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The Effect of Central Bank Credibility on Forward Guidance in an Estimated New Keynesian Model[*]

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Abstract

This paper examines the effectiveness of forward guidance in an estimated New Keynesian model with imperfect central bank credibility. We estimate credibility for the U.S. Federal Reserve with Bayesian methods exploiting survey data on interest rate expectations from the Survey of Professional Forecasters (SPF). The results provide important takeaways: (1) The estimate of Federal Reserve credibility in terms of forward guidance announcements is relatively high, which indicates a degree of forward guidance effectiveness, but still one that is below the fully credible case. Hence, anticipation effects are attenuated and, accordingly, output and inflation do not respond as favorably to forward guidance announcements. (2) Imperfect central bank credibility is an important feature to resolve the so-called "forward guidance puzzle," which the literature shows arises from the unrealistically large responses of macroeconomic variables to forward guidance statements in structural models with perfect credibility. (3) Imperfect monetary authority credibility can also explain the evidence of forecasting error predictability based on forecasting disagreement found in the SPF data. Thus, accounting for imperfect credibility is important to model the formation of expectations in the economy and to understand the transmission mechanism of forward guidance announcements.

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1 Introduction

Since the $2007 - 2009$ financial recession, central bank forward guidance has been an essential monetary policy tool. For instance, when short-term interest rates reached the zero lower bound (ZLB) in the aftermath of the financial recession, the U.S. Federal Reserve provided guidance on the future course of interest rates. The Federal Open Market Committee (FOMC) first implemented forward guidance in their December 2008 statement: "the Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time." This type of lower-for-longer policy is predicted to have beneficial effects on the economy as described by Eggertsson and Woodford (2003) and Woodford (2003). In addition, when interest rates are away from the ZLB, forward guidance can provide clarification and transparency about future monetary policy. As explained by Williams (2013), greater clarity about the future policy path from forward guidance can help households and businesses make better investment decisions and boost the economy.

The effectiveness of forward guidance, however, rests on the perceived credibility of the central bank to follow through with its statements. Standard macroeconomic models often consider the case of a fully credible monetary authority. However, this assumption ignores a key channel through which forward guidance can affect the economy. If the central bank is perceived as trustworthy, agents are likely to internalize how future statements about policy affect their decisions today. If not, the effect on the economy from forward guidance is not as strong. Indeed, Goodfriend and King (2016) recognize this stating that "forecasts, and policy, should not be based solely on forecasts from a model that assumes full credibility in the stated policy path." Thus, it is important to examine how the effectiveness of forward guidance depends on the credibility of the central bank.

This paper studies the effects of forward guidance with imperfect central bank credibility. A standard New Keynesian model augmented with standard macroeconomic persistence features (that is, with price stickiness, price indexation, habit formation, and interest rate inertia) is employed. Following Del Negro et al. (2012) and Laséen and Svensson (2011), forward guidance is implemented by adding anticipated or forward guidance shocks to the monetary policy rule. The model is estimated using Bayesian methods with data on expectations—including interest rate expectations—from the Survey of Professional Forecasters (SPF).

Private agents that believe the central bank announcements about forward guidance are assumed to follow the full-information rational expectations (FIRE) typically employed in the literature. Those agents who do not believe central bank announcements about forward guidance, instead, form forecasts based on a data-driven VAR(1) in output, inflation, and interest rates which effectively disregards all forward guidance announcements and only responds to the policy announced when it materializes.[1] A key parameter in our analysis is $0 \leq \tau \leq 1$, which defines the weight assigned by private agents to the belief that the monetary authority forward guidance commitments are credible and would be honored. If $\tau \to 1$, private agents believe the central bank to be perfectly credible and all announcements about forward guidance to be honored. Thus, in that limiting case, aggregate expectations follow FIRE and we are back in the standard setup in the literature (e.g., Del Negro et al. (2012)). If $\tau \to 0$, agents do not perceive the monetary authority to be credible and ignore forward guidance statements altogether. Aggregate expectations then do not contain forward guidance information and private agents do not react to it.

The results from our estimated model show a number of takeaways. First, a distinctive contribution of our paper regards our use of Bayesian estimation procedures and the SPF dataset to estimate the degree of central bank credibility in regards to forward guidance. We utilize expectations of the interest rate and other macro aggregates from the SPF to help identify forward guidance shocks. The estimate of the credibility parameter (i.e., τ) in terms of forward guidance announcements hovers around 0.8. Since the U.S. central bank is perceived as less than fully credible in its forward guidance announcements, there exists less immediate and overall anticipation effects on the economy from forward guidance than under the perfectly credible case. The impulse response functions and variance decomposition results in this paper show that the responses of output and inflation to forward guidance shocks do not respond as favorably relative to the scenario of a perfectly credible central bank.

Second, we also show that imperfect credibility is another important feature that can contribute to resolve the so-called "forward guidance puzzle". Del Negro et al. (2012) explain that the forward guidance puzzle arises because standard New Keynesian models produce unusually large responses to forward guidance news. The credibility estimate indeed is below the fully credible case and this dampens the power of forward guidance. Our evidence suggests that the attenuation that results from a forward guidance policy that is imperfectly credible can go a long ways in reconciling the standard New Keynesian workhorse model

[1] The structure of the model borrows from the axiomatic approach of Branch and McGough (2009) to represent aggregate expectations as a weighted sum of heterogenous private sector forecasts. The Appendix suggests that the mixture of forecasting strategies in the aggregate expectations can arise as an equilibrium from an evolutionary game-theoretic setup when incorporating central bank credibility and forward guidance. This offers an alternative take on the formation of expectations which highlights the importance of non-cooperative games between the central bank and the private sector in our understanding of expectations.

with the empirical evidence that points out more modest effects of forward guidance.

Third, we provide (compelling) evidence that our model captures central bank credibility in terms of forward guidance and aligns well with the data. For instance, the estimate of the monetary authority credibility parameter τ appears robust. The estimate of τ is similar to the benchmark results when we consider alternative forecasting models for those agents who do not believe forward guidance statements. The estimate of τ does not noticeably change when habits in consumption or price indexation are turned off. In addition, the model of imperfect central bank credibility cross-validates well with other important features of the SPF data—in particular, with the predictability of forecasting errors based on forecasting disagreements. In both simulated data from our model and in the SPF data, we compare the empirical relationship between forecast errors and forecast disagreements with a standard regression. Our model of imperfect credibility can display comovements between the previously mentioned variables at different horizons that are broadly consistent with the comovements implied by the SPF dataset. In contrast, the perfectly credible central bank scenario (i.e., $\tau = 1$) cannot capture those features of the SPF data.

Finally, we then examine additional alternative estimation strategies as a further robustness check. First, the main results do not substantially change if the model is estimated over our full sample ($1981:Q3-2018:Q4$), the non-ZLB subsample ($1981:Q3-2008:Q4$), or the Great Moderation period ($1985:Q1-2007:Q3$). In particular, the estimate of τ is high indicating a high level of trust in the central bank, but still below the fully credible case and similar to our baseline estimate for the full sample. Second, our results are largely robust to a more agnostic prior belief about τ. The prior distribution in the baseline exercise was centered around a high degree of central bank credibility. When a more agnostic prior belief is assumed, our estimate of τ does not noticeably change in relation to the benchmark case. Third, the results are robust if $t+1$ expectations correspond to the one-period ahead forecasts in the SPF instead of corresponding to the nowcast as in our benchmark mapping between the data and the model.

In summary, by using Bayesian estimation procedures and SPF expectations data, we provide an estimate of the U.S. Federal Reserve credibility in relation to forward guidance. While we obtain a high level of central bank credibility from our evidence, the Federal Reserve is perceived as less than fully credible. Thus, our paper shows that accounting for imperfect credibility is important to model the formation of expectations in the economy and particularly so for the transmission mechanism of forward guidance announcements.

1.1 Contribution to the Literature

There exists a growing strand of the monetary policy literature focused on the transmission mechanism through which forward guidance is thought to operate. This transmission channel relies on anticipation effects driven by a credible commitment to future policy. The evidence suggests that forward guidance moves expectations but only partially (Ferrero and Secchi (2009); Ferrero and Secchi (2010); Hubert (2014); Hubert (2015a); Hubert (2015b)). Mainstream theory suggests that the anticipation effects are simply too strong within the standard class of general equilibrium models to be consistent with the empirical evidence (the so-called "forward guidance puzzle" of Del Negro et al. (2012)).[2]

Some authors like McKay et al. (2016) have argued that capital market imperfections can be part of the story. Other papers have analyzed the expectations formation process of agents. Gauss (2015) and Andrade et al. (2019) show that heterogenous expectations in an economy can influence the power of forward guidance. Cole (2020a) and Cole (2020b) explain that the rational expectations assumption can overstate the benefits of forward guidance relative to a more realistic adaptive learning rule.

The effectiveness of forward guidance has also been analyzed via the communications channel. Campbell et al. (2019) find that FOMC forward guidance information has limited power at long horizons. De Graeve et al. (2014) argue that the effects of forward guidance on the economy can have more positive effects if its length is tied to the future condition of the economy (threshold-based forward guidance).[3]

The present paper is also related to prior research exploring the conduct of monetary policy when agents have imperfect information about the economy. Under an adaptive learning framework in which agents are uncertain about the true structure of the economy, Eusepi and Preston (2010) analyze different monetary policy communication strategies to ensure stable macroeconomic dynamics. Honkapohja and Mitra (2019) study central bank credibility in an adaptive learning framework when the monetary authority implements a price-level targeting policy. Ferrero and Secchi (2009) and Ferrero and Secchi (2010) show that if the central bank communicates to the public its projections of the output gap and inflation, more desirable and stable outcomes can occur in the economy. Orphanides and Williams (2004), Orphanides and Williams (2007), Gaspar et al. (2006), and Gaspar et al. (2010)

[2] Carlstrom et al. (2015) also show unusually large responses of the macroeconomic variables to interest rate pegs under a perfectly credible central bank.

[3] Campbell et al. (2012) also examine Odyssean forward guidance (commitment to a future path of the policy rate) and Delphic forward guidance (publication of the central bank's own forecasts) in the U.S and find that the FOMC has achieved some success in communicating Odyssean forward guidance.

study central bank behavior when agents have imperfect information about the parameters in the central bank's policy rule function or optimal monetary policy with adaptive learning.

Our paper is closest to Haberis et al. (2014), Goy et al. (2018), and Haberis et al. (2019) which also look at the role of monetary policy credibility.[4] The former shows that forward guidance can help escape the liquidity trap when central bank credibility is endogenous. The latter paper explains that interest rate pegs can produce more muted responses of the macroeconomic variables if agents in their model are allowed to perceive the central bank as not credible.[5]

Altogether we see our paper adding to the literature along the following dimensions: (1) We explicitly model the perceived credibility of the policy commitment on the part of the private agents tied to the formation of expectations. (2) We exploit SPF data on private sector expectations and Bayesian estimation techniques to analyze the effects of central bank credibility on the transmission of forward guidance shocks. Bayesian estimation procedures and survey data are employed to recover an estimate of central bank credibility in relation to forward guidance, which other studies have not estimated. None of the papers cited earlier explicitly estimates forward guidance and credibility jointly in a model as is done in our study. (3) The resulting outcomes from our estimation show a high but imperfectly credible central bank in the U.S. This result implies that credibility has dampened the effects of forward guidance on the economy relative to the perfectly credible central bank case. (4) To the best of our knowledge, we are also the first paper to cross-validate a macroeconomic model of central bank credibility and forward guidance on the forecastability of SPF forecasting errors.

In short, we argue that both macro data and expectations data are better described with a model that incorporates heterogeneous expectations and deviations from FIRE behavior resulting from an environment where policy commitments about the future path of the interest rate can be reneged by the central bank. Therefore, from our estimation, we conclude that the anticipation effects of forward guidance in the U.S. are attenuated—at least in part—because the central bank is perceived by private agents as unable to fully commit to honor the announced future path.

The remainder of the paper goes as follows: In Section 2 we discuss our baseline model with heterogenous expectations and central bank credibility. In Section 3 we introduce our

[4] Other papers related to ours include Eggertsson and Woodford (2003) who also discuss the importance of the management of expectations when the interest rate is constrained by the ZLB, and also Kiley (2016) and Swanson (2018) who explore forward guidance at the ZLB.

[5] Nakata and Sunakawa (2019) and Dong and Young (2019) examine time consistent policy in a model with forward guidance and credibility.

Bayesian estimation approach which is based on an expectations-augmented linearized version of the general equilibrium model. In Section 4 we present our main findings, while in Section 5 we provide additional robustness checks on our key estimate of central bank credibility. In Section 6 we conclude. We include in the Appendix an evolutionary game-theoretic motivation of our notion of central bank credibility and its connection to heterogenous expectations as well as all listed tables and figures.

2 Benchmark Model

We employ a standard New Keynesian model that follows from the workhorse framework laid out by Woodford (2003), Giannoni and Woodford (2004), Milani (2007), Cúrdia et al. (2015), and Cole and Milani (2017). The log-linear approximation that we bring to the data is derived from the optimizing behavior of households and firms. Our variant of the model includes four conventional sources of macroeconomic persistence—habit formation in consumption, price stickiness, price indexation, and interest rate inertia—to capture the dynamics of the macroeconomic data.

The model is completed with a Taylor (1993) interest rate feedback rule with inertia which describes the response of monetary policy to domestic economic conditions. We augment the standard monetary policy rule in one important dimension by explicitly distinguishing between unanticipated (surprises) and anticipated (forward guidance) shocks to monetary policy—a distinction that allows us to investigate the central bank's commitment to a future path of the nominal policy rate (forward guidance) through the lens of a general equilibrium model. We describe the monetary policy rule in greater detail in Subsection 2.2.

We depart from the full-information rational expectations (FIRE), homogeneous-beliefs paradigm embedded in the workhorse New Keynesian model.[6] Private agents are modeled as heterogeneous-beliefs rational households-firms that assign odds to whether the central bank will honor its forward guidance commitments or not. If the central bank's commitments are deemed credible, then rational expectations forecasts are used. If they are deemed not credible, then expectations are formed on the basis of standard VAR techniques used to fit the data. VAR techniques are fairly easy to implement, yet are immune to attempts to "manage expectations" on the part of the central bank through forward guidance announcements that

[6] Note that, as argued by Park (2018), monetary authorities typically employ macroeconomic models with rational expectations to forecast future economic activity as well as the future path of inflation and the policy rate—that is, models that do not incorporate the sort of heterogeneity that we capture here.

can be reneged.[7]

2.1 Main Structural Relationships

As in Cúrdia et al. (2015), the workhorse New Keynesian model can be described by the following pair of log-linearized equations:

$$\tilde{x}_t = \mathbb{E}_t \tilde{x}_{t+1} - (1-\beta\eta)(1-\eta)(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \quad (1)$$

$$\tilde{\pi}_t = \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \xi_p \left(\omega x_t + ((1-\beta\eta)(1-\eta))^{-1} \tilde{x}_t\right) + \mu_t, \quad (2)$$

where

$$\tilde{y}_t \equiv y_t - \eta y_{t-1} - \beta\eta \mathbb{E}_t (y_{t+1} - \eta y_t), \quad (3)$$

$$\tilde{y}_t^n \equiv y_t^n - \eta y_{t-1}^n - \beta\eta \mathbb{E}_t \left(y_{t+1}^n - \eta y_t^n\right), \quad (4)$$

$$\tilde{x}_t \equiv \tilde{y}_t - \tilde{y}_t^n = x_t - \eta x_{t-1} - \beta\eta \mathbb{E}_t (x_{t+1} - \eta x_t), \quad (5)$$

$$\tilde{\pi}_t \equiv \pi_t - \iota_p \pi_{t-1}. \quad (6)$$

Here, the one-period nominal interest rate (i_t) is the policy rate, inflation (π_t) is the first-difference on the consumption price level in logs, and the output gap (x_t) is defined as $x_t \equiv y_t - y_t^n$, i.e., as the log-deviation of actual output (y_t) from its potential level absent all nominal rigidities (y_t^n).

Equation (1), often referred to as the dynamic Investment-Savings (IS) equation, describes the aggregate demand of the economy arising from the optimal decisions (the intertemporal Euler equation) of households. Equation (1) together with (3) – (5) implies that the current output gap (x_t) depends on expected one-period and two-period ahead output gaps, the lagged output gap, the current nominal interest rate (i_t), the expected one-period ahead inflation rate $(\mathbb{E}_t(\pi_{t+1}))$, and the natural rate (r_t^n) which is the real rate of interest that would prevail absent all nominal rigidities. Here, the intertemporal rate of substitution is set to one. There exists habit formation in consumption given by the parameter $0 \leq \eta \leq 1$, and households' intertemporal discount rate is given by the parameter $0 < \beta < 1$.

Equation (2) denotes the New Keynesian Phillips Curve (NKPC) and follows from the

[7] A VAR model can be seen as a reduced-form representation of the solution to the rational expectations model without forward guidance announcements (Martínez-García (2018)), but it is more flexible than using the structural specification itself. This flexibility allows private agents to be agnostic about the policy rule (not just the commitments about the future path) and to form their expectations solely on the basis of the observed macro outcomes.

optimizing decision of firms. These firms are owned by the households and are operated in a monopolistically competitive environment with Calvo (1983) staggered price-setting behavior and Yun (1996) price indexation, similar to Christiano et al. (2005). Consequently, equation (2) shows that inflation (π_t) depends on lagged inflation, the expected one-period ahead inflation ($\mathbb{E}_t(\pi_{t+1})$), the current output gap (x_t), the lagged output gap, the expected one-period ahead output gap, and a cost-push shock (μ_t).

A fraction of firms given by the parameter $0 \leq \theta \leq 1$ are assumed to be unable to adjust their prices every period, while the remaining fraction $(1-\theta)$ of firms can. The non-reoptimizing firms index their prices to past inflation with the degree of indexation determined by the parameter $0 \leq \iota_p \leq 1$. Furthermore, the parameter $\omega > 0$ is the inverse of the Frisch elasticity of labor supply, while the composite coefficient ξ_p is defined as $\frac{(1-\theta\beta)(1-\theta)}{\theta}$ with β being the household's intertemporal discount factor and θ the constant fraction of non-reoptimizing firms per period.

We use (5) to re-express the system of equations given by (1) – (2) to describe the dynamics of the economy in terms of actual and potential output as follows:

$$\tilde{y}_t = \mathbb{E}_t\left(\tilde{y}_{t+1}\right) - \left(1-\beta\eta\right)\left(1-\eta\right)\left(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n\right) - \mathbb{E}_t\left(\Delta\tilde{y}_{t+1}^n\right), \tag{7}$$

$$\begin{aligned} \tilde{\pi}_t = {} & \xi_p\left(\omega y_t + \left(\left(1-\beta\eta\right)\left(1-\eta\right)\right)^{-1}\tilde{y}_t\right) + \beta\mathbb{E}_t\tilde{\pi}_{t+1} + \mu_t \\ & - \xi_p\left(\omega y_t^n + \left(\left(1-\beta\eta\right)\left(1-\eta\right)\right)^{-1}\tilde{y}_t^n\right). \end{aligned} \tag{8}$$

Based on the output potential transformation in (4), we can further re-write the system of equations in (1) – (2) to obtain that:

$$\begin{aligned} \tilde{y}_t = {} & \mathbb{E}_t\left(\tilde{y}_{t+1}\right) - \left(1-\beta\eta\right)\left(1-\eta\right)\left(i_t - \mathbb{E}_t\pi_{t+1} - r_t^n\right) \\ & - \left(\eta y_{t-1}^n - \left(1+\eta+\beta\eta^2\right)y_t^n + \left(1+\beta\eta+\beta\eta^2\right)\mathbb{E}_t\left(y_{t+1}^n\right) - \beta\eta\mathbb{E}_t\left(y_{t+2}^n\right)\right), \end{aligned} \tag{9}$$

$$\begin{aligned} \tilde{\pi}_t = {} & \xi_p\left(\omega y_t + \left(\left(1-\beta\eta\right)\left(1-\eta\right)\right)^{-1}\tilde{y}_t\right) + \beta\mathbb{E}_t\tilde{\pi}_{t+1} + \mu_t \\ & - \xi_p\left(\begin{array}{c} -\left(\left(1-\beta\eta\right)\left(1-\eta\right)\right)^{-1}\eta y_{t-1}^n + \left(\omega + \left(\left(1-\beta\eta\right)\left(1-\eta\right)\right)^{-1}\left(1+\beta\eta^2\right)\right)y_t^n \\ -\left(\left(1-\beta\eta\right)\left(1-\eta\right)\right)^{-1}\beta\eta\mathbb{E}_t\left(y_{t+1}^n\right) \end{array}\right), \end{aligned} \tag{10}$$

with the same structural relationships as the system of equations given by (7) – (8). This showcases that the dynamic IS and NKPC equations can be expressed in terms of three observable macro variables: output (y_t), inflation (π_t), and the policy rate (i_t), i.e., in terms of the three-variable vector $Y_t = \left[y_t, \pi_t, i_t\right]'$. Moreover, these equations also show that cost-push shocks (μ_t) as well as exogenously-driven shifts in the output potential (y_t^n) and the natural rate of interest (r_t^n) affect the dynamics of output (y_t) and inflation (π_t).

Frictionless Allocation. The potential output allocation (y_t^n) and the natural real rate (r_t^n) are important constructs in our analysis and represent the levels of output and of the real interest rate that would prevail absent all nominal rigidities. In that counterfactual scenario, output potential (y_t^n) evolves according to the following equation:

$$\begin{aligned}\omega y_t^n + \tfrac{1}{(1-\beta\eta)(1-\eta)}\left(y_t^n - \eta y_{t-1}^n\right) - \tfrac{\beta\eta}{(1-\beta\eta)(1-\eta)}\left(\mathbb{E}_t\left(y_{t+1}^n\right) - \eta y_t^n\right) \\ = \tfrac{\eta}{(1-\beta\eta)(1-\eta)}\left(\beta\mathbb{E}_t\left(\gamma_{t+1}\right) - \gamma_t\right).\end{aligned} \tag{11}$$

The previous equation follows from Cúrdia et al. (2015). Equation (11) implies that output potential is a linear combination of current, lagged, and future expected values of output potential as well as current and future expected values of exogenous productivity growth, $\gamma_t \equiv \Delta\ln(A_t)$ where A_t denotes total factor productivity (TFP). Given the efficient allocation of the output potential (y_t^n) in (11), the household's intertemporal Euler equation implies that the natural rate of interest (r_t^n) can be expressed as:

$$r_t^n = \mathbb{E}_t\left(\gamma_{t+1}\right) - \omega\mathbb{E}_t\left(\Delta y_{t+1}^n\right). \tag{12}$$

Equations (11) and (12) highlight the close connection between output potential and the natural rate of interest both of which respond to a common shock—the exogenous shock to productivity growth (γ_t).

Here, we observe that the natural rate of interest depends: (a) positively on the forecastable components of next period's exogenous productivity growth (γ_t), and (b) negatively on the forecastable component of next period's growth rate of output potential (Δy_{t+1}^n) which itself depends on the exogenous productivity growth (γ_t) through equation (11). Intuitively, point (b) captures the negative effect on the real interest rate of a higher expected growth rate of marginal utility which, under standard market clearing conditions, directly influences potential hours worked and in turn potential output.

Exogenous (Non-Monetary) Shock Processes. The exogenous shock to productivity growth (γ_t) and the cost-push shock (μ_t) are assumed to follow standard AR(1) processes:

$$\gamma_t = \rho_\gamma\gamma_{t-1} + \varepsilon_t^\gamma, \tag{13}$$

$$\mu_t = \rho_\mu\mu_{t-1} + \varepsilon_t^\mu, \tag{14}$$

where $\varepsilon_t^{\gamma} \overset{iid}{\sim} N\left(0, \sigma_{\gamma}^2\right)$ and $\varepsilon_t^{\mu} \overset{iid}{\sim} N\left(0, \sigma_{\mu}^2\right)$.[8] The persistence of the productivity growth and cost-push shocks is given by the parameters $0 < \rho_{\gamma} < 1$ and $0 < \rho_{\mu} < 1$, respectively. Similarly, the volatility of the productivity growth and cost-push shocks is given by $\sigma_{\gamma}^2 > 0$ and $\sigma_{\mu}^2 > 0$, respectively. We do not consider spillovers between productivity growth and cost-push shocks and assume that their respective innovations are uncorrelated at all leads and lags.

2.2 Monetary Policy

The monetary policy framework relies on the short-term nominal interest rate (i_t) as its policy instrument. A Taylor (1993)-type monetary policy rule is generally viewed as a simple and practical guide for the conduct of monetary policy in the U.S.[9] Henceforth, we assume that the central bank follows a variant of the Taylor (1993) rule whereby the nominal interest rate responds to inflation deviations from its zero-inflation target (π_t) and possibly also to fluctuations in the output gap ($x_t \equiv (y_t - y_t^n)$), i.e.,

$$i_t = \rho i_{t-1} + (1-\rho)\left(\chi_{\pi}\pi_t + \chi_x\left(y_t - y_t^n\right)\right) + \varepsilon_t^{MP}. \tag{15}$$

This policy rule ensures the determinacy of the equilibrium whenever the policy parameters satisfy the Taylor principle, that is, whenever $\chi_{\pi} > 1$ and $\chi_x \geq 0$. The rule also includes lagged interest rates with a smoothing parameter given by $0 \leq \rho < 1$ and a monetary policy shock term (ε_t^{MP}).

We introduce time-contingent forward guidance in the Taylor (1993) rule in the form of anticipated monetary policy shocks (news) following the approach of Del Negro et al. (2012), Cole (2020a), and Cole (2020b). Specifically, the monetary policy rule in (15) is augmented as follows:

$$i_t = \rho i_{t-1} + (1-\rho)\left[\chi_{\pi}\pi_t + \chi_x\left(y_t - y_t^n\right)\right] + \varepsilon_t^{MP} + \sum\nolimits_{l=1}^{L} \varepsilon_{l,t-l}^{FG}, \tag{16}$$

where the unanticipated (surprise) monetary policy shocks (ε_t^{MP}) are combined with forward guidance (news) shocks ($\varepsilon_{l,t-l}^{FG}$ for all $l = 1, ..., L$).[10] The length of the forward guidance horizon provided by the news shocks is defined by the horizon $1 \leq L < +\infty$ implying that

[8] In regards to equation (13), we have also considered a specification with a constant term. The results were largely robust and did not qualitatively change the main conclusions of this paper.

[9] A Taylor (1993)-type monetary policy rule tends to result in little loss of performance relative to an optimal discretionary rule as noted, e.g., by Dennis (2004).

[10] Schmitt-Grohé and Uribe (2012) utilize anticipated shocks and describe them as "news". However, they do not explicitly study forward guidance via monetary policy news shocks and its economic effects.

there is a finite number of L forward guidance shocks in the summation term in equation (16).

Monetary policy surprises and forward guidance shocks are assumed to be purely transitory or *i.i.d.*, i.e.,

$$\varepsilon_t^{MP} \overset{iid}{\sim} N\left(0, \sigma_{MP}^2\right), \tag{17}$$

$$\varepsilon_{l,t-l}^{FG} \overset{iid}{\sim} N\left(0, \sigma_l^{2,FG}\right), \ \forall l = 1, ..., L, \text{ and } 1 \leq L < +\infty. \tag{18}$$

Each $\varepsilon_{l,t-l}^{FG}$ in equation (16) represents anticipated or news shocks that private agents know about in period $t-l$ but do not affect the interest rate until l periods later, that is, until period t. The volatility of the unanticipated and anticipated monetary policy shocks is given by $\sigma_{MP}^2 > 0$ and $\sigma_l^{2,FG} > 0$ for all $l = 1, ..., L$, respectively. The innovations of anticipated and unanticipated monetary policy shocks are uncorrelated with each other and with the cost-push shock and productivity growth shock innovations at all leads and lags.

Following Laséen and Svensson (2011) and Del Negro et al. (2012), the following recursive representation is added to the model's system of equations to describe the news shocks:[11]

$$v_{1,t} = v_{2,t-1} + \varepsilon_{1,t}^{FG}, \tag{19}$$

$$v_{2,t} = v_{3,t-1} + \varepsilon_{2,t}^{FG}, \tag{20}$$

$$\vdots$$

$$v_{L,t} = \varepsilon_{L,t}^{FG}. \tag{21}$$

Each component of the vector $v_t = [v_{1,t}, v_{2,t}, \ldots, v_{L,t}]'$ represents all past and present central bank announcements to change the interest rate $1, 2, \ldots, L$ periods later that private agents know in period t. In addition, we define $\psi_t = \left[\varepsilon_{1,t}^{FG}, \varepsilon_{2,t}^{FG}, \ldots, \varepsilon_{L,t}^{FG}\right]'$ as the vector containing all current-period forward guidance shocks known today that affect the monetary policy rule $1, 2, \ldots, L$ periods later. Equations (19) $-$ (21) can be simplified to show that $v_{1,t-1}$ corresponds to the last term in equation (16), i.e., the summation of all anticipated monetary policy shocks realized at time t, $v_{1,t-1} = \sum_{l=1}^{L} \varepsilon_{l,t-l}^{FG}$.

[11] Laséen and Svensson (2011) argue that standard solution techniques apply when forward guidance is modeled as described here rather than as a peg on the future path of the policy rate. Moreover, this implementation also helps us avoid the indeterminacy issues which can arise when modeling central bank forward guidance as pegging the future path of interest rates to a certain value (see, e.g., Honkapohja and Mitra (2005) and Woodford (2005)). Indeed, the method used here based on anticipated monetary policy shocks (news) alleviates this concern.

Accordingly, the policy rule in (16) can be re-expressed more compactly as:

$$i_t = \rho i_{t-1} + (1-\rho)\left[\chi_\pi \pi_t + \chi_x \left(y_t - y_t^n\right)\right] + \varepsilon_t^{MP} + v_{1,t-1}. \tag{22}$$

The method of using equations (22) together with (17) − (21) provides a tractable way to incorporate anticipated monetary policy (forward guidance) shocks as well as conventional unanticipated monetary policy shocks into the policy framework. This monetary policy regime, therefore, involves forward guidance shocks which can be interpreted as the means by which the central bank communicates (announces) the time-contingent path of future policy rates.

Expectations Augmented Vector of Observable Variables. The state equations that describe the dynamics of the economy in (9) − (10), together with (3) − (4) and (6), pin down the solution to the vector of three observable macro variables given by $Y_t = [y_t, \pi_t, i_t]'$ which includes actual output (y_t), inflation (π_t), and the policy rate (i_t). However, with monetary policy shocks split into unanticipated (surprise) and anticipated (news) shocks, the vector of observable variables Y_t lacks fundamentalness in the sense of Hansen and Sargent (1980) and Martínez-García (2018). In other words, these three observable macro variables do not contain enough information to pin down the vector of unobserved structural shocks $\varepsilon_t = \left(\gamma_t, \mu_t, \varepsilon_t^{MP}, \left\{\varepsilon_{l,t-l}^{FG}\right\}_{l=1}^{L}\right)'$. Without additional observable variables, we can only recover residuals that are linear combinations of the underlying structural shocks.

Given the monetary policy rule in equation (16), we can show that the expected future path of the policy rate at time t can be written as follows:[12]

$$\mathbb{E}_t\left(i_{t+s}\right) = \begin{cases} \rho i_{t-1} + (1-\rho)\left[\chi_\pi \pi_t + \chi_x \left(y_t - y_t^n\right)\right] + \varepsilon_t^{MP} + v_{1,t-1}, \text{ for } s = 0, \\ \rho i_{t-1+s} + (1-\rho)\left[\chi_\pi \mathbb{E}_t\left(\pi_{t+s}\right) + \chi_x \mathbb{E}_t\left(y_{t+s} - y_{t+s}^n\right)\right] + v_{s+1,t-1} + \varepsilon_{s,t}^{FG}, \ \forall s \in \{1, 2, ..., L-1\}, \\ \rho i_{t-1+s} + (1-\rho)\left[\chi_\pi \mathbb{E}_t\left(\pi_{t+s}\right) + \chi_x \mathbb{E}_t\left(y_{t+s} - y_{t+s}^n\right)\right] + \varepsilon_{s,t}^{FG}, \text{ for } s = L, \\ \rho i_{t-1+s} + (1-\rho)\left[\chi_\pi \mathbb{E}_t\left(\pi_{t+s}\right) + \chi_x \mathbb{E}_t\left(y_{t+s} - y_{t+s}^n\right)\right], \ \forall s > L. \end{cases} \tag{23}$$

Hence, the expression in (23) shows that—consistent with expectations about inflation and economic activity—expectations on the future path of the interest rate should shift in response to announcements of anticipated (forward guidance) monetary policy shocks helping us tease them apart from unanticipated (surprise) monetary policy shocks. Given this, we adopt the identification strategy explored by Doehr and Martínez-García (2015) in a VAR

[12] In Subsection 2.3, we show how to model aggregate expectations that mixes FIRE and VAR-based expectations.

setting and employed by Cole and Milani (2017) within a DSGE model which consists in augmenting the vector of observables $Y_t = [y_t, \pi_t, i_t]'$ with expectations with which to disentangle anticipated from unanticipated monetary policy shocks.[13]

We expand the vector of observables Y_t with expectations as follows:

$$\overline{Y}_t = \left[y_t, \pi_t, i_t, \mathbb{E}_t\left(\Delta y_{t+1}\right), \mathbb{E}_t\left(\Delta y_{t+2}\right), \mathbb{E}_t\left(\pi_{t+1}\right), \mathbb{E}_t\left(i_{t+1}\right), ..., \mathbb{E}_t\left(i_{t+L}\right)\right]', \tag{24}$$

where $\Delta y_{t+j} = \Delta x_{t+j} + \Delta y^n_{t+j}$ denotes the growth rate of actual output in time period $t+j$ (for $j = 1, 2$) and, by analogy, we define $\Delta y^n_{t+j} = \left(y^n_{t+j} - y^n_{t+j-1}\right)$ to be the corresponding growth rate of output potential in time period $t+j$.[14] Given the structure of the economy described by equations (9)−(10), the non-monetary shock processes in (11)−(14), the Taylor (1993) rule in (22), and the unanticipated and anticipated monetary policy shocks given by (17) − (21), the vector of observables augmented with expectations $\overline{Y}_t$ in (24) suffices to ensure that we can identify all structural shocks $\varepsilon_t = \left(\gamma_t, \mu_t, \varepsilon^{MP}_t, \left\{\varepsilon^{FG}_{l,t-l}\right\}^L_{l=1}\right)'$.

2.3 Central Bank Credibility

Forward guidance opens up the possibility for central banks to manage expectations but is inherently time-inconsistent (Kydland and Prescott (1977)), that is, forward guidance are promises about future monetary policy that the central bank may find beneficial to renege from unless future policymakers could be bound somehow to credibly honor those commitments when the time comes.

This is partly because, while the vector of observables $Y_t = [y_t, \pi_t, i_t]'$ can be monitored with observable data, neither announcements about the expected future path of the policy rate (news shocks) nor the central bank's own public forecasts—if used to communicate the forward guidance policy—can be confirmed and validated with the observed current data Y_t

[13] Alternatively, we could also use the yield curve to help us identify the news shocks. Assuming the expectations hypothesis of the terms structure of interest rates holds, it follows from equations (16) and (23) that the long-term nominal interest at any given maturity $n \geq L+1$ (i^n_t) can be expressed as:

$$i^n_t = \rho\left(\tfrac{1}{n}\sum_{z=0}^{n-1}\mathbb{E}_t i_{t-1+z}\right) + (1-\rho)\left[\chi_\pi\left(\tfrac{1}{n}\sum_{z=0}^{n-1}\mathbb{E}_t\pi_{t+z}\right) + \chi_x\left(\tfrac{1}{n}\sum_{z=0}^{n-1}\mathbb{E}_t x_{t+z}\right)\right] + \tfrac{1}{n}\left(\varepsilon^{MP}_t + v_{1,t-1} + \sum_{l=1}^{L} v^{FG}_{l,t}\right).$$

Working directly with the expected path of the policy rate, as we do in this paper, lessens the concern that using longer maturity rates along the yield curve means that we are jointly testing the validity of our model and that of the expectations hypothesis of the term structure of interest rates too.

[14] In estimations, we use up to five periods ahead of the interest rate forecasts as part of our observables (i.e., up to $\mathbb{E}_t\left(i_{t+5}\right)$) given the data available in the SPF dataset. The corresponding observation equations will be described in more detail in Subsection 3.2.1.

or its lags at the time the announcement is made. It is also partly because central banks have incentives to deviate from those commitments.

Simply put, private agents realize that there is neither a full-proof verification mechanism nor a way to enforce those promises to ensure that the central bank delivers on the future policy path that has been announced and must form expectations accordingly. This implies that private agents have to factor the credibility of the central bank's forward guidance commitment in forming their own expectations about the future.

We assume that private agents that believe the central bank's commitments not to be credible form their expectations about the observables using a standard VAR model and committing themselves to forecast the future path of the economy in that way (ignoring all announcements until they materialize—if at all—at a later time). That is, private agents forecast the observable vector $Y_t = [y_t, \pi_t, i_t]'$ with the following parsimonious structural VAR(1) process in mind:

$$Y_t = A + BY_{t-1} + u_t, \tag{25}$$

which captures well the historical dynamics of Y_t in our sample. Here, A and B are reduced-form matrices of conforming dimensions, and u_t is a vector of (non-structural) residuals.[15]

Following on the footsteps of the axiomatic approach for heterogenous beliefs of Branch and McGough (2009), aggregate expectations $(\mathbb{E}_t(Y_{t+1}))$ are a weighted sum of expectations from private agents who believe the central bank to be credible and those who do not.[16] Specifically, we define aggregate expectations as follows:

$$\mathbb{E}_t(Y_{t+1}) = \tau \mathbb{E}_t^C(Y_{t+1}) + (1-\tau)\mathbb{E}_t^D(Y_{t+1}), \tag{26}$$

where $\mathbb{E}_t^C(Y_{t+1})$ represents the fully rational forecasts of private agents when the central bank's commitments are viewed as fully credible and $\mathbb{E}_t^D(Y_{t+1})$ denotes the expectations of private agents who believe the monetary authority's commitment to be not credible. As stated above, the latter form expectations based on equation (25). Equation (26) also follows Haberis et al. (2019) who model aggregate expectations as a weighted average of FIRE and VARs.

[15] In our estimation, the parameters of A and B in (25) would be recovered jointly as part of the full structural model. Therefore, the resulting estimates reflect the information available for the full sample. We leave the issue of learning about A and B, and its stability properties, for future research.

[16] We assume households own the firms and we often refer to the firm-owning households as private agents. This implies that, in our benchmark economy, the expectations of households will not differ from those of firms that enter into the aggregate demand and price-setting behavior equations in equilibrium. We leave for future research the exploration of richer environments where firms' expectations may differ from those of households.

The parameter $0 < \tau < 1$ determines the odds placed on $\mathbb{E}_t^C(Y_{t+1})$ or, alternatively, the share of private agents that forms FIRE expectations. We refer to τ as the credibility parameter of the model.[17] In the limiting case where $\tau \to 1$, all private agents in the economy believe the central bank to be perfectly credible, expectations are homogenous across agents, and $\mathbb{E}_t Y_{t+1} = \mathbb{E}_t\left(Y_{t+1}^C\right)$. In the opposite polar case where $\tau \to 0$, the monetary authority is considered not to be credible and homogenous beliefs imply that $\mathbb{E}_t(Y_{t+1}) = \mathbb{E}_t^D(Y_{t+1})$.

In general, aggregate expectations of private agents would be a convex combination weighed by a parameter τ that lies within the unit interval. This is the benchmark model we estimate in this paper and inevitably leads to an economy where forward guidance loses some of its power if τ is strictly less than one.

3 Bayesian Estimation Methods

The workhorse New Keynesian model with forward guidance and central bank credibility that we have laid out here includes equations for aggregate demand (the dynamic IS curve), the New Keynesian Phillips curve (NKPC), potential output, the efficient real interest rate (or natural rate), and the AR(1) processes for the productivity growth shock and for the cost-push shock. Moreover, the model is completed with a Taylor (1993) monetary policy rule with inertia and surprise monetary policy shocks, forward guidance (news shocks), and a recursive representation of the central bank's promises regarding changes to future interest rates (announcements). The private agents' expectations are based on a VAR if the central bank is not viewed as credible and based on FIRE expectations if credible, while heterogenous-beliefs aggregate expectations are weighted by a central bank's credibility parameter. In other words, the benchmark model that estimate includes equations (9), (10), (11), (12), (13), (14), (16), (19) – (21), (25), and (26). We implement our estimation strategy and approach using Dynare codes (Adjemian et al. (2011)).

[17] There could be alternative ways to motivate the credibility parameter $0 < \tau < 1$. In this paper, we suggest an interpretation based on the equilibrium of a game-theoretic framework between rational-expectations private agents and the central bank through the lens of an evolutionary-type "game of chicken" (Osborne and Rubinstein (1994). This credibility game is presented in the Appendix which provides all the necessary details on its structure and solution. However, for our analysis what is most crucial is that there is at least a fraction of agents in the economy making forecasts that abstract from monetary policy announcements as would be the case for those using VAR-based forecasts. Credibility in this more broad sense is something to be earned—if forward guidance commitments would be believed or thought to be effective, then private agents would be making systematic errors with VAR-based forecast that puts them at a disadvantage and more of them would recognize that like fully rational agents do.

3.1 Data Sources

We utilize Bayesian estimation techniques with U.S. macroeconomic time series variables at a quarterly frequency. Data for output, inflation, and interest rates correspond to U.S. real GDP growth, the growth rate in the GDP deflator, and the Federal Funds rate. The relevant acronyms are GDPC1, GDPDEF, and FEDFUNDS with the data retrieved from the FRED database of the Federal Reserve Bank of St. Louis. We also employ observations for expectations of future macroeconomic variables. Specifically, we utilize expectations regarding one-quarter and two-quarters ahead output growth, one-quarter ahead inflation, and one-quarter to five-quarters ahead interest rates. These forecast series are retrieved from the Survey of Professional Forecasters (SPF) database of the Federal Reserve Bank of Philadelphia (FRB of Philadelphia (2019)).[18] The relevant acronyms are RGDP, PGDP, and TBILL. In addition, our dataset spans $1981 : Q3$ through $2017 : Q3$.[19]

[18] We use the mean value across respondents.

[19] Forward guidance outside the explicit forward guidance statements that emanated in the aftermath of the $2007 - 2009$ financial recession can still be found in our dataset. Campbell et al. (2012) explain that the FOMC has issued implicit and explicit forward guidance long before the $2007 - 2009$ financial recession. Lindsey (2003) also discusses types of central bank communication in the 1980s in the U.S. Wynne (2013) explains how FOMC statements to the public have evolved from vague text in the early 1990s to more specific and clarifying statements post-2009. Contessi and Li (2013) also discuss FOMC statements containing elements of forward guidance in the early 2000s. BIS (2019) provides a detailed description an assessment of forward guidance and other monetary policy tools since the $2007 - 2009$ financial recession for the U.S. and across other countries with related experiences.

3.2 Estimation Strategy

3.2.1 Observation Equations

The observation equations mapping the model variables into the data are given by the following system of equations:

$$\begin{bmatrix} g_t^{obs} \\ \pi_t^{obs} \\ i_t^{obs} \\ \mathbb{E}_t^{obs}(g_{t+1}) \\ \mathbb{E}_t^{obs}(g_{t+2}) \\ \mathbb{E}_t^{obs}(\pi_{t+1}) \\ \mathbb{E}_t^{obs}(i_{t+1}) \\ \mathbb{E}_t^{obs}(i_{t+2}) \\ \mathbb{E}_t^{obs}(i_{t+3}) \\ \mathbb{E}_t^{obs}(i_{t+4}) \\ \mathbb{E}_t^{obs}(i_{t+5}) \end{bmatrix} = \begin{bmatrix} \Delta y_t \\ \pi_t \\ i_t \\ \mathbb{E}_t(\Delta y_{t+1}) \\ \mathbb{E}_t(\Delta y_{t+2}) \\ \mathbb{E}_t(\pi_{t+1}) \\ \mathbb{E}_t(i_{t+1}) \\ \mathbb{E}_t(i_{t+2}) \\ \mathbb{E}_t(i_{t+3}) \\ \mathbb{E}_t(i_{t+4}) \\ \mathbb{E}_t(i_{t+5}) \end{bmatrix} + \begin{bmatrix} \bar{\gamma}^g + \gamma_t \\ \bar{\gamma}^\pi \\ \bar{\gamma}^r \\ \bar{\gamma}^{g^1} + \mathbb{E}_t(\gamma_{t+1}) \\ \bar{\gamma}^{g^2} + \mathbb{E}_t(\gamma_{t+2}) \\ \bar{\gamma}^\pi \\ \bar{\gamma}^{r^1} \\ \bar{\gamma}^{r^2} \\ \bar{\gamma}^{r^3} \\ \bar{\gamma}^{r^4} \\ \bar{\gamma}^{r^5} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 8} \\ \mathbf{I}_{8\times 8} \end{bmatrix} \begin{bmatrix} o_t^{g_{t+1}} \\ o_t^{g_{t+2}} \\ o_t^{\pi_{t+1}} \\ o_t^{i_{t+1}} \\ o_t^{i_{t+2}} \\ o_t^{i_{t+3}} \\ o_t^{i_{t+4}} \\ o_t^{i_{t+5}} \end{bmatrix}, \tag{27}$$

where $g_t \equiv \Delta y_t$ represents the growth rate of output in time period t. Observations for expectations include an $i.i.d.$ measurement error term, i.e., $o_t^{g_{t+1}}$, $o_t^{g_{t+2}}$, $o_t^{\pi_{t+1}}$, $o_t^{i_{t+1}}$, $o_t^{i_{t+2}}$, $o_t^{i_{t+3}}$, $o_t^{i_{t+4}}$, and $o_t^{i_{t+5}}$. This mapping is similar to that of Cole and Milani (2017) and consistent with the expectations-augmented approach to disentangle between news and surprises about monetary policy proposed by Doehr and Martínez-García (2015).

It is important to clarify how the SPF expectations align with model implied expectations. From the SPF documentation (FRB of Philadelphia (2019)), the respondents of the SPF usually have to report their forecasts before the middle of the current quarter. For instance, in regards to forecasts for $Q1$, the deadline submission date is the second to third week of February. The nowcast is approximately a 2−month ahead forecast while the one-quarter ahead is a 5−month ahead forecast and so on. For that reason, a number of papers in the literature most closely connected to ours map the nowcast of the forecasted variables of the SPF as the one-quarter ahead forecast in the model (e.g., that of Cole and Milani (2017)). In our baseline analysis, we also treat the SPF nowcast as corresponding to the one-quarter ahead forecasts in our benchmark model to ensure comparability.[20] However, in Subsection

[20] As we utilize the nowcast for $t+1$ expectations, we have SPF data for the nowcast and up to four-quarters ahead for the estimation (which we exploit to its fullest). Hence, in the benchmark model, equation ((27)) includes up to five periods ahead interest rate expectations (i.e., $\mathbb{E}_t^{obs}(i_{t+1})$, $\mathbb{E}_t^{obs}(i_{t+2})$, $\mathbb{E}_t^{obs}(i_{t+3})$,

5.4, we also analyze the baseline results under a different (and plausible) timing convention using instead one-quarter ahead SPF forecasts as the one-quarter expectations in the model.

3.2.2 Choice of Priors

The choice of prior distributions on the structural parameters largely follows Cole and Milani (2017) and Smets and Wouters (2007).[21] The price indexation parameter ι_p is assumed to have a prior distribution of Beta. We select a Normal distribution centered over 1 for the prior distribution of ω. We also assume persistence in the productivity growth and cost-push shocks as these both have Beta prior distributions with mean of 0.50. To ensure positive values, the prior distributions on the standard deviations of the shocks are chosen to be Inverse Gamma.

The prior distribution of the policy parameters are also standard from prior studies. The priors on the χ_π and χ_x are both Normal centered over 1.5 and 0.125, respectively. We assume there exists a high degree of persistence *a priori* when the central bank adjusts the interest rate as ρ follows a Beta with mean 0.75. The prior assumptions on the previous three parameters follow from Smets and Wouters (2007). In addition, the value of the forward guidance horizon is chosen to be twelve periods, that is, $L = 12$. This assumption is based on the FOMC statement utilizing time-contingent forward guidance. Specifically, in September 2012, the FOMC stated "the Committee also ... anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." Thus, there are twelve quarters from September 2012 and "mid-2015" if the latter date is taken to be the end of quarter three of 2015.

The central parameter in our model is τ, which measures the degree of central bank credibility in the economy. As described above, all agents in the economy believe central bank statements to be perfectly credible whenever $\tau \to 1$. If $\tau \to 0$, the central bank is not perceived to be credible and agents do not factor forward guidance statements into their forecasts. In our benchmark estimation, we choose an informative prior distribution, a beta with mean 0.8. However, as this parameter τ is central to our analysis, we will conduct some robustness checks in Subsection 5.3. Specifically, we compare the baseline results to the case if one is more agnostic about the true value of τ and adopts a uniform distribution.

$\mathbb{E}_t^{obs}(i_{t+4})$, and $\mathbb{E}_t^{obs}(i_{t+5})$).

[21] The following parameters are fixed: the household's intertemporal discount factor β is set to 0.99, habit persistence η is fixed at 0.50, and the composite coefficient ξ_p is set to 0.0015. The latter two values roughly follow Cúrdia et al. (2015) and Giannoni and Woodford (2004), respectively. The constants in the observation equations in (27) are fixed to the historical mean of their respective series.

3.2.3 Reduced-Form Forecasting Model

The current paper assumes that expectations for the entire economy are composed of a weighted sum of FIRE expectations under perfect credibility and VAR-based expectations that simply ignore the forward guidance statements. As stated in equation (25), the latter type of agents form expectations via a VAR(1) process. However, it is important to justify the lag length of this forecasting model. To accomplish this task, we calculate the Bayes Information Criterion (BIC) for a VAR(1), a VAR(2), and a VAR(3) model on the vector of observables $Y_t^{obs} = \left[g_t^{obs}, \pi_t^{obs}, i_t^{obs}\right]$.[22] The BIC values for the three models are -31.16, -20.05, and 4.56, respectively. Thus, we utilize the VAR(1) as it has the lowest BIC.

4 Main Results

4.1 Estimates of Central Bank Credibility

We now proceed with our main empirical exercise to investigate the effects that central bank credibility has on the efficacy of forward guidance. When we estimate τ, we refer to this case as the not perfectly credible central bank scenario, denoted $\widehat{\tau}$. When we do not estimate τ and assume private agents perceive the monetary authority to be perfectly credible, we define this case setting $\tau = 1$ in the estimation. The results are shown via three channels: posterior point estimates, variance decomposition, and impulse response functions under both perfectly credible and imperfectly credible central banks. Table 1 and Table 2 display the posterior mean and 90% highest posterior density interval estimates. Table 3 shows the conditional variance decomposition upon impact of the structural and forward guidance shocks with parameter values at their posterior mean. The last line calculates the sum of all the variation in the macroeconomic variable due to the forward guidance shocks. Figure 1 and Figure 2 display the impulse response functions to forward guidance shocks at different horizons while Figure 3 displays the impulse response functions for the productivity growth, cost-push, and unanticipated monetary policy shocks. Each panel shows the mean response of the model-implied output and the other macro observables (inflation and interest rate). The solid line represents $\tau = 1$, while the dashed line denotes $\widehat{\tau}$.

We first examine the case in which the monetary authority is perceived to be perfectly credible. In Table 1 and Table 2 the first three columns under "Posterior Distribution" show that the estimates of the main structural parameters largely align with prior literature. The

[22] For this, we use data spanning 1985 : $Q1$ through 2007 : $Q3$. This period corresponds to the Great Moderation era in the U.S.

estimated value of the inverse of the Frisch elasticity of labor supply $\omega = 0.9709$ follows closely that of Cúrdia et al. (2015). The interest rate smoothing parameter is estimated to be high at 0.8817 which closely aligns the value found in Milani (2007) using a model with FIRE expectations. The estimates for the degree of productivity growth inertia (ρ_γ) and inflation indexation (ι_p) roughly follow the results found in Cúrdia et al. (2015) under their "W" rule.[23]

The solid lines in Figure 1 and Figure 2 show the mean impulse response under $\tau = 1$ to a one standard deviation increase in a shock. Specifically, given that agents are forward looking, news that the interest rate will increase 1, 4, 8, or 12 periods ahead affects agents' intertemporal decisions by (noticeably) lowering output and inflation on impact. When the shock is realized on the economy, output (roughly) reaches its trough. Since those who perceive the monetary authority to be perfectly credible form their expectations under FIRE, these agents completely understand the shock has already materialized, and thus, output, inflation, and interest rates proceed to return back to steady state.[24]

What are the predicted effects if the central bank is not assumed to be perfectly credible? To answer this question, we first analyze the posterior estimates in the last three columns in Table 1 and Table 2. Overall, the values of the main structural parameters do not drastically differ from the perfectly credible case, but do display a few slight differences. For instance, in the last three columns under "Not Perfectly Credible C.B.," the estimated value of the autoregressive parameter on the cost-push shock is relatively lower than in the $\tau = 1$ scenario. However, this lower persistence could instead be picked up by the higher estimates for the inflation indexation parameter relative to the perfectly credible central bank case.[25]

More importantly, when allowing private agents the option of not fully believing forward guidance statements about the path of interest rates, the estimate of the credibility parameter τ is 0.7696. This value indicates a certain level of trust in the U.S. central bank (i.e., the Federal Reserve) implying effectiveness of forward guidance on the economy.[26] However, the fact that this estimated value is not close to $\tau = 1$ suggests that private agents do not believe

[23] The posterior mean estimate for σ_γ is relatively lower than in Cúrdia et al. (2015). However, this low value could be due to the inclusion of forward guidance shocks and heterogenous expectations in the present paper that do not feature in their model.

[24] The relative smoothness of the output impulse responses could be due to a mixture of habits in consumption and the fact that when private agents believe the forward guidance statements to be fully credible before it materializes l periods later anticipate their decisions accordingly.

[25] The parameter ρ_γ is also estimated to be lower than under the $\tau = 1$ scenario. This result comes in hand with the fact tht the standard error of the productivity shock has a higher value under $\hat{\tau}$ than under $\tau = 1$.

[26] This result does agree with Swanson (2018) who finds that forward guidance has a degree of effectiveness when the economy is constrained by ZLB.

the monetary authority to be perfectly credible (or at least act in forming their expectations as if monetary policy announcements were not credible).

The ramifications of this result are a dampening of the power of forward guidance on the economy. Figure 1 and Figure 2 display this outcome. The impulse responses under $\widehat{\tau}$ (dashed line) follow similar paths as under $\tau = 1$ (solid line). However, the dashed line is not as reactive to central bank forward guidance as the solid line. Specifically, the initial impact of output and inflation to forward guidance news is larger under the perfectly credible case than under the imperfectly credible scenario. When the shock is realized on the economy l periods later, the responses of output and inflation are also overall larger under $\tau = 1$ than $\widehat{\tau}$. The reason for the discrepancies is that private agents believe central bank statements about future interest rates under the $\tau = 1$ scenario, and thus, fully internalize the effects of forward guidance. In contrast, agents who do not fully believe forward guidance commitments do not incorporate the full effects of forward guidance, and thus, macroeconomic variables are not as responsive.

Variance decomposition results also display the reduced effects of forward guidance on the economy under an imperfectly credible monetary authority. In Table 3 we compute the conditional variance decomposition upon impact of the shocks with parameter values at their posterior mean. The combined contribution of the forward guidance shocks to output and inflation is less under $\widehat{\tau}$ than $\tau = 1$. Under a central bank that is perceived as imperfectly credible, the total contribution of $\varepsilon_{1,t}^{FG}$, $\varepsilon_{2,t}^{FG}$,..., $\varepsilon_{12,t}^{FG}$ to output and inflation is 7.5346% and 0.0004%, respectively.[27] Under a monetary authority perceived as perfectly credible, the combined contribution is 23.1357% and 1.3579%. Thus, if a central bank is perceived as more credible, there exist greater immediate effects on the real economy from forward guidance but with little effect on prices. To put it another way, if a central bank is less credible, the immediate effects on output are not as great relative to the perfectly credible scenario.

The results show that modeling forward guidance credibility contributes to address the "forward guidance puzzle" of Del Negro et al. (2012). In the previously mentioned paper, the authors explain that a standard New Keynesian models similar to the one presented in Section 2 produces unusually large responses of the macroeconomic variables to forward guidance shocks. Those sizeable responses do not seem to reconcile well with the data. In contrast, our paper allows for private agents to not perceive the monetary authority as

[27] Without rounding the numbers, it should be noted that the contribution of each of the forward guidance shocks to inflation is a small, positive number. This low value agrees well with the response of inflation upon impact to forward guidance shocks found in the impulse response graphs (i.e., Figure 1 and Figure 2).

perfectly credible. As discussed in the previous paragraphs, the results show that the reaction of macroeconomic variables to forward guidance shocks is dampened and not as large under $\widehat{\tau}$ relative to $\tau = 1$.

Overall, the results of our main exercise suggest a number of takeaways. Our estimate of Federal Reserve credibility is high at $\widehat{\tau} = 0.7696$. However, this estimated value is below the fully credible case which leads to an attenuation of the power of forward guidance. If the central bank is perceived as less credible, there exist less immediate and overall effects on the economy from forward guidance. Hence, the integration of imperfect central bank credibility into a standard macroeconomic model can be another approach to resolve the forward guidance puzzle. Thus, accounting for imperfect credibility is important to model the formation of expectations in the economy and the transmission mechanism of forward guidance announcements.

4.2 Predictability of Forecasting Errors

Our benchmark model assumed that expectations in the economy are a weighted sum of private agents who believe the central bank to be perfectly credible and private agents who believe the monetary authority not to be credible. The former group forecasts macroeconomic variables under the assumption of FIRE expectations, while the latter uses a data-driven VAR(1) model. However, a natural question is whether aggregate expectations weighted by τ or the perfectly credible case is the more plausible method to model expectations?

Standard macroeconomic models often consider *only* the $\tau = 1$ case, that is, FIRE expectations. In this scenario, forecast errors will be random and on average be equal to zero, and thus, not dependent or correlated with forecasting disagreements among private agents (see, e.g., Sims (2002) and Cole and Milani (2017)). However, do the forecast errors from the data agree with those implications of the $\tau = 1$ scenario? Or does the imperfect credibility scenario with $\widehat{\tau}$ capturing the estimated central bank credibility fit the data better? To answer this, a further corroborating exercise involves comparing the forecast errors from our model with imperfect central bank credibility to what is found in the observed forecasting data.[28]

We proceed in the following manner. We define forecasting errors of the interest rate at the one-quarter ahead horizon as $FE_t^1 = \mathbb{E}_{t-1}(i_t) - i_t$. The remaining forecast er-

[28]Indeed, non-random forecast errors have been found in other settings. Andrade and Le Bihan (2013) examine the ECB Survey of Professional Forecasters and Czudaj and Beckmann (2018) study expectations for the G7 countries. Both papers find nonrandom forecasts errors in the data. Coibion et al. (2012) and Coibion et al. (2015) also test FIRE and show that information rigidities exist in the forecasts of agents.

rors are defined as $FE_t^2 = \mathbb{E}_{t-2}(i_t) - i_t$, $FE_t^3 = \mathbb{E}_{t-3}(i_t) - i_t$, $FE_t^4 = \mathbb{E}_{t-4}(i_t) - i_t$, and $FE_t^5 = \mathbb{E}_{t-5}(i_t) - i_t$ at the two, three, four, and five-quarters ahead horizons, respectively. Forecasting disagreement (DEV_t^1) at the one-quarter ahead horizon is specified as the difference between the 75^{th} and 25^{th} percentile (i.e., the interquartile range of the SPF data). The remaining forecasting disagreements are given by DEV_t^2, DEV_t^3, DEV_t^4, and DEV_t^5 at the two, three, four, and five-quarters ahead horizons, respectively.

We collect data from the U.S. economy. Forecasting errors are computed with respect to the mean forecast with SPF data. The relevant acronym is TBILL. Forecasting disagreements are measured with the interquartile range of the cross-sectional distribution of individual forecasts in order to make the empirical results less sensitive to outliers.[29] The data span $1981 : Q3 - 2018 : Q4$ for the baseline case implying 150 observations. We run separate regressions of FE_t^h on DEV_t^h at the one, two, three, four, and five-quarters ahead horizons $(h = 1, 2, 3, 4, 5)$:

$$FE_t^1 = \delta_0^1 + \delta_1^1 DEV_t^1 + e_t^1, \tag{28}$$
$$FE_t^2 = \delta_0^2 + \delta_1^2 DEV_t^2 + e_t^2, \tag{29}$$
$$FE_t^3 = \delta_0^3 + \delta_1^3 DEV_t^3 + e_t^3, \tag{30}$$
$$FE_t^4 = \delta_0^4 + \delta_1^4 DEV_t^4 + e_t^4, \tag{31}$$
$$FE_t^5 = \delta_0^5 + \delta_1^5 DEV_t^5 + e_t^5. \tag{32}$$

The usual regression error terms are described by e_t^1, e_t^2, e_t^3, e_t^4, and e_t^5 at the one, two, three, four, and five-quarters ahead horizons, respectively.

Furthermore, we perform the same exercise using the simulated counterparts from our model. Specifically, we simulate the model at the posterior mean for a time period of $2,000$ observations discarding the first 100 simulated observations. We then perform the same regressions given by equations (28) – (32) on a rolling window of 150 observations.[30] We compute our results on $10,000$ different draws of the simulated data.

The full range of rolling window estimates are represented in Figure 4, which shows a box-and-whisker plot that displays their min, max, median, and interquartile range. The point estimates obtained from the SPF data are also shown in Figure 4. The orange circles denote estimates of δ_1^1, δ_1^2, δ_1^3, δ_1^4, and δ_1^5 from the benchmark dataset (i.e., $1981 : Q3 - 2018 : Q4$).

[29] We have considered the same exercise where DEV is specified as the difference between the 90^{th} and 10^{th} percentile and similar results occurred.

[30] The forecasting disagreements like DEV_t^1 are computed the difference between the highest and lowest forecasts implied by $\mathbb{E}_{t-1}^C(i_t)$ and $\mathbb{E}_{t-1}^D(i_t)$.

The red diamonds represent estimates from the non-ZLB period of our dataset, that is, from the $1981 : Q3 - 2008 : Q4$ subsample.

We first examine what the data would postulate regarding the relationship between forecasting errors and forecasting disagreements. By using SPF data, estimates of δ_1^1, δ_1^2, δ_1^3, δ_1^4, and δ_1^5 show that forecasting errors are positively correlated with forecasting disagreement. The orange circles and red diamonds in Figure 4 are positive and notably display an upward trajectory as the forecast horizon increases. This challenges the assumption that forecasting errors ought to be unpredictable under the perfect credibility case with FIRE expectations of $\tau = 1$.

What occurs when private agents perceive the central bank as not perfectly credible? The simple answer is that the model under $\widehat{\tau}$ seems to match the data better than the $\tau = 1$ case. In Figure 4, the estimates δ_1^1, δ_1^2, δ_1^3, δ_1^4, and δ_1^5 using SPF data lie towards the median of their respective box-and-whisker plots under $\widehat{\tau}$. The median of the box-and-whisker plots are also in positive territory, which matches their counterparts in the data. In addition, under *both* SPF and simulated data, estimates of δ_1^1, δ_1^2, δ_1^3, δ_1^4, and δ_1^5 display an upward trend the longer the forecasting horizon. This result provides further external validation for our approach of modeling aggregate expectations with τ capturing central bank credibility.

In short, our model of central bank credibility matches the data well when utilizing SPF data to seek external cross-validation for our model's expectations framework. In the data, there exists notable disagreements among forecasters. The fully credible central bank model (i.e., the $\tau = 1$ scenario) cannot capture this feature of the SPF data. However, when we relax this assumption, and allow agents to perceive the central bank as not fully credible, the fit to the data substantially improves. This is not to say that monetary policy credibility explains all forecasting disagreements, but that it is a plausible reason for forecasting heterogeneity that can reconcile the model with the forecasting predictability that we find in the SPF data.

5 Robustness

A main result of our paper is that the higher the value of the central bank credibility parameter, τ, the greater the effects of forward guidance on the economy. In particular, τ is estimated to be 0.7696 for the U.S. economy. This high value implies a higher degree of Federal Reserve credibility as perceived by private agents in the economy, and thus, a robust level of effectiveness from forward guidance. However, the estimated value of τ is below the fully credible case (i.e., below $\tau = 1$). In the following subsections, we analyze the robustness of this result. Specifically, we examine the sensitivity of the benchmark outcomes to the time period used in the estimation, the presence of macroeconomic persistence features, the forecasting model used by private sector agents, the prior beliefs about τ, and the timing assumption matching SPF forecasts to our model's expectations.

5.1 Subsamples

5.1.1 Non-ZLB

Monetary policy was instrumented with the Fed Funds rate until hitting the ZLB in the aftermath of the $2007 - 2009$ financial recession. It is important to compare the effect of central bank credibility on forward guidance during the non-ZLB and ZLB time periods to assess whether our findings are sensitive across subsamples. Thus, in this subsection, we reestimate the model over the subsample $1981 : Q3 - 2008 : Q4$ and compare the results to our benchmark outcomes.

The "Non-ZLB" column in Table 4 and Table 5 displays the results. Even during an era where the interest rate does not bind at zero, our baseline result still holds. Specifically, the value of τ is estimated to be 0.7765, which is about the same as our benchmark estimate of 0.7696. The values of the other parameters do not considerably change either. Therefore, noticeable effects of forward guidance on the economy exist during the non-ZLB time period as a high degree of Federal Reserve credibility is estimated to exist. However, the estimate of τ still remains below the fully credible central bank scenario.

5.1.2 Great Moderation

Our full sample includes periods of relatively high volatility in the macroeconomic variables (i.e., pre-1985) and to some extent from the $2007 - 2009$ financial recession onward. Hence, it is also important to examine whether the effect of central bank credibility on forward guidance is the same or not during a stable time period. Thus, this subsection compares the benchmark estimation to the case in which we reestimate the model over the subsample $1985 : Q1 - 2007 : Q3$. This period has been called the "Great Moderation" in which the volatility in macroeconomic variables was relatively low (see, e.g., Clark (2009)).

The estimates of the structural and measurement error parameters of this exercise are displayed in the "Great Moderation" column of Table 4 and Table 5. The results show that the benchmark takeaway from Section 4 does not change. The estimate of our central bank credibility parameter is 0.7798, which is very similar to our baseline value. Thus, there exists a high degree of central bank credibility in the U.S. during a stable economic era, which provides further evidence that the effect of forward guidance appears largely unchanged over time in our full sample. And, once again, confirms that $\widehat{\tau}$ is below the fully credible central bank case.

5.2 Alternative Macro Persistence Features

The VAR(1) framework that private agents who do not believe the monetary authority to be credible utilize to construct forecasts involves inherent persistence in the macroeconomic variables. Because of this model's inertia in the macro variables, it is of interest to investigate if the estimated credibility parameter is affected because of these features. Intuitively, the hypothesis here is that perhaps the presence of macro inertia affects the relative advantages of one forecasting model over the other and, in doing so, affects the estimate of τ. Thus, we analyze the results when habits in consumption (η) and price indexation (ι_p) are turned off and compare it to the baseline outcomes.

The results shown in Table 6 and Table 7 provide further evidence that our estimate of τ capturing central bank credibility in terms of forward guidance statements is quite robust to alternative macro persistence features. First, when habits in consumption are shut off (i.e., $\eta = 0$), the results do not noticeably change. Under the "$\eta = 0$" column, Table 6 displays that the estimate for τ is 0.7637, which is virtually identical to our benchmark estimate of 0.7696. The posterior mean estimates of the other parameters are largely the same as under the benchmark $\hat{\tau}$ scenario in Table 1 and Table 2.

Second, when the degree of price indexation is turned off (i.e., $\iota_p = 0$), the posterior estimates of the parameters are largely unchanged. In particular, the "$\iota_p = 0$" column in Table 6 displays that the estimate of τ at 0.7789 is about the same as the benchmark case of price indexation existing. Thus, since the central bank credibility parameter does not seem to be reflecting a bias from lagged consumption or prices, the results of this subsection provide more corroboration of the robustness of τ capturing monetary authority credibility in terms of forward guidance.

5.3 Alternative Reduced-Form Forecasting Model

We also examine the estimation of the central bank credibility parameter τ under alternative specifications of the forecasting models used by the private sector. The benchmark case in Section 4 assumed private sector agents who believe the monetary authority to be not credible formed expectations from a VAR(1) based on (25). However, private agents who believe the central bank followed FIRE expectations. However, a natural question arises regarding whether the estimated value of τ depends on the information set in the forecasting model of private sector agents?

Here, we examine the case when private agents that do not believe the central bank know more about the true structure of the economy, that is, the case when not credible expectations $(\mathbb{E}_t\left(Y_{t+1}^D\right))$ become better informed. Private agents are assumed to know the AR(1) shock realizations, that is, $w_t = [a_t, \mu_t]'$, when formulating their expectations of future macroeconomic variables. Thus, equation (25) is replaced with:

$$Y_t = A + BY_{t-1} + Cw_t + e_t, \tag{33}$$

where the A, B, and C are coefficient matrices of appropriate dimensions, and e_t is a vector of white noise (non-structural) residual terms.

Table 8 and Table 9 produce two main takeaways. First, additional knowledge (or information) about the true structure of the economy seems to have minimal effect on the posterior estimates of the parameters. In Table 8, the estimate of our central bank credibility parameter is 0.7774. This previous value is approximately the same as our baseline estimate of 0.7696.[31] Thus, even if private agents utilize a forecasting model with more information, the results do not substantially change. In particular, there still are noticeable effects of forward guidance on the economy as private agents are estimated to believe the Fed to be highly credible, but still below the fully credible case of $\tau = 1$.

The second takeaway regards that, since the estimate of τ does not significantly change when private agents are more informed, τ does not seem to be capturing private agents' lack of knowledge about the true structure of the economy, that is, not knowing productivity growth and cost-push shocks. Thus, because the value of τ does not seem to depend on these other non-forward guidance elements, this latter result provides additional evidence that τ reflects central bank credibility.

5.4 Alternative Priors on the Credibility Parameter

Our baseline prior assumption for τ assumed a high degree of central bank credibility. This value seems reasonable and fits well with the Federal Reserve narrative. However, it is also important to examine the results under a different light, that is, when we adopt a more

[31] Altogether Table 8 and Table 9 display that the estimates of the other parameters in the model do not appreciably change.

agnostic view about the prior value of τ. Here, we examine the results when the prior distribution for τ changes to a Uniform distribution on the unit interval.

The outcomes of the estimation are displayed in Table 10 and Table 11. The results show that the estimate for τ in this case is 0.6034, which is somewhat smaller than our benchmark of 0.7696.[32] However, the former estimate of τ is still relatively high and within range of our benchmark estimate. Therefore, if one is agnostic about the true value of τ and adopts a $U(0,1)$ prior distribution, there continues to exist effects of forward guidance on the U.S. economy as the public largely believes the U.S. central bank to be credible. However, similar to the benchmark results in Subsection 4.1, the estimated value of τ is still below the fully credible case (i.e., $\tau = 1$).[33]

5.5 Alternative Mapping of SPF Forecasts

Section 3 described the data and observables that we included for the estimation of our model. In our benchmark analysis, we utilized the SPF nowcast for our model's one-quarter ahead expectations and so on. As explained in Subsection 3.2.1, we believe this assumption made sense given the actual submission dates and timing of SPF forecasters. However, a natural question can emerge. Specifically, what if SPF one-quarter ahead forecasts were used to correspond with our model's one-quarter ahead expectations instead of the nowcast?

This section performs a robustness check to analyze the results when the above possibility

[32] The estimates of the other parameters do not substantially change.

[33] A noticeable feature displayed in Table 10 and Table 11 is that the marginal likelihood is also higher under a uniform prior distribution on τ than under the beta prior distribution assumed in our benchmark (in Table 1 and Table 2). However, the estimate of the main parameter of interest, τ, is only somewhat smaller than the benchmark indicating a still high degree of central bank credibility. And, needless to say, the main results do not qualitatively change when utilizing an uninformative prior.

is taken into account. The model's observation equations are modified as follows:

$$\begin{bmatrix} g_t^{obs} \\ \pi_t^{obs} \\ i_t^{obs} \\ \mathbb{E}_t^{obs}\left(g_{t+1}\right) \\ \mathbb{E}_t^{obs}\left(g_{t+2}\right) \\ \mathbb{E}_t^{obs}\left(\pi_{t+1}\right) \\ \mathbb{E}_t^{obs}\left(i_{t+1}\right) \\ \mathbb{E}_t^{obs}\left(i_{t+2}\right) \\ \mathbb{E}_t^{obs}\left(i_{t+3}\right) \\ \mathbb{E}_t^{obs}\left(i_{t+4}\right) \end{bmatrix} = \begin{bmatrix} \Delta y_t \\ \pi_t \\ i_t \\ \mathbb{E}_t\left(\Delta y_{t+1}\right) \\ \mathbb{E}_t\left(\Delta y_{t+2}\right) \\ \mathbb{E}_t\left(\pi_{t+1}\right) \\ \mathbb{E}_t\left(i_{t+1}\right) \\ \mathbb{E}_t\left(i_{t+2}\right) \\ \mathbb{E}_t\left(i_{t+3}\right) \\ \mathbb{E}_t\left(i_{t+4}\right) \end{bmatrix} + \begin{bmatrix} \bar{\gamma}^g + \gamma_t \\ \bar{\gamma}^\pi \\ \bar{\gamma}^r \\ \bar{\gamma}^{g^1} + \mathbb{E}_t\left(\gamma_{t+1}\right) \\ \bar{\gamma}^{g^2} + \mathbb{E}_t\left(\gamma_{t+2}\right) \\ \bar{\gamma}^\pi \\ \bar{\gamma}^{r^1} \\ \bar{\gamma}^{r^2} \\ \bar{\gamma}^{r^3} \\ \bar{\gamma}^{r^4} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3\times 7} \\ \mathbf{I}_{7\times 7} \end{bmatrix} \begin{bmatrix} o_t^{g_{t+1}} \\ o_t^{g_{t+2}} \\ o_t^{\pi_{t+1}} \\ o_t^{i_{t+1}} \\ o_t^{i_{t+2}} \\ o_t^{i_{t+3}} \\ o_t^{i_{t+4}} \end{bmatrix}. \tag{34}$$

Two differences are apparent between the observation equations in this section (i.e., equation (34)) and the baseline observation equations (i.e., equation (27)). First, the $t+1$ timing in our model now corresponds to one-period ahead expectations in the SPF dataset. In addition, $\mathbb{E}_t^{obs}\left(i_{t+5}\right)$ is not in equation (34). Since this exercise utilizes one-quarter ahead SPF expectations (and not the nowcast) for $t+1$ expectations in our model, we only have data up to four-quarters ahead from the SPF. Table 12 and Table 13 display the estimated values of the structural and measurement error parameters of this exercise.

The main takeaway is that the benchmark results are robust to the timing assumption matching SPF forecasts to our model's expectations. The estimated value of our parameter of interest, τ, is 0.7761, which does not notably change relative to the benchmark value reported in Section 4. In addition, the estimates of the other parameters do not considerably change either. However, the value of the marginal likelihood is lower at 631.3625 compared to 901.1813 from the baseline case. Since the marginal likelihood depends on the data, the discrepancy could be due to this implementation using one less observable than that of Section 4 as described in previous paragraph.

6 Conclusion

The aftermath of the $2007-2009$ financial recession caused central banks around the world to more explicitly utilize monetary policy forward guidance as a policy tool. However, as we show here, its effectiveness rests on the credibility of the central bank. Thus, this paper examines the effectiveness of forward guidance in an estimated New Keynesian model with imperfect central bank credibility. We jointly model forward guidance and central bank credibility, exploiting interest rate expectations and other macro forecasts from the SPF, estimating credibility using Bayesian methods, and cross-validating the model in relation to the forecasting predictability arising from disagreement found in the SPF dataset.

The results show important takeaways. First, the estimate of central bank credibility in terms of forward guidance announcements is high for the Federal Reserve indicating a degree of effectiveness of forward guidance on the U.S. economy (particularly for output and less so for inflation). However, the estimated value is still below the fully credible case. Consequently, when the central bank is perceived to be less than perfectly credible, there exist less immediate and overall effects on the economy from forward guidance. Output and inflation does not respond as favorably to forward guidance relative to the fully credible case. Hence, we show that imperfect credibility is another feature that can contribute to resolve—at least partially—the forward guidance puzzle.

Second, we provide (compelling) evidence that our model's expectations framework reflects central bank credibility and aligns well with data. For instance, our model of imperfect central bank credibility cross-validates well with SPF data on the predictability of forecasting errors for the policy path. Furthermore, the results do not noticeably change when examining the following robustness scenarios: different sample periods, different assumptions on macro persistence, different forecasting models for private sector agents, different priors on the credibility parameter τ, and different timing assumptions regarding SPF forecasts.

Overall, we conclude that accounting for imperfect credibility is important to model the formation of expectations in the economy and the transmission mechanism of forward guidance announcements.

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7 Appendix. An Evolutionary Game of Central Bank Credibility

The private sector chooses between two different pure-strategies that affect how they form their expectations about the future—either they believe the central bank will honor its forward guidance commitments (C) or they disregard the promises that come from announcements about the future path of monetary policy and make forecasts solely on the basis of observable data (D). Similarly, the central bank concerns itself with two pure strategies—either to honor its commitments and deliver on the announced policy (C) or to renege from the existing commitments (D).

Conventionally, the literature on forward guidance has assumed the strategy pair (C, C) holds accepting that such an outcome could be sustained in equilibrium. Indeed, there are conditions on the payoffs of each player that would indeed support such an outcome as an evolutionarily stable strategy (ESS). However, without a payoff-based disciplining mechanism to sustain the strategy pair (C, C) in equilibrium, we must consider the broad range of plausible strategic implications of the non-cooperative (evolutionary) game that can arise between the central bank and the private sector.[34]

The general form of the central bank credibility game. We proceed by describing the credibility game between the central bank and private agents in general terms. The (evolutionary) game between the central bank and private agents consists of:

1. Two players referred as the central bank (cb) and the private sector (pa), i.e., $M = \{cb, pa\}$.

2. A strategy set S_i for each player $i \in M$ with two pure strategies which are to comply (C) or to deviate (D), i.e., $S_i = \{C, D\}$ for each $i \in M$.

3. A linear payoff function $u_i : S_i \to \mathbb{R}$, assigned to each player $i \in M$, which can be written in matrix form as $u_i(s_i) = Z_i s_i \in \mathbb{R}$ for any payoff matrix Z_i and strategy $s_i \in S_i$, for each player $i \in M$.

We define the strategy space of the game as $S = \Pi_{i \in M} S_i$ where each strategy pair is pin down as $s = (s_{pa}, s_{cb}) \in S$. Denoting $s_i \in S_i$ the strategy of player $i \in M$ and the strategy of the other player as $s_{-i} := (s_j) \in S_{-i} = \Pi_{j \in M, j \neq i} S_j$ where $j \neq i$ and $i, j \in M$, it follows that the strategy pair can be rewritten as $s := (s_i, s_{-i}) \in S_i \times S_{-i} = S = \Pi_{i \in M} S_i$ for all $i \in M$.

[34] In other words, there are payoffs (like in the well-known "game of chicken") where a mixed strategy equilibrium exists and is evolutionarily stable which supports the notion that there is some degree of imperfect central bank credibility.

From here, we define a best response for a given player in the following general terms:

Definition 1 *A strategy* $\widehat{s}_i \in S_i$ *is called a best response to strategy* $s_{-i} \in S_{-i}$ *iff* $u_i(\widehat{s}_i, s_{-i}) \geq u_i(s_i, s_{-i})$, $\forall i \in M$, $\forall s_i \in S_i$.

If every player chooses its best response, then no other strategy can increase the player's payoff. Hence, all players following their best response strategies constitutes a Nash equilibrium defined as follows:

Definition 2 *A pair of strategies* $s^* \in S$ *is called a Nash equilibrium iff* $u_i(s^*) = u_i\left(s_i^*, s_{-i}^*\right) \geq u_i\left(s_i, s_{-i}^*\right)$, $\forall i \in M$, $\forall s_i \in S_i$.

A Nash equilibrium is a strategy pair in the game that is a best response for both players simultaneously so no player can benefit from switching to play another alternative strategy. In other words, if player $i \in M$ were to choose the alternative strategy $s_i \neq s_i^*$ where $s_i \in S_i$ instead of the strategy s_i^* receives a payoff $u_i\left(s_i, s_{-i}^*\right) \leq u_i\left(s_i^*, s_{-i}^*\right)$, i.e., s_i^* does just as good or better than any other alternative strategy. However, a Nash equilibrium allows for the possibility that some alternative strategy may achieve the same payoff, i.e., there may be some $s_i \in S_i$ for which $u_i\left(s_i, s_{-i}^*\right) = u_i\left(s_i^*, s_{-i}^*\right)$. In turn, an evolutionary stable strategy (ESS) is a strategy that supports a stable solution that has the stronger property that, if the strategy is followed, no player who adopts a novel strategy can hope to successfully displace the ESS strategy. More precisely, a ESS strategy can be defined in the following terms:

Definition 3 *A strategy* $s_i^{ESS} \in S_i$ *for each* $i \in M$ *is an evolutionary stable strategy (ESS) if: either (a)* $u_i\left(s_i^{ESS}, s_{-i}^{ESS}\right) > u_i\left(s_i, s_{-i}^{ESS}\right)$, $\forall s_i \in S_i$ *and* $s_i \neq s_i^{ESS}$*; or (b)* $u_i\left(s_i^{ESS}, s_{-i}^{ESS}\right) = u_i\left(s_i, s_{-i}^{ESS}\right)$ *and* $u_i\left(s_i^{ESS}, s_{-i}\right) > u_i(s_i, s_{-i})$, $\forall (s_i, s_{-i}) \in S$ *and* $s_i \neq s_i^{ESS}$ *and* $s_{-i} \neq s_{-i}^{ESS}$.

The ESS concept is an equilibrium refinement to the Nash equilibrium. What this means is that a strategy pair $\left(s_i^{ESS}, s_{-i}^{ESS}\right)$ describes an ESS strategy for each player if: (a) the ESS strategy does strictly better than any alternative would do while playing against ESS; or (b) some alternative strategy does as well as ESS playing against ESS but ESS still does strictly better playing against the alternative strategy than it would do playing the alternative strategy against itself.

The linear payoff function $u_i : \{C, D\} \times \{C, D\} \rightarrow \mathbb{R}$ for both players (the central bank and the private sector) and two strategies (**C**omply or **D**eviate) can be described in normal form with the following payoff matrix:

		Private Agents	
		C	**D**
Central Bank	**C**	R_{cb}, R_{pa}	L_{cb}, T_{pa}
	D	T_{cb}, L_{pa}	P_{cb}, P_{pa}

To comply (C) means to commit to honor the policy announcements on the part of the central bank and to accept the credibility of such commitments on the part of the private agents, while to deviate (D) means to renege on the policy announcements and to rely on a forecasting model not containing policy announcements (e.g., equation (25)) to negate any credibility to such announcements respectively.

We assume that the payoff for the private sector and the central bank is tied to the social welfare achieved. **R** refers to the reward or social welfare that both players achieve jointly by each choosing C. If the two players deviate then each receives **P** which is the punishment payoff (the sub-optimal social welfare) that they achieve jointly by each choosing D. In our context, the social welfare that can be achieved when both players deviate is lower than if both comply, i.e., $P > R$.

When one player complies and the other deviates, **T** is the temptation payoff that the player that deviates (D) receives while **L** is the loser payoff received by the player that complies (C). In our context, the player that deviates (or cheats) in this game benefits at the expense of the player that complies, i.e., the social welfare perceived by the player that is cheated is lower than that of the cheater such that $L_i < T_i$, $\forall i \in M$.

For expositional tractability, we assume that the temptation and loser payoffs are symmetric for both players, i.e., $L_i = L$ and $T_i = T$, $\forall i \in M$. Similarly, the reward and punishment values are also symmetric for both players, i.e., $R_i = R$ and $P_i = P$, $\forall i \in M$. Given the symmetric payoff matrix that we describe here, the linear payoff function can be written in matrix form as $u_i(s_i) = Z_i s_i \in \mathbb{R}$ for any strategy $s_i \in S_i$ and for each player $i \in M$ with $Z_i = Z = \begin{bmatrix} R & L \\ T & P \end{bmatrix}$.

Now, depending on the ordering of R, T, L, and P, we can have significantly different games with different equilibrium outcomes. A well-known game, the Prisoner's Dilemma, requires the ordering to be $T > R > P > L$. We consider however two other orderings that stand out as most relevant for the interaction between the central bank and the private sector: the Game of Chicken which requires $T > R > L > P$ and the Trust Dilemma that requires instead that $R > T > L > P$.

Replicator dynamics. Let us consider $p_j(t)$ the frequency with which pure strategy $j = \{C, D\}$ is played and $p(t) = (p_C(t), p_D(t))^T$ the corresponding state vector, where t denotes the t-th replication of the same game. We postulate a law of motion for $p(t)$ that describes how the dynamics of the game evolve as players consider future generations (or replications) of the game at play. If players engage in a symmetric game with the payoff matrix Z, then $(Zp(t))_j$ is the expected payoff for strategy $j = \{C, D\}$ and $\left(p(t)^T Zp(t)\right)$ is the average payoff. Thus, the relative performance of the frequency vector $p_j(t)$ for each strategy $j = \{C, D\}$ is given by $\frac{(Zp(t))_j}{p(t)^T Zp(t)}$ if $p(t)^T Zp(t) \neq 0$.

We assume that the frequency $p_j(t)$ for each strategy $j = \{C, D\}$ is iteratively updated proportionally to its relative performance, i.e.,

$$\frac{p_j(t + \Delta t)}{p_j(t)} = \frac{(Zp(t))_j}{p(t)^T Zp(t)} \Delta t, \tag{35}$$

for $\Delta t > 0$ and for all $j = \{C, D\}$. Hence, $p_j(t + \Delta t) - p_j(t) = p_j(t) \frac{(Zp(t))_j - p(t)^T Zp(t)}{p(t)^T Zp(t)} \Delta t$. This, in turn, yields the following differential equation as $\Delta t \to 0$:

$$\dot{p}_j = p_j \frac{(Zp)_j - p^T Zp}{p^T Zp}, \tag{36}$$

for all $j = \{C, D\}$ with $\dot{p}_j$ denoting the derivative of $p_j(t)$ with respect to t.

A solution $q_j(t)$ to the simplified differential equation:

$$\dot{q}_j = q_j \left[(Zq)_j - q^T Zq \right], \tag{37}$$

suffices to describe the replicator dynamics of the game as (36) has the same trajectories as (37). That is because, according to the transformation of t given by $t(s) = \int_{s_0}^{s} p(t)^T Zp(t)$ with s_0 being the initial iteration, every solution $p_j(t)$ of (36) delivers a solution $q_j(s) := p_j(t(s))$ of the simplified differential equation (37).

Evolutionary stable strategies. Let us denote the frequency of strategy D with the parameter q and the frequency of strategy C as $1-q$ with $\widetilde{q}=(1-q,q)^T$. The replicator equation in (37) has two terms that depend on the payoff matrix Z. The first term depends on $Z\widetilde{q}$ which gives us that $\begin{pmatrix}(1-q)\,R+qL\\(1-q)\,T+qP\end{pmatrix}$. Since strategy D is ordered after C in the layout of the normal form of the game, we use the second component of $Z\widetilde{q}$ to describe $(Z\widetilde{q})_j$ when $j=D$. The second term $\widetilde{q}^T Z\widetilde{q}$ can be expressed as $(1-q)^2\,R+(1-q)\,q\,(L+T)+q^2P$. Thus, the replicator equation in (37) for strategy D is given by:

$$\dot{q}=q\left[(1-q)\,T+qP-(1-q)^2\,R-(1-q)\,q\,(L+T)-q^2P\right]. \tag{38}$$

By setting $\dot{q}=0$, i.e., by solving the equation:

$$q\left[T-R-(L-P+2\,(T-R))\,q+(L-P+T-R)\,q^2\right]=0, \tag{39}$$

we obtain the evolutionary states of the model. This holds trivially true for $q^{ES}=0$ and for $q^{ES}=1$. The mixed strategy solution can be pin down by factoring the roots from the quadratic function $q^2-\left(\frac{L-P+2(T-R)}{L-P+T-R}\right)q+\left(\frac{T-R}{L-P+T-R}\right)=0$ where we already know that one of the roots is $q^{ES}=1$. From that, we obtain that that the mixed strategy state of the model is $q^{ES}=\left(\frac{1}{1+\frac{L-P}{T-R}}\right)$.

To sum up:

Lemma 1 *The central bank credibility game has generically three states. Two states are in pure strategies where $q^{ES}=0$ implies playing C and $q^{ES}=1$ implies playing D. The mixed strategy state, if one exists, involves playing strategy D with a frequency of $q^{ES}=\left(\frac{1}{1+\frac{L-P}{T-R}}\right)$ and strategy C with a frequency of $1-q^{ES}=\left(\frac{\frac{L-P}{T-R}}{1+\frac{L-P}{T-R}}\right)$.*

The mixed strategy state is well-defined and satisfies $0\leq\left(\frac{1}{1+\frac{L-P}{T-R}}\right)\leq 1$ whenever $(L-P)+(T-R)\geq 0$ and $\frac{L-P}{T-R}\geq 0$. The Prisoner's Dilemma, as indicated before, requires the ordering to be $T>R>P>L$. Therefore, $T-R>0$ and $L-P<0$ violates the condition that $\frac{L-P}{T-R}\geq 0$ and for this case there are only two states based on pure strategies. Similarly, the Trust Dilemma that imposes instead that $R>T>L>P$ implies that $L-P>0$ and $T-R<0$. Therefore, for the Trust Dilemma, there are only two states in pure strategies as well. In turn, the Game of Chicken which requires $T>R>L>P$

implies that $L - P > 0$ and $T - R > 0$ and satisfies the conditions that insure a well-defined mixed strategy state exists.

Definition 4 *A strategy pair* $\left(1 - q^{ESS}, q^{ESS}\right)^T$ *is said to be an evolutionary stable strategy (ESS) if it's a locally convergent evolutionary state which is dynamically restored after a disturbance via the replicator equation in (37), provided the disturbance is not too large. That is,* $q^{ES} = 0$ *is an ESS if* $\dot{q} < 0$ *for* $q^0\,(> 0) \rightarrow 0$ *from the right and* $q^{ES} = 1$ *is an ESS if* $\dot{q} > 0$ *for* $q^0\,(< 1) \rightarrow 1$ *from the left. In turn,* $q^{ES} = \left(\frac{1}{1+\frac{L-P}{T-R}}\right)$ *is an ESS if* $\dot{q} < 0$ *for* $q^0 \left(> \frac{1}{1+\frac{L-P}{T-R}}\right) \rightarrow \frac{1}{1+\frac{L-P}{T-R}}$ *from the right and* $\dot{q} > 0$ *for* $q^0 \left(< \frac{1}{1+\frac{L-P}{T-R}}\right) \rightarrow \frac{1}{1+\frac{L-P}{T-R}}$ *from the left.*

When we explore the dynamics implied by the replicator equation in (37), it follows that given the orderings of the payoffs R, T, L, and P:

Proposition 1 *The Prisoner's Dilemma game has one ESS only, that is the state* $q^{ESS} = q^{ES} = 1$ *(which implies the player follows the pure strategy* D*). Similarly, the Trust Dilemma has one ESS only that corresponds to the other pure strategy* $q^{ESS} = q^{ES} = 0$ *(the player follows the pure strategy* C*). In turn, the only ESS of the central bank credibility game (the Game of Chicken between the central bank and the private sector) is the mixed strategy implied by* $q^{ESS} = q^{ES} = \left(\frac{1}{1+\frac{L-P}{T-R}}\right)$.

When we estimate our model with central bank credibility and forward guidance in Section 4, the data favor a mixed strategy equilibrium. Thus, Proposition 1 suggests that the Game of Chicken is better suited than the Prisoner's Dilemma or the Trust Dilemma to describe the central bank credibility game. In other words, the Game of Chicken where a mixed strategy equilibrium exists and is evolutionarily stable can support the notion that there is some degree of imperfect central bank credibility that we detect in the U.S. data.

We leave for future research the tasks of relaxing the symmetry of the payoff matrix used here for exposition and of incorporating those payoffs and dynamic learning via the replicator dynamics in our model estimation. That would help us endogenize the equilibrium credibility parameter and introduce learning in our framework.

8 Tables

Table 1: **Prior & Posterior Estimates of Structural Parameters**

	Prior Distr.	Posterior Distribution					
		Perfectly Credible C.B. ($\tau = 1$)			Not Perfectly Credible C.B. ($\hat{\tau}$)		
	Distr.	Mean	5%	95%	Mean	5%	95%
τ	B(0.80, 0.01)	-	-	-	0.7696	0.7524	0.7865
ω	N(1.00, 0.05)	0.9709	0.8881	1.0520	0.9954	0.9124	1.0770
ρ	B(0.75, 0.10)	0.8817	0.8601	0.9039	0.9669	0.9538	0.9801
χ_π	N(1.50, 0.10)	1.3276	1.1816	1.4745	1.4951	1.3329	1.6589
χ_x	N(0.125, 0.05)	0.0722	0.0415	0.1023	0.1331	0.0506	0.2146
ι_p	B(0.50, 0.15)	0.0660	0.0190	0.1110	0.6785	0.5434	0.8156
ρ_γ	B(0.50, 0.20)	0.9908	0.9862	0.9954	0.4987	0.4777	0.5200
ρ_μ	B(0.50, 0.20)	0.5037	0.4314	0.5786	0.0270	0.0030	0.0509
σ_γ	IG(0.30, 2.00)	0.0752	0.0639	0.0866	0.6026	0.5424	0.6623
σ_μ	IG(0.30, 2.00)	0.1058	0.0896	0.1215	0.1327	0.1189	0.1467
σ_{MP}	IG(0.30, 2.00)	0.1858	0.1669	0.2042	0.1916	0.1716	0.2110
σ_1^{FG}	IG(0.30, 2.00)	0.0748	0.0642	0.0855	0.0686	0.0579	0.0790
σ_2^{FG}	IG(0.30, 2.00)	0.0402	0.0356	0.0442	0.0492	0.0416	0.0565
σ_3^{FG}	IG(0.30, 2.00)	0.0387	0.0356	0.0418	0.0412	0.0359	0.0456
σ_4^{FG}	IG(0.30, 2.00)	0.0402	0.0356	0.0439	0.0424	0.0365	0.0476
σ_5^{FG}	IG(0.30, 2.00)	0.0610	0.0479	0.0738	0.0585	0.0464	0.0707
σ_6^{FG}	IG(0.30, 2.00)	0.0595	0.0470	0.0717	0.0584	0.0460	0.0702
σ_7^{FG}	IG(0.30, 2.00)	0.0584	0.0461	0.0705	0.0587	0.0461	0.0708
σ_8^{FG}	IG(0.30, 2.00)	0.0583	0.0461	0.0700	0.0584	0.0462	0.0707
σ_9^{FG}	IG(0.30, 2.00)	0.0581	0.0459	0.0698	0.0583	0.0462	0.0706
σ_{10}^{FG}	IG(0.30, 2.00)	0.0583	0.0461	0.0704	0.0583	0.0460	0.0701
σ_{11}^{FG}	IG(0.30, 2.00)	0.0588	0.0464	0.0709	0.0584	0.0461	0.0703
σ_{12}^{FG}	IG(0.30, 2.00)	0.0594	0.0468	0.0718	0.0585	0.0460	0.0706
	logMargL	599.0789			901.1813		

Note: C.B.: Central Bank, G: Gamma Distribution, N: Normal Distribution, B: Beta

Table 2: **Prior & Posterior Estimates of Measurement Errors**

	Prior Distr.	Posterior Distribution					
		Perfectly Credible C.B. ($\tau = 1$)			Not Perfectly Credible C.B. ($\hat{\tau}$)		
	Distr.	Mean	5%	95%	Mean	5%	95%
$\sigma^{me}_{g^1}$	IG(0.10, 2.00)	0.4504	0.4059	0.4942	0.3424	0.3088	0.3753
$\sigma^{me}_{g^2}$	IG(0.10, 2.00)	0.4500	0.4029	0.4960	0.1928	0.1737	0.2115
$\sigma^{me}_{\pi^1}$	IG(0.10, 2.00)	0.1668	0.1496	0.1834	0.1619	0.1457	0.1775
$\sigma^{me}_{i^1}$	IG(0.10, 2.00)	0.0327	0.0252	0.0403	0.0291	0.0232	0.0348
$\sigma^{me}_{i^2}$	IG(0.10, 2.00)	0.0179	0.0146	0.0211	0.0170	0.0139	0.0199
$\sigma^{me}_{i^3}$	IG(0.10, 2.00)	0.0153	0.0128	0.0178	0.0146	0.0124	0.0168
$\sigma^{me}_{i^4}$	IG(0.10, 2.00)	0.0157	0.0130	0.0183	0.0161	0.0132	0.0188
$\sigma^{me}_{i^5}$	IG(0.10, 2.00)	0.0209	0.0162	0.0252	0.0212	0.0166	0.0256
	logMargL	599.0789			901.1813		

Note: C.B.: Central Bank, G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 3: **Variance Decomposition Upon Impact**

	Perfectly Credible C.B. ($\tau = 1$)			Not Perfectly Credible C.B. ($\hat{\tau}$)		
	Output	Inflation	Interest Rate	Output	Inflation	Interest Rate
ε_t^{MP}	30.3333	0.4965	95.5377	83.2219	0.0028	99.6743
ε_t^{γ}	44.7020	12.7346	0.9890	0.0126	0.0039	0.0013
ε_t^{μ}	1.8289	85.4111	3.2918	9.2308	99.9928	0.3243
$\varepsilon_{1,t}^{FG}$	5.2008	0.0994	0.0245	6.3791	0.0003	0.0000
$\varepsilon_{2,t}^{FG}$	1.4658	0.0344	0.0075	1.0200	0.0001	0.0000
$\varepsilon_{3,t}^{FG}$	1.2690	0.0371	0.0072	0.1185	0.0000	0.0000
$\varepsilon_{4,t}^{FG}$	1.2507	0.0455	0.0079	0.0053	0.0000	0.0000
$\varepsilon_{5,t}^{FG}$	2.6102	0.1169	0.0182	0.0025	0.0000	0.0000
$\varepsilon_{6,t}^{FG}$	2.2434	0.1226	0.0174	0.0056	0.0000	0.0000
$\varepsilon_{7,t}^{FG}$	1.9449	0.1283	0.0167	0.0029	0.0000	0.0000
$\varepsilon_{8,t}^{FG}$	1.7389	0.1370	0.0165	0.0007	0.0000	0.0000
$\varepsilon_{9,t}^{FG}$	1.5523	0.1449	0.0163	0.0000	0.0000	0.0000
$\varepsilon_{10,t}^{FG}$	1.4022	0.1538	0.0163	0.0000	0.0000	0.0000
$\varepsilon_{11,t}^{FG}$	1.2829	0.1640	0.0164	0.0000	0.0000	0.0000
$\varepsilon_{12,t}^{FG}$	1.1746	0.1740	0.0165	0.0000	0.0000	0.0000
Total FG	23.1357	1.3579	0.1814	7.5346	0.0004	0.0000

Note: This table computes the conditional variance decomposition upon impact of the structural and forward guidance shocks with parameter values at their posterior mean. Each column displays the percentage contribution of each shock to model-implied output and observables (inflation and interest rates). Total FG denotes the sum of all of the forward guidance shocks. The measurement errors are not shown as their contribution concerns expected values of observables.

Table 4: **Prior & Posterior Estimates of Structural Parameters under Subsamples**

	Prior Distr.	Posterior Distribution					
		Non-ZLB			Great Moderation		
	Distr.	Mean	5%	95%	Mean	5%	95%
τ	B(0.80, 0.01)	0.7765	0.7592	0.7935	0.7798	0.7620	0.7975
ω	N(1.00, 0.05)	0.9972	0.9142	1.0785	0.9977	0.9155	1.0803
ρ	B(0.75, 0.10)	0.9527	0.9354	0.9696	0.9535	0.9316	0.9761
χ_π	N(1.50, 0.10)	1.4911	1.3269	1.6525	1.4996	1.3372	1.6626
χ_x	N(0.125, 0.05)	0.1356	0.0518	0.2169	0.1336	0.0520	0.2141
ι_p	B(0.50, 0.15)	0.6536	0.5099	0.8012	0.4032	0.2332	0.5787
ρ_γ	B(0.50, 0.20)	0.4944	0.4664	0.5223	0.4216	0.3514	0.4933
ρ_μ	B(0.50, 0.20)	0.0364	0.0039	0.0677	0.0795	0.0077	0.1518
σ_γ	IG(0.30, 2.00)	0.6490	0.5729	0.7224	0.5547	0.4734	0.6357
σ_μ	IG(0.30, 2.00)	0.1305	0.1145	0.1462	0.1241	0.1073	0.1409
σ_{MP}	IG(0.30, 2.00)	0.2191	0.1929	0.2449	0.1443	0.1251	0.1632
σ_1^{FG}	IG(0.30, 2.00)	0.0773	0.0636	0.0907	0.0636	0.0513	0.0758
σ_2^{FG}	IG(0.30, 2.00)	0.0554	0.0458	0.0648	0.0554	0.0457	0.0650
σ_3^{FG}	IG(0.30, 2.00)	0.0457	0.0387	0.0523	0.0446	0.0373	0.0513
σ_4^{FG}	IG(0.30, 2.00)	0.0467	0.0392	0.0537	0.0421	0.0357	0.0472
σ_5^{FG}	IG(0.30, 2.00)	0.0653	0.0503	0.0802	0.0691	0.0520	0.0857
σ_6^{FG}	IG(0.30, 2.00)	0.0653	0.0500	0.0801	0.0693	0.0518	0.0862
σ_7^{FG}	IG(0.30, 2.00)	0.0649	0.0499	0.0796	0.0691	0.0521	0.0859
σ_8^{FG}	IG(0.30, 2.00)	0.0654	0.0497	0.0803	0.0690	0.0517	0.0857
σ_9^{FG}	IG(0.30, 2.00)	0.0652	0.0501	0.0800	0.0692	0.0519	0.0859
σ_{10}^{FG}	IG(0.30, 2.00)	0.0653	0.0500	0.0801	0.0689	0.0521	0.0855
σ_{11}^{FG}	IG(0.30, 2.00)	0.0650	0.0498	0.0797	0.0691	0.0519	0.0858
σ_{12}^{FG}	IG(0.30, 2.00)	0.0651	0.0500	0.0797	0.0691	0.0522	0.0859
	logMargL	587.7405			601.7465		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 5: **Prior & Posterior Estimates of Measurement Errors under Subsamples**

	Prior Distr.	Posterior Distribution					
		Non-ZLB			Great Moderation		
	Distr.	Mean	5%	95%	Mean	5%	95%
$\sigma_{g^1}^{me}$	IG(0.10, 2.00)	0.3329	0.2959	0.3706	0.2892	0.2532	0.3249
$\sigma_{g^2}^{me}$	IG(0.10, 2.00)	0.2018	0.1788	0.2241	0.1763	0.1534	0.1985
$\sigma_{\pi^1}^{me}$	IG(0.10, 2.00)	0.1698	0.1502	0.1885	0.1657	0.1450	0.1862
$\sigma_{i^1}^{me}$	IG(0.10, 2.00)	0.0327	0.0251	0.0401	0.0360	0.0285	0.0435
$\sigma_{i^2}^{me}$	IG(0.10, 2.00)	0.0190	0.0152	0.0228	0.0178	0.0143	0.0213
$\sigma_{i^3}^{me}$	IG(0.10, 2.00)	0.0166	0.0136	0.0196	0.0172	0.0138	0.0204
$\sigma_{i^4}^{me}$	IG(0.10, 2.00)	0.0181	0.0146	0.0215	0.0177	0.0141	0.0212
$\sigma_{i^5}^{me}$	IG(0.10, 2.00)	0.0234	0.0178	0.0288	0.0208	0.0161	0.0254
	logMargL	587.7405			601.7465		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 6: **Prior & Posterior Estimates of Structural Parameters Without Frictions**

	Prior Distr.	Posterior Distribution					
		No Habit Formation in Consumption ($\eta = 0$)			No Price Indexation ($\iota_p = 0$)		
	Distr.	Mean	5%	95%	Mean	5%	95%
τ	B(0.80, 0.01)	0.7637	0.7476	0.7795	0.7789	0.7621	0.7960
ω	N(1.00, 0.05)	0.9898	0.9074	1.0722	0.9982	0.9151	1.0791
ρ	B(0.75, 0.10)	0.9621	0.9492	0.9753	0.9667	0.9534	0.9799
χ_π	N(1.50, 0.10)	1.4973	1.3322	1.6581	1.4955	1.3356	1.6582
χ_x	N(0.125, 0.05)	0.1442	0.0627	0.2257	0.1320	0.0501	0.2147
ι_p	B(0.50, 0.15)	0.6819	0.5503	0.8128	-	-	-
ρ_γ	B(0.50, 0.20)	0.1267	0.0506	0.1993	0.4972	0.4741	0.5207
ρ_μ	B(0.50, 0.20)	0.0277	0.0035	0.0513	0.7498	0.6525	0.8495
σ_γ	IG(0.30, 2.00)	0.7043	0.6114	0.7971	0.6031	0.5430	0.6632
σ_μ	IG(0.30, 2.00)	0.1385	0.1242	0.1526	0.0767	0.0637	0.0891
σ_{MP}	IG(0.30, 2.00)	0.1944	0.1741	0.2141	0.1919	0.1718	0.2119
σ_1^{FG}	IG(0.30, 2.00)	0.0671	0.0565	0.0776	0.0684	0.0577	0.0790
σ_2^{FG}	IG(0.30, 2.00)	0.0507	0.0430	0.0584	0.0484	0.0410	0.0555
σ_3^{FG}	IG(0.30, 2.00)	0.0427	0.0369	0.0481	0.0408	0.0356	0.0449
σ_4^{FG}	IG(0.30, 2.00)	0.0437	0.0373	0.0496	0.0421	0.0365	0.0472
σ_5^{FG}	IG(0.30, 2.00)	0.0601	0.0471	0.0728	0.0584	0.0463	0.0703
σ_6^{FG}	IG(0.30, 2.00)	0.0596	0.0468	0.0721	0.0583	0.0460	0.0702
σ_7^{FG}	IG(0.30, 2.00)	0.0592	0.0466	0.0714	0.0581	0.0459	0.0699
σ_8^{FG}	IG(0.30, 2.00)	0.0588	0.0462	0.0707	0.0583	0.0461	0.0701
σ_9^{FG}	IG(0.30, 2.00)	0.0587	0.0465	0.0709	0.0582	0.0461	0.0701
σ_{10}^{FG}	IG(0.30, 2.00)	0.0587	0.0464	0.0708	0.0583	0.0463	0.0704
σ_{11}^{FG}	IG(0.30, 2.00)	0.0587	0.0466	0.0708	0.0583	0.0462	0.0702
σ_{12}^{FG}	IG(0.30, 2.00)	0.0590	0.0465	0.0713	0.0582	0.0463	0.0702
	logMargL	635.9700			890.6997		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 7: **Prior & Posterior Estimates of Measurement Errors Without Frictions**

	Prior Distr.	Posterior Distribution					
		No Habit Formation in Consumption ($\eta = 0$)			No Price Indexation ($\iota_p = 0$)		
	Distr.	Mean	5%	95%	Mean	5%	95%
$\sigma^{me}_{g^1}$	IG(0.10, 2.00)	0.8504	0.7684	0.9318	0.3455	0.3118	0.3790
$\sigma^{me}_{g^2}$	IG(0.10, 2.00)	0.2608	0.2343	0.2867	0.1936	0.1741	0.2124
$\sigma^{me}_{\pi^1}$	IG(0.10, 2.00)	0.1660	0.1493	0.1821	0.1756	0.1581	0.1928
$\sigma^{me}_{i^1}$	IG(0.10, 2.00)	0.0306	0.0248	0.0363	0.0294	0.0236	0.0351
$\sigma^{me}_{i^2}$	IG(0.10, 2.00)	0.0172	0.0141	0.0202	0.0169	0.0139	0.0198
$\sigma^{me}_{i^3}$	IG(0.10, 2.00)	0.0154	0.0128	0.0179	0.0147	0.0123	0.0168
$\sigma^{me}_{i^4}$	IG(0.10, 2.00)	0.0178	0.0145	0.0210	0.0161	0.0133	0.0189
$\sigma^{me}_{i^5}$	IG(0.10, 2.00)	0.0216	0.0169	0.0261	0.0211	0.0166	0.0256
	logMargL	635.9700			890.6997		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 8: **Prior & Posterior Estimates of Struc-tural Parameters with Alternative Non-Credible Expectations**

	Prior Distr.	Posterior Distribution		
	Distr.	Mean	5%	95%
τ	B(0.80, 0.01)	0.7774	0.7606	0.7943
ω	N(1.00, 0.05)	0.9971	0.9148	1.0790
ρ	B(0.75, 0.10)	0.9791	0.9689	0.9896
χ_π	N(1.50, 0.10)	1.4945	1.3300	1.6575
χ_x	N(0.125, 0.05)	0.1291	0.0472	0.2120
ι_p	B(0.50, 0.15)	0.7114	0.5849	0.8460
ρ_γ	B(0.50, 0.20)	0.4687	0.4334	0.5046
ρ_μ	B(0.50, 0.20)	0.0299	0.0035	0.0557
σ_γ	IG(0.30, 2.00)	0.6007	0.5410	0.6606
σ_μ	IG(0.30, 2.00)	0.1406	0.1240	0.1566
σ_{MP}	IG(0.30, 2.00)	0.1919	0.1720	0.2107
σ_1^{FG}	IG(0.30, 2.00)	0.0701	0.0610	0.0786
σ_2^{FG}	IG(0.30, 2.00)	0.0437	0.0376	0.0495
σ_3^{FG}	IG(0.30, 2.00)	0.0393	0.0356	0.0427
σ_4^{FG}	IG(0.30, 2.00)	0.0388	0.0356	0.0421
σ_5^{FG}	IG(0.30, 2.00)	0.0580	0.0460	0.0699
σ_6^{FG}	IG(0.30, 2.00)	0.0578	0.0459	0.0695
σ_7^{FG}	IG(0.30, 2.00)	0.0578	0.0459	0.0695
σ_8^{FG}	IG(0.30, 2.00)	0.0580	0.0459	0.0700
σ_9^{FG}	IG(0.30, 2.00)	0.0580	0.0459	0.0695
σ_{10}^{FG}	IG(0.30, 2.00)	0.0580	0.0460	0.0700
σ_{11}^{FG}	IG(0.30, 2.00)	0.0581	0.0459	0.0700
σ_{12}^{FG}	IG(0.30, 2.00)	0.0578	0.0457	0.0697
	logMargL	928.8763		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 9: **Prior & Posterior Estimates of Mea-surement Errors with Alternative Non-Credible Expectations**

	Prior Distr.	Posterior Distribution		
	Distr.	Mean	5%	95%
$\sigma_{g^1}^{me}$	IG(0.10, 2.00)	0.3387	0.3051	0.3717
$\sigma_{g^2}^{me}$	IG(0.10, 2.00)	0.1880	0.1695	0.2064
$\sigma_{\pi^1}^{me}$	IG(0.10, 2.00)	0.1563	0.1401	0.1721
$\sigma_{i^1}^{me}$	IG(0.10, 2.00)	0.0199	0.0158	0.0238
$\sigma_{i^2}^{me}$	IG(0.10, 2.00)	0.0156	0.0130	0.0182
$\sigma_{i^3}^{me}$	IG(0.10, 2.00)	0.0141	0.0119	0.0159
$\sigma_{i^4}^{me}$	IG(0.10, 2.00)	0.0150	0.0126	0.0174
$\sigma_{i^5}^{me}$	IG(0.10, 2.00)	0.0188	0.0151	0.0224
	logMargL	928.8763		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 10: **Prior & Posterior Estimates of Structural Parameters under U(0,1) Prior on** τ

	Prior Distr.	Posterior Distribution		
	Distr.	Mean	5%	95%
τ	U(0, 1)	0.6034	0.5684	0.6389
ω	N(1.00, 0.05)	0.9976	0.9149	1.0792
ρ	B(0.75, 0.10)	0.9755	0.9632	0.9882
χ_π	N(1.50, 0.10)	1.5192	1.3585	1.6846
χ_x	N(0.125, 0.05)	0.1263	0.0448	0.2067
ι_p	B(0.50, 0.15)	0.5843	0.3912	0.7793
ρ_γ	B(0.50, 0.20)	0.4945	0.4794	0.5096
ρ_μ	B(0.50, 0.20)	0.0379	0.0047	0.0701
σ_γ	IG(0.30, 2.00)	0.6016	0.5426	0.6598
σ_μ	IG(0.30, 2.00)	0.1441	0.1279	0.1601
σ_{MP}	IG(0.30, 2.00)	0.2039	0.1820	0.2257
σ_1^{FG}	IG(0.30, 2.00)	0.0835	0.0687	0.0984
σ_2^{FG}	IG(0.30, 2.00)	0.0631	0.0522	0.0739
σ_3^{FG}	IG(0.30, 2.00)	0.0478	0.0404	0.0551
σ_4^{FG}	IG(0.30, 2.00)	0.0507	0.0421	0.0589
σ_5^{FG}	IG(0.30, 2.00)	0.0604	0.0474	0.0732
σ_6^{FG}	IG(0.30, 2.00)	0.0605	0.0475	0.0733
σ_7^{FG}	IG(0.30, 2.00)	0.0605	0.0475	0.0734
σ_8^{FG}	IG(0.30, 2.00)	0.0604	0.0474	0.0732
σ_9^{FG}	IG(0.30, 2.00)	0.0604	0.0476	0.0731
σ_{10}^{FG}	IG(0.30, 2.00)	0.0604	0.0473	0.0730
σ_{11}^{FG}	IG(0.30, 2.00)	0.0605	0.0474	0.0732
σ_{12}^{FG}	IG(0.30, 2.00)	0.0605	0.0473	0.0731
	logMargL	928.4596		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 11: **Prior & Posterior Estimates of Measurement Errors under U(0,1) Prior on τ**

	Prior Distr.	Posterior Distribution		
	Distr.	Mean	5%	95%
$\sigma_{g^1}^{me}$	IG(0.10, 2.00)	0.3401	0.3070	0.3731
$\sigma_{g^2}^{me}$	IG(0.10, 2.00)	0.1923	0.1732	0.2110
$\sigma_{\pi^1}^{me}$	IG(0.10, 2.00)	0.1517	0.1368	0.1662
$\sigma_{i^1}^{me}$	IG(0.10, 2.00)	0.0254	0.0202	0.0306
$\sigma_{i^2}^{me}$	IG(0.10, 2.00)	0.0173	0.0142	0.0203
$\sigma_{i^3}^{me}$	IG(0.10, 2.00)	0.0147	0.0124	0.0169
$\sigma_{i^4}^{me}$	IG(0.10, 2.00)	0.0167	0.0138	0.0196
$\sigma_{i^5}^{me}$	IG(0.10, 2.00)	0.0230	0.0178	0.0282
	logMargL	928.4596		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 12: **Prior & Posterior Estimates of Structural Parameters under Alternative SPF Timing**

	Prior Distr.	Posterior Distribution		
	Distr.	Mean	5%	95%
τ	B(0.8, 0.01)	0.7761	0.7588	0.7936
ω	N(1.00, 0.05)	0.9930	0.9120	1.0753
ρ	B(0.75, 0.10)	0.9525	0.9363	0.9687
χ_π	N(1.50, 0.10)	1.4874	1.3264	1.6482
χ_x	N(0.125, 0.05)	0.1374	0.0574	0.2172
ι_p	B(0.50, 0.15)	0.6806	0.5445	0.8186
ρ_γ	B(0.50, 0.20)	0.3784	0.3233	0.4353
ρ_μ	B(0.50, 0.20)	0.0313	0.0037	0.0579
σ_γ	IG(0.30, 2.00)	0.7045	0.6197	0.7882
σ_μ	IG(0.30, 2.00)	0.1362	0.1217	0.1501
σ_{MP}	IG(0.30, 2.00)	0.1931	0.1734	0.2131
σ_1^{FG}	IG(0.30, 2.00)	0.0898	0.0785	0.1010
σ_2^{FG}	IG(0.30, 2.00)	0.0447	0.0382	0.0510
σ_3^{FG}	IG(0.30, 2.00)	0.0412	0.0357	0.0456
σ_4^{FG}	IG(0.30, 2.00)	0.0621	0.0481	0.0755
σ_5^{FG}	IG(0.30, 2.00)	0.0620	0.0483	0.0754
σ_6^{FG}	IG(0.30, 2.00)	0.0617	0.0483	0.0751
σ_7^{FG}	IG(0.30, 2.00)	0.0618	0.0481	0.0752
σ_8^{FG}	IG(0.30, 2.00)	0.0619	0.0481	0.0753
σ_9^{FG}	IG(0.30, 2.00)	0.0617	0.0480	0.0748
σ_{10}^{FG}	IG(0.30, 2.00)	0.0619	0.0481	0.0753
σ_{11}^{FG}	IG(0.30, 2.00)	0.0618	0.0482	0.0750
σ_{12}^{FG}	IG(0.30, 2.00)	0.0618	0.0483	0.0755
	logMargL	631.3625		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

Table 13: **Prior & Posterior Estimates of Measurement Errors under Alternative SPF Timing**

	Prior Distr.	Posterior Distribution		
	Distr.	Mean	5%	95%
$\sigma^{me}_{g^1}$	IG(0.10, 2.00)	0.3189	0.2874	0.3501
$\sigma^{me}_{g^2}$	IG(0.10, 2.00)	0.1859	0.1666	0.2050
$\sigma^{me}_{\pi^1}$	IG(0.10, 2.00)	0.1818	0.1637	0.1993
$\sigma^{me}_{i^1}$	IG(0.10, 2.00)	0.0204	0.0160	0.0247
$\sigma^{me}_{i^2}$	IG(0.10, 2.00)	0.0153	0.0127	0.0177
$\sigma^{me}_{i^3}$	IG(0.10, 2.00)	0.0153	0.0127	0.0178
$\sigma^{me}_{i^4}$	IG(0.10, 2.00)	0.0204	0.0161	0.0247
	logMargL	631.3625		

Note: G: Gamma Distribution, N: Normal Distribution, B: Beta Distribution, U: Uniform Distribution, IG: Inverse-Gamma Distribution.

9 Figures

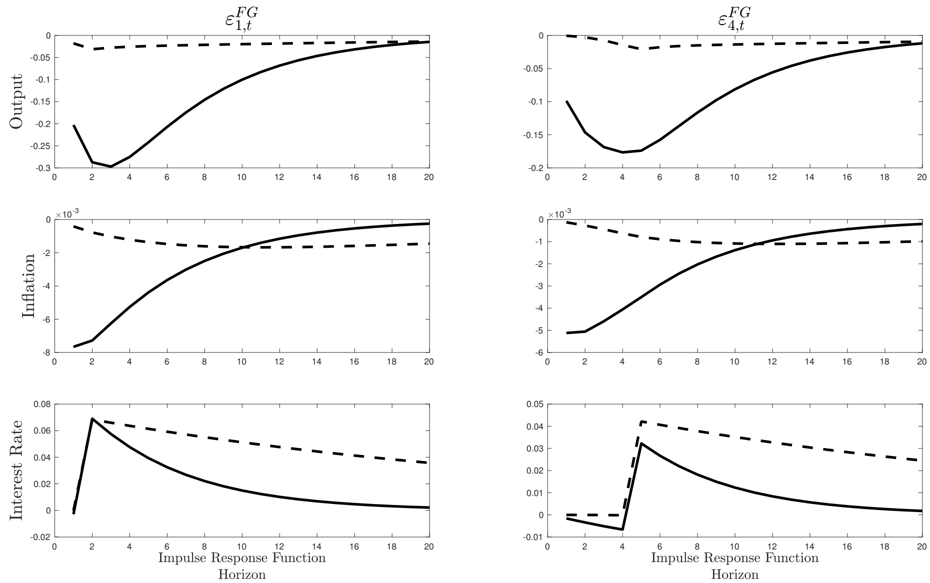

Figure 1: Impulse Response Functions. Mean response of model-implied output and observables (inflation and interest rate) to *one-period ahead forward guidance* and *four-period ahead forward guidance* shocks. Solid Line: Perfectly Credible C.B. (i.e., $\tau = 1$). Dashed line: Not Perfectly Credible C.B. (i.e., Benchmark $\hat{\tau}$). C.B.: Central Bank.

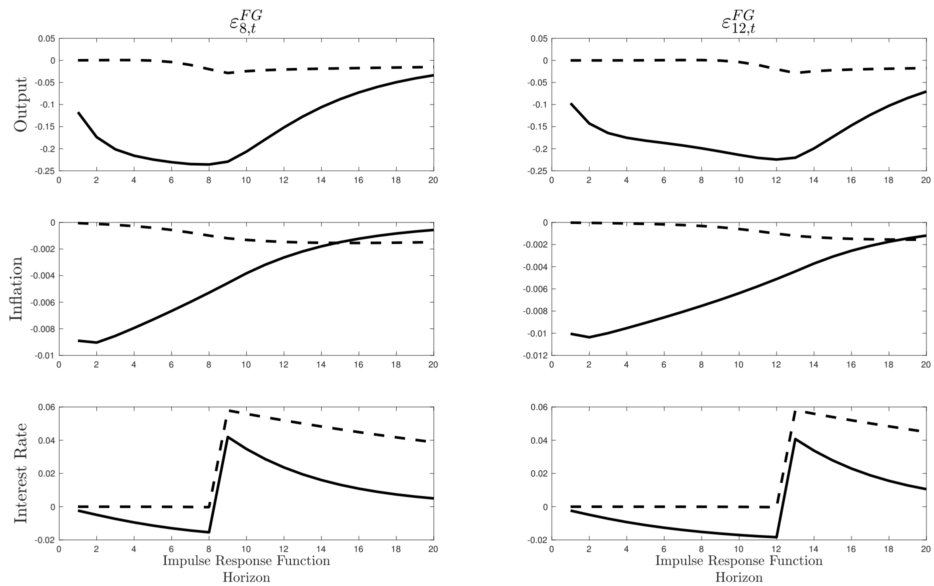

Figure 2: Impulse Response Functions. Mean response of model-implied output and observables (inflation and interest rate) to *eight-period ahead forward guidance* and *twelve-period ahead forward guidance* shocks. Solid Line: Perfectly Credible C.B. (i.e., $\tau = 1$). Dashed line: Not Perfectly Credible C.B. (i.e., Benchmark $\hat{\tau}$). C.B.: Central Bank.

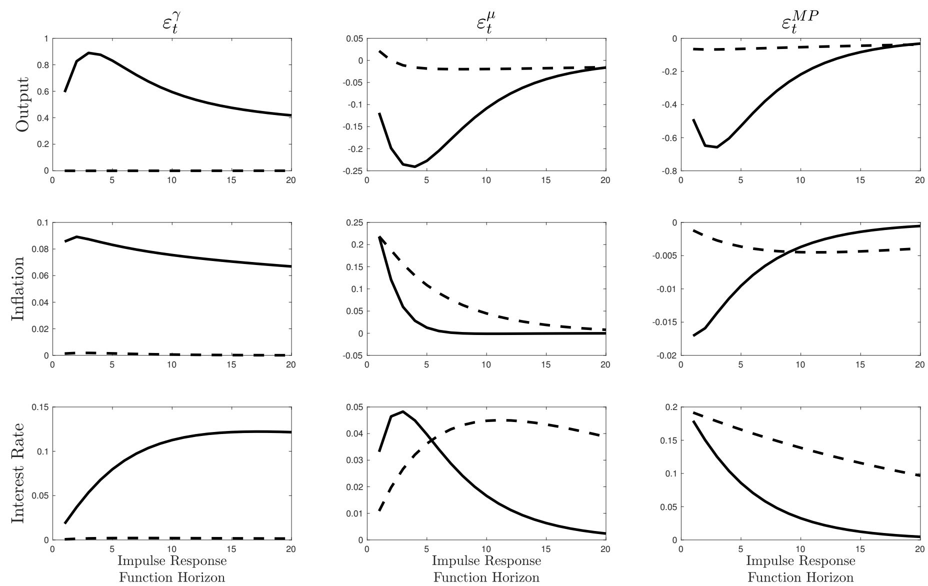

Figure 3: Impulse Response Functions. Mean response of model-implied output and observables (inflation and interest rate) to *productivity growth*, *cost-push*, and *unanticipated monetary policy* shocks. Solid Line: Perfectly Credible C.B. (i.e., $\tau = 1$). Dashed line: Not Perfectly Credible C.B. (i.e., Benchmark $\hat{\tau}$). C.B.: Central Bank.

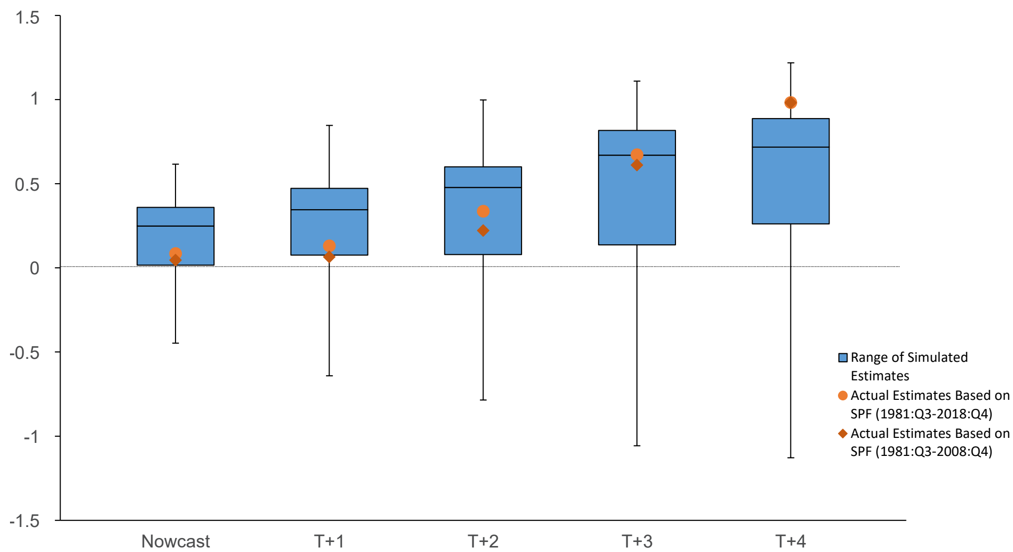

Figure 4: Estimates of Forecasting Errors Response to Forecasting Disagreement, Survey of Professional Forecasters (SPF) vs. Simulated Data

Note: Forecasting errors are computed with respect to the mean forecast with SPF data from 1981:Q3 until 2018:Q4 (150 observations). Forecasting disagreements are measured with the interquartile range of the cross-sectional distribution of individual forecasts in order to make the empirical results less sensitive to outliers. We regress forecasting errors on an intercept and this measure of forecasting disagreement for all available time horizons. We perform the same exercise on a rolling window of 150 observations from a simulated sample data of 2000 observations under $\hat{\tau}$. We represent the full range of rolling window estimates with a box-and-whisker plot that show their min, max, median, and interquartile range. SOURCES: Survey of Professional Forecasters (SPF), simulated data, author's calculations.