Federal Reserve Bank of Dallas

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Roberto Duncan and Enrique Martínez-García

Globalization Institute Working Paper 235

April 2015 (Revised December 2022)

Research Department

https://doi.org/10.24149/gwp235r1

Forecasting Inflation in Open Economies: What Can a NOEM Model Do?*

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April 2015
This Draft: December 6, 2022

Abstract

This paper evaluates the forecasting ability when inflation is viewed as an inherently global phenomenon through the lens of the workhorse New Open Economy Macro (NOEM) model. The NOEM model emphasizes the importance of cross-country spillovers arising through trade and its reduced form solution can be represented by a finite-order VAR which provides a tractable model of inflation forecasting. We use Bayesian techniques to estimate this VAR specification—we name it NOEM-BVAR—and pseudo-out-of-sample forecasts to assess its forecasting performance at different horizons in a diverse set of 18 countries. On average, the NOEM-BVAR specification produces a similar or even lower root mean square prediction error (RMSPE) than its standard competitors, which include both purely statistical models and theoretically-based forecasting models (e.g., Phillips-curve-type alternatives and others with global inflation measures). In a number of cases, the gains in smaller RMSPEs are statistically significant, especially at short horizons. The NOEM-BVAR model is also accurate in predicting the direction of change for inflation and is often better than its competitors along this dimension as well. Even though purely statistical models can be useful prediction tools, the NOEM-BVAR is attractive among those forecasting models motivated by economic theory.

JEL Classification: E31, F41, F42, F47.

Keywords: Inflation Forecasting, New Open Economy Macro model, Open-Economy Phillips Curves.

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*We would like to thank two anonymous referees, Nathan S. Balke, Christiane Baumeister, Claudio Borio, Efrem Castelnuovo, Mario J. Crucini, Michael B. Devereux, Charles Engel, Andrew Filardo, Marc Giannoni, Chulho Jung, Edward S. Knotek II, John B. Taylor, Patricia Toledo, Jian Wang, Benjamin Wong, Saeed Zaman, and many seminar and conference participants at Ohio University, the Central Bank of Chile, the 35th International Symposium on Forecasting, the 18th Central Bank Macroeconomic Modelling workshop, and the International Conference on Time Series Analysis for helpful suggestions and comments to this work and a previous version of the paper which circulated with the title "Forecasting Local Inflation with Global Inflation: When Economic Theory Meets the Facts." We also acknowledge the excellent research assistance provided by Valerie Grossman, Luke Miller, Ernest Ofori, Cheng Shen, Md Shah Jalal Uddin, and the support of Bradley Graves and Kelvinder Virdi. Data and codes to replicate the results of the paper can be found here: https://bit.ly/3UHEK1w. All remaining errors are ours alone. The views expressed in this paper do not necessarily reflect the views of the institutions to which the authors are affiliated.

†(Contacting author) Roberto Duncan, Department of Economics, Ohio University. Office: 349 Bentley Annex. Phone: +1 (740) 597-1264. E-mail: duncanr1@ohio.edu. Webpage: https://people.ohio.edu/duncanr1/.

‡Enrique Martínez-García, Federal Reserve Bank of Dallas. Correspondence: 2200 N. Pearl Street, Dallas, TX 75201. Phone: +1 (214) 922-5262. Fax: +1 (214) 922-5194. E-mail: emg.economics@gmail.com. Webpage: https://sites.google.com/view/emgeconomics.

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1 Introduction

The idea that domestic inflation may depend on international conditions is not new. A major risk from ignoring international developments is to misinterpret the effect of domestic economic conditions, and, as a result, pursuing suboptimal macroeconomic policies based on erroneous forecasts. Understanding the international links that affect inflation is, therefore, fundamental to develop better models—not just for policy analysis, but also for forecasting. Our paper presents a two-country New Keynesian model that explicitly links domestic inflation to global developments in a globally integrated world economy. We argue that the economic forces driving inflation in one country will permeate inflation everywhere else to a certain degree. Hence, we explore the importance of those international linkages on inflation forecasting along two dimensions: theoretical and empirical.

Our paper is related to the literature that uses dynamic stochastic general equilibrium (DSGE) models with explicit microfoundations and optimizing agents for forecasting. We study the dynamics of inflation in the context of the workhorse New Open Economy Macro Model (NOEM) that has become a cornerstone of modern international macro (see, e.g., Clarida et al. (2002), Martínez-García and Wynne (2010), and Martínez-García (2017)). Del Negro et al. (2007), Del Negro and Schorfheide (2013), and Martínez-García (2015) provide a thorough review of a class of New Keynesian DSGE model with nominal rigidities and a rich economic structure of which the NOEM model is the workhorse two-country variant.[1]

Microfounded DSGE models connect the deep parameters to the main features and relationships that characterize the economy and are more suitable for macroeconomic policy evaluation (see Lucas (1976)). However, in spite of the popularity of DSGE models, trade-offs between theoretical coherence and empirical fit persist as well as concerns about misspecification and omitted variables (Schorfheide (2013), Martínez-García and Wynne (2014)). Hence, what we do in our benchmark is to incorporate the key open-economy features of the NOEM model and map its reduced-form solution into a tractable finite-order VAR specification that captures the international linkages implied by the theory without imposing its cross-equation restrictions.

To exploit the reduced-form VAR representation of the workhorse NOEM model solution in order to more accurately forecast domestic inflation, we propose a Bayesian VAR (BVAR)—our benchmark NOEM-BVAR specification—which incorporates all relevant information pertaining to the dynamics of key domestic and rest-of-the-world variables (inflation and output).[2] Most of the existing DSGE model-based forecasting frameworks omit important channels for international spillovers, particularly so when looking at large economies such as the U.S. which tend to be treated as 'effectively' closed-economies. Our benchmark NOEM-BVAR offers us a parsimonious way to explicitly allow for non-trivial international linkages to play a

[1] Increasing efforts have been made to develop, assess, and use DSGE models as forecasting tools (Smets and Wouters (2003), Smets and Wouters (2004), and Edge and Gürkaynak (2010)). In fact, DSGE models have become an integral part of the toolkit for macroeconomic forecasting and policy analysis of many central banks. Examples include the DSGE models developed by the Sveriges Riksbank (Adolfson et al. (2007)), the European Central Bank (Christoffel et al. (2011)), and the Federal Reserve Board (Edge et al. (2010)). Small open-economy DSGE models are already pretty common for forecasting too—more recently, Marcellino and Rychalovska (2014). Our paper focuses, in turn, on inflation forecasting in relatively large and internationally-connected economies—for that purpose, we put the emphasis on modelling international spillovers explicitly for forecasting inflation (see also Martínez-García et al. (2012) and Martínez-García and Wynne (2014)). In our preferred multi-country framework, we incorporate explicitly the (direct) spillover effects on the domestic economy arising from foreign-originated shocks. We also recognize the (indirect) feedback effects that arise from domestic- or foreign-originated shocks impacting international relative prices (terms of trade) and the global economy and feeding back to the domestic economy.

[2] In our preferred two-country NOEM framework, we incorporate explicitly the (direct) spillover effects on the domestic economy arising from foreign-originated shocks. We also recognize the (indirect) feedback effects that arise from domestic- or foreign-originated shocks impacting international relative prices (terms of trade) and the global economy and feeding back to the domestic economy.

role in helping us improve the inflation forecasting performance of more standard (closed-economy) empirical forecasting models.

We provide novel pseudo-out-of-sample forecasting evidence suggesting those international linkages can be important for forecasting inflation when we look at 18 major, highly-interconnected economies (including the U.S.). We focus on headline CPI inflation as our measure of inflation—as it is less subject to revisions than, e.g., the GDP deflator—and run a very extensive model comparison exercise including up to 10 different specifications widely-used in the literature. We collect quarterly data on headline CPI inflation, real GDP, industrial production, and monetary aggregates (M2) for those 18 economies from the Federal Reserve Bank of Dallas' Database of Global Economic Indicators (DGEI)—whose sources are documented in greater detail by Grossman et al. (2014).[3] This sample provides a reasonable approximation of the global economic forces at play since the country selection we work with represents a large share of world output.

Our main results can be summarized as follows. First, we build on the existing literature on inflation forecasting and, particularly, on the seminal contributions of Atkeson and Ohanian (2001), Stock and Watson (2007), and Faust and Wright (2013), among others, as they explore the performance of theoretically-based models.[4] We argue that the NOEM-BVAR allows us to obtain a 4-variable specification accounting for cross-country spillovers, which is parsimonious as suggested by the best practices in forecasting.

Second, on average, the findings in our paper reveal that the NOEM-BVAR specification produces similar or lower root mean square prediction errors (RMSPEs) than its competitors. Our tests suggest that the NOEM-BVAR specification generally performs as good as the current crop of inflation forecasting models with factor components and even as good or better than standard closed-economy Phillips-curve-based specifications. In a number of cases, the gains in smaller RMSPEs are statistically significant. In particular, the NOEM-BVAR outperforms or shows a predictive ability similar to theoretically-based forecasting models.

Third, we also consider the performance of the NOEM-BVAR with an alternative measure of predictive success. The model produces success ratios—assessing the ability of the forecast to correctly anticipate the direction of change in inflation—that are comparable or higher than those of its competitors. For most countries, our findings suggest that the NOEM-BVAR produces statistically significant improvements in the accuracy of the direction of change forecast for inflation.

Finally, we also provide additional robustness checks comparing the predictions of the NOEM-BVAR against those of a random walk specification, a time-varying parameter model with stochastic volatility based on the closed-economy Phillips curve, and even against dynamic moving averaging. In all these robustness checks, we illustrate that overcoming some aspect or shortcoming of the competing forecasting models that we have considered does not fundamentally alter our findings about the NOEM-BVAR. While our forecasting evaluation is by no means exhaustive, the evidence on the NOEM-BVAR highlights the importance of modeling cross-country spillovers and incorporating them fully into our inflation forecasting models. We view the NOEM-BVAR as a flexible specification that presents us with a plausible benchmark for forecasting inflation allowing for non-negligible cross-country spillovers across a diverse group of countries around the world.

[3] The 18 economies included in our empirical work represent more than 50 percent of world output in the 1980s and even during most of the 1990s according to their PPP-adjusted shares of world GDP total from International Monetary Fund (IMF) data. Their combined share of world output has declined to around 40 percent since 2004, though, as emerging economies' share has grown rapidly. Key global indicators from the Federal Reserve Bank of Dallas' Database of Global Economic Indicators (DGEI) can be publicly accessed at: https://www.dallasfed.org/institute/dgei.

[4] Related contributions include Kishor and Koenig (2016), Medel (2016), and Monache and Petrella (2017).

We also find that the forecasting performance of the NOEM-BVAR is similar to that obtained using the semi-structural approach of Del Negro and Schorfheide (2004) or the structural method based on forecasting directly with the NOEM model. Both approaches incorporate more of the cross-equation restrictions imposed by theory than the NOEM-BVAR. While this does not imply that the NOEM model is the better description of those cross-country spillovers captured by the NOEM-BVAR, it suggests that theory can be used to complement the NOEM-BVAR forecasts for policy analysis. Having the theoretical model as a reference does not generally improve the forecasting performance as it imposes more restrictions on the relationships that characterize the data, but can be helpful to gain insight about what domestic and foreign sources explains the evolution of the inflation forecasts.

In Section 2, we formulate the theoretical case for the NOEM model and discuss its implications for forecasting. We derive the general state-space form solution of the NOEM model and show how such a solution can be represented with a finite-order VAR under general conditions. This VAR structure constitutes the basis of our NOEM-BVAR forecasting specification. In Section 3, we describe the data, formulate a broad range of competing models for inflation forecasting, and outline the forecast evaluation approach. In Section 4 we discuss the main results and robustness checks. Section 5 concludes with some final remarks. Appendix A provides all relevant tables, while Appendix B has all the additional technical details on the derivations of the NOEM model solution.[5]

2 The Workhorse New Open Economy Macro Model

The workhorse NOEM model (Martínez-García and Wynne (2010), Martínez-García (2019)) allows us, first, a better understanding of the role that global factors can play in domestic inflation and the international transmission channels; and, second, to derive a theoretically consistent, finite-order VAR representation of the model's reduced-form solution that can be easily estimated and used to forecast inflation. To be more precise, the reduced-form solution of the workhorse NOEM model can be cast in state-space form and approximated with a finite-order VAR which can then be exploited for inflation forecasting. In this section, we lay the groundwork for our investigation into inflation forecasting by describing the building blocks of the NOEM model.

2.1 The Model

The standard closed-economy New Keynesian model is given by a log-linearized system of three-equations—a closed-economy Phillips curve, a closed-economy investment-savings (IS) curve, and an interest rate-based monetary policy rule—that characterize the dynamics of output, inflation, and the short-term nominal interest rate around the deterministic zero-inflation steady state.[6] Goodfriend and King (1997), Clarida et al. (1999), and Woodford (2003), among others, contributed to developing this framework from explicitly optimizing behavior by households and price-setting firms in the presence of monopolistic competition and sticky prices (nominal rigidities) and from a rule-based description of the practice of central banking.

[5] There is also a not-for-publication Online Appendix which reports more detailed results (Duncan and Martínez-García (2022)).

[6] The assumption of a zero-inflation steady state rules out the existence of a long-run Phillips curve relating inflation to global economic activity. For a detailed discussion on the role of a non-zero steady state inflation rate within related New Keynesian models, the interested reader is referred to Ascari and Sbordone (2014).

Building on the earlier work of Clarida et al. (2002), Martínez-García and Wynne (2010) and Martínez-García (2019) show that the same basic structure of three log-linearized equations that constitutes the basis of the closed-economy New Keynesian framework can be generalized to a two-country setting to describe the dynamics of output, inflation, and the short-term policy rate and the trade linkages between a domestic and a foreign economy. The full details of the two-country New Open-Economy Macro (NOEM) specification and the key structural parameters can be found in Table 1 and Table 2.

Since the building blocks of the NOEM model are otherwise extensively discussed in Martínez-García and Wynne (2010) and Martínez-García (2019), here we put the emphasis instead on understanding the model implications for explaining and predicting inflation. We denote $\widehat{g}_t \equiv \ln G_t - \ln \overline{G}$ the deviation of a variable in logs from its steady-state. Hence, all variables are defined in log-deviations from steady-state. The open-economy Phillips curve can be written for each country as follows:

$$\widehat{\pi}_t = \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}\right) + \Phi \left(\varphi + \gamma\right) \left[\kappa \widehat{x}_t + (1-\kappa) \widehat{x}^*_t + \widehat{v}_t\right], \tag{1}$$

$$\widehat{\pi}^*_t = \beta \mathbb{E}_t \left(\widehat{\pi}^*_{t+1}\right) + \Phi \left(\varphi + \gamma\right) \left[(1-\kappa) \widehat{x}_t + \kappa \widehat{x}^*_t + \widehat{v}^*_t\right], \tag{2}$$

where $\widehat{\pi}_t \equiv \widehat{p}_t - \widehat{p}_{t-1}$ and $\widehat{\pi}^*_t \equiv \widehat{p}^*_t - \widehat{p}^*_{t-1}$ denote Home and Foreign inflation (that is, quarter-over-quarter changes in the consumption price index, CPI), and $\widehat{p}_t$ and $\widehat{p}^*_t$ denote the corresponding Home and Foreign CPI. We define $\widehat{x}_t \equiv \widehat{y}_t - \widehat{\overline{y}}_t$ and $\widehat{x}^*_t \equiv \widehat{y}^*_t - \widehat{\overline{y}}^*_t$ as the Home and Foreign output gaps defined as the deviation of local output ($\widehat{y}_t$ and $\widehat{y}^*_t$, respectively) from local output potential under flexible prices and perfect competition ($\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}^*_t$, respectively).

The composite coefficient $\Phi\left(\varphi+\gamma\right)$ where $\Phi \equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)$ is the common component of the slope of the open-economy Phillips curve (which equals the slope of the Phillips curve in the closed-economy case), $0 < \beta < 1$ is the intertemporal discount factor, $0 < \alpha < 1$ is the Calvo (1983) price stickiness parameter, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution, and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. The differences in slope coefficients for Home and Foreign slack that arise in (1) − (2) are related to the composite coefficient $\kappa \equiv (1-\xi)\left[1 - (\sigma\gamma - 1)\left(\frac{\gamma}{\varphi+\gamma}\right)\left(\frac{(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]$ which is itself a function of the elasticity of intratemporal substitution between Home and Foreign goods (or trade elasticity) $\sigma > 0$ and the share of imported goods $0 \leq \xi < \frac{1}{2}$.

We define the Home and Foreign cost-push shocks shifting the marginal costs for local producers as $\widehat{u}_t$ and $\widehat{u}^*_t$, respectively—these cost-push shocks enter weighted by the import share $0 \leq \xi < \frac{1}{2}$ into the Home Phillips curve as $\widehat{v}_t = (1-\xi)\widehat{u}_t + \xi \widehat{u}^*_t$ and into the Foreign Phillips curve as $\widehat{v}^*_t = \xi \widehat{u}_t + (1-\xi)\widehat{u}^*_t$. The stochastic process for the Home and Foreign cost-push shocks, $\widehat{u}_t$ and $\widehat{u}^*_t$, evolves according to the following bivariate autoregressive process:

$$\begin{pmatrix} \widehat{u}_t \\ \widehat{u}^*_t \end{pmatrix} = \begin{pmatrix} \delta_u & 0 \\ 0 & \delta_u \end{pmatrix} \begin{pmatrix} \widehat{u}_{t-1} \\ \widehat{u}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^u_t \\ \widehat{\varepsilon}^{u*}_t \end{pmatrix}, \tag{3}$$

$$\begin{pmatrix} \widehat{\varepsilon}^u_t \\ \widehat{\varepsilon}^{u*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_u & \rho_{u,u^*}\sigma^2_u \\ \rho_{u,u^*}\sigma^2_u & \sigma^2_u \end{pmatrix}\right). \tag{4}$$

The Home and Foreign cost-push shock innovations are labeled $\widehat{\varepsilon}^u_t$ and $\widehat{\varepsilon}^{u*}_t$, respectively. We assume a common volatility $\sigma^2_u \geq 0$ and a common autoregressive parameter $0 < \delta_u < 1$. We allow the cross-correlation of cost-push innovations between the two countries to be $-1 < \rho_{u,u^*} < 1$.

The open-economy IS equations in $(5)-(6)$ illustrate that the Home and Foreign output gaps, $\widehat{x}_t$ and $\widehat{x}_t^*$, are tied to shifts in consumption demand over time and across countries:

$$\gamma\left(\mathbb{E}_t\left[\widehat{x}_{t+1}\right]-\widehat{x}_t\right)=\Omega\left[\widehat{i}_t-\mathbb{E}_t\left(\widehat{\pi}_{t+1}\right)-\widehat{\overline{r}}_t\right]+(1-\Omega)\left[\widehat{i}_t^*-\mathbb{E}_t\left(\widehat{\pi}_{t+1}^*\right)-\widehat{\overline{r}}_t^*\right], \tag{5}$$

$$\gamma\left(\mathbb{E}_t\left[\widehat{x}_{t+1}^*\right]-\widehat{x}_t^*\right)=(1-\Omega)\left[\widehat{i}_t-\mathbb{E}_t\left(\widehat{\pi}_{t+1}\right)-\widehat{\overline{r}}_t\right]+\Omega\left[\widehat{i}_t^*-\mathbb{E}_t\left(\widehat{\pi}_{t+1}^*\right)-\widehat{\overline{r}}_t^*\right], \tag{6}$$

where $\Omega\equiv(1-\xi)\left(\frac{1-2\xi(1-\sigma\gamma)}{1-2\xi}\right)$. The real interest rates in the Home and Foreign country are defined by the Fisher equation as $\widehat{r}_t\equiv\widehat{i}_t-\mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$ and $\widehat{r}_t^*\equiv\widehat{i}_t^*-\mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]$ respectively, while $\widehat{i}_t$ and $\widehat{i}_t^*$ denote the Home and Foreign short-term nominal interest rates. The natural rates of interest—those that would prevail under flexible prices and perfect competition—are denoted $\widehat{\overline{r}}_t$ for the Home country and $\widehat{\overline{r}}_t^*$ for the Foreign country. As implied by $(5)-(6)$, the wedges between each country's real interest rate and its corresponding natural real rate of interest capture the distortionary effects of nominal rigidities on aggregate demand.[7]

The Home and Foreign Taylor (1993)-type monetary policy rules complete the NOEM model. Monetary policy pursues the goal of domestic stabilization responding solely to changes in local economic conditions as determined by each country's inflation and output gap in equations $(7)-(8)$:

$$\widehat{i}_t=\psi_\pi\widehat{\pi}_t+\psi_x\widehat{x}_t+\widehat{m}_t, \tag{7}$$

$$\widehat{i}_t^*=\psi_\pi\widehat{\pi}_t^*+\psi_x\widehat{x}_t^*+\widehat{m}_t^*, \tag{8}$$

where $\widehat{m}_t$ and $\widehat{m}_t^*$ are the Home and Foreign monetary policy shocks. The policy parameters $\psi_\pi>0$ and $\psi_x\geq 0$ represent the sensitivity of the monetary policy rule to deviations in inflation and the output gap, respectively.[8]

We introduce persistence through the monetary policy shocks themselves—a form of exogenous inertia in the policy rule consistent with the yield-curve evidence documented in Rudebusch (2006).[9] The stochastic process for the Home and Foreign monetary policy shocks, $\widehat{m}_t$ and $\widehat{m}_t^*$, in each country evolves according to the following bivariate autoregressive process:

$$\begin{pmatrix}\widehat{m}_t\\ \widehat{m}_t^*\end{pmatrix}=\begin{pmatrix}\delta_m & 0\\ 0 & \delta_m\end{pmatrix}\begin{pmatrix}\widehat{m}_{t-1}\\ \widehat{m}_{t-1}^*\end{pmatrix}+\begin{pmatrix}\widehat{\varepsilon}_t^m\\ \widehat{\varepsilon}_t^{m*}\end{pmatrix}, \tag{9}$$

$$\begin{pmatrix}\widehat{\varepsilon}_t^m\\ \widehat{\varepsilon}_t^{m*}\end{pmatrix}\sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix},\begin{pmatrix}\sigma_m^2 & \rho_{m,m^*}\sigma_m^2\\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2\end{pmatrix}\right). \tag{10}$$

[7] These real interest rate wedges describe the gap between the actual opportunity cost of consumption today versus consumption tomorrow in the presence of monopolistic competition and sticky prices and the opportunity cost that would prevail under flexible prices and perfect competition.

[8] A discussion of the condition on the policy parameters that ensures the existence and uniqueness of the solution to the NOEM model can be found in Martínez-García (2019).

[9] We adopt this particular representation (often referred as *extrinsic* monetary policy inertia), abstracting from partial adjustment arising from policy smoothing (*intrinsic* policy inertia). We argue that this is consistent with the evidence gathered by Rudebusch (2006) which points to fairly rapid central bank reactions to current information and news and provides little empirical support for the *intrinsic* policy inertia representation. Moreover, the *extrinsic* policy inertia representation also seems most in accord with the conventional policymakers' view that current policy actions are contingent largely on incoming current data.

The Home and Foreign monetary policy shock innovations are labeled $\widehat{\varepsilon}_t^m$ and $\widehat{\varepsilon}_t^{m*}$, respectively. We assume a common volatility $\sigma_m^2 \geq 0$, a common autoregressive parameter $-1 < \delta_m < 1$, and allow the cross-correlation of monetary innovations between the two countries to be $-1 < \rho_{m,m^*} < 1$.

The Home and Foreign natural rates of interest $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$ can be expressed as functions of expected changes in Home and Foreign potential output:

$$\widehat{\overline{r}}_t = \gamma \left[\Theta \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1} \right] - \widehat{\overline{y}}_t \right) + (1-\Theta) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1}^* \right] - \widehat{\overline{y}}_t^* \right) \right], \tag{11}$$

$$\widehat{\overline{r}}_t^* = \gamma \left[(1-\Theta) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1} \right] - \widehat{\overline{y}}_t \right) + \Theta \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1}^* \right] - \widehat{\overline{y}}_t^* \right) \right], \tag{12}$$

where the composite coefficient $\Theta \equiv (1-\xi) \left(\frac{\sigma\gamma - (\sigma\gamma-1)(1-2\xi)}{\sigma\gamma - (\sigma\gamma-1)(1-2\xi)^2} \right)$ is a function of: (a) the elasticity of intratemporal substitution between Home and Foreign goods (or trade elasticity) $\sigma > 0$ times the inverse of the intertemporal elasticity of substitution $\gamma > 0$, and (b) the import share $0 \leq \xi < \frac{1}{2}$. Home and Foreign potential output, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, can in turn be expressed as follows:

$$\widehat{\overline{y}}_t = \left(\frac{1+\varphi}{\gamma+\varphi} \right) \left[\Lambda \widehat{a}_t + (1-\Lambda) \widehat{a}_t^* \right], \tag{13}$$

$$\widehat{\overline{y}}_t^* = \left(\frac{1+\varphi}{\gamma+\varphi} \right) \left[(1-\Lambda) \widehat{a}_t + \Lambda \widehat{a}_t^* \right], \tag{14}$$

where the composite coefficient $\Lambda \equiv 1 + \frac{1}{2} \left(\frac{\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))} \right)$ depends on a number of deep structural parameters: the elasticity of intratemporal substitution between Home and Foreign goods (or trade elasticity) $\sigma > 0$, the inverse of the intertemporal elasticity of substitution $\gamma > 0$, the import share $0 \leq \xi < \frac{1}{2}$, and the inverse of the Frisch elasticity of labor supply $\varphi > 0$. Output potential changes in response to realizations of $\widehat{a}_t$ and $\widehat{a}_t^*$ which denote the corresponding Home and Foreign productivity shocks.

The stochastic process for Home and Foreign aggregate productivity, $\widehat{a}_t$ and $\widehat{a}_t^*$, evolves according to the following bivariate autoregressive process:

$$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix} = \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix}, \tag{15}$$

$$\begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix} \right). \tag{16}$$

The Home and Foreign productivity shock innovations are labeled $\widehat{\varepsilon}_t^a$ and $\widehat{\varepsilon}_t^{a*}$, respectively. We assume a common volatility $\sigma_a^2 \geq 0$, a common autoregressive parameter $0 < \delta_a < 1$, and a common spillover parameter $-1 < \delta_{a,a^*} < 1$.[10] We allow the cross-correlation of productivity innovations between the two countries to be $-1 < \rho_{a,a^*} < 1$.

[10] The eigenvalues corresponding to the matrix $\begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix}$ are $\delta_a - \delta_{a,a^*}$ and $\delta_a + \delta_{a,a^*}$. Hence, both eigenvalues lie inside the unit circle ensuring the stationarity of the productivity process whenever $\left|\delta_a + \delta_{a,a^*}\right| < 1$ and $\left|\delta_a - \delta_{a,a^*}\right| < 1$.

2.2 Reduced-Form Solution

The forcing processes of the NOEM model are the cost-push shocks, $\widehat{u}_t$ and $\widehat{u}_t^*$, in (3) – (4), the productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$, in (15) – (16), and the monetary policy shocks, $\widehat{m}_t$ and $\widehat{m}_t^*$, in (9) – (10). We replace the policy rules in (7) – (8) together with the definitions of the natural rates of interest in (11) – (12) and potential output in (13) – (14) into the structural relationships in (1) – (2) and (5) – (6) to express the generalized system of equations that describes the NOEM model as follows:

$$M(\theta)\,\widehat{Z}_t = N(\theta)\,\mathbb{E}_t\left[\widehat{Z}_{t+1}\right] + Q(\theta)\,\widehat{\varepsilon}_t, \tag{17}$$

where

$$\widehat{Z}_t = \left(\widehat{\pi}_t, \widehat{\pi}_t^*, \widehat{y}_t, \widehat{y}_t^*, \widehat{u}_{t-1}, \widehat{u}_{t-1}^*, \widehat{a}_{t-1}, \widehat{a}_{t-1}^*, \widehat{m}_{t-1}, \widehat{m}_{t-1}^*\right)^T, \tag{18}$$

$$\widehat{\varepsilon}_t = \left(\widehat{\varepsilon}_t^u, \widehat{\varepsilon}_t^{u*}, \widehat{\varepsilon}_t^a, \widehat{\varepsilon}_t^{a*}, \widehat{\varepsilon}_t^m, \widehat{\varepsilon}_t^{m*}\right)^T, \tag{19}$$

and $M(\theta)$, $N(\theta)$, and $Q(\theta)$ are conforming matrices whose entries are a function of the structural parameters given by the vector $\theta = (\beta, \gamma, \varphi, \alpha, \sigma, \xi, \psi_\pi, \psi_x, \delta_a, \delta_{a,a^*}, \sigma_a, \rho_{a,a^*}, \delta_u, \sigma_u, \rho_{u,u^*}, \delta_m, \sigma_m, \rho_{m,m^*})$. The mapping between $M(\theta)$, $N(\theta)$, and $Q(\theta)$ and the vector of structural parameters θ is derived explicitly in Appendix B.

For reasonable parameter values, the matrix $M(\theta)$ is invertible and (17) can be re-written as:

$$\widehat{Z}_t = \Gamma(\theta)\,\mathbb{E}_t\left[\widehat{Z}_{t+1}\right] + \Psi(\theta)\,\widehat{\varepsilon}_t, \tag{20}$$

where $\Gamma(\theta) = (M(\theta))^{-1} N(\theta)$ and $\Psi(\theta) = (M(\theta))^{-1} Q(\theta)$. Blanchard and Kahn (1980) provide general conditions under which a unique stable solution exists for (20). In the NOEM model case, we see that a close variant of the Taylor principle arises from the Blanchard and Kahn (1980) determinacy conditions for a wide range of plausible values of the structural and policy parameters.[11] We consider here only values of the parameter space for which uniqueness and existence can be guaranteed and abstract from further discussion of other scenarios where indeterminacy or no-solutions could emerge as an outcome.

We further partition $\widehat{Z}_t$ into two blocks in order to analyze the NOEM model: a block of observed variables $\widehat{Z}_{1t} = (\widehat{\pi}_t, \widehat{\pi}_t^*, \widehat{y}_t, \widehat{y}_t^*)^T$ and a block with the unobserved states $\widehat{Z}_{2t} = \left(\widehat{u}_{t-1}, \widehat{u}_{t-1}^*, \widehat{a}_{t-1}, \widehat{a}_{t-1}^*, \widehat{m}_{t-1}, \widehat{m}_{t-1}^*\right)^T$. Imposing $\lim_{\tau \to +\infty} (\Gamma(\theta))^\tau \mathbb{E}_t\left[\widehat{Z}_{t+\tau}\right] = 0$, we characterize the solution of the system of equations in (20) in state-space form as follows:

$$\widehat{Z}_{2t+1} = A(\theta)\,\widehat{Z}_{2t} + B(\theta)\,\widehat{\varepsilon}_t, \tag{21}$$

$$\widehat{Z}_{1t} = C(\theta)\,\widehat{Z}_{2t} + D(\theta)\,\widehat{\varepsilon}_t, \tag{22}$$

where $A(\theta)$, $B(\theta)$, $C(\theta)$, and $D(\theta)$ are conforming square matrices, and θ is the vector of structural parameters that enter those matrices.[12] Following on the footsteps of Martínez-García (2020), we show in

[11] Martínez-García (2019) provides an analytical derivation of the exact variant of the Taylor principle under which the solution to the NOEM model exists and is unique. The policy parameter ψ_π is key for determinacy, but we note that the lower bound on ψ_π above which the NOEM model attains determinacy depends on the policy parameter ψ_x.

[12] The solution in (21) – (22) shows that inflation and output in both countries, $\widehat{Z}_{1t}$, can be characterized as linear functions of the vector of state variables, $\widehat{Z}_{2t}$, and the vector of structural shock innovations, $\widehat{\varepsilon}_t$. Since the vector of structural shock

Appendix B that the NOEM solution given by $(21)-(22)$ can be represented under rather general conditions with a finite-order VAR specification of the following form:

$$\widehat{Z}_{1t} = \widetilde{A}(\theta)\,\widehat{Z}_{1t-1} + D(\theta)\,\widehat{\varepsilon}_t, \tag{23}$$

where $\widetilde{A}(\theta) = C(\theta)\,A(\theta)\,(C(\theta))^{+}$ and $D(\theta) = C(\theta)\,(A(\theta))^{-1}\,B(\theta)$.[13]

The theoretical mapping in (23) explicitly incorporates the interconnectedness between the domestic and foreign economies implied by the NOEM model. We should note that a richer specification with more complex dynamics for the shock processes driving the economy—which we have described with a VAR(1) structure—would likely imply a dynamic solution that requires a higher-order VAR representation. For that reason, in our empirical implementation we take the simple VAR(1) representation as a starting point but consider specifications of the VAR model with an order higher than one as well.

We should also note that the VAR(1) representation implied by (23) holds under general conditions (as discussed in Appendix B) and that this holds whether the NOEM solution is fundamental or not. The fact that we have more shock innovations (6 in total) than observables (only 4) in the benchmark NOEM model implies that the solution is non-fundamental in the sense that we cannot recover all structural shock innovations from the residuals of the conforming VAR(1) specification. However, although the residuals from a fitted VAR(1) are not structural for our benchmark specification of the NOEM model solution, we argue that such a VAR(1) specification as given in (23) can still be estimated and exploited for forecasting. This is the basis of our preferred empirical forecasting specification in the remainder of the paper.

3 The Forecasting Exercise

3.1 Data

We use end-of-quarter and seasonally-adjusted data for a sample of 18 countries (Australia, Austria, Belgium, Canada, France, Germany, Greece, Italy, Japan, South Korea, Netherlands, Portugal, Spain, Sweden, Switzerland, Taiwan, United Kingdom, and the United States) from the Federal Reserve Bank of Dallas' DGEI dataset over the 1980:Q1-2016:Q4 period. We focus on the quarter-on-quarter inflation rate, π_t, as measured with the headline Consumer Price Index (CPI). One reason to employ headline CPI rather than other price indices is that CPI revisions are relatively small compared to those of, for example, the GDP price deflator (see, e.g., Faust and Wright (2013)). To lighten the notation, we omit the country subscript for each variable used in this section. Thus, for every country in our sample, we define $\pi_t \equiv 400 \cdot (\ln CPI_t - \ln CPI_{t-1})$, for every quarter t. Table 3 reports the data sources and the transformations of variables. Further details on the variables used in each model are included in the next subsection.

For a given country, say i, variables that measure rest-of-the-world aggregates (e.g., π_t^*, y_t^* to be introduced below) are calculated as the arithmetic averages of the rest of the countries in our sample for

innovations is assumed normally distributed, then the Gaussian state-space representation of the solution in $(21)-(22)$ implies that inflation and output are also normally-distributed processes (see Hamilton (1994) for further discussion on the Gaussian state-space framework).

[13] $(C(\theta))^{+}$ denotes the Moore-Penrose left inverse of $C(\theta)$. See Appendix B for further details.

$j = 1, ..., 18$ and $j \neq i$. Whenever there is a missing value in a given period, averages are computed over all the available values.[14]

3.2 Null Model

There is evidence of (strong) comovement in the inflation data across countries. To quantify this, we follow the suggestion of David (1949) to compute a single summary statistic of the degree of comovement across all countries in our sample. If we let $corr_k$ denote the simple pairwise correlation between the series for country i and country j or the series for country i and an aggregate of all countries $-i$, then the proposed summary measure for all K pairwise correlations is given by:

$$corr \equiv \tanh \left(\frac{\sum_{k=1}^{K} T_k \tanh^{-1} (corr_k)}{\sum_{k=1}^{K} T_k} \right), \tag{24}$$

where $\tanh(\cdot)$ is the standard hyperbolic tangent function and T_k is the sample size for the k-th pairwise correlation $corr_k$. With the summary statistic given by (24), we can compute the aggregate correlation between the inflation rates across the 18 countries which comes at 0.61.

We can go a step further and compute the aggregate correlations of inflation across all 18 countries with respect to lagged domestic inflation (π_t), foreign inflation (π_t^*), domestic HP-detrended output (y_t), and foreign HP-detrended output (y_t^*) (see also next subsection for further details). The aggregate correlations indicate that $corr(\pi_t, \pi_{t-1}) = 0.75$, $corr(\pi_t, \pi_{t-1}^*) = 0.30$, $corr(\pi_t, y_{t-1}) = 0.21$, and $corr(\pi_t, y_{t-1}^*) = 0.19$. This evidence is purely descriptive and does not necessarily indicate causality, but it suggests that the comovement between domestic inflation and foreign measures of inflation and economic activity are important. Therefore, such foreign variables can potentially be exploited to improve inflation forecasting. That is, in fact, the rationale for our preferred forecasting specification based on the reduced-form VAR solution to the NOEM model in (23)—as this offers a way to incorporate the international spillovers into a simple forecasting framework.

In our robustness checks, we also incorporate more of the theoretical cross-equation restrictions of the NOEM model that motivated the exercise in the first place. That comes with the advantage that imposing the constraints arising from theory allows us to investigate causal relationships. Therefore, it permits us to use the framework for purposes other than forecasting, such as for policy analysis. In turn, doing so comes often at the expense of limiting the forecasting performance of the model if those cross-equation restrictions that theory brings forth are harder to reconcile with the true data-generating process for inflation. Those are the types of question that we investigate in the remainder of the paper.

The Hybrid NOEM-BVAR. Our null model, which we refer to as the NOEM-BVAR, is based on the VAR specification implied by the reduced-form solution of the NOEM model given in Section 2 (described by equation (23)). Omitting country subscripts, we define $\mathbf{x}_t = (\pi_t, \pi_t^*, y_t, y_t^*)^T$, where π_t is domestic inflation,

[14] We have fully balanced panels for CPI and the industrial production index and strongly balanced panels for the other macroeconomic indicators. There are some missing values at the beginning of period of study in the following series (countries): GDP (Taiwan (1980:Q1-1980:Q4)), M1 (Belgium (1980:Q1-1996:Q3), Netherlands (1980:Q1-1982:Q3), Sweden (1980:Q1-1997:Q4)), M2 (Australia (1980:Q1-1984:Q1), Belgium (1980:Q1-1996:Q3), Sweden (1980:Q1-1997:Q4), Switzerland (1980:Q1-1984:Q3)), and M3 (Belgium (1980:Q1-1996:Q3), Netherlands (1980:Q1-1982:Q3), Taiwan (1980:Q1-1996:Q4)). We do not have any missing values in our commodity prices data.

π_t^* is the rest-of-the-world inflation, y_t is domestic HP-detrended (logged) real GDP, and y_t^* is the rest-of-the-world HP-detrended (logged) real GDP. The VAR model for $\mathbf{x}_t$ can then be written as:

$$M_0: \quad \mathbf{x}_{t+h} = \Phi_0 + \Phi_1 \mathbf{x}_t + \boldsymbol{\epsilon}_{t+h}, \tag{25}$$

where Φ_0 is a column vector of parameters, and Φ_1 is a matrix of auto-regressive coefficients. We estimate the VAR using Bayesian techniques. Following Sims and Zha (1998), the VAR is estimated using Minnesota priors. In this model—as well as the competing BVAR models that use this sort of priors—the hyper-parameters adopted are $\mu_1 = 1$ (AR(1) coefficient dummies), $\lambda_1 = 0.5$ (overall tightness), $\lambda_2 = 1$ (relative cross-variable weight), and $\lambda_3 = 1$ (lag decay).

Some technical considerations on the implementation of the NOEM-BVAR model for forecasting:

- *Mapping data to theory*: We conform with the best practices in mapping data to theory for the estimation of linearized DSGE models adding an intercept in our VAR specification to demean inflation and removing the output trend with a standard two-sided HP-filter (see, e.g., Pfeifer (2018)).[15] Unlike the variables characterized by the NOEM model solution in (23), the observed data is not constructed in deviations from a constant (to account for the steady state) prior to estimation. Instead, we demean HP filtered-output and inflation during the estimation of the intercept-augmented VAR specification shown above ensuring in this way that the variables in theory and the observed data are consistent with each other.

- *Modelling choices*: We aim to balance theory, parsimony, and predictive accuracy using a hybrid approach. We borrow from the theoretical NOEM model investigated in Section 2 the following: (i) the VAR structure (the linearity and vector autoregressive nature of the specification), and (ii) the economically-relevant variables $\mathbf{x}_t = (\pi_t, \pi_t^*, y_t, y_t^*)^T$ that enter into the VAR specification. To the best of our knowledge, this simple four-variable VAR has not been proposed to forecast inflation across countries before. This particular specification is motivated by the theoretical model, but does not impose the cross-equation restrictions that arise from theory. Put differently, the key contribution of our NOEM-BVAR implementation is that our empirical approach explicitly recognizes the importance of the international linkages between domestic and foreign variables (inflation and output) for forecasting implied by theory. From this point onwards, we adopt an agnostic stand about parameter restrictions since our theoretical model remains a stylized description of the dynamics of an open economy in a multi-country setting. Furthermore, for the same reasons we also use the atheoretical normal-flat priors proposed by Sims and Zha (1998) in the robustness checks section.

- *Filtering the data*: We employ the standard two-sided HP filter on the data used in our experiments. The recursive implementation of our forecasting exercise means that for each forecast only the corresponding forecasting sample is HP-filtered. Hence, even without using the alternative one-sided HP

[15] The full NOEM model described in Section 2 is stationary, and evolves as a finite-order VAR process which describes the cyclical behavior of the economy for a vector of endogenous variables $\widehat{Z}_{1t}$ and innovations $\widehat{\varepsilon}_t$ around the deterministic zero-inflation steady state (equation (23)). In other words, the NOEM model describes the output and inflation of the domestic and rest-of-the-world economies in deviations from the output trend and zero-inflation steady state, respectively. Theory is, therefore, agnostic about long-run growth and long-run inflation which must be handled through an additional set or measurement equations (or removed from the data directly) in order to make the model variables comparable with the actual data that we observe. Pfeifer (2018) provides an extensive discussion on specifying observation equations for the estimation of DSGE or related models which we follow here in our implementation of the NOEM-BVAR forecasting benchmark.

filter, there is no information about the future that gets embedded into the forecast at time t. In this way (via the recursive implementation of the forecasting exercise), we are taking into account the well-known observation about the perils of filtering made in Stock and Watson (1999).

- *Aggregation measures*: For every country, say i, the rest-of-the-world aggregates π_t^* and y_t^* are calculated as the simple average of the inflation rates and detrended outputs, respectively, of the rest of the countries in our sample (i.e., for $j = 1, ..., 18$ and $j \neq i$).

3.3 Competing Models

We confront our null model with a wide variety of competitors. Most of these models are suggested by the literature on inflation forecasting. Aside from univariate specifications and frequentist techniques, we consider other elements and methods that have proved to be useful in inflation forecasting, such as factor components (Stock and Watson (2002), Ciccarelli and Mojon (2010)), Phillips-curve-type specifications and commodity price indexes (Stock and Watson (1999)), and Bayesian vector autoregressions (Doan et al. (1984), Litterman (1986)).

We employ the direct method to produce forecasts with most of our models. Ing (2003) finds that when the model is misspecified, the RMSPE of the direct approach is not greater than that of the iterated approach. In theory, iterated forecasts are more efficient if the one-period ahead model is correctly specified, but direct forecasts are more robust to model misspecification. This can be viewed as a sound argument for using the direct method whenever we suspect that the models we are using are misspecified. That said, Marcellino et al. (2006) argue that which approach is better remains a largely empirical matter. We explore the iterated method in one of our subsequent models because Marcellino et al. (2006) find that the iterated method tends to have smaller RMSPE when assessing the out-of-sample forecasting performance of U.S. macroeconomic time series. These authors also find that the relative performance of the iterated forecasts improves with the forecast horizon.

The set of competing forecasting models is the following:

1. **Recursive autoregression, AR(p) model (RAR).**

$$M_1: \quad \pi_t = \phi_0 + \sum_{i=1}^{p} \phi_i \pi_{t-i} + \epsilon_t, \tag{26}$$

   where p is the degree of the polynomial, ϕ_0 is the intercept, and ϕ_i for all $i = 1, ..., p$ the autoregressive coefficients. For every $h > 1$, the h-quarter-ahead forecast is computed by recursively iterating the one-step forecast forward. The objective here is to use the iterated method as an alternative to the direct method employed in the other models.

2. **Direct forecast, AR(p) model (DAR).**

$$M_2: \quad \pi_{t+h} = \phi_0 + \sum_{i=1}^{p} \phi_i \pi_{t-i+1} + \epsilon_{t+h}, \tag{27}$$

   It is straightforward to show that when $h = 1$ ($h > 1$), RAR and DAR provide the same (different) forecasts.

3. **Random Walk (RW-AO).**
$$M_3: \quad \pi_{t+h} = \frac{1}{4}\sum\nolimits_{i=1}^{4} \pi_{t-i+1}, \tag{28}$$
This is a variant of the random walk model along the lines of Atkeson and Ohanian (2001) and Faust and Wright (2013).

4. **Factor-Augmented AR(p) model (FAR).**
$$M_4: \quad \pi_{t+h} = \phi_0 + \sum_{i=1}^{p} \phi_i \pi_{t-i+1} + \sum_{i=1}^{p} \theta_i \widetilde{F}_{t-i+1} + \epsilon_{t+h}, \tag{29}$$
where $\widetilde{F}_t$ denotes an estimated static factor component based on the inflation rates of the countries in the sample except the country's inflation rate to be forecast. Here, ϕ_0 is the intercept, ϕ_i for all $i = 1, ..., p$ the autoregressive coefficient, and θ_i for all $i = 1, ..., p$ the coefficients on the current and lagged factor component.

5. **Bivariate BVAR (BVAR2).**
$$M_5: \quad \mathbf{x}_{t+h} = \Psi_0 + \Psi(L)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+h}, \tag{30}$$
where, in this case, $\mathbf{x}_t = \left(\pi_t, \widetilde{F}_t\right)^T$, Ψ_0 is a column vector of parameters, and $\Psi(L) = \Psi_1 + \Psi_2 L + ... + \Psi_p L^{p-1}$ denotes a 2×2 matrix of lag polynomials. Following Sims and Zha (1998), the VAR is estimated with Bayesian methods using Minnesota priors. The hyper-parameters used (in this and the next BVAR models) are $\mu_1 = 1$ (AR(1) coefficient dummies), $\lambda_1 = 0.5$ (overall tightness), $\lambda_2 = 1$ (relative cross-variable weight), and $\lambda_3 = 1$ (lag decay).

6. **Time-Varying Parameter (TVP) specification.**
$$M_6: \quad \pi_{t+h} = \phi_{0,t} + \phi_{1,t}\pi_t + \epsilon_{t+h}, \tag{31}$$
$$\phi_{0,t+h} = \phi_{0,t} + \nu_{0,t+h},$$
$$\phi_{1,t+h} = \phi_{1,t} + \nu_{1,t+h},$$
where $\phi_{0,t}$ and $\phi_{1,t}$ are random walk coefficients such that and $\nu_{0,t+h}$ and $\nu_{1,t+h}$ are uncorrelated i.i.d. shocks.

7. **Augmented Phillips Curve (APC).**
$$M_7: \quad \pi_{t+h} = \phi_0 + \Phi(L)\mathbf{x}_t + \epsilon_{t+h}, \tag{32}$$
where $\mathbf{x}_t = \left(\pi_t, \Delta IPI_t, \Delta M2_t, \Delta P_t^{Com}\right)^T$, with IPI, $M2$, and P^{Com} denoting the corresponding logarithms of the industrial production index, the monetary aggregate M2, and a commodity price index, respectively.[16] The latter is measured by a simple average of the price indexes of agricultural raw materials, beverages, metals, and crude oil produced by the IMF. In addition, we define $\Phi(L) =$

[16] We follow Stock and Watson (1999) here. They find that forecasts with a Phillips curve based on measures of real aggregate activity (e.g., the industrial production index) outperform those that use unemployment rates.

$(\phi(L), A(L), B(L), C(L))$, with $\phi(L) = \phi_1 + \phi_2 L + ... + \phi_p L^{p-1}$, $A(L) = a_1 + a_2 L + ... + a_p L^{p-1}$, $B(L) = b_1 + b_2 L + ... + b_p L^{p-1}$, and $C(L) = c_1 + c_2 L + ... + c_p L^{p-1}$, as lag polynomials.

8. **Augmented Phillips Curve in first differences (APC-FD).**

$$M_8: \quad \pi_{t+h} - \pi_t = \phi_0 + \Phi(L)\mathbf{x}_t + \epsilon_{t+h}, \tag{33}$$

where, in this case, $\mathbf{x}_t = \left(\Delta\pi_t, \Delta IPI_t, \Delta M2_t, \Delta P_t^{Com}\right)^T$ is a column vector of lag polynomials similar to the ones defined above.

9. **Multivariate BVAR (BVAR4).**

$$M_9: \quad \mathbf{x}_{t+h} = \Psi_0 + \Psi(L)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+h}, \tag{34}$$

where, in this case, $\mathbf{x}_t = \left(\pi_t, \Delta IPI_t, \Delta M2_t, \Delta P_t^{Com}\right)^T$, Ψ_0 is a column-vector of parameters, and $\Psi(L) = \Psi_1 + \Psi_2 L + ... + \Psi_p L^{p-1}$ denotes a 4×4 matrix of lag polynomials. The Bayesian VAR is estimated using Minnesota priors and the same hyper-parameter values as reported above.

10. **Bivariate BVAR with commodity price indexes (BVAR2-COM).**

$$M_{10}: \quad \mathbf{x}_{t+h} = \Psi_0 + \Psi(L)\mathbf{x}_t + \boldsymbol{\epsilon}_{t+h}, \tag{35}$$

where, in this case, $\mathbf{x_t} = \left(\pi_t, \Delta P_t^{Com}\right)^T$, Ψ_0 is a column-vector of parameters, and $\Psi(L)$ is a 2×2 matrix of lag polynomials.

In order to minimize the effect of the number of lags on the forecasting performance of the models, we set $p = 2$ in the baseline exercise for the competing models ($M_1 - M_{10}$) as in Faust and Wright (2013).

3.4 Forecast Comparison

Pseudo out-of-sample forecasts are constructed by estimating recursively each model. The forecast horizons are 1, 2, 4, and 12 quarters, similarly to those used by inflation-targeting central banks. For $h = 1$, for instance, the initial training sample is 1980:Q2-2008:Q3, whereas the last one corresponds to the 1980:Q2-2016:Q3 period. This implies that, for such a horizon, the first value in the forecasting sample is in 2008:Q4 and the last one appears in 2016:Q4. Hence, we are putting the light on the forecasting performance of the NOEM-BVAR and the competing models during the period of the $2007-09$ global financial crisis and its aftermath where inflation for many advanced economies tended to fall below target (as noted in Caldara et al. (2021)).

The prediction error is defined as the difference between actual and predicted values. Based on them, we compute the root mean squared prediction error (RMSPE) for each country, model, and forecast horizon. Then, we report the Theil-U statistic, that is, the ratio of the RMSPE of our NOEM-BVAR relative to the RMSPE of each competitor ($M_1 - M_{10}$). Values less than one imply that the NOEM-BVAR model has a lower RMSPE than does the competing alternative model. To assess the statistical significance of the difference of the Theil's U-statistics from one, we use a simple one-sided Diebold-Mariano-West test and adjust the statistic if the models are nested according to Clark and West (2007). In contrast to most of the previous studies on inflation forecasting with DSGE-related models, we use the adjustment proposed by Harvey et al. (1997) for small samples. The test statistics are constructed using heteroscedasticity and autocorrelation robust (HAC) standard errors. Values larger than 1.282 indicate that the null hypothesis of equal predictive accuracy is rejected at the 10% statistical level.

Contrasting with the literature that uses DSGE-related models to predict inflation, we also assess the directional accuracy of prediction by using the success ratio. This measure captures an estimate of the probability with which the forecast produced by a given model correctly anticipates the direction of change in inflation at a given forecast horizon. Tossing a fair coin on a sufficiently long sample already predicts the direction of change correctly about 50% of the time. Thus, a model needs to attain a success ratio greater than 0.5 to provide an improvement in directional accuracy over pure chance. The statistical significance of the directional accuracy relative to pure chance is assessed based on an implementation of the test of Pesaran and Timmermann (2009).

4 Empirical Findings

4.1 Main Results

The summary statistics of the relative RMSPEs by groups of model (purely statistical specifications and theoretically-based models) and forecast horizon (1, 2, 4, and 12) for our full sample of economies is reported in Table 4. A similar set of statistics for each of the ten models is displayed in Table 5.[17] Summary statistics related to the success ratios to assess the directional accuracy of the forecasts by groups of models are reported in Table 6. The same information disaggregated at the level of each model is provided in Table 7.[18]

Our main conclusions are the following:

- First, on average, the NOEM-BVAR model produces slightly lower median RMSPEs than its competitors (see last column, bottom panel of Table 4). That said, most of the gains in lower relative RMSPEs are not statistically significant. Notably, the gains in smaller RMSPEs appear more often among the very short-run forecasts, when $h = 1$ (the average of medians is about 0.943). Across countries and models, the median Theil's U-statistic favors, on average, the NOEM-BVAR in thirteen out of eighteen countries at the 1-quarter forecast horizon. This statistic—labeled as "$\# < 1$" in the tables—fluctuates between 8 and 11 at other horizons. On the other hand, the (average) number of economies that show a p-value lower than 0.1 for the null of equal predictive accuracy ranges between 2 ($h = 12$) and 5 ($h = 1$) as Table 4 shows. Statistically speaking, the null model's relative performance is more salient in the very short run. One possible reason is that when external spillovers are short-lived, one would expect that inflation models that incorporate such spillovers would tend to do better precisely at short horizons as is our case. We believe also that errors in forecasting inflation in the short-run may be quite important especially when we find ourselves at a turning point. So getting better accuracy in the short-run can be very helpful in order to predict those inflation turning points more accurately.

[17] Tables A1−A4 in the Online Appendix contain detailed information for each economy (Duncan and Martínez-García (2022)). In Table A1 we have nine (rather than ten) different forecasts because the iterated and direct methods are equivalent when $h = 1$. That is, M_1 and M_2 provide the same forecasts and, therefore, their relative RMSPEs are equal.

[18] Tables A5−A8 in the Online Appendix (Duncan and Martínez-García (2022)) report additional information at the country level for the four forecast horizons used in the exercise.

- Second, the NOEM-BVAR produces success ratios generally above the 0.5 threshold (see Table 6). Table 7 show that the differences between the ratios and the 0.5 cut-off value are often statistically significant.[19] The likelihood with which the null model correctly anticipates the direction of change in inflation tends to be comparable or better than that of its competitors. On average, the median success ratio of the null model is 0.657, whereas the average of the competitors' medians is about 0.63. The ratio of the number of statistically significant cases as a share of the total cases (net of the number of undefined cases) is 0.89. That is, 89% of all the relevant success ratios across countries and horizons are statistically above the 0.5-threshold value. Such a percentage is above the average across models (72%; see bottom panel, last column in Table 6)

- Third, regarding the average medians across models at all horizons, Table 5 shows that the NOEM-BVAR specification (slightly) outperforms models like the TVP (M_6), one of the versions of the augmented Phillips curve (M_8), and the BVAR4 model (M_9). The null model shows more similar predictive ability to the rest of models with the exception of the DAR (M_2). However, the evidence of statistically different cases is reduced (see Table 4 and Table 5, bottom panel).

- Fourth, in terms of directional accuracy, the NOEM-BVAR seems to be competitive or has a slight edge against all the models except for the FAR (M_4) and BVAR2 (M_5) as Table 7 (All horizons) suggests. Looking at the number of statistically significant cases as a share of the total cases, the null model is only beaten by the RAR (M_1; see Table 7). Notice that the FAR and BVAR2 models are the ones that include an estimated static factor component based on the inflation rates of the other countries in the sample—these are the models that come closest to the global inflation specifications documented empirically, among others, by Ciccarelli and Mojon (2010), Ferroni and Mojon (2016), and Kabukcuoglu and Martínez-García (2018). As argued by Martínez-García (2019), the open-economy aspects of the theoretical NOEM model imply that the gap between domestic and global inflation is stationary (mean-reverting) which explains the apparent role of global inflation as a pull for domestic inflation and the global inflation's value as a predictor of local inflation across countries and at varying horizons and sample periods.

- Fifth, if we look at the relative RMSPEs across economies, the NOEM-BVAR tends to outperform its competitors especially in Switzerland (25 statistically significant cases across all forecast horizons), Belgium (17 cases), Australia (12 cases), Portugal (12 cases), and Spain (11 cases), among others.[20] Regarding directional accuracy, the NOEM-BVAR's success ratio is, on average, particularly higher in Taiwan (an average of 0.74 across all forecast horizons), Canada (0.73), Japan (0.72), and Spain (0.70) than in other economies.[21]

Overall, our NOEM-BVAR model seems to have a slight relative advantage over the specifications inspired by economic theory. In turn, there is usually at least one purely statistical model—often RAR or DAR—that outperforms it. This suggests that the more parsimonious forecasting models still offer a tough benchmark to beat for more complex models that incorporate economic predictors and more complex structures such as our NOEM-BVAR during the period that we explore covering the aftermath of the $2007-09$ global financial

[19] See also Tables A5−A8 in the Online Appendix (Duncan and Martínez-García (2022)).
[20] See Tables A1−A4 in the Online Appendix (Duncan and Martínez-García (2022)).
[21] See Tables A5−A8 in the Online Appendix (Duncan and Martínez-García (2022)).

crisis. Even so, NOEM-BVAR is shown to be competitive among the alternative models under consideration and in some cases produces statistically significant gains.

4.2 Robustness Checks

We perform a number of robustness checks whose results are not always reported to economize on space, but are available upon request from the authors. Some conclusions from our robustness checks are nonetheless worth mentioning.

4.2.1 The Null Model: Detrending

We also run the forecasting horserace using a deterministic trend (DT) filter with a cubic polynomial and a first-difference (FD) filter for the real GDP series in our null model. The DT filter allows us to capture smooth shifts in the trend components. The FD transformation is the simplest filter and performs well especially in the presence of first-order integrated processes. It is well-known that the FD detrending is used when one seeks to remove a unit root component from a time series. Both filters do not take into account future values and, thus, we do not need to truncate it and lose observations.[22]

Overall, the results indicate that there are no substantial differences between employing an HP filter and those alternative filters. On average, we observe a slight deterioration in the predictive accuracy against the NOEM-BVAR model in terms of both the relative RMSPEs and the number of statistically significant cases when we use the DT filter. The differences are around 1 p.p. of the NOEM-BVAR's RMSPE.[23] The differences are of a smaller order (0.1 p.p.) when we compare the success ratios. That is, the directional accuracy of our model is virtually unchanged. Regarding the FD filter, we note a slight improvement in the number of statistically significant cases and the relative RMSPE in favor of the NOEM-BVAR model (1 p.p. approximately). Similarly, there is a modest increase in the average success ratio of our model. Note that in the case of directional accuracy statistics, the only ones that are affected by the use of alternative filters are those related to the NOEM-BVAR model since the other competing models do not use filtered variables.

4.2.2 The Null Model: Priors

Overall, the use of Sims and Zha (1998) normal-flat priors in both the NOEM-BVAR and the main BVAR specifications (M_5 and M_9) provides a small gain in the predictive ability of our preferred model. The use of such priors might be motivated by the shortcomings of the Minnesota priors, namely the forced posterior independence between equations and the fixed residual variance-covariance matrix, as highlighted by Kadiyala and Karlsson (1997).[24] Across all horizons and countries, we observe a modest reduction in the average median U-Theil ratio in favor of our model. This is related to the forecasting improvement with respect to the purely statistical models.[25] This is consistent with the results in directional accuracy that show an increased success ratio slightly above 67%.[26]

[22] The results from using these filters in our NOEM-BVAR can be seen in the Online Appendix Tables A9−A12 and A13−A16, respectively (Duncan and Martínez-García (2022)).
[23] See Table A9 of the Online Appendix (Duncan and Martínez-García (2022)).
[24] The Online Appendix Tables A17−A20 show the corresponding results (Duncan and Martínez-García (2022)).
[25] See Table A17 of the Online Appendix (Duncan and Martínez-García (2022)).
[26] See Table A19 of the Online Appendix (Duncan and Martínez-García (2022)).

4.2.3 The Null Model: Additional Predictors

The inclusion in the null model of an additional predictor to capture international shocks to commodity prices does not help achieve higher predictive power or directional accuracy. We estimate the NOEM-BVAR as shown in Subsection 3.2 but incorporating the average of the percent changes in the commodity price indices. We might see this as a possible extension of our theoretical model which could be relevant for small-open economies such as many of those in our sample.[27] Perhaps not surprisingly, we find a drastic deterioration of all the statistics at every forecast horizon. We interpret this result as a piece of evidence that backs the formulation of our original NOEM-BVAR framework that does not contain this additional source of inflation fluctuations.

We realize that the workhorse model that is the basis of our strategy may omit some important variables with relevant information content that could be exploited for forecasting purposes. However, adding more variables to our benchmark would imply a departure from the workhorse model. We should also point out that including one more endogenous variable does not necessarily imply that the VAR structure or solution that we got would fundamentally change unless we change the structural relationships of the model. An example of this is that the interest rate is an endogenous variable but it does not show up in the reduced-form VAR we use because it can be expressed as a function of the variables and the shocks that enter into the reduced-form solution indicated in the paper. Including interest rates among the observables, therefore, in general should not affect the forecasting performance of the model much.

Moreover, large models do not tend to produce better forecasts (see, for instance, the so-called KISS principle that Diebold (1998) mentions in his book).[28]

4.2.4 Training Sample

We looked at the results when we shrink the training sample by 20% to increase the number of predictions.[29] In general, our conclusions do not vary strongly. On the one hand, we observe a slight advantage in favor of the competing models if we look at the relative RMSPEs. Across horizons, countries, and models, the average median U-Theil ratio increases from 0.977 to 0.992.[30] This is consistent with a reduction in the average median success ratio from 0.657 to 0.641.[31] On the other hand, however, we note a rise in the number of statistically significant cases (average number of p-values below the significance level), especially when we compare the NOEM-BVAR with the theoretically-based models ($M_7 - M_{10}$). This might be related to the fact that here we are shortening the estimation window by excluding observations from the globalization and

[27] The results are summarized in Table A21 in the Online Appendix (Duncan and Martínez-García (2022)).

[28] In addition, we perform sensitivity analysis along other dimensions. In general, the results in terms of RMSPEs are qualitatively similar with just one lag in the NOEM-BVAR, which is in line with the lag order of the exogenous shocks usually assumed in the DSGE literature. A GDP-weighted average of the inflation rates as a measure of global inflation or rest-of-the-world inflation and a similarly constructed measure of GDP-weighted output for the global output can be used to estimate and forecast the NOEM-BVAR instead of the simple averages that we in our baseline specification. Doing so does not significantly change the main conclusions on the NOEM-BVAR outlined above. Forecasts with unrestricted VARs estimated with frequentist methods do not tend to provide lower RMSPEs than those with BVARs in our sample. Moreover, unrestricted VARs generally do not attain improvement in directional accuracy either. It is worth adding that in the Augmented Phillips Curve models (M_7, M_8), we evaluate other monetary aggregates apart from M2, but data availability for these alternative measures is an issue in some economies. We find that the specification with the M2 money aggregate mostly outperforms those with M1 or M3 in terms of RMSPE. Not surprisingly, our findings on the relative forecasting performance of the NOEM-BVAR are robust to both narrower and broader definitions of the monetary aggregates. These results are available from the authors upon request.

[29] Our findings are presented in Tables A22−A23 in the Online Appendix (Duncan and Martínez-García (2022)).

[30] See Table A22 in the Online Appendix (Duncan and Martínez-García (2022)).

[31] See Table A23 in the Online Appendix (Duncan and Martínez-García (2022)).

the Great Moderation era. Consequently, our model might carry less relevant information and yield more imprecise estimates. In turn, increasing the number of observations for the prediction statistics could reduce their variances, other things equal, and lead to more statistically significant cases.

4.3 Refinements and Other Competing Methods and Models

4.3.1 On Random Walk Models

We report the Faust and Wright (2013) version of the Atkeson and Ohanian (2001) model in M_3 because it usually outperforms the typical random-walk specification without drift in our sample and other studies (see Duncan and Martínez-García (2019)). That is, our findings on the relative forecasting performance of the NOEM-BVAR are not sensitive if we use this alternative version. Table 8 supports this claim. Moreover, in the majority of the cases, the NOEM-BVAR yields lower RMSPEs and most of them are statistically significant.

4.3.2 The Structural NOEM Model

We consider alternative specifications of the NOEM-BVAR model that incorporate more of the cross-equation restrictions inferred from the theoretical NOEM model. To do this, we implement two methods: (i) the semi-structural approach of Del Negro and Schorfheide (2004) that employs the NOEM model to form priors for a BVAR with the same structure as our NOEM-BVAR, and (ii) the structural NOEM model itself (our workhorse linearized DSGE model). Both methods produce iterative forecasts whose performance is reported in Table 8 together with the relative performance of a combination of the forecasts produced by both methods. As can be seen, in many cases there is not much difference in the statistical significance across all these methods—especially at shorter horizons—with the forecasting combination providing some improvement as is often the case.

The results in Table 9 have a two-fold interpretation. First, they suggest that specifications of the NOEM-BVAR that incorporate more of the cross-equation restrictions imposed by theory can still produce competitive forecasts in many cases, especially in the short run, comparable to those with the more agnostic specification that we use as our preferred benchmark. The advantage in these cases is that those specifications more closely tied to the causal relationships implied by theory can be used for forecasting, but also for policy analysis. Second, our agnostic specification of the NOEM-BVAR is more flexible because it is consistent with any theory that can be cast in the form of the null model. The NOEM model is one such theory that allows for cross-country spillovers to be captured by the dynamics described by the null model, but it does not necessarily have to be the correct data generating process. If the true data generating process is different than the NOEM model, then imposing more of the cross-equations restrictions that arise from theory could lead to a deterioration of its forecasting performance. In some cases, that might be what is happening. However, in general, the fact that the different specifications proposed in Table 8 appear to be competitive with the agnostic NOEM-BVAR does not necessarily prove but it is at least consistent with the theory behind the NOEM model. In that regard, the pseudo-out-of-sample forecasting performance presented here can be said to provide some support for the class of open-economy New Keynesian models that we are exploring by means of the workhorse NOEM model.

Note that if we consider our findings related to the structural DSGE model jointly with the main results (recall the fifth finding commented in Subsection 4.1), we can conclude that the NOEM-BVAR and its structural counterpart, the DSGE model, tend to outperform its competitors especially in the short run and in economies such as Australia, Belgium, Canada, and Spain (see Table 9).[32] In these economies, we can be confident that the use of the structural DSGE model to carry out counterfactual analysis of macroeconomic policies and out-of-sample prediction is appropriate and well supported by the empirical evidence collected in this section.

4.3.3 Time-Varying Parameters and Stochastic Volatility Model

Our NOEM-BVAR model is based on a reduced-form representation of the solution to the workhorse open-economy New Keynesian model. This specification explicitly allows for the possibility of endogenous cross-country spillovers arising through trade, and generally produces competitive inflation forecasts. In particular, the NOEM-BVAR model tends to outperform other theoretically-motivated models such as those models based on a (reduced-form) closed-economy version of the Phillips curve relationship.

As is well known, theoretical models pose a number of shortcomings when fitting the data that can also affect their forecasting performance. One of the possible explanations for the NOEM-BVAR forecasting performance relative to that of other model-based frameworks is that foreign factors could be proxying for unmodelled features of the true data-generating process such as structural breaks. One way in which we can investigate whether the relative forecasting performance of the NOEM-BVAR is just the result of comparing it with a parsimonious closed-economy Phillips curve model with constant parameters is if we allow for a more flexible representation of the Phillips curve model to capture time-variation in the parameters and stochastic volatility.

For that, we simply modify the Phillips curve forecasting model in the spirit of Chan et al. (2016). The time-varying parameter stochastic volatility (TVPSV) model that we estimate is then based on the following equations:

$$(\pi_t - \tau_t^{\pi}) = \rho_t^{\pi} \left(\pi_{t-1} - \tau_{t-1}^{\pi}\right) + \lambda_t \left(g_t - \tau_t^{g}\right) + \varepsilon_t^{\pi}, \tag{36}$$

$$(g_t - \tau_t^{g}) = \rho_1^{g} \left(g_{t-1} - \tau_{t-1}^{g}\right) + \rho_2^{g} \left(g_{t-2} - \tau_{t-2}^{g}\right) + \varepsilon_t^{g}, \tag{37}$$

$$\tau_t^{\pi} = \tau_{t-1}^{\pi} + \varepsilon_t^{\tau^{\pi}}, \tag{38}$$

$$\tau_t^{g} = \tau_{t-1}^{g} + \varepsilon_t^{\tau^{g}}, \tag{39}$$

$$\rho_t^{\pi} = \rho_{t-1}^{\pi} + \varepsilon_t^{\rho^{\pi}}, \tag{40}$$

$$\lambda_t = \lambda_{t-1} + \varepsilon_t^{\lambda}, \tag{41}$$

with

$$\varepsilon_t^{\pi} \sim N\left(0, e^{h_t}\right), \tag{42}$$

$$h_t = h_{t-1} + \varepsilon_t^{h}, \tag{43}$$

$$\varepsilon_t^{h} \sim N\left(0, \sigma_h^2\right), \tag{44}$$

[32] See also Tables A1−A9 in the Online Appendix (Duncan and Martínez-García (2022)).

and

$$\varepsilon_t^g \sim N\left(0, \sigma_g^2\right). \tag{45}$$

Equation (36) describes the reduced-form Phillips curve relationship where $\pi_t \equiv 400 \cdot \ln\left(\frac{CPI_t}{CPI_{t-1}}\right)$ is the approximate quarter-over-quarter (annualized) inflation rate and $g_t \equiv \Delta IPI_t = 400 \cdot \ln\left(\frac{IPI_t}{IPI_{t-1}}\right)$ the approximate quarter-over-quarter (annualized) growth rate in the industrial production index. Moreover, ρ_t^π defines the time-varying first-order autocorrelation of inflation, and λ_t is the time-varying (reduced-form) slope of the Phillips curve. The model allows the parameters of the Phillips curve relationship to vary over time as a random walk while it also permits the log-volatility h_t of the residual in that relationship ε_t^π to also behave as a random walk.

The specification presented here also introduces two more time-varying parameters, τ_t^π and τ_t^g, to capture the trend component of inflation and industrial production growth. Furthermore, the model is complemented with equation (37) which describes the dynamics of industrial production growth in deviations from its own trend component as an AR(2) process. In other words, detrended growth based on industrial production data is inferred within the model rather than exogenously added and for its estimation we adopt a flexible representation of the data-generating process given by equation (37).

Finally, the time-varying trend components on inflation and growth are bounded with:

$$\varepsilon_t^{\tau^\pi} \sim TN\left(a_\pi - \tau_{t-1}^\pi, b_\pi - \tau_{t-1}^\pi; 0, \sigma_{\tau^\pi}^2\right), \tag{46}$$

$$\varepsilon_t^{\tau^g} \sim TN\left(a_g - \tau_{t-1}^g, b_g - \tau_{t-1}^g; 0, \sigma_{\tau^g}^2\right), \tag{47}$$

as well as the time-varying parameters of the Phillips curve relationship with:

$$\begin{aligned} \varepsilon_t^{\rho^\pi} &\sim TN\left(-\rho_{t-1}^\pi, 1 - \rho_{t-1}^\pi; 0, \sigma_{\rho^\pi}^2\right), \\ \varepsilon_t^{\lambda} &\sim TN\left(a_\lambda - \lambda_{t-1}, b_\lambda - \lambda_{t-1}; 0, \sigma_\lambda^2\right). \end{aligned} \tag{48}$$

We also impose stationarity on equation (37) by assuming that $\rho_1^g + \rho_2^g < 1$, $\rho_2^g - \rho_1^g < 1$, and $|\rho_2^g| < 1$. We adopt here fairly lax bounds set at $a_\pi = 0$, $b_\pi = 10$, $a_g = -5$, $b_g = 5$, $a_\lambda = -5$, and $b_\lambda \to +\infty$.

As in Chan et al. (2016), the estimation is achieved using their extension of the MCMC sampler developed in Chan et al. (2013). For that, we adopt the following priors for the initial condition of every state equation:

$$\tau_1^\pi \sim TN\left(a_\pi, b_\pi; \tau_0^\pi, \omega_{\tau^\pi}^2\right), \tag{49}$$

$$\tau_1^g \sim TN\left(a_g, b_g; \tau_0^g, \omega_{\tau^g}^2\right), \tag{50}$$

$$\rho_1^\pi \sim TN\left(0, 1; \rho_0^\pi, \omega_{\rho^\pi}^2\right), \tag{51}$$

$$\lambda_1 \sim TN\left(a_\lambda, b_\lambda; \lambda_0, \omega_\lambda^2\right), \tag{52}$$

$$h_1 \sim TN\left(h_0, \omega_h^2\right), \tag{53}$$

where τ_0^π, $\omega_{\tau^\pi}^2$, τ_0^g, τ_{-1}^g, $\omega_{\tau^g}^2$, ρ_0^π, $\omega_{\rho^\pi}^2$, λ_0, ω_λ^2, h_0, and ω_h^2 are known constants. Furthermore, we choose the relatively non-informative values of $\tau_0^\pi = 3$, $\tau_0^g = \tau_{-1}^g = 3$, $h_0 = \rho_0^\pi = \lambda_0 = 0$, $\omega_{\tau^\pi}^2 = \omega_{\tau^g}^2 = \omega_h^2 = 5$ and $\omega_{\rho^\pi}^2 = \omega_\lambda^2 = 1$.

Before we proceed with the results of TVPSV, it is worth pointing out that our specification is similar to M_7. Unlike M_7, TVPSV does not include data from the monetary aggregate M2 and from a commodity price index, but the estimation permits variation in the parameters of the Phillips curve and adds trend inflation and stochastic volatility. However, in most country experiences the slope of the Phillips curve parameter λ_t becomes insignificant over time suggesting that industrial production growth ΔIPI_t tends to lose its predictive power for inflation.

Hence, the results generally suggest that the model in practice reduces to an AR(1) process for inflation with time-varying inertia and stochastic volatility (similar to M_6 but with stochastic volatility and trend inflation). The fact that domestic variables play such a limited role may, in turn, explain why a more complex specification like that of the NOEM-BVAR—even though its parameters are not time-varying—can do well in forecasting inflation by incorporating international spillovers when those have meaningful effects on domestic inflation that are not being fully captured by measures of domestic economic activity alone.

Table 10 presents our findings. We can see that the average median of the U-Theil statistic is relatively higher than one at short forecast horizons ($h = 1,\ 2$), and becomes smaller than one at longer horizons ($h = 8,\ 12$) favoring the NOEM-BVAR in the latter cases. On average, our NOEM-BVAR model shows its best predictive performance at the one-year-ahead forecasts ($h = 4$) with a median of 0.87 and relative RMSPEs below one in twelve economies. The largest number of statistically significant cases is reached when $h = 8$ as well. If we look at each economy individually, we observe significant gains particularly in Japan, Taiwan, and Switzerland.

4.3.4 Dynamic Model Averaging

In a forecasting exercise with many competing models like ours, it can be argued that a combination of all or a subset of predictors can provide a useful method to forecast inflation. Moreover, it is possible to think of a setup characterized by different sets of predictors inspired on economic theory with varying parameters that can capture instabilities over time. One alternative to deal with this general case is dynamic model averaging (DMA) which was originally proposed by Raftery et al. (2010) in an industrial application and later adapted for macroeconomic forecasting by Koop and Korobilis (2012). The idea of DMA is to capture both model uncertainty and parameter uncertainty in a relatively parsimonious way. This is an adequate alternative in our case because, as we have seen in previous sections, we have several possible exogenous variables (including lags of the inflation rate) and parameter instabilities are also common place since the 1980s.[33]

In this section, we follow the state-space specifications and notation proposed by Koop and Korobilis (2012) and Catania and Nonejad (2018). Consider a set of K models that can have a vector of potentially different predictors $x_t^{(k)}$ with $k = 1, 2, ..., K$. Then, we can represent the k-th dynamic linear model as:

$$\pi_t = x_t^{(k)}\theta_t^{(k)} + \epsilon_t^{(k)}, \qquad \epsilon_t^{(k)} \sim N\left(0, V_t^{(k)}\right), \tag{54}$$

$$\theta_t^{(k)} = \theta_{t-1}^{(k)} + \eta_t^{(k)}, \qquad \eta_t^{(k)} \sim N\left(0, W_t^{(k)}\right), \tag{55}$$

where $\theta_t^{(k)}$ is the vector of (stochastic) parameters for the model with k regressors. $V_t^{(k)}$ and $W_t^{(k)}$ are the corresponding conditional variances of the error terms $\epsilon_t^{(k)}$ and $\eta_t^{(k)}$, and these satisfy that $E[\epsilon_t^{(k)}\eta_t^{(k)}] = 0$. Note that all these elements of the system are allowed to vary over time.

[33]For additional technical details and the multiple applications of DMA in macroeconomics forecasting, see also Catania and Nonejad (2018) and Nonejad (2021).

The label of *dynamic model averaging* is justified because the state-space system above allows different models holding at each period, and those models are averaged using conditional probabilities, and the marginal effects of the predictors can change over time (Koop and Korobilis (2012)). For a computationally efficient and feasible estimation, Raftery et al. (2010) suggest an approximation within a Bayesian approach. Leaving some technical details aside, this approximation entails three hyper-parameters, $0 < \delta \leq 1$, $0 < \beta \leq 1$, and $0 < \alpha \leq 1$ that are known as forgetting factors. The first two govern the motion of $V_t^{(k)}$ and $W_t^{(k)}$ over time. For example, if $\delta = 1$, then $W_t^{(k)} = 0$ and, thus, $\theta_t^{(k)} = \theta_{t-1}^{(k)}$. That is, there is no time variation in the coefficients and we go back to a standard model with constant slopes. On the other hand, if δ is close to zero, then we induce (extremely) large time variation in the vector of slopes. Similarly, when $\beta = 1$, then $V_t^{(k)} = V^{(k)}$ and we recover the constant-variance model. Values of this parameter close to zero imply (extremely) large observational volatility. In turn, the parameter δ generates time variation in the full model set (Catania and Nonejad (2018); Nonejad (2021)). Values of this parameter close to zero induce (extremely) fast switches among models. Thus, practitioners tend to pick values close to one for all those parameters. Based on simulations, Catania and Nonejad (2018) suggest fixing δ close to 1 if the practitioner chooses $\beta < 1$.

For the purpose of our study, we assume $\beta = 0.96$ and $\alpha = 0.99$, while we fix a grid such that $\delta \in \{0.90, 0.91, ..., 1\}$ as in Catania and Nonejad (2018). For the vector of predictors we include an intercept, the first two lags of the inflation rate, the percent change in real GDP, the percent change in the industrial production index, the percent change in the monetary aggregate M2, the average percent change in the commodity price indices, and the average of the inflation rates in the rest of countries. This is a richer set of predictors than the subset of covariantes used with M_7. Given that we have $K = 7$ predictors, we consider a total of $2^7 = 128$ possible combinations at each point in time for a given value of δ, while contemporaneously allowing that their coefficients evolve over time. If we consider the values of the grid of δ, we have $2^7(11) = 1408$ possible model combinations including averaging over δ.

Table 11 displays the main outputs of this exercise. The average median suggests an important gain in the DMA's RMSPE especially when $h = 1$ and $h = 2$. The extreme flexibility of DMA to find the best predictor(s) in a parsimonious model plays a key role here. That said, the NOEM-BVAR tends to predict better, on average, at four-quarter-ahead forecasts ($h = 4$). When $h = 1$, our NOEM-BVAR model outperforms DMA in six economies and the gains are statistically significant in only three of them. Similarly, when $h = 2$, the NOEM-BVAR produces lower RMSPEs in five economies and the gains are statistically significant in only four of them. The average gains tend to vanish for both models at the longest forecast horizon ($h = 12$). Interestingly, across countries, the NOEM-BVAR leads to better predictive accuracy in Japan, Taiwan, and Switzerland, usually at one-, two- and four-quarter ahead horizons, with statistically significant gains in most of them.

The attentive reader would realize that the results and conclusions here are similar to those discussed in the previous subsection related to the TVPSV model. If we look at Table 10 and Table 11 we find similar results and patterns. This might be due to the flexibility of DMA and its ability to embed a system with time-varying parameters and variances like the TVPSV, especially during those periods when the main predictor in the latter (the growth of the industrial production index) has low forecasting power. Finally, note that the NOEM-BVAR outperforms DMA in forecasting the U.S. inflation rate at every forecast horizon with a statistically significant gain when $h = 2$.[34]

4.4 A Brief Comparison with Other DSGE Model-Based Studies

At this point, it could be useful to make a brief comparison between our study and other DSGE model-based forecasting studies. Table 12 summarizes the most important features of some of the well-known studies and ours. Let us begin highlighting certain important differences that allow us to start filling some voids in the literature. First, our forecasting exercise includes more economies (18) than any other study listed in the table. Even though we include a large economy like the U.S., we cover (seventeen) internationally connected and relatively small open economies that represent about 40% of world's PPP-adjusted GDP. The other articles usually focus on a single large economy, typically the U.S. or the Euro Zone. Second, we forecast CPI inflation instead of that of the GDP price deflator, which is the index commonly used by almost all the other articles. Third, we propose another metric to evaluate our forecasts—the success ratio—aside from the traditional RMSPE. The use of this metric coupled with the measure of CPI could be especially helpful for policymakers, such as central bankers, since they tend to target CPI inflation and might be also interested in the directional accuracy aside from standard predictive measures.

Likewise, a couple of other advantages are worth mentioning. Our forecasting exercise seems more challenging in terms of the number of competing models (10 specifications in the main exercise or about 15 if we consider all the robustness checks, extensions, and refinements we investigate) compared with a value that ranges between 1 and 7 (average of 3 approximately) in other DSGE model-based forecasting analyses. Moreover, our exercise might be seen as a more demanding one with respect to the forecast evaluation. Only very few works use the Diebold-Mariano-West test jointly with the Harvey et al. (1997) small-sample correction. We consider that this choice is important to perform a more rigorous inference.

5 Concluding Remarks

The workhorse open-economy New Keynesian model suggests that a simple VAR specification with domestic and rest-of-the-world inflation and output can be used to approximate the complex international linkages that influence local inflation dynamics and can also improve inflation forecasting performance. Our empirical findings, based on a varied cross-section of country experiences, show that a parsimonious VAR forecasting model of inflation that exploits those cross-country linkages as motivated by theory and estimated with Bayesian techniques—our preferred NOEM-BVAR specification—tends to perform as well as many other more conventional forecasting models of inflation.

Even though we find that certain purely statistical models can be very helpful prediction tools, the NOEM-BVAR offers an interesting new benchmark among those forecasting models derived from macroeconomic theory. The advantages of a theoretically-based and competitive forecasting model should not be undervalued. Theory can be used to gain insight on the inflation dynamics and forecasts and can help us

[34] We also computed RMSPEs for particular cases of DMA like Bayesian model averaging (DMA with $\alpha = \beta = \delta = 1$) and dynamic model selection. In the latter, the algorithm picks one, perhaps different, forecasting specification at each quarter (Koop and Korobilis (2012)). On average, none of these alternatives provides lower RMSPEs than does DMA. The results are available from the authors upon request.

evaluate counterfactual policies. Furthermore, with the aid of theory we can also recognize the role of openness and, in doing so, avoid the wrong inferences and policies that could come into play if we misunderstand the determinants of domestic inflation. For example, Martínez-García and Wynne (2014) warn us about the possibility of adopting a closed-economy specification which could lead to erroneous inferences about the effects of monetary policy. Martínez-García (2015) notes that ignoring the open-economy dimension can lead us to confound shocks that originate domestically with shocks that originate abroad which, in turn, makes the sources of business cycles murkier for policy analysis.

This paper suggests that understanding the global drivers of inflation and how global developments get incorporated into local inflation is important for accurately forecasting inflation and for the appropriate conduct of monetary policy as well. However, needless to say, more work still is needed to investigate further and more deeply the significance of modeling the international linkages highlighted in this paper for inflation forecasting. For that purpose, two notable avenues of future reserach stand out to us:

First, our preferred NOEM-BVAR is essentially equivalent to a global Bayesian VAR (GBVAR) in the spirit of Pesaran et al. (2004), Crespo Cuaresma et al. (2016), and Feldkircher and Huber (2016) when the economy is composed of just two countries and the foreign economy is an aggregate of the rest of the world. The difference in practice is that the GBVAR models, to the extent that they describe all trading partners individually, are better suited to account for third country effects—substitution across countries—in the international transmission of shocks while the NOEM-BVAR relies on direct linkages with the trading partners through aggregates. The advantage of the NOEM-BVAR approach is that it requires the estimation of fewer parameters and is more parsimonious, with a modest loss of generality so long as those third-country effects are of second-order importance. We leave for future research a more thorough exploration of third-country effects using the GBVAR approach for forecasting.

Finally, our robustness checks suggest that the use of methods that capture parameter uncertainty and model uncertainty like dynamic model averaging (DMA) could be useful to forecast inflation for certain economies and forecast horizons. On the one hand, the flexibility of DMA to find the best predictor(s) and capture parameter instabilities in a parsimonious model plays a key role here. On the other hand, there is a new vintage of New Keynesian models with non-zero steady-state inflation rates and endogenous Phillips-Curve parameters, such as those proposed by Ascari and Sbordone (2014), that might be extended to an open-economy setting with international linkages. We believe that a combination of dynamic model averaging and such theoretical extensions could result in another very interesting avenue for the future research in inflation forecasting.

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Appendix

A Tables

Table 1 - New Open-Economy Macro (NOEM) Model: Core Equations	
	Home Country
NKPC	$\widehat{\pi}_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}\right) + \Phi \left(\varphi + \gamma\right) \left[\kappa \widehat{x}_t + \left(1-\kappa\right) \widehat{x}^*_t + \widehat{v}_t\right]$ $\widehat{v}_t = \left(1-\xi\right) \widehat{u}_t + \xi \widehat{u}^*_t$
Dynamic IS	$\gamma \left(\mathbb{E}_t \left[\widehat{x}_{t+1}\right] - \widehat{x}_t\right) \approx \Omega \left[\widehat{r}_t - \widehat{\overline{r}}_t\right] + \left(1-\Omega\right) \left[\widehat{r}^*_t - \widehat{\overline{r}}^*_t\right]$
Monetary policy	$\widehat{i}_t \approx \psi_\pi \widehat{\pi}_t + \psi_x \widehat{x}_t + \widehat{m}_t$
Fisher equation	$\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t \left[\widehat{\pi}_{t+1}\right]$
Output decomposition	$\widehat{y}_t = \widehat{\overline{y}}_t + \widehat{x}_t$
Natural interest rate	$\widehat{\overline{r}}_t \approx \gamma \left[\Theta \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1}\right] - \widehat{\overline{y}}_t\right) + \left(1-\Theta\right) \left(\mathbb{E}_t \left[\widehat{\overline{y}}^*_{t+1}\right] - \widehat{\overline{y}}^*_t\right)\right]$
Potential output	$\widehat{\overline{y}}_t \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right) \left[\Lambda \widehat{a}_t + \left(1-\Lambda\right) \widehat{a}^*_t\right]$
	Foreign Country
NKPC	$\widehat{\pi}^*_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}^*_{t+1}\right) + \Phi \left(\varphi + \gamma\right) \left[\left(1-\kappa\right) \widehat{x}_t + \kappa \widehat{x}^*_t + \widehat{v}^*_t\right]$ $\widehat{v}^*_t = \xi \widehat{u}_t + \left(1-\xi\right) \widehat{u}^*_t$
Dynamic IS	$\gamma \left(\mathbb{E}_t \left[\widehat{x}^*_{t+1}\right] - \widehat{x}^*_t\right) \approx \left(1-\Omega\right) \left[\widehat{r}_t - \widehat{\overline{r}}_t\right] + \Omega \left[\widehat{r}^*_t - \widehat{\overline{r}}^*_t\right]$
Monetary policy	$\widehat{i}^*_t \approx \psi_\pi \widehat{\pi}^*_t + \psi_x \widehat{x}^*_t + \widehat{m}^*_t$
Fisher equation	$\widehat{r}^*_t \equiv \widehat{i}^*_t - \mathbb{E}_t \left[\widehat{\pi}^*_{t+1}\right]$
Output decomposition	$\widehat{y}^*_t = \widehat{\overline{y}}^*_t + \widehat{x}^*_t$
Natural interest rate	$\widehat{\overline{r}}^*_t \approx \gamma \left[\left(1-\Theta\right) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1}\right] - \widehat{\overline{y}}_t\right) + \Theta \left(\mathbb{E}_t \left[\widehat{\overline{y}}^*_{t+1}\right] - \widehat{\overline{y}}^*_t\right)\right]$
Potential output	$\widehat{\overline{y}}^*_t \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right) \left[\left(1-\Lambda\right) \widehat{a}_t + \Lambda \widehat{a}^*_t\right]$
	Exogenous, Country-Specific Shocks
Productivity shock	$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}^*_t \end{pmatrix} \approx \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_a & \rho_{a,a^*}\sigma^2_a \\ \rho_{a,a^*}\sigma^2_a & \sigma^2_a \end{pmatrix}\right)$
Cost-push shock	$\begin{pmatrix} \widehat{u}_t \\ \widehat{u}^*_t \end{pmatrix} \approx \begin{pmatrix} \delta_u & 0 \\ 0 & \delta_u \end{pmatrix} \begin{pmatrix} \widehat{u}_{t-1} \\ \widehat{u}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^u_t \\ \widehat{\varepsilon}^{u*}_t \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}^u_t \\ \widehat{\varepsilon}^{u*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_u & \rho_{u,u^*}\sigma^2_u \\ \rho_{u,u^*}\sigma^2_u & \sigma^2_u \end{pmatrix}\right)$
Monetary shock	$\begin{pmatrix} \widehat{m}_t \\ \widehat{m}^*_t \end{pmatrix} \approx \begin{pmatrix} \delta_m & 0 \\ 0 & \delta_m \end{pmatrix} \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^m_t \\ \widehat{\varepsilon}^{m*}_t \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}^m_t \\ \widehat{\varepsilon}^{m*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_m & \rho_{m,m^*}\sigma^2_m \\ \rho_{m,m^*}\sigma^2_m & \sigma^2_m \end{pmatrix}\right)$
	Composite Parameters
	$\Phi \equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)$ $\kappa \equiv (1-\xi)\left[1 - (\sigma\gamma - 1)\left(\frac{\gamma}{\varphi+\gamma}\right)\left(\frac{(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]$ $\Theta \equiv (1-\xi)\left[\frac{\sigma\gamma-(\sigma\gamma-1)(1-2\xi)}{\sigma\gamma-(\sigma\gamma-1)(1-2\xi)^2}\right] = (1-\xi)\left[\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right]$ $\Omega \equiv (1-\xi)\left(\frac{1-2\xi(1-\sigma\gamma)}{1-2\xi}\right)$ $\Lambda \equiv 1 + \frac{1}{2}\left[\frac{\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right]$

Table 2 - New Open-Economy Macro (NOEM) Model: Parameters	
Structural parameters	
Non-policy parameters	
$0 < \beta < 1$	Intertemporal discount factor
$\gamma > 0$	Inverse intertemporal elasticity of substitution
$\varphi > 0$	Inverse Frisch elasticity of labor supply
$0 < \alpha < 1$	Calvo (1983) price stickiness parameter
σ	Elasticity of substitution btw. Home and Foreign bundles of varieties
$0 \leq \xi < \frac{1}{2}$	Share of imported goods in the local consumption basket
Policy parameters	
$\psi_\pi > 0$	Policy response to inflation
$\psi_x \geq 0$	Policy response to the output gap
Shock parameters	
$-1 < \delta_a < 1$	Persistence parameter in productivity
$-1 < \delta_{a,a^*} < 1$	Spillover parameter in productivity (technological diffusion)
$\sigma_a \geq 0$	Std. deviation of productivity innovations
$-1 < \rho_{a,a^*} < 1$	Cross-correlation of productivity innovations
$-1 < \delta_u < 1$	Persistence parameter in the cost-push shock
$\sigma_u \geq 0$	Std. deviation of cost-push shock innovations
$-1 < \rho_{u,u^*} < 1$	Cross-correlation of cost-push shock innovations
$-1 < \delta_m < 1$	Persistence parameter in the monetary shock
$\sigma_m \geq 0$	Std. deviation of monetary shock innovations
$-1 < \rho_{m,m^*} < 1$	Cross-correlation of monetary shock innovations

This table reports a list of the 18 structural (policy and non-policy) and shock parameters of the NOEM model that influence its short-run dynamics. The shock process parameters are assumed to fall within the range that would be consistent with the stationarity of the shock processes.

Table 3 - Data Sources for the Different Forecasting Models

Concept	Data sources	Transformation
Headline CPI	National statistical offices and central banks; OECD; Grossman et al. (2014)	Quarter-over-quarter (%)
GDP	National statistical offices and central banks; OECD; Grossman et al. (2014)	Quarter-over-quarter (%)
Industrial production	National statistical offices and central banks; OECD; IMF; Grossman et al. (2014)	Quarter-over-quarter (%)
Money supply (M1, M2, M3)	Central banks; OECD; IMF	Quarter-over-quarter (%)
Commodity price index	IMF	Quarter-over-quarter (%)

This table reports the basic information about the data used in the forecasting exercise. The countries included in our forecasting exercises are: Australia, Austria, Belgium, Canada, France, Germany, Greece, Italy, Japan, Korea, Netherlands Portugal, Spain, Sweden, Switzerland, Taiwan, United Kingdom, and United States. The time series coverage spans the period between the first quarter of 1980 and the fourth quarter of 2016 across all variables and countries, with few exceptions. The monetary aggregates M1, M2, and M3 are, however, generally shorter time series.
The commodity price index is computed as a simple average of the price indexes of agricultural raw materials, beverages, metals and crude oil from the IMF. Country aggregation for the rest of the world is obtained with an arithmetic mean of the country variables. For model M7, we use the PPP-GDP weighted aggregates from Grossman et al. (2014). The data used to compute those weights for aggregation comes from the IMF.

All series are seasonally adjusted.

Table 4 - RMSPE of the NOEM-BVAR Model Relative to Competing Models (Averages of Groups of Models)

	Purely statistical models (Average of M1 - M6)	Theoretically based models (Average of M7-M10)	All models (Averages of M1-M10)
One-quarter ahead			
Mean	0.949	0.949	0.949
Median	0.945	0.940	0.943
#<1	13	14	13
#pv<.1	6	5	5
Two-quarter ahead			
Mean	0.976	0.963	0.971
Median	0.980	0.964	0.973
#<1	10	12	11
#pv<.1	3	4	4
Four-quarter ahead			
Mean	1.025	0.993	1.012
Median	1.024	0.992	1.011
#<1	7	10	8
#pv<.1	3	3	3
Twelve-quarter ahead			
Mean	0.981	0.956	0.971
Median	0.988	0.968	0.980
#<1	9	13	11
#pv<.1	3	1	2
Averages (all horizons)			
Mean	0.983	0.965	0.976
Median	0.984	0.966	0.977
#<1	10	12	11
#pv<.1	4	3	3

Notes: Rows for means and medians report the average/median ratio of root mean squared prediction error (RMSPE) from the NOEM-BVAR model relative to the RMSPE of competing forecasting models calculated over the 18 countries. Values less than one imply that the NOEM-BVAR model has a lower RMSPE than does the competitive benchmark. The row #<1 reports the number of economies that show relative RMSPE lower than 1 for a particular model. The row #pv<.1 reports the number of economies that show a p-value lower than 0.1 for the null of equal predictive accuracy measured by the RMSPEs of the NOEM-BVAR and the alternative model. We use the Diebold-Mariano-West statistic or the adjusted Clark-West statistic when models are nested. See Table 1 for the data sources.

Table 5 - RMSPE of the NOEM-BVAR Model Relative to Competing Models (Summary by Forecasting Model)

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
	RAR	DAR	RW-AO	FAR	BVAR2	TVP	APC	APC-FD	BVAR4	BVAR2-COM
One-quarter ahead										
Mean	0.969	0.969	0.896	0.977	0.982	0.903	0.958	0.894	0.967	0.979
Median	0.955	0.955	0.905	0.979	0.990	0.888	0.950	0.859	0.965	0.986
#<1	13	13	15	11	9	16	14	16	15	11
#pv<.1	4	4	7	4	4	10	2	10	2	4
Two-quarter ahead										
Mean	1.005	1.002	0.970	0.985	0.992	0.904	0.986	0.875	0.987	1.002
Median	1.010	1.012	0.958	0.995	1.003	0.901	0.982	0.898	0.975	1.000
#<1	8	7	13	10	8	12	10	18	12	9
#pv<.1	2	2	2	2	0	12	5	7	1	2
Four-quarter ahead										
Mean	1.052	1.072	1.057	1.043	1.024	0.903	1.049	0.878	1.021	1.025
Median	1.049	1.072	1.036	1.066	1.029	0.891	1.052	0.877	1.015	1.023
#<1	6	4	5	6	6	13	8	17	8	6
#pv<.1	1	1	1	1	1	11	2	8	1	1
Twelve-quarter ahead										
Mean	0.933	1.036	1.027	1.035	0.975	0.883	1.030	0.845	0.962	0.988
Median	0.934	1.045	1.032	1.060	0.975	0.879	1.047	0.876	0.968	0.981
#<1	10	6	7	6	12	14	7	16	15	13
#pv<.1	3	1	0	1	1	13	1	0	3	1
Averages (all horizons)										
Mean	0.990	1.020	0.987	1.010	0.993	0.898	1.006	0.873	0.984	0.998
Median	0.987	1.021	0.983	1.025	0.999	0.890	1.008	0.877	0.981	0.997
#<1	9	8	10	8	9	14	10	17	13	10
#pv<.1	3	2	3	2	2	12	3	6	2	2

Notes: Rows for means and medians report the average/median ratio of root mean squared prediction error (RMSPE) from the NOEM-BVAR model relative to the RMSPE of competing forecasting models calculated over the 18 countries. Values less than one imply that the NOEM-BVAR model has a lower RMSPE than does the competitive benchmark. The row #<1 reports the number of economies that show relative RMSPE lower than 1 for a particular model. The row #pv<.1 reports the number of economies that show a p-value lower than 0.1 for the null of equal predictive accuracy measured by the RMSPEs of the NOEM-BVAR and the alternative model. We use the Diebold-Mariano-West statistic or the adjusted Clark-West statistic when models are nested. See Table 1 for the data sources. RAR and DAR denote AR(2) model using the iterative and direct methods to forecast, RW-AO is the random walk model *á la* Atkeson and Ohanian (2001), FAR is the Factor-Augmented AR(2) model, APC is the Augmented Phillips Curve, APC-FD is an Augmented Phillips Curve in first differences, BVAR2 is the bivariate Bayesian VAR(2), BVAR4 is the 4-variable Bayesian VAR(2), BVAR2-COM is the bivariate Bayesian VAR(2) with commodity price indexes, and TVP is the time-varying parameter specification.

Table 6 - Directional Accuracy: Success Ratios (Averages of Groups of Models)

	NOEM-BVAR	Purely statistical models (Average of M1 - M6)	Theoretically based models (Average of M7-M10)	All models (Averages of M1-M10)
One-quarter ahead				
Mean	0.627	0.611	0.599	0.606
Median	0.620	0.594	0.587	0.591
#>0.5	17	17	15	16
StatSignif/Total	0.80	0.67	0.53	0.61
Two-quarter ahead				
Mean	0.649	0.649	0.619	0.637
Median	0.644	0.654	0.625	0.642
#>0.5	18	17	16	17
StatSignif/Total	0.83	0.81	0.67	0.75
Four-quarter ahead				
Mean	0.651	0.648	0.632	0.642
Median	0.651	0.659	0.637	0.650
#>0.5	18	17	17	17
StatSignif/Total	0.94	0.79	0.71	0.76
Twelve-quarter ahead				
Mean	0.700	0.624	0.664	0.640
Median	0.714	0.624	0.657	0.637
#>0.5	18	16	17	16
StatSignif/Total	1.00	0.74	0.79	0.76
Averages (all horizons)				
Mean	0.657	0.633	0.629	0.631
Median	0.657	0.633	0.626	0.630
#>0.5	18	17	16	17
StatSignif/Total	0.89	0.75	0.67	0.72

Notes: Rows for means and medians report the average and median ratio of success in directional accuracy over the 18 countries. The row #>0.5 reports the number of economies that show a success ratio higher than 0.5 for a particular model. StatSignif/Total represents the ratio of the number of statistically significant cases as a share of the total number of cases net of the number of undefined cases. See Table 1 for the data sources.

Table 7 - Directional Accuracy: Success Ratios (Summary by Forecasting Model)

	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
	NOEM-BVAR	RAR	DAR	RW-AO	FAR	BVAR2	TVP	APC	APC-FD	BVAR4	BVAR2-COM
One-quarter ahead											
Mean	0.627	0.620	0.620	0.620	0.633	0.645	0.530	0.593	0.569	0.603	0.630
Median	0.620	0.609	0.609	0.609	0.609	0.609	0.522	0.598	0.554	0.576	0.620
#>0.5	17	17	17	18	17	18	13	13	13	16	17
StatSignif/Total	0.80	0.80	0.72	0.78	0.78	0.83	0.11	0.50	0.33	0.50	0.78
Two-quarter ahead											
Mean	0.649	0.672	0.660	0.640	0.683	0.685	0.556	0.640	0.563	0.626	0.648
Median	0.644	0.656	0.656	0.644	0.700	0.711	0.556	0.656	0.578	0.633	0.633
#>0.5	18	18	18	17	18	18	15	16	13	17	18
StatSignif/Total	0.83	0.94	0.89	0.89	0.83	0.78	0.50	0.72	0.39	0.78	0.78
Four-quarter ahead											
Mean	0.651	0.650	0.678	0.658	0.677	0.680	0.548	0.659	0.578	0.643	0.649
Median	0.651	0.651	0.686	0.663	0.686	0.709	0.558	0.686	0.570	0.651	0.640
#>0.5	18	17	18	16	18	18	15	16	15	18	18
StatSignif/Total	0.94	0.93	0.94	0.78	0.89	0.94	0.28	0.78	0.44	0.78	0.82
Twelve-quarter ahead											
Mean	0.700	0.565	0.643	0.652	0.638	0.652	0.592	0.659	0.616	0.665	0.717
Median	0.714	0.543	0.657	0.657	0.657	0.629	0.600	0.657	0.614	0.643	0.714
#>0.5	18	13	17	18	15	17	16	17	16	17	18
StatSignif/Total	1.00	1.00	1.00	0.67	0.82	0.87	0.11	0.94	0.35	0.86	1.00
Averages (all horizons)											
Mean	0.657	0.627	0.650	0.642	0.658	0.666	0.556	0.638	0.581	0.634	0.661
Median	0.657	0.615	0.652	0.643	0.663	0.664	0.559	0.649	0.579	0.626	0.652
#>0.5	18	16	18	17	17	18	15	16	14	17	18
StatSignif/Total	0.89	0.92	0.89	0.78	0.83	0.86	0.25	0.74	0.38	0.73	0.84

Notes: Rows for means and medians report the average and median ratio of success in directional accuracy over the 18 countries. The row #>0.5 reports the number of economies that show a success ratio higher than 0.5 for a particular model. StatSignif/Total represents the ratio of the number of statistically significant cases as a share of the total number of cases net of the number of undefined cases. See Table 1 for the data sources. RAR and DAR denote AR(2) model using the iterative and direct methods to forecast, RW-AO is the random walk model á la Atkeson and Ohanian (2001), FAR is the Factor-Augmented AR(2) model, APC is the Augmented Phillips Curve, APC-FD is an Augmented Phillips Curve in first differences, BVAR2 is the bivariate Bayesian VAR(2), BVAR4 is the 4-variable Bayesian VAR(2), BVAR2-COM is the bivariate Bayesian VAR(2) with commodity price indexes, and TVP is the time-varying parameter specification.

Table 8 - RMSPE of the NOEM-BVAR Model Relative to the Naïve Random Walk Model				
	h=1	h=4	h=8	h=12
Australia	**0.798**	**0.852**	**0.874**	**0.892**
Austria	**0.839**	**0.883**	0.892	**0.887**
Belgium	**0.877**	**0.835**	**0.859**	**0.879**
Canada	**0.856**	**0.794**	**0.813**	**0.871**
France	1.026	0.987	**0.873**	**0.806**
Germany	**0.847**	**0.829**	0.920	**0.882**
Greece	0.983	1.000	0.939	1.015
Italy	1.053	0.961	**0.875**	**0.788**
Japan	**0.812**	**0.811**	**0.774**	**0.914**
Korea	0.931	0.990	1.109	**0.887**
Netherlands	**0.821**	**0.801**	**0.880**	**0.908**
Portugal	0.978	0.965	**0.886**	0.899
Spain	0.964	**0.919**	**0.867**	0.919
Sweden	0.916	**0.901**	**0.884**	**0.890**
Switzerland	0.922	**0.848**	**0.805**	**0.889**
Taiwan	**0.774**	**0.710**	**0.770**	0.907
United Kingdom	**0.889**	**0.868**	0.940	**0.882**
United States	**0.832**	**0.743**	**0.794**	**0.898**
Mean	0.895	0.872	0.875	0.890
Median	0.883	0.860	0.875	0.889
#<1	16	17	17	17
#pv<.1	10	13	13	10

Notes: Columns report the ratio of root mean squared prediction error (RMSPE) from the NOEM-BVAR model relative to the RMSPE of naive random walk. Values less than one imply that the NOEM-BVAR model has a lower RMSPE than does the RW. Values in bold indicate that the null hypothesis of equal predictive accuracy is rejected at 10% level. See Table 1 for the data sources.

Table 9 - RMSPE of the NOEM-BVAR Model Relative to the DSGE BVAR, DGSE Model, and a Forecast Combination

	Mean DSGE BVAR				Mean DSGE				Forecast combination			
	h=1	h=4	h=8	h=12	h=1	h=4	h=8	h=12	h=1	h=4	h=8	h=12
Australia	0.980	0.918	0.799	0.652	0.980	0.923	0.821	0.677	0.980	0.921	0.810	0.664
Austria	0.986	1.015	1.163	0.837	0.990	1.019	1.203	0.880	0.988	1.017	1.183	0.858
Belgium	0.987	1.142	1.215	0.991	0.987	1.154	1.239	1.023	0.987	1.149	1.227	1.007
Canada	1.005	0.976	0.908	0.739	1.013	0.958	0.905	0.747	1.009	0.968	0.907	0.743
France	0.978	0.841	0.629	0.543	0.952	0.809	0.618	0.546	0.966	0.825	0.624	0.545
Germany	0.933	0.963	1.000	0.845	0.941	0.940	1.008	0.867	0.938	0.952	1.004	0.856
Greece	0.815	0.463	0.365	0.341	0.777	0.446	0.363	0.344	0.796	0.454	0.364	0.343
Italy	0.835	0.583	0.486	0.415	0.796	0.556	0.480	0.417	0.816	0.569	0.483	0.416
Japan	0.880	0.934	1.005	0.947	0.884	0.938	1.019	0.967	0.882	0.936	1.012	0.957
Korea	0.544	0.758	0.888	0.822	1.021	0.694	0.621	0.549	0.902	1.088	1.150	0.902
Netherlands	0.982	0.990	0.956	0.961	0.988	0.983	0.967	0.984	0.985	0.987	0.961	0.972
Portugal	0.876	0.538	0.496	0.443	0.839	0.511	0.488	0.443	0.858	0.524	0.492	0.443
Spain	0.976	0.771	0.632	0.534	0.958	0.753	0.631	0.539	0.967	0.762	0.632	0.537
Sweden	0.968	0.866	0.780	0.527	0.957	0.841	0.776	0.533	0.963	0.853	0.778	0.530
Switzerland	1.034	0.901	0.697	0.626	1.027	0.885	0.696	0.634	1.032	0.893	0.697	0.630
Taiwan	0.848	0.904	0.931	0.807	0.849	0.909	0.944	0.824	0.849	0.907	0.938	0.815
United Kingdom	1.057	1.195	1.008	0.832	1.063	1.224	1.071	0.890	1.060	1.210	1.039	0.861
United States	0.930	1.013	0.994	0.970	0.939	1.028	1.019	1.007	0.934	1.021	1.007	0.988
Mean	0.923	0.876	0.831	0.713	0.942	0.865	0.826	0.715	0.940	0.891	0.850	0.726
Median	0.972	0.911	0.898	0.773	0.958	0.916	0.863	0.712	0.965	0.929	0.923	0.779
#<1	15	14	13	18	14	14	12	16	15	13	11	17
#>1	3	4	5	0	4	4	6	2	3	5	7	1

Notes: Rows for means and medians report the average/median of the root mean squared prediction error (RMSPE) from the NOEM-BVAR model relative to the RMSPE of the DSGE BVAR, DGSE, or a forecast combination calculated over the 18 countries. Values less than one imply that the NOEM-BVAR model has a lower RMSPE than does the competitive benchmark. The row #<1 (#>1) reports the number of economies that show relative RMSPE lower (higher) than 1 for a particular forecast horizon (*h*).

Table 10 - Relative RMSPE of the NOEM-BVAR Model Relative to the Time-Varying Parameter with Stochastic Volatility Model

	h=1	h=2	h=4	h=12
Australia	0.955	1.024	1.397	1.317
Austria	0.995	1.007	2.211	1.532
Belgium	1.675	0.889	0.914	1.309
Canada	1.288	1.089	1.143	1.155
France	1.390	2.281	1.113	0.586
Germany	0.989	1.058	0.814	1.049
Greece	1.447	1.005	0.792	0.823
Italy	2.053	1.515	1.018	**0.646**
Japan	**0.585**	**0.643**	**0.499**	0.598
Korea	1.493	0.995	**0.501**	0.920
Netherlands	1.465	1.290	**0.784**	0.903
Portugal	1.421	1.292	1.010	0.875
Spain	1.422	1.131	0.885	0.789
Sweden	1.384	1.140	0.858	0.934
Switzerland	0.898	**0.630**	**0.540**	**0.664**
Taiwan	**0.418**	**0.354**	**0.688**	1.147
United Kingdom	1.673	1.352	0.991	1.168
United States	0.699	0.630	0.600	0.946
Mean	1.236	1.074	0.931	0.964
Median	1.387	1.041	0.872	0.927
#<1	7	6	12	11
#pv<.1	2	3	5	2

Notes: Columns report the ratio of root mean squared prediction error (RMSPE) from the NOEM-BVAR model relative to the RMSPE of the time-varying parameter model with stochastic volatility (TVPSV). Values less than one imply that the NOEM-BVAR model has a lower RMSPE than does the TVPSV model. Values in bold indicate that the null hypothesis of equal predictive accuracy is rejected at 10% level. See Table 1 for the data sources.

Table 11 - Relative RMSPE of the NOEM-BVAR Model Relative to Dynamic Model Averaging

	h=1	h=2	h=4	h=12
Australia	**0.912**	1.025	1.446	1.408
Austria	0.975	1.074	2.331	1.555
Belgium	1.621	0.909	1.011	1.361
Canada	1.335	1.159	1.253	1.318
France	1.450	2.344	1.159	0.675
Germany	1.071	1.171	0.850	1.067
Greece	1.478	1.084	1.003	1.187
Italy	1.986	1.664	1.230	0.793
Japan	**0.586**	**0.689**	**0.515**	0.599
Korea	1.477	1.042	**0.548**	1.108
Netherlands	1.429	1.267	**0.795**	0.986
Portugal	1.409	1.366	1.074	1.079
Spain	1.348	1.193	0.896	1.007
Sweden	1.309	1.148	0.898	1.021
Switzerland	0.896	**0.680**	**0.583**	0.804
Taiwan	**0.480**	**0.359**	**0.665**	1.248
United Kingdom	1.549	1.301	1.012	1.409
United States	0.691	**0.586**	0.604	0.949
Mean	1.222	1.114	0.993	1.087
Median	1.341	1.116	0.951	1.073
#<1	6	5	9	6
#pv<.1	3	4	5	0

Notes: Columns report the ratio of root mean squared prediction error (RMSPE) from the NOEM-BVAR model relative to the RMSPE of dynamic model averaging (DMA). Values less than one imply that the NOEM-BVAR model has a lower RMSPE than does the DMA. Values in bold indicate that the null hypothesis of equal predictive accuracy is rejected at 10% level. See Table 1 for the data sources.

Table 12 - A Comparison with Other Studies on Inflation Forecasting with DSGE Models

Study	Price index used	SOE?	No. of competing models	DMW test and HLN correction?	Directional accuracy?	Forecast horizons	Countries or regions
Adolfson et al. (2007)	GDPDef, CDef, IDef	Yes	2	No	No	1-8 Q	Sweden
Christoffel et al. (2011)	GDPDef	No	6	No	No	1-8 Q	Euro Area
Del Negro and Schorfheide (2013)	GDPDef	No	1	No	No	1-8 Q	US
Del Negro et al. (2015)	GDPDef	No	2	No	No	1-20 Q	US
Dib et al. (2008)	GDPDef	Yes	1	Yes	No	1-8 Q	Canada
Edge and Gürkaynak (2010)	GDPDef	No	2	No	No	1-8 Q	US
Edge et al. (2010)	GDPDef	No	5	No	No	1-8 Q	US
Kolasa and Rubaszek (2015)	GDPDef	No	3	No	No	1-16 Q	US
Kolasa et al. (2012)	GDPDef	No	2	Yes	No	0-4 Q	US
Korenok and Swanson (2005)	CPI, GDPDef	No	5	No	No	4, 8, 12 Q	US
Liu et al. (2009)	GDPDef	Yes	3	No	No	1-4 Q	South Africa
Marcellino and Rychalovska (2014)	CPI	Yes	3	No	No	1, 4, 8 Q	Luxembourg
Rubaszek and Skrzypczynski (2008)	GDPDef	No	3	No	No	1-4 Q	US
Schorfheide et al. (2010)	PCE	No	2	Yes	No	1, 2, 4, 12 Q	US
Smets and Wouters (2004)	GDPDef	No	7	No	No	1-4 Q	Euro Area
Wieland and Wolters (2011)	GDPDef	No	6	No	No	1-4 Q	US
Wolters (2011)	GDP/GNPDef	No	5	No	No	0-5 Q	US
This study	CPI	Yes	10 (15)	Yes	Yes	1, 2, 4, 12 Q	17 SOEs & US

Notes: SOE denotes small open economy, GDPDef/CDef/IDef denote GDP/consumption/investment price deflator. PCE denotes the personal consumption expenditures index. DMW refers to the Diebold-Mariano-West test. HLN denotes the Harvey, Leybourne, and Newbold (1997) correction. Our study includes 10 competing models and about 15 if we consider the robustness checks and extensions.

B Characterizing the NOEM Model Solution

Structural Relationships Implied by the NOEM Model. Using the definitions of the Home and Foreign natural rates of interest and the Home and Foreign potential output in (11) − (14) together with the bivariate stochastic process driving the productivity shocks in (15) − (16), we derive the following mapping between the corresponding endogenous variables and the productivity shocks:

$$\begin{pmatrix} \widehat{\overline{y}}_t \\ \widehat{\overline{y}}_t^* \end{pmatrix} = \left(\frac{1+\varphi}{\gamma+\varphi}\right) \begin{pmatrix} \Lambda & (1-\Lambda) \\ (1-\Lambda) & \Lambda \end{pmatrix} \begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix}, \tag{56}$$

$$\begin{pmatrix} \widehat{\overline{r}}_t \\ \widehat{\overline{r}}_t^* \end{pmatrix} = \gamma \begin{pmatrix} \Theta & (1-\Theta) \\ (1-\Theta) & \Theta \end{pmatrix} \begin{pmatrix} \mathbb{E}_t \left(\widehat{\overline{y}}_{t+1} - \widehat{\overline{y}}_t\right) \\ \mathbb{E}_t \left(\widehat{\overline{y}}_{t+1}^* - \widehat{\overline{y}}_t^*\right) \end{pmatrix} \tag{57}$$

$$= \gamma \left(\frac{1+\varphi}{\gamma+\varphi}\right) ((\delta_a - 1) + \delta_{a,a^*}) \begin{pmatrix} F & 1-F \\ 1-F & F \end{pmatrix} \begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix}, \tag{58}$$

$$F \equiv \Theta \left(\frac{\Lambda (\delta_a - 1) + (1-\Lambda)\,\delta_{a,a^*}}{(\delta_a - 1) + \delta_{a,a^*}}\right) + (1-\Theta) \left(\frac{(1-\Lambda)(\delta_a - 1) + \Lambda \delta_{a,a^*}}{(\delta_a - 1) + \delta_{a,a^*}}\right),$$

where

$$\begin{pmatrix} \mathbb{E}_t \left(\widehat{\overline{y}}_{t+1} - \widehat{\overline{y}}_t\right) \\ \mathbb{E}_t \left(\widehat{\overline{y}}_{t+1}^* - \widehat{\overline{y}}_t^*\right) \end{pmatrix} = \left(\frac{1+\varphi}{\gamma+\varphi}\right) \begin{pmatrix} (\Lambda (\delta_a - 1) + (1-\Lambda)\,\delta_{a,a^*}) & ((1-\Lambda)(\delta_a - 1) + \Lambda \delta_{a,a^*}) \\ ((1-\Lambda)(\delta_a - 1) + \Lambda \delta_{a,a^*}) & (\Lambda (\delta_a - 1) + (1-\Lambda)\,\delta_{a,a^*}) \end{pmatrix} \begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix}. \tag{59}$$

The key equations of the NOEM model—that is, equations (1) − (16) in the paper (Table 1)—can be summarized with the following stochastic system of equations:

$$M_{10\times 10}(\theta)\,\widehat{Z}_t = N_{10\times 10}(\theta)\,\mathbb{E}_t\left[\widehat{Z}_{t+1}\right] + Q_{10\times 6}(\theta)\,\widehat{\varepsilon}_t, \tag{60}$$

$$\widehat{\varepsilon}_t \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & \rho_{u,u^*}\sigma_u^2 & 0 & 0 & 0 & 0 \\ \rho_{u,u^*}\sigma_u^2 & \sigma_u^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 & 0 & 0 \\ 0 & 0 & \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ 0 & 0 & 0 & 0 & \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{pmatrix}\right), \tag{61}$$

which describes the dynamics of the vector of variables $\widehat{Z}_t = \left(\widehat{\pi}_t, \widehat{\pi}_t^*, \widehat{y}_t, \widehat{y}_t^*, \widehat{u}_{t-1}, \widehat{u}_{t-1}^*, \widehat{a}_{t-1}, \widehat{a}_{t-1}^*, \widehat{m}_{t-1}, \widehat{m}_{t-1}^*\right)^T$ in relation to the vector of innovations $\widehat{\varepsilon}_t = \left(\widehat{\varepsilon}_t^u, \widehat{\varepsilon}_t^{u*}, \widehat{\varepsilon}_t^a, \widehat{\varepsilon}_t^{a*}, \widehat{\varepsilon}_t^m, \widehat{\varepsilon}_t^{m*}\right)^T$ in the following terms:

$$M_{10\times 10}(\theta)\begin{pmatrix} \widehat{\pi}_t \\ \widehat{\pi}_t^* \\ \widehat{y}_t \\ \widehat{y}_t^* \\ \widehat{u}_{t-1} \\ \widehat{u}_{t-1}^* \\ \widehat{a}_{t-1} \\ \widehat{a}_{t-1}^* \\ \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} = N_{10\times 10}(\theta)\,\mathbb{E}_t\begin{pmatrix} \widehat{\pi}_{t+1} \\ \widehat{\pi}_{t+1}^* \\ \widehat{y}_{t+1} \\ \widehat{y}_{t+1}^* \\ \widehat{u}_t \\ \widehat{u}_t^* \\ \widehat{a}_t \\ \widehat{a}_t^* \\ \widehat{m}_t \\ \widehat{m}_t^* \end{pmatrix} + Q_{10\times 6}(\theta)\begin{pmatrix} \widehat{\varepsilon}_t^u \\ \widehat{\varepsilon}_t^{u*} \\ \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \\ \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix}. \tag{62}$$

We can define the matrices $M_{10\times 10}(\theta)$, $N_{10\times 10}(\theta)$, and $Q_{10\times 6}(\theta)$ of this system as follows:

$$M_{10\times 10}(\theta) = \begin{bmatrix} \mathbf{M}_{4\times 4}^a(\theta) & \mathbf{0}_{4\times 6} \\ \mathbf{0}_{6\times 4} & -\mathbf{M}_{6\times 6}^d(\theta) \end{bmatrix},\ N_{10\times 10}(\theta) = \begin{pmatrix} \mathbf{N}_{4\times 4}^a(\theta) & \mathbf{N}_{4\times 6}^b(\theta) \\ \mathbf{0}_{6\times 4} & -\mathbf{I}_{6\times 6} \end{pmatrix},\ Q_{10\times 6}(\theta) = \begin{pmatrix} \mathbf{0}_{4\times 6} \\ \mathbf{I}_{6\times 6} \end{pmatrix}, \tag{63}$$

and

$$\mathbf{M}_{4\times 4}^a(\theta) \equiv \begin{pmatrix} 1 & 0 & -\Phi(\varphi+\gamma)\kappa & -\Phi(\varphi+\gamma)(1-\kappa) \\ 0 & 1 & -\Phi(\varphi+\gamma)(1-\kappa) & -\Phi(\varphi+\gamma)\kappa \\ -\Omega\psi_\pi & -(1-\Omega)\psi_\pi & -(\Omega\psi_x+\gamma) & -(1-\Omega)\psi_x \\ -(1-\Omega)\psi_\pi & -\Omega\psi_\pi & -(1-\Omega)\psi_x & -(\Omega\psi_x+\gamma) \end{pmatrix}, \tag{64}$$

$$\mathbf{M}_{6\times 6}^d(\theta) \equiv \begin{pmatrix} \delta_u & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_u & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_a & \delta_{a,a^*} & 0 & 0 \\ 0 & 0 & \delta_{a,a^*} & \delta_a & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_m & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_m \end{pmatrix}, \tag{65}$$

$$\mathbf{N}_{4\times 4}^a(\theta) \equiv \begin{pmatrix} \beta & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ -\Omega & -(1-\Omega) & -\gamma & 0 \\ -(1-\Omega) & -\Omega & 0 & -\gamma \end{pmatrix}, \tag{66}$$

$$\mathbf{N}_{4\times 6}^b(\theta) \equiv \begin{pmatrix} \Phi(\varphi+\gamma)(1-\xi) & \Phi(\varphi+\gamma)\xi & -\Phi(1+\varphi)K & -\Phi(1+\varphi)(1-K) & 0 & 0 \\ \Phi(\varphi+\gamma)\xi & \Phi(\varphi+\gamma)(1-\xi) & -\Phi(1+\varphi)(1-K) & -\Phi(1+\varphi)K & 0 & 0 \\ 0 & 0 & -\left(\frac{1+\varphi}{\gamma+\varphi}\right)\psi_x\Sigma & -\left(\frac{1+\varphi}{\gamma+\varphi}\right)\psi_x(1-\Sigma) & \Omega & (1-\Omega) \\ 0 & 0 & -\left(\frac{1+\varphi}{\gamma+\varphi}\right)\psi_x(1-\Sigma) & -\left(\frac{1+\varphi}{\gamma+\varphi}\right)\psi_x\Sigma & (1-\Omega) & \Omega \end{pmatrix}, \tag{67}$$

where $\mathbf{0}$ is a conforming (possibly non-square) matrix filled with zeroes and $\mathbf{I}$ is a square identity matrix with ones on the main diagonal and zeroes elsewhere. We define the corresponding composite parameters for matrices $M_{10\times10}(\theta)$, $N_{10\times10}(\theta)$, and $Q_{10\times6}(\theta)$ as follows:

$$
\begin{aligned}
K &\equiv \kappa\Lambda + (1-\kappa)(1-\Lambda),\\
\Sigma &\equiv \Omega\Lambda + (1-\Omega)(1-\Lambda) - \frac{\gamma}{\psi_x}(2\Lambda-1)(\Omega+\Theta-2\Omega\Theta)\left((\delta_a-1)-\delta_{a,a^*}\right),\\
\Phi &\equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right),\\
\kappa &\equiv (1-\xi)\left(1-(\sigma\gamma-1)\left(\frac{\gamma}{\varphi+\gamma}\right)\left(\frac{(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right),\\
\Theta &\equiv (1-\xi)\left(\frac{\sigma\gamma-(\sigma\gamma-1)(1-2\xi)}{\sigma\gamma-(\sigma\gamma-1)(1-2\xi)^2}\right),\\
\Omega &\equiv (1-\xi)\left(\frac{1-2\xi(1-\sigma\gamma)}{1-2\xi}\right),\\
\Lambda &\equiv 1+\frac{1}{2}\left(\frac{\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right).
\end{aligned}
$$

From here, it follows that if the matrix $M_{10\times10}(\theta)$ is invertible, the solution of the NOEM model in (60) together with the corresponding transversality conditions (that is, $\lim_{\tau\to+\infty}(\Gamma_{10\times10}(\theta))^{\tau}\,\mathbb{E}_t\left[\widehat{Z}_{t+\tau}\right]=0$) can be re-written as:

$$
\widehat{Z}_t = \Gamma_{10\times10}(\theta)\,\mathbb{E}_t\left[\widehat{Z}_{t+1}\right] + \Psi_{10\times6}(\theta)\,\widehat{\varepsilon}_t, \tag{68}
$$

where $\Gamma_{10\times10}(\theta) = (M_{10\times10}(\theta))^{-1} N_{10\times10}(\theta)$ and $\Psi_{10\times6}(\theta) = (M_{10\times10}(\theta))^{-1} Q_{10\times6}(\theta)$. Given the particular form of $M_{10\times10}(\theta)$, whenever $\mathbf{M}^a_{4\times4}(\theta)$ and $\mathbf{M}^d_{6\times6}(\theta)$ are invertible, the inverse matrix $(M_{10\times10}(\theta))^{-1}$ reduces to:

$$
(M_{10\times10}(\theta))^{-1} = \begin{bmatrix} \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} & \mathbf{0}_{4\times6} \\ \mathbf{0}_{6\times4} & -\left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \end{bmatrix}, \tag{69}
$$

$$
\left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \equiv \begin{pmatrix}
\frac{1}{\delta_u} & 0 & 0 & 0 & 0 & 0\\
0 & \frac{1}{\delta_u} & 0 & 0 & 0 & 0\\
0 & 0 & \frac{\delta_a}{\delta_a^2-\delta_{a,a^*}^2} & -\frac{\delta_{a,a^*}}{\delta_a^2-\delta_{a,a^*}^2} & 0 & 0\\
0 & 0 & -\frac{\delta_{a,a^*}}{\delta_a^2-\delta_{a,a^*}^2} & \frac{\delta_a}{\delta_a^2-\delta_{a,a^*}^2} & 0 & 0\\
0 & 0 & 0 & 0 & \frac{1}{\delta_m} & 0\\
0 & 0 & 0 & 0 & 0 & \frac{1}{\delta_m}
\end{pmatrix}. \tag{70}
$$

The corresponding matrices $\Gamma_{10\times10}(\theta)$ and $\Psi_{10\times6}(\theta)$ of the solution in (68) can then be expressed as:

$$\Gamma_{10\times10}(\theta) \equiv (M_{10\times10}(\theta))^{-1} N_{10\times10}(\theta) = \begin{pmatrix} \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} & \mathbf{0}_{4\times6} \\ \mathbf{0}_{6\times4} & -\left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{N}^a_{4\times4}(\theta) & \mathbf{N}^b_{4\times6}(\theta) \\ \mathbf{0}_{6\times4} & -\mathbf{I}_{6\times6} \end{pmatrix}$$
$$= \begin{pmatrix} \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^a_{4\times4}(\theta) & \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^b_{4\times6}(\theta) \\ \mathbf{0}_{6\times4} & \left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \end{pmatrix}, \tag{71}$$
$$\Psi_{10\times6}(\theta) \equiv (M_{10\times10}(\theta))^{-1} Q_{10\times6}(\theta) = \begin{pmatrix} \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} & \mathbf{0}_{4\times6} \\ \mathbf{0}_{6\times4} & -\left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \end{pmatrix} \begin{pmatrix} \mathbf{0}_{4\times6} \\ \mathbf{I}_{6\times6} \end{pmatrix} = \begin{pmatrix} \mathbf{0}_{4\times6} \\ -\left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \end{pmatrix}. \tag{72}$$

The Solution of the NOEM Model in VAR Form. We can further partition $\widehat{Z}_t$ into two blocks defined as $\widehat{Z}_{1t} = (\widehat{\pi}_t, \widehat{\pi}^*_t, \widehat{y}_t, \widehat{y}^*_t)^T$ for the observable variables and $\widehat{Z}_{2t} = \left(\widehat{u}_{t-1}, \widehat{u}^*_{t-1}, \widehat{a}_{t-1}, \widehat{a}^*_{t-1}, \widehat{m}_{t-1}, \widehat{m}^*_{t-1}\right)^T$ for the unobserved states of the NOEM model. Following on the footsteps of Fernández-Villaverde et al. (2007), we note that the state-space representation of the NOEM model solution can be given as:

$$\widehat{Z}_{2t+1} = A_{6\times6}(\theta)\,\widehat{Z}_{2t} + B_{6\times6}(\theta)\,\widehat{\varepsilon}_t, \tag{73}$$
$$\widehat{Z}_{1t} = C_{4\times6}(\theta)\,\widehat{Z}_{2t} + D_{4\times6}(\theta)\,\widehat{\varepsilon}_t. \tag{74}$$

Here, $A_{6\times6}(\theta)$, $B_{6\times6}(\theta)$, $C_{4\times6}(\theta)$, and $D_{4\times6}(\theta)$ are conforming matrices, and $\theta = (\beta, \gamma, \varphi, \alpha, \sigma, \xi, \psi_\pi, \psi_x, \delta_a, \delta_{a,a^*}, \sigma_a, \rho_{a,a^*}, \delta_u,$ is the vector of the structural parameters that enter those matrices.

Accordingly, the system of equations described in (68) can be re-written as:

$$\begin{pmatrix} \widehat{Z}_{1t} \\ \widehat{Z}_{2t} \end{pmatrix} = \begin{pmatrix} \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^a_{4\times4}(\theta) & \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^b_{4\times6}(\theta) \\ \mathbf{0}_{6\times4} & \left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{E}_t\left[\widehat{Z}_{1t+1}\right] \\ \mathbb{E}_t\left[\widehat{Z}_{2t+1}\right] \end{pmatrix} + \begin{pmatrix} \mathbf{0}_{4\times6} \\ -\left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \end{pmatrix} \widehat{\varepsilon}_t, \tag{75}$$

or, given that $\mathbb{E}_t\left[\widehat{Z}_{2t+1}\right] = \widehat{Z}_{2t+1}$, simply as:

$$\widehat{Z}_{1t} = \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^a_{4\times4}(\theta)\,\mathbb{E}_t\left[\widehat{Z}_{1t+1}\right] + \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^b_{4\times6}(\theta)\,\widehat{Z}_{2t+1}, \tag{76}$$
$$\widehat{Z}_{2t} = \left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \widehat{Z}_{2t+1} - \left(\mathbf{M}^d_{6\times6}(\theta)\right)^{-1} \widehat{\varepsilon}_t. \tag{77}$$

The state-space solution of the NOEM model in (76) – (77) can be described in canonical state-space form with equations (73) – (74). A straightforward manipulation of condition (77) implies that the conforming matrices $A_{6\times6}(\theta)$ and $B_{6\times6}(\theta)$ for the state-space solution must be given by:

$$A_{6\times6}(\theta) = \mathbf{M}^d_{6\times6}(\theta),\ B_{6\times6}(\theta) = \mathbf{I}_{6\times6}. \tag{78}$$

Then, replacing the functional form of the solution given in (74) into the system of equations for the NOEM model in (76) implies that:

$$\widehat{Z}_{1t} = \left[\left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^a_{4\times4}(\theta)\,C_{4\times6}(\theta) + \left(\mathbf{M}^a_{4\times4}(\theta)\right)^{-1} \mathbf{N}^b_{4\times6}(\theta)\right] \widehat{Z}_{2t+1}, \tag{79}$$

which, given the form of the matrices $A_{6\times 6}(\theta)$ and $B_{6\times 6}(\theta)$ derived in (78), becomes:

$$\widehat{Z}_{1t} = \left[\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\,C_{4\times 6}(\theta) + \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\right]\mathbf{M}^d_{6\times 6}(\theta)\,\widehat{Z}_{2t} + ... \tag{80}$$

$$\left[\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\,C_{4\times 6}(\theta) + \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\right]\widehat{\varepsilon}_t. \tag{81}$$

By the method of undetermined coefficients, matching (80) and (74), it now follows that the conforming matrices $C_{4\times 6}(\theta)$ and $D_{4\times 6}(\theta)$ must satisfy the following conditions:

$$C_{4\times 6}(\theta) = \left[\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\,C_{4\times 6}(\theta) + \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\right]\mathbf{M}^d_{6\times 6}(\theta), \tag{82}$$

$$D_{4\times 6}(\theta) = \left[\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\,C_{4\times 6}(\theta) + \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\right] = C_{4\times 6}(\theta)\left(\mathbf{M}^d_{6\times 6}(\theta)\right)^{-1}, \tag{83}$$

or, simply,

$$C_{4\times 6}(\theta) - \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\,C_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta) = \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta), \tag{84}$$

$$D_{4\times 6}(\theta) = C_{4\times 6}(\theta)\left(\mathbf{M}^d_{6\times 6}(\theta)\right)^{-1}. \tag{85}$$

The existence and uniqueness of a solution to $C_{4\times 6}(\theta)$ in (84) suffices to ensure the existence and uniqueness of the NOEM model's solution in (73) – (74).

Re-writing equation (84) as:

$$C_{4\times 6}(\theta) = \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\,C_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta) + \left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta), \tag{86}$$

and applying the relevant properties for the vectorization of a matrix it follows that:[35]

$$\begin{aligned} vec\left(C_{4\times 6}(\theta)\right) &= vec\left(\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\,C_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta)\right) + vec\left(\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta)\right) \\ &= \left(\left(\mathbf{M}^d_{6\times 6}(\theta)\right)^{T}\otimes\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\right)vec\left(C_{4\times 6}(\theta)\right) + vec\left(\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta)\right). \end{aligned} \tag{87}$$

Further re-arranging, we can show that the solution for $C_{4\times 6}(\theta)$ becomes:

$$\left[\mathbf{I}_{24\times 24} - \left(\left(\mathbf{M}^d_{6\times 6}(\theta)\right)^{T}\otimes\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\right)\right]vec\left(C_{4\times 6}(\theta)\right) = vec\left(\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta)\right), \tag{88}$$

and

$$vec\left(C_{4\times 6}(\theta)\right) = \left[\mathbf{I}_{24\times 24} - \left(\left(\mathbf{M}^d_{6\times 6}(\theta)\right)^{T}\otimes\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\right)\right]^{-1} vec\left(\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^b_{4\times 6}(\theta)\,\mathbf{M}^d_{6\times 6}(\theta)\right), \tag{89}$$

where $\mathbf{I}_{24\times 24}$ is the conforming identity matrix which in this case must be a 24×24 square matrix. Hence, the existence and uniqueness of the solution for $C_{4\times 6}(\theta)$ requires the invertibility of $\mathbf{M}^a_{4\times 4}(\theta)$ but also the invertibility of $\mathbf{I}_{24\times 24} - \left(\left(\mathbf{M}^d_{6\times 6}(\theta)\right)^{T}\otimes\left(\mathbf{M}^a_{4\times 4}(\theta)\right)^{-1}\mathbf{N}^a_{4\times 4}(\theta)\right)$.

[35] The vectorization is a linear transformation that converts a given matrix into a column-vector. The relevant properties are: (a) $vec\left(Y_{j\times i} + Z_{j\times i}\right) = vec\left(Y_{j\times i}\right) + vec\left(Z_{j\times i}\right)$ for any given conforming matrices of dimension $j\times i$; and (b) $vec\left(X_{k\times l}Y_{l\times m}Z_{m\times n}\right) = \left(Z^T_{n\times m}\otimes X_{k\times l}\right)vec\left(Y_{l\times m}\right)$, where $\otimes$ refers to the Kronecker product, for any given matrices $X_{k\times l}$, $Y_{l\times m}$, and $Z_{m\times n}$ of dimensions $k\times l$, $l\times m$, and $m\times n$, respectively.

Finally, the state-space solution for the NOEM model in $(73)-(74)$ can be expressed as follows: Equation (73) becomes

$$\widehat{Z}_{2t+1} = \mathbf{M}^{d}_{6\times 6}(\theta)\,\widehat{Z}_{2t} + \widehat{\varepsilon}_t, \tag{90}$$

and, replacing out the vector $\widehat{Z}_{2t} = \left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{Z}_{2t+1} - \left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{\varepsilon}_t$ derived from (73) whenever $\mathbf{M}^{d}_{6\times 6}(\theta)$ is invertible into (74), we obtain that:

$$\widehat{Z}_{1t} = C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{Z}_{2t+1} + \left[D_{4\times 6}(\theta) - C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\right]\widehat{\varepsilon}_t = D_{4\times 6}(\theta)\,\widehat{Z}_{2t+1}, \tag{91}$$

where it holds by (85) that $D_{4\times 6}(\theta) = C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}$.

Hence, pre-multiplying both sides of (90) with $D_{4\times 6}(\theta) = C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}$ and replacing out $D_{4\times 6}(\theta)\,\widehat{Z}_{2t+1} = \widehat{Z}_{1t}$ according to (91), it follows that:

$$\begin{aligned}
\widehat{Z}_{1t} &= D_{4\times 6}(\theta)\,\widehat{Z}_{2t+1} = D_{4\times 6}(\theta)\,\mathbf{M}^{d}_{6\times 6}(\theta)\,\widehat{Z}_{2t} + D_{4\times 6}(\theta)\,\widehat{\varepsilon}_t \\
&= C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\mathbf{M}^{d}_{6\times 6}(\theta)\,\widehat{Z}_{2t} + C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{\varepsilon}_t \\
&= C_{4\times 6}(\theta)\,\mathbf{M}^{d}_{6\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{Z}_{2t} + C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{\varepsilon}_t.
\end{aligned} \tag{92}$$

Whenever a Moore-Penrose left inverse of $C_{4\times 6}(\theta)$ exists (i.e., whenever $C_{6\times 4}(\theta)^{+}$ exists such that $C_{6\times 4}(\theta)^{+}\,C_{4\times 6}(\theta) = \mathbf{I}_{6\times 6}$), shifting back one period equation (91) and replacing out appropriately, we obtain:

$$\begin{aligned}
\widehat{Z}_{1t} &= C_{4\times 6}(\theta)\,\mathbf{M}^{d}_{6\times 6}(\theta)\,C_{6\times 4}(\theta)^{+}\,C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{Z}_{2t} + C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{\varepsilon}_t \\
&= C_{4\times 6}(\theta)\,\mathbf{M}^{d}_{6\times 6}(\theta)\,C_{6\times 4}(\theta)^{+}\,\widehat{Z}_{1t-1} + C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}\widehat{\varepsilon}_t.
\end{aligned} \tag{93}$$

In particular, when $C_{4\times 6}(\theta)$ has linearly independent columns (and thus the matrix $\left(C_{6\times 4}(\theta)^{*}\,C_{4\times 6}(\theta)\right)$ is invertible), the Moore-Penrose left inverse matrix $C_{6\times 4}(\theta)^{+}$ can be expressed as:

$$C_{6\times 4}(\theta)^{+} = \left(C_{6\times 4}(\theta)^{*}\,C_{4\times 6}(\theta)\right)^{-1} C_{6\times 4}(\theta)^{*}, \tag{94}$$

where $C_{6\times 4}(\theta)^{*}$ is the Hermitian of $C_{4\times 6}(\theta)$ (the conjugate transpose or Hermitian transpose $C_{6\times 4}(\theta)^{*}$ is obtained from $C_{4\times 6}(\theta)$ by taking the transpose and then taking the complex conjugate of each entry).

Hence, it follows from (93) that the NOEM model solution takes the VAR(1) form indicated in equation (23) in the paper as follows:

$$\widehat{Z}_{1t} = \widetilde{A}_{4\times 4}(\theta)\,\widehat{Z}_{1t-1} + D_{4\times 6}(\theta)\,\widehat{\varepsilon}_t, \tag{95}$$

where $\widetilde{A}_{4\times 4}(\theta) = C_{4\times 6}(\theta)\,\mathbf{M}^{d}_{6\times 6}(\theta)\,C_{6\times 4}(\theta)^{+}$ and $D_{4\times 6}(\theta) = C_{4\times 6}(\theta)\left(\mathbf{M}^{d}_{6\times 6}(\theta)\right)^{-1}$. As a result, the NOEM model solution has a precise finite-order VAR representation whose characterization depends on the invertibility of $\mathbf{M}^{d}_{6\times 6}(\theta)$ and the existence and uniqueness of a Moore-Penrose left-invertible solution for the matrix $C_{4\times 6}(\theta)$ defined by condition (84) above. This is a special case of a more general result discussed by Martínez-García (2020).