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Sudden Stops and Optimal Foreign Exchange Intervention*

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Abstract

This paper shows how foreign exchange intervention can be used to avoid a sudden stop in capital flows in a small open emerging market economy. The model is based around the concept of an under-borrowing equilibrium defined by Schmitt-Grohe and Uribe (2020). With a low elasticity of substitution between traded and non-traded goods, real exchange rate depreciation may generate a precipitous drop in aggregate demand and a tightening of borrowing constraints, leading to an equilibrium with an inefficiently low level of borrowing. The central bank can preempt this deleveraging cycle through foreign exchange intervention. Intervention is effective due to frictions in private international financial intermediation. Reserve accumulation has ex ante benefits by reducing the risk of a sudden stop, while intervention has ex-post benefits by limiting inefficient deleveraging. But intervention itself faces constraints. When the central bank's stock of reserves is low, even foreign exchange intervention cannot prevent a sudden stop.

Keywords: Central bank, sudden stops, foreign exchange reserves, capital controls

JEL: E50, E30, F40

* The views presented here are those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

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1 Introduction

Emerging economies are prone to 'sudden stops'. As described by Calvo et al. (2008), Bianchi and Mendoza (2020) and others, in a sudden stop there is a sharp forced reversal in net capital inflows, leading to a fall in net external debt, a fall in imports, and a depreciation in the real exchange rate. The typical experience of sudden stops involves a tightening of borrowing constraints, leading to a fall in the real exchange rate which further constrains borrowing, generating a downward spiral in net external debt, consumption and a large fall in welfare.

Neumeyer and Perri (2005) argue that changes in external borrowing costs are one of the main factors driving emerging market business cycles. They argue that increases in interest rates cause a fall in GDP and lead to net capital outflows (the current account) that are strongly counter-cyclical. Uribe and Yue (2006) expand on this and argue that an increase in the U.S. interest rate has both a direct effect on emerging market business cycles as well as an indirect effect by widening the spread between a country's borrowing cost and the U.S. interest rate. In the wake of the financial crisis in 2008 and the subsequent recovery a new literature has emerged focusing on the "Global Financial Cycle". In this view, global push factors, including advanced economy monetary policy and global risk aversion, drive swings in capital inflows and outflows into many emerging markets (see for instance Bruno and Shin, 2015b and Miranda-Agrippino and Rey, 2020).[1]

This paper presents a framework to study how central bank foreign exchange intervention can be used to prevent a sudden stop in a small open economy that is subject to exogenous shocks in its cost of borrowing. The paper follows the model of an underborrowing equilibrium from Schmitt-Grohe and Uribe (2020). Schmitt-Grohe and Uribe show how under certain parameterizations, and when external debt passes a high enough threshold, multiple equilibria are possible in the standard small open economy model of Bianchi (2011). In one equilibrium, consumption and external debt remains high, sustaining a high value of the real exchange

[1]See Forbes and Warnock (2012), Fratzscher (2012), Rey (2015), Ghosh et al. (2014), Bruno and Shin (2015a), Rey (2016), Eichengreen and Gupta (2016), Fratzscher et al. (2018), Rogers et al. (2018), Jorda et al. (2019), Davis et al. (2020b).

rate, and this ensures that value of collateral is high enough so that the borrowing constraint is slack. But in other equilibria, a low consumption level (equivalently a reduction in borrowing) leads to a real depreciation, a falling value of collateral, and a binding borrowing constraint.

While Schmitt-Grohe and Uribe (2020) show how borrowing constraints that depend on the real exchange rate can give rise to multiple welfare ranked equilibria in an emerging economy, there is an additional message of their paper that emphasizes the instability in the current account of such economies. Even if agents beliefs coalesce around the 'best' equilibrium, the presence of pecuniary externalities and non-linearity in the tightness of borrowing constraints can lead to sharp collapses in borrowing capacity in response to small external shocks to the economy. In our paper, we show that the 'best' stable competitive equilibrium may be associated with precipitous falls in consumption, borrowing and the real exchange rate in response to very minor increases in external borrowing costs. Our paper highlights this instability, and shows how foreign exchange intervention can be used to avoid such collapses.

The mechanics of a sudden stop in our model are similar to those in Schmitt-Grohe and Uribe (2020), but the role for policy is not based on an equilibrium selection motive, but rather due to the interaction of the Fisherian 'debt-deflation' process as described by Mendoza (2002), and the role of pecuniary externalities. When the economy has an external debt level that puts it in the range where multiple equilibria are possible, a small rise in the world real interest rate can eliminate the high borrowing/ high consumption equilibrium. In that case, since agents don't internalize the impact of their deleveraging on the price of collateral, the economy enters a vicious spiral and the only stable equilibrium involves a large drop in consumption and the real exchange rate.

This pecuniary externality gives a role for policy. The policy maker internalizes the effect of deleveraging on the price of collateral. When the constraint is not binding this externality doesn't matter. But at the point where further deleveraging would cause the borrowing constraint to bind, the policy maker will act to prevent further deleveraging and thus keep the price of collateral high. As in Devereux et al. (2019) and others, the policy maker could achieve this through the use of capital controls. To prevent a sudden stop a policy maker can raise capital outflow taxes and thus increase domestic absorption. However, Eichengreen and Rose (2014) and Fernandez et al. (2015) show that capital controls in the data are acyclical and are typically not deployed in response to booms and busts in capital flows. On the other hand, as we describe below, there is ample evidence of countries using foreign exchange intervention as a cyclical policy instrument. Moreover, Davis et al. (2020a) show that under certain conditions the equilibrium from optimal central bank foreign exchange intervention is identical to the equilibrium from an optimal tax on capital flows.

An optimal policy response in our model can be achieved through sterilized foreign exchange intervention, which involves selling some of the central bank's stock of foreign bonds, and buying domestic bonds from the private sector with the proceeds. If the borrowing constraint was slack, and the central bank and private sector had equal access to international capital markets, this intervention policy would have no real effects, since, by Ricardian equivalence, the private sector would fully offset the central bank intervention by a one-for-one purchase of foreign bonds. But a central part of our paper is the presence of intermediary frictions in private capital markets, which we model as in Gabaix and Maggiori (2015), that prevent the private sector from fully offsetting a central bank foreign exchange intervention. When the economy is near the critical region where a sudden stop is precipitated, the private sector begins to delever by reducing its debt position through the financial intermediaries. But the central bank can more than offset this by selling some of their stock of foreign bonds, thus increasing domestic absorption and keeping the economy in the region where the borrowing constraint does not bind. Thus at a time when the private sector wishes to delever and increase their net asset position, the central bank can prevent an underborrowing equilibrium by doing the opposite and selling foreign assets.

But there is a natural limit to central bank foreign exchange intervention, central bank foreign exchange reserves cannot fall below zero. If the initial stock of reserves is low then the central bank may not have the reserves necessary to sell to keep domestic absorption and the price of collateral high. At some point, reserves may fall to zero and the central bank no longer has the ability to prevent the sudden stop equilibrium. This non-negativity constraint to foreign exchange reserves gives the central bank an incentive to accumulate reserves before a crisis. The marginal benefit of holding an extra unit of reserves is simply the welfare loss of a sudden stop multiplied by the reductuion in the probability of a sudden stop gaind by holding that extra unit of reserves.

But the same intermediary friction which gives foreign exchange intervention traction during a crisis makes the accumulation of foreign exchange reserves distortionary before a crisis. Just as the central bank sale of foreign exchange reserves would lead the economy to consume and borrow more, the central bank purchase of foreign exchange reserves would force the economy to consume less and save more. This distortion of consumer's optimal spending plans represents the marginal cost to acquiring reserves. This, together with the marginal benefit of reserves will pinpoint the optimal stock of central bank foreign exchange reserves.[2]

This paper contributes to a growing literature on foreign exchange intervention. We review some of the recent empirical and theoretical work in this literature in Section 2. The model is presented in Section 3. The determination of net external assets in the model is presented in Section 4, where we discuss both the properties of the steady state as well as the short-run instability in net external assets due to the endogeneity of the borrowing constraint. Section 5 discusses the mechanics of a sudden stop following a shock to the world interest rate and the optimal policy response. Numerical results from a global solution of the model and a numerical solution for optimal policy is presented in Section 6. Finally Section 7 concludes.

[2] Rodrik (2006) uses the spread between domestic interest rates in emerging market economies and the yield on U.S. treasuries to quantify the cost of holding reserves. He concludes that the cost of holding the observed stock of central bank reserves amounted to about 1% of GDP. But he concludes that compared to the cost of a sudden stop, this "insurance premium" is not excessively high.

2 Empirical and theoretical literature on foreign exchange intervention

Our paper is motivated by the recent empirical literature on central bank foreign exchange intervention and contributes to the theoretical literature in this area. In the empirical literature two major themes stand out: one has to do with evidence on the effectiveness of using reserves, either to prevent a crisis or to prevent depreciation in the exchange rate, the second relates to the precautionary accumulation of reserves ex-ante as insurance against a crisis.

With respect to the first theme, Fratzscher et al. (2019), using daily data on sterilized foreign exchange intervention, test to see if intervention has the desired effect on the exchange rate (they ask whether daily intervention moved the exchange rate in the desired direction or if the exchange rate intervention successfully stabilized the exchange rate). They find it does, and thus FXI is an effective tool to use for exchange rate stabilization. Forbes and Klein (2015) argue that foreign exchange intervention is an effective policy tool to prevent currency depreciation in the face of shocks to the foreign interest rate. Ghosh et al. (2016) estimate a policy reaction function for central bank foreign exchange accumulation and find that emerging market central banks engage in foreign exchange intervention to smooth fluctuations in the real exchange rate, selling foreign reserves to prevent depreciation and buying reserves to prevent appreciation. Obstfeld et al. (2009) show that countries with a larger stock of reserves in 2007 had less exchange rate depreciation during the crisis of 2008.

Closely related to the fact that the central bank can use foreign exchange intervention to stabilize the exchange rate is the fact that the central bank can use intervention to prevent a crisis. Gourinchas and Obstfeld (2012) run logit regressions of crisis incidence on a number of variables including the stock of reserves. They find that the stock of reserves is highly negatively associated with increased probability of a crisis. Ahmed et al. (2017) show that emerging market countries with stronger fundamentals, including a higher stock of central bank reserves to GDP and a lower ratio of short-term external debt to reserves, outperformed their emerging market peers on a number of financial indicators during the “taper tantrum” episode of 2013, including having less currency depreciation, lower stock market losses, and a smaller increase in EMBI or CDS spreads. These findings are related to a number of other papers documenting the early warning signs of an emerging market crisis, including Bussiere and Fratzscher (2006), Rose and Spiegel (2011), Frankel and Saravelos (2012).

Obstfeld et al. (2010) regress reserve stocks on financial openness, and find that countries hold more reserves when they become more financially open and thus more vulnerable to external crises. Arce et al. (2019) have similar findings, and show that in many emerging market countries central bank reserve accumulation is positively correlated with private external liability accumulation. Jeanne and Sandri (2020) report how in early stages of financial integration central banks increase their holdings of foreign exchange reserves as foreign liabilities increase, but as a country develops and the private sector begins to hold more external assets, these begin to replace central bank reserves as the main component of the external asset portfolio. Aizenman and Hutchison (2012) and Aizenman and Sun (2012) discuss the "fear of losing international reserves" where during the crisis of 2008-2009 many emerging market countries chose to allow their exchange rate to depreciate instead of having to sell reserves.

In the recent theoretical literature on foreign exchange intervention, Jeanne and Ranciere (2011) model reserves as an insurance contract to prevent sudden stops. Durdu et al. (2009) also model reserve accumulation as insurance against a sudden stop resulting from domestic shocks. Chang et al. (2015) and Cavallino (2019) look at optimal foreign exchange intervention in a linear-quadratic New Keynesian model. In these models foreign exchange intervention is an additional tool that helps to stabilize the economy in the presence of portfolio shocks. Fanelli and Straub (2016) model optimal foreign exchange intervention in a setting where the central bank tries to manipulate the terms-of-trade for distributional considerations. Hur and Kondo (2016) and Bianchi et al. (2018) both consider the use of reserves to mitigate the rollover risk, which is characterized by a sudden stop in foreign capital inflows precipitated by foreign investors refusing to rollover existing debts. Amador et al. (2020) study foreign exchange intervention as a policy intervention when the nominal interest rate is at the zero lower bound. They use this to explain how central bank foreign exchange intervention at the zero bound was responsible for the deviations from Covered Interest Parity observed during the global financial crisis.

In a stylized three-period model, Cespedes et al. (2017) and Bocola and Lorenzoni (2020) develop models with multiple equilibria ex-post. The central bank could eliminate bad equilibria by implementing a lending of last resort policy if the central bank has accumulated a sufficient stock of reserves. Jeanne and Sandri (2020) explore the accumulation for foreign exchange reserves in a model where the private sector can also acquire liquid foreign assets. Both private and central bank foreign assets can serve as insurance against a sudden stop in foreign liabilities, but private agents do not internalize the insurance role of their stock of foreign assets, and thus the central bank, which does internalize the insurance role of their stock of foreign assets, will acquire reserves and as a result the economy will have a higher level of liquid external assets than in the laissez-faire equilibrium. In Cespedes and Chang (2020), the central bank acquires reserves ex-ante to lend to banks following a shock to banks' collateral constraint.

In Arce et al. (2019) the fact that private agents don't internalize the effect of their own external borrowing on the likelihood of a sudden stop crisis represents a pecuniary externality that leads the private sector to borrow more than is efficient. By acquiring foreign exchange reserves the central bank can force the economy as a whole to save more and thus lead to the efficient level of borrowing. While Arce et al. (2019) model foreign exchange intervention in a different way than in our paper, there are some similarities. We also find that the accumulation of foreign exchange reserves leads to reduced borrowing for the economy as a whole, and we discuss how this is one benefit of the accumulation of reserves. But we focus on the role of reserves as insurance against the risk of a sudden stop caused by a spike in external interest rates, and the way in which the central bank can use foreign exchange intervention to respond to this shock.

3 Model

We construct a small open economy model with an infinite horizon. The country consists of a representative household, a financial sector, and a central bank. Households derive utility from the consumption of a tradable good x and a non-tradable good y in period t. Households begin each period with an initial stock of debt. They face a borrowing constraint limiting debt to a fraction of the market value of their endowment in a given period. In line with the empirical literature on the global financial cycle, the only source of exogenous variation in the model is a shock to the country's external borrowing cost.

3.1 Households

Households maximize utility, described as follows:

$$U = E_t \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{1}$$

where $u\left(c\right) = \frac{c^{1-\sigma}}{1-\sigma}$.

$$c_t = \left[\alpha\left(c_{t,X}\right)^{\frac{\xi-1}{\xi}} + \left(1-\alpha\right)\left(c_{t,Y}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \tag{2}$$

where $c_{t,X}$ denotes the amount of traded good and $c_{t,Y}$ for the amount of non-traded good. The budget constraint for households is written as follows:

$$c_{t,X} + p_t c_{t,Y} + B_t = x + p_t y + R_{t-1}B_{t-1} + T_{t-1} + \Pi_{t-1} \tag{3}$$

where B_t represents the household's holdings of domestic bonds (which are held by households, the financial sector, and the central bank) and R_t is the interest rate on domestic bonds. x and y denote the endowment of traded and nontraded goods, respectively. The central bank earns a net return T_{t-1} on their bond portfolio which is rebated lump-sum to savers (more on this later). The financial sector also earns net interest income on their bond portfolio which is rebated to households in lump sum Π_{t-1}.

Combining the first order conditions for traded and non-traded goods gives you the price of non-traded goods p_t:

$$p_t = \frac{1-\alpha}{\alpha}\left(\frac{c_{t,X}}{c_{t,Y}}\right)^{\frac{1}{\xi}} \tag{4}$$

Due to limited enforcement of debt contracts, home country borrowers face a borrowing constraint given by:

$$-B_t \leq \kappa\left(x + p_t y\right) \tag{5}$$

The multiplier on the borrowing constraint is μ_t.

The first order condition with respect to B_t is:

$$\lambda_t - \mu_t = E_t \beta \lambda_{t+1} R_t \tag{6}$$

Where λ_t is the marginal utility of traded goods consumption:

$$\lambda_t = c_t^{\frac{1-\sigma\xi}{\xi}} \alpha c_{t,X}^{-\frac{1}{\xi}} \tag{7}$$

3.2 Financial Intermediaries

A key feature of the model is the presence of frictions in international financial markets. Empirical evidence strongly supports the fact that private households in emerging markets do not have access to international financial markets on the same terms as banks, central banks, or governments. To capture this feature, we assume that private households in our model do not directly hold foreign bonds, but must trade with financial intermediaries who can borrow and lend on international financial markets subject to enforcement costs. The financial sector is made up of a continuum of identical atomistic financiers, indexed $i \in [0,1]$. When private household issue domestic bonds, financiers issue bonds on the international market, and use the proceeds to buy home bonds from domestic households. Financiers begin each period with zero net worth. They then issue $F_t^{fs}(i)$ international bonds and purchase $B_t^{fs}(i)$ domestic bonds. By intermediating the borrowing from domestic households, financiers thus take a positive position in B_t^{fs} and a negative position in F_t^{fs}, where $B_t^{fs}(i) + F_t^{fs}(i) = 0$. After aggregating across all atomistic financiers the balance sheet for the financial sector is given by:

$$B_t^{fs} + F_t^{fs} = 0 \tag{8}$$

where $F_t^{fs} = \int_0^1 F_t^{fs}(i)\,di$ and $B_t^{fs} = \int_0^1 B_t^{fs}(i)\,di$.

Note that both international bonds and domestic bonds are denominated in units of the traded good. Thus, unanticipated movements in real exchange rates have no impact on financier's balance sheets through currency mismatches. But, as we establish below, since financiers act as an intermediary between households and international financial markets, this drives a wedge between the domestic interest rate, R_t, and the exogenous world interest rate R_t^W. Both returns are denominated in traded goods.

The net interest income from the financier's domestic and international bond portfolio is:

$$\Pi_t(i) = R_t^W F_t^{fs}(i) + R_t B_t^{fs}(i) \tag{9}$$

where $\Pi_t = \int_0^1 \Pi_t(i)\,di = R_t^W F_t^{fs} + R_t B_t^{fs}$, is remunerated lump-sum to households.

Each atomistic financier is in operation for a single period, and their objective is to maximize the discounted net interest income from bonds purchased in period t:

$$\beta \Pi_t(i) = \beta \left(R_t^W F_t^{fs}(i) + R_t B_t^{fs}(i) \right) = \beta \left(R_t^W - R_t \right) F_t^{fs}(i) \tag{10}$$

Financiers do not operate without constraints however. As in Gabaix and Maggiori (2015) we assume that financiers have an incentive to divert the funds they receive from issuing foreign bonds. After taking the position $F_t^{fs}(i) < 0$, the financier can divert a share $\Gamma \left| F_t^{fs}(i) \right|$ of their credit position $\left| F_t^{fs}(i) \right|$, where Γ is a non-negative constant. If the financier diverts the funds their firm is unwound and the proceeds are returned to the creditor. Since creditors correctly anticipate the ability and motivation of the financier to divert funds, financiers are subject to the following incentive compatibility constraint:

$$\beta \Pi_t (i) \geq \Gamma \left| F_t^{fs} (i) \right| \times \left| F_t^{fs} (i) \right| = \Gamma \left(F_t^{fs} (i) \right)^2 \tag{11}$$

The financiers maximization problem is to choose $F_t^{fs} (i)$ to maximize $\Pi_t (i)$ subject to this incentive compatibility constraint. Since the value of the financier's firm, $\Pi_t (i)$, is linear in $F_t^{fs} (i)$ and the right hand side of this constraint is convex in $F_t^{fs} (i)$, the constraint always binds. Thus:

$$\beta \left(R_t^W - R_t \right) = \Gamma F_t^{fs} (i) \tag{12}$$

And thus the stock of foreign bonds held by the financial sector is:

$$F_t^{fs} = \frac{1}{\Gamma} \beta \left(R_t^w - R_t \right) \tag{13}$$

which can also be written as:

$$R_t = R_t^W - \frac{\Gamma}{\beta} F_t^{fs} \tag{14}$$

If $\Gamma = 0$ then the financial sector is simply a veil and the equilibrium condition for foreign bond holding is $R_t = R_t^W$, exactly as it would be if households could borrow directly from foreigners and faced no frictions. But when $\Gamma > 0$, and $F_t^{fs} < 0$, the domestic interest rate will be higher than the foreign interest rate, inducing households to save more and consume less. Note that this equilibrium condition in the market for foreign bonds can also be derived in a reduced form by adding a quadratic adjustment cost to holding foreign bonds in the household budget constraint, as in Schmitt-Grohe and Uribe (2003). This reduced form approach was the preferred way of adding intermediary frictions in models with central bank foreign exchange intervention in Chang et al. (2015) and Davis et al. (2020a)

3.3 Central Bank and Market Clearing

The central bank also holds a stock of domestic and international bonds. It can vary the composition of that bond portfolio.

$$B_t^{cb} + F_t^{cb} = 0 \tag{15}$$

By the fact that they participate in both the domestic and international bond markets, the central bank is similar to the financial sector. But the central bank does not face the intermediation friction Γ. This assumption is realistic. The central banks of emerging countries are long-lived institutions which must maintain their reputations on international financial markets. Nevertheless, we impose the constraint that central banks must maintain a non-negative stock of international reserves. This is also a realistic assumption. As witnessed in many episodes of sudden stops in emerging economies, central banks have no recourse to international debt markets when reserves are depleted.

Note that by equation (15), any central bank foreign exchange intervention, the buying or selling of F_t^{cb}, is *sterilized* foreign exchange intervention. This is where the central bank buys or sells foreign bonds while at the same time selling or buying an equal quantity of domestic bonds in order to keep the total size of their asset portfolio constant. If the central bank were to buy or sell foreign bonds without an off-setting sale or purchase of domestic bonds, that would be an *unsterilized* foreign exchange intervention.

The central bank earns a net return T_{t-1} on that portfolio which is rebated lump-sum to savers.

$$T_{t-1} = R_{t-1}^{W} F_{t-1}^{cb} + R_{t-1} B_{t-1}^{cb} \tag{16}$$

Domestic bonds B are held by three agents, households, the financial sector, and the central bank. The domestic bond market clearing condition is given by:

$$B_t + B_t^{fs} + B_t^{cb} = 0 \tag{17}$$

The world interest rate, R_t^W, is taken as given in this small open economy model (and hence there is no market clearing condition for foreign bonds).

Finally, market clearing conditions in the market for non-traded goods are given by:

$$c_{t,Y} = y \tag{18}$$

3.4 Balance of payments identity

If we substitute the financial sector and central bank net interest income in equations (9) and (16), the financial sector and central bank balance sheets in equations (8) and (15), and the domestic bond market clearing and non-traded goods market clearing conditions in equations (17) and (18), into the household's budget constraint in (3), we are left with the economy-wide budget constraint:

$$c_{t,X} = x - F_t^{fs} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb} + R_{t-1}^W F_{t-1}^{cb} \tag{19}$$

This condition can be rearranged into the familiar balance of payments identity where the current account is equal to the capital account plus the change in central bank foreign exchange reserves:

$$CA_t = \Delta F_t^{fs} + \Delta F_t^{cb} \tag{20}$$

where

$$CA_t = x - c_{t,X} + F_{t-1}^{cb}\left(R_{t-1}^W - 1\right) + F_{t-1}^{fs}\left(R_{t-1}^W - 1\right) \tag{21}$$

The current account, CA_t, is equal to net exports: $x - c_{t,X}$ plus interest income from international bonds purchased in $t-1$: $F_{t-1}^{cb}\left(R_{t-1}^W - 1\right) + F_{t-1}^{fs}\left(R_{t-1}^W - 1\right)$. The capital and financial account , ΔF_t^{fs}, is equal to net international bond purchases by financiers, $F_t^{fs} - F_{t-1}^{fs}$. The change in reserves, ΔF_t^{cb}, is equal to net international bond purchases by the central bank, $F_t^{cb} - F_{t-1}^{cb}$.

Thus in this model, net borrowing from abroad, a current account deficit $CA_t < 0$, can be financed either by net private capital inflows, a negative capital and financial account $\Delta F_t^{fs} < 0$, or the sale of central bank foreign exchange reserves $\Delta F_t^{cb} < 0$. The two types of financing, public and private, are not equal, since private financiers face an intermediary friction $\Gamma > 0$. As in Gabaix and Maggiori (2015) this friction allows the central bank to use the purchase or sale of foreign exchange reserves as an instrument to adjust the current account and thus the economy's total external debt. A central bank sale of foreign exchange reserves reduces the current account, while a purchase of foreign reserves has the opposite effect. A formal proof is presented in the appendix, but the intuition is as follows.

Suppose the central bank increases their holding of foreign exchange reserves, $\Delta F_t^{cb} > 0$. Through the central bank's balance sheet, $\Delta B_t^{cb} < 0$, as the central bank finances this purchase by issuing domestic bonds. This puts upward pressure on the domestic interest rate and creates an arbitrage opportunity for the financial sector to buy domestic bonds and finance this by selling foreign bonds, $\Delta B_t^{fs} > 0$ and $\Delta F_t^{fs} < 0$.

If the intermediary friction $\Gamma = 0$ then private financiers can fully exploit this arbitrage opportunity and the increase in domestic bond sales by the central bank is exactly offset by the increase in domestic bond purchases by the financial sector, $\Delta B_t^{cb} = -\Delta B_t^{fs}$ and the increase in the central banks holdings of foreign bonds is offset by the reduction in the financial sector's holdings, so that $\Delta F_t^{fs} = -\Delta F_t^{cb}$. The total stock of debt in the economy is unaffected and the equilibrium condition in the market for foreign bonds ensures that $R_t = R_t^W$. The financial sector is a veil and central bank foreign exchange intervention has no effect on aggregate macroeconomic variables, as in Obstfeld (1981), Backus and Kehoe (1989), Gabaix and Maggiori (2015), and Davis et al. (2020a).

But if the intermediary friction $\Gamma > 0$ then when the central bank purchases foreign bonds, $\Delta F_t^{cb} > 0$, creating an arbitrage opportunity between foreign and domestic bonds, the intermediary friction means that this opportunity is not fully exploited by financiers. As financiers sell foreign bonds to take advantage of the arbitrage opportunity provides by the central banks purchase of foreign bonds, the intermediary friction tends to push up the domestic interest rate above the world interest rate, reducing the private sector's incentive to sell domestic bonds in the same volume as their purchase of bonds from the central bank. As a result, from the domestic bond market clearing condition, household holdings of domestic bonds must increase, $\Delta B_t > 0$, i.e. the central bank purchase of foreign bonds led to increased saving and decreased consumption for the economy as a whole. This entire sequence can be run in the opposite direction to show why the central bank sale of foreign bonds will decrease savings and increase consumption.

4 Determination of net external assets

Here we examine the determination of the economy's net external debt in the model. Define a country's net external assets as $F_t = F_t^{fs} + F_t^{cb}$, and thus $-F_t$ represents net external debt. In the next section we'll derive optimal policy for central bank foreign exchange intervention, so until then, we set $F_t^{cb} = 0$. We begin with a discussion of the steady state level of external debt, we then move to the determination of the equilibrium level of external debt in the short run.

4.1 Steady State

A steady state of the model is defined by constant values of consumption, domestic and foreign bond holdings, and domestic interest rates. We can describe a steady state assuming a constant value of the world gross interest rate R^W. Here, and for the rest of the paper, we assume that in a steady state $\beta R^W < 1$. So domestic agents are more impatient than the rest of the world.

Using (4) -(7) and (14) we can describe a steady state of the model by the conditions

$$1 = \beta R^W - \Gamma(F) + \frac{\mu}{\lambda} \tag{22}$$

$$-F \leq \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + \left(R^W - 1 \right) F \right)^{\frac{1}{\xi}} \right) \tag{23}$$

$$c_X = x + (R^W - 1)F \tag{24}$$

where the domestic interest rate $R_t = \frac{\lambda_t - \mu_t}{\beta E_t \lambda_{t+1}}$. Later when presenting the numerical solution to the model, the global solution will of course incorporate the fact that the stochastic steady state value of $E_t(\lambda_{t+1})$ is greater than the steady state value of λ_t. But for now in this analytical exposition, we make the simplification that the steady state value of the domestic interest rate $R = \frac{1}{\beta} - \frac{\mu}{\beta\lambda}$.

Equations (22)-(24) describe steady state values of μ, F and c_X. The steady state can be of two types, depending on whether the borrowing constraint is binding or not. The steady state borrowing constraint can be plotted in a chart with total external debt along both the horizontal and vertical axis in Figures 1 and 2. In these figures the right hand side of the inequality in equation (23) is represented by the blue downward sloping line.

Any equilibrium must lie along the 45 degree line. The steady state equilibrium level of external debt is determined by the parameters in the model. If the parameters of the model imply that the steady state level of external debt is less than the steady state borrowing constraint then this is a steady state with a non-binding constraint. Graphically this is represented by a point along the 45 degree line to the left of the point where the blue borrowing constraint crosses the 45 degree line. If instead the parameters of the model and equation (22) imply a steady state level of external debt that is higher than the steady state borrowing constraint in (23), then the equilibrium level of external debt is determined by the point of equality in the borrowing constraint in (23) and the household's Euler equation in (22) determines the multiplier $\mu > 0$. We denote this maximum level of external debt by $-\bar{F}$. Graphically this is represented by the intersection of the blue line and the 45 degree line.[3]

Note that if financial intermediation is costless, so that $\Gamma = 0$, then if a steady state exists, the borrowing constraint will always bind in a steady state, and $F = \bar{F}$. Intuitively, if impatient agents could borrow at a world interest rate that was less that their subjective

[3]Note, if $\Gamma = 0$ and there were no borrowing constraints, then there would be no steady state level of debt or consumption, since with $\beta R^W < 1$ agents would borrow as much as is consistent with their long run budget constraint in the first period, and consumption would be declining and debt rising over time to its natural limit.

discount factor, agents will always borrow up to the limit implied by the borrowing constraint, so that the steady state external debt is again defined by (23), while (22) determines μ. However, if intermediation isn't costless, $\Gamma > 0$, then that intermediation friction creates a spread between the steady state domestic interest rate and the steady state world interest rate. If the parameters of the model determine that the steady state level of external debt is less than $-\bar{F}$, then $\mu = 0$ and $\beta R = 1$. So while domestic agents are more impatient than the rest of the world and $\beta R^W < 1$, the spread between the domestic and world interest rates created by the financial intermediation friction can still lead to a non-binding steady state equilibrium where $\beta R = 1$.

4.2 Short-run

In this subsection, following Schmitt-Grohe and Uribe (2020) we illustrate the possibility of multiple non-steady state equilibria borrowing levels in the economy constrained by the borrowing constraint (5). Recall that in the steady state the borrowing constraint in terms of the economy's total external debt is given by (23). But while this represents the long-run borrowing constraint when $F = F_t = F_{t-1}$, in the short-run this borrowing constraint is given by:

$$-F_t \leq \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W_{t-1} F_{t-1} - F_t \right)^{\frac{1}{\xi}} \right) \tag{25}$$

The right hand side of this short-term borrowing constraint in equation (25) is *increasing* in the choice of debt in period t, $-F_t$. This is due to the fact that given F_{t-1}, increased borrowing in period t raises the price of the non-traded good and thus the value of collateral. The slope of this borrowing constraint with respect to $-F_t$ is

$$\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(x + R^W_{t-1} F_{t-1} - F_t \right)^{\frac{1}{\xi} - 1} \tag{26}$$

If the elasticity of substitution between traded and non-traded goods $\xi < 1$ and we assume that the consumption of traded goods is positive, $x + R^W_{t-1} F_{t-1} - F_t > 0$, then this slope is positive and increasing in $-F_t$. This short term borrowing constraint is given by the red upward sloping convex line in Figures 1 and 2. The difference between the two figures is that the initial debt $-F_{t-1}$ is higher in the second figure. As we can see the short term borrowing constraint in the right hand side of equation (25) is shifted down and to the right as debt carried over from the last period $-F_{t-1}$ increases.

Starting from a steady state, this initial level of debt is represented by the point where the steady state and short-term borrowing constraints cross, denoted point A in both figures. As long as the parameters in the model imply a non-binding steady state, then the equilibrium level of external debt at point A, $-F^A$, is given by the Euler equation in (22) where $\mu = 0$.

Notice that the short-term borrowing constraint for a low initial stock of debt does not intersect with the 45 degree line. But for a higher level of initial debt the short-term borrowing constraint does intersect the 45 degree line. The initial level of debt $-\tilde{F}$, which separates the low and high levels of initial debt is represented by the point where the short-term borrowing constraint is tangent to the 45 degree line. At this point, the slope of the short-term borrowing constraint is unity, and the short-term borrowing constraint is binding, so that a) $\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(x + R^W \tilde{F} - F_t \right)^{\frac{1-\xi}{\xi}} = 1$ and b) $-F_t = \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W \tilde{F} - F_t \right)^{\frac{1}{\xi}} \right)$.

A little algebra shows that these two conditions are satisfied when:

$$-\tilde{F} = \frac{1}{R^W} \left(x(1+\kappa) + (\frac{\kappa(1-\alpha)}{\alpha\xi})^{\frac{\xi}{\xi-1}} (\xi - 1) \right)$$

When starting from the steady state where $F_t = F_{t-1} = F$, we can see from the two figures, as $-F_{t-1}$ increases the convex short-term borrowing constraint shifts to the right, as shown in Figure 2. As long as $-F$ is less than the maximum steady state debt limit there will still be a non-binding steady state equilibrium as shown by point A in the figure where $-F_t = -F_{t-1} = -F^A$. However, if $-F^A > -\tilde{F}$, the short-term borrowing constraint intersects the 45 degree line twice. At this point there are three equilibria. The non-binding equilibria, and two where the short-term constraint is binding in period t, $-F_t = -F^B$ and $-F_t = -F^C$, where $-F^C < -F^B < -F^A$ and:

$$
\begin{aligned}
-F^B &= \kappa\left(x+\frac{1-\alpha}{\alpha}\left(x+R^W F^A-F^B\right)^{\frac{1}{\xi}}\right) \\
-F^C &= \kappa\left(x+\frac{1-\alpha}{\alpha}\left(x+R^W F^A-F^C\right)^{\frac{1}{\xi}}\right)
\end{aligned}
$$

Figure 2 is constructed using the benchmark parameter values in the model, so obviously from the figure the short-term borrowing constraint crosses the 45 degree line twice and there are three equilibria under our benchmark parameterization. In the appendix we present a formal proof to derive under that conditions the short-term borrowing constraint crosses the 45 degree line twice and thus under what conditions will the model have three equilibria.

As we show in the appendix, the key to the existence of multiple equilibria is that the slope of the borrowing constraint in equation (26) is greater than one when the economy is in a non-binding steady state equilibrium with $-F_t = -F^A$. When the slope is greater than one, each additional unit of debt will have a direct effect of tightening the borrowing constraint by one unit but an indirect effect of loosening the borrowing constraint by more than one unit since additional debt leads to additional traded goods consumption and thus a higher relative price of non-traded goods. A large part of Schmitt-Grohe and Uribe (2020) is devoted to showing under what combinations of parameters this will hold. In our model we adopt the same parameterization, but later when discussing the parameters we use in the numerical model we will highlight the key paramters for this result.

Each equilibrium can be sustained by self-confirming beliefs. Notice however that equilibrium B is unstable in a traditional sense, since if debt is just below (above) the level indicated by point B, the borrowing constraint would be violated (slack), and debt would be forced to fall (would be increasing) over time.

Which equilibrium will prevail? For this, we need an equilibrium selection rule. In the equilibrium selection rule we use, if the non-binding equilibrium is possible, agents' beliefs will always coalesce around this non-binding equilibrium. But if the non-binding equilibrium is not possible then the only possible equilibrium is the stable binding equilibrium, point C. Note that this selection rule eliminates the possibility of self-fulfilling deleveraging. Movement from a "good" non-binding equilibrium to a "bad" binding equilibrium is driven by fundamental shocks, which in this model are shocks to the country's cost of external borrowing, the world interest rate R_t^W.

Maintaining this assumption on equilibrium selection, the rest of the paper will analyze the impact of shocks to the world interest rate which leads to sudden stops in capital flows. We will show that this economy faces a risk of highly unstable capital flows, not as a result of multiple equilibrium and self-fulfilling beliefs in themselves, but due to the fact that the pecuniary externality associated with the effect of the real exchange rate on the value of collateral allows for sudden discrete collapses in borrowing capacity for the economy. The main objective of the rest of the paper is to explore how foreign exchange intervention can be used to offset these externalities and to prevent the occurrence of sudden stops in capital flows.

Figure 1: Single Non-binding equilibrium for a low level of debt.

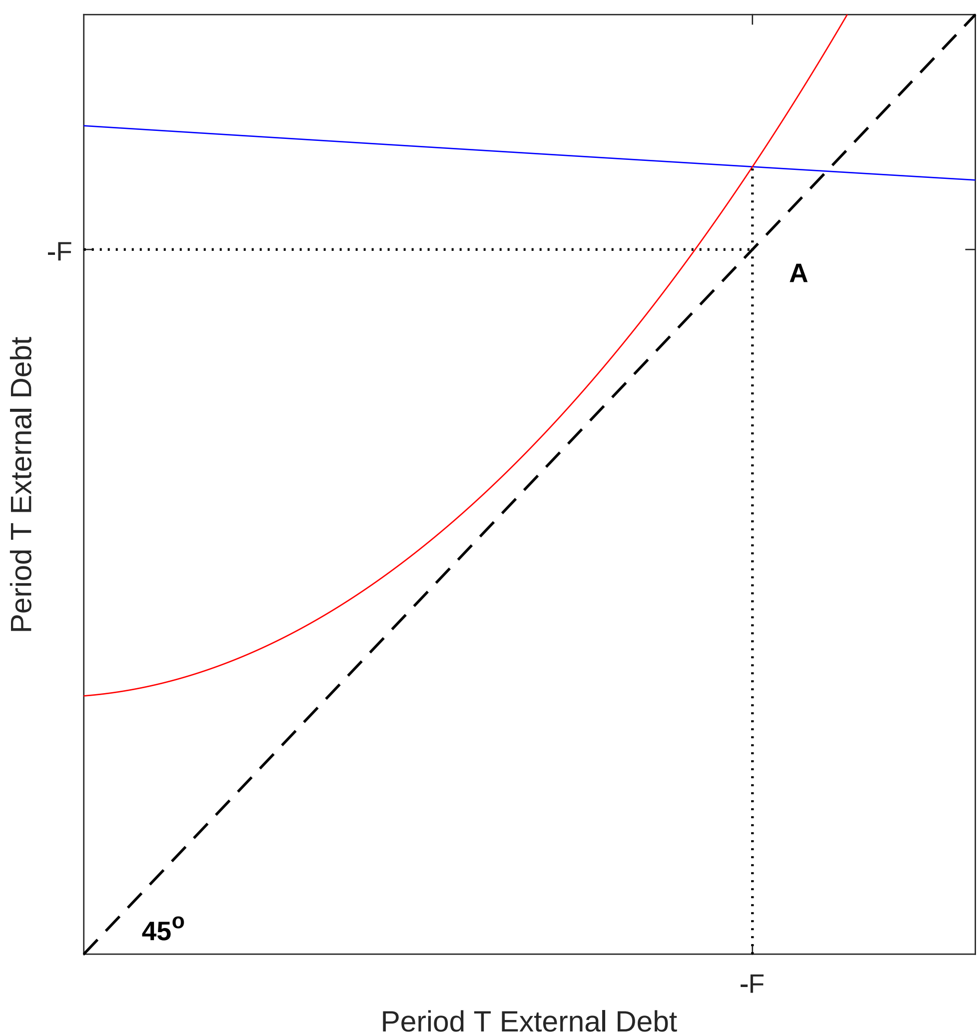
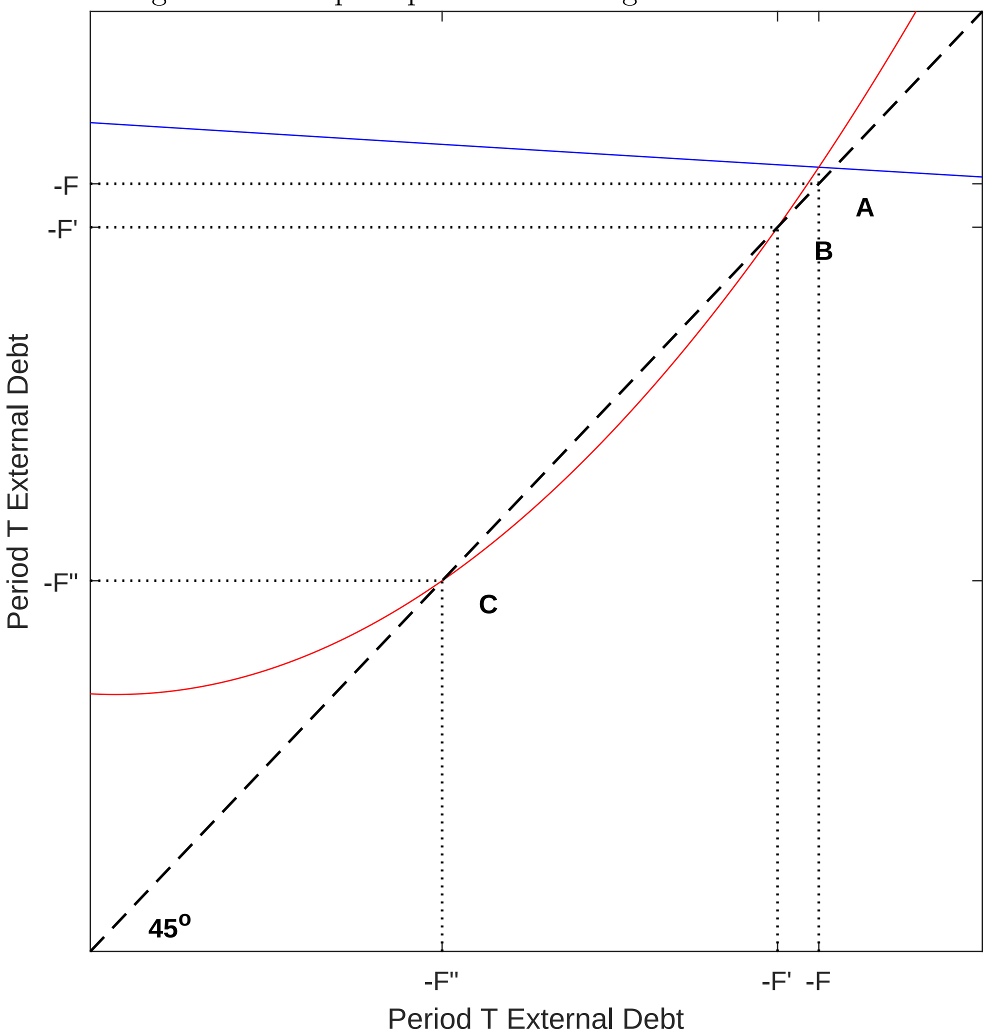


Figure 2: Multiple equilibria for a higher level of debt.



5 Sudden stops with and without policy intervention

5.1 Competitive equilibrium without intervention

Following on the discussion of the previous section, we can describe how the competitive equilibrium without policy intervention evolves. Assume first that $F_t^{cb} = 0$. Assume also that the steady state debt level is $-F^A$, so that the borrowing constraint is non-binding in the steady state. In this case, the first order condition with respect to external debt is

$$\frac{\lambda_t}{\beta E_t \left(\lambda_{t+1}\right)} = R_t^W - \frac{\Gamma}{\beta}\left(F_t\right) \tag{27}$$

As in the last section this steady state is represented graphically by point A in Figure 2. Assume that the initial debt level is at F^A, which means that the point where the short-term borrowing constraint crosses the long-run borrowing constraint represents the (non-binding) steady state debt level F^A. Following a shock to R_t^W, agents will adjust their desired debt levels according to the first order condition above. Specifically, for a given sequence of current and expected future world interest rates $\left\{R_t^W\right\}_{t=1}^{\infty}$ agent's will pick a sequence of external borrowing $-\left\{F_t\right\}_{t=1}^{\infty}$ to satisfy the first-order condition with respect to external debt in equation (27), subject to the economy wide budget constraint in (19) and the borrowing constraint in (5), taking as given the prices $\{p_t, R_t\}_{t=0}^{\infty}$ that clear the corresponding nontradable good market and domestic bond market.

Following a positive shock to R_t^W, agents' desired level of external borrowing $-F_t$ will fall from (27). This can be represented as a movement left along the 45-degree line in Figure 2. After the initial shock, if R_t^W is stationary then external borrowing $-F_t$ will gradually converge back to the steady state at point A.

A small positive shock to R_t^W will lead agents to delever to a point on the 45-degree line to the left of point A and then gradually return to the steady state. For a small shock, the desired delevering leads to a debt level to the right of point B, and so there still exists an equilibrium where the borrowing constraint does not bind. But there is a critical value of R_t^W where the equilibrium condition in equation (27) is satisfied at $-F_t = -F'$ and the borrowing constraint is just on the margin of binding, so $\mu_t = 0$. This is the debt level indicated at point B. Past this, for any further increases in R_t^W, agents will delever to $-\hat{F}$ where $-\hat{F} < -F^B$. Thus $-\hat{F}$ is in the region where the short-term borrowing constraint lies below the 45 degree line:

$$-\hat{F} > \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R_{t-1}^W F^A - \hat{F} \right)^{\frac{1}{\xi}} \right)$$

But this point is not an equilibrium, since it violates the borrowing constraint. Agents are forced to delever further to $-\hat{F}' = \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R_{t-1}^W F^A - \hat{F} \right)^{\frac{1}{\xi}} \right)$. But this deleveraging causes further fall in the price of collateral and agents are again forced to delever if the short-term borrowing constraint lies below the 45 degree line at $-\hat{F}'$, where $-\hat{F}' > \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R_{t-1}^W F - \check{F}' \right)^{\frac{1}{\xi}} \right)$. It is easy to see that this process will continue until agents delever to the point $-F^C$ where the short run borrowing constraint crosses the 45 degree line, point C in the figure.

The shock to R_t^W thus causes agents to delever, but they do not internalize the fact that deleveraging leads to a fall in the price of the non-traded good. Past point B in the figure, the constraint becomes binding and the dynamics lead to a forced deleveraging multiplier with falling collateral values reinforcing the deleveraging pressure.

Define $\bar{R}_t^w$ as the level of the foreign interest rate where agents delever to $-F_t = -F^B$ in period t before gradually moving back to the original steady state. This is represented graphically by point B in Figure 2. Starting from the steady state point A, where external debt is equal to $-F^A$, this cutoff value $\bar{R}_t^w$ is found from the equilibrium condition in equation (27):

$$\bar{R}_t^W = \frac{\lambda_t^B}{\beta E_t \left(\lambda_{t+1}^B \right)} + \frac{\Gamma}{\beta} \left(F^B \right) \tag{28}$$

where λ_t^B and λ_{t+1}^B are the marginal utilities of current and next period's consumption when $c_{X,t}^B = x + R_{t-1}^W F^A - F_t^B$ and $c_{X,t+1}^B = x + R_t^W F_t^B - F_{t+1}^B$. Therefore, if the world interest rate shock exceeds the value $\bar{R}_t^W$, the country experiences a sudden stop which leads external debt to fall to $-F^C$ in the Figure.

Note while the description of the deleveraging here suggests a sequential adjustment downwards in debt, consumption, and the real exchange rate, in effect the adjustment takes place instantaneously. Thus, the sudden stop in capital flows is discrete and precipitous. While our analysis does not involve a self-fulfilling sudden stop driven collapse in beliefs, the process has all the hallmarks of such a process. This is because beginning at a value of the world interest rate just above $\bar{R}_t^W$, a very small increase in $\bar{R}_t^W$ which pushes debt downwards by a marginal amount can precipitate a large collapse in capital flows, driven by a classic debt deflation process similar to that in Mendoza and others.[4]

5.2 Equilibrium with Policy Intervention

We now allow F_t^{cb} to be the instrument of a benevolent policy maker. The policy maker can either buy or sell reserves, subject to a non-negativity constraint on reserves. Recall from section 3 that, starting from a position where the borrowing constraint is not binding, if the central bank sells foreign exchange reserves, this leads to an increase in the economy's total external debt, while if the central bank were to buy foreign exchange reserves, that will decrease the economy's total external debt. Both steps are relevant for minimizing the probability of a sudden stop. Ex-post selling reserves can increase total external debt, and thus traded goods consumption, the price of the non-traded good, and the value of collateral, in effect stemming the process of deleveraging. Ex-ante buying reserves will reduce the economy's total external debt next period, thus making a potential crisis less likely, and give the central bank a stock of reserves to use in the case of a crisis next period.

Agents take the non-traded goods price p_t as given and do not internalize the effect of their own deleveraging on this price. This leads to the possibility sudden stop as described in the previous section. The central bank however takes account of the effect of their actions on the price of the non-traded good. Thus the central bank perceives that both buying reserves

[4] In Mendoza (2010) and Bianchi (2011), there is a unique equilibrium, and while the presence of pecuniary externalities generates a debt deflation multiplier, there is no equivalent threshold whereby very small shocks given rise to large discrete collapse in capital inflows.

ex-ante and selling reserves ex-post affect the non-traded goods price and the probability of a sudden stop.

The central bank chooses F_t^{cb} to maximize welfare in equation (1) subject to the economy wide budget constraint in equation (19), financier's incentive compatibility condition in (14), and the borrowing constraint in (5), where $p_t = \frac{1-\alpha}{\alpha}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}} = \frac{1-\alpha}{\alpha}\left(\frac{x_t+R_{t-1}^W F_{t-1}^{cb}-F_t^{cb}+R_{t-1}^W F_{t-1}^{fs}-F_t^{fs}}{y}\right)^{\frac{1}{\xi}}$. The central bank is subject to the additional constraint that its holdings of foreign exchange reserves can't be negative. The planner's problem becomes:

$$\max U = E_t \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma}\left[\alpha\left(c_{t,X}\right)^{\frac{\xi-1}{\xi}} + (1-\alpha)\left(y\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}(1-\sigma)}$$

subject to:

$$c_{t,X} = x + R_{t-1}^W F_{t-1}^{cb} - F_t^{cb} + R_{t-1}^W F_{t-1}^{fs} - F_t^{fs}$$

$$\frac{\lambda_t}{\beta E_t \lambda_{t+1}} \geq R_t^W - \frac{\Gamma}{\beta} F_t^{fs}$$

$$-F_t^{cb} - F_t^{fs} \leq \kappa\left(x + \frac{1-\alpha}{\alpha}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}} y\right)$$

$$F_t^{cb} \geq 0$$

The central bank's optimality condition is given by:

$$\begin{aligned}\beta E_t\left\{\lambda_{t+1}\right\}\left(R_t - R_t^W\right) \quad = \quad & -\mu_t \kappa \frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}-1} + \beta R_t^W E_t\left\{\mu_{t+1}\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t+1,X}}{y}\right)^{\frac{1}{\xi}-1}\right\} \\ & +\phi_t\left(1 - \frac{1}{\Gamma E_t \lambda_{t+1}}\frac{\partial \lambda_t}{\partial c_{t,X}}\right) + \beta R_t^W E_t\left\{\frac{\phi_{t+1}}{\Gamma E_{t+1}\lambda_{t+2}}\frac{\partial \lambda_{t+1}}{\partial c_{t+1,X}}\right\} \qquad (29)\end{aligned}$$

where ϕ_t is the multiplier on the non-negativity constraint for reserves, and μ_t captures the social shadow price of binding credit constraint. The full derivation of this condition is presented in the appendix.

The left hand side of this expression, $\beta E_t(\lambda_{t+1})\left(R_t - R_t^W\right)$ represents the marginal cost of acquiring one extra unit of reserves. In general, since the economy is a debtor, R_t will exceed R_t^W, so buying reserves involves the central bank borrowing at rate R_t and lending at a lower rate. Equivalently, buying reserves distorts agents' optimal consumption paths. Buying reserves in period t reduces the economy's total external debt, thus forcing agents to save more than they otherwise would have. If agents have a discount factor β where $\beta R^W < 1$, then by acquiring foreign exchange reserves, the central bank is forcing relatively impatient agents to save at a low world interest rate.[5] The marginal cost of acquiring one unit of reserves is the discounted value of next period's marginal utility of consumption multiplied by $R_t - R_t^W$, the spread between the domestic interest rate and the world interest rate.

The first term on the right hand side of this expression, $-\mu_t \kappa \frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t,X}}{y}\right)^{-\frac{1}{\xi}}$, represents the effect of acquiring reserves at time t on the tightness of the borrowing constraint at time t. Equivalently, the negative of this term represents the welfare gain from foreign exchange intervention and a sale of foreign exchange reserves during a crisis in period t. Since this is a foreign exchange intervention in time t in response to a crisis in time t, we will refer to this as the ex-post benefits of intervention. The ex-post benefit of selling one unit of reserves in a crisis is the multiplier on the borrowing constraint in the case of a sudden stop, μ_t, multiplied by $\kappa \frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}-1}$, which tells us how much the value of collateral changes when there is a change in the economy's total external debt, and thus tradable consumption in period t. Notice that this is simply the slope of the borrowing constraint given in the red convex curve in Figure 2.

The second term on the right hand side of this expression represents the ex-ante benefits of foreign exchange intervention. Acquiring one extra unit of reserves in period t has two ex-ante benefits on period $t+1$. First, it reduces the economy's total external debt at the beginning of period $t+1$ and thus reduces the risk of a sudden stop crisis (recall that the red convex borrowing constraint in Figure 2 shifts to the left as external debt decreases,

[5] As a practical example, when a developing country central bank acquires a large stock of foreign exchange reserves, they are forcing domestic savings to be channeled into relatively low yielding assets like T-bonds instead of investing in higher yielding domestic investments.

causing the critical point B where a sudden stop is triggered to move to the left, meaning that for a larger share of the exogenous shock space, the economy does not fall into the sudden stop equilibrium). The second ex-ante benefit of acquiring reserves in period t is that it gives the central bank an extra R_t^W units of reserves that they can sell in period $t+1$ in a foreign exchange intervention in the event of a crisis in period $t+1$. This is then multiplied by the expected marginal benefit of a foreign exchange intervention in period $t+1$, $E_t\left(\mu_{t+1}\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t+1,X}}{y}\right)^{\frac{1}{\xi}-1}\right)$, to give the expected ex-ante benefit of acquiring reserves.

The third and fourth terms are related to the shadow value of reserves when reserves are at their lower bound, ϕ_t and ϕ_{t+1}. [6]

First consider ex-post foreign exchange intervention. This is foreign exchange intervention in response to a crisis in the current period, where $\mu_t > 0$ and $E_t(\mu_{t+1}) = 0$. In this case, the central bank would sell reserves to raise total external debt and current consumption in order to raise the price of the non-traded good and eliminate the balance of payments crisis. The left hand side of (29) then would measure the benefit of reserve sales. In addition, there would be a direct current benefit coming from a relaxation of the collateral constraint. If there was no lower bound on reserves, $\phi_t = \phi_{t+1} = 0$, then this central bank selling of reserves would continue up to the point where $R_t^W - R_t \geq 0$, and thus central bank debt equals or exceeds the economy's total external debt. If however the central bank faces a non-negativity constraint on reserves then the central bank will sell reserves until they hit their lower bound, at this point the multiplier on the non-negativity constraint, and thus the value of one extra unit of reserves is equal to:

$$\phi_t\left(1-\frac{1}{\Gamma E_t\lambda_{t+1}}\frac{\partial\lambda_t}{\partial c_{t,X}}\right) = \mu_t\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}-1} + \beta E_t(\lambda_{t+1})\left(R_t - R_t^W\right) -$$

$$\beta R_t^W E_t\left\{\frac{\phi_{t+1}}{\Gamma E_{t+1}\lambda_{t+2}}\frac{\partial\lambda_{t+1}}{\partial c_{t+1,X}}\right\}$$

[6] Note that $\phi_t = \Gamma\gamma_t\mathbb{E}_t\lambda_{t+1}$, where γ_t is the Lagrange multiplier for the private consumption Euler equation as in the appendix. Therefore the term $-\frac{1}{\Gamma E_t\lambda_{t+1}}\frac{\partial\lambda_t}{\partial c_{t,X}}$ captures an additional marginal benefit of rising current consumption when reserves are bounded below.

where $\frac{\partial \lambda_t}{\partial c_{t,X}} < 0$. If the central bank runs out of reserves in the current period the value of one extra unit of reserves today would be a weighted average of the net benefit of using one more unit of reserves this period, $\mu_t \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_{t,X}}{y} \right)^{\frac{1}{\xi}-1} + \beta E_t \left(\lambda_{t+1} \right) \left(R_t - R_t^W \right)$, and the discounted shadow value of reserves next period. One extra unit of reserves today could be held till the next period when it would be R_t^W extra units of reserves. Thus an extra unit of reserves today has a value to the central bank as insurance against running out of reserves next period, and like any insurance, its value is based on the curvature of the utility function, $\frac{\partial \lambda_{t+1}}{\partial c_{t+1,X}}$, and is agents were risk nuetral and $\frac{\partial \lambda_{t+1}}{\partial c_{t+1,X}} = 0$, this insurance value is zero.

Next consider ex-ante foreign exchange intervention. This is foreign exchange intervention in response to the possibility of a crisis next period, $\mu_t = 0$ and $E_t \left(\mu_{t+1} \right) > 0$. In this case the first term of the first-order condition is negative, the second is zero, and the third is positive, and if the central bank holds a positive stock of reserves then the multiplier on the non-negativity constraint in the current period is zero. The optimal quantity of reserves is the point where:

$$\beta E_t \left\{ \lambda_{t+1} \right\} \left(R_t - R_t^W \right) = \beta R_t^W E_t \left(\mu_{t+1} \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_{t+1,X}}{y} \right)^{\frac{1}{\xi}-1} + \frac{\phi_{t+1}}{\Gamma E_{t+1} \lambda_{t+2}} \frac{\partial \lambda_{t+1}}{\partial c_{t+1,X}} \right)$$

The optimal quantity of reserves is a trade-off between the distortion caused by an extra unit of reserves this period $\beta E_t \left\{ \lambda_{t+1} \right\} \left(R_t - R_t^W \right)$, and the expectation of the ex-post benefit of those reserves next period $\beta R_t^W E_t \left(\mu_{t+1} \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_{t+1,X}}{y} \right)^{\frac{1}{\xi}-1} \right)$, plus a term related to the private consumption Euler equation $\frac{\phi_{t+1}}{\Gamma E_{t+1} \lambda_{t+2}} \frac{\partial \lambda_{t+1}}{\partial c_{t+1,X}}$ next period. If agents were risk neutral, $\frac{\partial \lambda_{t+1}}{\partial c_{t+1,X}} = 0$, then the benefit of holding reserves only comes from relaxing the future credit constraint.

The distortion, the spread between the home and foreign interest rate, is positive and increasing in F_t^{cb} since F_t^{fs} is negative and decreasing in F_t^{cb}. The expectation of the ex-post benefits side if positive but decreasing in F_t^{cb} since with each additional unit of reserves that the central bank holds at the end of period t, the probability of a crisis, and thus $E_t \left(\frac{\mu_{t+1}}{\lambda_{t+1}} \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(\frac{c_{t+1,X}}{y} \right)^{\frac{1}{\xi}-1} \right)$ falls. If the probability of a crisis in period $t+1$ is a continuously declining function of F_t^{cb}, then as long as $\Gamma > 0$, when the central bank is holding the optimal stock of reserves the probability of a crisis next period is positive. Conceptually it is easy to see why this is true. Once the probability of a crisis is zero the marginal benefit of an additional unit of reserves falls to zero. So as long as the marginal cost is positive, the stock of reserves that completely eliminates the possibility of a crisis is never optimal. As we will see when discussing numerical results, this optimal stock of reserves reduces the probability of a crisis, but is never large enough to reduce the probability of a crisis to zero.

Finally consider the case where there is not a crisis this period or any probability of a crisis next period, $\mu_t = E_t(\mu_{t+1}) = 0$. In this case the central bank's sole focus is on eliminating the spread between the domestic and foreign interest rates. If there was no non-negativity constraint for reserves then the central bank would hold a negative stock of reserves that is exactly equal to the economy's total external debt and completely supplant private borrowing. If the central bank did face a non-negativity constraint for reserves then the central bank would hold zero reserves and the shadow value of reserves would be proportional to the positive spread between the home and foreign interest rates.

6 Numerical results for the optimal stock of reserves

In this section we present the numerical results from the infinite-horizon dynamic model to show the mechanisms and the optimal stock of central bank foreign exchange reserves. We focus on the optimal policy under discretion.

6.1 Parameters and Calibration

The top panel of Table 1 presents the parameter values that we use to calculate the numerical results. Following Schmitt-Grohe and Uribe (2020), $\alpha = 0.31$, $\xi = 0.5$, $\beta = 0.91$, $R^W = 1.04$, and $\sigma = 2$. One of the key parameters for the existence of multiple equilibria is the elasticity of substitution between traded and non-traded goods. If this elasticity is too high, Schmitt-Grohe and Uribe (2020) show that there are no multiple equilibria. Bianchi

(2011) uses $\xi = 0.83$, and at this level there are not multiple equilibria. Benigno et al. (2013) follow the empirical estimate from Ostry and Reinhart (1992) of $\xi = 0.76$. Stockman and Tesar (1995) use $\xi = 0.44$. Akinci (2017) surveys the empirical literature estimating this elasticity and finds that estimates of this elasticity vary between 0.43 and 1.50 depending on the estimation methodology and the countries sampled. Akinci argues that empirical estimates tend to be lower for the emerging markets, and estimates using a few different methodologies put the elasticity around 0.5 in emerging market countries like Argentina and Uruguay.

Table 1: Parameter values

Parameter	Value	Description
		Structural Parameters
R^W	1.04	Annual gross world interest rate
σ	2	Inverse of intertemporal elasticity of consumption
κ	$0.27R^w$	Parameter in borrowing constraint
β	0.91	Subjective discount factor
x	1	endowment of traded goods
y	1	endowment of non-traded goods
α	0.31	Weight on traded goods in CES aggregator
ξ	0.5	Elasticity of substitution between traded/non-traded goods
Γ	0.05	Financial intermediation friction
		Discretization of State Space
$\left[\ln \frac{R^W_{\min}}{R^W} \quad \ln \frac{R^W_{\max}}{R^W}\right]$	$[-0.071, 0.071]$	Range for the world interest rate
$[-F_{\min} \quad -F_{\max}]$	$[0.2, 1.0]$	Range for total external debt
$\left[F^{cb}_{\min} \quad F^{cb}_{\max}\right]$	$[0, 0.5]$	Range for foreign exchange reserves
n_{Rw}	11	number of grid points for $\ln \frac{R^W_t}{R^W}$, equally spaced
n_F	300	number of grid points for $-F_t$, equally spaced
n_{Fcb}	300	number of grid points for F^{cb}_t, equally spaced

We make one adjustment to the calibration in Schmitt-Grohe and Uribe (2020) in order to ensure that a sudden stop crisis happens with sufficient frequency. While Schmitt-Grohe and Uribe (2020) focus on endowment shocks, our model is driven by shocks to the world interest rate. With the different shock process the probability of a crisis is different, and in order to ensure that the probability of a crisis is around 5% in the competitive equilibrium without policy intervention, we lower the coefficient in the borrowing constraint from $\kappa = 0.32R^W$ in Schmitt-Grohe and Uribe to $\kappa = 0.27R^W$.

We have no prior for the value of the financial intermediation friction Γ, and thus Γ is calibrated to match a certain value for the steady state level of private external borrowing:

$$\frac{\lambda}{\beta E(\lambda)} = R^W - \frac{\Gamma}{\beta} F^{fs}$$

where $\frac{\lambda}{E(\lambda)} < 1$ depends on the amount of uncertainty in the economy (we will discuss shocks shortly). If the value of Γ is too low then the steady state level of external debt is above the maximum limit and thus the borrowing constraint binds in the steady state. If it is too high then starting from the steady state equilibrium level of $-F^{fs}$ the short-term borrowing constraint does not cross the 45 degree line, and thus sudden stops are very rare and only occur after a sequence of negative shocks to R_t^W which raise the economy's external debt which is then followed by a large positive shock to R_t^W, triggering a sudden stop. But in order to have a meaningful probability of a sudden stop, we set $\Gamma = 0.05$, which makes the steady state level of external debt high enough to make a sudden stop possible following a sufficiently large shock when starting from a steady state and low enough that the constraint is not binding in the steady state.

Exogenous shocks to the model are shocks to the world interest rate, R_t^W. These shocks follow an AR(1) process with persistence coefficient 0.572 and standard deviation 0.02. To approximate the equilibrium, we use a time iteration procedure over a discretized state space, and the bottom panel of Table 1 provides information for the discretization of the state space. We discretize the interest rate shock into 11 grid points, the endogenous state $-F_t$ into 300 grid points and central bank reserves F_t^{cb} into 300 grid points. For ease of exposition, we denote the median R^W as the 'steady state R^W'.

The way that the model is written, there is only one endogenous state variable in the model, the total external debt $-F_{t-1}$. It is important to note that for a given value of the endogenous state in period $t-1$, $-F_{t-1}$, the policy variable F_t^{cb} is a choice variable with the constraint that $F_t^{cb} \geq 0$. The choice of F_t^{cb} will then affect the endogenous state variable in period t, $-F_t$, but in period $t+1$, it is the state variable $-F_t$ that matters.

Note that foreign reserves in themselves are not a state variable in the model, but represent a control variable at the discretion of the central bank (see the functional problem in appendix E). Since reserves cannot be negative, central banks must choose in advance the reserves that would be used to offset a sudden stop in the event of a crisis. This implies that the central bank must hold a stock of reserves in periods when there is a probability of a crisis, and this stock must be accumulated in periods when a future crisis becomes more likely.

As discussed in section 5 above, while the model admits multiple expectational equilibrium, we maintain a particular equilibrium selection criterion, and focus on the role of shocks to fundamentals to generate sudden stops. Following a shock to R_t^W, if agents' first-order conditions and the other equilibrium conditions in the model are satisfied at a level of external debt $-\hat{F}$ where the borrowing constraint is not binding (to the right of point B in Figure 2), we pick this as the equilibrium. If on the other hand agents' first-order conditions and the other equilibrium conditions in the model are satisfied at a level of external debt $-\hat{F}$ where the borrowing constraint is binding (to the left of point B), the equilibrium at the lower level of external debt (point C).

6.2 Numerical results

The policy function for the optimal choice of F_t^{cb} as a function of the endogenous state, $-F_{t-1}$, and the exogenous state, R_t^W, is presented in Figure 3. The blue solid line shows the central bank's optimal choice of F_t^{cb}, and the red dashed line plots the multiplier on the borrowing constraint, which changes from 0 to a positive number when the sudden stop occurs. All quantity variables like total external debt and the stock of central bank reserves are presented as a percent of GDP.

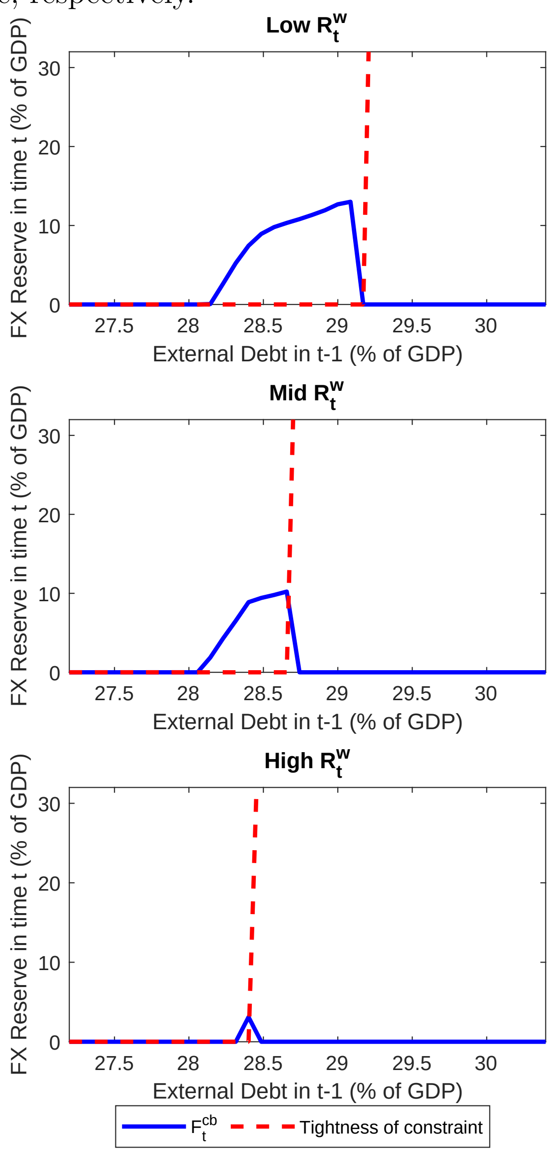
The optimal choice of F_t^{cb} is based on the marginal benefit of holding reserves, the reduction of the probability of a crisis in period $t+1$, and the marginal cost of holding reserves, is captured by the reduction in the economy's total external debt in period t, forcing the economy as a whole to save more than it otherwise would.

Begin with the middle panel in the figure, this panel plots the optimal choice of F_t^{cb} as a function of $-F_{t-1}$ when R_t^W is equal to its steady state value. The figure shows that as long as the initial stock of external debt is less than 28.5 percent of GDP, a crisis does not occur in period t. This means that the equilibrium $-F_t$ along the 45 degree line in Figure 2 remains to the right of point B when the initial stock of external debt is less than 28.5% of GDP. The policy function for F_t^{cb} for a low stock of initial external debt, say external debt less than 28% shows that when debt is this low, the probability of a sudden stop in period $t+1$ is zero and thus the central bank sees no need to distort the economy today by buying reserves as insurance against a possible crisis tomorrow. But the figure shows that for an external debt limit greater than 28% but less than 28.5% the central bank will start acquiring reserves F_t^{cb} as precaution against a large positive shock to the world interest rate in period $t+1$. As the initial stock of external debt increases the probability of a sudden stop increases, and thus the marginal benefit of F_t^{cb} increases. The central bank's optimal F_t^{cb} increases right up to the point where the level of external debt is high enough that a sudden stop crisis would have happened in period t. At the highest point, central bank reserves are about 10% of GDP.

The top panel of the figure shows the same policy function when R_t^W is below its steady state, and the bottom panel shows the same policy functions when R_t^W is above its steady state. First, notice that when R_t^W is below its steady state value a crisis in period t occurs at a higher level of initial external debt, while when R_t^W is above its steady state value the crisis occurs starting at a lower level of initial external debt. This is not surprising since a high value of R_t^W simply means that agents will delever to a point further to the left along the 45 degree line in Figure 2 and thus there is a greater chance that agents will delever to the left of point B. What is more interesting is the policy function for F_t^{cb} is much higher when the current shock is low than when it is high. Recall that the exogenous world interest rate R_t^W follows an AR(1) process. So a low value of R_t^W today implies that is is likely to go higher in the next period, and a high value of R_t^W implies that it is likely to be lower in the next period. When the shock is currently low an initial level of external debt in excess of 29% may not lead to a crisis in period t, but it may in the future when the interest rate mean reverts. Thus the central bank will seek to reduce the probability of a crisis next period by acquiring reserves, and at its height, when R_t^W is low, the central bank will buy reserves F_t^{cb}

up to 13% of GDP. Likewise when the current world interest rate is high, if the initial level of external debt is low enough that a crisis was not triggered in period t even when the interest rate was high, the probability of a crisis next period as the interest rate mean reverts is low, and as a consequence the policy function for F_t^{cb} remains close to zero.

Figure 3: The policy function for reserve accumulation (blue solid line) and the multiplier on the borrowing constraint (red dashed line) as a function of external borrowing and the exogenous state. Low, Mid and High R_t^W denote the lowest level, middle level and highest level of world interest rate, respectively.



In Figure 3 we focus on the benefits of the central bank holding F_t^{cb}, where the policy function for F_t^{cb} would increase in the region of the state space where the probability of a crisis next period was higher. In Figure 4 we instead focus on the costs of holding F_t^{cb}. In this figure we simulate the model over $T = 10^6$ periods and plot the density of the distribution of total external debt, $-F_t$, over these simulations. The blue solid line plots the density when the central bank does not engage in foreign exchange intervention and $F^{cb} = 0$, the red dashed line plots the density when the central bank engages in optimal foreign exchange intervention, described by the policy functions in Figure 3.

Focus initially on the blue solid line. When the central bank does not engage in foreign exchange intervention the density of $-F_t$ has a large mass around 28.5 percent of GDP and a long left tail. With no policy intervention the economy, after a long string of shock realizations of zero, the economy would settle to a steady state level of external debt a little less than 29%. Negative shocks to R_t^W would lead agents to hold more debt and positive shocks to R_t^W would lead agents to hold a little less debt. But as the density figure shows, at a point around an external debt level of 28% the density drops. This is where the sudden stop occurs at point B in Figure 2.[7] If the shock is large enough to trigger a sudden stop then the economy's total external debt falls to less than 18 percent of GDP and then begins a slow process of releveraging as the economy returns to the steady state.

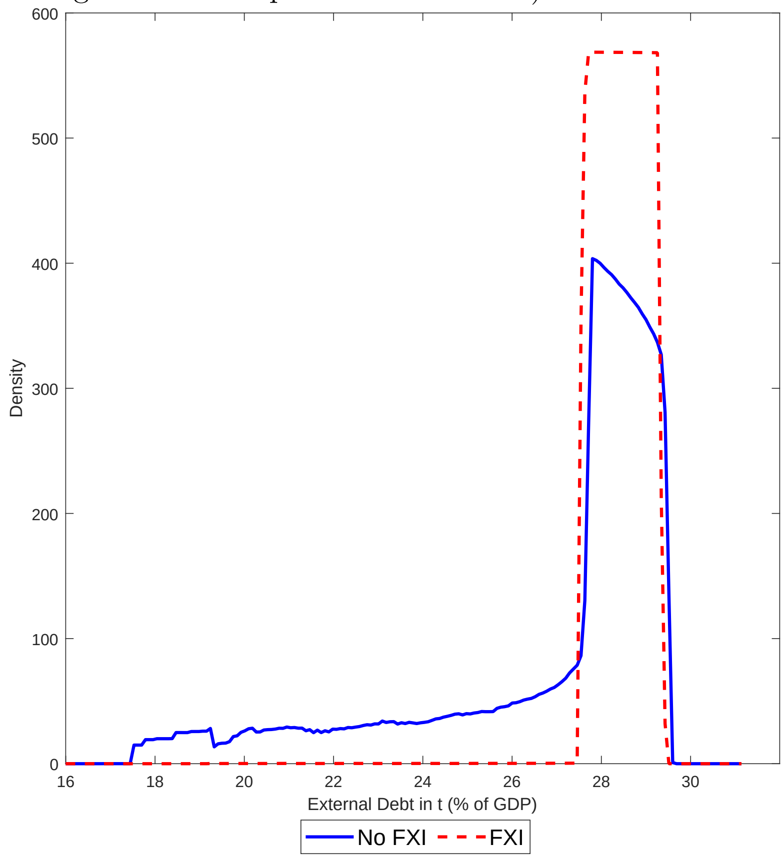
The density of external debt under optimal foreign exchange intervention shows that optimal policy nearly eliminates the probability of a sudden stop. While it is difficult to see the scale on this graph, but there is a small weight in the left tail in the optimal FXI density, but the weight is very close to zero. As discussed earlier, this probability is never zero, but as we will show in a later table, in these simulations the probability is small.

But the density plots in Figure 4 show that the density in the non-binding region, where the economy is not in a sudden stop, is shifted to the left under optimal FXI. As discussed earlier, optimal foreign exchange accumulation is insurance against a crisis, but the cost is that it forces the economy to save more than it otherwise would. In the absence of sudden stops, this would clearly be welfare reducing, since households are less patient than the rest of the world (as reflected in R^W), and desire to front load consumption. But the optimal policy

[7]The convex curve in Figure 2 is drawn when the initial level of external debt (and thus how far the convex curve shifts to the right) is at its steady state level. In this density, the economy is not always starting from the steady state level of initial external debt, and so the past history of shocks to R_t^W will affect the initial level of external debt and thus the exact level of external debt that triggers a sudden stop.

ensures that the benefits of foreign exchange intervention in response to foreign interest rate shocks more than offsets the costs of reduced external debt, so that expected utility is higher than would be the case under a competitive equilibrium.

Figure 4: The density of external debt in period t in the model without foreign exchange intervention and in the model with optimal foreign exchange intervention. The smooth parameter is 20 (average of between period $t-19$ and t).



We now turn to event analysis to examine the dynamics of a typical financial crisis in the model to see how foreign exchange intervention reduces the probability of a sudden stop. We construct an event analysis as follows. In a simulation path with length $T = 10^6$, a crisis is defined as a binding credit constraint $\mu_t > 0$ in period $t = 0$ in the competitive equilibrium with no foreign exchange intervention, and an event is a window of 11 periods from $t = -5$ to $t = 5$. We average all such events along the simulated path above. Figure 5 presents the path of the world interest rate, total external debt, the central bank's stock of foreign exchange reserves, tradable consumption, the price of non-tradables, the current account, the domestic interest rate, the multiplier on the borrowing constraint, and the spread between the domestic interest rate and the world interest rate during the average of these events. The solid blue line in the figure plots the event in economy with no foreign exchange intervention, and the dashed red line presents the responses of the same variables to the same path of the exogenous world interest rate and conditional on identical external debt in the first period of the event in the economy with optimal foreign exchange intervention.

The exogenous shock that triggers a crisis event is an increase in the world interest rate. The figure shows that the average crisis is triggered by an increase in the world interest rate from 3% to 7%. It is interesting to note that in the period before the crisis the world interest rate was slightly less than the steady state world interest rate of 4%. This indicates that at least some of the crisis episodes occur following a period of low interest rates which lead to a build-up external debt (shown in the middle figure in the top tow). This period of low borrowing costs is also reflected in the domestic interest rate panel of the figure. This shock to the world interest rate is enough to cause agents to de-lever to a point to the left of point B in Figure 2, triggering a sudden stop and leading to a rapid deleveraging to a much lower level of external debt. This deleveraging implies a sharp fall in traded goods consumption a large depreciation of the real exchange rate, and a sharp improvement in the current account.

These events, a rapid deleveraging and a sharp fall in traded goods consumption, a fall in the price of non-tradables, and an increase in the current account, describe the typical sudden stop in an emerging market economy.

The red dashed line plots the same variables in the same events (the window of ± 5 periods around a crisis in the economy without foreign exchange intervention), but now allowing the central bank to implement the optimal foreign exchange intervention, as show in the policy functions in Figure 3. Under optimal foreign exchange intervention, external debt is lower in the periods leading up to the crisis. The same size foreign interest rate shock now causes some deleveraging, but not enough to trigger a crisis. Thus there is some fall in traded goods consumption and the real exchange rate depreciation, but it is substantially ameliorated relative to the outcome without policy intervention.

The response of the domestic interest rate and the spread over the world interest rate provides an interesting insight into the optimal FXI response. Prior to the crisis there is a spread between the domestic interest rate and the world interest rate of about 4.5% in the economy without FXI and 6% in the economy with FXI. The central bank's holdings of foreign exchange reserves keeps domestic interest rates higher than in the equilibrium without policy intervention. The interest rate spread is higher due to the fact that financier's issue of foreign bonds $-F^{fs}$ is larger than in the case without policy intervention, but despite that, external net debt is lower. When the economy is hit by the interest rate shock, in the case without intervention, the domestic interest rate rises, but by less than the world rate, as the large deleveraging reduces the spread over the world rate. But contrast, with foreign exchange intervention, the private deleveraging is substantially mitigated (the fall in $-F^{fs}$ is much less), and the domestic interest spread is relatively unchanged.

Finally, the graph of the central bank's optimal stock of reserves in the bottom right hand corner shows that in the periods leading up to the crisis the central bank adds to the stock of reserves as a precautionary measure in the face of low world interest rates. Then in the period of the high interest rate shock the central bank sells some of their stock of reserves. Although it is interesting to note that the central bank does not need to sell a large part of their stock to prevent a crisis. Much of the work of preventing the sudden stop is done ex-ante and by holding a stock of reserves the central bank reduces the economy's total external debt, and thus makes a crisis less likely.

In the discussion above we explained why it was not optimal for the central bank to use foreign reserve accumulation and intervention to completely eliminate the possibility of a sudden stop. We show below that while optimal FXI dramatically reduces the probability of a sudden stop, the probability does not go to zero. Figure 6 illustrates the characteristics of a sudden stop event under optimal central bank foreign exchange intervention. The construction of the average event is similar as before.

The first point to notice from the figure is that the size of the average shock that precipitates a crisis event in the economy with FXI is much larger than the corresponding shock size in the economy without policy intervention. With optimal FXI, the crisis is triggered after a rise in the world interest rate from 2% to 10%, whereas without policy intervention, a crisis occurred after an increase in the interest rate from 3% to 7%. The bottom right hand graph in the figure is the central bank's stock of foreign exchange reserves. Prior to the shock, the optimal stock of central bank reserves is right around 8% of GDP, but immediately following the interest rate shock the central bank reduces its stock of reserves to zero. But as we can see from the behaviour of the multiplier on the borrowing constraint and the other variables like total external debt, tradable goods consumption, and the relative price of non-tradeables, this sale of central bank reserves was not enough to prevent a crisis, and despite optimal central bank reserve holding, the central bank ran out of reserves to sell and the economy fell into a sudden stop.

Table 2: Crisis probability, mean, and steady state levels of external debt in the model with and without optimal foreign exchange intervention.

	No FXI	Optimal FXI
Crisis Probability	0.0481	0.0003
'Steady State' $-F_t$	28.6	28.4
Mean $-F_t$	26.9	28.4

Notes: This table reports the probability of a crisis and mean and median levels of external debt under the competitive equilibrium with no policy intervention and under optimal foreign exchange intervention. The simulation period is $T = 10^6$.

Table 2 calculates the probability of a crisis, the 'steady state' level of external debt, and the mean level of external debt as a percent of GDP in the model with and without FXI calculated from the same simulation over $T = 10^6$ periods.[8] As discussed earlier, the coefficient in the borrowing constraint κ is calibrated to ensure that in the model without policy intervention the probability of a crisis is around 5%. Then using the same parameters and the same shock process the probability of a crisis falls to 0.03% when the central bank engages in optimal foreign exchange intervention. The 'steady state' level of external debt

[8] The dynamic models have a stationary distribution of external debts. To facilitate comparison with the deterministic steady state without shocks, we still use the term 'steady state' to describe the situation in which the economy stays when world interest rate is at its middle level for a long time given policy functions obtained in a dynamic model. This situation is sometimes called 'risky steady state' (Coeurdacier et al. (2011)).

is lower under optimal FXI than it is under the competitive equilibrium without policy intervention, indicating that by holding the optimal stock of foreign exchange reserves, the central bank forces 'steady state' excess savings. However, the mean level of external debt is lower in the model without policy intervention due to the strong left skew on the distribution of external debt in this case. Put differently, the economy's external debt in normal times is reduced as a result of the optimal foreign exchange policy, but by avoiding crises which involve large episodes of de-leveraging, the unconditional average level of external debt is actually higher due to the optimal policy. This offers another sense in which optimal intervention policy avoids an underborrowing outcome in the small economy.

7 Summary and Conclusion

This paper presents a simple model where a sudden stop can arise in a small open economy. There is a pecuniary externality in that agents do not internalize the effect of their deleveraging on the price of collateral, and because of this, the economy call fall into a self-fulfilling sudden stop equilibrium. A policy maker that does internalize the effect of deleveraging on the price of collateral will set policy to keep the economy out of this sudden stop equilibrium.

When the tools available to the policy maker are the central bank purchases and sales of foreign bonds, foreign exchange intervention, some interesting questions arise. The central bank's holdings of foreign bonds can't be negative, so in order to sell bonds to prevent a sudden stop during a crisis, the central bank must buy a stock of bonds before the crisis. This central bank purchase of foreign bonds is distortionary since it forces economy wide savings to be higher than it otherwise would be. This paper attempts to provide a framework for assessing the marginal costs and marginal benefits of holding a stock of reserves in order to pinpoint the optimal stock of foreign bonds held by the central bank to safeguard against a sudden stop.

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Not For Publication Technical Appendix

A Variables and Equations

Variables in the model:

$c_{t,X},\ c_{t,Y},\ R_t,\ B_t^{cb}, B_t, B_t^{fs},\ F_t^{fs}, \lambda_t,\ \mu_t,\ p_t,\ T_t, \Pi_t, F_t^{cb}, \tau_t$ taking $B_0,\ F_0^{fs},\ F_0^{cb}$ and R_t^W as given

Equations:

$c_{t,X}$:

$$\left(\left[\alpha\left(c_{t,X}\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_{t,Y}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}\right)^{-\sigma}\times$$

$$\left[\alpha\left(c_{t,X}\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_{t,Y}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}-1}\alpha\left(c_{t,X}\right)^{\frac{-1}{\xi}}=\lambda_t$$

$c_{t,Y}$:

$$\left(\left[\alpha\left(c_{t,X}\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_{t,Y}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}\right)^{-\sigma}\times$$

$$\left[\alpha\left(c_{t,X}\right)^{\frac{\xi-1}{\xi}}+(1-\alpha)\left(c_{t,Y}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}-1}(1-\alpha)\left(c_{t,Y}\right)^{\frac{-1}{\xi}}=p_t\lambda_t$$

$R_t : B_t + B_t^{cb} + B_t^{fs} = 0$
$B_t^{cb} : B_t^{cb} + F_t^{cb} = 0$
$B_t : \frac{\lambda_t-\mu_t}{R_t} = \beta\lambda_{t+1}$
$B_t^{fs} : B_t^{fs} + F_t^{fs} = 0$
$F_t^{fs} : R_t = R_t^W - \frac{\Gamma}{\beta}F_t^{fs}$
$\lambda_t : c_{t,X} + p_t c_{t,Y} + B_t = x_t + p_t y_t + R_{t-1}B_{t-1} + T_t + \Pi_t$
$\mu_t : \mu_t = 0$ or $-B_t = \kappa\left(x_t + p_t y_t\right)$
$p_t : c_{t,Y} = y_t$
$T_t : T_t = R_{t-1}^W F_{t-1}^{cb} + R_{t-1}B_{t-1}^{cb} - F_t^{cb} - B_t^{cb}$
$\Pi_t : \Pi_t = R_{t-1}^W F_{t-1}^{fs} + R_{t-1}B_{t-1}^{fs} - F_t^{fs} - B_t^{fs}$
$F_t^{cb} : policy$

B Proof of effectiveness of foreign exchange intervention when $\Gamma > 0$

To see how central bank foreign exchange intervention can have a macroeconomic effect when the economy is in a non-binding equilibrium and $\Gamma > 0$, start from the non-binding equilibrium where private sector demand for international bonds is given by F_t^{fs}, the central bank does not change their stock of international bonds $F_t^{cb} = F_{t-1}^{cb}$, and thus traded goods consumption in periods t and $t+1$ is given by:

$$\begin{aligned} c_{t,X} &= x - F_t^{fs} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb} + R_{t-1}^W F_{t-1}^{cb} \\ c_{t+1,X} &= x - F_{t+1}^{fs} + R_t^W F_t^{fs} - F_{t+1}^{cb} + R_t^W F_t^{cb} \end{aligned}$$

Now suppose instead the central bank engages in a sale of some of their stock of international bonds, $F_t^{cb\prime} < F_{t-1}^{cb}$. Following this central bank intervention the private sector changes their demand for international bonds from F_t^{fs} to $F_t^{fs\prime}$. The new consumption level following this central bank intervention and private sector response is:

$$\begin{aligned} c'_{t,X} &= x - F_t^{fs\prime} + R_{t-1}^W F_{t-1}^{fs} - F_t^{cb\prime} + R_{t-1}^W F_{t-1}^{cb} \\ c'_{t+1,X} &= x - F_{t+1}^{fs} + R_t^W F_t^{fs\prime} - F_{t+1}^{cb} + R_t^W F_t^{cb\prime} \end{aligned}$$

We want to prove that when $\Gamma > 0$, the central bank sale of international bonds led to higher external debt, $-F_t^{fs} - F_t^{cb} < -F_t^{fs\prime} - F_t^{cb\prime}$ and thus by extension, higher traded goods consumption in period t.

If $F_t^{fs\prime} < F_t^{fs}$, the proof is trivial (this would be where the central bank sold foreign bonds, and as a result the private sector also sold foreign bonds). Turning to the case where $F_t^{fs\prime} > F_t^{fs}$: combine the household's first order condition with respect the F in equation (6) with the equilibrium condition for foreign borrowing in equation (14).

$$\frac{\lambda_t - \mu_t}{\beta \lambda_{t+1}} = R_t^W - \frac{\Gamma}{\beta}\left(F_t^{fs}\right) \tag{B.1}$$

Thus in a non-binding equilibrium:

$$\frac{\lambda_t}{\beta \lambda_{t+1}} + \frac{\Gamma}{\beta}\left(F_t^{fs}\right) = \frac{\lambda'_t}{\beta \lambda'_{t+1}} + \frac{\Gamma}{\beta}\left(F_t^{fs\prime}\right) \tag{B.2}$$

where λ_t and λ_{t+1} are the marginal utilities of consumption when traded goods consumption is $c_{t,X}$ and $c_{t+1,X}$, and λ'_t and λ'_{t+1} are the marginal utilities of consumption when traded goods consumption is $c'_{t,X}$ and $c'_{t+1,X}$. If $F_t^{fs\prime} > F_t^{fs}$ and $\Gamma > 0$ then the above equality becomes:

$$\frac{\lambda_t}{\beta \lambda_{t+1}} > \frac{\lambda'_t}{\beta \lambda'_{t+1}} \tag{B.3}$$

which implies that the central bank intervention by selling international bonds lowers the domestic interest rate. This implies:

$$c_{t,X} - c_{t+1,X} < c'_{t,X} - c'_{t+1,X}$$

which implies that $-F_t^{fs} - F_t^{cb} < -F_t^{fs\prime} - F_t^{cb\prime}$. Note that the switch from the equality in equation (B.2) to the inequality in equation (B.3) relied on the fact that $\Gamma > 0$. If there were no intermediary frictions $\Gamma = 0$, then the inequality in equation (B.3) would be an equality and $-F_t^{fs} - F_t^{cb} = -F_t^{fs\prime} - F_t^{cb\prime}$. Any central bank sale of international bonds would be exactly offset by a private sector purchase of international bonds, leaving total external debt unchanged.

C Proof of multiple equilibria

We want to prove that for an initial level of external debt of $-F^A$, where $-\tilde{F} < -F^A < -\bar{F}$. The short term borrowing constraint given by:

$$-F_t \leq \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W_{t-1} F^A - F_t \right)^{\frac{1}{\xi}} \right)$$

will have two points $-F^B$ and $-F^C$ where:

$$\begin{aligned} -F^B &= \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W F^A - F^B \right)^{\frac{1}{\xi}} \right) \\ -F^C &= \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W F^A - F^C \right)^{\frac{1}{\xi}} \right) \end{aligned}$$

We know that when $-F^A < -\bar{F}$, the borrowing constraint is not binding when $-F_t = -F^A$

$$-F^A < \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W F^A - F^A \right)^{\frac{1}{\xi}} \right)$$

We know that if $-\tilde{F} < -F^A$, the point $-\hat{F}$ where the slope of the borrowing constraint is equal to one, $\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(x + R^W_{t-1} F^A - \hat{F} \right)^{\frac{1}{\xi}-1} = 1$ must violate the borrowing constraint:

$$-\hat{F} > \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W F^A - \hat{F} \right)^{\frac{1}{\xi}} \right)$$

Furthermore we know that when $\xi < 1$, the borrowing constraint, $\kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W_{t-1} F^A - F_t \right)^{\frac{1}{\xi}} \right)$ it convex. If at the point $-F_t = -F^A$, the slope of the borrowing constraint is greater than one, $\kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(x + R^W_{t-1} F^A - F^A \right)^{\frac{1}{\xi}-1} > 1$, and the slope of the same borrowing constrain is equal to one when $-F_t = -\hat{F}$, then it must be that $-\hat{F} < -F^A$. Thus is the borrowing constraint is non-binding when $-F_t = -F^A$ and yet it is violated when $-F_t = -\hat{F}$, then by the intermediate value theorem there must be a point $-F_t = -F^B$ where $-\hat{F} < -F^B < -F^A$ where the borrowing constraint holds with equality.

At the other end, at the maximum amount of deleveraging (i.e. the minimum $-F_t$), traded goods consumption in period t is zero, $x + R^W F^A - F_t = 0$. Thus at the point where $-F_t = -x - R^W F^A$, the short term collateral constraint is equal to κx, under what parameterization is the constraint not binding at the minimum amout of traded goods consumption, $\kappa x > -x - R^W F^A$. Thus we ask, what is the minimum set of parameter values where $-R^W F^A < (1+\kappa)\, x$ holds?

We know that $-F^A < -\bar{F}$ where:

$$-\bar{F} = \kappa \left(x + \frac{1-\alpha}{\alpha} \left(x + R^W_{t-1} \bar{F} - \bar{F} \right)^{\frac{1}{\xi}} \right)$$

Define $a = \kappa \frac{1-\alpha}{\alpha} \frac{1}{\xi} \left(x + R^W_{t-1} \bar{F} - \bar{F} \right)^{\frac{1}{\xi}-1}$, then we can rearrange the above equality $-\bar{F} =$

$\kappa x + a\xi \left(x + R^W_{t-1}\bar{F} - \bar{F}\right)$, or $-\bar{F} = \frac{(a\xi+\kappa)x}{\left(a\xi R^W_{t-1}+(1-a\xi)\right)}$. Since $-F^A < -\bar{F}$ then $-F^A < \frac{(a\xi+\kappa)x}{\left(a\xi R^W_{t-1}+(1-a\xi)\right)}$. From this it should be obvious that $-R^W F^A < (1+\kappa)\,x$ will also hold as long as $a\xi < 1$, where $a\xi = \kappa\frac{1-\alpha}{\alpha}\left(x + R^W_{t-1}F^A - F^A\right)^{\frac{1}{\xi}-1}$. We know that $x + R^W_{t-1}F^A - F^A < 1$, so as long as $\kappa < \frac{\alpha}{1-\alpha}\left(x + R^W_{t-1}F^A - F^A\right)^{1-\frac{1}{\xi}} < \frac{\alpha}{1-\alpha}$ in our benchmark parameterization $\alpha = 0.31$, so as long as $\kappa < 0.44$ the constraint is non-binding at point where traded goods consumption is zero. (we use $\kappa = 0.27R^W$ where $R^W = 1.04$)

Furthermore we know that at this point since traded goods consumption is zero, $x + R^W F^A - F_t = 0$, the slope of the borrowing constraint $\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(x + R^W_{t-1}F^A - F_t\right)^{\frac{1}{\xi}-1} = 0$. Since the slope of the borrowing constraint is one when $-F_t = -\hat{F}$, then $-x - R^W F^A < -\hat{F}$. By the intermediate value theorem there must be a point $-x - R^W F^A < -F^C < -\hat{F}$ where the borrowing constraint holds with equality.

D Social planner's problem

The social planner's problem can be written as

$$\begin{aligned}
\mathcal{L}_{\{c_{t,X},F_t,F^{cb}_t\}} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ & U(c_{t,X}) + \eta_t \left(x + R^W_{t-1}F_{t-1} - F_t - c_{t,X}\right) \\
& + \gamma_t \left[\lambda_t - \beta\lambda_{t+1}\left(R^W_t - \frac{\Gamma}{\beta}(F_t - F^{cb}_t)\right)\right] \\
& + \mu_t \left[\kappa \left[x + y\frac{1-\alpha}{\alpha}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}}\right] + F_t\right] \\
& + \phi_t \left(F^{cb}_t\right)
\end{aligned}$$

given F_{-1} and shocks $\{R^W_t\}_0^\infty$. Lagrange multipliers $\eta_t \geq 0$, $\gamma_t \geq 0$, $\mu_t \geq 0$ and $\phi_t \geq 0$.

The optimality conditions can be written as follows. For traded consumption good, $c_{t,X}$:

$$\eta_t = U_x(c_{t,X}) + \mu_t\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}-1} + \gamma_t\frac{\partial\lambda_t}{\partial c_{t,X}} - \gamma_{t-1}R_{t-1}\frac{\partial\lambda_t}{\partial c_{t,X}} \tag{D.1}$$

For net external asset position F_t:

$$\eta_t = \mathbb{E}_t\beta R^W_t\eta_{t+1} + \mu_t + \Gamma\gamma_t\mathbb{E}_t\lambda_{t+1} \tag{D.2}$$

For reserve F^{cb}_t:

$$\phi_t = \Gamma\gamma_t\mathbb{E}_t\lambda_{t+1} \tag{D.3}$$

And complementarity conditions:

$$\eta_t\left(x + R^W_{t-1}F_{t-1} - F_t - c_{t,X}\right) = 0, \eta_t \geq 0$$

$$\gamma_t\left[\lambda_t - \beta\mathbb{E}_t\lambda_{t+1}\left(R^W_t - \frac{\Gamma}{\beta}(F_t - F^{cb}_t)\right)\right] = 0, \gamma_t \geq 0$$

$$\mu_t \left[\kappa\left[x + y\frac{1-\alpha}{\alpha}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}}\right] + F_t\right] = 0, \mu_t \geq 0 \tag{D.4}$$

$$\phi_t\left(F_t^{cb}\right) = 0, \phi_t \geq 0$$

Consider the planner's policy under discretion, therefore $\gamma_{t-1} = 0$. By definition, $\lambda_t = U_x(c_{t,X})$.

From equation (D.1), the social marginal utility of traded consumption consists of three parts: marginal utility of traded goods consumption λ_t, the benefit of relaxing credit constraint when credit constraint is binding $\mu_t > 0$, and the intertemporal reallocation of consumption $\gamma_t \frac{\partial \lambda_t}{\partial c_{t,X}} \leq 0$.

Equation (D.2) shows the social optimal Euler equation for external saving/borrowing. Social marginal cost of saving is the foregone consumption η_t, while the social marginal benefit of saving consists of relaxing current credit constraint, increasing future consumption via saving in abroad and the intertemporal shifting of consumption, captured by γ_t.

Changes in reserves F_t^{cb} will change domestic interest rate, $\partial R_t/\partial F_t^{cb} = \Gamma/\beta$. Equation (D.3) shows that when the reserve hits its lower bound below and reserves are decreased by one unit, the marginal cost is captured by the shadow price ϕ_t (public borrowing when negative), while the marginal benefit is the product of the change of domestic interest rate (Γ), private marginal utility of consumption λ_{t+1}, and the shadow price of shifting consumption intertemporally (λ_t).

Substituting out $\gamma_t = \frac{\phi_t}{\Gamma \mathbb{E}_t \lambda_{t+1}}$ from the FOCs, and the social Euler equation can be written as

$$\begin{aligned} & \lambda_t + \mu_t\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}-1} + \frac{\phi_t}{\Gamma\mathbb{E}_t\lambda_{t+1}}\frac{\partial\lambda_t}{\partial c_{t,X}} \\ = \; & \mathbb{E}_t\beta R_t^W\left\{\lambda_{t+1} + \mu_{t+1}\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t+1,X}}{y}\right)^{\frac{1}{\xi}-1} + \frac{\phi_{t+1}}{\Gamma\mathbb{E}_{t+1}\lambda_{t+2}}\frac{\partial\lambda_{t+1}}{\partial c_{t+1,X}}\right\} + \mu_t + \phi_t \end{aligned}$$

which can be rewritten as

$$\mathbb{E}_t\beta\left\{(R_t^W - R_t)\lambda_{t+1}\right\} - \mu_t\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t,X}}{y}\right)^{\frac{1}{\xi}-1}$$

$$+\mathbb{E}_t\beta R_t^W\left\{\mu_{t+1}\kappa\frac{1-\alpha}{\alpha}\frac{1}{\xi}\left(\frac{c_{t+1,X}}{y}\right)^{\frac{1}{\xi}-1} + \frac{\phi_{t+1}}{\Gamma\mathbb{E}_{t+1}\lambda_{t+2}}\frac{\partial\lambda_{t+1}}{\partial c_{t+1,X}}\right\} + \phi_t\left(1 - \frac{1}{\Gamma\mathbb{E}_t\lambda_{t+1}}\frac{\partial\lambda_t}{\partial c_{t,X}}\right) = 0$$

E Optimal FXI in an infinite horizon model

This section presents the policy maker's problem under discretion in an infinite horizon model. As usual, the policy maker's problem is defined as follows: a path of reserve holding $\{F_t^{cb}\}_{t=0}^{\infty}$ that maximizes the representative domestic household's objective function, subject to the constraints in the competitive equilibrium defined in the main text. In order to make the expressions of constraints more concisely, we make the following equivalent changes of variables. Let the exogenous variable be $s_t \equiv R_t^w$, total external borrowing at the beginning of period t, $f_{t-1} \equiv -R_{t-1}^W F_{t-1}$ and reserve holding at the beginning of period t, $f_{t-1}^{cb} \equiv F_{t-1}^{cb} R_{t-1}^W$.

Therefore the end-of-period portfolio can be written as $F_t = -f_t/R_t^W$ and $F_t^{cb} = f_t^{cb}/R_t^W$. The Bellman equation for the policy maker's problem can be written as,

$$\mathbf{P_1} : v(f_{t-1}, s_t) = \max_{\{c_{t,X}, f_t, f_t^{cb}\}} \{u(c_t) + \beta \mathbb{E}_t [v(f_t, s_{t+1})]\} \tag{E.1}$$

subject to

$$c_{t,X} = x_t - f_{t-1} + \frac{f_t}{R_t^w} \tag{E.2}$$

$$u_x(c_t)(1-\mu_t) = \beta R_t \mathbb{E}_t u_x(c_{t+1}) \tag{E.3}$$

$$\frac{f_t}{R_t^w} \leq \kappa (x_t + p_t y_t) \tag{E.4}$$

$$\mu_t \left[\kappa (x_t + p_t y_t) - \frac{f_t}{R_t^w}\right] = 0, 0 \leq \mu_t < 1$$

$$p_t = \frac{1-\alpha}{\alpha} \left(\frac{c_{t,X}}{y_t}\right)^{\frac{1}{\xi}} \tag{E.5}$$

$$R_t = R_t^w + \frac{\Gamma}{\beta} \frac{f_t + f_t^{cb}}{R_t^w} \tag{E.6}$$

where aggregate consumption and marginal utility can be written as

$$c_t = \left[\alpha (c_{t,X})^{\frac{\xi-1}{\xi}} + (1-\alpha)(y_t)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$$

$$u_x(c_t) = \alpha c_t^{-\sigma} (c_t/c_{t,X})^{\frac{1}{\xi}}$$

Note that central bank policy f_t^{cb} only appears in equation (E.6) as a control variable and could change domestic bond interest rate R_t, which can facilitate computing an optimal allocation substantially.

Figure 5: An event in a typical financial crisis when there is no foreign exchange intervention (blue solid lines) and the corresponding event when there's foreign exchange intervention (red dashed lines) along a simulated period of $T = 10^6$. A crisis occurs at period $t = 0$ in the competitive equilibrium without policy intervention. Interest rates, Lagrange multiplier and nontraded good price are expressed in level and other variables are expressed as percentage point of GDP.

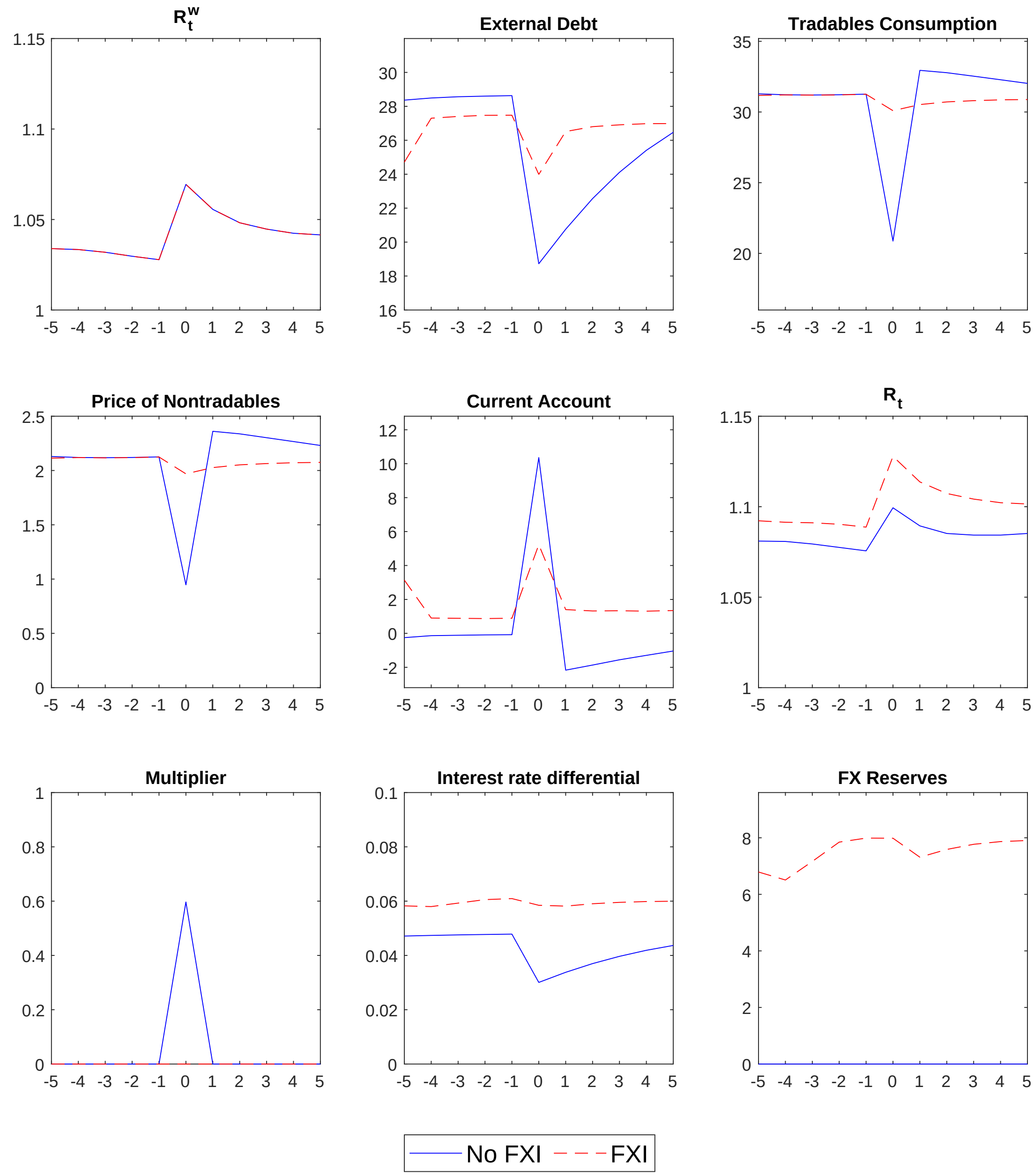

Figure 6: An event in a typical financial crisis that following a shock that is large enough to cause a crisis when foreign exchange intervention is used (red dashed lines) and the corresponding event in the competitive equilibrium without foreign exchange intervention (blue solid lines) along a simulated period of $T = 10^6$. A crisis occurs at period $t = 0$ in the economy with optimal policy intervention. Interest rates, Lagrange multiplier and nontraded good price are expressed in level and other variables are expressed as percentage point of GDP.