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Ties that Bind: Estimating the Natural Rate of Interest for Small Open Economies*

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Abstract

This paper estimates the natural rate of interest for six small open economies (Australia, Canada, South Korea, Sweden, Switzerland, and the U.K.) with a structural New Keynesian model using Bayesian techniques. Our empirical analysis establishes the following four main findings: First, we show that the open economy framework provides a better fit of the data than its closed economy counterpart for the six countries we investigate. Second, we also show that, in all six countries, a Taylor (1993)-type monetary policy rule that tracks the Wicksellian short-term natural rate fits the data better than a rule that does not. Third, we show that the natural interest rates of all six countries have shifted downwards and strongly comoved with each other over the past 35 years. Fourth, our findings illustrate that foreign output shocks (spillovers from the rest of the world) are a major contributor to the dynamics of the natural rate in these six small open economies and that those natural rates strongly comove also with the existing U.S. natural rate estimates.

JEL Classification: C11, C13, E43, E58, F41.

Keywords: Small Open Economy Model, Monetary Policy, Natural Rate, Bayesian Estimation.

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1 Introduction

The concept of the natural rate of interest and its role as a benchmark for monetary policy dates back at least to Wicksell (1898), who defined it as the level of "the [real] rate of interest on loans which is neutral with respect to commodity prices and tends neither to increase nor to decrease them." The concept was familiar to mid-twentieth century monetary economists—it played a key role in Milton Friedman's proposal of an analogous natural rate of unemployment—but had largely disappeared from academic discourse since the latter part of the century. After having been out of fashion in mainstream macroeconomics for decades, the notion of the natural rate resurfaced as a central concept in modern monetary policy thinking and practice at the turn of the century, mostly due to two developments.

The first was purely academic and had to do with the success of the reinterpretation of the concept by Woodford (2003) within the canonical New Keynesian framework. This framework has since become very influential in central bank research departments. The second was the extended period of declining interest rates around the world since at least the 1980s-1990s (Bernanke (2015a), Borio et al. (2017)). The low interest rate levels, achieved without creating inflation, led many scholars and policymakers to argue that the natural rate of interest must have declined, either because of the lingering impact of globalization (the savings glut hypothesis), because of slower productivity growth (the secular stagnation hypothesis), or because of lower labor force growth due to aging populations (the demographics hypothesis) (Bernanke (2015b), Bernanke (2015c), and Bernanke (2015d)).

In this paper we contribute to the ongoing debate on the role of the natural rate in monetary policy deliberations by investigating the propagation of foreign shocks and their contribution to the natural rate in small open economies. We develop a variant of the business cycle model of Galí and Monacelli (2005) and Lubik and Schorfheide (2007) that incorporates exogenous domestic technological progress (trend growth) and a more flexible representation of preferences for domestic households (including a domestic preference shock). Then, we estimate the domestic natural interest rate within our reference model for a sample of six small open-economies that includes Australia, Canada, South Korea, Sweden, Switzerland, and the U.K. Our country sample encompasses different geographic regions, levels of development, and notable differences in the degree of openness and economic size, all of which introduce distinct sources of cross-sectional variation in the analysis.

With this, we focus on empirically identifying the foreign determinants of the natural rate in small open-economies and on assessing the significance of international spillovers from the rest of the world. Estimating our reference model with standard Bayesian techniques, yields the following four major findings:

First, we show by means of Bayesian model comparison that the small open economy model provides a more empirically plausible framework with which to describe the data than the closed economy model for all six countries in our sample.

Second, we show, also by way of Bayesian model comparison, robust evidence that monetary policy within the general class of linear Taylor (1993)-type rules tracks the (Wicksellian) short-term natural rate of interest rather than the steady state (long-run) rate of Taylor (1993)'s original specification. This is the case for all countries in our sample (similar evidence is reported by Cúrdia et al. (2015) for the U.S.), highlighting the fact that the short-term natural rate is significant not just to assess the path (and the stance) of monetary policy but also for its implementation.

Third, we find that the synchronization of nominal interest rates across all countries in our sample is mostly attributable to the strong comovement in the natural rate, while the cross-country comovement of the gap between the real interest rate and the natural interest rate (a measure of the stance of monetary policy) is fairly weak.

Forth, foreign output shocks are a very important drivers of the natural interest rate (accounting for 13% to 60% of the short-term variation and over 60% of the long-run variation) across all six countries. Our findings suggest that foreign determinants are a major contributor to the natural rate in small open-economies. Existing evidence suggests that the natural rate of the leading advanced economies has declined to historically low and even negative levels over the last two decades (e.g., Holston et al. (2017) and Del Negro et al. (2019)).[1] Similarly, our evidence suggests that the natural rates of all six countries comove positively and quite strongly with a broad range of measures of the U.S. natural rate, providing external validation of international spillovers.[2]

The remainder paper is structured as follows. Section 2 outlines our benchmark small open economy model based on the modeling work of Galí and Monacelli (2005) and Lubik and Schorfheide (2007). Section 3 discusses the Bayesian estimation methodology we use in our empirical analysis as well as the choice of relevant priors and data sources. Section 4 reports our main findings regarding the robustness of the small open economy specification and the variance decomposition of the natural rate into its different domestic and global exogenous drivers. We also discuss evidence suggestive of cross-country spillovers related to the declining U.S. natural rate. Finally, Section 5 concludes by putting our empirical exercise into perspective in relation to the ongoing debate on the declining natural rates around the world. A more detailed discussion of the existing literature on estimating the natural rate and how our paper contributes to it can be found in the Literature Appendix. The derivations of the building blocks of the model can be found in the Model Appendix while further information on our data sources is available in the Data Appendix.

2 Small Open Economy Framework

Our benchmark model closely follows the small open economy setup of Galí and Monacelli (2005) and Lubik and Schorfheide (2007). We refer to the Model Appendix for the technical details on the derivation of the log-linearized equilibrium conditions of the small open economy framework that we more succinctly summarize here. In our setup, we introduce an underlying non-stationary technological productivity shock process A_t to account for growth—albeit exogenously—while still retaining the structural representation of business cycles that the small open economy model provides. We assume that aggregate productivity, A_t, grows at a rate, $z_t \equiv \ln\left(\frac{A_t}{A_{t-1}}\right)$, which follows an exogenous AR(1) process of the following form:

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t},\ \varepsilon_{z,t} \sim N\left(0, \sigma_z\right), \tag{1}$$

where $-1 < \rho_z < 1$ denotes the productivity persistence parameter and $\sigma_z > 0$ its volatility. In order to guarantee the stationarity of the model, all quantity variables are re-expressed relative to a transformation of aggregate productivity A_t.

[1] Holston et al. (2017) and Del Negro et al. (2019), among others, suggest that estimated natural rates in a variety of countries have exhibit similar declining patterns over the past two decades while Wynne and Zhang (2018b) show the same appears to be the case for the world natural rate.

[2] To be precise, we use the most recent available estimates of the U.S. natural rate of interest from Laubach and Williams (2003), Lubik and Matthes (2015), Holston et al. (2017), Johannsen and Mertens (2018), and Kiley (2020).

The evolution of the small open economy is determined specifically by the following equations. First, a small open economy dynamic IS curve that is obtained after log-linearizing the Euler equation that arises from the optimization problem of the domestic household:

$$x_t = \mathbb{E}_t (x_{t+1}) - \frac{1}{(1-\alpha)\tau_\alpha} (i_t - \mathbb{E}_t (\pi_{t+1}) - r_t^n), \tag{2}$$

where $\tau_\alpha \equiv (\tau + \alpha (2-\alpha)(\eta - \tau))^{-1}$, $0 < \alpha < 1$ measures the degree of openness (the import share in steady state), $\tau > 0$ denotes the intertemporal elasticity of substitution, and $\eta > 0$ determines the degree of substitutability between domestic and imported goods. Here, all variables are expressed in log-deviations from steady state, i_t is the nominal one-period domestic interest rate, π_t is the domestic inflation rate, $y_{A,t} \equiv y_t - \frac{\tau+\tau\varphi}{1+\tau\varphi} a_t$ is the domestic real output (y_t) stationarized with respect to productivity (a_t), and $y_{A,t}^n \equiv y_t^n - \frac{\tau+\tau\varphi}{1+\tau\varphi} a_t$ is the stationarized domestic potential output under flexible prices and perfect competition (y_t^n). The dynamic IS equation states that the current output gap, $x_t \equiv y_{A,t} - y_{A,t}^n$, depends on the expected future output gap, $\mathbb{E}_t (x_{t+1})$, and on the wedge between the *ex-ante* one-period real interest rate, $i_t - \mathbb{E}_t (\pi_{t+1})$, and its natural level, r_t^n. Equation (2) reduces to its closed economy variant when $\alpha = 0$.

Second, the optimal price-setting behavior of domestic firms under monopolistic competition and staggered pricing à la Calvo (1983) leads to the following small open economy Phillips curve:

$$\pi_t = \beta \mathbb{E}_t (\pi_{t+1}) + \alpha\beta \mathbb{E}_t (\Delta q_{t+1}) - \alpha \Delta q_t + (\tau_\alpha + \varphi) \kappa x_t + u_t, \tag{3}$$

where $\varphi > 0$ measures the inverse of the Frisch elasticity of labor supply and the composite parameter $\kappa \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ influencing the slope of the Phillips curve is a function of the intertemporal discount factor $0 < \beta < 1$ and of the Calvo (1983) price stickiness parameter $0 < \theta < 1$. The Phillips curve in (3) implicitly uses the fact that international *perfect risk-sharing* holds across countries in order to relate current inflation, π_t, to expected future inflation, $\mathbb{E}_t (\pi_{t+1})$, the domestic output gap, x_t, and the terms of trade growth, $\Delta q_t \equiv q_t - q_{t-1}$. Here, the terms of trade q_t are defined as the price of home goods in terms of the foreign goods. The small open economy Phillips curve also includes a cost-push shock (an exogenous shifter), u_t, arising from the time-varying domestic price markup that follows an AR(1) process of the following form:

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t},\ \varepsilon_{u,t} \sim N(0, \sigma_u), \tag{4}$$

where $-1 < \rho_u < 1$ denotes the persistence parameter and $\sigma_u > 0$ the volatility. Again, the closed economy variant obtains when $\alpha = 0$.

Third, we work with two supplementary equations for consistency. Log-linearizing the definition of the domestic inflation around the symmetric, zero-inflation steady state, we obtain that:

$$\pi_t = \Delta s_t + (1-\alpha) \Delta q_t + \pi_t^f, \tag{5}$$

with s_t denoting the nominal exchange rate, $\Delta s_t \equiv s_t - s_{t-1}$ the first differences of the nominal exchange rate, and π_t^f the foreign inflation. The foreign inflation is treated as a residual and estimated with the rest of the model assuming that it follows an AR(1) process:

$$\pi_t^f = \rho_{\pi^f} \pi_{t-1}^f + \varepsilon_{\pi^f,t},\ \varepsilon_{\pi^f,t} \sim N(0, \sigma_{\pi^f}), \tag{6}$$

where $-1 < \rho_{\pi^f} < 1$ denotes the persistence parameter and $\sigma_{\pi^f} > 0$ the volatility which ought to conform with the autoregressive patterns of foreign inflation. Following Lubik and Schorfheide (2007), we recognize that the terms of trade are largely determined in international markets—that is, we recognize that the domestic conditions of the small open economy will be affected by the terms of trade but that domestic conditions would have no significant influence on neither the global economy nor on international relative prices (the terms of trade). We then add a reduced-form law of motion for the growth rate of the terms of trade, $\Delta q_t \equiv q_t - q_{t-1}$, to be estimated with the rest of the model as another consistency check that assumes the following AR(1) representation for the terms of trade growth, Δq_t:

$$\Delta q_t = \rho_q \Delta q_{t-1} + \varepsilon_{q,t},\ \varepsilon_{q,t} \sim N\left(0, \sigma_q\right), \tag{7}$$

where $-1 < \rho_q < 1$ denotes the persistence parameter and $\sigma_q > 0$ the volatility.

Fourth, we refer to the natural rate of interest, r_t^n, and the stationarized potential output, $y_{A,t}^n$, of the small open economy as the real interest rate and level of economic activity that would prevail in the domestic economy absent all nominal rigidities (that is, under perfect competition and flexible prices in the domestic markets). The frictionless allocation of the domestic economy is determined with replacing the price-setting behavior described by (3) derived with nominal rigidities with an alternative price-setting equation that implies price markups (the marginal cost of domestic production over the local price of domestic production) are set to zero in every period. The frictionless allocation takes all variables determined in international markets as given: that includes the terms of trade growth, Δq_t, the stationarized rest-of-the-world output, $y_{A,t}^f$, and the rest-of-the-world inflation, π_t^f.

In this frictionless equilibrium, the stationarized potential output of the domestic economy, $y_{A,t}^n$, can be shown to depend on the unobserved movements of the stationarized rest-of-the-world output, $y_{A,t}^f$, as follows:

$$y_{A,t}^n = -\Gamma_* y_{A,t}^f, \tag{8}$$

where $\Gamma_* \equiv \frac{1-\tau\tau_\alpha}{\tau\tau_\alpha+\tau\varphi}$ is non-negative as long as $\eta > \tau$ and equals $\Gamma_* = 0$ when $\alpha = 0$. We interpret (8) as saying that domestic stationarized potential output fluctuations that are not the result of fluctuations in productivity comove with stationarized foreign output. However, the business cycle synchronization that emerges in the frictionless allocation breaks down when we compare output between the domestic and foreign economies in the presence of nominal rigidities (that is, when looking at $y_{A,t}$ and $y_{A,t}^f$). We assume that the stationarized rest-of-the-world (foreign) output, $y_{A,t}^f$, is exogenous and follows an AR(1) process of the following form:

$$y_{A,t}^f = \rho_{y^f} y_{A,t-1}^f + \varepsilon_{y^f,t},\ \varepsilon_{y^f,t} \sim N\left(0, \sigma_{y^f}\right), \tag{9}$$

where $-1 < \rho_{y^f} < 1$ denotes the persistence parameter and $\sigma_{y^f} > 0$ the volatility.

Given the stationarized potential output, $y_{A,t}^n$, the intertemporal Euler equation in the frictionless case implies that the domestic natural interest rate, r_t^n, which is the focal point of our analysis, evolves according to:

$$r_t^n = \mathbb{E}_t\left(\Delta g_{t+1}\right) + \left(\frac{1+\varphi}{1+\tau\varphi}\right)\mathbb{E}_t\left(z_{t+1}\right) + \left[\frac{1}{\tau} - (1-\alpha)\,\tau_\alpha\left(\Gamma_* + 1\right)\right]\mathbb{E}_t\left(\Delta y_{A,t+1}^f\right), \tag{10}$$

where $\Delta g_t \equiv g_t - g_{t-1}$ is the change in intertemporal tastes (or preferences). Furthermore, we assume that the unobservable preference shock, g_t, follows an AR(1) process of the following form:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t},\ \varepsilon_{g,t} \sim N(0, \sigma_g), \tag{11}$$

where $-1 < \rho_g < 1$ denotes the persistence parameter and $\sigma_g > 0$ the volatility. From equation (10), we observe that the natural rate, r_t^n, depends positively on the forecastable components of next period's productivity growth, $\mathbb{E}_t(z_{t+1})$, the expected changes in preferences, $\mathbb{E}_t(\Delta g_{t+1})$, and the expected stationarized world output growth, $\mathbb{E}_t\left(\Delta y_{A,t+1}^f\right)$. Given (1), (11), and (9), the domestic natural rate from equation (10) can be re-expressed as:

$$r_t^n = \left(\rho_g - 1\right) g_t + \left(\frac{1+\varphi}{1+\tau\varphi}\right)\rho_z z_t + \left[\frac{1}{\tau} - (1-\alpha)\tau_\alpha(\Gamma_* + 1)\right]\left(\rho_{y^f} - 1\right) y_{A,t}^f. \tag{12}$$

Intuitively, an increase in households' desire to consume early, which is captured by a decline in g_t, puts upward pressure on the domestic natural interest rate (since $\rho_g < 1$), so as to dissuade domestic households from acting on their desire to anticipate consumption. Similarly, higher productivity growth, measured by z_t, requires steeper consumption profiles, and hence a higher natural rate in equilibrium (if $0 < \rho_z < 1$). Finally, the last term in (12) captures the negative effect on the natural rate of a higher stationarized foreign output as measured by $y_{A,t}^f$ whenever the composite coefficient $\left[\frac{1}{\tau} - (1-\alpha)\tau_\alpha(\Gamma_* + 1)\right]$ is positive (since $\rho_{y^f} < 1$).[3] The contribution of stationarized foreign output depends on several of the structural parameters of the model including the degree of openness $0 < \alpha < 1$, the inverse elasticity of labor supply $\varphi > 0$, the elasticity of intertemporal substitution $\tau > 0$, and the elasticity of substitution between home and foreign goods $\eta > 0$. Notice that stationarized foreign output, $y_{A,t}^f$, drops out from the domestic natural interest rate in (12) if the degree of openness is set to $\alpha = 0$.

Finally, the model also includes an interest rate feedback rule capturing the response of monetary policy to domestic economic developments. We consider different specifications of the policy reaction function while retaining the same equilibrium conditions to describe the behavior of the private sector to investigate the role that short-term natural rate and the steady-state (long-run) real interest rate play in anchoring the conduct of monetary policy among small open economies.[4] We start with a standard Taylor (1993)-type interest rate rule (T rule), where the central bank adjusts its policy instrument, i_t, in response to deviations in domestic inflation rate, π_t, and the domestic output gap, x_t:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)(\psi_x x_t + \psi_\pi \pi_t) + \varepsilon_{i,t},\ \varepsilon_{i,t} \sim N(0, \sigma_i), \tag{13}$$

where the smoothing term $0 < \rho_i < 1$ introduces inertia in the policy rule while the response to inflation and output gap deviations is given by the parameters $\psi_\pi > 0$ and $\psi_x > 0$. Here, $\varepsilon_{i,t}$ is an exogenous i.i.d. monetary policy shock which can be interpreted as the non-systematic component of monetary policy rule with $\sigma_i > 0$ being its volatility. The specification in (13) is in deviations from the steady state, so it implicitly

[3] The coefficient $\left[\frac{1}{\tau} - (1-\alpha)\tau_\alpha(\Gamma_* + 1)\right]$ is not unambiguously positive. However, for a wide range of conventional values of the corresponding structural parameters, it will indeed be positive.

[4] We consider here a range of plausible monetary policy rule specifications that are broadly consistent with inflation targeting and with each central bank's statement of goals and objectives (stated *de facto* and generally also *de jure* in terms of domestic inflation and of some measure of domestic real economic activity as well). A review of the inflation targeting policy framework for the small open economies in our dataset can be found in Hammond (2012).

sets the long-run real rate as the reference rate for policymaking.[5]

Another policy rule we study is a variant of the Taylor (1993) rule that tracks directly the short-term natural interest instead of the long-run real rate targeted by (13). This is referred as the Wicksellian rule (W rule) in the terminology of Cúrdia et al. (2015). Following the W rule, the policy rate responds to the domestic natural interest rate, r_t^n, and to domestic inflation, π_t, with some inertia, as in:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(r_t^n + \psi_\pi \pi_t\right) + \varepsilon_{i,t} \ \varepsilon_{i,t} \sim N\left(0, \sigma_i\right). \tag{14}$$

In here, policymakers respond solely to inflation but the natural rate is the relevant target for the actual policy rate. As in Cúrdia et al. (2015), we also estimate the model with a more general rule ($W\&T$ rule) to explore the possibility that central banks track the short-term natural rate, r_t^n, and aim both to close the output gap, x_t, and tame inflation, π_t, in order to stabilize the domestic economy. The $W\&T$ rule is of the form:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left(r_t^n + \psi_x x_t + \psi_\pi \pi_t\right) + \varepsilon_{i,t}, \ \varepsilon_{i,t} \sim N\left(0, \sigma_i\right). \tag{15}$$

Under the W and $W\&T$ rules, when the natural rate rises, say because of a significant adjustment in foreign demand, the actual policy rate tends to follow, so as to close the gap with the natural rate and hence keep output close to its potential level and prices stable.

The dynamic IS equation in (2), the New Keynesian Phillips curve in (3), the domestic natural rate equation in (10) and the domestic potential output in (8) describing the frictionless equilibrium together with one of the policy rule specifications in (13) − (15) describe the behavior of the small open economy model.[6] All of that forms a linear rational expectations model subject to the domestic productivity growth shock, z_t, in (1), the domestic cost-push shock, u_t, in (4), the domestic preference shock, g_t, in (11), the domestic i.i.d. monetary policy shock, $\varepsilon_{i,t}$, common to all monetary policy rules in (13) − (15), and the rest-of-the-world stationarized output shock, $y_{A,t}^f$, in (9). The model is also augmented with two residual shock processes for the rest-of-the-world inflation, π_t^f, in (6) tied to observable variables through the definition of inflation in (5) and for the observable terms of trade in first-differences, Δq_t, given by (7). The corresponding innovations of the stochastic processes denoted by $\varepsilon_{z,t}$, $\varepsilon_{u,t}$, $\varepsilon_{i,t}$, $\varepsilon_{g,t}$, $\varepsilon_{y^f,t}$, $\varepsilon_{\pi^f,t}$, and $\varepsilon_{q,t}$ are assumed to be uncorrelated with each other at all leads and lags. The model is solved using the method described in Sims (2002) and estimated using standard Bayesian techniques as in Martínez-García et al. (2012) and Schorfheide (2013). To evaluate the fit of the three different policy rules we compare the marginal data density (or posterior probabilities) for the corresponding models where the only difference is the specification of the monetary policy rule as in Martínez-García and Wynne (2014). Bayesian model comparison techniques help us then examine the role of the natural interest rate in monetary policy decisions in the small open economy models we estimate for the six countries in our sample.

[5] The symmetric, zero-inflation steady state of the frictionless model and that of the benchmark model with nominal rigidities is the same. Hence, the steady state real rate and the steady state natural rate of interest are the same and can be computed as $r^{ss} = -400 \ln(\beta)$ in percent terms, annualized.

[6] The closed economy specification is nested in the small open economy model because the log-linearized equilibrium conditions that describe the dynamics of our small open economy framework reduce to their closed economy counterparts when the openness parameter, α, is set to zero. Moreover, the three monetary policy rules in (13) − (15) are also nested given that they only vary on the weight each one places on the domestic short-term natural rate and on the domestic output gap.

3 Empirical Estimation Strategy

We proceed with a discussion of our econometric methodology which uses standard Bayesian techniques. We also describe the construction of the datasets that we use in our estimation and describe our choice of prior distributions for the Bayesian analysis.[7]

3.1 Methodology

The focus of our empirical exercise is to recover estimates of the domestic natural interest rates and to explore how the foreign output shock helps explain the dynamics of the natural rate through the lens of the structural model described in Section 2. In addition, we are interested in evaluating the role of trade openness in the propagation of foreign shocks and in determining the impact of foreign shocks on the domestic natural rate—something for which our framework is well suited for small and fairly open economies. We also investigate whether central banks track their domestic natural rate of interest when making monetary policy decisions by comparing models with competing policy rules (to be specific, by comparing the model under the T rule formulated in equation (13), the W rule described in equation (14), and the $W\&T$ rule expressed in equation (15)). To achieve the goals described above, we estimate six alternative models associated with small open economy ($\alpha \geq 0$) and closed economy ($\alpha = 0$) specifications under the three different monetary policy rules noted before for each of the economies we investigate in this paper.

In our empirical analysis, similar to Lubik and Schorfheide (2007), the vector of observables, Y_t, includes the output growth (quarter-over-quarter), the CPI inflation rate (quarter-over-quarter), the annualized nominal short-term interest rate, the rate of change of the nominal exchange rate (quarter-over-quarter), and the rate of change in the terms of trade (quarter-over-quarter) where the terms of trade are computed as the ratio between the export price index and the import price index. All data is demeaned over the full sample prior to the estimation. The vector of observations is the related to the following vector of model variables:

$$Y_t = \left[\Delta y_{A,t} + \frac{\tau + \tau\varphi}{1 + \tau\varphi} z_t, \pi_t, 4i_t, \Delta s_t, \Delta q_t\right]^T . \tag{16}$$

Recall that the stationarized foreign output variable, $y_{A,t}$, is defined as the log of the ratio of domestic output Y_t over the function of domestic aggregate productivity given by $A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}$. Hence, observed output growth must correspond to $\Delta y_{A,t}$ adjusted by scaled productivity growth $\frac{\tau+\tau\varphi}{1+\tau\varphi} z_t$. Apart from that adjustment, the only other model variable that needs attention is the nominal interest rate which needs to be scaled by 4 to make it consistent with the convention that observed short-term interest rates are reported on a *per annum* basis.

We estimate the model with a Bayesian Markov Chain Monte Carlo (MCMC) approach which permits us to develop our analysis along three important dimensions. First, the Bayesian approach gives us the opportunity to incorporate prior information from ancillary sources other than the observed data that we use for the estimation. This might relieve potential weak-identification problems that can occur in structural DSGE models like ours as shown in Martínez-García et al. (2012). Second, the Bayesian approach also facilitates our evaluation of the uncertainty associated with the recovered natural interest rate and parameter

[7] The data and codes to replicate our estimation and the results reported in the paper can be found here: https://bit.ly/3aXY72u.

estimates which is an important concern for monetary policy risk management as noted by Evans et al. (2016).[8] Third, the Bayesian method enables us to evaluate the fit of different models by computing the marginal data densities (or posterior probability) of each model using the modified harmonic mean estimator proposed by Geweke (1999). This is helpful for model comparison and can help shed light in deciding whether the small open economy or the closed economy specifications explain the data better and which policy rule specification is most favored by the observed macro data we use in our estimation.

We construct the likelihood using the Kalman filter based on the state-space representation of the linear rational expectations solution of each model under consideration, setting to zero the prior probability of the configurations of values of the parameter space that imply indeterminacy.[9] An optimization algorithm is used to obtain an initial estimate of the mode when implementing the MCMC procedure. We start the maximization algorithm from 20 different random starting values before launching the MCMC chains and check that the optimization routine always converges to the same value. This is a useful diagnostic for the presence of identification problems, conditional on a given set of priors. Indeed, our experience is that this is crucial to identify local modes which may achieve almost identical values of the posterior with sometimes rather different configurations of coefficients.

Having ensured a unique mode for the baseline model, the Hessian matrix from the optimization routine is used as a proposal density, properly scaled to yield a target acceptance rate between 20% and 30%. For the MCMC results, four chains of 250000 draws each were initialized by randomly selecting starting values (using an over-dispersed normal density centered at the mode with a scaled-up Hessian as the variance-covariance matrix). For each chain, following a burn-in phase of 62500 draws, convergence is monitored using the trace plots, the R-statistic of Gelman and Rubin (1992), and the SPM test of Geweke (1992). To compare the fit across models, we calculate Kass and Raftery (1995) ratios (henceforth, KR ratios), defined as two times the log of the Bayes factor. The Bayes factor of one model against another is defined as the ratio of their marginal likelihoods. Kass and Raftery (1995) recommend this measure of relative fit as its scale is the same as that of a class Likelihood Ratio statistic. They suggest that values of KR above 10 can be considered "very strong" evidence in favor of a model. Furthermore, they indicate that values between 6 and 10 represent "strong" evidence in favor of a model, between 2 and 6 "positive" evidence, while values below 2 are "not worth more than a bare mention."

3.2 Data Description

We apply our estimation technique to six typical small open-economies from four different continents: Australia, Canada, South Korea, Sweden, Switzerland, and the U.K., with quarterly data from 1983:Q1 to 2018:Q1. All of these countries are market economies and exhibit a high degree of economic and financial integration with the rest of the world so that domestic business fluctuations are likely to have a substantial

[8] Given a prior, the posterior density is proportional to the product of the likelihood and the prior. As described by Schorfheide (2013), posterior draws for this density can be generated using a random-walk metropolis algorithm and the state-space representation implied by the solution of the linear rational expectations model and the Kalman filter. Measures of location and scatter are obtained from the draws by computing, for instance, the median and standard deviations as well as posterior probability bands. Furthermore, given the draws, it is possible to characterize the posterior distribution of any given function of the variables of interest by computing the corresponding functional for each of the draws.

[9] We also investigated the model allowing for indeterminacy in the solution with the technique of Bianchi and Nicolò (2019). However, in that exercise, the evidence continued to favor a joint distribution that puts most of its mass on the determinacy region. Results on those additional exercises are available from the authors upon request.

international component.[10] As noted before, we use demeaned observations on output growth, inflation, nominal interest rates, exchange rate changes, and terms of trade growth at a quarterly frequency in our empirical analysis.

Output growth rates are computed as log first differences of the real GDP series from the OECD Quarterly National Accounts (OECDNAQ) database and scaled by 100 to convert them into quarter-to-quarter percentages. For the real GDP, we also collect the real-time series available from the OECD Dataset: MEI Original Release Data and Revisions corresponding to the fourth quarter (December release) of the years 2006, 2007, 2008, 2009, 2010, and 2011. These fourth quarter (December release) vintages include data up to third quarter of the corresponding year and are similarly transformed in log-growth rates. We use the real-time growth rate in one of our robustness checks where we consider whether the shifts in the natural rate of interest rate around the 2007-09 Global Financial Crisis could have been identified also with the data available to policymakers and scholars at the time.

Inflation rates are defined as log first differences (scaled by 100) of the core CPI excluding food and energy indices from the OECD Main Economic Indicators (OECDMEI) database. We calculate the changes in the nominal exchange rate by taking log first differences (scaled by 100) of the Bank for International Settlements (BIS) nominal narrow effective exchange rate computed as a geometric trade-weighted average of the bilateral nominal exchange rate of the country with its major trading partners. The terms of trade series are measured by the ratio between the export prices of goods and services index and the corresponding import price index from the OECD Quarterly National Accounts (OECDNAQ) database, except for Sweden. The import and export prices of Sweden are from the International Monetary Fund's International Financial Statistics (IFS) database given that the relevant OECDNAQ indices only start from 1993 and are highly correlated with the respective IFS data. We take log first differences (scaled by 100) to convert the terms of trade series into their growth rates.

Consistent with Wynne and Zhang (2018b), the policy rate is proxied by the official cash rate for Australia from the Reserve Bank of Australia (G10 database), by the central bank rate for Canada from the OECD Main Economic Indicators (OECDMEI) database, by the central bank discount rate for South Korea from the International Monetary Fund's IFS (IFS) database, by the overnight money rate for Sweden from the OECD Main Economic Indicators (OECDMEI) database, by the central bank policy rate for Switzerland from the International Monetary Fund's IFS (IFS) database, and by the official bank rate for the U.K. from the Bank of England (G10 database).

All of the series are obtained from Haver Analytics except the real-time vintages of real GDP that are obtained directly from OECD (2020). The Haver mnemonics are listed in the Data Appendix together with some additional information on the data sources to facilitate the replication of our results.

3.3 Choice of Priors

Table 1 and Table 2 report our selection of priors for all the structural parameters of the reference small open economy model and all the parameters that characterize the dynamics of the exogenous shock processes. The priors for the parameters are the same for all countries, except for the degree of openness parameter, α, and are described in Table 1. To evaluate the interdependence between the small open-economies and

[10] Grossman et al. (2014) explore the degree of trade openness concluding that South Korea, Sweden, Switzerland, and Canada have traditionally been relatively more open to trade than Australia and the U.K., a pattern consistent with what we observe in our own data.

the rest of the world, we set country-specific prior for the openness parameter, α, tightly centered around the average import share of the corresponding country over our sample period from 1983:Q1 to 2018:Q1, as reported in Table 2.

Similar to Justiniano and Preston (2010), we impose fairly loose Gamma priors, with large tails, for the inverse of the Frisch elasticity of labor supply, φ, aware of the wide range of estimates emerging from macro and micro studies. The elasticity of substitution between home and foreign goods, η, imposes a Gamma prior centered at 1 which corresponds to the special case of a Cobb-Douglas consumption aggregator. In our monetary policy rule, the prior on the inertia parameter, ρ_i, has a Beta prior with dispersion wide enough to encompass most existing estimates. The Gamma priors for the feedback coefficients on inflation, ψ_π, and the output gap, ψ_x, are centered around the original Taylor (1993) values of 1.5 and 0.5, respectively.

The prior distributions for the other structural parameters of the reference model are consistent with those used by Lubik and Schorfheide (2007). To be specific, we employ a Gamma distribution with a large standard deviation for the steady state of the annualized real interest rate implying that the discount factor, $\beta \equiv \exp\left(-\frac{r^{ss}}{400}\right)$. We center the prior mean for the discount factor, β, around 0.994 therefore implying that the annualized real rate in steady state, r^{ss}, is around 2.5%. The prior for the slope coefficient in the Phillips curve, $\kappa \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$, is set with a loose Gamma prior centered at 0.5, which is consistent with values reported elsewhere in the literature (Rotemberg and Woodford (1998); Galí and Gertler (1999); Sbordone (2002); Zhang (2019)). We also restrict the intertemporal elasticity of substitution τ to lie inside the interval $[0, 1)$ with a Beta prior with a mean of 0.5. This choice is consistent with the range of plausible micro and macro estimates of the intertemporal elasticity of substitution found in the literature as surveyed in Havránek (2015) and Thimme (2017).[11]

We specify the priors for the rest-of-world output shocks, $y^f_{A,t}$, the rest-of-world inflation shock, π^f_t, the domestic technology growth shock, z_t, and the terms of trade growth shock, Δq_t, as in Lubik and Schorfheide (2007). These authors based their prior choices on their pre-sample analysis using data from 1970:Q1 to 1982:Q4. The priors for the parameters of the rest-of-world output shock process, $y^f_{A,t}$, are set by averaging the country-specific estimates of AR(1) models for the ratio of the U.S. GDP to domestic GDP. Similarly, the priors for parameters of the rest-of-world inflation shock, π^f_t, are obtained by fitting an AR(1) process to the U.S. CPI inflation series. For the priors on the domestic technology growth shock, z_t, and the terms of trade shock, Δq_t, these authors allow differences across countries and center their prior based on values obtained after estimating separate AR(1) models of the output growth and the terms of trade growth for each country. Given that they impose loose priors with roughly the same mean for these two shocks across countries, we select the same prior for all of the six countries by replicating the prior these authors propose for the U.K. (which assumes a Beta prior distribution for the persistence and an Inverse Gamma prior for the volatility).

[11] The intertemporal elasticity of substitution τ corresponds to the inverse of the coefficient of relative risk aversion. Therefore, the range $[0, 1)$ on τ implies that the coefficient of relative risk aversion is bounded below by one (i.e., $\frac{1}{\tau} > 1$ with $\frac{1}{\tau} = 1$ corresponding to the log-utility on consumption case).

Table 1. Prior Distributions for the Structural Parameters

Name	Description	Range	Density	Mean	Std. Dev	90% Interval
φ	Inverse Frisch	$\mathbb{R}^+$	Gamma	1.50	0.50	[0.78, 2.41]
η	Elasticity H-F goods	$\mathbb{R}^+$	Gamma	1.00	0.20	[0.70, 1.35]
ψ_π	Monetary rule, inflation	$\mathbb{R}^+$	Gamma	1.50	0.50	[0.78, 2.41]
ψ_x	Monetary rule, output	$\mathbb{R}^+$	Gamma	0.50	0.25	[0.17, 0.97]
ρ_i	Monetary rule, smoothing	[0,1)	Beta	0.50	0.20	[0.17, 0.83]
r^{ss}	Steady state real interest	$\mathbb{R}^+$	Gamma	2.50	1.00	[1.11, 4.34]
κ	Phillips curve, slope	$\mathbb{R}^+$	Gamma	0.50	0.25	[0.17, 0.97]
τ	Inter. elast. of substitution.	[0,1)	Beta	0.50	0.20	[0.17, 0.83]
ρ_q	Terms of trade	[0,1)	Beta	0.40	0.20	[0.10, 0.75]
ρ_z	Technology	[0,1)	Beta	0.20	0.10	[0.06, 0.39]
ρ_g	Preference	[0,1)	Beta	0.20	0.10	[0.06, 0.39]
ρ_{y^f}	World output	[0,1)	Beta	0.90	0.05	[0.81, 0.97]
ρ_{π^f}	World inflation	[0,1)	Beta	0.80	0.10	[0.61, 0.94]
ρ_u	Cost-push shock	[0,1)	Beta	0.80	0.10	[0.61, 0.94]
σ_i	sd monetary rule	$\mathbb{R}^+$	InvGamma	0.50	0.20	[0.29, 0.86]
σ_q	sd terms of trade	$\mathbb{R}^+$	InvGamma	1.50	0.55	[0.90, 2.50]
σ_z	sd technology	$\mathbb{R}^+$	InvGamma	1.00	0.35	[0.61, 1.64]
σ_{y^f}	sd world output	$\mathbb{R}^+$	InvGamma	1.50	0.35	[1.05, 2.15]
σ_{π^f}	sd world inflation	$\mathbb{R}^+$	InvGamma	0.55	0.20	[0.33, 0.92]
σ_u	sd cost-push	$\mathbb{R}^+$	InvGamma	0.55	0.20	[0.33, 0.92]
σ_g	sd preference	$\mathbb{R}^+$	InvGamma	0.25	0.10	[0.14, 0.43]

Note: Except for the degree of openness α, we set identical priors for all six countries. The effective priors are truncated at the boundary of the determinacy region.

Table 2. Country-Specific Priors for the Degree of Openness (α)

Name	Range	Density	Mean	Std. Dev.	90% Interval
Australia	[0,1)	Beta	0.15	0.05	[0.08, 0.24]
Canada	[0,1)	Beta	0.25	0.05	[0.17, 0.34]
South Korea	[0,1)	Beta	0.30	0.05	[0.22, 0.39]
Sweden	[0,1)	Beta	0.27	0.05	[0.19, 0.36]
Switzerland	[0,1)	Beta	0.27	0.05	[0.19, 0.36]
U.K.	[0,1)	Beta	0.20	0.05	[0.12, 0.29]

Note: The prior mean of the degree of openness is set to the average of the corresponding country's average import share over the period from 1983:Q1 to 2018:Q1, except for Sweden where due to data availability limitations we compute the average import share over the shorter period from 1993:Q1 till 2018:Q1.

Unlike Lubik and Schorfheide (2007), we also consider a domestic preference shock, g_t, and a domestic cost-push shock, u_t, both unobserved. With the prior belief that the domestic households' preference does not shift substantially over our sample period, we center the Beta prior for the persistence parameter of the preference shock, ρ_g, at 0.2, and locate the Inverse Gamma prior mean for the standard deviation of the preference shock, σ_g, at 0.25.[12] Finally, we set the priors for the cost-push shock, u_t, by adopting the same priors as for the foreign inflation shock. Specifically, our Beta prior for the persistence of the cost-push shock, ρ_u, is centered at 0.8, and its standard deviation, σ_u, comes from an Inverse Gamma prior distribution centered around 0.55. For all these priors, we allow a degree of uncertainty with the specification of the prior standard deviations.

4 Empirical Findings

Our structural model imposes the cross-equation restrictions arising from the New Keynesian small open economy framework. Here, we bring those to bear in order to help us with identification. More specifically, to help us recover a structural estimate of the natural rate for each one of the six economies in our sample and to help us determine its domestic and foreign drivers.[13]

4.1 Bayesian Model Comparison

We begin by fitting our small open economy model to the data for all of six economies in our sample. We evaluate the fit between the small open economy ($\alpha > 0$) and closed economy ($\alpha = 0$) specifications of the model under all three different monetary policy rules: the conventional Taylor (1993)-type rule or T rule in equation (13), the W rule in equation (14), and the $W\&T$ rule in equation (15). Figure 1 displays the Bayesian KR ratios illustrating the small open economy vs. closed economy comparison for each monetary policy rule for each country. The red bar is when the central bank follows the Taylor (1993)-type rule (the T rule) in both the small open economy and closed economy cases. The yellow bar shows a similar model comparison where monetary policy is assumed to follow the Wicksellian rule (the W rule), while the blue bar does the comparison assuming monetary policy tracks the short-term natural rate but also responds to domestic inflation and the domestic output gap (the $W\&T$ rule).

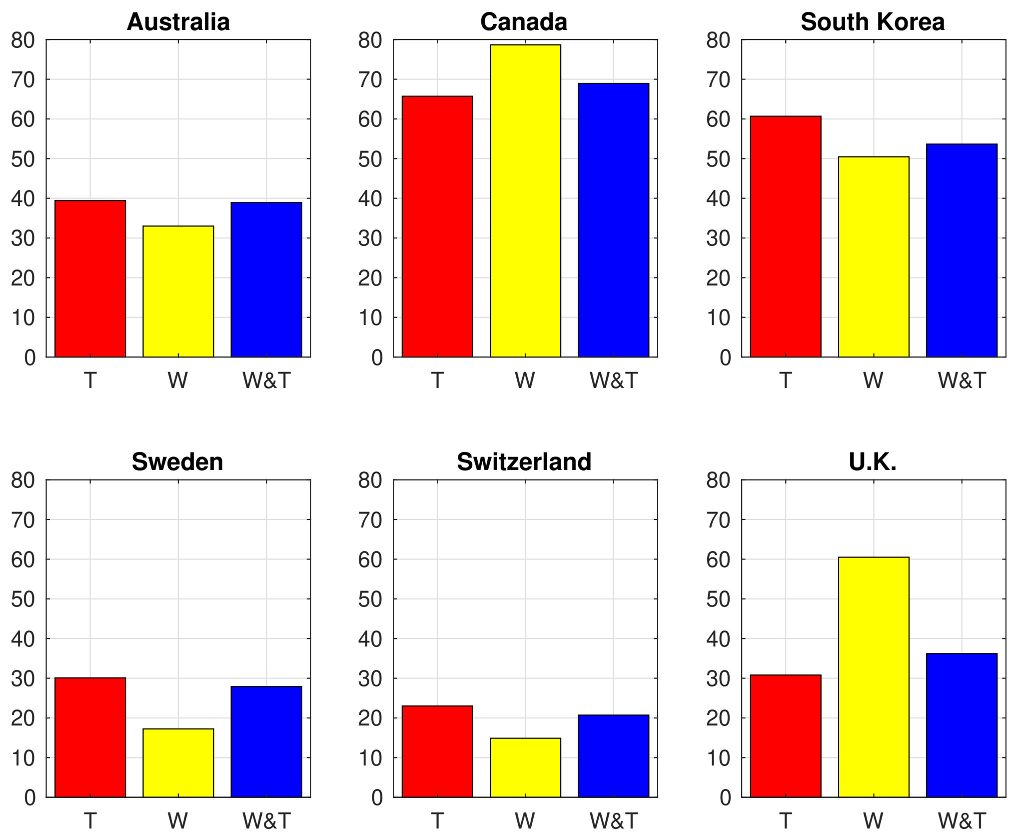
For all six countries across all three monetary policy rules, the KR ratios shown in Figure 1 far exceed 10. For Australia, Canada, South Korea, and the U.K., the KR ratios are in fact above 30. According to Kass and Raftery (1995), KR ratios above 10 indicate "very strong" evidence that the reference model (the small open economy specification) fits the data better than its alternative (the closed economy counterpart) for these countries, even though the closed economy specification is more parsimonious and has fewer parameters to estimate.[14]

[12] If the prior mean of ρ_g is higher or with greater uncertainty, there could be convergence (bimodal) problems in the estimation with the MCMC algorithm for some of the countries in our sample. Our numerical optimization routine will have difficulties finding the maximum of the posterior density.

[13] For a discussion of how our empirical work relates more broadly to the literature on estimating the natural rate of interest, see Literature Appendix.

[14] Martínez-García and Wynne (2014) show that Bayesian model comparison strongly penalizes overfitting which can lead to favor a less parameterized model against the true data-generating process when the two become arbitrarily close to each other. In light of that, the evidence here in favor of the small open economy model is particularly strong.

Figure 1. KR Ratio: Open vs. Closed Economy



Note: This figure shows the Kass and Raftery (1995) KR ratio (defined as $KR = 2(ML_A - ML_B)$, where ML_A is the log of the marginal likelihood of the estimated model A (the small open economy model) and ML_B is the log of the marginal likelihood of the estimated model B (the closed economy specification where $\alpha = 0$). Model A is compared against model B under alternative specifications of the monetary policy rule: the Taylor rule ("T"), the Wicksell rule ("W"), and the generalized Wicksell rule ("W&T"). Values of the KR ratio in excess of 10 are considered "very strong" evidence in favor of model A, i.e., in favor of the small open economy specification. Values of the KR ratio between 6 and 10 provide "strong" evidence and values between 2 and 6 offer some "positive" support in favor of model A. Values below 2 show no evidence in favor of model A.

Table 3. Bayesian Model Comparison

Name	Policy Rule (i_t^*)	KR (Open)	KR (Closed)	ML (W&T, Open)
Australia				-1243.62
W&T	$r_t^n + \psi_\pi \pi_t + \psi_x x_t$	0	-38.94***	
W	$r_t^n + \psi_\pi \pi_t$	-6.50**	-39.59***	
T	$\psi_\pi \pi_t + \psi_x x_t$	-4.92*	-44.34***	
Canada				-976.78
W&T	$r_t^n + \psi_\pi \pi_t + \psi_x x_t$	0	-68.94***	
W	$r_t^n + \psi_\pi \pi_t$	-1.90	-80.58***	
T	$\psi_\pi \pi_t + \psi_x x_t$	-12.50***	-78.22***	
South Korea				-1226.80
W&T	$r_t^n + \psi_\pi \pi_t + \psi_x x_t$	0	-53.66***	
W	$r_t^n + \psi_\pi \pi_t$	-24.40***	-74.86***	
T	$\psi_\pi \pi_t + \psi_x x_t$	-4.58*	-65.26***	
Sweden				-1060.49
W&T	$r_t^n + \psi_\pi \pi_t + \psi_x x_t$	0	-27.90***	
W	$r_t^n + \psi_\pi \pi_t$	-25.82***	-43.06***	
T	$\psi_\pi \pi_t + \psi_x x_t$	-5.32*	-35.42***	
Switzerland				-743.35
W&T	$r_t^n + \psi_\pi \pi_t + \psi_x x_t$	0	-20.72***	
W	$r_t^n + \psi_\pi \pi_t$	-16.52***	-31.40***	
T	$\psi_\pi \pi_t + \psi_x x_t$	-10.12***	-33.14***	
U.K.				-981.72
W&T	$r_t^n + \psi_\pi \pi_t + \psi_x x_t$	0	-36.20***	
W	$r_t^n + \psi_\pi \pi_t$	-12.50***	-73.00***	
T	$\psi_\pi \pi_t + \psi_x x_t$	-4.68*	-35.50***	

Note: Each panel shows the pairwise model comparison results for all six countries in our dataset. Small open economy and closed economy specifications are considered as well as three different specifications of the monetary policy rule: the Taylor rule ("T"), the Wicksell rule ("W"), and the generalized Wicksell rule ("W&T").

The second column of the table describes the target policy rate (i_t^*) of the corresponding variant of the policy rule. Monetary policy contains a smoothing component and is implemented as follows: $i_t = \rho_i i_{t-1}^* + (1 - \rho_i) i_t^* + \varepsilon_{i,t}$.

For each country, the table reports the log of the marginal likelihood (ML_A) for the reference small open economy model under the W&T monetary policy rule in the fifth column. Moreover, the table includes the Kass and Raftery (1995) KR ratio for each other possible modeling combination as well. The KR ratio is defined as $KR = 2(ML_B - ML_A)$ where ML_B is the log of the marginal likelihood of the estimated variant B of the model. Then, the three monetary policy variants under the small open economy setup relative to the small open economy specification with the W&T monetary policy rule are compared in the third column. Similarly, the KR ratio for the three monetary policy variants under the closed economy setup relative to the small open economy specification with the W&T monetary policy rule are compared in the fourth column. Values of the KR ratio in excess of 10 are considered "very strong" evidence in favor of model A, i.e., in favor of the small open economy specification. Given the definition of the KR ratio used here, values of $-$KR in excess of 10 provide "very strong" evidence in favor of model A (the small open economy model with the W&T policy rule) which we mark with three asterisks, between 6 and 10 provide "strong" evidence which we mark with two asterisks, between 2 and 6 offer some "positive" support which we mark with one asterisks, and below 2 shows no favorable evidence (with no asterisks).

We also evaluate whether the central bank in all six countries tracks the domestic (short-term) natural rate when making monetary policy and whether it responds to domestic inflation and the domestic output gap or only to domestic inflation. Table 3 summarizes our key findings comparing the performance of the reference small open economy model under each alternative specification. Each panel shows the log-marginal likelihood (ML) for the relevant small open economy model under the $W\&T$ rule, and the KR ratio for the other models relative to the reference model (that is, relative to the small open economy specification with the $W\&T$ rule). Consistent with the findings in Figure 1, the fourth column of Table 3 (column "KR (Closed)") shows that the reference model fits the data better than the closed economy specification irrespective of the monetary policy rule that the central bank is thought to be following in the closed economy case.

In the third column of Table 3 (column "KR (Open)"), we compare the different monetary policy rules under the small open economy specification. Given that all the numbers in the column "KR (Open)" are negative, the results show that the reference model with the $W\&T$ rule fits the data better than the same small-open economy model does under either the W rule or the T rule. When comparing the $W\&T$ rule and the T rule in the small open economy setting, we find that there is clear evidence in favor of the reference model featuring the $W\&T$ rule ("very strong" for Canada and Switzerland ($-$KR$>$ 10) and "positive" ($2 < -$KR$<$ 6) for Australia, Canada, South Korea, and the U.K.). This finding suggests that the central banks of these six countries appear to have tracked the short-term natural interest rate when making monetary policy decisions. In other words, the evidence presented here suggests that all six small open-economies track the short-term natural rate rather than the long-run (steady-state) as in the original Taylor (1993) monetary policy rule.

Furthermore, the reference model with the $W\&T$ rule also fits the data better than its counterpart under the W rule. When comparing the goodness of fit between the $W\&T$ rule and the W rule, there is "strong" evidence ($-$KR$>$ 6) in favor of the model associated with the $W\&T$ rule for Australia and "very strong" evidence ($-$KR$>$ 10) for South Korea, Sweden, Switzerland, and the U.K. Interestingly, the KR ratio for Canada equals just 1.9, a low value similar to what Cúrdia et al. (2015) report in their closed economy study of the U.S. This is borderline "positive" evidence ($2 < -$KR$<$ 6) in favor of the reference model according to the criteria laid out in Kass and Raftery (1995). All considered, the evidence indicates that central banks of these small open-economies respond to inflation deviations as well as to the output gap when making monetary policy decisions, with the weakest favorable evidence found with the Canadian data. As a result, in the remainder of the paper, we focus on the results from the reference small open economy model under the $W\&T$ monetary policy rule specification.[15]

4.2 Parameter Estimates

The Bayesian estimates of the structural parameters of the reference model for all six countries can be found in Table 4. In addition to 90% posterior probability intervals, we report posterior means as point estimates. The results are by and large consistent with the literature. The posterior mean of the degree of openness, α, ranges from 0.12 for the U.K. to 0.30 for South Korea. For all six countries, α is significantly above 0 and statistically different from it, which conforms with our previous findings on model comparison suggesting that it is better to estimate the natural rate for these countries using a small open economy framework (see

[15] The estimation results for other specifications not reported elsewhere in the paper are available from the authors upon request.

Figure 1 and Table 3). The estimates of all other structural parameters also fall within plausible ranges.

There is also a reasonably high degree of interest rate inertia (smoothing) by the central banks in all six countries, with the estimate of ρ_i ranging from 0.53 for Australia to 0.75 for the U.K. We find that countries such as Switzerland, Canada, and the U.K. pursue a slightly more anti-inflationary policy—with posterior means for ψ_π around 1.90—than countries such as Australia, South Korea, and Sweden—where the posterior means for ψ_π are all around 1.40. The estimates of the parameters of the monetary policy rule in all six countries lie comfortably within the determinacy region over our sample period.[16] The central bank of Switzerland is more responsive to the output gap, with ψ_x close to 0.70, than other countries, where estimates of ψ_x range between 0.38 and 0.55. Although the posterior means for the monetary policy parameters (ψ_π, ψ_x, and ρ_i) for all countries do not differ markedly from their priors, the data appear informative as the posterior distributions for the policy parameters are more concentrated than the prior distributions. As a robustness check, we also consider alternative priors on ψ_π and ψ_x which we discuss later where we find further evidence that the data is informative about these policy parameters.

The range of estimates of the elasticity of intertemporal substitution, τ, goes from 0.26 to 0.73, which is within the range of estimates found in previous micro and macro studies as surveyed in Havránek (2015) and Thimme (2017). As noted in discussing the impact of foreign (stationarized) shocks on the domestic potential output and the natural rate of interest in equations (8) and (12), the parameter τ is crucial to determine the magnitude and even the sign of those effects. For that reason, we also conduct a robustness check based on setting an alternative uniform prior distribution for τ. This alternative prior is uninformative, but our findings in regards to τ are nonetheless quite robust in this case too as we discuss later.

Our estimates of the parameter φ range between 1.53 (Switzerland) and 3.07 (South Korea), implying Frisch elasticity estimates of between 0.33 and 0.65. This range of estimates is more in keeping with the values reported elsewhere in the micro literature (usually between 0 and 0.5) than with the values commonly used to calibrate general equilibrium macro models like ours.[17] Our estimates of the elasticity of substitution between home and foreign goods, η, range between 0.63 and 1.22. These estimates are somewhat lower than the range of macro estimates reported by Feenstra et al. (2018) (which range from 1.2 to 4.1). However, our estimates are within the range of Armington trade elasticities reported by Gallaway et al. (2003) at the detailed industry level (the short-term trade elasticities being between 0.15 and 4.85, with an average estimate of 0.95, while the range of long-run elasticities is between 0.52 and 4.83, with an average of 1.55). Similarly, our estimates are also consistent with those of Blonigen and Wilson (1999) who find an average Armington trade elasticity of 0.81.

All our estimates of the slope of the Phillips curve, κ, are positive and statistically significant—ranging from 0.38 (the U.K.) to 1.87 (South Korea). Our point estimates of the slope of the Phillips curve, κ, are largely consistent with the estimates reported by other authors based on a range of techniques (see, e.g., the values estimated by Lubik and Schorfheide (2007) for Canada, those of Holmberg (2006) for Sweden, and the estimated values of Abbas et al. (2016) for Australia). We consider an alternative prior specification centered at $\kappa = 0.085$ to capture the hypothesis that the Phillips curve may be flatter. However, the estimated results are fairly close even in this case as we argue later when discussing our robustness checks.

[16] This is robust even when we estimate the reference model allowing for indeterminacy with the technique of Bianchi and Nicolò (2019).

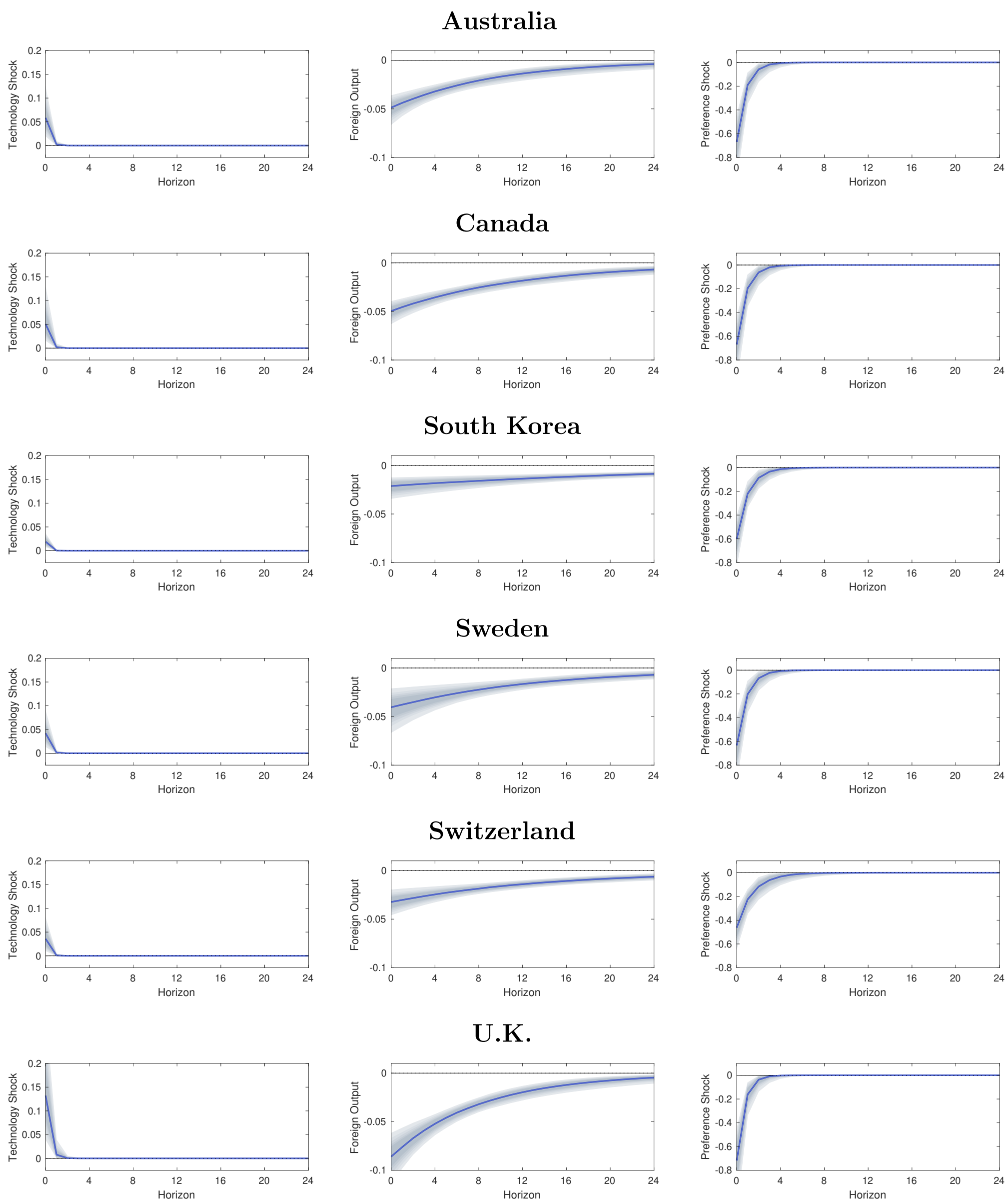
[17] See Chetty et al. (2011), Keane and Rogerson (2012), and Peterman (2016) for a discussion of how to reconcile the micro and macro estimates.

Table 4. Parameter Estimation Results

Name	Prior	Australia	Canada	South Korea	Sweden	Switzerland	UK
φ	1.50 [0.78, 2.41]	2.70 [1.60, 4.00]	2.22 [1.20, 3.44]	3.07 [2.04, 4.26]	2.30 [1.40, 3.40]	1.53 [0.83, 2.42]	2.67 [1.55, 3.94]
η	1.00 [0.70, 1.35]	0.71 [0.47, 0.98]	0.65 [0.46, 0.88]	1.22 [0.92, 1.56]	0.98 [0.71, 1.29]	1.01 [0.74, 1.34]	0.63 [0.41, 0.91]
ψ_π	1.50 [0.78, 2.41]	1.38 [1.07, 1.91]	1.95 [1.44, 2.60]	1.40 [1.09, 1.84]	1.41 [1.10, 1.90]	2.00 [1.51, 2.60]	1.83 [1.37, 2.41]
ψ_x	0.50 [0.17, 0.97]	0.48 [0.17, 0.90]	0.38 [0.16, 0.67]	0.53 [0.21, 0.94]	0.47 [0.19, 0.85]	0.70 [0.36, 1.12]	0.55 [0.27, 0.91]
ρ_i	0.50 [0.17, 0.83]	0.53 [0.38, 0.68]	0.61 [0.48, 0.72]	0.67 [0.56, 0.77]	0.70 [0.60, 0.79]	0.64 [0.51, 0.75]	0.75 [0.67, 0.82]
α	$--$ $[--, --]$	0.20 [0.11, 0.29]	0.23 [0.16, 0.31]	0.30 [0.22, 0.39]	0.28 [0.20, 0.37]	0.13 [0.08, 0.18]	0.12 [0.07, 0.18]
r^{ss}	2.50 [1.11, 4.34]	2.46 [1.09, 4.31]	2.48 [1.10, 4.31]	2.50 [1.11, 4.35]	2.49 [1.10, 4.34]	2.51 [1.11, 4.36]	2.50 [1.11, 4.34]
κ	0.50 [0.17, 0.97]	0.95 [0.48, 1.55]	0.65 [0.36, 1.02]	1.87 [1.27, 2.56]	1.12 [0.70, 1.64]	1.05 [0.62, 1.60]	0.38 [0.20, 0.67]
τ	0.50 [0.17, 0.83]	0.48 [0.28, 0.70]	0.44 [0.29, 0.63]	0.71 [0.54, 0.87]	0.63 [0.46, 0.81]	0.44 [0.30, 0.61]	0.26 [0.14, 0.47]
ρ_q	0.40 [0.10, 0.75]	0.53 [0.41, 0.65]	0.42 [0.30, 0.55]	0.12 [0.03, 0.23]	0.34 [0.21, 0.47]	0.10 [0.02, 0.20]	0.06 [0.01, 0.13]
ρ_z	0.20 [0.06, 0.39]	0.04 [0.01, 0.08]	0.04 [0.01, 0.08]	0.02 [0.01, 0.03]	0.03 [0.01, 0.07]	0.02 [0.01, 0.05]	0.07 [0.02, 0.14]
ρ_g	0.20 [0.06, 0.39]	0.30 [0.11, 0.51]	0.31 [0.12, 0.52]	0.39 [0.18, 0.59]	0.33 [0.13, 0.54]	0.51 [0.28, 0.71]	0.24 [0.08, 0.46]
ρ_{y^f}	0.90 [0.81, 0.97]	0.90 [0.85, 0.94]	0.92 [0.89, 0.95]	0.96 [0.94, 0.98]	0.93 [0.88, 0.96]	0.93 [0.90, 0.96]	0.89 [0.84, 0.92]
ρ_{π^f}	0.80 [0.61, 0.94]	0.35 [0.25, 0.46]	0.33 [0.23, 0.44]	0.32 [0.22, 0.43]	0.44 [0.33, 0.55]	0.34 [0.23, 0.45]	0.38 [0.27, 0.49]
ρ_u	0.80 [0.61, 0.94]	0.95 [0.91, 0.98]	0.92 [0.88, 0.96]	0.97 [0.95, 0.99]	0.97 [0.95, 0.99]	0.96 [0.94, 0.98]	0.94 [0.91, 0.97]
σ_i	0.50 [0.29, 0.86]	0.35 [0.28, 0.44]	0.30 [0.24, 0.37]	0.24 [0.19, 0.29]	0.26 [0.21, 0.31]	0.18 [0.15, 0.22]	0.23 [0.20, 0.27]
σ_q	1.50 [0.90, 2.50]	2.66 [2.41, 2.92]	1.66 [1.51, 1.83]	2.48 [2.25, 2.73]	1.34 [1.22, 1.48]	1.09 [0.99, 1.21]	1.25 [1.13, 1.38]
σ_z	1.00 [0.61, 1.64]	0.81 [0.67, 0.98]	0.64 [0.52, 0.79]	1.50 [1.32, 1.71]	0.99 [0.86, 1.16]	0.62 [0.50, 0.77]	0.65 [0.50, 0.84]
σ_{y^f}	1.50 [1.05, 2.15]	3.19 [1.49, 5.95]	2.63 [1.42, 4.50]	1.72 [1.19, 2.44]	1.75 [1.18, 2.56]	1.43 [1.03, 2.00]	2.15 [1.15, 4.674]
σ_{π^f}	0.55 [0.33, 0.92]	4.11 [3.72, 4.55]	2.33 [2.11, 2.57]	5.09 [4.60, 5.62]	2.59 [2.35, 2.86]	2.06 [1.87, 2.28]	2.95 [2.67, 3.26]
σ_u	0.55 [0.33, 0.92]	0.71 [0.43, 1.13]	0.73 [0.47, 1.09]	0.52 [0.33, 0.79]	0.59 [0.38, 0.90]	0.44 [0.31, 0.64]	0.48 [0.33, 0.70]
σ_g	0.25 [0.14, 0.43]	0.18 [0.12, 0.26]	0.18 [0.12, 0.25]	0.16 [0.11, 0.22]	0.18 [0.12, 0.26]	0.18 [0.12, 0.28]	0.21 [0.13, 0.33]

Note: The table reports posterior means and 90% confidence intervals (in brackets) which are computed from 4 MCMC chains of 250,000 draws each of which 62,500 draws were used in the initial burn-in phase. Convergence diagnostics were assessed using trace plots, R statistics, and SPM tests. The prior mean of the degree of openness, α, varies from country to country as reported in Table 2.

Figure 2. Impulse Response Functions (IRFs)



Note: This figure plots the Bayesian impulse response functions for the estimated natural rates in response to a standard deviation of the technology (productivity growth), foreign output (stationarized), and preference shocks. The impulse response functions are reported for each of the six countries in our dataset.

The estimates of the parameters of the stochastic processes for the shocks capture the substantial degree of persistence found in the data, most of which comes from the persistence of the foreign (stationarized) output shock ($\rho_{y^f} > 0.89$ for all countries) and the cost-push shock ($\rho_u > 0.92$ for all countries). The preference shock exhibits more moderate persistence, with estimates for ρ_g ranging between 0.24 and 0.51. The domestic technological productivity growth shock exhibits even less persistence, with $\rho_z < 0.07$.

Figure 2 shows the impulse responses of the natural interest rates and the associated 90% confidence intervals to a one standard deviation innovation from the three shocks that affect the natural rate (the technological productivity growth, stationarized foreign output, and preference shocks) for all the six countries. The signs of all the impulse responses are consistent with the implications of the natural rate equation derived in (12). The technology growth shock causes a rise in the natural interest rate. The stationarized foreign output shock and the preference shock are assumed to be stationary and generated by an exogenously-given AR(1) stochastic process. As a result, positive level shocks in foreign output and in preferences tend to have the opposite effect and, thus, tend to reduce the natural rate of interest.

As can also be seen in Figure 2, we find that the technology growth shock causes a transitory increase in the natural interest rate whose magnitude appears to be larger in the Bayesian impulse response of the U.K.'s natural interest rate than in the responses recorded for any of the other countries. In turn, the estimated effects of a preference shock tend to be much larger in magnitude than those of other shocks, while the responses to the (stationarized) foreign output shocks generally exhibit more persistence.

4.3 Natural Rate Estimates

Figure 3 shows time series of our estimates of the natural rate of interest in deviations from the steady state for the six countries we investigate in this paper. This figure illustrates the estimates recovered from the small open economy model under the $W\&T$ monetary policy rule given by (15) (blue solid line) and the 90% posterior probability bands associated with it (blue shaded area). The confidence bands generally are tighter than those estimated with semi-structural models (e.g., Holston et al. (2017) and Fries et al. (2018)) as we use more of the cross-equation restrictions that arise from theory. We also include the demeaned real policy rate for each country computed as the nominal policy rate less inflation over the previous four quarters in deviations from its historical mean (black solid line). Furthermore, we assume that monetary policy also follows the $W\&T$ rule in the more parsimonious closed economy case and plot the recovered closed economy natural rate (green dashed line) as well.

The wedge between the natural rate estimate obtained from the reference model and its closed economy counterpart illustrates the substantial bias that can arise whenever the estimated model ignores the strong trade linkages of economies such as the ones we investigate in this paper. On the one hand, the cases of Australia, South Korea, and Sweden show how misspecification resulting from looking at the evidence through the lens of a closed economy model can distort the recovered natural rates and, in doing so, affect our inferences about the stance of monetary policy. In fact, it is interesting to note that the misspecified closed economy estimates of the natural rate of interest for those three countries would lead us to conclude that their respective domestic natural rates have been largely insulated from the pattern of decline seen among many major advanced economies while the small open economy estimates favored by the data would strongly suggest the opposite.

On the other hand, even when the discrepancy between the closed economy and small open economy estimates of the natural rate is less pronounced—as is the case for Canada, Switzerland, and the U.K.—the closed economy specification may still distort our inferences about monetary policy in other ways. For starters, it tends to magnify the decline of the natural rate of interest when contrasted against the small open economy estimates. Moreover, the observed fall in the natural rate under the misspecified closed economy model would attribute all of the decline entirely to domestic factors while the small open economy model tells us otherwise. The reference model tells us that domestic shocks are part of the explanation, but not the full story. Hence, the evidence presented in Figure 3 underscores the importance of taking into account the degree of openness of these economies when making empirical inferences about their natural rate of interest.

The estimated natural rates in Figure 3 follow the same broad downward trend over the past 35 years in all six countries, and all fall significantly below their steady state around the year 2000, with the fall being most pronounced in the aftermath of the 2007-09 Global Financial Crisis. The extent to which the *ex post* real policy rate tracks the estimates of the natural rate varies substantially across countries, though. Figure 3, in fact, illustrates how the *ex post* real rates in Australia, Canada, and the U.K. seem to track the small open economy estimates of the natural rate reasonably closely, while for South Korea, Sweden, and Switzerland we find episodes where the *ex post* real policy rate deviates by a noticeable amount from the natural rate estimated under the reference model. Moreover, we also find that the *ex post* real interest rate has been well below the small open economy estimates of the natural rate in all six countries in the aftermath of the 2007-09 Global Financial Recession. By the end of our sample in 2018:Q1, the *ex post* real rates remained quite below their natural rates in Canada, Sweden, and Switzerland while the gap between the two had largely closed in Australia, South Korea, and the U.K.

4.3.1 Natural Rate Comovement, Monetary Policy Divergence

In order to further explore the implications of all of this for monetary policy, we plot our cross-country evidence in levels in Figure 4: Panel A shows the nominal policy rates of all countries in our sample, Panel B includes their natural rate in levels constructed combining the smoothed natural rate series in deviations from the steady state (as seen in Figure 3) with the corresponding point estimate of the steady state real interest rate (the r^{ss} in Table 4), while Panel C showcases their monetary policy stance (that is, the real interest rate gap) in terms of the difference between the *ex ante* real interest rate and the natural rate.

Panel A shows the history of the nominal policy rates in the six countries and illustrates its pattern of comovement, while Panel B highlights on a comparable scale in levels the estimated natural rates recovered under our reference model (a small open economy specification under the $W\&T$ rule).[18] Panel B illustrates the similarly robust pattern of comovement displayed by the natural rate of interest across all countries in our sample, but also shows the significant downward drift of the natural rates over time (particularly since the 2007-09 Global Financial Crisis). A common observation for both the nominal policy rates and the natural interest rates that emerges in our evidence is that they both display a notable degree of synchronization and a significant downward shift in the late 2000s.

Moreover, Panel B suggests that the recent history of natural rates can be divided into two/three distinct phases depending on the country. From the early 1980s through the mid-1990s, natural rates were generally higher in Australia, Canada, and the U.K. From the mid-1990s through 2007 or so, natural rates remained stable while from 2008 onwards they have taken a nosedive falling into negative territory (although at levels

[18] The natural rate in levels is computed as the median of the real interest rate in steady state, r^{ss}, plus the median of the recovered natural rate of interest in deviations from the steady state, r_t^n.

that on average are not too far below from zero). For Switzerland, Sweden, and South Korea, the evolution of our estimates of the natural rate of interest is qualitatively similar but quantitatively more muted. While the patterns of the natural rate estimates of those three countries did not change much in the 80s and 90s, their natural rate estimates still experienced a modest downward shift in the late 2000s.

Panel B also shows the significant impact of the 2007-09 Global Financial Crisis on the recovered natural rates of interest and the divergence that has emerged since then. Indeed, Panel B shows that the natural rates were quite similar across all six countries in levels during the 90s and early 2000s but started to diverge since the 2007-09 Global Financial Crisis as natural rates held up better in Switzerland, Sweden, and South Korea than in Australia, Canada, and the U.K. Either way, we should also note that the decline in the natural rate in all six countries has been quite sticky over time irrespective of the magnitude of the decline. Indeed, we observe limited signs of mean-reversion by the end of our sample in 2018:Q1 particularly in Australia, Canada, and the U.K.

Finally, Panel C plots the time series of the real interest rate gaps (i.e., the *ex ante* real interest rate minus the natural rate implied by the reference model) over our sample period. Panel C showcases the pattern of variability (volatility) of these real interest rate gaps and also, in contrast with what we observed with the natural rates, their weak comovement. These real interest rate gaps provide an indication of the stance of monetary policy. Hence, Panel C comes to indicate a very diverse set of monetary policy experiences across the six countries in our sample. We argue that those differences in the monetary policy stance have to do with the cross-country differences reflected in our discussion of the estimates and performance of the reference model in Figure 1, Figure 2, and Figure 3 and well as in Table 3 and Table 4. This is because adjusting monetary policy—in particular, adjusting policy rates in the face of significant declines in the natural rate, in particular during the 2007-09 Global Financial Crisis—provides contrast. We view this as having helped elicit more clearly some of the distinctive features of the structural, particularly in regards to the importance of tracking the natural rate of interest in the implementation of monetary policy and the impact of international spillovers on the path of the natural rate.

4.4 Foreign Spillovers

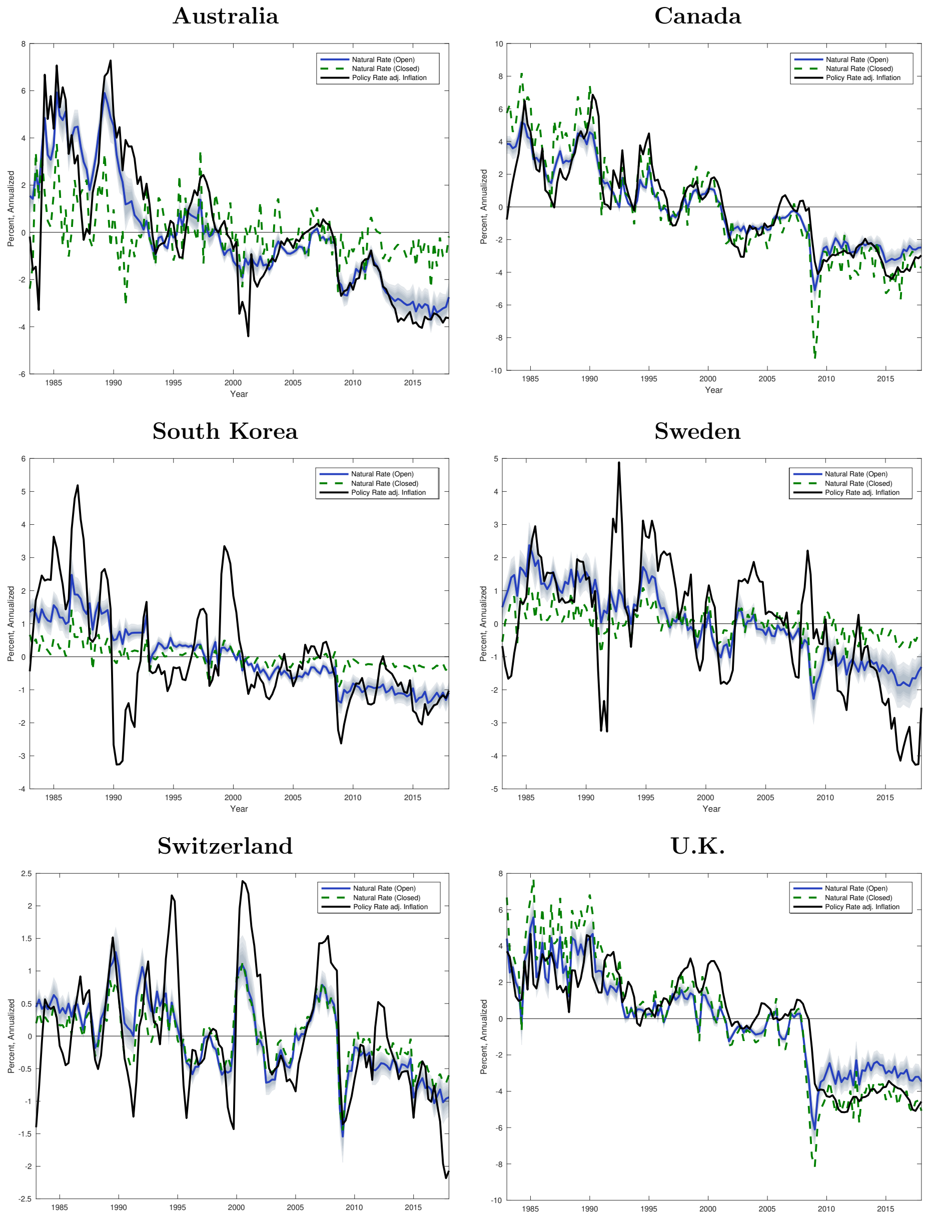
Though a decrease in inflation expectations can explain part of the fall in nominal interest rates, there is also a clear downward trend in real interest rates over the past 35 years among many leading advanced economies, with a further fall to historically low levels since the 2007-09 Global Financial Crisis (e.g., Holston et al. (2017) and Fries et al. (2018)). Our findings suggest similar patterns for small open-economies where declines in the real rate have followed the declines in the natural rate of interest. To quantify this, we follow the suggestion of David (1949) to compute a single summary statistic of the degree of comovement across all countries in our sample. With six countries there are $C_6^2 = 15$ simple pairwise correlations for a given variable. If we let $corr_k$ denote the simple pairwise correlation between country i and country j, $i, j = 1, ..., 6$ and $i \neq j$, then the proposed summary measure is given by:

$$corr \equiv \tanh\left(\frac{\Sigma_{k=1}^{15} T_k \tanh^{-1}(corr_k)}{\Sigma_{i=k}^{15} T_k}\right), \tag{17}$$

where $\tanh(\cdot)$ is the standard hyperbolic tangent function and T_k is the sample size for the k-th pairwise correlation $corr_k$.

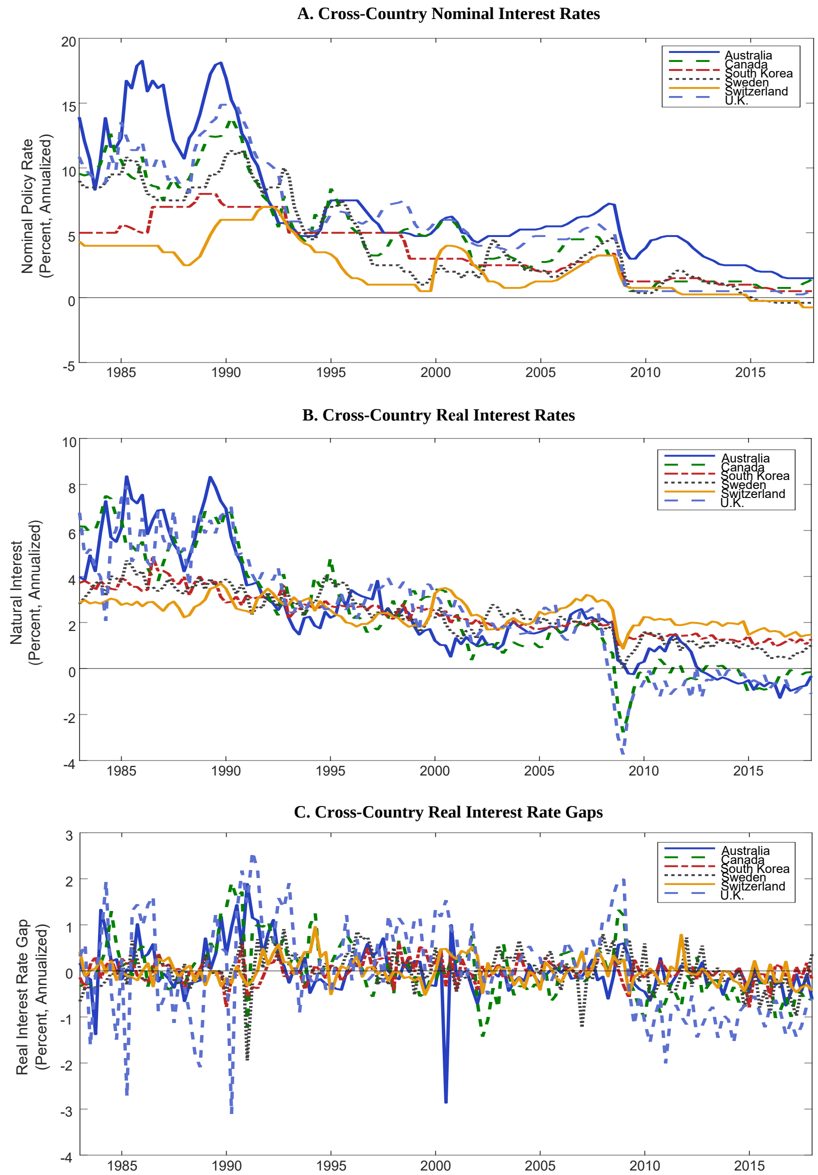
Figure 3. Natural Interest Rates: Open vs. Closed Economy

(In Deviations from Steady State)



Note: The figure plots the recovered natural rate estimates from the reference model (a small open economy model with a W&T monetary policy rule that tracks the natural rate and responds to inflation and output gap fluctuations) with its corresponding 90% confidence intervals (blue line and shaded area) for all six countries in our dataset. Together with this, the figure also plots the estimated natural rate of interest under the same W&T monetary policy rule but assuming a closed economy specification instead (green dashed lines). The comparison between the estimates of the reference model and the closed economy model illustrates the bias in the estimation of the natural rate of interest that arises whenever the estimated model is misspecified by imposing that $\alpha = 0$. The figure also plots the (observed) *ex post* real interest rate computed as the nominal policy rate less inflation over the previous four quarters (black solid lines). The real rates are demeaned and the natural rates reported in deviations from their respective steady states, and all of them are expressed in percentages at an annualized rate.

Figure 4. Cross-Country Comparison: Nominal Interest Rates, Natural Interest Rates, and Real Interest Rate Gaps



Note: The subplot at the top of the figure shows the nominal interest rates of the six small open-economies in our dataset. The subplot in the middle of the figure plots the smoothed posterior median estimates of the natural rates of interest in levels by adding the median estimate of the steady state real interest rate to the estimates of the natural rate in deviations from the steady state shown in Figure 3. These estimates are recovered from the reference model (a small open economy under the W&T monetary policy rule where the central bank tracks the natural rate while responding to domestic inflation and the domestic output gap) for each of the six small open-economies in our dataset. The subplot at the bottom of the figure plots the smoothed posterior median estimates of the real interest rate gap (i.e., the difference between the real rate and the natural rate estimates) in deviations from the same steady state, also based on the reference model (the small open economy under the W&T monetary policy rule).

In Figure 4, we illustrate the comovement between the nominal interest rates, the natural rates of interest, and the *ex ante* real interest rate gaps in all six small open-economies in our sample. With the summary statistic given by (17), we can quantify the comovement for each one of them. The aggregate correlation between nominal interest rates in the six countries is 0.87, while the aggregate correlation between our estimates of the natural rate is 0.84. However, the aggregate correlation between the real interest rate gaps of all the countries is only 0.18. These aggregate correlations, therefore, confirm the strong comovement among natural rates of interest in all six small open-economies in our sample which is so apparent by simple visual inspection—and similarly confirms the weak correlation on the stance of monetary policy that we have noted before.

We also explore the patterns of comovement of the natural rate estimates of each country with the estimates of the natural rate for the U.S. using the same aggregate correlation formula. Rather than take a stand on a specific natural rate estimate for the U.S., we compute the aggregate correlation with respect to 5 different U.S. estimates (those of Laubach and Williams (2003), Lubik and Matthes (2015), Holston et al. (2017), Johannsen and Mertens (2018), and Kiley (2020)) for which there are 15 different pairwise combinations for each one of the six countries.[19] The results are shown in Table 5 together with a number of country characteristics that describe the degree of openness of each country (measured by the import share over GDP), their economic size (measured by the share of PPP-adjusted GDP over that of the U.S.), and their level of development (measured by their GDP per capita relative to that of the U.S.).[20]

Our natural rate estimates for all six countries are highly correlated with the estimates of the U.S. natural rate, suggesting a high degree of international comovement and an important role for foreign output shocks as drivers of the natural rate in these small open-economies. Moreover, this evidence of a strong aggregate comovement ranging from a low of 0.61 (Switzerland) to a high of 0.86 (U.K.) is even more notable given the significant differences in the degree of openness, economic size, and level of development of these countries seen in Table 5.

The strong correlation between the domestic natural interest rate in these six countries and the U.S. natural rate is consistent with the view that international spillovers play a significant role among small open-economies. In order to dig deeper into the foreign drivers of the natural rate, we report the results of a variance decomposition exercise with our reference model (a small open economy model with the $W\&T$ monetary policy rule) in Table 6. We decompose the fluctuations of our estimates of the short-term natural rate in each of the six countries into the components attributable to technology growth shocks, preference shocks, and foreign output shocks at horizons of one quarter, eight quarters (two years), twenty quarters (five years), as well as at an infinite horizon.

We observe here that at an infinite horizon, the foreign output shock accounts for well over half of the variation in the natural rate of every country, and about $70-85\%$ in Australia, Canada, and the U.K. These three countries are the ones where the decline in the natural rate shown in Panel B of Figure 4 is more

[19] The U.S. natural rate estimated by Laubach and Williams (2003) is obtained from https://www.frbsf.org/economic-research/files/Laubach_Williams_updated_estimates.xlsx; the estimates of Holston et al. (2017) are from https://www.frbsf.org/economic-research/files/Holston_Laubach_Williams_estimates.xlsx; and those of Lubik and Matthes (2015) are available at https://www.richmondfed.org/publications/research/economic_brief/2015/eb_15-10. The U.S. natural rates of interests estimated by Johannsen and Mertens (2018) and Kiley (2020) were obtained directly from the authors upon request—we thank them for sharing their data with us.

[20] A recent paper by Fernández et al. (2015) classifies the degree of openness/closedness of the capital account of different countries as being either "open," having some controls that amount to a "gate," or having many controls that amount to a "wall." According to their measure, Canada, Sweden, and the U.K. are considered "open" countries while Australia, South Korea, and Switzerland are "gate" countries.

stark, as discussed earlier. Even on impact, foreign output shocks are important, especially for the same three countries (Australia, Canada, and the U.K.) Therefore, the variance decomposition exercise shows more than comovement; it clearly shows that foreign shocks are a major factor driving the domestic natural rates of these six small open-economies.

But to what extent is this result driven by the priors instead of the observed data? When setting all the parameters at their prior means while allowing the openness parameter to be equal to the prior mean for South Korea, i.e. imposing $\alpha = 0.30$, only around 9.70% of the volatility in the natural interest rate is attributable to the foreign output shock at short horizons which increases to 35.17% at the infinite horizon. On the contrary, the contribution of the domestic technology growth shock and the preference shock is, respectively, 66.51% and 23.80% at short horizons, dropping to 47.74% and 17.08% at the infinite horizon. The foreign output shock contributes even less to the natural interest rate fluctuations in the other five countries given that South Korea is by far the country with the highest import share α among the small open-economies in our sample (as can be seen in Table 5). This indicates, not just that our data is informative about the role of foreign shocks, but most notably that foreign spillovers play a major role—in fact, a dominant role—on the domestic natural rate.

All of this, coupled with the rest of the evidence presented earlier, paints a very distinct picture about the natural rate in small open-economies. It suggests that weakened domestic factors (and, in particular, the perceived domestic productivity growth slowdown) are not necessarily what lies behind the declining natural rates in these countries. The totality of our findings point straight at spillovers from the rest of the world as the main factor behind the observed decline (most starkly so in the English-speaking countries of Australia, Canada, and the U.K. that traditionally have had stronger economic and cultural ties with the U.S.).

And this, needless to say, has major implications for monetary policy which through the natural rate becomes quite dependent on developments abroad over which the country's central bank has no influence even when domestic economic conditions remain largely unchanged. In plain English, one could say that weakening economic prospects abroad can be powerful enough to drag with them the natural rates (and monetary policy) on countries that are otherwise not subject to the same domestic malaïse.

4.5 Robustness Checks

We have considered a number of robustness checks to explore the sensitivity of our estimates to a number of assumptions. Most notably, we have considered different prior specifications. We replace the prior on τ with an uninformative uniform prior to allow greater flexibility for the data to guide the estimation. This parameter plays an important role in shaping the magnitude and even the sign of the foreign output effect on the domestic natural rate as seen in (12). We have also considered a prior that centers the parameter κ at 0.085 which is at the lower end of the estimates we obtain in Table 4 in order to ascertain the extent to which priors on the slope of the Phillips curve affect the recovered estimates of the natural rate of interest, particularly when one's prior views are of a much flatter Phillips curve.We have also investigated lowering the prior mean of the parameter in the monetary policy reaction function that sets the response to the output gap, ψ_x, to 0.3 and 0.1. This would be the case if our prior belief was that monetary policy responds to inflation and pays little attention to anything else. In a similar vein, we have also analyzed the case where the prior mean of the parameter that defines the response to inflation, ψ_π, is increased to 1.75 and even 2.

Table 5. Comovement of the Natural Rate with the U.S. Estimates

	Australia	Canada	South Korea	Sweden	Switzerland	U.K.
$corr\left(r^n_{i,t}, \left\{r^n_{U.S.,t}\right\}\right)$	0.74	0.82	0.81	0.79	0.61	0.86
Some Country Characteristics (Average $1983-2017$)						
Import Share (%)	14.79	25.11	29.76	27.27	26.67	20.20
PPP-Based Share of U.S. GDP (%)	5.70	9.17	7.51	2.70	2.96	15.86
GDP per capita relative to U.S. (%)	81.32	83.28	45.57	83.29	113.47	73.41

Note: Entries in the first row are calculated computing the aggregate correlation between the natural rate of each small open-economy and the five U.S. natural rate estimates for which we have data available over the period between 1983:Q1 and 2016:Q3. The aggregate correlation is computed using the the Fisher transformation method described in David (1949) as in (17). The remaining three rows of the table provide an average over the period from 1983 till 2017 of three basic country characteristics of each economy: the degree of openness measured with the imports as a share of GDP (for which the Swedish data is only available since 1993:Q1), the economic size of the country's economy relative to that of the U.S. based on PPP-adjusted GDP numbers, and the relative development of the country's economy with the ratio of the GDP per capita (constant prices, 2011 international $) of the country over that of the U.S.

Table 6. Shock Contribution to the Natural Interest Rate

Name	Australia	Canada	South Korea	Sweden	Switzerland	UK
Period 1						
Technology	7.12 [0.57, 19.77]	5.12 [0.33, 15.86]	9.11 [1.36, 24.35]	11.20 [0.89, 31.45]	6.95 [0.50, 20.01]	14.10 [1.00, 39.23]
Foreign Output	57.34 [34.92, 78.67]	52.21 [31.12, 72.29]	13.87 [3.56, 31.67]	26.97 [6.93, 54.33]	22.77 [7.44, 43.31]	49.89 [23.85, 74.37]
Preference	35.54 [16.58, 58.87]	42.67 [22.83, 64.07]	77.02 [56.69, 91.83]	61.83 [35.31, 84.60]	70.29 [48.36, 87.56]	36.01 [15.00, 61.04]
Period 8						
Technology	2.61 [0.19, 7.69]	1.85 [0.11, 6.47]	5.18 [0.67, 13.60]	5.83 [0.42, 17.17]	3.39 [0.21, 10.00]	6.41 [0.36, 20.59]
Foreign Output	83.00 [68.47, 92.86]	81.43 [67.01, 91.35]	42.22 [17.59, 67.89]	58.05 [28.55, 82.00]	50.19 [25.17, 72.30]	77.06 [55.47, 90.76]
Preference	14.39 [5.83, 28.25]	16.72 [7.64, 29.76]	52.60 [28.62, 76.63]	36.11 [15.18, 63.61]	46.42 [25.64, 70.56]	16.53 [6.37, 33.10]
Period 20						
Technology	2.21 [0.16, 6.59]	1.48 [0.08, 5.37]	4.05 [0.49, 10.81]	4.84 [0.34, 14.30]	2.84 [0.17, 8.54]	5.73 [0.32, 18.80]
Foreign Output	85.58 [72.47, 94.03]	85.20 [73.24, 93.36]	54.66 [29.53, 77.76]	65.23 [38.20, 85.44]	58.28 [34.50, 77.90]	79.60 [60.10, 91.90]
Preference	12.20 [4.99, 24.25]	13.32 [5.97, 24.57]	41.29 [20.48, 65.28]	29.93 [12.47, 54.71]	38.88 [20.07, 61.79]	14.67 [5.60, 29.33]
Period Infinity						
Technology	2.21 [0.16, 6.52]	1.43 [0.08, 5.25]	3.45 [0.40, 8.98]	4.58 [0.32, 13.40]	2.70 [0.16, 8.17]	5.67 [0.32, 18.63]
Foreign Output	85.86 [72.74, 94.22]	85.75 [74.06, 93.69]	61.39 [40.52, 80.89]	67.02 [43.25, 85.80]	60.36 [37.50, 79.11]	79.80 [60.34, 92.01]
Preference	11.97 [4.83, 24.01]	12.82 [5.62, 23.82]	35.16 [17.35, 55.51]	28.41 [11.91, 50.46]	36.94 [18.93, 58.60]	14.53 [5.55, 29.01]

Note: Each entry decomposes the forecast error variance of the natural rates of interest of each of the six countries in our dataset into percentages due to each shock at different forecast horizons. The table reports posterior means and 90% confidence intervals of each shock contribution (in brackets) one-quarter ahead, eight-quarters ahead (two-years ahead), twenty-quarters ahead (five-years ahead), and for the limit case where the forecasting horizon tends to infinity.

In results available upon request we show that the estimates we obtain and the recovered natural rates are very similar to those reported for the reference model in all these alternative scenarios about the priors. Some of the few noteworthy differences that we detect are for Australia and Canada in the case where the alternative prior centers the policymakers response to the output gap at a very low $\psi_x = 0.1$. Even in this case, however, the differences between the recovered natural rate of interest and that from our reference model are generally about 10 basis points or less.[21] This gives us more confidence on the recovered estimates of the natural rate for the six small open-economies shown in Figure 3 and Figure 4.

In addition, we evaluate how our estimates of the natural interest rate perform with real-time data in Figure 5. The real GDP data is usually released with some lags and then revised periodically. We are particularly interested in the real-time analysis around the 2007-09 Global Financial Crisis period as at the time the initial GDP releases may have been less accurate and, therefore, so would be the estimates of the natural rate crucial for policymaking. Figure 5 compares the final estimates of the natural interest rate (in the black line) with the real-time estimates based on data available in the fourth quarter (December release) of each year from 2006 to 2011. Because of the lags in the data publication all these historical vintages which we get from OECD (2020) contain time series observations only up to the third quarter of the year.

The analysis in Figure 5 shows that the perceptions about the natural rate of interest have shifted to some degree with more recent vintages of data, particularly in regards to the magnitudes. For example, the real-time estimates of the natural interest rates are lower in South Korea than the final estimates by around 30 basis points. The real-time estimates are also lower for Australia in most periods while somewhat higher for Sweden in 2010 and 2011. For other countries, there are negligible differences between the real-time and final estimates. However, the evidence shows that the fall of the natural rate of interest during the 2007-09 Global Financial Crisis should have been apparent pretty quickly with real-time data too. It should have also been quite apparent that the natural rates continued to be below their pre-recession levels even as the recovery started to take on a more solid footing since 2010. Moreover, the differences that emerge among some countries between the real-time estimates at the time and our current view of their natural rates based on the most recent vintage of data available to us (that of our full sample) suggest that, if anything, the magnitude of the natural interest rate fall may have been underpredicted in some cases as estimates of the pre-recession natural rate tend to be a touch higher now for most of the countries in our sample.[22]

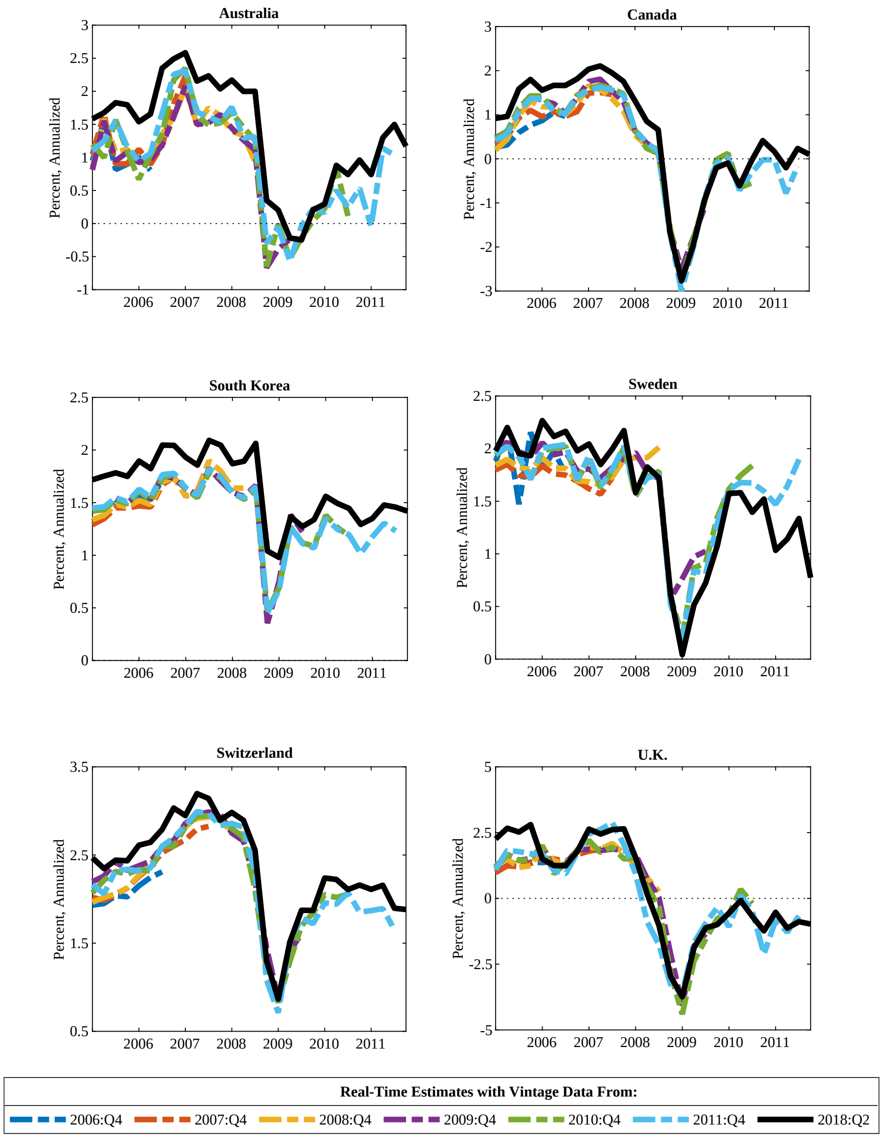
To explore the question a bit further, we also investigated whether this was the result of data revisions occuring between the first-release (the real-time vintage) of the data and the most recent vintage that we use to derive our full sample estimates. To do this, we compared the real-time estimates across vintages with the estimates of the natural rate obtained using a subsample of our most recent vintage (the full-sample dataset) over the same period of time. While differences exist, they are significantly smaller than 10 basis points on average for all countries in our sample.[23] We interpret this as evidence that the modest re-assessment of the natural rate of interest shown in Figure 5, particularly before the 2007-09 Global Financial Crisis, is not primarily the result of data revisions but the result of having more observations in the sample.

[21] Table 3 also suggests that the evidence in favor of the $W\&T$ monetary policy rule when compared against the W rule under the small open economy specification is weaker for Australia and Canada. Our robustness checks here therefore indicate that the apparent proximity between both specifications in part reflects the fact that the central bank's response to the output gap does not seem to have much effect on the equilibrium or on our assessment of the natural interest rate for these two countries.

[22] We leave the exploration of questions such as the implications for monetary policy of underestimating the magnitude of the decline of the natural rate in real-time for future research.

[23] These results are available upon requests from the authors.

Figure 5. Real-Time Estimates of the Natural Rate During the 2007-09 Great Financial Crisis



Note: The figure plots the smoothed posterior median estimates of the natural rate of interest in levels adding the median estimate of the steady state real interest rate to the country's estimates of the natural rate in deviations from the steady state as in Figure 4. These estimates are those recovered from the reference model (a small open economy under the W&T monetary policy rule where the central bank is seen as tracking the natural rate while responding to domestic inflation and the domestic output gap) for each of the six small open-economies in our dataset. The figure illustrates the estimates of the natural rate around the 2007-09 Great Financial Crisis covering the period from 2005:Q1 till 2011:Q4 across different vintages of the data. The black line corresponds to the data available as of 2018:Q2 which is the same vintage of data that we use for all other results in the paper. All the dashed lines refer to the estimates of the model that are obtained using the same reference model but only the data actually available to policymakers and scholars at the time. The real-time vintages included correspond to the fourth quarter (December release) of each year from 2006 till 2011.

In other words, our full sample estimates (black line in Figure 5) have the benefit of being derived with a sample size one third larger than that of the real-time estimates (dashed lines in Figure 5) and with a great deal more information about the aftermath of the 2007-09 Global Financial Crisis. In spite of that, the qualitative patterns in the data were already apparent in real-time. What we have learned over time since the 2007-09 Global Financial Crisis as more data points were observed has more to do about the magnitude of the fall and about the persistence of those low natural rate estimates.

5 Concluding Remarks

This paper provides new estimates of the natural rate of interest for six small open-economies and quantifies the relative importance of domestic vs. foreign factors in driving fluctuations in the natural rate over the past three and a half decades. We extend the small open economy framework of Galí and Monacelli (2005) and Lubik and Schorfheide (2007), generalizing it to allow for exogenous technological progress and a more flexible representation of preferences (including shocks to preferences). To begin, we assess the significance of the cross-equation restrictions imposed by the model to fit the data by comparing our benchmark model against more parsimonious, alternative specifications that abstract from the key features of the model (the international linkages that arise through trade and the implementation of monetary policy tracking the short-term natural rate). From this evidence, we conclude that there is strong evidence in favor of the small open economy specification and the assumption that policymakers' track the short-term (rather than the long-run) natural rate of interest when implementing monetary policy.

Taking the setup favored by the data as our reference model, we find robust evidence that the natural rates have been trending down in all six countries over time. We also show that they also tend to comove strongly with each other and with natural rate estimates for the U.S., consistent with our empirical evidence of significant international spillovers and a large role for foreign output shocks in driving the natural rate of interest among small open-economies. Indeed, our variance decomposition exercise finds that foreign output shocks account for between 60% and 85% of the movement in natural rates at infinite horizons. Hence, we provide supportive evidence showing that foreign drivers are the dominant force explaining the fluctuations of the natural rate of interest for the six small open-economies we investigate here. We therefore conclude that international spillovers are one very important propagation channel that has contributed much more to drag natural interest rates in these economies than domestic forces have. In other words, international spillovers from major advanced economies appears to be the key factor at play.

The analysis in this paper could usefully be extended along a number of dimensions. To begin with, the framework of Galí and Monacelli (2005) which forms the basis for our analysis abstracts from potentially important dimensions of globalization, such as trade in intermediate inputs that are used in production. Rumler (2007) and Leith and Malley (2007) show how the small open economy can be extended to incorporate such trade. Also, we believe that allowing for capital accumulation would greatly enrich the dynamics of the model and potentially sharpen our insights into the drivers of the natural rate. We leave those avenues open for future research.

References

Abbas, S., P. Bhattacharya, D. Mallick, and P. Sgro (2016). The New Keynesian Phillips Curve in a Small Open Economy: Empirical Evidence from Australia. *Economic Record 92*(298), 409–434. https://doi.org/10.1111/1475-4932.12262.

Andrade, P., J. Galí, H. Le Bihan, and J. Matheron (2018). The Optimal Inflation Target and the Natural Rate of Interest. Technical report, National Bureau of Economic Research No. 24328. https://doi.org/10.3386/w24328.

Andrés, J., J. D. López-Salido, and E. Nelson (2009). Money and the Natural Rate of Interest: Structural Estimates for the United States and the Euro Area. *Journal of Economic Dynamics and Control 33*(3), 758–776. https://doi.org/10.1016/j.jedc.2008.01.011.

Armelius, H., M. Solberger, and E. Spånberg (2018). Is the Swedish Neutral Interest Rate Affected by International Developments? *Sveriges Riksbank Economic Review 1*, 22–37. https://goo.gl/E44euB.

Barsky, R., A. Justiniano, and L. Melosi (2014). The Natural Rate of Interest and Its Usefulness for Monetary Policy. *American Economic Review 104*(5), 37–43. https://doi.org/10.1257/aer.104.5.37.

Berger, T. and B. Kempa (2014). Time-Varying Equilibrium Rates in Small Open Economies: Evidence for Canada. *Journal of Macroeconomics 39*, 203–214. https://doi.org/10.1016/j.jmacro.2013.12.002.

Bernanke, B. S. (2015a). Why Are Interest Rates So Low? Brookings Institution, Blog Post, March 30, 2015. https://goo.gl/kjrAzN.

Bernanke, B. S. (2015b). Why Are Interest Rates So Low, Part 2: Secular Stagnation. Brookings Institution, Blog Post, March 31, 2015. https://goo.gl/mYat3D.

Bernanke, B. S. (2015c). Why Are Interest Rates So Low, Part 3: The Global Savings Glut. Brookings Institution, Blog Post, April 1, 2015. https://goo.gl/cRpQmo.

Bernanke, B. S. (2015d). Why Are Interest Rates So Low, Part 4: Term Premiums. Brookings Institution, Blog Post, April 13, 2015. https://goo.gl/CcUVpF.

Bianchi, F. and G. Nicolò (2019). A Generalized Approach to Indeterminacy in Linear Rational Expectations Models. *Finance and Economics Discussion Series 2019-033*. Washington: Board of Governors of the Federal Reserve System. https://doi.org/10.17016/FEDS.2019.033.

Blonigen, B. A. and W. W. Wilson (1999). Explaining Armington: What Determines Substitutability Between Home and Foreign Goods? *The Canadian Journal of Economics 32*(1), 1–21. https://doi.org/10.2307/136392.

Borio, C., P. Disyatat, M. Juselius, and P. Rungcharoenkitkul (2017). Why So Low For So Long? A Long-Term View of Real Interest Rates. *BIS Working Papers 685, Bank for International Settlements*. https://www.bis.org/publ/work685.htm.

Calvo, G. A. (1983). Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics 12*(3), 383–398. https://doi.org/10.1016/0304-3932(83)90060-0.

Chetty, R., A. Guren, D. Manoli, and A. Weber (2011). Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. *American Economic Review Papers and Proceedings 101*(3), 471–475. https://doi.org/10.1257/aer.101.3.471.

Clarida, R. H. (2019). The Global Factor in Neutral Policy Rates: Some Implications for Exchange Rates, Monetary Policy, and Policy Coordination. *International Finance 22*(1), 2–19. https://doi.org/10.1111/infi.12345.

Clark, T. E. and S. Kozicki (2005). Estimating Equilibrium Real Interest Rates in Real Time. *The North American Journal of Economics and Finance 16*(3), 395–413. https://doi.org/10.1016/j.najef.2005.04.002.

Coibion, O. and Y. Gorodnichenko (2015). Is the Phillips Curve Alive and Well after All? Inflation Expectations and the Missing Disinflation. *American Economic Journal: Macroeconomics 7*(1), 197–232. https://doi.org/10.1257/mac.20130306.

Cúrdia, V., A. Ferrero, G. C. Ng, and A. Tambalotti (2015). Has U.S. Monetary Policy Tracked the Efficient Interest Rate? *Journal of Monetary Economics 70*, 72–83. https://doi.org/10.1016/j.jmoneco.2014.09.004.

David, F. N. (1949). The Moments of the z and F Distributions. *Biometrika 36*(3/4), 394–403. https://doi.org/10.2307/2332676.

Del Negro, M., D. Giannone, M. P. Giannoni, and A. Tambalotti (2017). Safety, Liquidity, and the Natural Rate of Interest. *Brookings Papers on Economic Activity 2017*(1), 235–316. https://doi.org/10.1353/eca.2017.0003.

Del Negro, M., D. Giannone, M. P. Giannoni, and A. Tambalotti (2019). Global Trends in Interest Rates. *Journal of International Economics 118*(C), 248–262. http://dx.doi.org/10.1016/j.jinteco.2019.01.010.

Eggertsson, G. B., N. R. Mehrotra, and J. A. Robbins (2019). A Model of Secular Stagnation: Theory and Quantitative Evaluation. *American Economic Journal: Macroeconomics 11*(1), 1–48. https://doi.org/10.1257/mac.20170367.

Evans, C., J. Fisher, F. Gourio, and S. Krane (2016). Risk Management for Monetary Policy Near the Zero Lower Bound. *Brookings Papers on Economic Activity 2015*(1), 141–219. https://doi.org/10.1353/eca.2016.0003.

Feenstra, R. C., P. Luck, M. Obstfeld, and K. Russ (2018). In Search of the Armington Elasticity. *The Review of Economics and Statistics 100*(1), 135–150. https://doi.org/10.1162/rest_a_00696.

Fernández, A., M. W. Klein, A. Rebucci, M. Schindler, and M. Uribe (2015). Capital Control Measures: A New Dataset. *National Bureau of Economic Research No. 20970*. https://doi.org/10.3386/w20970.

Fiorentini, G., A. Galesi, G. Pérez-Quirós, and E. Sentana (2018). The Rise and Fall of the Natural Interest Rate. *Banco de Espana Working Paper No. 1822*. https://goo.gl/bwuoDf.

Fries, S., J.-S. Mésonnier, S. Mouabbi, and J.-P. Renne (2018). National Natural Rates of Interest and the Single Monetary Policy in the Euro Area. *Journal of Applied Econometrics 33*(6), 763–779. https://doi.org/10.1002/jae.2637.

Gagnon, E., B. K. Johannsen, and D. Lopez-Salido (2016). Understanding the New Normal: The Role of Demographics. *Finance and Economics Discussion Series 2016-080*. https://doi.org/10.17016/feds.2016.080.

Gallaway, M. P., C. A. McDaniel, and S. A. Rivera (2003). Short-Run and Long-Run Industry-Level Estimates of U.S. Armington Elasticities. *North American Journal of Economics and Finance 14*(1), 49–68. https://doi.org/10.1016/s1062-9408(02)00101-8.

Galí, J. and M. Gertler (1999). Inflation Dynamics: A Structural Econometric Analysis. *Journal of Monetary Economics 44*(2), 195–222. https://doi.org/10.1016/s0304-3932(99)00023-9.

Galí, J. and T. Monacelli (2005). Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *The Review of Economic Studies 72*(3), 707–734. https://doi.org/10.1111/j.1467-937x.2005.00349.x.

Gelman, A. and D. B. Rubin (1992). Inference From Iterative Simulation Using Multiple Sequences. *Statistical Science 7*(4), 457–472. https://doi.org/10.1214/ss/1177011136.

Geweke, J. (1992). Evaluating the Accuracy of Sampling-Based Approaches to the Calculations of Posterior Moments. *Bayesian Statistics 4*, 641–649.

Geweke, J. (1999). Using Simulation Methods for Bayesian Econometric Models: Inference, Development, and Communication. *Econometric Reviews 18*(1), 1–73. https://doi.org/10.1080/07474939908800428.

Grossman, V., A. Mack, and E. Martínez-García (2014). A New Database of Global Economic Indicators. *The Journal of Economic and Social Measurement 39*(3), 163–197. https://www.doi.org/10.3233/JEM-140391. The database can be accessed at: https://www.dallasfed.org/institute/dgei.

Hamilton, J. D., E. S. Harris, J. Hatzius, and K. D. West (2016). The Equilibrium Real Funds Rate: Past, Present, and Future. *IMF Economic Review 64*(4), 660–707. https://doi.org/10.1057/s41308-016-0015-z.

Hammond, G. (2012). State of the Art of Inflation Targeting - 2012. *Centre for Central Banking Studies. Handbook - No. 29*. Bank of England. https://urlzs.com/xJvF.

Havránek, T. (2015). Measuring Intertemporal Substitution: The Importance of Method Choice and Selective Reporting. *Journal of the European Economic Association 13*(6), 1180–1204. https://doi.org/10.1111/jeea.12133.

Hirose, Y. and T. Sunakawa (2017). The Natural Rate of Interest in a Nonlinear DSGE Model. *CAMA Working Papers 2017-38*. Centre for Applied Macroeconomic Analysis (CAMA), Crawford School of Public Policy, The Australian National University. https://doi.org/10.2139/ssrn.2967137.

Holmberg, K. (2006). Derivation and Estimation of a New Keynesian Phillips Curve in a Small Open Economy. *Sveriges Riksbank Working Paper No. 197*. https://doi.org/10.2139/ssrn.985902.

Holston, K., T. Laubach, and J. C. Williams (2017). Measuring the Natural Rate of Interest: International Trends and Determinants. *Journal of International Economics 108*, S59–S75. https://doi.org/10.1016/j.jinteco.2017.01.004. Data available at: https://nyfed.org/3aNOmAI.

Hristov, A. (2016). Measuring the Natural Rate of Interest in the Eurozone: A DSGE Perspective. In *CESifo Forum*, Volume 17, pp. 86–91. München: Ifo Institut–Leibniz Institut für Wirtschaftsforschung an der Universität München. https://www.cesifo-group.de/DocDL/forum-2016-1-hristov-march.pdf.

Johannsen, B. K. and E. Mertens (2018). A Time Series Model of Interest Rates with the Effective Lower Bound. *BIS Working Papers 715, Bank for International Settlements*. https://www.bis.org/publ/work715.htm.

Juselius, M., C. Borio, P. Disyatat, and M. Drehmannb (2017). Monetary Policy, the Financial Cycle, and Ultra-Low Interest Rates. *International Journal of Central Banking 13*(3), 55–89. https://www.ijcb.org/journal/ijcb17q3a2.pdf.

Justiniano, A. and B. Preston (2010). Monetary Policy and Uncertainty in an Empirical Small Open-Economy Model. *Journal of Applied Econometrics 25*(1), 93–128. https://doi.org/10.1002/jae.1153.

Kara, E. and L. von Thadden (2016). Interest Rate Effects of Demographic Changes in a New Keynesian Life-Cycle Framework. *Macroeconomic Dynamics 20*(1), 120–164. https://doi.org/10.1017/s1365100514000273.

Kass, R. E. and A. E. Raftery (1995). Bayes Factors. *Journal of the American Statistical Association 90*(430), 773–795. https://doi.org/10.1080/01621459.1995.10476572.

Keane, M. and R. Rogerson (2012). Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom. *Journal of Economic Literature 50*(2), 464–476. https://doi.org/10.1257/jel.50.2.464.

Kiley, M. T. (2020). What Can the Data Tell Us About the Equilibrium Real Interest Rate? *International Journal of Central Banking*, 181–209. June 2020. https://www.ijcb.org/journal/ijcb20q2a5.pdf.

Laubach, T. and J. C. Williams (2003). Measuring the Natural Rate of Interest. *Review of Economics and Statistics 85*(4), 1063–1070. https://doi.org/10.1162/003465303772815934. Data available at: https://nyfed.org/3aNOmAI.

Leith, C. and J. Malley (2007). Estimated Open Economy New Keynesian Phillips Curves for the G7. *Open Economies Review 18*(4), 405–426. https://doi.org/10.1007/s11079-007-9008-x.

Lubik, T. A. and C. Matthes (2015). Calculating the Natural Rate of Interest: A Comparison of Two Alternative Approaches. *Richmond Fed Economic Brief (Oct.)*, 1–6. https://goo.gl/ngg94o. Data available at: https://bit.ly/2KQXwSk.

Lubik, T. A. and F. Schorfheide (2007). Do Central Banks Respond to Exchange Rate Movements? A Structural Investigation. *Journal of Monetary Economics 54*(4), 1069–1087. https://doi.org/10.1016/j.jmoneco.2006.01.009.

Martínez-García, E. (2015). The Global Content of Local Inflation: Revisiting the Empirical Content of the Global Slack Hypothesis. In W. Barnett and F. Jawadi (Eds.), *Monetary Policy in the Context of the Financial Crisis: New Challenges and Lessons*, pp. 51–112. Emerald Group Publishing Limited. https://doi.org/10.1108/s1571-038620150000024016.

Martínez-García, E. (2019). Good Policies or Good Luck? New Insights on Globalization and the International Monetary Policy Transmission Mechanism. *Computational Economics 54*(1), 419–454. https://doi.org/10.1007/s10614-017-9746-9.

Martínez-García, E. (2020). Get the Lowdown: The International Side of the Fall in the U.S. Natural Rate of Interest. *Globalization Institute Working Paper no. 403*. October. https://doi.org/10.24149/gwp403.

Martínez-García, E., D. Vilán, and M. A. Wynne (2012). Bayesian Estimation of NOEM Models: Identification and Inference in Small Samples. *Advances in Econometrics 28*, 137–199. DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments (Advances in Econometrics, Volume 28). https://doi.org/10.1108/s0731-9053(2012)0000028007.

Martínez-García, E. and M. A. Wynne (2010). The Global Slack Hypothesis. Federal Reserve Bank of Dallas *Staff Papers, 10*. September. https://goo.gl/uqSXM1.

Martínez-García, E. and M. A. Wynne (2014). Assessing Bayesian Model Comparison in Small Samples. *Advances in Econometrics 34*, 71–115. Bayesian Model Comparison (Advances in Econometrics, Volume 34). https://doi.org/10.1108/s0731-905320140000034006.

Mavroeidis, S., M. Plagborg-Møller, and J. H. Stock (2014). Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve. *Journal of Economic Literature 52*(1), 124–188. https://doi.org/10.1257/jel.52.1.124.

Mesonnier, J.-S. and J.-P. Renne (2007). A Time-Varying "Natural" Rate of Interest for the Euro Area. *European Economic Review 51*(7), 1768–1784. https://doi.org/10.1016/j.euroecorev.2006.11.006.

Nelson, E. (2002). Direct Effects of Base Money on Aggregate Demand: Theory and Evidence. *Journal of Monetary Economics 49*(4), 687–708. https://doi.org/10.1016/s0304-3932(02)00118-6.

Neri, S. and A. Gerali (2018). Natural Rates Across the Atlantic. *Journal of Macroeconomics*. Available online 24 April 2018. https://doi.org/10.1016/j.jmacro.2018.04.007.

Neto, A. R. and O. Candido (2018). Measuring the Neutral Real Interest Rate in Brazil: A Semi-Structural Open Economy Framework. *Empirical Economics*, 1–17. First Online: 05 September 2018. https://doi.org/10.1007/s00181-018-1550-4.

OECD (2020). Dataset: MEI Original Release Data and Revisions. *Organisation for Economic Co-operation and Development, OECD Statistics*. https://stats.oecd.org/Index.aspx?DataSetCode=MEI_ARCHIVE.

Pescatori, A. and J. Turunen (2015). Lower for Longer: Neutral Rates in the United States. *IMF Working Paper WP/2015/135*. https://doi.org/10.5089/9781513508382.001.

Peterman, W. B. (2016). Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity. *Economic Inquiry 54*(1), 100–120. https://doi.org/10.1111/ecin.12252.

Rotemberg, J. J. and M. Woodford (1998). An Optimization-Based Econometric Model for the Evaluation of Monetary Policy. In B. S. Bernanke and J. J. Rotemberg (Eds.), *NBER Macroeconomics Annual 1997, vol. 12*, Volume 12, pp. 297–346. Cambridge, Massachusetts, USA: MIT University Press. https://doi.org/10.2307/3585236.

Rumler, F. (2007). Estimates of the Open Economy New Keynesian Phillips Curve for Euro Area Countries. *Open Economies Review 18*(4), 427–451. September. https://doi.org/10.1007/s11079-007-9028-6.

Sbordone, A. M. (2002). Prices and Unit Labor Costs: A New Test of Price Stickiness. *Journal of Monetary Economics 49*(2), 265–292. https://doi.org/10.1016/s0304-3932(01)00111-8.

Schorfheide, F. (2013). Estimation and Evaluation of DSGE Models: Progress and Challenges. In D. Acemoglu, M. Arellano, and E. Dekel (Eds.), *Advances in Economics and Econometrics. Tenth World Congress*, Volume Volume 3, Econometrics, pp. 184–230. Cambridge University Press. https://doi.org/10.1017/cbo9781139060035.006.

Sims, C. A. (2002). Solving Linear Rational Expectations Models. *Computational Economics 20*(1), 1–20. https://doi.org/10.1023/A:1020517101123.

Taylor, J. B. (1993). Discretion versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214. https://doi.org/10.1016/0167-2231(93)90009-l.

Thimme, J. (2017). Intertemporal Substitution in Consumption: A Literature Review. *Journal of Economic Surveys 31*(1), 226–257. https://doi.org/10.1111/joes.12142.

Trehan, B. and T. Wu (2007). Time-Varying Equilibrium Real Rates and Monetary Policy Analysis. *Journal of Economic Dynamics and Control 31*(5), 1584–1609. https://doi.org/10.1016/j.jedc.2006.04.009.

Wicksell, K. (1898). Interest and Prices (Geldzins und Güterpreise): A Study of the Causes Regulating the Value of Money. *Translated from the German by R.F. Kahn*. Published in behalf of the Royal Economic Society. (London: Macmillan. 1936. Pp. xxxi, 219. 2s. 6d.).

Woodford, M. (2003). *Interest and Prices. Foundations of a Theory of Monetary Policy*. Princeton, New Jersey: Princeton University Press.

Wynne, M. A. and R. Zhang (2018a). Estimating the Natural Rate of Interest in an Open Economy. *Empirical Economics 55*(3), 1291–1318. https://doi.org/10.1007/s00181-017-1315-5.

Wynne, M. A. and R. Zhang (2018b). Measuring the World Natural Rate of Interest. *Economic Inquiry 56*(1), 530–544. https://doi.org/10.1111/ecin.12500.

Zhang, R. (2019). News Shocks and the Effects of Monetary Policy. *SSRN Working Paper No. 3348466*. http://dx.doi.org/10.2139/ssrn.3348466.

Appendix

A Literature Review

Over the last 15 years or so, various econometric approaches have been proposed to estimate the unobserved (and time-varying) natural rate of interest, which can be broadly divided into three strands: pure time-series methods, semi-structural methods, and structural methods. Each approach has its advantages and disadvantages.

The *first strand* relies on pure multivariate time-series models such as the univariate local level model, the time-varying-parameter vector autoregression model, and the multivariate trend-cycle decomposition approach.[24] Compared to other more structural models, the pure time-series models tend to impose fewer restrictions, thereby letting the data speak for itself, which generally results in a more flexible estimation of the unobserved natural rate. However, the pure time-series models also arguably result in less precise estimates of the natural rate and produce estimates that rely on atheoretical or non-structural identifying assumptions. This is a particularly significant limitation for our purposes because by leaving largely unmodeled the structural relationships between natural interest rates, inflation, and output, makes it difficult to identify the drivers behind the movements in the natural rate. Therefore, our approach in this paper is structural to allow us to incorporate the theoretical cross-equation restrictions to help us with identification.

A *second strand* of the literature uses semi-structural econometric models in which the natural rate is a latent variable that depends on the trend growth rate of potential output and a unit root process which captures other determinants. The natural interest rate is inferred from the structural relationships that link the output gap, inflation, and the deviation of the real interest rate from its natural level implied by an empirical IS curve and an empirical Phillips curve. This burgeoning strand of the literature was pioneered by the seminal work of Laubach and Williams (2003) which spawned a large subsequent literature.[25] Most of the early groundwork in this area, such as Laubach and Williams (2003) but also Mesonnier and Renne (2007) and Trehan and Wu (2007), adopts a closed economy framework. More recent studies have become more aware and recognize the crucial role that international factors play in determining the domestic natural interest rate. Notably, Holston et al. (2017) uncover a substantial international comovement among the closed economy natural interest rate estimates of four advanced economies (Canada, the Euro area, the U.S., and the U.K.). This substantial comovement suggests an important role for global factors in shaping trend growth and the natural rates of interest.

Wynne and Zhang (2018a) extend the closed economy Laubach and Williams (2003) model to a two-country setup in order to explore the international interaction in the determination of the natural interest

[24] Lubik and Matthes (2015) estimate a simple time-varying-parameter vector autoregresion model for three macroeconomic variables (GDP growth, inflation, and the real interest rate) and measure the natural interest rate as the conditional long-horizon forecast of the observed real rate. Hamilton et al. (2016) interpret the natural interest rate as a long-run or steady-state safe real rate and infer the time-series measure of the natural rate with a ten-year moving average of the real interest rates. Fiorentini et al. (2018) adopt a local-level model with stochastic volatility to estimate the natural interest rates for 17 advanced economies with over a century of data. Del Negro et al. (2017) and Del Negro et al. (2019) infer the natural interest rate from a spectrum of bond yields with a trend-cycle decomposition approach, while Johannsen and Mertens (2018) adopt a similar methodology but use shadow rates to take into account the period when policy rates became in many countries constrained at their effective-lower bound (ELB) (from 2008 to 2015 or later in some countries).

[25] See, e.g., Clark and Kozicki (2005), Trehan and Wu (2007), Pescatori and Turunen (2015), Juselius et al. (2017), and Kiley (2020) for the U.S.; Mesonnier and Renne (2007) and Fries et al. (2018) for the euro area; Berger and Kempa (2014) for Canada; Armelius et al. (2018) for Sweden; Neto and Candido (2018) for Brazil; Wynne and Zhang (2018a) for the U.S. and Japan; Holston et al. (2017) for Canada, the euro area, the U.S., and the U.K; and Wynne and Zhang (2018b) for the world.

rates in Japan and the U.S. Neto and Candido (2018) adopt a similar two-country model and measure the contribution of global factors to the natural interest rate in Brazil. With a multi-country setup, Fries et al. (2018) estimate the natural interest rates for the four largest economies in the euro area (France, Germany, Italy, and Spain), assuming that each national economy is linked to the other three via a trade channel and a productivity channel. Berger and Kempa (2014) and Armelius et al. (2018) apply a semi-structural small open economy model to estimate the natural interest rate in Canada and Sweden, respectively.

Compared to pure time-series models, the semi-structural approach explicitly models the link between the natural rate and macro fundamentals and, arguably, in doing so is better suited to recovering the natural rate. As noted in Fiorentini et al. (2018), however, the precision of the unobservable natural rate estimates depends on whether the IS curve or the Phillips curve is flat or not.[26] The high degree of uncertainty around the natural rate estimates leads to risk management problems for central banks when making monetary policy as addressed in Evans et al. (2016). Apart from that, semi-structural approaches may not suffice to identify the structural driving forces behind the estimated natural rates. For the above-mentioned reasons, we pursue in this paper a fully structural approach instead as it is more suitable for our purpose.

The *third strand* of the literature, the one our where paper fits in, relies on structural models which tend to be either dynamic stochastic general equilibrium (DSGE) models in the New Keynesian tradition or overlapping generations (OLG) models.[27] If properly specified, DSGE models are also convenient in evaluating the underlying forces that contribute to the fluctuations of the natural rate without sacrificing additional precision in the estimation of the natural rate.[28] DSGE models are also convenient in evaluating the optimal monetary policy response when faced with an increase or a decrease in the natural rate, unlike the pure time-series and even the semi-structural models (Andrade et al. (2018)). We should note, however, that DSGE models generally estimate the business-cycle frequency component in the natural interest rate. This is different from pure time-series models and from the semi-structural methods that, in turn, focus on low-frequency movements of the natural rate. However, the existing studies in the DSGE literature tend to estimate the natural interest rate within a closed economy setup and only for large economies such as the U.S., the euro area, and Japan.

To our knowledge, our paper is among the first attempts to investigate the determinants of the natural rate with a full-fledged, structural small open economy in line with that of Galí and Monacelli (2005) and Lubik and Schorfheide (2007). Our paper also makes a novel contribution to the structural literature estimating the natural rate of interest for six countries with a small open economy model that explicitly incorporates domestic growth by modeling exogenous domestic technological progress. Furthermore, ours is also the first paper in the literature to evaluate the extent to which the central banks of a wide range of small open economies tracks the natural interest rate in policymaking (along the lines of what Cúrdia et al. (2015) does for the U.S.).

[26] Fiorentini et al. (2018)'s survey of the literature highlights that the slope of the IS curve and that of the Phillips curve are usually estimated to be flat. The same conclusion is reached also in a number of the studies mentioned here and even with other econometrics models unrelated to the natural interest rate literature (see, e.g., Nelson (2002), Mavroeidis et al. (2014), and Coibion and Gorodnichenko (2015),).

[27] See, e.g., Gagnon et al. (2016), Kara and von Thadden (2016), and Eggertsson et al. (2019) for OLG models; and Andrés et al. (2009), Martínez-García and Wynne (2010), Justiniano and Preston (2010), Barsky et al. (2014), Martínez-García (2015), Cúrdia et al. (2015), Hristov (2016), Del Negro et al. (2017), Hirose and Sunakawa (2017), Neri and Gerali (2018), Martínez-García (2019), and Martínez-García (2020) for DSGE models.

[28] Martínez-García (2019) and Clarida (2019) show, with a two-country workhorse New Keynesian (DSGE) model, that foreign factors are important determinants of the natural interest rate in open economies. However, they do not provide any quantitative estimates of the natural interest rate.

B Derivations of the Small Open Economy Model

Our benchmark model discussed in Section 2 is a variant of the standard small open economy model described in Galí and Monacelli (2005) and Lubik and Schorfheide (2007). We deviate from those two papers by adding time-varying price markups (which introduce a cost-push shock into the Phillips curve) to better capture the dynamics of inflation and a preference shock as an additional exogenous driver that can directly impact the domestic natural rate of interest. We also consider monetary policy rule specifications that are different from those studied in both seminal papers and incorporate a non-stationary technological productivity shock process to properly account for economic growth while still maintaining a flexible structural representation of the business cycles. The model setup we estimate is also more general than that of Lubik and Schorfheide (2007) who also apply a Bayesian approach to estimate a small open economy model with (exogenous) domestic technological growth. Lubik and Schorfheide (2007) also bring the small open economy framework to the data with Bayesian techniques. However, unlike them, we estimate the inverse of the Frisch elasticity parameter instead of setting it to 0 and also estimate the elasticity of substitution between home and foreign goods instead of assuming it is equal to 1. We also investigate different monetary policy rule specifications than Lubik and Schorfheide (2007) do.

Here we present the key derivations of the benchmark model from first principles.

B.1 Domestic Households

Each household in the home country (the small open economy) solves the following lifetime utility maximization problem:

$$\mathbb{E}_0 \sum\nolimits_{t=0}^{\infty} \beta^t e^{-g_t} \left[\frac{C_t^{1-\frac{1}{\tau}}}{1-\frac{1}{\tau}} - \frac{N_t^{1+\varphi}}{1+\varphi} \right], \tag{18}$$

where N_t refers to the hours worked and C_t is the aggregate consumption index defined as:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} \left(C_{H,t}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(C_{F,t}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{19}$$

which bundles up a sub-index of domestic goods $C_{H,t}$ given by:

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}, \tag{20}$$

where $C_{H,t}(j)$ denotes the locally-produced variety $j \in [0,1]$ in the domestic good consumption bundle and an analogous sub-index of imported goods $C_{F,t}$ given by:

$$C_{F,t} \equiv \left(\int_0^1 C_{F,t}(j^*)^{\frac{\epsilon_t - 1}{\epsilon_t}} dj^* \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}, \tag{21}$$

where $C_{F,t}(j^*)$ refers to the foreign variety $j^* \in [0,1]$ in the imported good consumption bundle.

The parameter $\tau > 0$ denotes the intertemporal elasticity of substitution, $\varphi > 0$ represents the inverse of the Frisch elasticity of labor supply, $\eta > 0$ measures the substitutability between home and foreign goods from the viewpoint of the domestic consumer, $0 < \alpha < 1$ measures the degree of openness (the steady state import share in the domestic consumption basket), and $\epsilon_t > 1$ is the (time-varying) elasticity of substitution between varieties produced within each country. The parameter $0 < \beta < 1$ refers to the intertemporal discount factors that holds in steady state with the aggregate preference shock, g_t, shifting the discount factor exogenously in the short-run. This preference shock follows a stationary AR(1) process given by:

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}, \ \varepsilon_{g,t} \sim N\left(0, \sigma_g\right), \tag{22}$$

where $-1 < \rho_g < 1$ denotes the persistence parameter and $\sigma_g > 0$ the volatility.

The household lifetime utility maximization of (18) is subject to the following per-period nominal budget constraint:

$$\int_0^1 P_{H,t}\left(j\right) C_{H,t}\left(j\right) dj + \int_0^1 P_{F,t}\left(j^*\right) C_{F,t}\left(j^*\right) dj^* + \mathbb{E}_t\left(\Lambda_{t,t+1} D_{t+1}\right) \leqslant W_t N_t + D_t + T_t + Pr_t, \tag{23}$$

where $P_{H,t}\left(j\right)$ is the nominal price of the domestically-produced variety j, $P_{F,t}\left(j^*\right)$ is the price of the foreign-produced variety j^* in units of the domestic currency, W_t denotes the nominal wage rate, and Pr_t denotes the nominal profits (or losses) accrued from full ownership of the domestic firms. Furthermore, D_t refers to the nominal domestic households' holdings of Arrow-Debreu securities spanning over all possible events of nature (domestic and foreign), $\Lambda_{t,t+1}$ is the state-contingent price of these Arrow-Debreu securities, and T_t is a nominal lump-sum tax raised by the government of the home country.[29]

The optimal allocation of consumption expenditures for each individually produced domestic good or imported good yields the following demand functions:

$$C_{H,t}\left(j\right) = \left(\frac{P_{H,t}\left(j\right)}{P_{H,t}}\right)^{-\epsilon_t} C_{H,t}, \tag{24}$$

$$C_{F,t}\left(j^*\right) = \left(\frac{P_{F,t}\left(j^*\right)}{P_{F,t}}\right)^{-\epsilon_t} C_{F,t}, \tag{25}$$

for all $j, j^* \in [0,1]$ where $P_{H,t} \equiv \left(\int_0^1 P_{H,t}(j)^{1-\epsilon_t} dj\right)^{\frac{1}{1-\epsilon_t}}$ is the price sub-index of the domestically-produced good and $P_{F,t} \equiv \left(\int_0^1 P_{F,t}(j^*)^{1-\epsilon_t} dj^*\right)^{\frac{1}{1-\epsilon_t}}$ is the price sub-index of the imported good. Furthermore, the optimal allocation of consumption expenditures between domestic and imported goods can be expressed as:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \tag{26}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \tag{27}$$

where $P_t \equiv \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the price level of the domestic bundle of consumption goods, C_t.

Accordingly, total consumption expenditures are given by $\int_0^1 P_{H,t}\left(j\right) C_{H,t}\left(j\right) dj + \int_0^1 P_{F,t}\left(j^*\right) C_{F,t}\left(j^*\right) dj^* =$

[29] T_t funds a (time-invariant) labor employment subsidy scheme that cancels out the monopolistic competition price markup charged by intermediate goods producers in steady state which we discuss in more detail later when we describe the pricing behavior of firms.

$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_tC_t$. Therefore, the nominal budget constraint in (23) can be re-expressed as:

$$P_tC_t + \mathbb{E}_t\left(\Lambda_{t,t+1}D_{t+1}\right) \leqslant W_tN_t + D_t + T_t + Pr_t. \tag{28}$$

Solving the domestic household's optimization problem in (18) subject to the nominal budget constraint in (28) leads to the following first-order condition on hours worked (the labor supply equation):

$$C_t^{-\frac{1}{\tau}}W_t = N_t^{\varphi}P_t, \tag{29}$$

and the following intertemporal optimality condition pricing the Arrow-Debreu securities:

$$\Lambda_{t,t+1} = \beta e^{-\Delta g_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\tau}}\frac{P_t}{P_{t+1}}. \tag{30}$$

Taking conditional expectations on both sides of (30) we obtain a standard Euler equation:

$$C_t^{-\frac{1}{\tau}}e^{-g_t} = \beta\mathbb{E}_t\left[\frac{(1+i_t)}{(1+\pi_{t+1})}C_{t+1}^{-\frac{1}{\tau}}e^{-g_{t+1}}\right], \tag{31}$$

where $1+\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the domestic CPI inflation rate and $1+i_t \equiv \frac{1}{\mathbb{E}_t(\Lambda_{t,t+1})}$ is the domestic short-term nominal interest rate which is equal to the reciprocal of the expected intertemporal marginal rate of substitution one-quarter ahead, i.e., the reciprocal of $\mathbb{E}_t\left(\Lambda_{t,t+1}\right) = \beta\mathbb{E}_t\left[e^{-\Delta g_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\tau}}\frac{P_t}{P_{t+1}}\right]$. As can be seen from (29) and (31), the preference shock, g_t, shifts the intertemporal allocation of consumption without directly affecting the intratemporal margin between leisure and consumption.

B.2 Domestic Firms

A typical firm in the home country produces a differentiated variety j with a linear-in-labor technology represented by the production function:

$$Y_t(j) = A_tN_t(j), \tag{32}$$

where A_t measures the aggregate TFP productivity and $N_t(j)$ denotes the labor employed by the $j-$th firm. We assume that aggregate productivity, A_t, grows at a rate, $z_t \equiv \ln\left(\frac{A_t}{A_{t-1}}\right)$, and that the exogenous growth rate displays some persistence such that:

$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t},\ \ \varepsilon_{z,t} \sim N\left(0,\sigma_z\right), \tag{33}$$

where $-1 < \rho_z < 1$ denotes the productivity growth persistence parameter and $\sigma_z > 0$ its volatility. The marginal cost in units of the domestically-produced good, MC_t, is defined as:

$$MC_t \equiv \frac{W_t}{P_{H,t}A_t}. \tag{34}$$

Letting $Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon_t-1}{\epsilon_t}}dj\right)^{\frac{\epsilon_t}{\epsilon_t-1}}$ represent the aggregate domestic production index constructed analogous to the CES consumption aggregator in (24), then the production of domestic goods in the small open economy where each firm uses the linear-in-labor technology given by (32) can be described as:

$$N_t \equiv \int_0^1 N_t(j)\,dj = \frac{Y_t Z_t}{A_t}, \tag{35}$$

where N_t is the aggregate hours worked across all firms and $Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t} dj$ is a measure of output dispersion across firms.

The $j-$th firm sets prices on a staggered basis as in Calvo (1983) where $0 < \theta < 1$ is the constant probability that a firm keeps its price fixed in a given period. We also assume firms operate under producer-currency pricing meaning that firms set the same price in units of the domestic currency on all their production irrespective of whether the goods are sold locally or exported. This assumption implies that the law of one price holds at the variety level, i.e., $P_t(j) = S_t P_t^f(j)$ where S_t is the bilateral exchange rate while $P_t(j)$ and $P_t^f(j)$ are the prices charged for variety j in local currency units for the domestic market and in foreign currency units for the export market, respectively. Firms that are able to optimally update their price in period t choose the reset price $P_{H,t}^{\#}(j)$ to maximize their expected present value of current and future profits as follows:

$$\mathbb{E}_t \sum\nolimits_{s=0}^{\infty} \theta^s \Lambda_{t,t+s} \left\{ Y_{t+s}(j) \left[P_{H,t}^{\#}(j) - (1-\phi) MC_{t+s} P_{H,t+s} \right] \right\}, \tag{36}$$

with $\Lambda_{t,t+s} \equiv \beta^s e^{-\frac{g_{t+s}}{g_t}} \left(\frac{C_{t+s}}{C_t}\right)^{-\frac{1}{\tau}} \frac{P_t}{P_{t+s}}$ and ϕ referring to a labor subsidy funded by the domestic government. The re-optimizing firms' maximization problem is subject to the following sequence of demand constraints:

$$Y_{t+s}(j) \leq \left(\frac{P_{H,t}^{\#}(j)}{P_{H,t+s}}\right)^{-\epsilon_t} \left(C_{H,t+s} + C_{H,t+s}^f\right) \equiv Y_{t+s}^d \left(P_{H,t}^{\#}(j)\right), \tag{37}$$

where $C_{H,t}^f$ refers to the foreign consumption of domestic goods while $C_{H,t}$ is the domestic consumption of domestic goods and MC_t is the real marginal cost given in (34). An interior solution to the firms' optimization problem implies that the sequence of demand constraints holds with equality. Under Calvo (1983) pricing, $P_{H,t+s}(j) = P_{H,t}^{\#}(j)$ with probability θ^s for $s = 1, 2, ...$ Since all firms resetting prices in any given period choose the same price, we henceforth drop the j subscript.

The first-order condition for $P_{H,t}^{\#}$ implies that:

$$\sum\nolimits_{s=0}^{\infty} (\theta\beta)^s \mathbb{E}_t \left\{ e^{-g_{t+s}} (C_{t+s})^{-\frac{1}{\tau}} Y_{t+s} \frac{P_{H,t-1}}{P_{t+s}} \left[\frac{P_{H,t}^{\#}}{P_{H,t-1}} - \mu_{t+s} MC_{t+s} \frac{P_{H,t+s}}{P_{H,t-1}} \right] \right\} = 0, \tag{38}$$

where $\mu_t \equiv (1-\phi)\frac{\epsilon_t}{\epsilon_t - 1}$ is the monopolistic competition markup net of the labor subsidy on employment. The domestic government sets the labor subsidy to compensate the price markup in steady state, i.e., it implies $\mu \equiv (1-\phi)\frac{\epsilon}{\epsilon-1} = 1$. Hence, the markups can vary over time in the short-run but do not have any effect on the long-run (in steady state). We define the log of the price markup adjusted to account for the employment subsidy as, $u_t \equiv \ln(\mu_t)$. This follows an AR(1) process of the following form:

$$u_t = \rho_u u_{t-1} + \varepsilon_{u,t},\ \varepsilon_{u,t} \sim N(0, \sigma_u), \tag{39}$$

where $-1 < \rho_u < 1$ denotes the persistence parameter and $\sigma_u > 0$ the volatility. The optimal pricing equation can then be re-written here as:

$$\frac{P^{\#}_{H,t}}{P_{H,t-1}} = \frac{\sum_{s=0}^{\infty} (\theta\beta)^s \mathbb{E}_t \left\{ e^{-g_{t+s}} (C_{t+s})^{-\frac{1}{\tau}} Y_{t+s} \frac{P_{H,t-1}}{P_{t+s}} \left[\mu_{t+s} \frac{P_{H,t+s}}{P_{H,t-1}} MC_{t+s} \right] \right\}}{\sum_{s=0}^{\infty} (\theta\beta)^s \mathbb{E}_t \left\{ e^{-g_{t+s}} (C_{t+s})^{-\frac{1}{\tau}} Y_{t+s} \frac{P_{H,t-1}}{P_{t+s}} \right\}}, \tag{40}$$

or, alternatively, as:

$$\frac{P^{\#}_{H,t}}{P_{H,t-1}} = \mu_t \frac{P_{H,t}}{P_{H,t-1}} \frac{C_t^{-\frac{1}{\tau}} Y_t MC_t + \sum_{s=1}^{\infty} (\theta\beta)^{s-1} \mathbb{E}_t \left\{ e^{-\Delta g_{t+s}} (C_{t+s})^{-\frac{1}{\tau}} Y_{t+s} \frac{P_t}{P_{t+s}} \frac{\mu_{t+s}}{\mu_t} \frac{P_{H,t+s}}{P_{H,t}} MC_{t+s} \right\}}{C_t^{-\frac{1}{\tau}} Y_t + \sum_{s=1}^{\infty} (\theta\beta)^{s-1} \mathbb{E}_t \left\{ e^{-\Delta g_{t+s}} (C_{t+s})^{-\frac{1}{\tau}} Y_{t+s} \frac{P_t}{P_{t+s}} \right\}}. \tag{41}$$

Again by Calvo (1983) price-setting, the law of large numbers implies that the domestic price inflation $\pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}} - 1$ evolves according to:

$$(1 + \pi_{H,t})^{1-\theta} = (1 - \alpha) \left(1 + \pi^{\#}_{H,t} \right)^{1-\theta}, \tag{42}$$

where $\pi^{\#}_{H,t} \equiv \frac{P^{\#}_{H,t}}{P_{H,t-1}} - 1$ is the reset inflation rate.

B.3 Other Equilibrium Conditions

Perfect risk-sharing between the home and foreign country holds under the assumption of complete domestic and international asset markets (whereby the set of Arrow-Debreu securities traded internationally spans all possible states of nature). This implies that the intertemporal marginal rate of substitution equalizes across countries and across all possible states of nature when expressed in units of the same currency. Hence, it follows that $\Lambda_{t,t+1} = \Lambda^f_{t,t+1} \left(\frac{S_t}{S_{t+1}} \right)$ where S_t denotes the level of the bilateral nominal exchange rate and $\Lambda^f_{t,t+s}$ is the foreign intertemporal marginal rate of substitution while $\Lambda_{t,t+1}$ is the domestic intertemporal marginal rate of substitution in (30). Assuming symmetric preferences in both countries and also implicitly assuming that the preference shock, g_t, is common in both the home and foreign country, it follows that:

$$\beta e^{-\Delta g_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\tau}} \frac{P_t}{P_{t+1}} = \beta e^{-\Delta g_{t+1}} \left(\frac{C^f_{t+1}}{C^f_t} \right)^{-\frac{1}{\tau}} \frac{P^f_t}{P^f_{t+1}} \left(\frac{S_t}{S_{t+1}} \right), \tag{43}$$

where the superscript f denotes the counterpart of a given variable in the foreign country. Then, (43) can be re-written recursively as:

$$C_t = \upsilon C^f_t (RER_t)^{\tau}, \tag{44}$$

where $RER_t \equiv \frac{P^f_t S_t}{P_t}$ defines the bilateral real exchange rate and υ is a constant that depends on the initial conditions. The terms of trade Q_t are defined as $Q_t \equiv \frac{S_t P^f_{H,t}}{P_{F,t}} = \frac{P_{H,t}}{P_{F,t}}$ which follows because the law of one price holds under producer-currency pricing and that implies $P_{H,t} = S_t P^f_{H,t}$ and $P_{F,t} = S_t P^f_{F,t}$. This implies that the terms of trade can be computed simply as the ratio between the export price of goods over the corresponding import price index. We assume that the initial conditions are consistent with a deterministic, zero-inflation steady state satisfying power purchasing parity ($P_H = P_F$ implying) and $C = C^f$ such that the constant υ would be equal to one.[30]

Goods market clearing in the domestic economy requires that:

$$Y_t(j) = C_{H,t}(j) + C^f_{H,t}(j), \text{ for all } j. \tag{45}$$

Given the domestic demand for the domestic variety j, $C_{H,t}(j)$, derived in (24) and (26) and the foreign demand for the domestic variety j, $C^f_{H,t}(j)$, constructed under the assumption of symmetry in the preferences of domestic and foreign households, we obtain that:

$$\begin{aligned} Y_t(j) &= (1-\alpha)\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon_t}\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P^f_{H,t}(j)}{P^f_{H,t}}\right)^{-\epsilon_t}\left(\frac{P^f_{H,t}}{P^f_t}\right)^{-\eta} C^f_t \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon_t}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P^f_{H,t}}{P^f_t}\right)^{-\eta} C^f_t\right], \end{aligned} \tag{46}$$

since the law of one price holds, i.e. $P_{H,t}(j) = S_t P^f_{H,t}(j)$, and by extension that implies $P_{H,t} = S_t P^f_{H,t}$. Plugging this into the definition of aggregate domestic output (analogous to the domestic-goods CES aggregator in (20)), we get that:

$$\begin{aligned} Y_t &\equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon_t-1}{\epsilon_t}} dj\right)^{\frac{\epsilon_t}{\epsilon_t-1}} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P^f_{H,t}}{P^f_t}\right)^{-\eta} C^f_t \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) C_t + \alpha\left(\frac{P_{H,t}}{P_t}\right)^{\eta}\left(\frac{P^f_{H,t}}{P^f_t}\right)^{-\eta} C^f_t\right] \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) C_t + \alpha\left(\frac{S_t P^f_t}{P_t}\right)^{\eta} C^f_t\right] \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) C_t + \alpha (RER_t)^{\eta} C^f_t\right], \end{aligned} \tag{47}$$

where again the real exchange rate is defined as $RER_t \equiv \frac{P^f_t S_t}{P_t}$. The foreign country is assumed to be a nearly closed economy so that aggregate foreign consumption is approximately equal to aggregate foreign production, i.e., $C^f_t \simeq Y^f_t$.

Monetary policy is set according to three variants of the Taylor (1993)-type interest rate rule that we discuss in log-linear form below.

B.4 Log-Linearized Equilibrium Conditions

Unless otherwise noted, from here onwards, we use lower-case letters in place of capital letters in order to denote the variables in log-deviations from the symmetric, zero-inflation steady state when log-linearizing the equilibrium conditions of the model.

[30] The conditions under which PPP holds in the model are discussed in Galí and Monacelli (2005). PPP holding in the steady state implies that the differential between the domestic and foreign real interest rate reverts to a zero mean.

B.4.1 Stationarizing the Model

Given that domestic aggregate productivity is assumed to contain a stochastic trend, we stationarize the model by defining some of the variables in relation to productivity A_t as follows: $Y_{A,t} \equiv \frac{Y_t}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $Y^f_{A,t} \equiv \frac{Y^f_t}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $C_{A,t} \equiv \frac{C_t}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $C_{AH,t} \equiv \frac{C_{H,t}}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $C^f_{AH,t} \equiv \frac{C^f_{H,t}}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$, $N_{A,t} \equiv \frac{N_t}{A_t^{\frac{\tau-1}{1+\tau\varphi}}}$, and $W_{A,t} \equiv \frac{W_t}{A_t}$.[31] We then transform the first-order conditions arising from the domestic household's optimization problem ((29) and (31)) as follows:

$$C_{A,t}^{-\frac{1}{\tau}} W_{A,t} = N^{\varphi}_{A,t} P_t, \tag{48}$$

$$\left(C_{A,t}\right)^{-\frac{1}{\tau}} e^{-g_t} = \beta \mathbb{E}_t \left[\frac{(1+i_t)}{(1+\pi_{t+1})} \left(C_{A,t+1}\right)^{-\frac{1}{\tau}} e^{-g_{t+1}} \left(e^{-\left(\frac{1+\varphi}{1+\tau\varphi}\right) z_t} \right) \right]. \tag{49}$$

The real marginal cost equation in (34) becomes:

$$MC_t \equiv \frac{W_{A,t}}{P_{H,t}}. \tag{50}$$

Similarly, we can transform the optimal pricing equation in (41) in the following form:

$$\frac{P^{\#}_{H,t}}{P_{H,t-1}} = \mu_t \frac{P_{H,t}}{P_{H,t-1}} \frac{C_{A,t}^{-\frac{1}{\tau}} Y_{A,t} MC_t + \sum_{s=1}^{\infty} (\theta\beta)^{s-1} \mathbb{E}_t \left\{ e^{-\Delta \overline{g}_{t+s}} \left(C_{A,t+s}\right)^{-\frac{1}{\tau}} Y_{A,t+s} \frac{P_t}{P_{t+s}} \frac{\mu_{t+s}}{\mu_t} \frac{P_{H,t+s}}{P_{H,t}} MC_{t+s} \right\}}{C_{A,t}^{-\frac{1}{\tau}} Y_{A,t} + \sum_{s=1}^{\infty} (\theta\beta)^{s-1} \mathbb{E}_t \left\{ e^{-\Delta \overline{g}_{t+s}} \left(C_{A,t+s}\right)^{-\frac{1}{\tau}} Y_{A,t+s} \frac{P_t}{P_{t+s}} \right\}}, \tag{51}$$

where $\Delta \overline{g}_{t+s} \equiv \Delta g_{t+s} + \frac{(1-\tau)(1+\varphi)}{1+\tau\varphi} \sum_{j=1}^{s} z_{t+j}$ for all $s > 0$.

The perfect risk-sharing condition in (44) can be stationarized analogously as:

$$C_{A,t} = \upsilon C^f_{A,t} \left(RER_t\right)^{\tau}. \tag{52}$$

We can stationarize the domestic market clearing condition in (47) in the following terms:

$$Y_{A,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) C_{A,t} + \alpha \left(RER_t\right)^{\eta} C^f_{A,t} \right]. \tag{53}$$

The foreign country is assumed to be a nearly closed economy so, when stationarized, it holds that the foreign market clearing condition becomes $C^f_{A,t} \simeq Y^f_{A,t}$. We can similarly re-write the domestic output production in (35) as:

$$Y_{A,t} Z_t = N_{A,t}, \tag{54}$$

where the output dispersion is $Z_t \equiv \int_0^1 \frac{Y_t(j)}{Y_t} dj = \int_0^1 \frac{Y_{A,t}(j)}{Y_{A,t}} dj$.

[31] Analogously, we definte the potential output that arises absent all frictions as Y^n_t and stationarize it as $Y^n_{A,t} \equiv \frac{Y^n_t}{A_t^{\frac{\tau+\tau\varphi}{1+\tau\varphi}}}$.

B.4.2 Definitions: CPI and Terms of Trade

Log-linearizing the definition of the domestic CPI price level corresponding to (19) (satisfying the purchasing power parity condition in steady state, $P_H = P_F$), it follows that:

$$\begin{aligned} p_t &= (1-\alpha)\, p_{H,t} + \alpha p_{F,t} \\ &= p_{H,t} - \alpha q_t, \end{aligned} \tag{55}$$

where we define the log of terms of trade as $q_t = p_{H,t} - p_{F,t}$. From here, it follows that the domestic inflation rate of consumption goods, π_t, can be expressed as a weighted average of the inflation rate of the domestically produced goods, $\pi_{H,t}$, and the inflation rate of the imported goods, $\pi_{F,t}$, with the degree of openness α as the weight associated with $\pi_{F,t}$:

$$\pi_t = (1-\alpha)\, \pi_{H,t} + \alpha \pi_{F,t}, \tag{56}$$

and, alternatively, that the domestic CPI inflation rate is a function of the domestic producer price inflation, $\pi_{H,t}$, and the growth rate of the terms of trade, Δq_t, as follows:

$$\pi_t = \pi_{H,t} - \alpha \Delta q_t, \tag{57}$$

where $\pi_t \equiv p_t - p_{t-1}$, $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$, and $\Delta q_t \equiv q_t - q_{t-1}$.

We have been assuming that the law of one price holds for all individual varieties at all times (both for import and export prices), then it easily follows that:

$$\begin{aligned} p_{F,t} &= s_t + p^f_{F,t} \\ &= s_t + p^f_t, \end{aligned} \tag{58}$$

where the second equality holds because the price of foreign goods in the foreign marketplace, $p^f_{F,t}$, is approximately equal to the foreign CPI, p^f_t, given that the foreign economy is approximately closed and foreign consumption is nearly the same as foreign production. In turn, this implies that:

$$\pi_{F,t} = \Delta s_t + \pi^f_t, \tag{59}$$

where $\pi^f_t \equiv p^f_t - p^f_{t-1}$, $\pi_{F,t} \equiv p_{F,t} - p_{F,t-1}$, and $\Delta s_t \equiv s_t - s_{t-1}$.

Rewriting the definition of the terms of trade, $q_t = p_{H,t} - p_{F,t}$, in growth rates and combining it with equations (57) and (59), we obtain the following expression:

$$\begin{aligned} -\Delta q_t &= \pi_{F,t} - \pi_{H,t} \\ &= \left(\Delta s_t + \pi^f_t\right) - (\pi_t + \alpha \Delta q_t) \\ &= \Delta s_t + \pi^f_t - \pi_t - \alpha \Delta q_t. \end{aligned} \tag{60}$$

This implies that the domestic CPI inflation rate, π_t, can be formalized as:

$$\pi_t = \Delta s_t + (1-\alpha)\,\Delta q_t + \pi_t^f, \tag{61}$$

which is a linear combination of the change in the nominal exchange rate, Δs_t, the growth rate of the terms of trade, Δq_t, and the foreign CPI inflation, π_t^f. The foreign inflation is treated as unobservable and assumed to follow an AR(1) process:

$$\pi_t^f = \rho_{\pi^f}\pi_{t-1}^f + \varepsilon_{\pi^f,t},\ \varepsilon_{\pi^f,t} \sim N\left(0, \sigma_{\pi^f}\right), \tag{62}$$

where $-1 < \rho_{\pi^f} < 1$ denotes the persistence parameter and $\sigma_{\pi^f} > 0$ the volatility.

Moreover, the definition of the real exchange rate, $RER_t \equiv \frac{P_t^f S_t}{P_t}$, can also be expressed in logs as follows:

$$rer_t = s_t + p_t^f - p_t. \tag{63}$$

Then, using (55) and (58), we infer that:

$$\begin{aligned} rer_t &= p_{F,t} - (p_{H,t} - \alpha q_t) \\ &= -(1-\alpha)\,q_t, \end{aligned} \tag{64}$$

which implies that the real exchange rate, rer_t, is proportional to the terms of trade, q_t. This comes to show that although purchasing power parity does not hold in the short-run (albeit it does in steady state), its movements are proportional to those of the terms of trade. Hence, terms of trade alone summarizes all relevant information on the international relative prices in this model.

B.4.3 Market Clearing Conditions

If we log-linearize the perfect risk-sharing equation in (52) around its steady state, we obtain the following risk-sharing condition in log terms:

$$\begin{aligned} c_{A,t} &= c_{A,t}^f + \tau rer_t \\ &= c_{A,t}^f - \tau (1-\alpha)\,q_t, \end{aligned} \tag{65}$$

where the second equality follows from (64). If we also log-linearize the market clearing condition for domestic goods in (53) in deviations from the steady state, it follows that:

$$y_{A,t} = -\eta\,(p_{H,t} - p_t) + (1-\alpha)\,c_{A,t} + \eta\alpha rer_t + \alpha c_{A,t}^f, \tag{66}$$

which holds because in the symmetric, zero-inflation steady state satisfying purchasing power parity ($P_H = P_F$) it also occurs that $RER = Q = 1$ and, whenever the initial conditions of the problem are consistent with the steady state ($\upsilon = 1$), that $C_A = C_A^f$.

Then, using (55) and (64) we can derive the following more compact expression for (66):

$$\begin{aligned} y_{A,t} &= -\eta\alpha q_t + (1-\alpha)\,c_{A,t} - \eta\alpha\,(1-\alpha)\,q_t + \alpha c_{A,t}^f \\ &= (1-\alpha)\,c_{A,t} - \eta\alpha\,(2-\alpha)\,q_t + \alpha c_{A,t}^f. \end{aligned} \tag{67}$$

Replacing out $c^f_{A,t}$ with the perfect-risk sharing condition in (65), it follows from here that:

$$y_{A,t} = c_{A,t} - (\eta\alpha + (\eta - \tau)\,\alpha\,(1-\alpha))\,q_t. \tag{68}$$

The foreign economy behaves approximately as a closed economy, therefore it is the case that $c^f_{A,t} \simeq y^f_{A,t}$. Given that approximation together with equations (65) and (68), we infer that:

$$y_{A,t} + (\eta\alpha + (\eta - \tau)\,\alpha\,(1-\alpha))\,q_t = c_{A,t} = y^f_{A,t} - \tau\,(1-\alpha)\,q_t, \tag{69}$$

or, simply, that:

$$y_{A,t} = y^f_{A,t} - \frac{1}{\tau_\alpha} q_t, \tag{70}$$

where $\tau_\alpha \equiv \frac{1}{\tau + \alpha(2-\alpha)(\eta-\tau)}$.

The terms of trade, q_t, is determined endogenously as the international relative price that clears international goods markets and, accordingly, is proportional to the scarcity of foreign goods relative to domestic ones. In terms of growth rates, the relationship can be written as:

$$\Delta q_t = \tau_\alpha \left(\Delta y^f_{A,t} - \Delta y_{A,t}\right). \tag{71}$$

An increase in world output raises the demand for the domestically-produced goods so that the terms of trade (i.e., its relative price) improves, while a decline in domestic output has the opposite effect. In this framework, the domestic conditions of the small open economy are not thought to have a significant effect on the path of the terms of trade which is determined in international markets. Hence, we follow Lubik and Schorfheide (2007) estimating the dynamics of the (observable) growth rate in terms of trade, Δq_t, which take the following AR(1) form:

$$\Delta q_t = \rho_q \Delta q_{t-1} + \varepsilon_{q,t},\ \ \varepsilon_{q,t} \sim N\left(0, \sigma_q\right), \tag{72}$$

where $-1 < \rho_q < 1$ denotes the persistence parameter and $\sigma_q > 0$ the volatility. This preserves the small open economy feature that terms of trade are largely exogenous from domestic economic conditions.

B.4.4 The Phillips Curve

An approximate log-linear aggregate production function can be derived from (54) as follows:

$$y_{A,t} = n_{A,t}, \tag{73}$$

given that the output dispersion across firms, $\ln Z_t \equiv \ln\left(\int_0^1 \frac{Y_t(j)}{Y_t} dj\right) = \ln\left(\int_0^1 \frac{Y_{A,t}(j)}{Y_{A,t}} dj\right)$, around the symmetric, zero-inflation steady state is only of second-order importance. The labor supply equation in (48) can be log-linearized as:

$$w_{A,t} - p_t = \varphi n_{A,t} + \frac{1}{\tau} c_{A,t}. \tag{74}$$

Hence, if we log-linearize the real marginal cost equation (50) and then substitute in it equations (55), (73), and (74), we obtain that:

$$\begin{aligned} mc_t &= (w_{A,t} - p_t) + (p_t - p_{H,t}) \\ &= \varphi n_{A,t} + \frac{1}{\tau} c_{A,t} - \alpha q_t \\ &= \varphi y_{A,t} + \frac{1}{\tau} c_{A,t} - \alpha q_t. \end{aligned} \tag{75}$$

Putting this together with equation (44) and $c^f_{A,t} \simeq y^f_{A,t}$, it follows that:

$$mc_t = \varphi y_{A,t} + \frac{1}{\tau} y^f_{A,t} - q_t. \tag{76}$$

Finally, replacing out the terms of trade q_t with equation (70), we can express the real marginal cost function as:

$$mc_t = (\tau_\alpha + \varphi)\, y_{A,t} + \left(\frac{1}{\tau} - \tau_\alpha\right) y^f_{A,t}. \tag{77}$$

In the frictionless economy, the real marginal cost of the domestic economy, mc^n_t, is similarly derived as:

$$mc^n_t = (\tau_\alpha + \varphi)\, y^n_{A,t} + \left(\frac{1}{\tau} - \tau_\alpha\right) y^f_{A,t}. \tag{78}$$

In this scenario, given that prices are flexible and perfect competition holds, the price markup (and the labor subsidy applied to remove the steady state price markup) $\mu_t \equiv (1-\phi)\, \frac{\epsilon_t}{\epsilon_t - 1}$, drops out and the natural marginal cost is constant for all t and equal to one. Therefore, the natural marginal cost in logs is simply $mc^n_t = 0$. Thus, it follows that:

$$y^n_{A,t} = -\Gamma_* y^f_{A,t}, \tag{79}$$

where $\Gamma_* \equiv \frac{1-\tau\tau_\alpha}{\tau\tau_\alpha + \tau\varphi}$. The measure of foreign output, $y^f_{A,t}$, is exogenous and follows an AR(1) process of the following form:

$$y^f_{A,t} = \rho_{y^f} y^f_{A,t-1} + \varepsilon_{y^f,t},\ \ \varepsilon_{y^f,t} \sim N\left(0, \sigma_{y^f}\right), \tag{80}$$

where $-1 < \rho_{y^f} < 1$ denotes the persistence parameter and $\sigma_{y^f} > 0$ the volatility.

The difference between the marginal cost in (77) and the natural marginal cost in (78) equals:

$$mc_t - mc^n_t = (\tau_\alpha + \varphi)\left(y_{A,t} - y^n_{A,t}\right), \tag{81}$$

where $x_t \equiv y_{A,t} - y^n_{A,t}$ is a measure of the domestic output gap in the small open economy. Notice that given that both output and potential output are stationarized with the same transformation of productivity, then it should be the case that the output gap is the same even if expressed as the difference between the output and potential output not stationarized, i.e., as $x_t \equiv y_t - y^n_t$. Log-linearizing the domestic firms' price-setting equilibrium condition in (51) and (42) yields an equation determining domestic-goods inflation (or producer-price index), $\pi_{H,t}$, as a function of the deviations of the real marginal cost from its natural counterpart, $mc_t - mc_t^n$, which can be written as:

$$\begin{aligned}\pi_{H,t} &= \beta \mathbb{E}_t\left(\pi_{H,t+1}\right) + \kappa\left(mc_t - mc_t^n\right) + u_t \\ &= \beta \mathbb{E}_t\left(\pi_{H,t+1}\right) + \kappa\left(\tau_\alpha + \varphi\right) x_t + u_t, \end{aligned} \tag{82}$$

where the slope of the Phillips curve is related to the composite coefficients $\kappa \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ and $\tau_\alpha \equiv \frac{1}{\tau+\alpha(2-\alpha)(\eta-\tau)}$. Substituting into (82) the condition (57) that relates domestic-goods inflation, $\pi_{H,t}$, to CPI inflation, π_t, and the terms of trade growth, Δq_t, the small open economy Phillips curve for CPI inflation can be expressed as:

$$\pi_t = \beta \mathbb{E}_t\left(\pi_{t+1}\right) + \alpha\beta \mathbb{E}_t\left(\Delta q_{t+1}\right) - \alpha \Delta q_t + \kappa(\tau_\alpha + \varphi) x_t + u_t, \tag{83}$$

where $u_t \equiv \ln\left(\mu_t\right)$ is the exogenous shock that refers to the log of the price markup adjusted to account for the employment subsidy.

B.4.5 Dynamic IS Curve

If we log-linearize the Euler equation (49) around the symmetric, zero-inflation steady state, we have:

$$c_{A,t} = \mathbb{E}_t\left(c_{A,t+1}\right) - \tau\left[i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \frac{1+\varphi}{1+\tau\varphi}\mathbb{E}_t\left(z_{t+1}\right)\right]. \tag{84}$$

Replacing $c_{A,t}$ with (67) into the Euler equation in (84) and using the foreign country goods market equilibrium condition ($c_{A,t}^f \simeq y_{A,t}^f$), it follows that:

$$y_{A,t} = \mathbb{E}_t\left(y_{A,t+1}\right) - \alpha \mathbb{E}_t\left(\Delta y_{A,t+1}^f\right) + \eta\alpha\left(2-\alpha\right)\mathbb{E}_t\left(\Delta q_{t+1}\right) - \tau\left(1-\alpha\right)\left[i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \left(\frac{1+\varphi}{1+\tau\varphi}\right)\mathbb{E}_t\left(z_{t+1}\right)\right]. \tag{85}$$

Equation (71) then can be used to re-write this equation as:

$$\begin{aligned} y_{A,t} \quad = \quad & \mathbb{E}_t\left(y_{A,t+1}\right) - \alpha \mathbb{E}_t\left(\Delta y_{A,t+1}^f\right) + \eta\alpha\left(2-\alpha\right)\tau_\alpha \mathbb{E}_t\left(\Delta y_{A,t+1}^f - \Delta y_{A,t+1}\right) - ... \\ & \tau\left(1-\alpha\right)\left[i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \frac{1+\varphi}{1+\tau\varphi}\mathbb{E}_t\left(z_{t+1}\right)\right], \end{aligned} \tag{86}$$

or simply as:

$$\begin{aligned} \left[1-\eta\alpha\left(2-\alpha\right)\tau_\alpha\right] y_{A,t} \quad = \quad & \left[1-\eta\alpha\left(2-\alpha\right)\tau_\alpha\right]\mathbb{E}_t\left(y_{A,t+1}\right) + \left[\eta\left(2-\alpha\right)\tau_\alpha - 1\right]\alpha \mathbb{E}_t\left(\Delta y_{A,t+1}^f\right) - ... \\ & \tau\left(1-\alpha\right)\left[i_t - \mathbb{E}_t\left(\pi_{t+1}\right) - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \frac{1+\varphi}{1+\tau\varphi}\mathbb{E}_t\left(z_{t+1}\right)\right]. \end{aligned} \tag{87}$$

Hence, it follows that:

$$\begin{aligned}
y_{A,t} &= \mathbb{E}_t\left(y_{A,t+1}\right) + \frac{\left(\eta\left(2-\alpha\right)\tau_\alpha - 1\right)\alpha}{1-\eta\alpha\left(2-\alpha\right)\tau_\alpha}\mathbb{E}_t\left(\Delta y^f_{A,t+1}\right) - \frac{\left(1-\alpha\right)\tau}{1-\eta\alpha\left(2-\alpha\right)\tau_\alpha}\left[r_t - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \frac{1+\varphi}{1+\tau\varphi}\mathbb{E}_t\left(z_{t+1}\right)\right] \\
&= \mathbb{E}_t\left(y_{A,t+1}\right) + \frac{\left(1-\alpha\right)\tau}{1-\eta\alpha\left(2-\alpha\right)\tau_\alpha}\left(\frac{\left(\eta\left(2-\alpha\right)\tau_\alpha - 1\right)\alpha}{\left(1-\alpha\right)\tau}\right)\mathbb{E}_t\left(\Delta y^f_{A,t+1}\right) - \ldots \\
&\quad \frac{\left(1-\alpha\right)\tau}{1-\eta\alpha\left(2-\alpha\right)\tau_\alpha}\left[r_t - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \frac{1+\varphi}{1+\tau\varphi}\mathbb{E}_t\left(z_{t+1}\right)\right] \\
&= \mathbb{E}_t\left(y_{A,t+1}\right) - \frac{1}{\left(1-\alpha\right)\tau_\alpha}\left[r_t - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \left(\frac{1+\varphi}{1+\tau\varphi}\right)\mathbb{E}_t\left(z_{t+1}\right) - \left(\frac{1}{\tau} - (1-\alpha)\tau_\alpha\right)\mathbb{E}_t\left(\Delta y^f_{A,t+1}\right)\right] \\
&= \mathbb{E}_t\left(y_{A,t+1}\right) - \frac{1}{\left(1-\alpha\right)\tau_\alpha}\left[r_t - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \left(\frac{1+\varphi}{1+\tau\varphi}\right)\mathbb{E}_t\left(z_{t+1}\right) - \left(\frac{1}{\tau} - (1-\alpha)\tau_\alpha\right)\mathbb{E}_t\left(\Delta y^f_{A,t+1}\right)\right], \quad (88)
\end{aligned}$$

where $\tau_\alpha \equiv \frac{1}{\tau+\alpha(2-\alpha)(\eta-\tau)}$. Here, the Fisher equation allows us to write the real interest rate as $r_t = i_t - \mathbb{E}_t\left(\pi_{t+1}\right)$ as well.

Equation (88) can be derived similarly in the frictionless equilibrium case so as to obtain that:

$$y^n_{A,t} = \mathbb{E}_t\left(y^n_{A,t+1}\right) - \frac{1}{\left(1-\alpha\right)\tau_\alpha}\left[r^n_t - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \left(\frac{1+\varphi}{1+\tau\varphi}\right)\mathbb{E}_t\left(z_{t+1}\right) - \left(\frac{1}{\tau} - (1-\alpha)\tau_\alpha\right)\mathbb{E}_t\left(\Delta y^f_{A,t+1}\right)\right], \quad (89)$$

where $y^n_{A,t}$ denotes the stationarized potential output and r^n_t is the corresponding natural (real) rate of interest. Taking the difference between (88) and (89), we can rewrite after some algebra the small open economy dynamic IS curve in terms of the domestic output gap, x_t, as follows:

$$x_t = \mathbb{E}_t(x_{t+1}) - \frac{1}{\left(1-\alpha\right)\tau_\alpha}\left(i_t - \mathbb{E}_t(\pi_{t+1}) - r^n_t\right). \quad (90)$$

Furthermore, given the domestic potential output equation in (79), we can replace $y^n_{A,t}$ out of (89) in order to derive a formula for the natural rate of interest, r^n_t, as follows:

$$0 = -\left(1-\alpha\right)\tau_\alpha\Gamma_*\mathbb{E}_t\left(\Delta y^f_{A,t}\right) - \left[r^n_t - \mathbb{E}_t\left(\Delta g_{t+1}\right) - \left(\frac{1+\varphi}{1+\tau\varphi}\right)\mathbb{E}_t\left(z_{t+1}\right) - \left(\frac{1}{\tau} - \left(1-\alpha\right)\tau_\alpha\right)\mathbb{E}_t\left(\Delta y^f_{A,t+1}\right)\right], \quad (91)$$

which formalizes the domestic natural interest rate, r^n_t, as:

$$r^n_t = \mathbb{E}_t\left(\Delta g_{t+1}\right) + \left(\frac{1+\varphi}{1+\tau\varphi}\right)\mathbb{E}_t\left(z_{t+1}\right) + \left[\frac{1}{\tau} - \left(1-\alpha\right)\tau_\alpha\left(\Gamma_* + 1\right)\right]\mathbb{E}_t\left(\Delta y^f_{A,t+1}\right), \quad (92)$$

where $\Gamma_* \equiv \frac{1-\tau\tau_\alpha}{\tau\tau_\alpha+\tau\varphi}$. Notice that the natural rate of interest, r^n_t, can be re-written in levels of the preference shock, g_t, the productivity growth shock, z_t, and the stationarized foreign output, $y^f_{A,t}$, using (22), (33), and (80) to obtain that:

$$r^n_t = \left(\rho_g - 1\right)g_t + \left(\frac{1+\varphi}{1+\tau\varphi}\right)\rho_z z_t + \left[\frac{1}{\tau} - \left(1-\alpha\right)\tau_\alpha\left(\Gamma_* + 1\right)\right]\left(\rho_{y^f} - 1\right)y^f_{A,t}, \quad (93)$$

which comes to show that the natural rate of interest increases in response to a positive productivity shock, z_t, if $0 < \rho_z < 1$. It also shows that the natural rate decreases in response to a positive preference shock, g_t, and (whenever $\left[\frac{1}{\tau}-(1-\alpha)\,\tau_{\alpha}\left(\Gamma_{*}+1\right)\right]>0$) to a positive stationarized foreign output shock, $y_{A,t}^{f}$, given that $\rho_{g}, \rho_{y^{f}}<1$.

B.4.6 Monetary Policy

Monetary policy is implemented with a standard Taylor (1993)-type interest rate rule (T rule) where the central bank adjusts its policy instrument in deviations from the steady state, i_t, in response to deviations on the domestic CPI inflation rate, π_t, and the domestic output gap, x_t:[32]

$$i_{t}=\rho_{i} i_{t-1}+\left(1-\rho_{i}\right)\left(\psi_{x} x_{t}+\psi_{\pi} \pi_{t}\right)+\varepsilon_{i,t},\ \varepsilon_{i,t} \sim N\left(0, \sigma_{i}\right), \tag{94}$$

or with a Wicksellian rule (W rule) that tracks the domestic natural interest rate, r_t^n, and responds to domestic inflation, π_t:

$$i_{t}=\rho_{i} i_{t-1}+\left(1-\rho_{i}\right)\left(r_{t}^{n}+\psi_{\pi} \pi_{t}\right)+\varepsilon_{i,t}\ \varepsilon_{i,t} \sim N\left(0, \sigma_{i}\right), \tag{95}$$

or with a more general form of the Wicksellian rule ($W\&T$ rule) that tracks the domestic natural interest rate, r_t^n, and responds to domestic inflation, π_t, and domestic slack, x_t:

$$i_{t}=\rho_{i} i_{t-1}+\left(1-\rho_{i}\right)\left(r_{t}^{n}+\psi_{x} x_{t}+\psi_{\pi} \pi_{t}\right)+\varepsilon_{i,t},\ \varepsilon_{i,t} \sim N\left(0, \sigma_{i}\right), \tag{96}$$

where $0<\rho_i<1$ is an inertia parameter while the response to inflation and output gap deviations is given by the parameters $\psi_\pi>0$ and $\psi_x>0$.[33] Moreover, $\varepsilon_{i,t}$ is an exogenous i.i.d. policy shock with $\sigma_i>0$ being its volatility.

[32] The symmetric, zero-inflation steady state of the frictionless model and that of the benchmark model with nominal rigidities is the same. Hence, the steady state real rate and the steady state natural rate of interest are the same and can be computed as $r^{ss}=-400\ln(\beta)$ in percent terms, annualized.

[33] The three monetary policy rules in (94) – (96) are also nested given that they only vary on the weight each one places on the short-term natural rate and output gap. The following formula describes the family of policy rules under consideration:

$$i_{t}=\rho_{i} i_{t-1}+\left(1-\rho_{i}\right)\left(\psi_{r} r_{t}^{n}+\psi_{x} x_{t}+\psi_{\pi} \pi_{t}\right)+\varepsilon_{i,t},\ \varepsilon_{i,t} \sim N\left(0, \sigma_{i}\right). \tag{97}$$

Hence, the $W\&T$ rule in (96) is nothing more than estimating the policy rule above imposing a prior with mass one on ψ_r being equal to one. The (95) would be equal to estimating the policy rule imposing a prior with mass one on ψ_r being equal to one as before and a prior with mass one on ψ_x being equal to zero. And, analogously, (94) is equivalent to imposing a prior with mass one on ψ_r being equal to zero in that same specification of the monetary policy reaction function.

C Data Sources

Our data are all from Haver Analytics. To facilitate replication of our results, the Haver mnemonics used to construct our data set are listed as follows:

Real GDP: Australia: B193GDPC@OECDNAQ; Canada: B156GDPC@OECDNAQ; South Korea: B542GDPC@OECDNAQ; Sweden: B144GDPC@OECDNAQ; Switzerland: B146GDPC@OECDNAQ; U.K.: B112GDPC@OECDNAQ.

Core CPI: Australia: SA(C193CZCN@OECDMEI); Canada: SA(C156CZCN@OECDMEI); South Korea: SA(C542CZCN@OECDMEI); Sweden: SA(C144CZCN@OECDMEI); Switzerland: SA(C146CZCN@OECDMEI); U.K.: SA(C112CZCN@OECDMEI), where "SA" denotes the series was seasonally adjusted by Haver Analytics.

Policy Rate: Australia: N193RTAR@G10; Canada: C156FROS@OECDMEI; South Korea: C542IFC@IFS; Sweden: C144FRUO@OECDMEI; Switzerland: C146IC@IFS; U.K.: N112RTAR@G10.

Nominal Narrow Effective Exchange Rate (constructed by the Bank for International Settlements (BIS)): Australia: B193XNN@BIS; Canada: B156XNN@BIS; South Korea: B542XNN@BIS; Sweden: B144XNN@BIS; Switzerland: B146XNN@BIS; U.K.: B112XNN@BIS.

Export Price: Australia: C193XPI@OECDNAQ; Canada: C156XPI@OECDNAQ; South Korea: C542XPI@OECDNAQ; Sweden: C144CP@IFS; Switzerland: C146XPI@OECDNAQ; U.K.: C112XPI@OECDNAQ.

Import Price: Australia: C193MPI@OECDNAQ; Canada: C156MPI@OECDNAQ; South Korea: C542MPI@OECDNAQ; Sweden: C144CPX@IFS; Switzerland: C146MPI@OECDNAQ; U.K.: C112MPI@OECDNAQ.

We compute the terms of trade series as the ratio between the export price of goods and services index and the corresponding import price index.

Real-time Data. Almost all the series which we use in our estimation are largely inmune to data revisions except the real GDP. For the real GDP, we collect the real-time series available from the OECD's real-time database known as Dataset: MEI Original Release Data and Revisions. We estimate the reference model replacing the full sample collected with vintage data from 2018:Q2 with the corresponding real-time series from the OECD dataset on fourth quarter (December release) of the years 2006, 2007, 2008, 2009, 2010, and 2011. The 2018:Q2 vintage data covers the full sample period from 1983:Q1 till 2018:Q1 while the fourth quarter (December release) vintages include data up to third quarter of the corresponding year. All OECD real-time data can be accessed publicly at OECD (2020).

Country Characteristics. We also collect other data not directly used in the observable set in our estimations, but employed to describe the characteristics of each one of the six small open-economies in our sample (degree of openness, economic size, and level of development):

Import Value (fob) as a Percent of GDP: Australia: C193GMCP@OECDMEI; Canada: C156GMCP@OECDMEI; South Korea: C542GMCP@OECDMEI; Sweden: C144GMCP@OECDMEI; Switzerland: C146GMCP@OECDMEI; U.K.: C112GMCP@OECDMEI.

GDP Based on PPP Share of World Total: Australia: R193GPPS@IMFWEO; Canada: R156GPPS@IMFWEO; South Korea: R542GPPS@IMFWEO; Sweden: R144GPPS@IMFWEO; Switzerland: R146GPPS@IMFWEO; U.K.: R112GPPS@IMFWEO; U.S.: R111GPPS@IMFWEO.

GDP Per Capita, Constant Prices (2011 International $): Australia: R193GPPC@IMFWEO; Canada: R156GPPC@IMFWEO; South Korea: R542GPPC@IMFWEO; Sweden: R144GPPC@IMFWEO; Switzerland: R146GPPC@IMFWEO; U.K.: R112GPPC@IMFWEO; U.S.: R111GPPC@IMFWEO.

We use the import share as a measure of the degree of openness based on OECD Main Economic Indicators (MEI) database. For Sweden the import share is available since 1993:Q1 only, while all other countries have data over the full sample from 1983:Q1 till 2018:Q1. A measure of the relative economic size of each small open economy is constructed as the ratio of the GDP based on PPP share of world total for each country over ther GDP based on PPP share of world total for the U.S. Similarly, we construct a measure of the relative level of economic development of each small open economy based on the ratio of the GDP per capita (constant prices, 2011 international $) of a given country relative to that of the U.S. All the data we use on PPP-based GDP shares and GDP per capita is reported at an annual frequency in the IMF's World Economic Outlook (WEO) database.