Federal Reserve Bank of Dallas

Central Bank Digital Currency: Financial Inclusion vs. Disintermediation

Jeremie Banet and Lucie Lebeau

Working Paper 2218 **September 2022**

Research Department

https://doi.org/10.24149/wp2218

Central Bank Digital Currency: Financial Inclusion vs. Disintermediation*

Jeremie Banet† and Lucie Lebeau‡

September 2022

Abstract

An overlapping-generations model with income heterogeneity is developed to analyze the impact of introducing a Central Bank Digital Currency (CBDC) on financial inclusion, and its potential adverse effect on bank funding. We highlight the role of two design parameters: the fixed cost of CBDC usage and the interest rate it pays, and derive principles for maximum inclusion and for mitigating the inclusion-intermediation trade-off. Agents' choice of money instrument is endogenously driven by income heterogeneity. Pre-CBDC, wealthier agents adopt deposits, while poorer agents adopt cash and remain unbanked. CBDCs with low fixed costs (and low interest rates) are adopted by cash holders and directly increase inclusion. CBDCs with high fixed costs (and high interest rates) are adopted by deposit holders and increase inclusion by raising deposit rates. The former allows for more favorable inclusion-intermediation trade-offs. We calibrate the model to match the US income distribution and aggregate share of unbanked households. A CBDC 50% cheaper (30% more expensive) than bank deposits decreases financial exclusion by 93% (71%) without impacting intermediation. In comparison, making the deposit market perfectly competitive would only decrease exclusion by 45%.

Keywords: Central Bank Digital Currency; Financial Inclusion; Payments; Monetary Policy

JEL Classification: E42; E51; E58; G21

---

* The views expressed in this paper are those of the authors and are not necessarily reflective of the views of the Federal Reserve Bank of Dallas or the Federal Reserve System.

† Jeremie Banet, University of California, Irvine, jbanet@uci.edu.

‡ Lucie Lebeau, Federal Reserve Bank of Dallas, lucie.lebeau@dal.frb.org.

There has been a growing interest in investigating the potential of a Central Bank Digital Currency (CBDC) to support financial inclusion. While the share of unbanked households has decreased steadily in the US over the last decade, it still remains significant, as displayed in the left panel of Figure 1. Beyond the US, Demirguc-Kunt et al. (2022) reports that despite some significant improvements as COVID-19 boosted the adoption of digital financial services, 24% of the global population does not own a bank account (this share rises to 29% in developing countries). In its January 2022 report examining the pros and

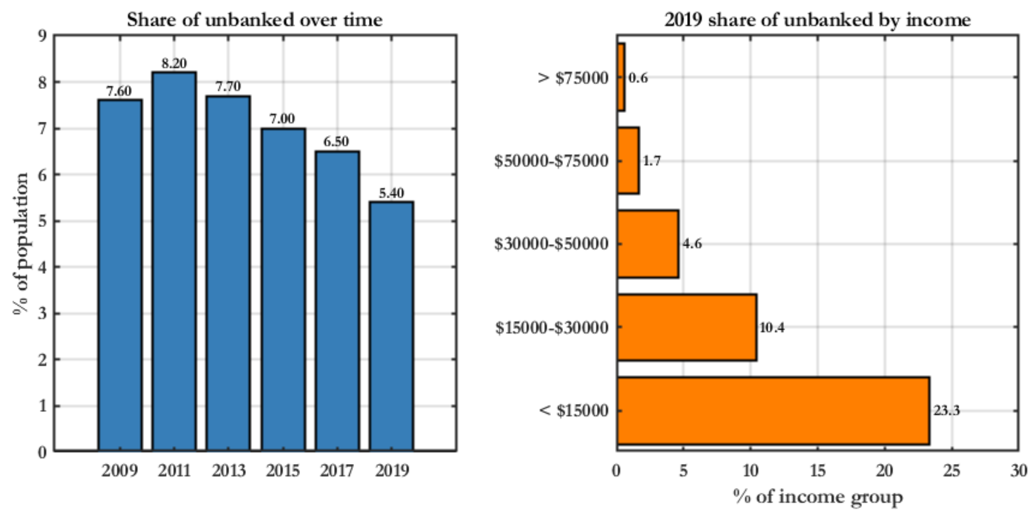


Figure 1: Left panel: Share of unbanked US households over time. Right panel: Share of US unbanked households by yearly income in 2019. Source: Federal Deposit Insurance Corporation (2020).

cons of a potential US CBDC, the Federal Reserve Board highlighted that, among other benefits, a CBDC could "provide households and businesses a convenient, electronic form of central bank money, with the safety and liquidity that would entail," and "expand consumer access to the financial system."[1] The inclusion benefits of a CBDC have also been brought forward on the political stage. In a Senate hearing that took place on June 21, 2021, Senator Elizabeth Warren stated that "Nearly 33 million Americans have been locked out of the traditional banking system[.] CBDC [...] has great promise. Legitimate digital public money could help drive out bogus digital private money, while improving financial inclusion, efficiency, and the safety of our financial system—if that digital public money is

[1]See Board of Governors of the Federal Reserve (2022).

well-designed and efficiently executed." In that same hearing, Neha Narula noted that "a Bank for International Settlements survey of 65 central banks found that 86% are actively engaging in some sort of work on CBDC, for reasons including improving payment efficiency and robustness, facilitating financial inclusion, and maintaining financial stability."

Despite these promises, the introduction of a CBDC carries various concerns. In particular, a potential pitfall raised in the Federal Reserve Board report comes from the destabilizing impact that a CBDC may have on bank funding. Indeed, digital money backed by the central bank would stand as a more direct form of competition to bank deposits than paper money, which could lead to a substitution away from deposits. Quoting the report, "this substitution effect could reduce the aggregate amount of deposits in the banking system, which could in turn increase bank funding expenses, and reduce credit availability[.]"

Our goals in this paper are twofold. First, we set to investigate how a CBDC could impact financial inclusion. In particular, we aim to understand which design principles are paramount for a CBDC to have the maximal effect on giving access to electronic payments to unbanked households, taking into account that unbanked households are over-represented in the lower deciles of the income distribution (see right panel of Figure 1).[2] Second, we aim to assess the potential trade-off between increased financial inclusion thanks to an easy to access CBDC, and an increased risk of disintermediation. Does designing a CBDC to increase financial inclusion necessarily imply disintermediation? Are there ways to mitigate the disintermediation impact without compromising the effects on inclusion?[3] To answer these questions, one of the main contributions of this paper is to produce a model where demand for paper money and deposits, as well as CBDC once introduced, is endogenously driven by income heterogeneity, reproducing the pattern we observe in the right panel of

[2] While there are many other layers to financial inclusion, we exclusively focus on access to digital payment in this paper. As noted in Auer et al. (2022), "digital payments are often the entry point for digital financial services and provide the infrastructure or "rails" through which additional products and use cases can be developed (e.g., credit, insurance, savings products)."

[3] These questions echo many of the questions that the Federal Reserve Board report brings forward as avenues for research, e.g., "Could a CBDC affect financial inclusion? Would the net effect be positive or negative for inclusion? [...] Could a CBDC adversely affect the financial sector? [...] What tools could be considered to mitigate any adverse impact of CBDC on the financial sector? Would some of these tools diminish the potential benefits of a CBDC? [...] Are there additional design principles that should be considered? Are there trade-offs around any of the identified design principles, especially in trying to achieve the potential benefits of a CBDC?" This paper provides elements of answers to many of these questions.

Figure 1.

Following Freeman and Huffman (1991), who draw on Sargent and Wallace (1982), we study an overlapping-generations (OLG) model where workers receive heterogeneous endowments. Combined with the assumption that opening a bank account carries fixed costs, this wealth heterogeneity results in the coexistence of outside and inside money (paper money and bank deposits), and an endogenous sorting of households' choice of money instrument based on their income. Richer households prefer to hold deposits, which carry a higher interest, while poorer households prefer to hold paper money, despite carrying a zero nominal interest rate, for it carries no fixed cost. Banks provide intermediation services. Specifically, they are able to lend the deposits made by households to entrepreneurs, who have access to productive investment projects. We then introduce a CBDC, characterized by two design parameters: its fixed costs and the interest it pays. The two parameters could apply to a central bank cryptocurrency as imagined by Bech and Garratt (2017) or an account-based CBDC as discussed in Auer and Böhme (2020). We study the CBDC's impact on financial inclusion, measured by the mass of workers that carry digital money (deposits or CBDC), and on disintermediation, measured by the quantity of investment financed by banks. We study both economies where the banking sector is perfectly competitive and economies where banks compete in the deposit market a la Cournot (following Chiu et al. (2019)).[4] While the goal of increasing financial inclusion is first taken as given, we develop an extension that rationalizes one of its benefits: we assume that digital means of payment give their holders access to a wider range of goods and services, which they value. Finally, we calibrate the model to the US economy, matching both the US income distribution and the aggregate share of unbanked households. This allows us to quantify the theoretical results obtained in the first part of the paper.

We first find that a CBDC helps inclusion if, conditional on its fixed use cost, its interest rate is high enough. The increase in inclusion can occur through two channels. When the CBDC fixed cost is low, the currency is attractive to agents in the middle of the income

[4]Empirical support for banking concentration and for the non-competitive pricing of deposits go back to Berger and Hannan (1989). More recent evidence is reported, for example, in Drechsler et al. (2021). Choi and Rocheteau (2021) provide theoretical micro-foundations.

distribution, including some agents that uniquely held paper money until then. This channel directly benefits inclusion. When the fixed cost is high, wealthier agents (formerly deposit holders) adopt the CBDC first. Then, its positive impact on inclusion is primarily due to poaching deposits away from banks, which raises the deposit rate, making deposits attractive to the wealthiest paper money holders. We also show that the minimum nominal interest rate required for the CBDC to be competitive against paper money need not be positive if the latter carries an illiquidity premium (e.g., if we assume that some transactions cannot be settled in paper money).

Second, we find that a CBDC that increases inclusion does not necessarily decrease intermediation if the banking sector is not perfectly competitive. Indeed, in line with previous research (e.g., Chiu et al. (2019)), we find that the introduction of a CBDC provides additional competition, forcing banks to decrease their margins, thereby reducing the pass-through between the deposit and the loan rates. The novelty is that we can examine the simultaneous impact on inclusion, and show that when this force is large enough, inclusion and intermediation can increase simultaneously.

Third, an important and novel result is that it is possible to obtain a more favorable intermediation-inclusion trade-off with a CBDC that has a low fixed cost and a low interest rate compared to a CBDC that has a higher fixed cost and a higher interest rate. Because richer agents have larger quantities of real balances, they are more sensitive than poorer agents to interest rates, and less sensitive to fixed costs. Then, a CBDC with a low fixed cost allows to attract a large quantity of poorer workers, formerly paper money holders (which increases inclusion), all the while minimizing the mass of richer workers switching from deposits to CBDCs (which mitigates disintermediation).

Quantitatively, we find that a CBDC with a fixed cost 50% lower than the fixed cost of bank deposits would impact the economy as long as its nominal interest rate is above -3.46%. This lower bound on the nominal rate raises to 0.60% with a CBDC characterized by a fixed cost 30% larger than the bank deposits fixed cost. In addition, with a CBDC 50% cheaper than deposits, it is possible to reduce the fraction of unbanked workers by 93% with zero impact on intermediation. Were the CBDC 30% more expensive than deposits, the fraction of unbanked workers could be reduced by 71% without harming intermediation. With that same design, a 5% fall in investment would allow for an additional 14 percentage points decrease in the percentage of unbanked workers. Alternatively, it would also be possible to achieve a more modest 45% decrease in exclusion all the while increasing investment by 5.2%. Note that this latter outcome is close to what would be achieved by making banking competition perfect. In comparison, tripling the number of banks (from 8 to 24) would reduce the fraction of unbanked workers by 30%.

Related literature There has been a growing theoretical literature studying the impact of CBDCs on bank deposits and investment.[5] The formalization of the intermediation side of our environment shares similarities with three of them in particular: Andolfatto (2021), Keister and Sanches (2022), and Chiu et al. (2019).

Addressing the concern of disintermediation posed by CBDCs, Andolfatto (2021) uses an OLG environment very similar to that developed in the present paper, based on Freeman and Huffman (1991).[6] A main difference is that he investigates the case where deposits are issued by a monopolistic bank. In this framework, the introduction of a CBDC reduces the monopoly rent by forcing the monopolistic bank to make deposits more attractive to match the rate of return on the CBDC. One of the paper's main conclusions, drawing from Klein (1971) and Monti (1972), is that the optimal loan rate is independent of the characteristics of the deposit market. Therefore, a higher deposit rate does not translate into a higher loan rate, and the CBDC does not impact intermediation.[7]

Keister and Sanches (2022) build a model drawing on Lagos and Wright (2005) and the New-Monetarist framework. The banking sector is assumed to be competitive. Their model points to a link between the introduction of a CBDC, disintermediation, and eventually lower

[5] Note that another significant line of research about CBDCs focuses on bank runs and financial stability, building off the seminal work of Diamond and Dybvig (1983). Examples include Chiu et al. (2020), Fernández-Villaverde et al. (2021), Schilling et al. (2020) and Williamson (2022).

[6] Chapter 11 of Champ et al. (2022) provides a simple exposition of the model in Freeman and Huffman (1991).

[7] Chapter 3 and Chapter 8 of Freixas and Rochet (2008) review the conditions for this result to hold. For example, when management costs associated with deposits and loans are not additive, and there are some synergies between loans and deposits—this could be due to a better knowledge of borrowers who hold accounts with the bank—the independence result would not hold anymore. Introducing liquidity risk into the Monti-Klein model would also break the impermeability of loans to deposits.

aggregate investments. They conclude that a CBDC designed as a deposit with the central bank could improve welfare if there were not enough private investment opportunities to back the deposits of private banks. By increasing money liquidity, a CBDC would displace unproductive investment and increase trade. Nevertheless, because the deposit market is perfectly competitive, the aggregate impact on investment is always negative. Chiu et al. (2019) use a similar environment but introduce market power in the deposits market. Introducing CBDC acts as an outside option for depositors. The more concentrated the banking sector, the more the outside option offered by the CBDC forces banks to reduce their margins and make deposits more attractive. Depending on the banks' market power, the competing effect from CBDC can reverse the disintermediation impact obtained in Keister and Sanches (2022).

Note that in the three papers described above, the CBDC is designed either as a perfect substitute to deposits or to paper money (or both), so that it is generically not held in equilibrium (its existence as an outside option is sufficient to impact the equilibrium). In our model, the coexistence of CBDC with other means of payment comes from its usage cost being different from that of paper money and bank deposits (in equilibrium, its interest rate will thus be different as well). Then, not only can it be held by a non-zero mass of households generically, but its appeal will differ across households depending on their wealth. For example, we show that it is possible to design the CBDC so that it is held by poorer workers who were previously unbanked, thereby directly impacting financial inclusion (rather than indirectly, through its impact on deposit rates).

Nyffenegger (2022) also studies the impact of a CBDC on intermediation, but combines an OLG model with a Lagos and Wright (2005) model so as to distinguish the role that money may play as a means of exchange from its role as a savings vehicle. Monnet et al. (2021) focus on the impact of a CBDC on the efficiency and risk-taking behavior of banks in an environment with risky investment projects that can be monitored at a cost. Agur et al. (2022) also study the optimal design of a CBDC in terms of two parameters. Like us, the first parameter is the interest rate the CBDC may bear. The second parameter however is the location of the CBDC on the anonymity-security frontier (as opposed to its usage cost). A more anonymous CBDC competes with paper money, while a more secure CBDC competes bank deposits. This matters since bank deposits increase investment and because households have heterogeneous preferences over payment instruments.[8]

Theoretical literature related to financial inclusion is scarcer. Ait Lahcen and Gomis-Porqueras (2021) propose a theoretical model of endogenous financial inclusion also based on the notion of fixed banking costs, and study the impact of financial exclusion on inequality and the effectiveness of monetary policy.[9] Gomis-Porqueras (2001) investigates the impact of regulations that generate differential access to saving products between rich and poor households on inequality and inflation. Other papers formalizing the link between financial inclusion and inflation include Cysne et al. (2005) and Menna and Tirelli (2017).

Finally, the potential impact of innovation in digital payments, decentralized finance, and CBDC on financial inclusion has been discussed in policy reports such as Maniff and Marsh (2017), Maniff and Wong (2020), Maniff (2020a), Maniff (2020b), CPMI and World Bank (2020), and Auer et al. (2022).

The remainder of the paper is organized as follows. In Section 1, we lay out the physical environment of the model. In Section 2, we describe the worker's problem and the resulting aggregate demand of money and financial inclusion. In Section 3, we present the investor's and the banker's problems, and the resulting level of intermediation. Section 4 describes the equilibrium and the impact of a CBDC on inclusion and intermediation. Section 5 provides two theoretical extensions. In Section 6, we calibrate the model and quantify the theoretical results obtained in previous sections. Finally, Section 7 contains concluding remarks.

[8]In our environment, agents do not have reduced-form preferences over payment instruments—the liquidity properties of different instruments dictate households' demand for each type of instrument in equilibrium.

[9]In terms of formalization, while we obtain endogenous exclusion by assuming that agents are heterogeneous in wealth (so that it is not worth it for the poorest agents to bear the banking costs in equilibrium), Ait Lahcen and Gomis-Porqueras (2021) obtain exclusion by assuming that the cost of accessing the banking sector is heterogeneous among agents (so that agents with a high access cost may prefer to remain excluded). Another important difference is that in their model, financial inclusion is synonym with access to the credit sector, while we focus on inclusion as access to digital payments and the wider of array of goods and services associated with it.

1 Environment

Time is indexed by $t = 1, 2, \ldots, \infty$. The economy is populated by three types of two-period-lived agents: workers, entrepreneurs, and bankers. Every period, a continuum of measure one of each type of agent is born. We denote "young" ("old") an agent in the first (second) period of his life. At $t = 1$, there exists an initial old generation of each type of agents that only lives one period.

A worker's lifetime utility from consumption is given by $u(c_1) + \beta c_2$, where c_1 is her consumption when young, c_2 her consumption when old, and $\beta \in (0, 1)$ her discount factor. We assume that u is twice-continuously differentiable, with $u' > 0$, $u'' < 0$, and $u'(0) = \infty$. The lifetime utility of a banker or an entrepreneur is given by their consumption when old.

Young workers are endowed a heterogeneous quantity of goods, y_i, at birth.[10] The distribution of endowments is described by the cumulative density function $\Theta(y)$, with support $[y_\ell, y_h]$, where $y_h > y_\ell > 0$. Workers do not have their own storage technology, so that the good entirely perishes if it stays in their hands from one period to another.

Bankers and entrepreneurs do not receive endowments. However, they each have exclusive access to a special technology. Entrepreneurs have access to projects that transform k units of good invested at time t into $f(k)$ units at time $t + 1$. We assume that $f' > 0$, $f" < 0$, $f'(0) = \infty$ and $f'(\infty) = 0$. As for bankers, they have access to a commitment and enforcement technology. More precisely, young bankers can commit to repay any liability once they are old and can enforce repayment of any debt they extended. We assume that agents who make a deposit with a banker incur a fixed disutility cost $\gamma^d > 0$.

Finally, the monetary authority issues two assets: paper money and CBDC. Both are costless to produce, recognized by all, and cannot be counterfeit. CBDC differs from paper money in that (1) it is digital, (2) it may pay a nominal interest rate i^e, and (3) agents who make a CBDC deposit incur a disutility cost $\gamma^e > 0$. The nominal stocks of paper money and outstanding CBDC are respectively denoted M_t^c and M_t^e. At time $t = 1$, the initial stock of

[10] This is equivalent to endowing workers with labor and assuming that their productivity is heterogeneous. Workers with a low (high) endowment are low-productivity (high-productivity) workers.

paper money is endowed homogeneously among the old generation of workers.

The total nominal outstanding liabilities issued by the central bank, $M_t \equiv M_t^c + M_t^e$, grow at a constant rate π. This rate of growth, as well as the servicing of the central bank liabilities, is obtained through lump-sum taxes or transfers to old workers.

Notes on the environment. Before moving on to the equilibrium, we quickly discuss two features of the model: the fixed costs of bank and CBDC accounts, and the role of money.

(i) Fixed banking and CBDC usage costs. One interpretation of the fixed effort cost of using a bank account is that these costs represent the cost of identifying oneself with the bank (cost of gathering documentation and visiting a branch, privacy cost, etc.), as described in Freeman and Huffman (1991). Fixed fees charged by banks, who pass through their account management costs, are another hurdle faced by customers desiring to open a bank account. While both identification requirements and fees seem pertinent in the context of studying financial inclusion, we chose to model the former for simplicity: contrary to fees, they do not impact banks' profits directly. Evidence in support of the relationship between financial inclusion and high fixed costs in the form of fees, access costs, and physical distance can be found in Allen et al. (2016) and Demirguc-Kunt et al. (2018), among others. Regarding CBDCs, we are agnostic as to whether to interpret the fixed cost parameter as a policy instrument. One can see it as the usage cost determined by the current state of technology or see it as a parameter that policy-makers could influence ex-ante when designing the CBDC. Indeed, appealing to unbanked populations is only one of many factors that may matter to policy-makers when designing a CBDC. The accessibility of a CBDC, in practice, may therefore depend on the weight of that factor in the objective function of policy-makers. Taking this modeling approach precludes us from making normative arguments regarding the desirability of a cheap CBDC for inclusion: obviously, in our model, the cheaper the CBDC, the higher the financial inclusion. What we focus on is the trade-off that exists between inclusion and bank intermediation, which we examine for any possible CBDC cost.

(ii) The role of money. Although we use the "young/old" nomenclature typical of OLG models, we do not interpret the generations in our model as literal generations. In line

with Andolfatto (2021), we argue that, in our model, money takes the role of a medium of exchange rather than a store of value. Andolfatto (2014) explains it best by equating the OLG model of money to an infinite version of the Wicksell triangle (A wants to consume bread in the morning, but can only produce it in the afternoon; B wants to consume bread in the afternoon, but can only produce it at night; C wants to consume bread at night, but can only produce it in the morning). Assuming that bread is perishable and that there is a lack of trust and enforcement among agents, the absence of double-coincidence of wants precludes bilateral gains from trade. The introduction of a monetary instrument solves this problem. Hence, money mitigates the same frictions in the OLG model of money as it does in other broadly-used models—such as search models of money—where money is valued endogenously.[11]

2 Workers and financial inclusion

The problem faced by a young worker with endowment y_i is

$$\max_{\{c_{1,i},c_{2,i},z_i^c,z_i^d,z_i^e\}} u_1(c_{1,i}) + \beta c_{2,i} - \gamma^d \mathbb{1}_{z_i^d>0} - \gamma^e \mathbb{1}_{z_i^e>0} \tag{1}$$

$$\text{s.t. } c_{1,i} \leq y_i - z_i^c - z_i^d - z_i^e \tag{2}$$

$$\text{and } c_{2,i} \leq R^c z_i^c + R^d z_i^d + R^e z_i^e - \tau. \tag{3}$$

The young worker chooses how to allocate her endowment between consumption, $c_{1,i}$, and money demand, which comprise paper money balances, z_i^c, bank deposits, z_i^d, and CBDC deposits, z_i^e (all expressed in real terms). The worker takes as given the real gross rates of return on the three money instruments, respectively R^c, R^d and R^e. Consumption when old is bound above by the gross real return on the money balances carried into the period, net of lump-sum taxes τ.

Let $\mathcal{S} = \{c, d, e\}$. The solution to the worker's maximization problem requires that $u'(c_1) \geq$

[11] For that matter, our model could easily be transposed to resemble Chiu et al. (2019), who use the Lagos and Wright (2005) framework, with the addition of heterogeneity in money demand across agents (e.g., due to heterogeneity in their marginal value of wealth). Our main insights would remain identical.

βR^j for all $j \in \mathcal{S}$. This condition holds at equality if $z^j > 0$, in which case the marginal cost of carrying one extra unit of instrument j, on the left-hand side, must equal its marginal benefit, on the right-hand side.

Lemma 1 (Optimal quantity of money holdings)**.** *If $z^{k\in\mathcal{S}}(y) > 0$, then*

$$z^j(y) = \begin{cases} y - (u')^{-1}(\beta R^j) & \textit{for } j = k \\ 0 & \textit{for } j \in \mathcal{S} \setminus k. \end{cases} \tag{4}$$

Also, $\partial z^k(y)/\partial y > 0$ and $\partial z^k(y)/\partial R^k > 0$.

This lemma first stipulates that in equilibrium, workers never carry more than one type of money. Second, it describes the optimal quantity of holdings for a given money, which increases with its return and with the holder's endowment.

Denote $W(y, R, \gamma)$ the lifetime utility of an agent with endowment y carrying a currency with return R and redemption cost γ in the amount given by the first line of (4) if it is greater than zero, and the utility from consuming all of her endowment in the first period otherwise. We can then write

$$W(y, R, \gamma) = \begin{cases} u[u'^{-1}(\beta R)] + \beta R[y - u'^{-1}(\beta R)] - \gamma - \beta\tau & \text{if } y > (u')^{-1}(\beta R) \\ u(y) - \beta\tau & \text{otherwise.} \end{cases} \tag{5}$$

Lemma 2 (Income monotonicity of money type choices)**.** *If $R^1 > R^2$ and $W(y, R^1, \gamma^1) > W(y, R^2, \gamma^2)$, then $W(y', R^1, \gamma^1) > W(y', R^2, \gamma^2) \quad \forall \quad y' \geq y$.*

According to Lemma 2, if a young worker prefers a given type of money over another one with a lower return, another young worker with a greater endowment will have the same preference ordering. The intuition goes as follows. Including transaction costs, the average rate of return on a money of type $k \in \mathcal{S}$, given an endowment y, is $R^k - \gamma^k/z^k(y)$. It is increasing in an agent's money holdings, which are increasing in the agent's endowment (Lemma 1). Indeed, while the holding cost, γ, is independent of income, the benefit increases with income since returns are proportional to money demand. This makes the currency with a higher return even more appealing to a richer individual.

For the remaining of the paper, we preclude equilibrium outcomes where it would be optimal for a young worker to consume all of her endowment in the first period by imposing $u'(y_\ell) < \beta R^c$, in which case $W(y, R^c, 0) > u(y) - \beta\tau$ for any $y \geq y^\ell$. Under this assumption, any young worker would prefer to carry at least a marginal amount of paper money rather than nothing. Then, following Lemma 1, a young worker will carry the type of money $k = \operatorname{argmax}_{\mathcal{S}} W(y, R^k, \gamma^k)$. We now derive results to help us rank $W(y, R^c, 0)$, $W(y, R^d, \gamma^d)$, and $W(y, R^e, \gamma^e)$ for any young worker.

Lemma 3 (Money type thresholds). *(i) Let $y^* \equiv \{y > 0 : W(y, R^c, 0) = W(y, R^e, \gamma^e)\}$. It exists if and only if $R^e > R^c$. Then, it is unique and independent of R^d. (ii) Let $\tilde{y}(R^d) \equiv \{y > 0 : W(y, R^c, 0) = W(y, R^d, \gamma^d)\}$. It exists if and only if $R^d > R^c$. Then it is unique given R^d, and $\partial\tilde{y}(R^d)/\partial R^d < 0$. (iii) Let $\hat{y}(R^d) \equiv \{y > 0 : W(y, R^e, \gamma^e) = W(y, R^d, \gamma^d)\}$. When $\gamma^d > \gamma^e$, $\tilde{y}(R^d)$ exists if and only if $R^d > R^e$. Then it is unique given R^d, and $\partial\hat{y}(R^d)/\partial R^d < 0$. When $\gamma^d = \gamma^e$, $\tilde{y}(R^d)$ exists if and only if $R^d = R^e$. Then it is equal to $[y^*, \infty]$. When $\gamma^d < \gamma^e$, $\tilde{y}(R^d)$ exists if and only if $R^d < R^e$. Then it is unique given R^d, and $\partial\hat{y}(R^d)/\partial R^d > 0$. (iv) Let $R^{d*} \equiv \{R^d : y^* = \tilde{y}(R^d) = \hat{y}(R^d)\}$. It exists as long as y^* exists, in which case it is unique and inversely related to y^*.*

Lemma 3 defines the income thresholds that determine the type of money holdings chosen by a young worker. While a young worker with endowment y^* is indifferent between paper money and CBDC, a young worker with endowment $\tilde{y}$ is indifferent between paper money and bank deposits, and a young worker with endowment $\hat{y}$ is indifferent between CBDC and bank deposits. Note, that the latter two thresholds depend on the return on bank deposits, R^d, which is determined in equilibrium. Since we know that preferences over money holdings are monotone in income from Lemma 2, we can then use the thresholds defined in Lemma 3 to map out the choice of money type as a function of income and R^d.

The different regions are plotted in Figure 2, where it is assumed that $R^e > R^c$ and that y_ℓ and y_h are such that $y^* \in [y_\ell, y_h]$, as will be the case from now on. We can observe a stratification of the population, which self sorts into different means of payment as a function of their endowment. Paper money is always held by the poorest workers. Then, the choice between CBDC and bank deposits depends on parameters. If either of these two types of

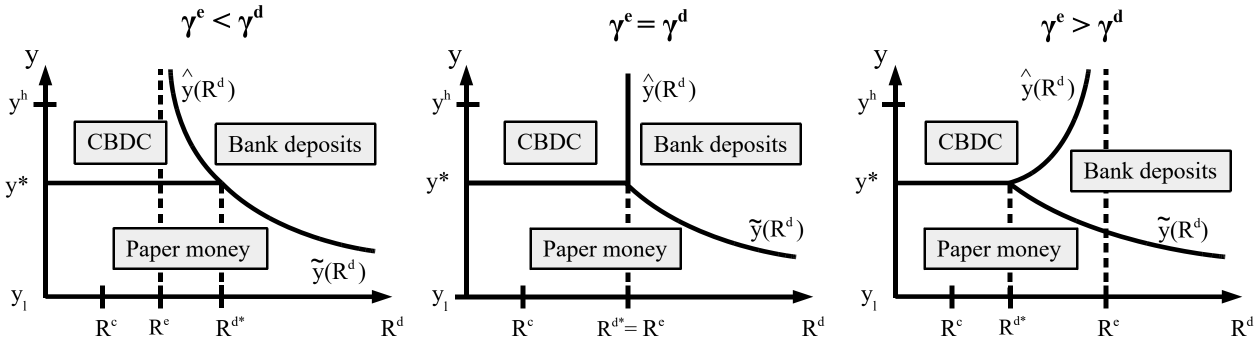


Figure 2: Optimal type of money holding as a function of a worker's endowment, y, and the return on bank deposits, R^d. The three panels corresponds to different assumptions about the magnitude of the fixed cost of bank deposits, γ^d, relative to the fixed cost of CBDC, γ^e.

money enjoys both a lower fixed cost and a higher return, all of the wealthier workers pick it. Otherwise, the wealthiest workers pick bank deposits and the middle-income workers pick CBDC when $\gamma^e < \gamma^d$, while the opposite is true when $\gamma^e > \gamma^d$. Intuitively, workers with the highest money demands can amortize fixed costs on larger real balances, and enjoy higher returns in proportion to their savings. Meanwhile, workers with lower desired real balances find moneys with lower fixed costs, albeit linked with lower returns, more attractive. Note that when $\gamma^e = \gamma^d$, CBDC and bank deposits are perfect substitutes, and thus only coexist if their return are equal. In this case, agents who hold one are indifferent with the other one—i.e., they do not correspond to different income levels within the population. This is the case studied in Andolfatto (2021).

We can now introduce our notion of financial inclusion and derive preliminary results. Workers who carry bank deposits or CBDC are considered financially included. Conversely, workers who exclusively carry paper money are not financially included.

Definition 1 (Financial inclusion)**.** *Financial inclusion, Inc, is defined as the mass of financially-included old workers. Formally,*

$$Inc(R^d) = \begin{cases} 1 - \Theta(y^*) & \text{if } R^d \leq R^{d*} \\ 1 - \Theta(\tilde{y}(R^d)) & \text{otherwise.} \end{cases} \tag{6}$$

Note that

$$\frac{\partial Inc(R^d)}{\partial R^d} = \begin{cases} 0 & \text{if } R^d \leq R^{d*} \\ \frac{z^d(\tilde{y}(R^d))}{R^d - R^c}\Theta'(\tilde{y}(R^d)) > 0 & \text{otherwise.} \end{cases} \tag{7}$$

When $R^d \leq R^{d*}$, the margin between financial inclusion and exclusion corresponds to the choice of paper money versus CBDC. In this case, the return on deposits does not impact financial inclusion. When $R^d > R^{d*}$, the margin of inclusion lies between paper money and bank deposits. Then a higher return on bank deposits increases financial inclusion. The larger the slope of the cumulative density function of income at the marginal income level, $\Theta'(\tilde{y}(R^d))$, the larger the impact of a change in R^d on inclusion.

Finally, we can formally define the aggregate demands for paper money, bank deposits, and CBDC as a function of R^d. The demand for paper money is given by

$$Z^c(R^d) = \begin{cases} \int_{y_\ell}^{y^*} z^c(y)d\Theta(y) & \text{if } R^d \leq R^{d*} \\ \int_{min(y_\ell,\tilde{y}(R^d))}^{\tilde{y}(R^d)} z^c(y)d\Theta(y) & \text{if } R^d > R^{d^*}. \end{cases} \tag{8}$$

The demands for bank deposits and CBDC depend on the relative magnitude of γ^e and γ^d. When $\gamma^e < \gamma^d$, they are respectively given by

$$Z^d(R^d) = \begin{cases} 0 & \text{if } R^d \leq R^e \\ \int_{min(y^h,\hat{y}(R^d))}^{y^h} z^d(y)d\Theta(y) & \text{if } R^e < R^d \leq R^{d*} \\ \int_{min(y^h,\tilde{y}(R^d))}^{y^h} z^d(y)d\Theta(y) & \text{if } R^d > R^{d^*}, \end{cases} \tag{9}$$

and

$$Z^e(R^d) = \begin{cases} \int_{y^*}^{y^h} z^e(y)d\Theta(y) & \text{if } R^d \leq R^e \\ \int_{y^*}^{min(y^h,\hat{y}(R^d))} z^e(y)d\Theta(y) & \text{if } R^e < R^d \leq R^{d*} \\ 0 & \text{if } R^d > R^{d^*}. \end{cases} \tag{10}$$

Money demands for the cases $\gamma^e = \gamma^d$ and $\gamma^e > \gamma^d$ are provided in the appendix.

3 Entrepreneurs, banks, and financial intermediation

A young entrepreneur chooses how much to borrow from banks, l^d. The loan can be used to finance capital investment, k, so as to maximize consumption when old. This problem can be written as

$$\max_{k,l^d} f(k) - R^l l^d \text{ s.t. } l^d \geq k, \tag{11}$$

where the borrowing cost, R^l, is taken as given by the entrepreneur. The problem is increasing in k and decreasing in l^d, so the constraint must be binding in equilibrium, $k = l^d$. Second, since $f'(0) = \infty$, the solution must be satisfy the first-order condition for an interior condition,

$$l^d(R^l) = f'^{-1}(R^l). \tag{12}$$

The demand for funds from entrepreneurs is a decreasing function of the cost of funds.

Finally, bankers choose how many deposits to take, d, and how many loans to give out, l^s, taking the loan rate and the deposit rate, R^l and R^d, as given. Their problem can be written as

$$\max_{l^s,d} R^l l^s - R^d d \text{ s.t. } l^s \leq d. \tag{13}$$

We can directly see that, in equilibrium, a bank's supply of loans must be equal to the amount of deposits it holds, $l^s = d$. Given d must be both positive and finite, the equilibrium loan and deposit rates must be equal, $R^d = R^l$.

Definition 2 (Financial intermediation)**.** *Financial intermediation, Int, is defined as the aggregate amount of loans supplied to entrepreneurs, so that $Int \equiv l^s$.*

Due to market clearing, $Int = k = d$. Note that

$$\frac{\partial Int(R^d)}{\partial R^d} = \frac{1}{f''(k)} < 0. \tag{14}$$

An increase in the deposit rate negatively impacts financial intermediation. Given the market is perfectly competitive, such an increase translates to an identical change in the loan rate, which reduces the entrepreneurs' incentive to borrow.[12]

[12] Section 5.1 describes how this result would change were the banking competition imperfect.

4 Equilibrium

We restrict our analysis to deterministic stationary equilibria where the aggregate real balances issued by the government are constant through time. That is, we require $M_t/p_t = M_{t-1}/p_{t-1}$.[13] Then, the inflation rate must be equal to the money growth rate,

$$\frac{p_{t+1}}{p_t} = \frac{M_{t+1}}{M_t} = 1 + \pi. \tag{15}$$

This pins down the equilibrium returns on paper money and CBDC as a function of the money growth rate and the nominal interest on CBDC. Consider an agent who carries z_t^c real balances of paper money at the end of period t. Her real balances are worth $p_t z_t^c/p_{t+1}$ in period $t+1$. The gross real return on paper money is therefore $p_t/p_{t+1} = 1/(1+\pi) \equiv R^c$. Consider instead an agent who carries z_t^e real balances of CBDC at the end of period t. The following period, these balances are worth $(1+i^e)p_t z_t^e/p_{t+1}$. The gross real return is $(1+i^e)p_t/p_{t+1} = (1+i^e)/(1+\pi) \equiv R^e$. We consider M_1^c, i^e, and π policy instruments of the central bank. The price level and the composition of the central bank liabilities are, on the other hand, endogenously determined in equilibrium.

The equilibrium deposit rate is determined by market clearing in the banking sector,

$$Z^d(R^d) = (f')^{-1}(R^d), \tag{16}$$

where $Z^d(R^d)$ is given by (9) when $\gamma^e < \gamma^d$, (A1) when $\gamma^e = \gamma^d$, and (A3) when $\gamma^e > \gamma^d$. Given the equilibrium deposit rate, the price level is then determined by market clearing for paper money,

$$p_t = \frac{M_t^c}{Z^c(R^d)}, \tag{17}$$

where $M_t^c = (1+\pi)^{t-1}M_1^c$, and $Z^c(R^d)$ is given by (8).[14] The model is closed by ensuring the balance of the government's budget through the appropriate taxes ($\tau > 0$) or transfers

[13] We assume that the central bank can implement a one-to-one convertibility of paper currency and CBDC, ensuring that the two assets have the same value and allowing us to define a single price level.

[14] Then, the nominal stock of CBDC is given by $M_t^e = Z^e(R^d)p_t$, where $Z^e(R^d)$ is given by (10) when $\gamma^e < \gamma^d$, (A2) when $\gamma^e = \gamma^d$, and (A4) when $\gamma^e > \gamma^d$.

($\tau < 0$) to old workers,

$$M_t + p_t \tau_t = M_{t-1} + i^e M^e_{t-1}. \tag{18}$$

A stationary equilibrium where paper money is valued exists as long as y^ℓ is low enough for some workers to prefer paper money over deposits. Then, it is unique. In addition, in any equilibrium, intermediation cannot be null since the demand for loans from entrepreneurs when $k = 0$ is infinite. Whether the CBDC is adopted depends on the parameters related to its design, γ^e and R^e.

4.1 CBDC adoption

Lemma 4 (Minimum interest rate on the CBDC)**.** *Consider an economy without a CBDC. There exists $\bar{R}^e(\gamma^e)$ such that introducing a CBDC with fixed cost $\gamma^e > 0$ impacts equilibrium outcomes in the unique stationary monetary equilibrium if and only if $R^e > \bar{R}^e(\gamma^e)$. Also, $\partial \bar{R}^e(\gamma^e)/\partial \gamma^e > 0$.*

Lemma 4 highlights that introducing a CBDC will impact the equilibrium if the interest rate it pays is sufficiently high. Otherwise, the CBDC is not competitive enough relative to paper money or bank deposits, and it has no bite on the equilibrium. The larger the fixed cost that CBDC holders must pay, the higher the minimum interest it must carry to become a competitive means of payment.

We can now compare intermediation and inclusion in an economy where a CBDC is available and the same economy without a CBDC.

Proposition 1 (Impact of a CBDC on inclusion and intermediation)**.** *Denote respectively Int^1 and Inc^1 intermediation and financial inclusion in a monetary economy pre-CBDC, and Int^2 and Inc^2 these outcomes after the introduction of a CBDC with parameters $\gamma^e > 0$ and $R^e > \bar{R}^e(\gamma^e)$. Then $Int^2 < Int^1$ and $Inc^2 > Inc^1$.*

If a CBDC impacts the pre-CBDC equilibrium, then it increases inclusion and decreases intermediation. The channel through which this occurs depends on the sign of $(\gamma^e - \gamma^d)$.

When $\gamma^e < \gamma^d$, both the richest paper money holders and the poorest bank depositors switch to the CBDC. In partial equilibrium, keeping R^d at its initial level, this directly increases inclusion since the workers who switched from paper money to CBDC now have access to digital payments. The decrease in deposits implies that the demand for loans is higher that what banks can provide, and therefore R^d must increase in general equilibrium. This has no additional impact on financial inclusion: the margin of inclusion now lies between paper money and CBDC, and is therefore independent of the interest rate on deposits. However, the increase in R^d is exactly matched by an increase in R^ℓ, and investment goes down.

When $\gamma^e > \gamma^d$, the richest depositors are the first to switch to the CBDC.[15] In partial equilibrium, this switch from bank deposits to CBDC has no impact on financial inclusion since these workers were already included. In general equilibrium, however, R^d must increase since once again the supply of deposits is now lower than the demand for loans. Like in the pre-CBDC economy, the margin for inclusion still lies between paper money and deposits. Therefore, the increase in R^d decreases the income level at which holding deposits becomes preferable to paper money, $\tilde{y}(R^d)$, and some paper money holders switch to deposits. Thus, although the CBDC is not held by workers that used to be financially excluded, it still benefits inclusion through its impact on the deposit rate. As was the case earlier, the corresponding increase in R^ℓ necessarily implies a decrease in investment.

4.2 Inclusion-Intermediation trade-off

Proposition 2 (GE impact of broader inclusion)**.** *Consider a monetary equilibrium with $R^e > \bar{R}^e$. Then,*

$$\frac{\partial Inc}{\partial \gamma^e} < 0, \frac{\partial Inc}{\partial R^e} > 0, \frac{\partial Int}{\partial \gamma^e} > 0, \frac{\partial Int}{\partial R^e} < 0. \tag{19}$$

Proposition 2 states that everything else equal, while a decrease (increase) in the cost (the interest rate) of a CBDC broadens inclusion, it comes at the cost of diminished financial intermediation. The intuition for these results is the same as the intuition behind the impact of introducing a CBDC. When the CBDC fixed cost is low, so that it is held by workers in the middle of the income distribution, increasing R^e or decreasing γ^e both make the CBDC more attractive to the marginal paper money holder (increasing inclusion) and the marginal

[15] Meaning that a worker with endowment y_i would switch only if all workers with endowments greater than y_i had also switched to CBDC.

bank depositor (decreasing investment). When the CBDC fixed cost is high, so that it is held by workers at the top of the income distribution, increasing R^e or decreasing γ^e attracts more of the richest bank depositors. This puts upwards pressure on the deposit rate R^d (decreasing investment), which indirectly makes deposits more attractive to paper money holders (increasing inclusion).[16]

Proposition 3 (Optimal design conditional on intermediation)**.** *Consider a CBDC design* (γ_1^e, R_1^e) *such that there exists an equilibrium where paper money and CBDC are both held by workers. Let* $R_1^d \equiv \bar{R}$*. Consider another CBDC design,* (γ_2^e, R_2^e)*, where* $\gamma_1^e < \gamma_2^e$ *and* $R_2^e > R_1^e$*, such that* $R_2^d = \bar{R}$*. Therefore,* $Int^1 = Int^2$*. If* $\gamma_2^e < \gamma^d$*, then* $Inc^1 \geq Inc^2$*. If* $\gamma_1^e > \gamma^d$*, then* $Inc^1 = Inc^2$*.*

This proposition shows that there does not necessarily exist a trade-off between financial inclusion and intermediation when it comes to *designing* a CBDC.

On one hand, when the CBDC fixed cost is high, and the CBDC is held by the richest workers, there is a one-to-one mapping between investment and financial inclusion. Indeed, in this case, inclusion is driven by R^d, which also directly drives investment since $R^d = R^\ell$. Thus, the specific design (γ^e, R^e) of the CBDC has no impact on inclusion conditional on the level of investment. On the other hand, when the CBDC fixed cost is low, and the CBDC is held by workers with intermediate income, two CBDC designs that implement the same level of investment do not necessarily implement the same level of financial inclusion. In other words, the design of the CBDC matters for the inclusion-intermediation trade-off. For example, it is possible to keep the same deposit rate (and thus the same amount of intermediation) all the while increasing inclusion by designing a CBDC with a lower fixed cost (proportionally offset by a lower interest rate). Figure 3 provides a graphical illustration. Decreasing R^e shifts $\hat{y}(R^d)$ to the left, while decreasing γ^e rotates it counter-clockwise. The new $\hat{y}(R^d)$ curve, in orange, intersects with the former, in black, at the original equilibrium deposit rate $\bar{R}$. The demand for deposits remains unchanged. However, the threshold income

[16] Of course, our results regarding the impact of the CBDC on intermediation hinge on the assumption of perfect competition in the banking sector, and thus are to be understood as an extreme case. While we consider this simplifying assumption helpful to fix ideas and formalize the idea of a trade-off between financial inclusion and disintermediation, we relax it in Section 5.1 and in our quantitative exercises, allowing us to provide some nuance.

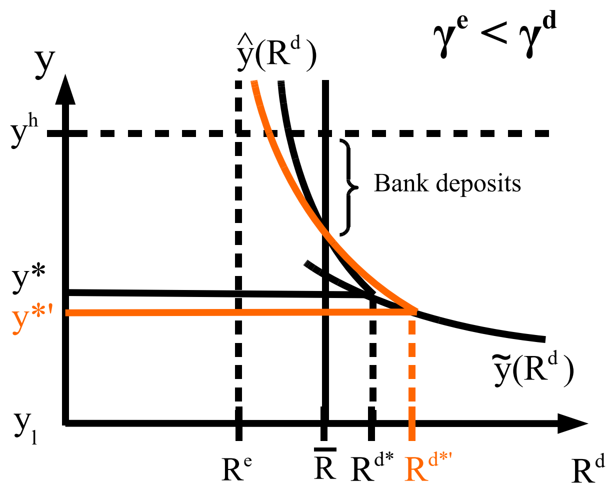


Figure 3: Example of simultaneous decreases in R^e and γ^e that leave the equilibrium deposit rate, $\bar{R}$, unchanged. Financial intermediation remains the same while financial inclusion increases.

delineating preferences between paper money and CBDC, y^*, goes down, increasing financial inclusion.

The intuition behinds this result is that in order to mitigate the disintermediation effect of more inclusion, the CBDC must be designed to attract financially-excluded workers at the margin between paper money and CBDC without "poaching" richer workers previously holding deposits. It is possible to do so by taking advantage of the different responses of these two groups to changes in γ^e and R^e. As described earlier, the utility of the richer workers that may switch from bank deposits is relatively more impacted by R^e, while that of poorer workers, whom we want to include, are more impacted by γ^e. Thus, decreasing both γ^e and R^e, keeping the interest rate R^d intact, would allow for relatively more inclusion without impacting deposits (and therefore investment).

5 Extensions

In this section we develop two extensions to the model. First, we relax the assumption of perfect competition in the deposit market. A finite number of banks compete a la Cournot, and earn non-zero profits from their intermediation services. This adds a counterweight to the negative impact of a CBDC on the demand for deposit money, as CBDC acts as additional competition and puts downwards pressure on banks profits. Second, we endogenize the benefits of financial inclusion with the addition of a continuum of varieties of consumption goods. We assume that financially-excluded workers have access to a smaller range of goods, making paper money a less attractive medium of exchange than bank deposits and CBDC.

5.1 Imperfect competition in the deposit market

Until now we assumed the deposit and loan markets were perfectly competitive. In this section we instead assume banks engage in Cournot competition in the deposit market (keeping competition perfect in the loan market).[17]

The problem of young banker j can be written as

$$\max_{l_j^s, d_j} R^l l_j^s - \hat{R}^d(D_{-j} + d_j)d_j \text{ s.t. } l_j^s \leq d_j. \tag{20}$$

Young banker j chooses the amount of loans to supply, l_j^s, and the amount of deposits to take in, d_j. The constraint simply precludes the bank from investing beyond its funding capacity. Note that it is assumed that the bank can hold neither paper money nor CBDC. Since the loan market is competitive, the return on loans, R^l, is taken as given by the banker. In contrast, the gross interest paid on deposits can be impacted by the banker's supply of deposits (in addition to the supply of deposits by all other bankers, $D_{-j} \equiv \sum_{i \neq j} d_i$). It is given by the inverse demand function for deposits,

$$\hat{R}^d(x) = (\hat{Z}^d)^{-1}(x). \tag{21}$$

Similarly to the competitive case, we can directly see that the balance sheet constraint must be binding in equilibrium, $l_j^s = d_j$. We focus on symmetric, pure-strategy equilibrium, whereby each bank supplies the same amount of deposits, hence $d_i = d = l^s$ for all i. An interior solution must satisfy

$$\hat{R}^{d\prime}(Nd)d + \hat{R}^d(Nd) = R^l. \tag{22}$$

[17]This market structure is similar to that used in Chiu et al. (2019).

We can rearrange and interpret this equation as follows,

$$\frac{R^l}{R^d} = 1 + \frac{\text{market share}}{\varepsilon(R^d)} > 1, \tag{23}$$

where $\varepsilon(R^d)$ is the elasticity of the demand for deposit money from young workers, $Z^d(R^d)$, with respect to the deposit rate. Under Cournot competition, the loan rate is now greater than the deposit rate as long as the number of banks and the demand elasticity are finite. The equilibrium condition previously given by (16) becomes

$$Z^d(R^d) = (f')^{-1}\left[R^d\left(1 + \frac{\text{market share}}{\varepsilon(R^d)}\right)\right]. \tag{24}$$

Because the right-hand side is lower, for any deposit rate, than it was under perfect competition, the equilibrium deposit rate must also lower.

We now explain how relaxing the assumption of perfect competition matters for the introduction of the CBDC. The left-hand-side of the equilibrium condition is unchanged, since the degree of competition has no impact on the demand for deposit money conditional on the deposit rate. Since inclusion is entirely driven by money demands, whether competition is perfect or not does not matter for inclusion conditional on the deposit rate. However, the impact of a decrease in demand for deposit money on the loan rate, and therefore on investment, is impacted by the degree of competition, as we can see from the right-hand-side of the equilibrium condition. It is easy to show that introducing a CBDC increases the elasticity of demand to R^d. The intuition is that a CBDC makes the marginal depositor richer, and that the richer a worker, the more sensitive she is to the interest rate. From the first-order condition, we can directly see that an increase in the demand elasticity implies smaller bank margins—the spread between R^d and R^ℓ diminishes given R^d. In other words, the passthrough from R^d to R^ℓ is reduced, which mitigates the negative impact that upwards presssure on R^d had on investment in the perfectly competitive setup. Note that at the limit, the decrease in margins can be sufficient for R^ℓ to decrease, in which case the introduction of a CBDC can in fact increase intermediation. This result, while perhaps surprising, is described in more details in Chiu et al. (2019).

5.2 Imperfect acceptability of paper money

Up to this point, increasing financial inclusion was taken as an exogenous goal for the policy maker. We now endogenize the benefit of financial inclusion by introducing frictions in the acceptability of different means of payment, in the spirit of Chiu et al. (2019) and Keister and Sanches (2022). Specifically, we introduce a taste for diversity and assume that carrying digital means of payment (bank deposits or CBDC) broadens the variety of goods and services that individuals can purchase.

Assume that the consumption good comes in a continuum of varieties of measure 1, indexed by $j \in [0,1]$. An agent who consumes the bundle $\{x_j\}_{j\in[0,1]}$ enjoys utility $u(\mathbf{x})$, where

$$\mathbf{x} = \left(\int_0^1 x_j^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}}, \tag{25}$$

and $\eta > 1$. Futher assume that sellers can change the variety of their product at no cost. Then, all varieties must bear the same equilibrium price. We can then easily show that the first best is for workers to consume $x_j = x$ for all j. In words, agents are variety-loving due to the concavity of preferences, and therefore prefer to consume all varieties in equal amounts.

Next, assume that while varieties $k \in [0, \alpha]$, where $\alpha \in (0,1)$, can be purchased with any means of payment, varieties $k \in (\alpha, 1]$ can only be purchased with bank deposits or CBDC. Then, an agent who carries Rz units of bank deposits or CBDC splits his real balances available for consumption, $(Rz - \tau)$, equally between all varieties, and consumes

$$c_2 = \left(\int_0^1 (Rz-\tau)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}} = Rz - \tau. \tag{26}$$

An agent who carries Rz units of paper money (and no other types of money) splits her available real balances equally between the paper-money-accessible varieties and consumes

$$c_2 = \left(\int_0^\alpha \left(\frac{Rz-\tau}{\alpha}\right)^{\frac{\eta-1}{\eta}} dj\right)^{\frac{\eta}{\eta-1}} = \alpha^{\frac{1}{\eta-1}}(Rz-\tau) < Rz - \tau. \tag{27}$$

As η goes to infinity, $\alpha^{\frac{1}{\eta-1}}$ goes to 1. As η goes to 1, $\alpha^{\frac{1}{\eta-1}}$ goes to 0.

Hence, with the same amount of real balances, more utility can be enjoyed by an agent who can purchase all varieties of goods than by an agent limited to varieties that can be purchased with paper money. The parameter η determines the degree of substitution between varieties. The higher η, the higher the substitutability, and the smaller the loss suffered by paper-money-only agents. Financial inclusion then becomes endogenously valuable by giving access to the larger variety of goods to workers.

Note that for all types of agents, carrying different types of money is always dominated, so that Lemma 1 still holds. Bank depositors and CBDC holders have access to all varieties, so there is no need to carry another type of money. Paper money holders could access more varieties by switching to bank deposits or CBDC. If doing so was optimal for them, holding any quantity of paper money, which has a lower return, would then be dominated.

We assume that endowments are uniformly distributed among all varieties of goods. Then, the lifetime utility of agents who carry CBDC or bank deposits is not impacted by the introduction of α, and their maximization problem remains the same. Workers who choose to carry paper money, however, now maximize

$$W(y, R^c, 0) = u(c_1) + \beta R^c \alpha^{\frac{1}{\eta-1}}(y - c_1) - \beta\tau. \tag{28}$$

The first-order condition is

$$u'(c_1) = \beta R^c \alpha^{\frac{1}{\eta-1}}. \tag{29}$$

This is similar to the first-order condition derived in the baseline model, although with an additional factor scaling down the return on paper savings on the right-hand-side. Therefore, the equilibrium can be defined similarly to the equilibrium defined earlier in the paper, swapping R^c with $R^{c\prime} \equiv R^c \alpha^{\frac{1}{\eta-1}}$. Provided $R^{c'} < R^c$, paper money suffers an extra discount on top of its exposure to inflation, due to the limited liquidity services it provides compared to bank deposits and CBDC.[18]

[18] Lack of access to a range of goods and services is only one of the reasons why paper money is not a perfect substitute to digital currencies. Its storage cost, as well as the possibility of theft, could also rationalize discounting paper money in excess of inflation.

One interesting implication is that this feature reduces the interest rate that must be paid on a CBDC in order for it to be adopted. At the extreme, a CBDC may even be adopted with a negative interest rate, despite carrying a higher fixed cost, thanks to its superior liquidity.

6 Quantitative results

In this section we calibrate the extended model to the US economy to quantify the inclusion-intermediation impact of a CBDC as a function of its design.

6.1 Calibration

While it was convenient to assume that the utility of older workers was linear in their consumption so as to derive analytical results, we now assume that the utility functions in both the first and the second period of a worker's life are concave, taking a CRRA form,

$$U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \frac{c_2^{1-\sigma}}{1-\sigma}. \tag{30}$$

The production function is Cobb-Douglas,

$$f(k) = Ak^{\nu}. \tag{31}$$

Externally-calibrated parameters The model is calibrated at the yearly frequency. The discount factor, β, is calibrated to match a 4% annual discount rate. The production function elasticity is calibrated to equate the elasticity of commercial loans with respect to the prime rate. The real gross return on paper money, R^c, corresponds to a 1.52% inflation rate. The share of consumption good varieties that can be purchased with both paper money and deposits, α, is calibrated using 2016 data from the Survey of Consumer Payment Choice (Greene and Stavins (2018)) and the Diary of Consumer Payment Choice (Greene and Schuh (2017)), following the method in Chiu et al. (2019). The elasticity of substitution between varieties is set to 5 following Dolfen et al. (2019). The income distribution is assumed

	Externally calibrated	
Parameter	**Value**	**Target**
β	0.96	4% risk-free rate
ν	0.66	Elasticity of commercial loans w.r.t. prime rate
R_c	0.9850	2014-19 avg. annual inflation 1.52%
(y_l, y_h)	(500,200000)	
(μ_y, σ_y)	(11.03,0.82)	US Income distribution (2017 CPS)
α	0.7314	SCPC and DCPC
η	5	Dolfen et al. (2019)
	Internally calibrated	
Parameter	**Value**	**Target**
σ	0.1580	2007 M1/GDP
N	8	Deposit rate
A	50.55	Spread between deposit and loan rates
γ^d	120	6.5% share of unbanked households

Table 1: Calibrated parameters.

to be log-normal. The mean and standard deviations, (μ_y, σ_y) are set using data on the income distribution in the US from the 2017 CPS. Following the method described in Schield (2018), the mean is approximated by the log of the empirical median, and the variance is approximated by doubling the log of the ratio of the empirical mean to the empirical median. Finally, the distribution is truncated to exclude both the lowest and highest earners ($y < 500$ and $y > 200000$).

Internally-calibrated parameters The remaining parameters, $\{\sigma, N, A, \gamma^d\}$, are jointly calibrated. The elasticity of substitution between consumption at period t and $t+1$, σ, is set to match the ratio of M1/GDP in 2007 (equal to 0.2583), where the model counterparts are calculated as

$$M1 = Z^d + Z^c \text{ and } GDP = \int_{y_\ell}^{y^h} y d\Theta(y) + f(Z^d). \tag{32}$$

The number of banks, N, and the productivity parameter, A, are set to match a target deposit rate of 0.3049% and a target loan rate of 3.6947% (thus, a spread of 339 basis points). Finally, γ^d is set to match the aggregate unbanked share of population in 2017 of 6.5%. Calibrated parameter values are summarized in Table 1. The stock of money supply

Financial inclusion threshold, $\tilde{y}$	\$17, 253
Loan rate, $i^l \equiv (1+\pi)R^l - 1$	3.6824%
Deposit rate, $i^d \equiv (1+\pi)R^d - 1$	0.3559%
Investment/GDP	25.30%
Investment/Aggregate endowment	41.20%

Table 2: Aggregate outcomes in calibrated model pre-CBDC.

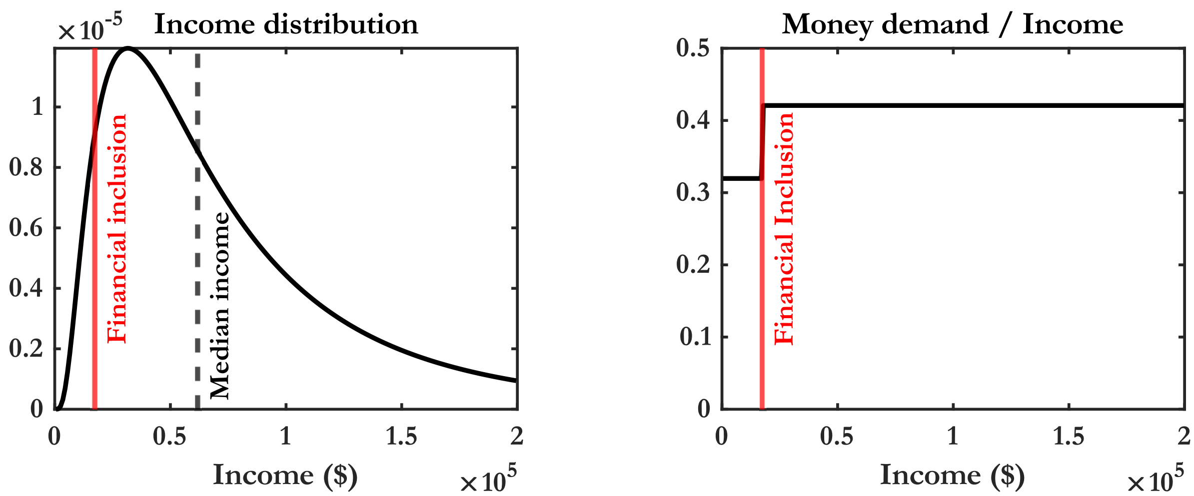


Figure 4: Distributional outcomes in calibrated model pre-CBDC.

is set so that the price level can be interpreted as US dollars.

Pre-CBDC equilibrium Aggregate outcomes of the pre-CBDC economy are displayed in Table 2. The financial inclusion income thresholds, loan rate and deposit rate were directly targeted. The investment over GDP ratio and investment over aggregate endowment ratios were not targeted and can be used as a check on the calibration. For example, the former was equal to 22.4% on average between 1947 and 2021 in the US, which provides some external validation. Figure 4 shows calibrated outcomes along the distribution of income. The left panel displays the fitted income distribution, its median, and the income threshold under which workers are unbanked. The right panel shows the share of income that young workers use for transactions in the next period. Due to the CRRA preferences, this share is constant across the workers who carry the same type of money. It is lower for unbanked workers. Note that $R^{c\prime} = 0.925R^c$, meaning that paper money carries a 7.5% illiquidity discount.

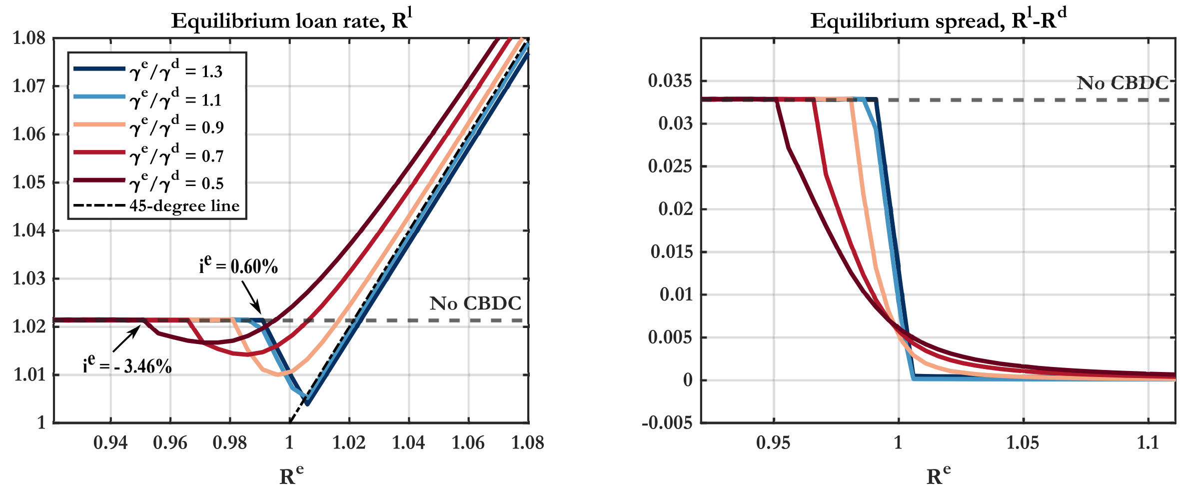


Figure 5: Left panel: Equilibrium loan rate. Right panel: equilibrium spread. Colors represent different γ^e/γ^d ratios, under Cournot competition.

6.2 Quantitative impact of a CBDC

We now study the steady-state impact of introducing a CBDC, comparing outcomes across different designs. We consider a range of fixed costs by setting the relative value of γ^e to γ^d to $\{0.5, 0.7, 0.9, 1.1, 1.3\}$. We consider interest rates $R^e \in [R^{c\prime} + 0.01, R^{c\prime} + 0.2]$. For a given R^e, a design with a lower ratio γ^e/γ^d can be seen as more "accessible" since holding the CBDC is then cheaper. Note that while the results we provide below were obtained in the model with imperfect competition in the banking sector, where $N = 8$, we provide figures that compare these outcomes to those obtained when N goes to infinity (perfect competition) in the appendix.

Equilibrium banking rates The left panel of Figure 5 shows the loan rate, R^l, while the right panel shows the spread $R^l - R^d$. In both panels, a slope of zero corresponds to the CBDC having no impact on the equilibrium. In this case, R^d, R^l and the spread remain at their pre-CBDC levels. As noted earlier, a positive nominal rate on the CBDC is not necessarily required for it to be a threat to paper money, because the latter also carries an illiquidity discount. For example, when the CBDC fixed cost is 50% lower than the deposit fixed cost, the CBDC still matters with a nominal interest as low as -3.46%. If the fixed cost is 30% larger than the deposit fixed cost, then the minimum nominal interest rate that the CBDC must carry is 0.60%.

Next, we can see that for all the fixed costs we consider, there exists a range for R^e where the introduction of a CBDC pushes the loan rate down. As R^e increases, we can observe two types of convergence. First, R^d and R^l converge towards each other (i.e., the spread converges to zero). This can be interpreted as the banking sector behaving in a more competitive way. In addition, both rates converge towards R^e (represented as the dotted-dashed 45-degree line in the top panels). This can be seen as the CBDC acting as a more and more competitive alternative monetary instrument as the interest it pays gets higher and higher. In addition, we can see that the slope of the spread as R^e increases is steeper the higher γ^e/γ^d. Indeed, the least "accessible" the design, the less important the role played by the fixed cost γ^e, and the more important that played by the interest rate R^e. In other words, when γ^e/γ^d is high, banking rates become very sensitive to R^e, especially in the region close to the minimum rates required for the CBDC to be adopted. Assuming that the government cannot exactly assess γ^e/γ^d, it therefore faces some uncertainty as towards which curve is the correct one. Then, decreasing the sensitivity of outcomes to R^e by thriving to design a CBDC with the lowest γ^e possible may be particularly desirable in terms of policy-making.

Financial inclusion and intermediation Next, we turn to our two main outcomes of interest, financial inclusion and intermediation. We study the former by plotting the share of unbanked workers (left panel of Figure 6), and the latter by plotting aggregate investment, which equals aggregate deposits by market clearing (right panel of Figure 6).

Confirming our theoretical results, when R^e is high enough for the CBDC to be held, the share of unbanked workers decreases in R^e and increases in γ^e. It tends to zero (i.e., full financial inclusion) as R^e tends to infinity. As for investment, we can see on the right panel that for all γ^e considered, there exists a region where investment increases in R^e. Note that this region starts at the lowest R^e that can sustain an equilibrium with CBDC adoption, so that investment not only increases in R^e—it also is higher than investment in the pre-CBDC equilibrium. Also, the higher γ^e, the steeper the growth of investment and the higher the maximum amount of investment that can be achieved. The intuition is similar to that delineated earlier to explain the slope of the spread $R^l - R^d$ with respect to R^e. The higher γ^e, the more important R^e is in determining behaviors and the more sensitive the economy

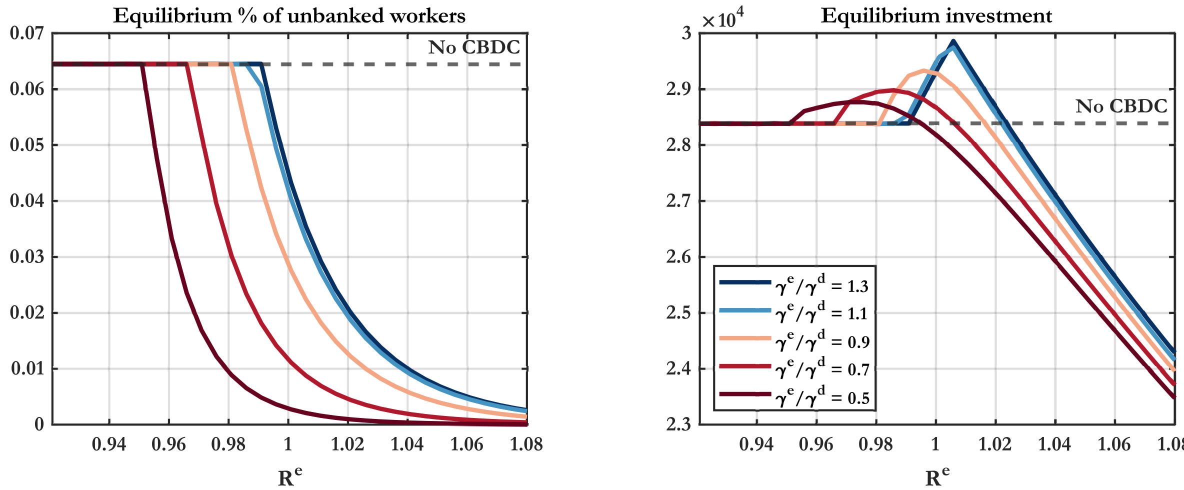


Figure 6: Left panel: Percentage of unbanked workers. Right panel: Aggregate investment. Colors represent different γ^e/γ^d ratios, under Cournot competition.

to R^e.

Financial inclusion-intermediation trade-off Figure 7 plots the financial inclusion-intermediation trade-off by representing $Inc-Int$ frontiers. For a given γ^e/γ^d ratio, a frontier represents all the combinations of changes in financial inclusion and changes in intermediation that can be obtained after introducing a CBDC, where the variation comes from varying R^e. More precisely, the x-axis represents the percent change in investment, and the y-axis reprensents the percentage point increase in financial inclusion, both compared to the pre-CBDC equilibrium. Thus, outcomes towards the top and to the right are unambiguously better. Note that the change in financial inclusion can go from 0 percentage point (no change) to 6.5 percentage points (full inclusion).

CBDC designs such that the curve is upwards-sloping are strictly dominated by other CBDC designs that can deliver better outcomes along at least one margin without sacrificing the other ones. For example, on the pink curve ($\gamma^e/\gamma^d = 0.9$), the design that generates a 2.2 pp increase in inclusion with a 3% increase in investment is dominated by the design that generates the same increase in investment but with a 3.8 pp increase in inclusion. In this case, there is no trade-off: one can increase both inclusion and intermediation at the same time by increasing R^e. A trade-off appears when the curve is downwards sloping.

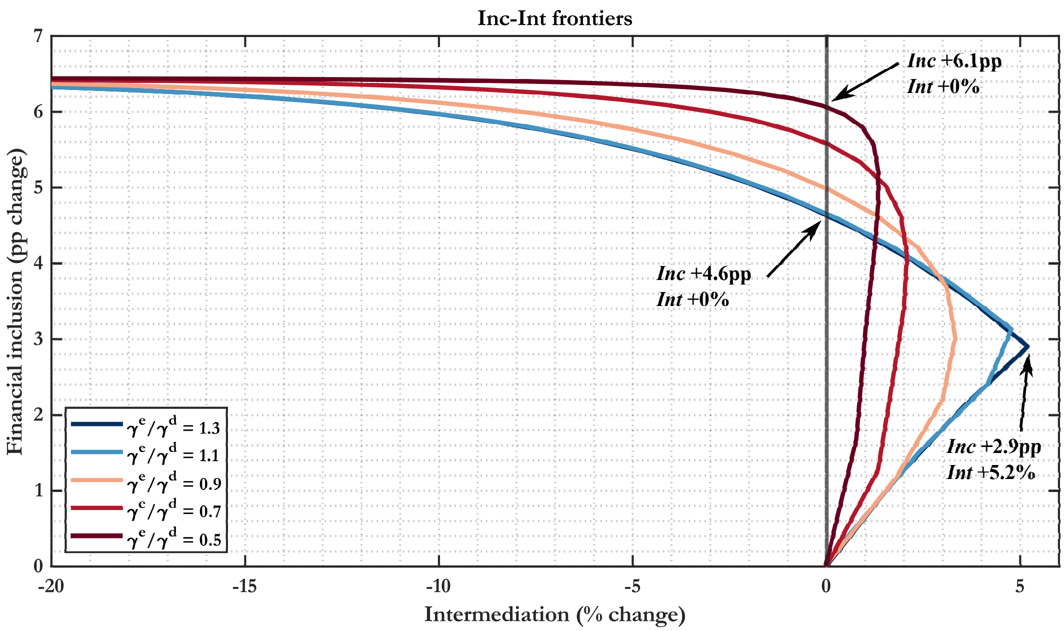


Figure 7: Inclusion-intermediation frontiers. Colors represent diffferent γ^e/γ^d ratios under Cournot competition.

The intersection of the gray vertical line with the frontiers indicates the amount of inclusion that can be achieved without impacting intermediation (in comparison to the pre-CBDC equilibrium). In line with the analytical results described earlier, the lower γ^e, the higher the financial inclusion for a given level of intermediation. With a CBDC 50% cheaper than deposits ($\gamma^e/\gamma^d = 0.5$), the mass of unbanked workers can decrease by 93% with zero impact on intermediation. At the other extreme, with a CBDC 30% more expensive than deposits ($\gamma^e/\gamma^d = 1.3$), the mass of unbanked workers can decrease by 71% at most if the policymaker wishes not to harm intermediation. Were the policymaker willing to decrease investment by 5%, financial exclusion could become virtually nonexistent if the CBDC usage cost was the smallest considered ($\gamma^e/\gamma^d = 0.5$), and it could be reduced to less than 1% of the population (an 85% decrease in the mass of unbanked workers) with the highest cost considered ($\gamma^e/\gamma^d = 1.3$). Alternatively, with that latter design, it would also be possible to achieve a more modest 45% decrease in exclusion all the while increasing investment by 5.2%.

Could an increase in banking competition be more effective? Finally, we explore in Figure 8 whether increasing competition in the banking sector, which always benefits

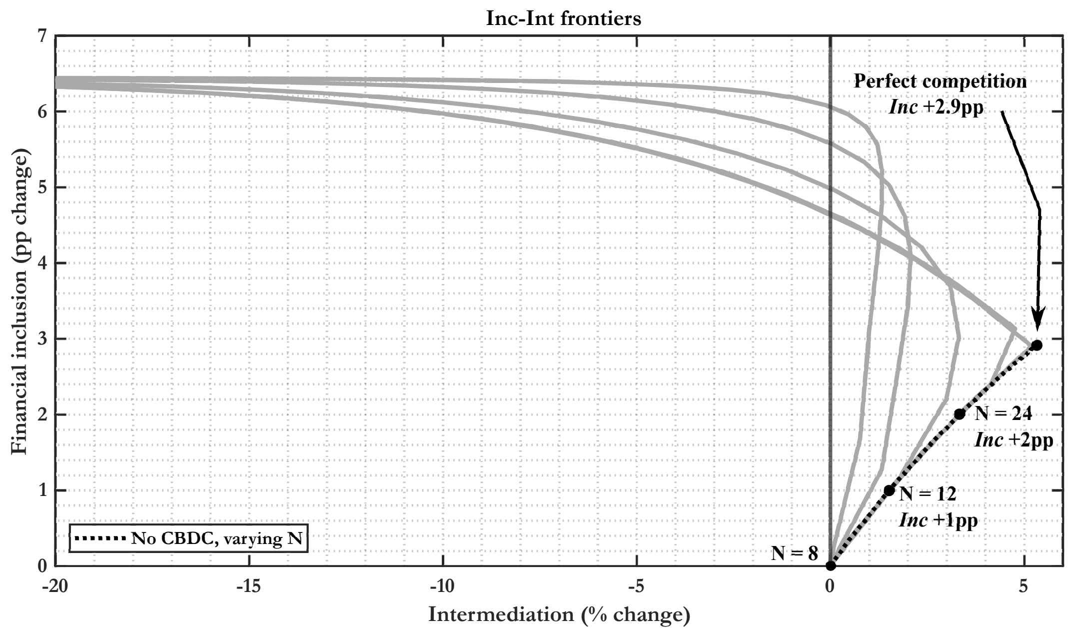


Figure 8: Inclusion-intermediation frontiers. The dashed line represents the possible pairs of inclusion-intermediation obtained in the pre-CBDC economy by varying the number of banks from 8 to infinity.

investment, could be more beneficial for inclusion than introducing a CBDC. First, we can see that the impact on inclusion obtained by increasing competition is relatively modest. Tripling the number of banks, from 8 to 24, reduces the fraction of unbanked workers by 31%. At the limit, perfect competition would reduce it by 45%. In addition, we can see that the intermediation-inclusion trade-off obtained by increasing competition is very similar to those obtained in the upwards sloping parts of the curves that correspond to an expensive CBDC ($\gamma^e/\gamma^d > 1$). Thus, it appears possible to construct a CBDC such that the inclusion-intermediation outcomes strictly dominate those obtained by increasing competition.

7 Concluding remarks

We examined the trade-off between financial inclusion (defined as the access to digital payments) and financial intermediation (defined as investment funded by bank deposits) in a model where financial inclusion is endogenously driven by income heterogeneity. We highlighted that the channel through which a CBDC could impact inclusion depends on its usage cost relative to that of bank deposit accounts. While the former need not necessarily be cheaper than the latter so as to improve inclusion, we show that CBDC designs with lower usage costs (and lower interest rates) make the inclusion-intermediation trade-off more favorable. Indeed, such designs allow for the CBDC to be adopted by poorer agents, who are otherwise financially excluded, rather than attract the bank deposits of wealthier depositors. The calibrated model allowed us to quantify the intermediation-inclusion trade-off faced by policy makers as a function of both the interest paid on the CBDC and its usage cost.

While we focused our quantitative analysis on the US, we believe that the mechanisms described in the paper could directly translate to emerging markets, where access to a transaction account is even less prevalent (Demirguc-Kunt et al. (2022)), and where the impact of financial inclusion on poverty is larger (see, e.g., Omar and Inaba (2020) and Park and Mercado (2021)). Indeed, according to the results of the 2021 BIS survey on CBDC, reported in Kosse and Mattei (2022), financial inclusion remains the main driver of engagement for exploring the potential of a retail CBDC in emerging countries. Quantifying the potential trade-off between inclusion and disintermediation in these countries may then be particularly relevant.[19]

[19]For example, when examining some of the implications of a potential Colombian CBDC, Vargas (2022) notes that related issues include both "whether a CBDC would support or hinder financial inclusion" and "whether the issuance of a CBDC would reduce financial intermediation and hamper credit supply."

References

Itai Agur, Anil Ari, and Giovanni Dell'Ariccia. 2022. Designing central bank digital currencies. *Journal of Monetary Economics* 125 (2022), 62–79. https://doi.org/10.1016/j.jmoneco.2021.05.002

Mohammed Ait Lahcen and Pedro Gomis-Porqueras. 2021. A model of endogenous financial inclusion: Implications for inequality and monetary policy. *Journal of Money, Credit and Banking* 53, 5 (2021), 1175–1209. https://doi.org/10.1111/jmcb.12808

Franklin Allen, Asli Demirguc-Kunt, Leora Klapper, and Maria Soledad Martinez Peria. 2016. The foundations of financial inclusion: Understanding ownership and use of formal accounts. *Journal of financial Intermediation* 27 (2016), 1–30. https://doi.org/10.1016/j.jfi.2015.12.003

David Andolfatto. 2014. What is the OLG model of money good for? *Macro-Mania, Blog post* (2014). http://andolfatto.blogspot.com/2014/01/what-is-olg-model-of-money-good-for.html

David Andolfatto. 2021. Assessing the impact of central bank digital currency on private banks. *The Economic Journal* 131, 634 (2021), 525–540. https://doi.org/10.1093/ej/ueaa073

Raphael Auer, Holti Banka, Nana Yaa Boakye-Adjei, Ahmed Faragallah, Jon Frost, Harish Natarajan, and Jermy Prenio. 2022. Central bank digital currencies: a new tool in the financial inclusion toolkit? *FSI Insights* 41 (2022).

Raphael Auer and Rainer Böhme. 2020. The technology of retail central bank digital currency. *BIS Quarterly Review, March* (2020).

Morten L Bech and Rodney Garratt. 2017. Central bank cryptocurrencies. *BIS Quarterly Review, September* (2017).

Allen N Berger and Timothy H Hannan. 1989. The price-concentration relationship in banking. *The Review of Economics and Statistics* (1989), 291–299. https://doi.org/10.2307/1926975

Board of Governors of the Federal Reserve. 2022. Money and Payments: The US Dollar in the Age of Digital Transformation. *Report* (2022).

Bruce Champ, Scott Freeman, and Joseph H Haslag. 2022. *Modeling monetary economies* (fifth ed.). Cambridge University Press. https://doi.org/10.1017/CBO9781316535486

Jonathan Chiu, Seyed Mohammadreza Davoodalhosseini, Janet Hua Jiang, and Yu Zhu. 2019. Bank market power and central bank digital currency: Theory and quantitative assessment. *SSRN working paper No. 3331135* (2019). https://doi.org/10.2139/ssrn.3331135

Jonathan Chiu, Seyed Mohammadreza Davoodalhosseini, Janet Hua Jiang, and Yu Zhu. 2020. Safe payments. *Bank of Canada Staff Working Paper* (2020). https://doi.org/10.34989/swp-2020-53

Michael Choi and Guillaume Rocheteau. 2021. Foundations of market power in monetary economies. *SSRN working paper No. 3763587* (2021). https://doi.org/10.2139/ssrn.3763587

CPMI and World Bank. 2020. Payment aspects of financial inclusion in the fintech era. *Report* (2020).

Rubens P Cysne, Wilfredo L Maldonado, and Paulo Klinger Monteiro. 2005. Inflation and income inequality: A shopping-time approach. *Journal of Development Economics* 78, 2 (2005), 516–528. https://doi.org/10.1016/j.jdeveco.2004.09.002

Asli Demirguc-Kunt, Leora Klapper, Dorothe Singer, and Saniya Ansar. 2018. *The Global Findex Database 2017: Measuring financial inclusion and the fintech revolution.* World Bank Publications.

Asli Demirguc-Kunt, Leora Klapper, Dorothe Singer, and Saniya Ansar. 2022. *The Global Findex Database 2021: Financial Inclusion, Digital Payments, and Resilience in the Age of COVID-19.* World Bank Publications.

Douglas W Diamond and Philip H Dybvig. 1983. Bank runs, deposit insurance, and liquidity. *Journal of Political Economy* 91, 3 (1983), 401–419. https://doi.org/10.1086/261155

Paul Dolfen, Liran Einav, Peter J Klenow, Benjamin Klopack, Jonathan D Levin, Laurence Levin, and Wayne Best. 2019. Assessing the gains from e-commerce. *NBER Working Paper No. w25610* (2019). https://doi.org/10.3386/w25610

Itamar Drechsler, Alexi Savov, and Philipp Schnabl. 2021. Banking on deposits: Maturity transformation without interest rate risk. *The Journal of Finance* 76, 3 (2021), 1091–1143. https://doi.org/10.1111/jofi.13013

Federal Deposit Insurance Corporation. 2020. How America banks: Household use of banking and financial services. *2019 FDIC Survey* (2020).

Jesús Fernández-Villaverde, Daniel Sanches, Linda Schilling, and Harald Uhlig. 2021. Central bank digital currency: Central banking for all? *Review of Economic Dynamics* 41 (2021), 225–242. https://doi.org/doi={https://doi.org/10.1016/j.red.2020.12.004}

Scott Freeman and Gregory W Huffman. 1991. Inside money, output, and causality. *International Economic Review* (1991), 645–667. https://doi.org/10.2307/2527112

Xavier Freixas and Jean-Charles Rochet. 2008. *Microeconomics of banking*. MIT press.

Pere Gomis-Porqueras. 2001. When should bank regulation favor the wealthy? *Journal of economic theory* 99, 1-2 (2001), 327–337. https://doi.org/10.1006/jeth.2000.2676

Claire Greene and Scott D Schuh. 2017. The 2016 diary of consumer payment choice. *Federal Reserve Bank of Boston Research Data Reports* 17-7 (2017).

Claire Greene and Joanna Stavins. 2018. The 2016 and 2017 surveys of consumer payment choice: summary results. *Federal Reserve Bank of Atlanta Discussion Paper* 18-3 (2018).

Todd Keister and Daniel R Sanches. 2022. Should central banks issue digital currency? *The Review of Economic Studies* forthcoming (2022). https://doi.org/10.1093/restud/rdac017

Michael A Klein. 1971. A theory of the banking firm. *Journal of Money, Credit and Banking* 3, 2 (1971), 205–218. https://doi.org/10.2307/1991279

Anneke Kosse and Ilaria Mattei. 2022. Gaining momentum: Results of the 2021 BIS survey on central bank digital currencies. *BIS Papers* (2022).

Ricardo Lagos and Randall Wright. 2005. A unified framework for monetary theory and policy analysis. *Journal of political Economy* 113, 3 (2005), 463–484. https://doi.org/10.1086/429804

Jesse Maniff and Paul Wong. 2020. Comparing means of payment: what role for a central bank digital currency? *FEDS Notes* 2020-08 (2020), 13–2. https://doi.org/10.17016/2380-7172.2739

Jesse Leigh Maniff. 2020a. Inclusion by Design: Crafting a Central Bank Digital Currency to Reach All Americans. *Federal Reserve Bank of Kansas City, Payments System Research Briefing* (2020).

Jesse Leigh Maniff. 2020b. Motives Matter: Examining Potential Tension in Central Bank Digital Currency Designs. *Federal Reserve Bank of Kansas City, Payments System Research Briefing* (2020).

Jesse Leigh Maniff and W Blake Marsh. 2017. Banking on Distributed Ledger Technology: Can It Help Banks Address Financial Inclusion? *Federal Reserve Bank of Kansas City, Economic Review* 102, 3 (2017), 53–77. https://doi.org/10.18651/er/3q17maniffmarsh

Lorenzo Menna and Patrizio Tirelli. 2017. Optimal inflation to reduce inequality. *Review of Economic Dynamics* 24 (2017), 79–94. https://doi.org/10.1016/j.red.2017.01.004

Cyril Monnet, Asgerdur Petersdottir, and Mariana Rojas-Breu. 2021. Central Bank Account For All: Efficiency and Risk Taking. *Working paper* (2021).

Mario Monti. 1972. *Deposit, credit and interest rate determination under alternative bank objective function.* North-Holland/American Elsevier.

Remo Nyffenegger. 2022. Central bank digital currency with heterogeneous bank deposits. *University of Zurich, Department of Economics, Working Paper* 409 (2022). https://doi.org/10.2139/ssrn.4055352

Md Abdullah Omar and Kazuo Inaba. 2020. Does financial inclusion reduce poverty and income inequality in developing countries? A panel data analysis. *Journal of Economic Structures* 9, 1 (2020), 1–25. https://doi.org/10.1186/s40008-020-00214-4

Cyn-Young Park and Rogelio V Mercado. 2021. Financial inclusion: New measurement and cross-country impact assessment 1. In *Financial Inclusion in Asia and Beyond*. Routledge, 98–128.

Thomas J Sargent and Neil Wallace. 1982. The real-bills doctrine versus the quantity theory: A reconsideration. *Journal of Political economy* 90, 6 (1982), 1212–1236. https://doi.org/10.1086/261118

Milo Schield. 2018. Statistical Literacy and the Lognormal Distribution. *Working paper* (2018).

Linda Schilling, Jesús Fernández-Villaverde, and Harald Uhlig. 2020. Central bank digital currency: When price and bank stability collide. *NBER working paper No. 28237* (2020). https://doi.org/10.2139/ssrn.3753147

Hernando Vargas. 2022. Some thoughts about the issuance of a retail CBDC in Colombia. *BIS Papers* (2022), 49.

Stephen D. Williamson. 2022. Central bank digital currency and flight to safety. *Journal of Economic Dynamics and Control* 142 (2022). https://doi.org/10.1016/j.jedc.2021.104146

Appendix A - Proofs

Preliminary material: Money demands when $\gamma^e = \gamma^d$ and $\gamma^e > \gamma^d$. The demand for paper money, $Z^c(R^d)$, is the same as when $\gamma^e < \gamma^d$. However, demands for bank deposits and CBDC differ. When $\gamma^e = \gamma^d$, money demands are given by

$$Z^d(R^d) = \begin{cases} 0 & \text{if } R^d < R^e \\ \int_{\tilde{y}(R^d)}^{y^h} z^d(y)d\Theta(y) & \text{if } R^d \geq R^e, \end{cases} \tag{A1}$$

$$Z^e(R^d) = \begin{cases} \int_{y^*}^{y^h} z^e(y)d\Theta(y) & \text{if } R^d < R^e \\ 0 & \text{if } R^d \geq R^e, \end{cases} \tag{A2}$$

where we assumed that workers indifferent between CBDC and bank deposits pick bank deposits. When $\gamma^e > \gamma^d$, they are given by

$$Z^d(R^d) = \begin{cases} 0 & \text{if } R^d < R^{d*} \\ \int_{\tilde{y}(R^d)}^{min(y^h, \hat{y}(R^d))} z^d(y)d\Theta(y) & \text{if } R^{d*} \leq R^d < R^e \\ \int_{\tilde{y}(R^d)}^{y^h} z^d(y)d\Theta(y) & \text{if } R^d \geq R^e, \end{cases} \tag{A3}$$

$$Z^e(R^d) = \begin{cases} \int_{y^*}^{y^h} z^e(y)d\Theta(y) & \text{if } R^d < R^{d*} \\ \int_{min(y^h, \hat{y}(R^d))}^{y^h} z^e(y)d\Theta(y) & \text{if } R^{d*} \leq R^d < R^e \\ 0 & \text{if } R^d \geq R^e. \end{cases} \tag{A4}$$

Proof of Lemma 1. Assume that a worker simultaneously carries moneys of types j and h. From the first-order condition derived from the worker's maximization problem, this implies $R^j = R^h \equiv R$. Then, the worker enjoys a lifetime utility $u(y - z^j - z^h) + \beta R(z^j + z^h) - \gamma^j - \gamma^h - \beta\tau$, where $\gamma^c = 0$. The worker could increase her lifetime utility by γ^h by selling all her units of instrument h against exactly as many units of instrument j, for a net lifetime benefit equal to γ^h. Thus, a worker would never carry two types of money. Second, making use of this result in the first-period budget constraint, which holds at equality, we obtain $z_i^j = y_i - c_{1,i} = y - (u')^{-1}(\beta R)$. An increase in income leads to one-to-one increase in money holdings. Finally, $\partial z_i^j/\partial R = -\beta/u''(c_{1,i}) > 0$ since $u''(c) < 0$ by assumption. $\square$

Proof of Lemma 2. Let $W(y, R^1, \gamma^1) > W(y, R^2, \gamma^2)$ and $R^1 > R^2$. Then, for any $y' > y$, it must be that $\beta R^1(y'-y)+W(y, R^1, \gamma^1) > \beta R^2(y'-y)+W(y, R^2, \gamma^2)$. Using the definition of W given by (5), this is equivalent to $W(y', R^1, \gamma^1) > W(y', R^2, \gamma^2)$. $\square$

Proof of Lemma 3. First note that $\lim_{y\to 0^+} W(y, R, \gamma) = \gamma - \beta\tau$, $\lim_{y\to\infty} W(y, R, \gamma) = \infty$, and that

$$\frac{\partial W(y, R, \gamma)}{\partial y} = \begin{cases} \beta R & \text{if } y > (u)'^{-1}(\beta R) \\ 0 & \text{otherwise.} \end{cases} \tag{A5}$$

Part (i). We show that there exists an income threshold $y^* > 0$ such that young workers with endowments larger than y^* would prefer to hold CBDC over paper money if and only if $R^e > R^c$. Making use of the preliminary results above, we know that $W(0, R^c, 0) > W(0, R^e, \gamma^e)$. If $R^c \geq R^e$, then $\partial W(y, R^c, 0)/\partial y \geq \partial W(y, R^e, \gamma^e)/\partial y$. Then, $W(y, R^c, 0) > W(y, R^e, \gamma^e)$ for all $y \geq 0$, implying that no young worker would prefer to hold the CBDC over paper money. Indeed, in this case, CBDC has both a higher fixed cost and a lower interest rate than paper money, making it strictly dominated for any endowment level. If $R^c < R^e$, $\partial W(y, R^c, 0)/\partial y \leq \partial W(y, R^e, \gamma^e)/\partial y$, with the inequality being strict for $y > (u')^{-1}(\beta R^e)$. Thus $W(y, R^c, 0)$ and $W(y, R^e, \gamma^e)$ cross uniquely for some $y > 0$. Because both objects are independent from R^d, the threshold y^* must also be independent of R^d.

Part (ii). We show that there exists an income threshold $\tilde{y}(R^d) > 0$ such that young workers with endowments larger than $\tilde{y}(R^d)$ would prefer to hold bank deposits over paper money if and only if $R^d > R^c$. Making use of the preliminary results above, we know that $W(0, R^c, 0) > W(0, R^d, \gamma^d)$. If $R^c \geq R^d$, then $\partial W(y, R^c, 0)/\partial y \geq \partial W(y, R^d, \gamma^d)/\partial y$. Then, $W(y, R^c, 0) > W(y, R^d, \gamma^d)$ for all $y \geq 0$, implying that no young worker would prefer to hold bank deposits over paper money since the former carries a higher fixed cost and a lower interest rate. If $R^c < R^d$, $\partial W(y, R^c, 0)/\partial y \leq \partial W(y, R^e, \gamma^e)/\partial y$, with the inequality being strict for $y > (u')^{-1}(\beta R^d)$. Thus, for a given R^d, $W(y, R^c, 0)$ and $W(y, R^d, \gamma^d)$ cross uniquely for some $y > 0$. Furthermore, going back to the definition of $\tilde{y}(R^d)$ and using total differentiation, we obtain

$$\frac{\partial \tilde{y}(R^d)}{\partial R^d} = -\frac{\tilde{y} - c_1^d}{R^d - R^c} \tag{A6}$$

when $\tilde{y}$ exists. So $\tilde{y}(R^d)$ is a decreasing function of R^d as long as $R^d > R^c$, which is required for $\tilde{y}$ to exist. Also, note that as R^d goes to infinity, $\tilde{y}$ goes to 0, while as R^d goes to R^c, $\tilde{y}$ goes to infinity.

Part (iii). We omit the proof of existence of $\hat{y}(R^d)$ as is it similar to (ii) above. Note that when $\hat{y}$ exists and $R^e \neq R^d$,

$$\frac{\partial \hat{y}(R^d)}{\partial R^d} = \frac{\hat{y} - c_1^d}{R^e - R^d}. \tag{A7}$$

Thus, $\hat{y}(R^d)$ is increasing when $\gamma^e > \gamma^d$ and decreasing when $\gamma^e < \gamma^d$, with an asymptote at $R^d = R^e$.

Part (iv). First, we claim that $\tilde{y}(R^d) = \hat{y}(R^d)$ if and only if $\tilde{y}(R^d) = y^*$. Once again the intuition is simple and follows from transitivity: if a worker is indifferent between CBDC and deposits ($y = \hat{y}(R^d)$), and also indifferent between paper money and deposits ($y = \tilde{y}(R^d)$), then he must be indifferent between paper money and CBDC ($y = y^*$). Formally, the proof goes as follows. Assume that $\tilde{y}(R^d) = y^*$. Then, by the definition of $\tilde{y}$, $W(y^*, R^c, 0) = W(\tilde{y}, R^d, \gamma^d)$. By the definition of y^*, we then have $W(y^*, R^e, \gamma^e) = W(\tilde{y}, R^d, \gamma^d)$. This implies $y^* = \hat{y}$, and therefore $\hat{y} = \tilde{y}$. This proves that $\tilde{y}(R^d) = y^* \Rightarrow \tilde{y}(R^d) = \hat{y}(R^d)$. Now assume that $\tilde{y}(R^d) = \hat{y}(R^d)$. Using the definition of $\tilde{y}$, we must have $W(\tilde{y}, R^c, 0) = W(\hat{y}, R^d, \gamma^d)$. Then, applying the definition of $\hat{y}$, it must be that $W(\tilde{y}, R^c, 0) = W(\hat{y}, R^e, \gamma^e)$. Finally, this means that $\hat{y} = y^*$ from the definition of y^*. So we proved $\tilde{y}(R^d) = \hat{y}(R^d) \Rightarrow \tilde{y}(R^d) = y^*$, and the proof is complete. Since y^* is independent of R^d and $\tilde{y}(R^d)$ is decreasing, R^{d*} exists, is unique, and inversely related to y^*. Finally, we can easily show that

$$R^{d*} \begin{array}{c} > \\ = \\ < \end{array} R^e \text{ if } \gamma^d \begin{array}{c} > \\ = \\ < \end{array} \gamma^e \tag{A8}$$

□

Proof of Lemma 4. We start from an economy without CBDC. The equilibrium interest rate on deposits, $\bar{R}^d$ must satisfy equilibrium condition (16), where the right-hand side is given by

$$Z^d(\bar{R}^d) = \int_{\tilde{y}(\bar{R}^d)}^{y^h} z^d(y) d\Theta(y). \tag{A9}$$

Now consider the introduction of a CBDC. If $R^e \leq R^c$, then the CBDC is strictly dominated by paper money for all workers and cannot impact the equilibrium. From now on, we assume $R^e > R^c$. Consider the case $\gamma^e < \gamma^d$. From (9), we can see that $Z^d(\bar{R}^d)$ will be impacted if $\bar{R}^d < R^{d*}$. By definition, this is equivalent to $y^* < \tilde{y}(\bar{R}^d)$. Using total differentiation, we can show that

$$\frac{\partial y^*}{\partial R^e} = -\frac{y^* - c_1^e}{R^e - R^c} < 0, \tag{A10}$$

so that the left-hand side of the inequality decreases in R^e. Since the right-hand side is independent of R^e, there exists a unique R^e such that $y^* = \tilde{y}(\bar{R}^d)$, which we denote $\bar{R}^e$. In addition, we can show that

$$\frac{\partial y^*}{\partial \gamma^e} = \frac{1}{\beta(R^e - R^c)} > 0, \tag{A11}$$

while $\tilde{y}(\bar{R}^d)$ is independent of γ^e. Thus, $\bar{R}^e$ must strictly increase in γ^e. The case when $\gamma^e = \gamma^d$ is derived in Andolfatto (2021). Finally, we skip the proof for the case when $\gamma^e > \gamma^d$ since the method is similar to the case when $\gamma^e < \gamma^d$. Intuitively, we can see on the right-most panel of Figure 2 that when $R^e < \bar{R}^d$, the introduction of a CBDC does not impact the demand for deposits. As R^e increases, there is eventually a point when $\hat{y}(\bar{R}^d) < y^h$, at which point the wealthiest individuals switch from deposits to CBDC, impacting equilibrium deposit demand. As γ^e increases, so do y^* and $\hat{y}(R^d)$, raising the threshold $\bar{R}^e$. □

Proof of Proposition 1. Denote $Z_1^d(R^d)$ the aggregate demand for deposits before the introduction of a CBDC. It is given by

$$Z_1^d(R^d) = \int_{min(y^h, \tilde{y}(R^d))}^{y^h} z^d(R^d) d\Theta(y). \tag{A12}$$

The equilibrium deposit rate in the pre-CBDC economy, R_1^d, is given by $Z_1^d(R^d) = (f')^{-1}(R^d)$. The left-hand side is increasing in R^d while the right-hand side is decreasing in R^d. Denote $Z_2^d(R^d)$ the aggregate demand for deposits after the introduction of a CBDC with $R^e > \bar{R}^e$ and $\gamma^e > 0$.

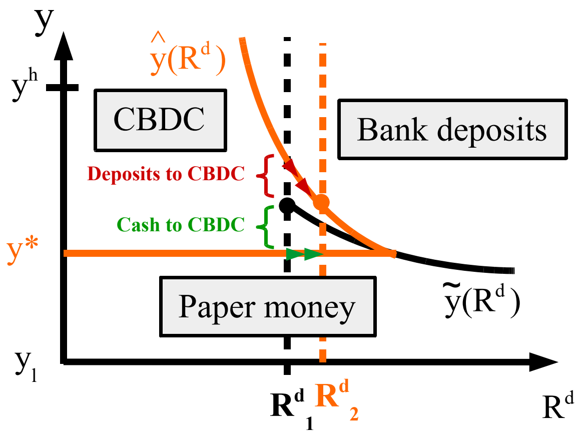


Figure A1: Impact of the introduction of a CBDC with $R^e > \bar{R}^e$ when $\gamma^e < \gamma^d$.

Assume that $\gamma^e < \gamma^d$. Then $Z_2^d(R^d)$ is given by (9). The equilibrium deposit rate after the introduction of the CBDC, R_2^d, is given by $Z_2^d(R^d) = (f')^{-1}(R^d)$. Compared to the original equilibrium equation, the right-hand side is unchanged. While the left-hand side is still increasing in R^d, it is now smaller given R^d. Formally, $Z_2^d(R^d) \leq Z_1^d(R^d)$ for any R^d, and the inequality is strict at $R^d = R_1^d$ since $R_1^d \leq R^{d*}$ by definition. Thus, the unique equilibrium must be such that $R_2^d > R_1^d$. Since $Int(R^d)$ is a decreasing function of R^d, we directly obtain that $Int^2 < Int^1$. Inclusion pre-CBDC was given by $Inc^1 = 1 - \Theta[\tilde{y}(R_1^d)]$. Post-CBDC it is given by $Inc^2 = 1 - \Theta(y^*)$. We showed earlier that $R^e > \bar{R}^e$ implies $y^* < \tilde{y}(R_1^d)$. Thus, $\Theta(y^*) < \Theta[\tilde{y}(R_1^d)]$, and therefore $Inc^2 > Inc^1$. The intuition behind this proof can be seen graphically in Figure A1. Starting from the pre-CBDC equilibrium in black, we can see that the introduction of a CBDC (in orange) reduces the demand for deposits at the original deposit rate R_1^d. Indeed, the poorest agents among those who used to carry bank deposits switch to CBDC. The deposit rate must then increase for the market to clear, as represented by the green and red arrows. This decreases intermediation. Pre-CBDC, workers with an endowment smaller than $\tilde{y}(R_1^d)$ were financially excluded. Post-CBDC, it is only the case for workers with an endowment smaller than y^*, so that inclusion expanded.

Now assume that $\gamma^e > \gamma^d$. Then $Z_2^d(R^d)$ is given by (A3), which increases in R^d. The equilibrium deposit rate after the introduction of the CBDC, R_2^d, must still satisfy $Z_2^d(R^d) = (f')^{-1}(R^d)$. Following the same reasoning as in the previous section, the deposit rate post-CBDC is higher than pre-CBDC, $R_2^d > R_1^d$, and financial intermediation decreases, $Int^2 < Int^1$. Inclusion is given by $Inc^2 = 1 - \Theta[\tilde{y}(R_2^d)]$. The threshold $\tilde{y}(R^d)$ decreases in R^d, so

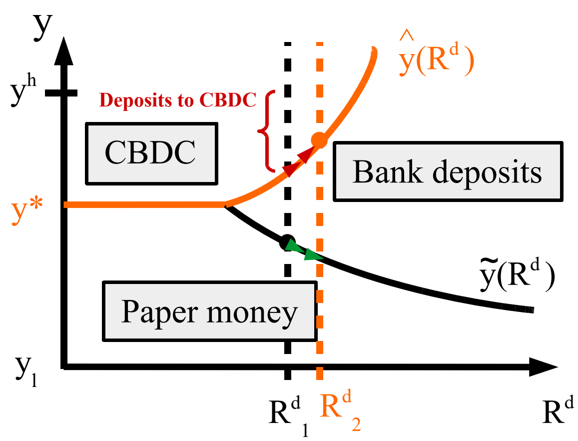


Figure A2: Impact of the introduction of a CBDC with $R^e > \bar{R}^e$ when $\gamma^e > \gamma^d$.

$\tilde{y}(R_2^d) < \tilde{y}(R_1^d)$, and therefore $Inc^2 > Inc^1$. Figure A2 provides a graphical illustration of the intuition behind this proof. Again, starting from the pre-CBDC equilibrium in black, we can see that the introduction of a CBDC (in orange) reduces the demand for deposits at the original deposit rate R_1^d. In this case, the reason is that the wealthiest agents among those who used to carry bank deposits switch to CBDC. The deposit rate must then increase for the market to clear, as represented by the green and red arrows. This decreases intermediation. Pre-CBDC, workers with an endowment smaller than $\tilde{y}(R_1^d)$ were financially excluded. Post-CBDC, it is only the case for workers with an endowment smaller than $\tilde{y}(R_2^d)$, which is a lower threshold, so that inclusion expands.

We omit the proof for the case $\gamma^e = \gamma^d$ as it corresponds to the case studied in Andolfatto (2021). $\square$

Proof of Proposition 2. We start from a CBDC equilibrium with $R^e > \bar{R}^e$. Also, assume for now that $\gamma^e < \gamma^d$. The aggregate demand schedule for deposits, $Z^d(R^d)$, is given by (9), and the equilibrium deposit rate must satisfy $Z^d(R^d) = (f')^{-1}(R^d)$. Consider a marginal increase in R^e (or similarly, a marginal decrease in γ^e). The right-hand side of the equilibrium condition is not impacted given R^d. The demand for deposits, on the left-hand-side, is impacted uniquely through $\hat{y}(R^d)$ (given R^d). Indeed, using total differentiation on the equation that defines $\hat{y}$, we obtain

$$\frac{\partial \hat{y}}{\partial R^e} = \frac{\hat{y} - c_1^e}{R^d - R^e} > 0 \text{ and } -\frac{\partial \hat{y}}{\partial \gamma^e} = \frac{1}{\beta(R^d - R^e)} > 0. \tag{A13}$$

Since $\hat{y}(R^d)$ is higher for any R^d after an increase in R^e (or a decrease in γ^e), aggregate demand for deposits must also be lower for any R^d (and strictly so around the original deposit rate). As a result, the equilibrium deposit rate strictly increases. Investment decreases with the deposit rate, so intermediation, Int, strictly decreases. Inclusion is given by $Int = 1 - \Theta(y^*)$. Following the proof for Lemma 4, we know that y^* strictly decreases following a marginal increase in R^e or a marginal decrease in γ^e. Therefore, inclusion must strictly increase.

Now consider the case when $\gamma^e < \gamma^d$. The aggregate demand schedule for deposits is given by (A3). Again, a marginal change in R^e or γ^e only impacts it through $\hat{y}$ given R^d, but the direction is reversed since now $R^d < R^e$:

$$\frac{\partial \hat{y}}{\partial R^e} = \frac{\hat{y} - c_1^e}{R^d - R^e} < 0 \text{ and } -\frac{\partial \hat{y}}{\partial \gamma^e} = \frac{1}{\beta(R^d - R^e)} < 0. \tag{A14}$$

Hence, a marginal increase in R^e or decrease in γ^e lower the threshold $\hat{y}$ for any R^d, leading to a decrease in the aggregate demand for deposits given R^d (strictly so around the original equilibrium deposit rate). Then, the equilibrium deposit rate strictly increases, and intermediation strictly decreases. Inclusion is given by $1 - \Theta[\tilde{y}(R^d)]$. We showed earlier that $\tilde{y}(R^d)$ strictly decreases in R^d, therefore inclusion must strictly increase.

We omit the proof for the case $\gamma^e = \gamma^d$ as it corresponds to the case studied in Andolfatto (2021). $\square$

Proof of Proposition 3. By construction, $R_1^d = R_2^d = \bar{R}$. It follows directly that $Int_1 = Int_2 = (f')^{-1}(\bar{R})$.

If $\gamma_1^e > \gamma_d$, then $\gamma_d^e < \gamma_1^e < \gamma_2^e$, and inclusion is given by $Inc_1 = Inc_2 = 1 - \Theta[\tilde{y}(\bar{R})]$.

If $\gamma_2^e < \gamma_d$, then $\gamma_1^e < \gamma_2^e < \gamma^d$. Denote $Z_1^d(\bar{R})$ and $Z_2^d(\bar{R})$ the aggregate demands for deposits under the two different CBDC designs. By market clearing we must have $Z_1^d(\bar{R}) = Z_2^d(\bar{R}) = (f')^{-1}(\bar{R})$. Since the aggregate demand schedule is given by (9), the previous equality implies that $\hat{y}(\bar{R}; \gamma_1^e, R_1^e) = \hat{y}(\bar{R}; \gamma_2^e, R_1^2)$. We know from earlier derivations that in the regime currently studied, an increase in γ^e decreases $\hat{y}$ everything else being equal. We also know that an increase in R^e increases $\hat{y}$ everything else equal. Hence, it must be that $R_2^e > R_1^e$. Now recall that when $\hat{y}$ exists and $R^e \neq R^d$,

$$\frac{\partial \hat{y}(R^d;\gamma^e,R^e)}{\partial R^d} = -\frac{\hat{y}(R^d;\gamma^e,R^e) - c_1^d(R^d)}{R^d - R^e}. \tag{A15}$$

We showed earlier that $\hat{y}(\bar{R};\gamma_1^e,R_1^e) = \hat{y}(\bar{R};\gamma_2^e,R_2^e)$. Thus, the expression for the slope of $\hat{y}(R^d)$ evaluated at $R^d = \bar{R}$ is the same under the two CBDC designs except for the value of R^e in the denominator. Since $R_2^e > R_1^e$, $\hat{y}$ is steeper under the second CBDC. This implies that the intersection between $\hat{y}(R^d)$ and $\tilde{y}(R^d)$ shifts up and to the left when going from the first to the second design, and thus y_2^* is higher than y_1^*. Because inclusion is given by $1 - \Theta(y^*)$, we finally obtain $Inc^2 \leq Inc^1$. $\square$

Appendix B - Quantitative Exercise

We derive the system of equation necessary to solve for the general equilibrium under the set of assumptions used for the quantitative exercise. In particular, in contrast to the assumptions used in the main text, preferences are assumed identical in the first and second period of a worker's life, following a CRRA specification, the deposit market runs according to a Cournot competition, and paper money limits the variety of goods that can be purchased (see details in Section 5).

Individual Money demands. Money demands are given by

$$z(R) = \frac{y}{1 + \beta^{-\frac{1}{\sigma}} R^{\frac{\sigma-1}{\sigma}}}. \tag{B1}$$

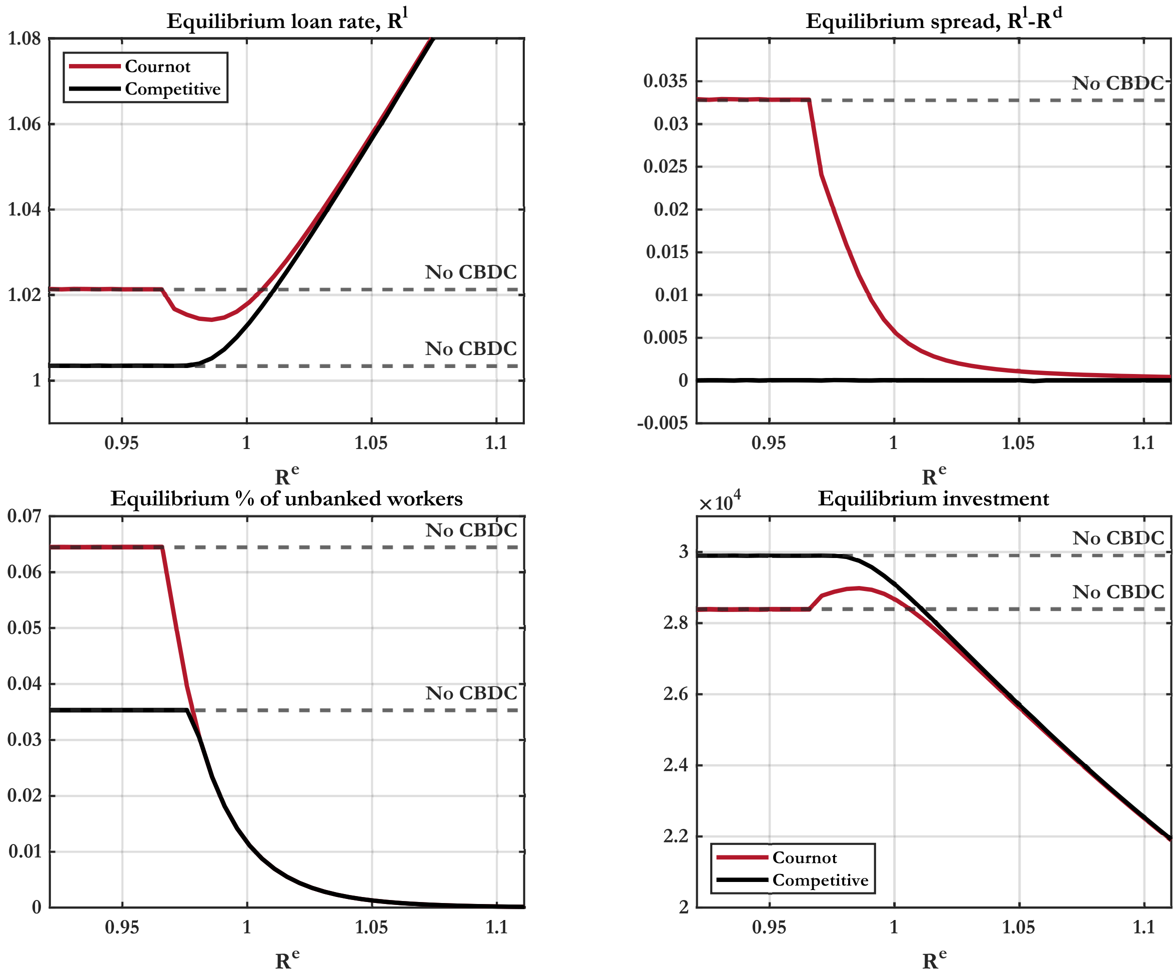
Money type thresholds. The definitions of y^*, $\tilde{y}(R^d)$ and $\hat{y}(R^d)$ remain identical to those denoted in Lemma 3, with the difference that the lifetime utility function W now takes the form

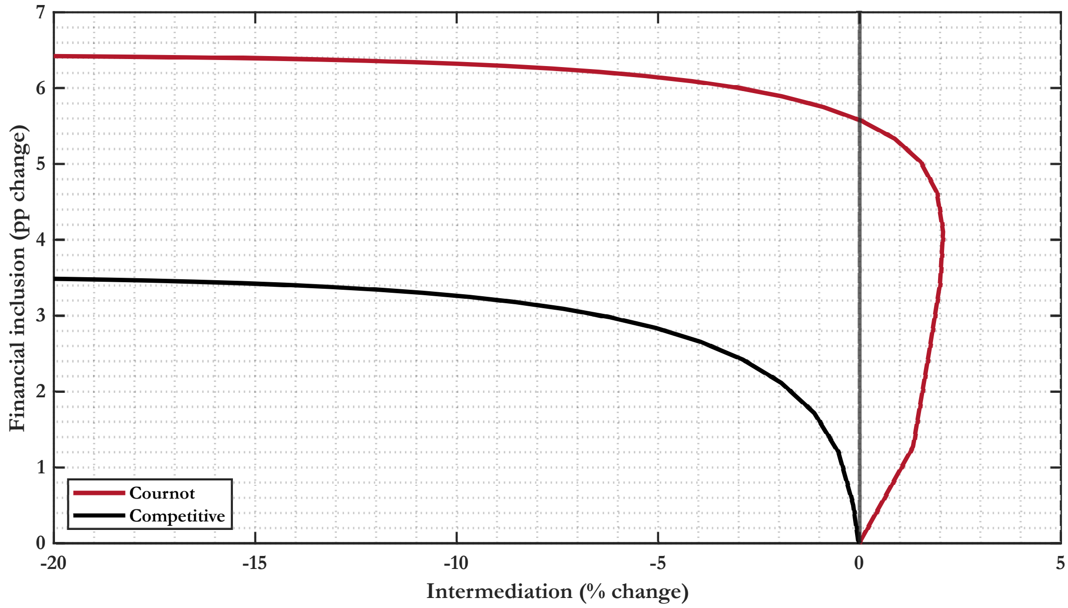
$$W(y,R,\gamma) = \frac{(y-z)^{1-\sigma}}{1-\sigma} + \beta\frac{(Rz)^{1-\sigma}}{1-\sigma} - \gamma - \beta\tau, \tag{B2}$$

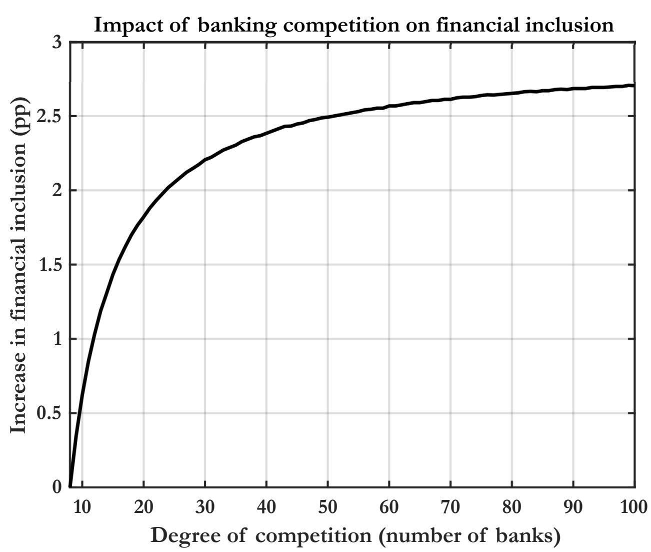
and that we plug in for $R = R^{c\prime}$ to compute the lifetime value of paper money.

Appendix C - Additional figures

Comparison between competition and Cournot equilibrium. The fixed cost ratio, γ^e/γ^e, is fixed to 0.6




Financial inclusion (pp change)
Intermediation (% change)
Cournot
Competitive
7
6
5
4
3
2
1
0
-20
-15
-10
-5
0
5


Impact of banking competition on financial inclusion
Increase in financial inclusion (pp)
Degree of competition (number of banks)
3
2.5
2
1.5
1
0.5
0
10
20
30
40
50
60
70
80
90
100