.25	-0.13	3.80	1.50	0.92	0.65	68.7	85.4	89.7	91.6	67.3	84.0	89.0	91.2
100	-2.27	-0.62	-0.27	-0.15	3.14	1.15	0.68	0.47	61.1	81.0	88.1	91.4	60.0	80.4	87.3	91.0
200	-2.20	-0.60	-0.25	-0.14	2.63	0.92	0.50	0.35	45.2	75.8	85.9	89.2	43.9	74.7	85.2	88.8
	SPMG estimator								Standard				Robust bootstrapped			
17	0.00	-0.06	-0.01	-0.02	6.69	2.41	1.48	1.02	60.7	81.2	85.3	88.5	86.3	92.5	93.4	94.4
30	-0.07	-0.10	-0.03	-0.02	4.71	1.78	1.07	0.76	60.2	80.9	87.0	88.3	86.3	92.3	93.3	94.2
50	0.01	0.04	0.02	0.01	3.60	1.32	0.79	0.58	61.1	80.0	86.3	88.2	86.7	93.8	94.5	94.0
100	0.08	-0.02	-0.02	-0.01	2.51	0.95	0.57	0.40	58.9	79.3	86.2	89.1	85.6	92.1	93.7	94.5
200	0.04	-0.01	-0.01	0.00	1.69	0.66	0.39	0.28	62.0	80.6	86.7	89.0	87.1	92.3	94.1	94.1

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n \backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-4.78	-2.42	-1.60	-1.16	6.80	3.50	2.27	1.66	76.8	78.3	80.3	80.8
30	-4.74	-2.41	-1.55	-1.14	6.01	3.01	1.97	1.43	67.5	69.4	72.2	72.6
50	-4.75	-2.33	-1.50	-1.10	5.55	2.71	1.76	1.29	54.7	56.7	58.4	59.8
100	-4.66	-2.33	-1.52	-1.12	5.08	2.53	1.65	1.21	32.6	31.6	32.2	32.2
200	-4.63	-2.31	-1.49	-1.09	4.83	2.42	1.56	1.14	9.0	8.4	9.1	9.4
	PDOLS estimator, leads and lags order $p = 4$											
17	-2.61	-1.20	-0.78	-0.56	6.61	2.91	1.79	1.30	83.7	88.4	89.3	91.0
30	-2.52	-1.20	-0.74	-0.54	5.22	2.23	1.42	1.01	82.7	86.8	87.8	88.6
50	-2.55	-1.14	-0.71	-0.51	4.37	1.85	1.16	0.83	78.8	83.8	84.9	85.9
100	-2.46	-1.15	-0.73	-0.53	3.48	1.55	0.97	0.70	73.4	76.2	76.2	77.6
200	-2.48	-1.13	-0.71	-0.51	2.99	1.36	0.85	0.61	60.6	59.7	61.3	61.2
	PDOLS estimator, leads and lags order $p = 8$											
17	-1.30	-0.54	-0.36	-0.25	11.06	3.29	1.82	1.27	71.1	88.3	90.6	92.6
30	-1.25	-0.56	-0.33	-0.23	8.04	2.34	1.37	0.95	72.6	90.2	91.8	92.5
50	-1.41	-0.51	-0.30	-0.20	6.21	1.83	1.07	0.74	72.4	90.0	90.8	92.2
100	-1.32	-0.53	-0.32	-0.23	4.41	1.36	0.79	0.54	74.8	87.8	89.3	91.2
200	-1.41	-0.50	-0.30	-0.21	3.25	1.01	0.59	0.41	73.4	85.8	87.7	88.9
	MGMW estimator, $q = 5$											
17	-5.70	-1.87	-0.92	-0.54	10.92	5.46	3.44	2.65	87.6	91.7	92.9	92.2
30	-5.69	-1.96	-0.96	-0.58	9.10	4.27	2.82	2.00	83.2	90.1	92.0	93.6
50	-5.57	-1.82	-0.91	-0.56	7.68	3.45	2.15	1.57	79.7	88.6	91.2	93.5
100	-5.51	-1.90	-0.95	-0.56	6.77	2.81	1.70	1.16	66.1	83.4	88.6	91.0
200	-5.36	-1.91	-0.88	-0.54	6.02	2.42	1.31	0.92	46.3	72.0	84.9	88.2

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0$, and no cross section dependence of errors. Errors are heteroskedastic across both, time and cross section, dimensions. See Section S.3.4 for details of the data generating process. Description of the PMG, 2-step Breitung, SPMG, and MGMW estimators, and the description of bootstrapping procedures are provided in Sections S.2.1-S.2.3 PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). The number of Monte Carlo replications is $R_{MC} = 2000$. Bootstrapped critical values are computed in each of the Monte Carlo replication as described in Sections S.2.1-S.2.3, based on $R_b = 2000$ bootstrap replications.

Table S40: MC results for the estimation of LR coefficient $\theta_0 = 1$ in the baseline experiment (b) with with GARCH effects

(Non-Gaussian errors, $x \leftrightarrow y$, $\pi = 0.2$ and factor+SAR CS dependence of errors)

A: PMG, SPMG estimators using conventional and bootstrapped confidence intervals, and 2-step Breitung estimator using 2S-OLS and 2S-robust confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI coverage rate (x100)**							
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PMG estimator								Standard				Robust bootstrapped			
17	-2.01	-0.96	-0.67	-0.49	4.49	2.05	1.39	1.00	53.1	61.9	63.0	66.0	87.4	89.3	89.5	90.9
30	-1.71	-0.83	-0.57	-0.43	3.43	1.62	1.05	0.79	50.1	58.0	59.9	61.4	85.8	88.5	89.2	88.8
50	-1.56	-0.82	-0.54	-0.41	2.91	1.38	0.89	0.68	45.7	49.5	55.4	54.8	83.2	84.7	87.0	86.7
100	-1.57	-0.77	-0.51	-0.40	2.41	1.14	0.76	0.57	35.7	40.2	42.7	43.0	76.6	81.3	81.5	80.8
200	-1.53	-0.77	-0.53	-0.39	2.12	1.06	0.70	0.51	25.7	29.7	28.2	28.8	71.1	73.4	73.0	74.1
	2-step Breitung's estimator								2S-OLS				2S-robust			
17	-4.39	-2.73	-2.25	-2.15	10.72	8.39	6.59	6.90	52.3	57.1	60.8	61.6	62.7	68.9	70.4	70.1
30	-4.13	-2.47	-1.97	-2.06	8.61	7.97	5.38	5.27	48.7	57.2	59.5	59.3	60.8	67.4	68.3	67.4
50	-3.82	-2.44	-2.04	-1.92	6.57	4.83	4.44	4.16	44.0	50.6	56.0	57.0	56.3	61.6	65.0	65.2
100	-3.96	-2.70	-2.01	-1.96	6.05	7.33	3.61	3.46	34.2	46.1	52.5	50.3	43.5	56.0	60.8	59.1
200	-4.04	-2.49	-2.01	-1.88	5.33	3.52	3.81	3.31	22.1	36.1	38.8	42.2	31.0	45.9	47.0	49.7
	SPMG estimator								Standard				Robust bootstrapped			
17	0.02	0.02	-0.02	0.00	5.12	1.83	1.16	0.82	51.5	64.5	68.8	73.6	88.5	91.9	93.4	94.7
30	-0.04	0.04	-0.01	0.00	3.23	1.35	0.83	0.62	52.7	63.3	69.5	68.7	88.1	92.4	94.1	93.9
50	0.07	-0.02	0.01	0.00	2.62	1.01	0.63	0.47	48.9	62.9	70.0	70.3	86.8	92.2	94.3	94.1
100	-0.05	0.02	0.01	0.01	1.76	0.70	0.44	0.33	48.8	64.2	68.6	68.7	88.2	94.0	94.6	94.3
200	0.01	-0.01	0.00	0.00	1.24	0.51	0.32	0.23	48.4	62.3	65.7	67.1	88.3	92.2	94.8	94.6

B: PDOLS and MGMW estimators using standard asymptotic confidence intervals

	Bias (×100)				**RMSE (×100)**				**95% CI cov.r. (x100)**			
$n\backslash T$	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**	**50**	**100**	**150**	**200**
	PDOLS estimator, leads and lags order $p = 1$											
17	-5.17	-3.51	-3.08	-2.76	8.74	6.41	5.79	5.25	75.2	81.0	81.7	85.1
30	-4.97	-3.41	-2.76	-2.62	7.48	5.34	4.34	4.30	71.1	80.1	82.1	84.0
50	-4.83	-3.33	-2.75	-2.54	6.60	4.56	3.93	3.66	67.7	75.2	74.4	79.3
100	-4.92	-3.42	-2.73	-2.55	6.09	4.33	3.54	3.24	56.5	61.9	65.6	67.6
200	-4.96	-3.35	-2.86	-2.49	5.77	3.92	3.35	2.95	43.3	47.3	45.4	48.5
	PDOLS estimator, leads and lags order $p = 4$											
17	-4.51	-3.11	-2.80	-2.52	9.81	6.83	6.00	5.37	75.1	83.6	84.9	87.2
30	-4.29	-3.02	-2.46	-2.41	8.12	5.48	4.36	4.33	74.1	83.4	86.4	87.3
50	-4.28	-2.99	-2.47	-2.32	6.96	4.56	3.87	3.62	71.5	81.4	81.7	85.3
100	-4.30	-3.08	-2.46	-2.33	6.06	4.23	3.42	3.13	63.7	73.7	77.2	77.5
200	-4.35	-3.01	-2.60	-2.28	5.55	3.70	3.17	2.80	55.3	62.6	61.6	63.5
	PDOLS estimator, leads and lags order $p = 8$											
17	-4.47	-3.01	-2.76	-2.48	13.97	7.79	6.50	5.70	58.2	80.4	83.8	86.9
30	-3.58	-2.89	-2.39	-2.37	10.84	6.12	4.67	4.56	57.2	81.9	86.4	87.3
50	-3.91	-2.94	-2.41	-2.28	8.96	5.01	4.10	3.77	56.3	80.9	83.7	85.8
100	-3.91	-3.01	-2.40	-2.27	7.40	4.55	3.54	3.20	52.0	75.5	79.6	81.4
200	-4.00	-2.93	-2.55	-2.23	6.37	3.84	3.24	2.84	47.6	67.1	66.7	69.9
	MGMW estimator, $q = 5$											
17	-2.51	-1.62	-1.28	-1.08	9.07	6.73	5.78	5.74	84.1	87.1	89.7	92.5
30	-1.94	-1.38	-1.13	-0.95	6.83	5.26	4.51	4.24	84.5	87.6	90.4	91.6
50	-1.86	-1.27	-1.00	-1.02	5.84	4.02	3.69	3.29	83.7	87.8	87.4	89.9
100	-1.92	-1.28	-0.95	-0.93	4.57	3.36	2.79	2.55	78.1	81.9	86.0	86.4
200	-1.87	-1.18	-1.05	-0.89	3.82	2.62	2.26	2.05	72.4	78.8	81.3	81.9

Notes: This table reports findings for the estimation of long run coefficient $\theta_0 = 1$ in experiments featuring Gaussian errors, LR causality $x \rightarrow y$, $\pi = 0$, and no cross section dependence of errors. Errors are heteroskedastic across both, time and cross section, dimensions. See Section S.3.4 for details of the data generating process. Description of the PMG, 2-step Breitung, SPMG, and MGMW estimators, and the description of bootstrapping procedures are provided in Sections S.2.1-S.2.3 PDOLS is the panel dynamic OLS estimator by Mark and Sul (2003). The number of Monte Carlo replications is $R_{MC} = 2000$. Bootstrapped critical values are computed in each of the Monte Carlo replication as described in Sections S.2.1-S.2.3, based on $R_b = 2000$ bootstrap replications.

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