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# Entry and Exit, Unemployment, and the Business Cycle[*]

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**Abstract**

Establishment entry and exit is strongly correlated with output and unemployment. This paper examines how these linkages affect business cycle dynamics through the lens of a search and matching model augmented to include multi-worker establishments that endogenously enter and exit. Analytical results show cyclical entry and exit cause reallocation of inputs that amplifies and skews business cycle dynamics. When the model is calibrated to the data, it generates realistic asymmetry in output and unemployment, data-consistent counter-cyclical endogenous uncertainty, and a 55% higher welfare cost than the model without entry and exit.

**Keywords:** Unemployment; Firm Dynamics; Nonlinear; Skewness; Tail Risk; Uncertainty

**JEL Classifications**: E24; E32; E37; J63; L11

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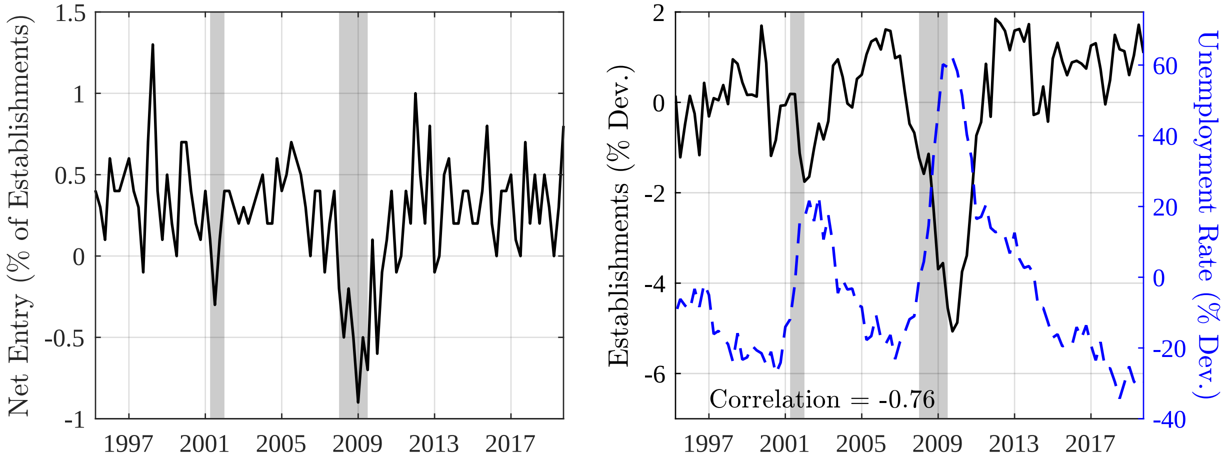


Figure 1: The left panel plots establishment net entry from the Business Employment Dynamics database as a percent of all establishments. The right panel plots the unemployment rate and establishments as percent deviations from trend. Both trends are computed using a Hamilton (2018) filter with an 8-quarter window.

## 1 Introduction

Data suggest that establishment entry and exit is an important feature of business cycles. For example, data from the Business Employment Dynamics database shows establishment entry and exit regularly occurs, the declines are particularly pronounced during the Great Recession, and the number of establishments is strongly correlated with the unemployment rate (Figure 1). Furthermore, data from the Business Dynamics Statistics database shows establishment entry on average accounts for $35.9\%$ of total job creation and establishment exit on average accounts for $33.4\%$ of total job destruction per year.[1] This paper develops a business cycle model that accounts for these features and shows cyclical entry and exit significantly amplifies and skews business cycle dynamics.

To conduct our analysis, we augment a textbook search and matching model with risk-averse households and capital to include multi-worker firms that endogenously enter and exit.[2] As in the firm dynamics literature, each firm faces decreasing returns to scale in production, which generates non-zero profits and creates motives for entry and exit in equilibrium.[3] This straightforward model extension permits analytical insights and a global nonlinear solution, which is essential to accurately quantify the effects of entry and exit on the business cycle. Our results are also directly comparable to simpler business cycle models that abstract from entry and exit or search frictions.

We first examine a special case of our model that allows us to derive an expression linking the number of firms and aggregate output. This uncovers three endogenous effects of entry and exit. First, entry and exit amplify the effects of aggregate productivity shocks on output. Second, entry

[1] Similarly, Haltiwanger (2012) finds the entry and exit shares of job creation and job destruction are $37\%$ and $31\%$.

[2] A "firm" refers to an establishment where multiple workers are employed (e.g., an office, factory, or headquarters).

[3] An alternative, but qualitatively identical approach, is to assume each firm produces a unique good in a monopolistically competitive environment. See Jaimovich and Floetotto (2008) and Bilbiie et al. (2012) for recent examples.

and exit amplify movements in the marginal product of labor and exacerbate the effects of labor market frictions. Third, job destruction due to firm exit creates negatively skewed output dynamics.

Our analytical results follow from the reallocation of inputs driven by the cyclical entry and exit of firms. For example, a negative productivity shock lowers output and firm profits. This causes some firms to exit, which further lowers productivity as inputs are reallocated to surviving firms who face decreasing returns to scale. With fewer firms, each with lower productivity, output falls by more than in a model without entry and exit. A similar feedback loop amplifies the labor market frictions. These dynamics are driven by the marginal product of labor, which inherits the amplified dynamics of output. Finally, negatively skewed output dynamics are due to endogenous feedback from firm exit that only occurs during recessions. When firms exit, it destroys jobs, decreasing employment and output. Lower employment then leads to lower profits and more exit in equilibrium.

We then turn to the quantitative properties of our model. To credibly assess the strength of the mechanisms, we solve the model nonlinearly by extending the algorithm in Richter et al. (2014) and calibrate the parameters governing entry and exit to target the shares of job creation and job destruction due to entry and exit in the data. The other parameters are set to match a range of benchmark moments. As validation, we confirm our model is consistent with the volatility of net entry and correlations between firm dynamics and the macroeconomy. To isolate the roles of labor market frictions, entry and exit, and their interaction, we compare the volatility and skewness of real activity and unemployment in our model with simpler frameworks that remove entry and exit or search frictions. When we remove search frictions, we model aggregate labor supply using employment lotteries over an indivisible labor choice at the worker level (Hansen, 1985; Rogerson, 1988).

Our calibrated model generates realistic asymmetries in output. In the data, the skewness of detrended output is $-0.59$, which implies that the economy is further below trend during recessions than it is above trend during expansions. For example, output is $6.2\%$ below trend at the $5$th percentile but only $4.5\%$ above trend at the $95$th percentile of its empirical distribution. Our baseline model generates skewness close to the data ($-0.49$). Entry and exit is crucial for this asymmetry, since it amplifies the nonlinearities of the search and matching frictions and generates negative skewness through endogenous job destruction when firms exit. Without these channels, the model only generates half of the empirical skewness ($-0.31$). The amplification channel alone generates a $20\%$ increase in output volatility, and it does not depend on search and matching frictions. Finally, the baseline model also generates data-consistent counter-cyclical endogenous uncertainty about future output (Jurado et al., 2015; Ludvigson et al., 2020). In the simpler models, the volatility of uncertainty is much smaller, and in the models without search frictions uncertainty is procyclical.

In addition to output, our model is consistent with two well-documented asymmetries in the labor market: peaks in the unemployment rate are further from trend than troughs (Dupraz et al., 2019; Sichel, 1993), and the unemployment rate rises much faster than it declines (Ferraro, 2018;

Neftci, 1984). Empirically, these asymmetries are measured by skewness in detrended levels ($0.6$) and growth rates ($1.3$). Our baseline model accounts for all of the skewness in detrended levels and $40\%$ of the skewness in growth rates. The same model without entry and exit generates about $75\%$ of the asymmetry in levels but none of the asymmetry in growth rates. This finding further highlights the importance of the interaction between entry and exit and unemployment dynamics.

Our model's departures from normality increase the welfare cost of business cycles. Following Lucas (1987, 2003), we find households would be willing to give up $0.42\%$ of lifetime consumption to forgo business cycle fluctuations. In contrast, the welfare cost is $0.27\%$ in our model without entry and exit and only $0.05\%$ in Lucas (2003) due to the much weaker skewness in consumption.

**Related Literature** The literature on search and matching frictions in a business cycle setting is extensive. Our analysis maintains the quantitative tradition of the early literature (Andolfatto, 1996; Den Haan et al., 2000; Merz, 1995), while incorporating the insights of the recent literature that abstracts from capital and risk aversion (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017; Mortensen and Nagypal, 2007; Shimer, 2005). Our results also capture the nonlinear congestion externality in Petrosky-Nadeau and Zhang (2017) and Petrosky-Nadeau et al. (2018).[4]

A few papers examine entry and exit in models without labor market search. Campbell (1998) shows entry and exit arise due to technological obsolescence and is a significant source of business cycle fluctuations. Jaimovich and Floetotto (2008) find cyclical variation in the number of competing firms leads to counter-cyclical markups and pro-cyclical variation in productivity. Bilbiie et al. (2012) show that endogenous product entry amplifies real business cycles in a linear setting. Empirically, Gourio et al. (2016) provide direct evidence that net firm entry is an important propagation mechanism for aggregate shocks. Relative to these two literatures, we are the first to combine establishment entry and exit with labor market frictions in a real business cycle model.[5]

To quantify the nonlinear interactions in our model, we compute the volatility, skewness, and uncertainty of key macro variables. These statistics complement the literature that documents empirical departures from normality. Neftci (1984) was the first to provide formal evidence that the U.S. unemployment rate rises faster in recessions than it declines in expansions. Sichel (1993) finds there is not only asymmetry in unemployment growth (steepness asymmetry), but also in the level of unemployment (deepness asymmetry). Acemoglu et al. (2017) find the distribution of U.S. output growth rates is negatively skewed and features more mass in the tails than a normal distribution implies. Finally, Bekaert and Popov (2019) document that similar patterns hold outside of the U.S.

Recent papers have emphasized various mechanisms that generate the non-normal features of macro data. For example, McKay and Reis (2008) explain that contractions in employment are

[4]Search and matching models in which firms employ one worker technically feature entry and exit. However, this setting is difficult to map to the data, and it is silent on how the number of firms interacts with labor market frictions.

[5]Shao and Silos (2013) use a model with firm entry and unemployment. However, they abstract from endogenous exit and focus on the cyclical costs of vacancy creation and its implications for income shares using a linear solution.

briefer and more violent than expansions using a business cycle model with asymmetric employment adjustment costs and endogenous technology adoption. Ilut et al. (2018) show the volatility and skewness in aggregate employment dynamics follow from asymmetry in how firms respond to information. Ferraro (2018) explains the asymmetry in unemployment dynamics through the lens of a labor search model with heterogeneity in workers' productivity. Dupraz et al. (2019) show downward wage rigidity generates negative skewness in employment and asymmetry in the speeds of recessions and recoveries.[6] We contribute to this growing literature by proposing a new source of non-normality driven by the combination of search and matching frictions and firm entry and exit.

Our focus on business cycles complements a large literature that builds on Hopenhayn (1992) and Hopenhayn and Rogerson (1993) and studies business dynamism using models with heterogeneous firms.[7] Those frameworks include a cross-sectional distribution of firm activity, but abstract from cyclical dynamics for numerical tractability. One notable exception is Clementi and Palazzo (2016). They also find that entry and exit propagate aggregate shocks, but their solution relies on a log-linear approximation of aggregate dynamics. This precludes analysis of higher-order moments, which is essential for understanding business cycles. They also abstract from labor market frictions, which we find have important interactions with firm dynamics. To account for these features, we model the distribution of firm activity as degenerate, and discipline its interaction with search frictions by calibrating to the shares of job creation and job destruction due to entry and exit.

The paper proceeds as follows. Section 2 lays out our model. Section 3 analytically derives the qualitative effects of entry and exit on output and employment. Section 4 describes our calibration strategy and solution method. Section 5 documents our quantitative findings. Section 6 concludes.

## 2 ENVIRONMENT

A representative household is populated by a unit mass of workers who are either employed or unemployed, so the fraction of employed workers coincides with the aggregate employment rate. Following Merz (1995), Andolfatto (1996), and Den Haan et al. (2000), workers pool their incomes together before choosing consumption, $C_t$, and investment, $I_t$. Given the specification in Jermann (1998), investment is subject to a capital adjustment cost, so the capital stock evolves according to

$$K_t = (1-\delta)K_{t-1} + \left(a_1 + \frac{a_2}{1-1/\nu}\left(\frac{I_t}{K_{t-1}}\right)^{1-1/\nu}\right)K_{t-1}, \tag{1}$$

where $0 < \delta \leq 1$ is the capital depreciation rate, $\nu > 0$ determines the size of the capital adjustment cost, and $a_1 = \delta/(1-\nu)$ and $a_2 = \delta^{1/\nu}$ are chosen so there are no adjustment costs in steady state.

[6] Andolfatto (1997) was one of the first to document that a simple model with a search and matching mechanism can generate the observed asymmetries in the unemployment rate because it features an asymmetric job destruction rate.

[7] At the frontier of the heterogenous firm literature, Bilal et al. (2019) estimate a heterogenous firm model with search frictions and decreasing returns to scale using rich cross-sectional data on establishment and worker dynamics.

The representative household solves

$$J_t^H = \max_{C_t, I_t} \ \ln C_t + \beta E_t J_{t+1}^H$$

subject to (1) and

$$C_t + I_t = w_t^n N_t + r_{k,t} K_{t-1} + D_t,$$

where $\beta$ is the discount factor, $w_t^n$ is the wage rate, $N_t$ is employment, $r_{k,t}$ is the return on capital, $D_t$ is dividends net of start up costs for entering firms and rebates of fixed production costs, and $E_t$ is an expectation operator conditional on information in period $t$. The optimality conditions imply

$$\frac{1}{a_2}\left(\frac{I_t}{K_{t-1}}\right)^{1/\nu} = E_t\left[x_{t+1}\left(r_{k,t+1} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_t}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_t}\right)\right], \quad (2)$$

where $x_{t+1} = \beta(C_t/C_{t+1})$ is the household's stochastic discount factor. This condition says the marginal cost of investing in period $t$ equals the discounted marginal benefit in period $t+1$, which includes the return on capital, the undepreciated capital stock, and the foregone adjustment costs.

**Production** The supply side of the economy consists of a mass $Z_t$ of competitive firms. Each firm $j \in [0, Z_t]$ produces the final good using the technology $y_{j,t} = a_t(k_{j,t}^{\alpha} n_{j,t}^{1-\alpha})^{\vartheta}$, where $k_{j,t}$ and $n_{j,t}$ are the capital and labor inputs employed by firm $j$. This specification follows the firm dynamics literature and uses decreasing returns to scale as a source of profits, which are necessary to generate motives for entry and exit (Bilal et al., 2019; Carvalho and Grassi, 2019; Clementi and Palazzo, 2016; Sedlacek, 2020). $a_t$ is total factor productivity (TFP), which is common across firms and follows

$$\ln a_{t+1} = (1-\rho_a)\ln \bar{a} + \rho_a \ln a_t + \sigma_a \varepsilon_{a,t+1}, \ 0 \leq \rho_a < 1, \ \varepsilon_a \sim \mathbb{N}(0,1). \quad (3)$$

Profit maximization implies all active firms choose the same amounts of capital and labor. Aggregating over $Z_t$ active firms yields the aggregate production function and input prices given by

$$Y_t = a_t Z_t^{1-\vartheta}(K_{t-1}^{\alpha} N_t^{1-\alpha})^{\vartheta}, \quad (4)$$

$$w_t = (1-\alpha)\vartheta Y_t / N_t, \quad (5)$$

$$r_{k,t} = \alpha\vartheta Y_t / K_{t-1}, \quad (6)$$

where $K_t = Z_t k_t$ is the aggregate capital stock and $N_t = Z_t n_t$ is aggregate employment. Similar to Clementi and Palazzo (2016), (4) shows that the number of active firms is an input to production. Equivalently, measured TFP $a_t Z_t^{1-\vartheta}$ has an endogenous component driven by firm entry and exit.

The optimality conditions for capital and labor, (5) and (6), yield an expression for aggregate flow profits $(1-\vartheta)Y_t/Z_t - \psi_y$, where $\psi_y$ is a fixed payment that each active firm must make to sell its final good. We assume these payments are rebated each period to the representative household.

**Firm Entry and Exit** Using the expression for flow profits, the value of an active firm satisfies

$$J^F_{A,t} = \max_{\xi_{t+1}\in[0,1]} (1-\vartheta)Y_t/Z_t - \psi_y + E_t[x_{t+1}\xi_{t+1}J^F_{A,t+1}], \tag{7}$$

where $\xi_{t+1}$ is the probability the firm chooses to remain active next period. We assume firms that decide to exit have no scrap value, which is a normalization given the fixed operating cost, $\psi_y > 0$.

A mass of inactive firms may enter the production sector after paying a one-time fixed cost $\psi_n \geq 0$. Therefore, the value of an inactive firm is $J^F_{I,t} = \max\{0, J^F_{A,t} - \psi_n\}$. Free entry implies $J^F_{A,t} \leq \psi_n$, while free exit of active firms implies $J^F_{A,t} \geq 0$. Hence $J^F_{I,t} = 0$ and $J^F_{A,t} \in [0, \psi_n]$. This means firm entry only occurs when $J^F_{A,t} = \psi_n$, firm exit only occurs when $J^F_{A,t} = 0$, and (7) implies

$$Z_t = \begin{cases} \frac{(1-\vartheta)Y_t}{\psi_y+\psi_n-E_t[x_{t+1}\xi_{t+1}J^F_{A,t+1}]}, & J^F_{A,t} = \psi_n, \\ \frac{(1-\vartheta)Y_t}{\psi_y-E_t[x_{t+1}\xi_{t+1}J^F_{A,t+1}]}, & J^F_{A,t} = 0, \\ Z_{t-1}, & J^F_{A,t} \in (0, \psi_n), \end{cases} \tag{8}$$

so $Z_t$ is endogenous and time-varying. The probability a firm chooses to remain active is given by

$$\xi_{t+1} = \mathbb{I}\{J^F_{A,t+1} > 0\} + (Z_{t+1}/Z_t)\mathbb{I}\{J^F_{A,t+1} = 0\}, \tag{9}$$

where $\mathbb{I}$ is an indicator function that equals 1 if the condition is true and 0 otherwise. This says that if $J^F_{A,t+1} > 0$, then $\xi_{t+1} = 1$ because all active firms remain active. If $J^F_{A,t+1} = 0$, then $\xi_{t+1} = Z_{t+1}/Z_t$, which is the fraction of active firms that choose to remain active in the next period.

**Search and Matching** Entering period $t$, there are $N_{t-1}$ employed workers and $U_{t-1}$ unemployed workers. Then a constant fraction $\bar{s}$ of the employed workers exogenously lose their jobs. The remaining fraction $1-\bar{s}$ endogenously lose their jobs with probability $1-\xi_t$, which is the probability an active firm exits the product market. Therefore, the endogenous job separation rate is given by

$$s_t = \bar{s} + (1-\bar{s})(1-\xi_t). \tag{10}$$

Newly separated workers search for a job in the same period as their job loss, but these workers have less time to search for a job than those who became unemployed in a previous period. Let $\chi \in [0,1]$ denote the fraction of a period that newly unemployed workers search for work in the same period as their job loss. Then, the number of unemployed searching for work in period $t$ is given by

$$U^s_t = U_{t-1} + \chi s_t N_{t-1}. \tag{11}$$

Shimer (2005) sets $\chi$ equal to 0.5 when constructing a measure of the monthly job finding rate in the data. We follow Bernstein et al. (2020) and calibrate $\chi$ to match the average unemployment rate.

Following Den Haan et al. (2000), the number of new matches in period $t$ is given by

$$M_t = U_t^s V_t/((U_t^s)^\iota + V_t^\iota)^{1/\iota},$$

where $\iota > 0$ determines the curvature of the matching function and $V_t$ is vacancy postings. Define $\theta_t \equiv V_t/U_t^s$ as labor market tightness. Then job-filling rate, $q_t$, and job-finding rate, $f_t$, are given by

$$q_t = M_t/V_t = 1/(1+\theta_t^\iota)^{1/\iota}, \tag{12}$$

$$f_t = M_t/U_t^s = \theta_t q_t. \tag{13}$$

Following Blanchard and Galí (2010), we assume newly matched workers begin employment in the same period they are matched with a firm, so aggregate employment evolves according to

$$N_t = (1-s_t)N_{t-1} + q_t V_t. \tag{14}$$

The unemployment rate $U_t$ includes anyone who is not employed in period $t$, so it is given by

$$U_t \equiv U_t^s - M_t = 1 - N_t. \tag{15}$$

**Employment Agencies** Employment agencies facilitate labor market operations by posting vacancies on behalf of firms and by selling the labor services of the representative household.[8] They sell each unit of labor to active firms at the competitive rate $w_t$ and then pay workers a wage $w_t^n$ determined by Nash Bargaining (described below). The representative employment agency solves

$$J_t^E = \max_{N_t, V_t}\ (w_t - w_t^n)N_t - \kappa V_t + E_t[x_{t+1}J_{t+1}^E] \tag{16}$$

subject to (14) and $V_t \geq 0$, where $\kappa > 0$ is the per-period vacancy posting cost. Optimality implies

$$\lambda_{N,t} = (\kappa - \lambda_{V,t})/q_t, \tag{17}$$

$$\lambda_{N,t} = w_t - w_t^n + E_t[x_{t+1}(1-s_{t+1})\lambda_{N,t+1}], \tag{18}$$

where $\lambda_{N,t}$ and $\lambda_{V,t}$ are the Lagrange multipliers on (14) and the inequality constraint. Thus, $\lambda_{N,t}$ is the marginal surplus value of a new match to the agency at time $t$. Combining (17) and (18) yields

$$\tfrac{\kappa-\lambda_{V,t}}{q_t} = w_t - w_t^n + E_t[x_{t+1}(1-s_{t+1})\tfrac{\kappa-\lambda_{V,t+1}}{q_{t+1}}], \tag{19}$$

which determines vacancy creation by equating the marginal cost of posting an additional vacancy with the marginal benefit of an additional worker. The benefit includes the time $t$ profit from the new match plus the present value of the foregone vacancy posting cost net of time $t+1$ separations.

[8] If active firms directly posted vacancies, the vacancy posting decision would depend on each firm's entire history.

**Wages** As noted by Hall (2005), the match surplus created by search and matching frictions leads to wage indeterminacy in the absence of additional model structure. Furthermore, Ljungqvist and Sargent (2017) emphasize that the choice and calibration of the wage determination mechanism has strong implications for the volatility of unemployment. In light of these issues, we follow the bulk of the literature and assume wages are determined via Nash bargaining between an employed worker and the employment agency. We then calibrate the parameters of this wage setting protocol to ensure that the model generates realistic labor market volatility. To operationalize Nash bargaining, we define the total surplus of a new match as $\Lambda_t = \lambda_{N,t} + V_{E,t} - V_{U,t}$, where $V_{E,t}$ and $V_{U,t}$ satisfy

$$V_{E,t} = w_t^n + E_t[x_{t+1}((1 - s_{t+1}(1 - \chi f_{t+1}))V_{E,t+1} + s_{t+1}(1 - \chi f_{t+1})V_{U,t+1})],$$
$$V_{U,t} = b + E_t[x_{t+1}(f_{t+1}V_{E,t+1} + (1 - f_{t+1})V_{U,t+1})].$$

$V_{E,t}$ is the capitalized value of employment for the worker, while $V_{U,t}$ and $b$ define the worker's highest credible payoff from walking away from the negotiation (i.e., the worker's outside option).

The equilibrium wage rate maximizes $(V_{E,t} - V_{U,t})^\eta \lambda_{N,t}^{1-\eta}$, where $\eta \in [0, 1]$ is the household's bargaining weight. Optimality implies $V_{E,t} - V_{U,t} = \eta\Lambda_t$ or, equivalently, $\lambda_{N,t} = (1 - \eta)\Lambda_t$. To derive the equilibrium wage rate, plug (18) in for $\lambda_{N,t}$ and combine the two conditions to obtain

$$w_t^n = \eta(w_t + \kappa E_t[x_{t+1}(1 - \chi s_{t+1})\theta_{t+1}]) + (1 - \eta)b. \tag{20}$$

The household's wage rate in period $t$ is a weighted average of the firm's value of a new match and the worker's outside option $b$. The firm's value of a new worker includes the additional output produced plus the discounted expected value of the worker net of separations that occur in period $t+1$.

**Equilibrium** After imposing dividends implied by (7) and (16), the resource constraint is given by

$$C_t + I_t + \kappa V_t = Y_t. \tag{21}$$

A competitive equilibrium includes sequences of prices $\{\lambda_{N,t}, \lambda_{V,t}, w_t^n, w_t, r_{k,t}\}_{t=0}^\infty$, quantities $\{C_t, N_t, U_t, U_t^s, V_t, I_t, K_t, Y_t, J_{A,t}^F, Z_t, \xi_t, q_t, f_t, s_t\}_{t=0}^\infty$, and exogenous variables $\{a_t\}_{t=1}^\infty$ that satisfy (1)-(15), (18)-(21) and $\lambda_{V,t}V_t = 0$, given the initial state $\{N_{-1}, Z_{-1}, K_{-1}, a_0\}$ and shocks $\{\varepsilon_{a,t}\}_{t=1}^\infty$.

## 3 UNDERSTANDING THE MECHANISM

This section analytically uncovers three channels through which endogenous firm entry and exit affect the macroeconomy: (1) Entry and exit dynamics amplify the transmission of aggregate technology shocks to output; (2) Entry and exit amplify the effects of labor market frictions; (3) The destruction of jobs caused by endogenous firm exit creates negatively skewed output dynamics.

Figure 2 shows how these mechanisms create endogenous feedback loops in the economy. To

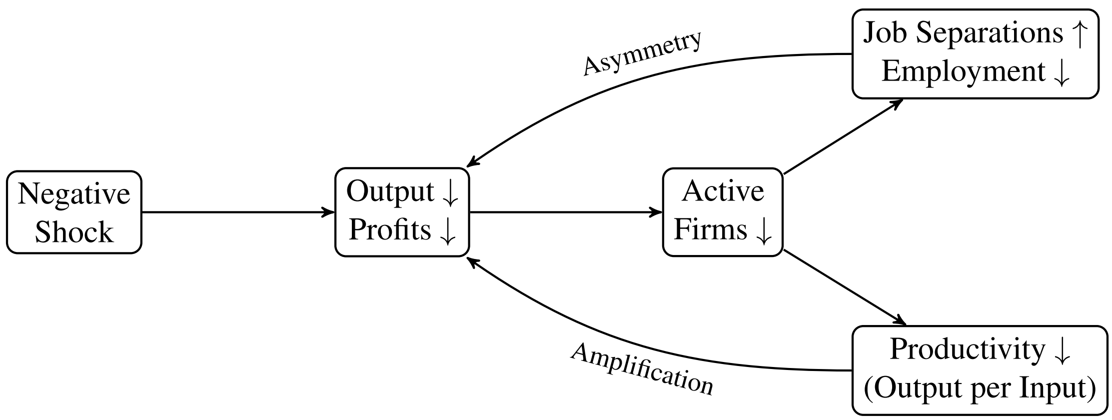


Figure 2: Endogenous sources of amplification and asymmetry due to firm entry and exit.

tractably analyze the amplification and asymmetry channels, we consider a special case of our economy in which there is no capital depreciation or investment and the entry cost is zero, $\psi_n = 0$. In this case, the capital stock is fixed at $\bar{K}$ and entry and exit ensure the value of active firms is zero, $J^F_{A,t} \equiv 0$. Applying these conditions to the active firm value function (7) yields an expression for the mass of active firms, $Z_t = (1-\vartheta)Y_t/\psi_y$, so the response of $Z_t$ to a change in $Y_t$ is given by

$$d\ln Z_t = d\ln Y_t. \tag{22}$$

Intuitively, the number of active firms is increasing in profits, which scale with aggregate output.

### 3.1 Output Amplification

To see the effects of $Z_t$ on output, differentiate (4) to obtain

$$d\ln Y_t = d\ln a_t + (1-\vartheta)d\ln Z_t + \vartheta(1-\alpha)d\ln N_t. \tag{23}$$

All else equal, (23) shows output is increasing in the number of active firms, and output responds more when there are stronger decreasing returns to scale (i.e., a lower $\vartheta$). To understand the microeconomic foundation of this relationship, consider the case of firm entry. When new firms enter, some capital and labor are reallocated from incumbents to entrants. Since each firm faces decreasing returns to scale, the decline in inputs per firm causes an increase in firm productivity and a boost in production, holding aggregate inputs fixed. This mechanism is similar to models with heterogeneous firms (Clementi and Palazzo, 2016) in which average firm productivity is endogenously pro-cyclical. It also finds support in Gourio et al. (2016), who show that an increase in net firm entry leads to an increase output and productivity when controlling for economic conditions.

We then substitute for $d\ln Z_t$ using (22) and compare to the model without entry and exit (NE):

$$d\ln Y_t = (1/\vartheta)d\ln a_t + (1-\alpha)d\ln N_t, \tag{24}$$

$$d\ln Y^{NE}_t = d\ln a_t + \vartheta(1-\alpha)d\ln N_t. \tag{25}$$

These results demonstrate that endogenous firm entry and exit cause output to respond more aggressively to changes in productivity and aggregate employment. Intuitively, (22) implies that firm entry responds positively to increases in output. Since output is also increasing in the number of active firms by (23), a positive feedback loop amplifies the dynamics of output relative to the case without entry and exit. Quantitatively, (24) shows the amount of amplification due to entry and exit is governed by $\vartheta$, which controls the share of aggregate output attributable to firm profits.

### 3.2 LABOR MARKET AMPLIFICATION

Firm entry and exit is linked to changes in unemployment through a frictional labor market à la Diamond-Mortensen-Pissarides. At the center of that market is the vacancy creation equation (19), which describes how vacancies (and hence the job filling rate) respond to the marginal product of labor, $w_t$. When $w_t$ increases, the marginal benefit of an employment match increases, and employment agencies post more vacancies in equilibrium.

To see the effect on $w_t$, differentiate (5) and substitute for output using (24) and (25) to obtain

$$d\ln w_t = (1/\vartheta) d\ln a_t - \alpha d\ln N_t, \tag{26}$$

$$d\ln w_t^{NE} = d\ln a_t - (1-\vartheta(1-\alpha)) d\ln N_t. \tag{27}$$

Comparing these results shows entry and exit amplify the dynamics of $w_t$, similar to how it affects aggregate output dynamics. Entry and exit strengthen the response to changes in productivity and weaken the offsetting response to changes in aggregate employment. Intuitively, $w_t$ inherits the amplified dynamics of output. Since $w_t$ governs the payoff to vacancy creation, entry and exit amplify the dynamics of the frictional labor market between employment agencies and households.

### 3.3 OUTPUT ASYMMETRY

An important feature of our economy is that the endogenous exit of firms naturally leads to the destruction of jobs and an increase in unemployment, all else equal. In contrast, firm entry does not lead to a direct drop in unemployment since new firms must still hire labor through the frictional search and matching process. This asymmetric relationship between firm entry and exit and unemployment leads to negatively skewed output dynamics in equilibrium.

From (10), the response of the job separation rate to a change in the number of firms is given by

$$ds_t = -(1-\bar{s})(Z_t/Z_{t-1})\mathbb{I}\{d\ln Z_t < 0\} d\ln Z_t, \tag{28}$$

where the indicator function equals one when firms exit (i.e., when $d\ln Z_t < 0$) and zero otherwise.

Combining (28) with the law of motion for aggregate employment (14) imples

$$d\ln N_t = (1-\bar{s})(n_{t-1}/n_t)\mathbb{I}\{d\ln Z_t < 0\} d\ln Z_t + d(q_t V_t)/N_t, \tag{29}$$

where $n_t = N_t/Z_t$ is employment per firm and $d(q_t V_t)$ measures the change in new hires due to vacancy creation, which is discussed below. This expression shows firm exit causes declines in employment through endogenous separations while firm entry does not directly affect employment.

Combining (22), (24), and (29) shows the asymmetric response of aggregate output is given by

$$d\ln Y_t = \frac{1}{1-(1-\alpha)(1-\bar{s})\frac{n_{t-1}}{n_t}\mathbb{I}\{d\ln Z_t < 0\}}\left(\frac{1}{\vartheta}d\ln a_t + (1-\alpha)\frac{d(q_tV_t)}{N_t}\right). \tag{30}$$

This shows firm exit endogenously strengthens the response of output to changes in productivity and new hires, unlike firm entry. Since firm exit occurs when output is declining, output is negatively skewed. Intuitively, job destruction that occurs when firms exit amplifies the decline in employment and output during recessions. Quantitatively, (30) shows the amount of asymmetry due to entry and exit is sensitive to $\alpha$, which controls the share of aggregate output attributable to labor.

Our frictional labor market is also subject to the nonlinearities generated by congestion externalities as explained in Petrosky-Nadeau and Zhang (2017) and Petrosky-Nadeau et al. (2018). We emphasize that these nonlinearities operate in tandem with the entry and exit mechanisms in the model. As our quantitative results will show, the combination of these frictions with entry and exit is necessary to generate data-consistent higher-order moments in real activity and unemployment.

## 4 CALIBRATION AND SOLUTION METHOD

This section explains how we calibrate and solve our nonlinear model. It first describes the empirical targets we use to discipline the parameters and then plots cross-sections of our model solution.[9]

**4.1 CALIBRATION** The model is calibrated at a monthly frequency to capture employment flows in the data. The discount factor $\beta$ is set to $0.9983$, which implies a $2\%$ average annual real interest rate. We calibrate the rest of the parameters to match data from 1955-2019. The capital depreciation rate, $\delta$, is set to $0.0077$ to match the annual average rate on private fixed assets and durable goods. Table 1 summarizes the other parameter values and their corresponding empirical targets.

The parameters governing the returns to scale, $\vartheta$, and the weight on capital in production, $\alpha$, are set to $0.8656$ and $0.2889$ to match the average profit and labor shares of income in the non-farm business sector. We view these choices as conservative. Using lower values of $\vartheta$ or $\alpha$ would generate greater amplification and asymmetries, as Section 3 shows. Our value of $\vartheta$ lies in the middle of the range of micro-estimates of returns to scale and is higher than Clementi and Palazzo (2016). The Fernald (2012) average estimate of $0.33$ for the capital share of income maps to an $\alpha$ of $0.2260$.

To discipline the amount of entry and exit in our model, we appeal to the shares of job creation and job destruction attributable to establishment entry and exit, computed using annual data from the Business Dynamics Statistics database (1978-2018). Over rolling ten year horizons, we find establishment entry on average accounts for $35.9\%$ of total job creation and establishment exit on

[9] Appendix A describes our data sources and how the time series are transformed to construct our empirical targets.

| Model Parameter | | Value | Empirical Target | Data | Model |
|---|---|---|---|---|---|
| Returns to Scale | $\vartheta$ | 0.8656 | Average Profit Share of Income | 13.44 | 13.44 |
| Curvature in Production | $\alpha$ | 0.2889 | Average Labor Share of Income | 61.55 | 61.55 |
| Intra-Period Search Duration | $\chi$ | 0.5195 | Average Unemployment Rate | 5.89 | 5.93 |
| Vacancy Posting Cost | $\kappa$ | 0.1139 | Average Job-Finding Rate | 42.14 | 42.38 |
| Exogenous Separation Rate | $\bar{s}$ | 0.0322 | Average Job Separation Rate | 3.27 | 3.25 |
| Nash Bargaining Weight | $\eta$ | 0.0990 | Wage-Productivity Elasticity* | 0.47 | 0.47 |
| Outside Option | $b$ | 0.9702 | Unemployment Standard Deviation* | 22.29 | 22.22 |
| Matching Function Curvature | $\iota$ | 0.6914 | Vacancy Standard Deviation* | 23.03 | 22.77 |
| Investment Adjustment Cost | $\nu$ | 7.1206 | Investment Growth Standard Deviation | 2.13 | 2.48 |
| Firm Entry Cost | $\psi_n$ | 0.0680 | Entry Share of Job Creation | 35.92 | 35.95 |
| Fixed Production Cost | $\psi_y$ | 0.2060 | Exit Share of Job Destruction | 33.38 | 33.76 |
| Productivity Persistence | $\rho_a$ | 0.9473 | Output Growth Autocorrelation | 0.31 | 0.22 |
| Productivity Shock SD | $\sigma_a$ | 0.0045 | Output Growth Standard Deviation | 0.86 | 0.95 |

Table 1: Calibrated parameter values and empirical targets. Monthly time series are averaged to a quarterly frequency. An asterisk denotes a moment based on detrended data, where the empirical trend is computed using a Hamilton (2018) filter with an 8-quarter window and the model trend is equal to the simulated mean.

average accounts for $33.4\%$ of total job destruction. In the model, we first set $\psi_y$ to $0.206$, so firm value is normalized to $0$ in the deterministic steady state. Given $\psi_y$ and the exit boundary at $0$, $\psi_n$ then determines the dynamics of $Z_t$ and the entry and exit shares of job creation and job destruction. To match the shares in the data, we calibrate $\psi_n$ to $0.068$, which equals $0.4\%$ of annual output in the deterministic steady state. We validate this choice using several untargeted moments (described below) and argue that it provides a conservative account of entry and exit in the data.[10]

Following Bernstein et al. (2020), we calibrate $\chi, \kappa, \bar{s}, \eta, b$, and $\iota$ to target six labor market moments.[11] Specifically, we set $\chi$ and $\kappa$ to target an average unemployment rate of $5.89\%$ and an average job finding rate of $42.14\%$. We set $\bar{s}$ to target an average job separation rate of $3.27\%$ and set $\eta$ to match the wage-productivity elasticity of $0.47$. We calibrate $b$ and $\iota$ to target the standard deviations of detrended unemployment and vacancies, which equal 22.29 and 23.03, respectively.

Finally, we set the autocorrelation of productivity, $\rho_a$, and the standard deviation of the productivity shock, $\sigma_a$, to target the autocorrelation and standard deviation of per capita output. Similarly, the curvature of the investment adjustment cost function, $\nu$, is set to target the standard deviation of per capita investment. Given that we are interested in detrended levels and growth rates, we strike a balance between targeting both sets of moments, instead of perfectly matching one set. Our calibration of $\sigma_a$ is smaller than in other papers in the literature (e.g., Gertler and Trigari, 2009; Hagedorn and Manovskii, 2008) because firm entry and exit endogenously generates volatility in the model.

[10] Our calculation of the empirical entry and exit shares is robust to the horizon and consistent with Haltiwanger (2012). Appendix B describes how we compute the shares in the model and then provides support for our methodology. It also shows our model is consistent with the amount net job creation due to the net entry of establishments in the data.

[11] Recent work shows the Hamilton (2018) filter is more accurate than a Hodrick and Prescott (1997) filter in time series that are first-difference stationary, such as the unemployment rate (Hodrick, 2020). Therefore, we detrend firm and labor market data using a Hamilton (2018) filter rather than the more common Hodrick and Prescott (1997) filter.

| Moment | Data | Model | Moment | Data | Model |
|---|---|---|---|---|---|
| $CORR(Z, U)$ | −0.76 | −0.91 | $CORR(U, s)$ | 0.44 | 0.33 |
| $CORR(Z, Y)$ | 0.67 | 0.89 | $CORR(Y, s)$ | −0.60 | −0.33 |
| $CORR(Z, s)$ | −0.33 | −0.13 | $SD(\Delta\tilde{Z})$ | 0.35 | 0.44 |

(a) Quarterly moments. Establishment data is from the Business Employment Dynamics database.

| Moment | Data | Model | Moment | Data | Model |
|---|---|---|---|---|---|
| $CORR(Z, U)$ | −0.76 | −0.92 | $CORR(U, s)$ | 0.40 | 0.42 |
| $CORR(Z, Y)$ | 0.64 | 0.91 | $CORR(Y, s)$ | −0.46 | −0.42 |
| $CORR(Z, s)$ | −0.28 | −0.19 | $SD(\Delta\tilde{Z})$ | 1.41 | 1.06 |

(b) Annual moments. Establishment data is from Business Dynamics Statistics database.

Table 2: Model validation. $SD$ and $CORR$ denote standard deviation and correlation across time. Monthly values are averaged to a quarterly or annual frequency. $\Delta\tilde{Z}_t \equiv (Z_t - Z_{t-1})/((Z_t + Z_{t-1})/2)$ to match the definition of net entry in the data. The data in levels is detrended, where the empirical trend is computed using a Hamilton (2018) filter with an 8-quarter window and the model trend is equal to the simulated mean.

**Validation of Firm Dynamics** The interaction between entry and exit and labor market dynamics is crucial for our results. While we calibrated the parameters so the model is consistent with the share of employment variation due to entry and exit, it is important to examine other dimensions.

Table 2 reports the empirical and model-implied volatility of net entry and the correlations between cyclical variation in the number of active firms and a range of outcomes: the unemployment rate, output, and the job separation rate. To examine the feedback from job destruction, we also report the correlations between the job separation rate and both the unemployment rate and output. The empirical moments are computed using data from the quarterly Business Employment Dynamics database (1992-2019) and the annual Business Dynamics Statistics database (1978-2018).

In all cases, the model-implied correlations are the same sign as in the data. The model reproduces the strong negative correlation between the number of active firms and the unemployment rate used to motivate our study (see Figure 1) as well as the positive correlation with output. The model also generates similar correlations with the job separation rate.[12] Finally, while the model slightly overstates the volatility of net entry at a quarterly frequency, it understates the annual volatility. Taken together, these results provide strong evidence that our model accurately reflects the relationships between establishments, labor markets, and real activity in the data and that our calibration does not overstate the role of firm entry and exit in generating cyclical net job creation.

### 4.2 Solution Method and Policy Functions

Endogenous firm entry and exit create inherent nonlinearities that endogenously generate higher-order moments in our model. To accurately capture these effects, we solve the model globally by adapting the policy function iteration

[12] Crane et al. (2020) show permanent and temporary establishment exit is positively correlated with unemployment. They also find evidence that firm exit rose in 2020 due to the pandemic, although official statistics are not yet available.

algorithm in Richter et al. (2014) to our setting. The algorithm minimizes the Euler equation errors on every node in the discretized state space. It then computes the maximum distance between the policy functions on any node and iterates until that distance falls below a given tolerance criterion.[13]

We make two modifications to the numerical algorithm since our model has inequality constraints that are absent from standard real business cycle models. The first stems from the nature of endogenous firm entry and exit. Let $\Delta Z_t = Z_t - Z_{t-1}$ denote the net entry of active firms. Entry ($\Delta Z_t > 0$) occurs when $J^F_{A,t} = \psi_n$, while exit ($\Delta Z_t < 0$) occurs when $J^F_{A,t} = 0$. Therefore, the solution must satisfy the following two complementary slackness conditions: $J^F_{A,t} \min\{0, \Delta Z_t\} = 0$ and $(\psi_n - J^F_{A,t}) \max\{0, \Delta Z_t\} = 0$. We numerically impose these constraints on the firm by setting

$$J^F_{A,t} = \min\{\max\{0, \mu_{A,t}\}, \psi_n\}, \quad \Delta Z_t = \mu_{A,t} - J^F_{A,t}$$

where $\mu_{A,t}$ is an auxiliary variable that is continuous in the state of the economy. $\mu_{A,t}$ equals the value of active firms when $J^F_{A,t}$ is between the entry and exit boundaries, so the number of firms is unchanged from the last period. At the boundaries, the gap $\mu_{A,t} - J^F_{A,t}$ determines net firm entry.

The second modification stems from the constraint on vacancies, $V_t \geq 0$. Following Garcia and Zangwill (1981), we impose this constraint using a second auxiliary variable $\mu_{V,t}$ that satisfies

$$V_t = \max\{0, \mu_{V,t}\}^2, \quad \lambda_{V,t} = \max\{0, -\mu_{V,t}\}^2.$$

$\mu_{V,t}$ maps into vacancies when $V_t > 0$ and the Lagrange multiplier, $\lambda_{V,t}$, when $V_t = 0$. The two conditions are squared to guarantee that they are sufficiently smooth for the algorithm to converge.

To highlight the influence of entry and exit, Figure 3 plots cross-sections of the policy functions for the active firm value, $J^F_{A,t}$, and net entry of active firms, $\Delta Z_t$, as a function of the initial number of active firms and initial level of productivity, which are shown in percent derivations from their steady states. The initial levels of employment and the capital stock are fixed at their steady states.

The nonlinearities in the model are apparent. When $\mu_{A,t}$ exceeds the entry cost, $\psi_n = 0.068$, there is sufficient firm entry to prevent further increases in the value of active firms. This is shown in the upper plateau of the left panel and the upward sloping portion of the right panel. Intuitively, the solution indicates that firm entry is increasing in productivity but decreasing in the number of active firms. Similarly, when $\mu_{A,t}$ is negative, firms exit to prevent a negative value of active firms, creating the lower plateau and downward sloping portions of the two policy functions. The amount of firm exit is decreasing in productivity but increasing in the mass of active firms. Between $0$ and $\psi_n$, the value of the firm is free to adjust, so there is no change in the number of active firms. This is represented by the upward sloping portion of the left panel and the flat portion of the right panel.

[13] We approximate the productivity process with an $N$-state Markov chain following Rouwenhorst (1995). Given the updated state of the economy, we use piecewise linear interpolation to calculate the period-$t+1$ policy functions and the transition matrix to numerically integrate. See Appendix C for a more detailed description of the solution method.

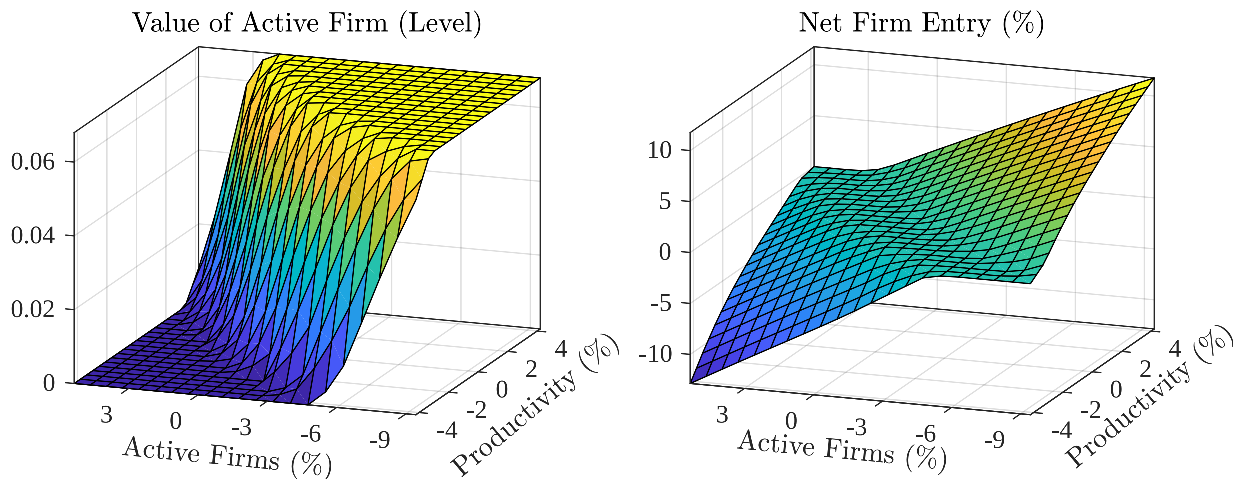


Figure 3: Policy functions for the active firm value and net firm entry. The initial conditions are shown in percent deviations from the deterministic steady state. Employment and capital are fixed at their steady states.

The fact that net firm entry only occurs for sufficiently high or low firm values is crucial for generating non-normalities in model simulations. Intuitively, when we initialize the economy with a firm value equal to its long run value $\bar{J}_A^F \in (0, \psi_n)$, small shocks do not change it enough to generate entry or exit. As a result, the model behaves like a standard real business cycle model with search and matching frictions. However, in response to larger shocks, firm value hits one of its bounds, and entry or exit occurs. Thus, amplification and asymmetry occur exactly when the economy has already entered a recession or boom phase. We find this creates additional volatility and skewness that are consistent with the data and help us understand extreme macroeconomic events.

# 5 QUANTITATIVE RESULTS

We begin by using impulse responses to show the quantitative significance of the amplification and asymmetry generated by entry and exit.[14] We then report model moments to provide a comparison to the higher-order properties of the data. Finally, we examine the welfare cost of business cycles.

**5.1 IMPULSE RESPONSES** Figure 4 plots nonlinear impulse responses to a positive and negative productivity shock.[15] To demonstrate the amplification and asymmetry resulting from firm entry and exit, we initialize the simulations at the ergodic mean and consider a shock that is big enough to generate firm exit. For ease of comparison, we multiply the responses to a positive shock by $-1$. We consider three variants of the model. The first column shows the responses in the model without

[14] To see these mechanisms another way, Appendix E.1 plots the ergodic distributions of output and unemployment.

[15] Following Koop et al. (1996), the response of variable $x_{t+h}$ over horizon $h$ is given by $\mathcal{G}_t(x_{t+h}|\varepsilon_{a,t+1} = 2SD(\hat{a}), \mathbf{z}_t) = E_t[x_{t+h}|\varepsilon_{a,t+1} = 2SD(\hat{a}), \mathbf{z}_t] - E_t[x_{t+h}|\mathbf{z}_t]$, where $\mathbf{z}_t$ is a vector of initial states and $2SD(\hat{a})$ is the size of the shock. The expectations are computed based on the mean path from 20,000 simulations of the model.

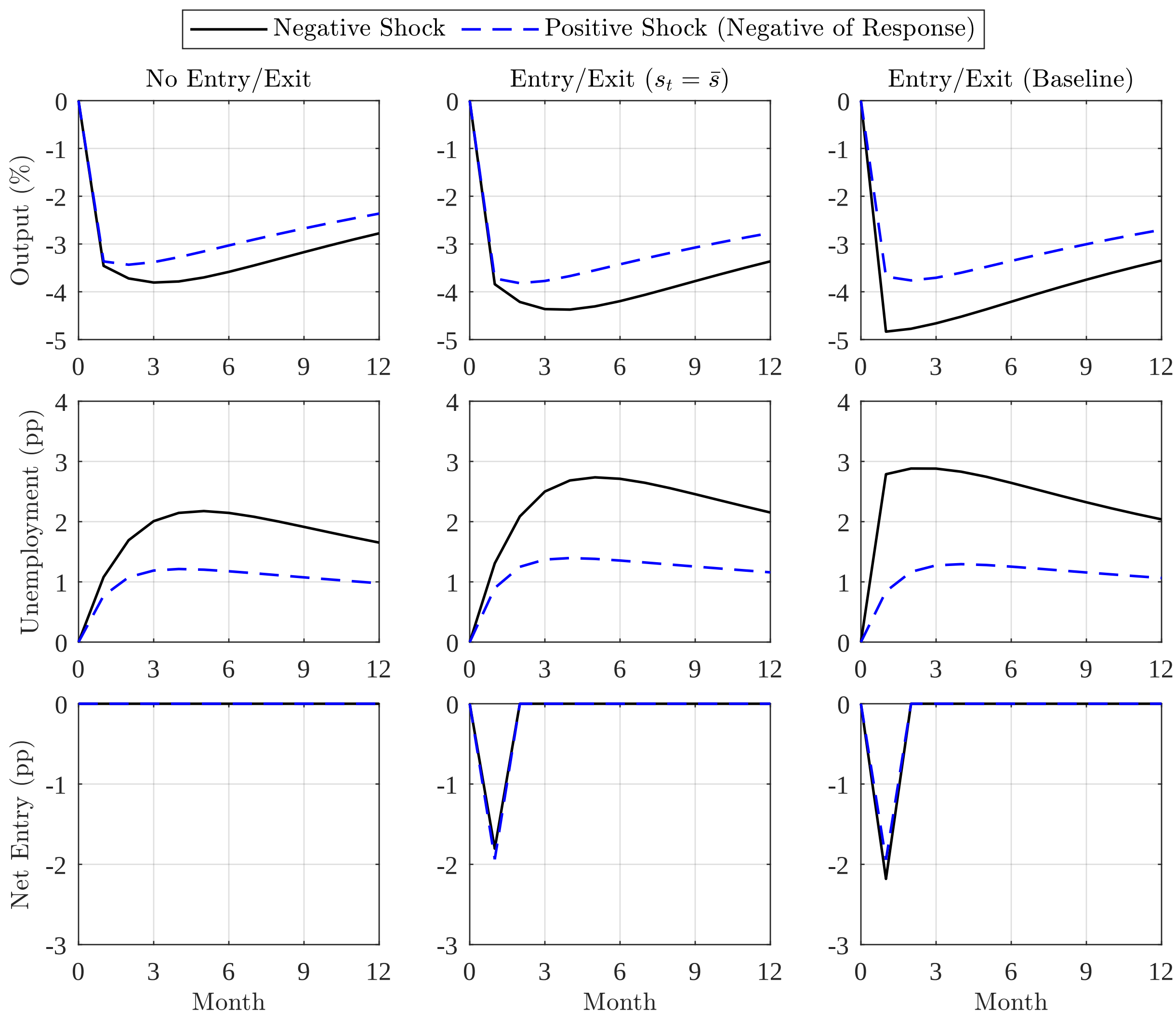


Figure 4: Generalized impulse responses to a positive and negative productivity shock. The responses are based on the method in Koop et al. (1996) and reported in deviations from a simulation without the shock.

entry and exit. The second column features entry and exit but removes the link between firm exit and job destruction by setting $s_t = \bar{s}$. The final column plots the responses of the baseline model.

Consider first the responses from the model without entry and exit. The output responses remain almost symmetric, while the unemployment responses feature asymmetry that favors negative shocks. This pattern is a consequence of the congestion externality highlighted by Petrosky-Nadeau and Zhang (2017) and Petrosky-Nadeau et al. (2018). For example, when productivity declines the marginal benefit of vacancy creation falls, so employment agencies post fewer vacancies and unemployment rises. This raises the ratio of unemployed searching to vacancies, which lowers the sensitivity of the job filling rate to further declines in vacancy postings. As a result, a larger decline in vacancies is necessary to lower the marginal cost of vacancy creation. Despite this mechanism, the model without entry and exit does not generate much asymmetry in output because the value of active firms responds to changes in productivity, violating the entry and exit boundaries.

We now turn to the second column, which includes entry and exit but sets $s_t = \bar{s}$ to shut down the endogenous link between firm exit and job destruction. The net entry response is far from zero. As emphasized in Section 3, entry and exit strengthen the fluctuations in the marginal product of labor, which amplify the responses of unemployment relative to the no entry and exit case. The amplification is stronger for the negative shock, which creates asymmetry.[16] It also feeds into output, which exhibits larger responses than in the no entry and exit case and more pronounced asymmetry.

Finally, the third column shows the responses from the baseline model in which $s_t$ positively responds to firm exit. As shown in Section 3, the link between firm exit and job destruction directly creates an asymmetry in which recessions are more powerful than expansions. All responses exhibit a sizable asymmetry. In line with theory, the asymmetry only amplifies the negative productivity responses, leaving the positive responses close to their counterparts in the second column.

**State-Dependence** The nonlinearities in our baseline model generate significant state-dependance in the transmission of aggregate shocks. Figure 5 illustrates the state-dependance by comparing the responses to a productivity shock starting in different initial states of the economy: bad times ($Z_0 = 0.97$; $U_0 = 8\%$), normal times ($Z_0 = 1$; $U_0 = 6\%$), and good times ($Z_0 = 1.03$; $U_0 = 4\%$).[17]

Across all states, negative shocks are more powerful than positive shocks. This omnipresent asymmetry highlights the quantitative importance of the interaction between firm exit and endogenous job destruction. Moving from good times to bad times, the magnitude and asymmetry in the responses increase.[18] The amplification follows from the interaction of entry and exit dynamics with the congestion externality endemic to labor market search frictions (Petrosky-Nadeau et al., 2018). This interaction generates larger changes in firm profits, greater net entry, and stronger feedback effects between the real economy and the mass of active firms. Greater amplification translates into more asymmetry because a higher probability of exit leads to more job destruction.

## 5.2 SIMULATED MOMENTS

We now compare data and model-implied moments of several key variables in both levels and growth rates. To obtain the cyclical component of the data in levels, we detrend using a Hamilton (2018) filter with an 8-quarter window. The empirical moments are estimated using a 2-step Generalized Method of Moments estimator that applies a Newey and West (1987) weighting matrix with 5 lags. For each moment, we report the estimated mean and standard error (SE) in the data. The model-implied moments are based on the mean of $20{,}000$ simulations that are initialized with a draw from the ergodic distribution and span 780 months—the same number of observations as our monthly data sample. The model simulations are aggregated to a quar-

[16] Ferraro (2018) shows unemployment skewness is due to job loss rather than quits, consistent with our mechanism.

[17] We set the state by averaging periods from a simulation when unemployment is within 0.25pp of a specified target.

[18] These results are consistent with Pizzinelli et al. (2020), who find that aggregate productivity shocks have a much larger effect on the unemployment rate in periods of low productivity using a threshold vector autoregression model.

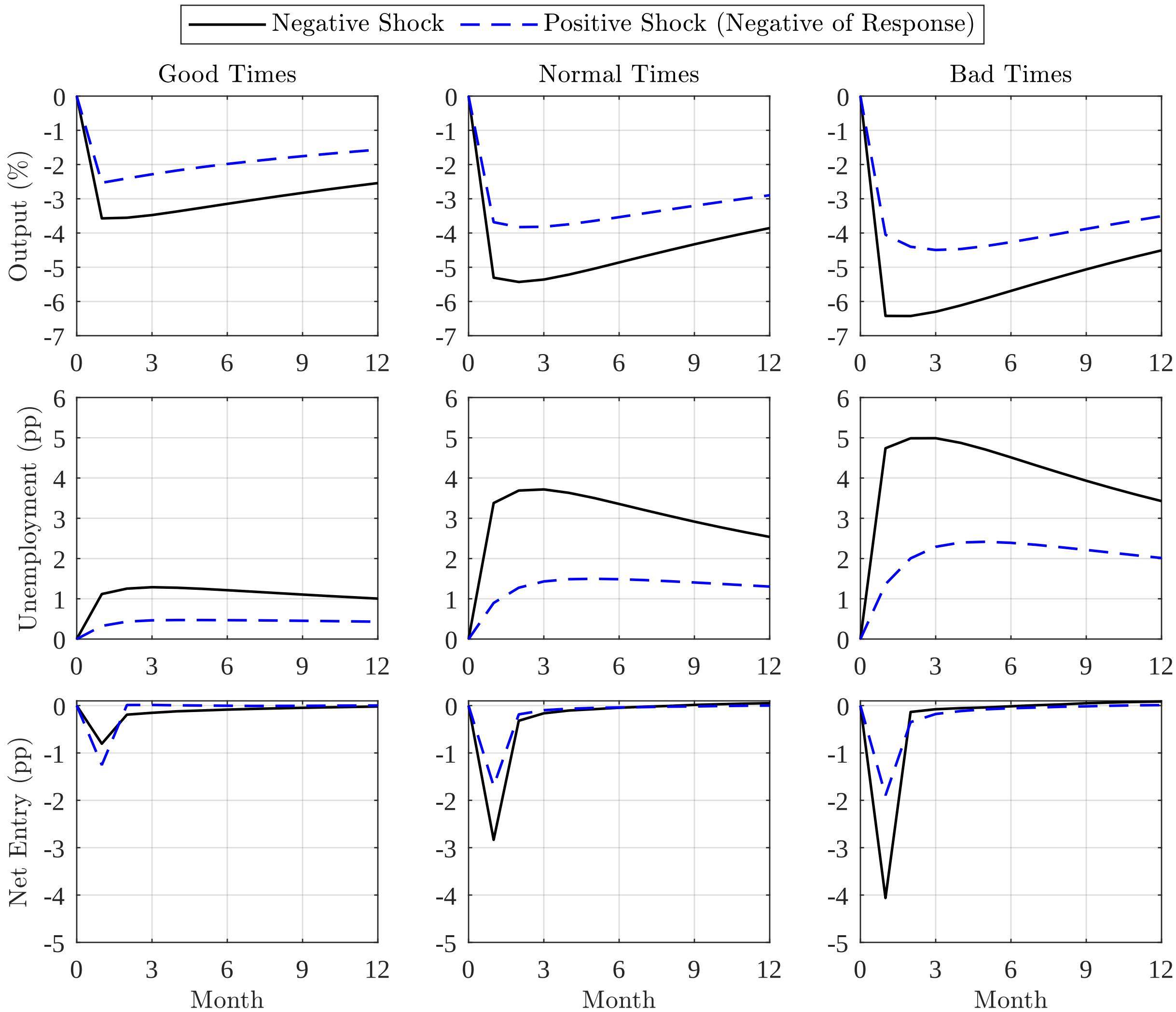


Figure 5: Generalized impulse responses to a positive and negative productivity shock. The responses are based on the method in Koop et al. (1996) and reported in deviations from a simulation without the shock.

terly frequency by averaging across the monthly values to match the frequency of GDP releases.

To quantify the amplification and asymmetry mechanisms, we report the standard deviation and skewness of output, consumption, and investment. To better understand the roles of labor market frictions, entry and exit, and their interaction, we report moments for four versions of the model that remove entry and exit and search frictions in all possible combinations. When we remove search frictions, we model aggregate labor supply using employment lotteries over an indivisible labor choice at the worker level (Hansen, 1985; Rogerson, 1988). To draw as close of a comparison as possible, we recalibrate the capital adjustment cost and productivity parameters so the models without entry and exit generate the same volatilities of output and investment. The entry cost is set so the two models with entry and exit generate the same shares of job creation and job destruction.[19]

[19] Appendix D shows the nonlinear equilibrium system for the indivisible labor model and the calibrated parameters.

| | Data | | Employment Lotteries | | Search and Matching | |
|---|---|---|---|---|---|---|
| Moment | Mean | SE | No Entry/Exit | Entry/Exit | No Entry/Exit | Entry/Exit |
| $SD(Y)$ | 3.17 | 0.26 | 2.21 | 2.61 | 2.20 | 2.67 |
| $SKEW(Y)$ | −0.59 | 0.20 | 0.01 | 0.01 | −0.31 | −0.49 |
| $SD(\Delta \log Y)$ | 0.86 | 0.06 | 0.62 | 0.66 | 0.86 | 0.95 |
| $SKEW(\Delta \log Y)$ | −0.47 | 0.25 | 0.00 | 0.01 | 0.03 | −0.15 |
| $SD(C)$ | 2.00 | 0.16 | 1.57 | 1.90 | 1.37 | 1.71 |
| $SKEW(C)$ | −0.42 | 0.16 | 0.00 | 0.01 | −0.30 | −0.41 |
| $SD(\Delta \log C)$ | 0.51 | 0.04 | 0.32 | 0.34 | 0.42 | 0.49 |
| $SKEW(\Delta \log C)$ | −0.87 | 0.48 | −0.01 | −0.01 | −0.02 | −0.27 |
| $SD(I)$ | 8.92 | 0.77 | 5.19 | 5.96 | 5.21 | 6.22 |
| $SKEW(I)$ | −0.81 | 0.21 | −0.03 | −0.03 | −0.38 | −0.69 |
| $SD(\Delta \log I)$ | 2.13 | 0.18 | 1.79 | 1.92 | 2.27 | 2.48 |
| $SKEW(\Delta \log I)$ | −0.95 | 0.33 | −0.00 | −0.01 | 0.05 | −0.20 |

Table 3: Real activity moments. $SD$ and $SKEW$ denote standard deviation and skewness across time. Output, consumption, and investment in the model are converted to a quarterly frequency by summing the monthly values. The data in levels is reported as a percent deviation from trend. The data trend is based on a Hamilton (2018) filter with an 8-quarter window. The model-implied trend is equal to the simulated mean.

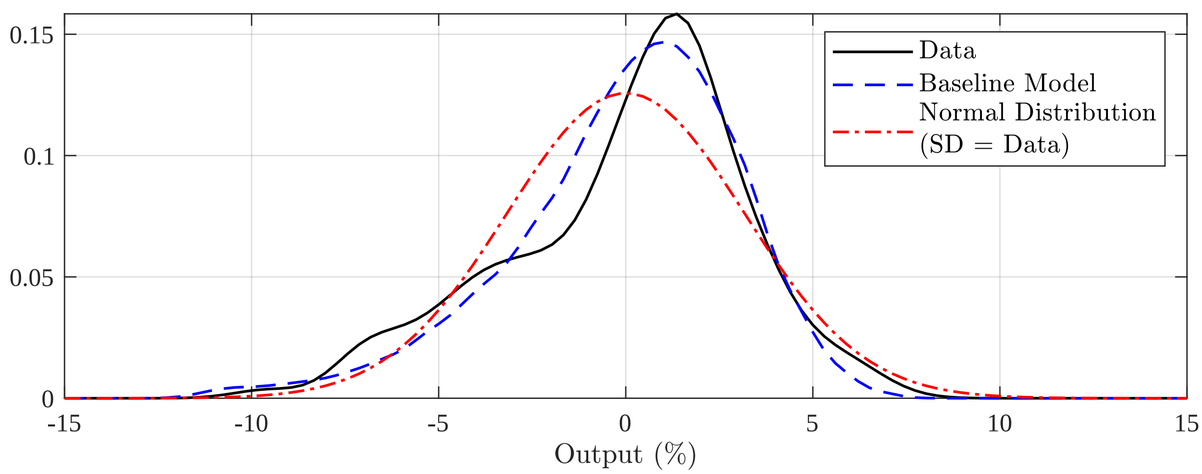


Figure 6: Detrended output probability density function. The data and model are kernel-smoothed estimates.

**Real Activity** Table 3 reports the data and model-implied moments for the four versions of the model. The final column contains results for our baseline model with search frictions and entry and exit. First consider the statistics for the detended level of output. Recall that the productivity process is calibrated to ensure that the baseline model generates similar values for the standard deviation of output in the data. When we apply the same process to the model with search frictions but without entry and exit, the standard deviation falls by $18\%$ from $2.67$ to $2.20$. There is a comparable $19\%$ decline in volatility in the model without search frictions. The amplification is driven by a $16\%$ increase in the standard deviation of measured TFP, relative to the models without entry and exit.

Turning to asymmetry, the baseline model generates an average skewness of $-0.49$, which accords well with the skewness in the data ($-0.59$). When we remove entry and exit, the only source of nonlinearity is the congestion externality in labor markets. As a result, skewness in output falls by $37\%$ to $-0.31$. The asymmetry in the $5$th and $95$th percentiles of the empirical and model distributions provides some context for the differences in skewness. In the data, the $(5, 95)$ percentiles are $(-6.2, 4.5)$, compared with $(-4.9, 3.9)$ in our baseline model and $(-3.9, 3.4)$ in the model with entry and exit. These findings show that the interaction of entry and exit with this congestion effect is a quantitatively important channel through which recessions are more severe than booms. Unlike volatility, the model without search features no amplification of skewness. Without entry and exit, the model does not generate asymmetry, so there is nothing for entry and exit to amplify.

A similar story applies to the moments for the growth rate of output. Entry and exit amplifies growth rate volatility, increasing the standard deviation from $0.86$ to $0.95$ in the model with search and matching frictions. It is useful to note that the model without search generates a much lower standard deviation of output growth due to the lack of endogenous persistence in aggregate employment. In addition, only our baseline model is capable of generating the negative skew of output growth in the data ($-0.47$). This highlights the importance of endogenous job destruction driven by firm exit. Without this channel, the model generates slightly *positive* skewness in output growth.[20]

The quantitative success of our model is easily identified from a kernel density plot. Figure 6 shows the density for detrended output in the model and data. For reference, we also plot a normal distribution with the same standard deviation as the data. Relative to the normal distribution, the data clearly exhibits negative skewness, as it is noticeably different from the normal distribution. In line with the simulated moments, the model density closely approximates the empirical density.

The same insights apply to the moments for consumption and investment. The baseline model amplifies the standard deviations of consumption and investment by $25\%$ and $19\%$ relative to the search and matching model without entry and exit, while similar amplification occurs in the model without labor market frictions. The interaction of entry and exit with search and matching frictions is also essential to generate the negative skewness in consumption and investment that is in the data, in detrended levels and growth rates. Overall, these results demonstrate the importance of both entry and exit and search frictions for generating the empirical non-normalities in real activity.

**Unemployment and Vacancies** Table 4 reports the moments for the unemployment and vacancy rates. Consider first the unemployment rate in levels. There is significant positive skewness in the data, showing that troughs are larger in magnitude than peaks (i.e., deepness asymmetry). This is

[20] In contrast to skewness, detrended output displays little excess kurtosis in the data and model. Interestingly, this is not true for growth rates. Appendix E.2 shows our model can simultaneously generate little excess kurtosis in levels, but significant excess kurtosis in growth rates. Following Acemoglu et al. (2017), we also report tail risk, defined as the probability an outcome falls more than 1.96 standard deviations above (right tail risk) or below (left tail risk) its mean. Our baseline model consistently generates more tail risk than models without search frictions or entry and exit.

| | Data | | Search and Matching | |
|---|---|---|---|---|
| Moment | Mean | SE | No Entry/Exit | Entry/Exit |
| $SD(U)$ | 22.29 | 1.85 | 18.02 | 22.22 |
| $SKEW(U)$ | 0.60 | 0.20 | 0.45 | 0.64 |
| $SD(\Delta \log U)$ | 5.56 | 0.57 | 6.17 | 7.37 |
| $SKEW(\Delta \log U)$ | 1.30 | 0.26 | $-0.02$ | 0.51 |
| $SD(V)$ | 23.03 | 1.96 | 20.51 | 22.77 |
| $SKEW(V)$ | $-0.65$ | 0.19 | $-0.32$ | $-0.27$ |
| $SD(\Delta \log V)$ | 6.27 | 0.50 | 12.39 | 11.94 |
| $SKEW(\Delta \log V)$ | $-0.54$ | 0.28 | 0.06 | 0.14 |
| $CORR(U, V)$ | $-0.78$ | 0.04 | $-0.92$ | $-0.94$ |
| $CORR(\Delta \log U, \Delta \log V)$ | $-0.80$ | 0.03 | $-0.68$ | $-0.65$ |

Table 4: Labor market moments. $SD$, $SKEW$, and $CORR$ denote standard deviation, skewness, and cross-correlation across time. The vacancy and unemployment rates are converted to a quarterly frequency by averaging across the monthly values. The data in levels is reported as a percent deviation from trend. The data trend is based on a Hamilton (2018) filter with an 8-quarter window. The model-implied trend is equal to the simulated mean. The lotteries model is excluded since it does not include unemployment or vacancies.

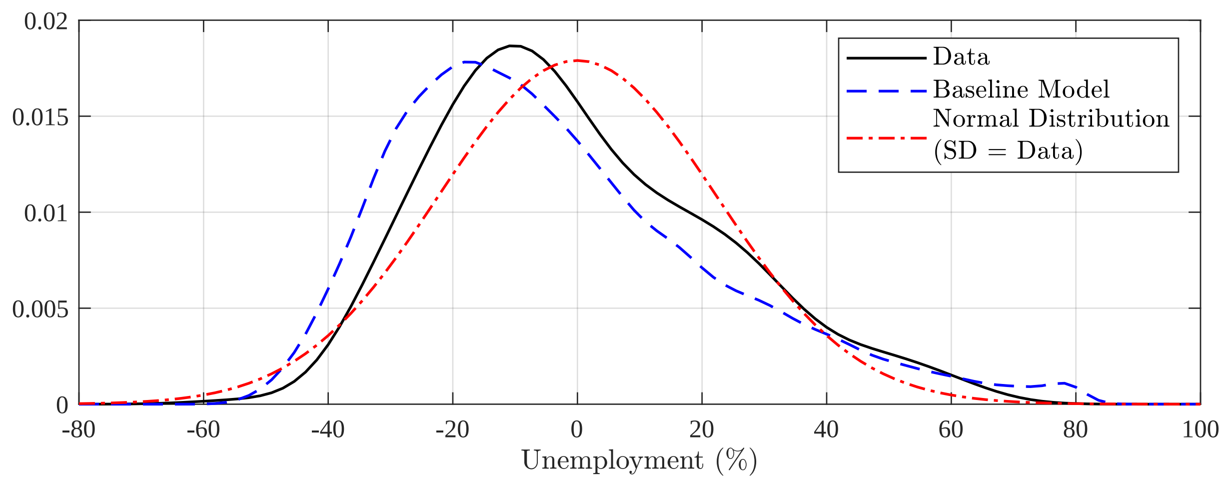


Figure 7: Unemployment probability density function. The data and model are kernel-smoothed estimates.

consistent with the findings in Sichel (1993), Ferraro (2018), and Dupraz et al. (2019). The baseline model provides a strong account of the data, almost exactly matching the standard deviation and positive skewness. Without entry and exit, there is a 30% reduction in skewness and a 19% fall in the standard deviation. These results again highlight the importance of entry and exit in providing an additional source of amplification and asymmetry that complements the congestion externality.

Recent work has also examined asymmetries in the growth rate in unemployment (steepness asymmetry), which capture the fast rises and slow declines of unemployment in the data. Ferraro (2017) shows that a standard search and matching model cannot generate this pattern. Consist with his results, we find the model without entry and exit generates no steepness asymmetry. However, once we introduce entry and exit, the model is able to explain $40\%$ of the skewness in the data.[21]

Figure 7 plots the kernel density of the unemployment rate. Just like in Figure 6, we also plot the density of the data and a normal distribution with the same standard deviation as the data. Relative to the normal distribution, the data density clearly exhibits positive skewness. In line with the simulated moments, the model density is close to the data and far away from the normal density.

The entry and exit mechanism has a muted effect on the vacancy rate. Both models generate a strong negative correlation between unemployment and vacancy rates. While there is still some amplification of the vacancy rate in detrended levels, it is weaker than the unemployment rate, and the model is unable to generate more skewness than the model without entry and exit. Both models also overstate the volatility of vacancy growth and predict positive skewness, in contrast with the data. Overall, firm entry and exit has little effect on the model's ability to match vacancy dynamics.

| | Data | | Employment Lotteries | | Search and Matching | |
|---|---|---|---|---|---|---|
| Moment | Mean | SE | No Entry/Exit | Entry/Exit | No Entry/Exit | Entry/Exit |
| $SD(\Delta \log Y)$ | 0.86 | 0.20 | 0.62 | 0.66 | 0.86 | 0.95 |
| $SD(\mathcal{U}^Y)$ | 5.68 | 0.56 | 0.28 | 1.05 | 3.10 | 6.94 |
| $AC(\mathcal{U}^Y)$ | 0.84 | 0.42 | 0.93 | 0.75 | 0.93 | 0.78 |
| $CORR(\Delta \log Y, \mathcal{U}^Y)$ | −0.37 | 0.26 | 0.29 | 0.08 | −0.17 | −0.42 |

Table 5: Uncertainty moments. $AC$ and $SD$ denote autocorrelation and standard deviation across time. Output in the model is converted to a quarterly frequency by summing the monthly values and then transformed into a growth rate. $SD(\mathcal{U}^Y_{t,t+1})$ is normalized by $SD(\Delta \ln Y)$ to match the units of uncertainty in the data.

**Endogenous Uncertainty** Another useful way to quantitatively assess the higher-order properties of our model is the through the conditional distribution of future output growth. Given the quantitatively significant nonlinearities in our model, this distribution will depend on the state of the economy. State-dependence generates time-varying endogenous uncertainty about future output growth and allows us to relate our calibrated model to a burgeoning empirical literature that estimates the endogenous response of macroeconomic uncertainty to exogenous first moment shocks.

In line with recent empirical work (Jurado et al., 2015; Ludvigson et al., 2020), we define uncertainty as the expected volatility of the $h$-month ahead forecast error for output growth given by

$$\mathcal{U}^Y_{t,t+h} = \sqrt{E_t[(\Delta \ln Y_{t+h} - E_t[\Delta \ln Y_{t+h}])^2]}.$$

We measure uncertainty in the data as the quarterly average of the monthly real uncertainty series ($h = 1$) from Ludvigson et al. (2020). This series is a sub-index of the macro uncertainty series from Jurado et al. (2015) that accounts for 73 real activity variables. Repeated simulations of a factor augmented vector autoregression are used to obtain estimates of uncertainty for each real vari-

[21] Following Dupraz et al. (2019), Appendix E.3 also shows the speed and durations of expansions and contractions.

able and then averaged to generate the aggregate real uncertainty series. Before estimating, most variables are transformed into growth rates and standard normalized. To make the units from our model comparable to the real uncertainty series, we define $SD(\mathcal{U}^Y) \equiv SD(\mathcal{U}^Y_{t,t+1})/SD(\Delta \ln Y_t)$.

Table 5 compares uncertainty moments in the data to our four model variants. In the data, uncertainty is volatile, persistent, and strongly counter-cyclical. Our baseline model closely matches these features, without the aid of exogenous volatility shocks that are common in the uncertainty literature (e.g., Bloom, 2009; Fernández-Villaverde et al., 2011; Leduc and Liu, 2016). Intuitively, the amplification and asymmetry created by cyclical entry and exit imply that the distribution of future output is more dispersed and biased downward in states where firm exit has reduced the number of firms. Without entry and exit, the volatility of uncertainty significantly undershoots the data. Without search frictions, the correlation of uncertainty with output growth is positive. These results are consistent with Ludvigson et al. (2020), who show empirically that uncertainty about real activity is often an endogenous response to business cycles rather than an exogenous propagation.

### 5.3 WELFARE COST OF BUSINESS CYCLES

Our baseline model generates much larger departures from normality than models without entry and exit. Given these results, we revisit the welfare cost of business cycles by implementing the experiment in Lucas (1987, 2003). First, we compute the representative household's lifetime utility in an economy without shocks in which consumption always equals its stochastic steady state, $\tilde{C}$. Second, we compute expected welfare in the stochastic economy. Finally, we compute the percentage of stochastic consumption $\lambda$ households would require to make them indifferent between the two consumption paths. Formally, we compute $\lambda = 100 \times (\exp(\ln \tilde{C} - \frac{1-\beta}{1-\beta^{T-1}} \frac{1}{N_E} \sum_{j=1}^{N_E} E_0[\sum_{t=0}^{T} \beta^t \ln C_{j,t} | \mathbf{z}_{j,0}]) - 1)$, where $T = 3000$, $\mathbf{z}_{j,0}$ is a draw from the ergodic distribution with consumption path $C_{j,t}$, and $N_E$ is the number of draws.

Consistent with Hairault et al. (2010) and Jung and Kuester (2011), the nonlinearities induced by search and matching frictions alone create a larger welfare cost of business cycles ($\lambda = 0.27\%$) than is reported in Lucas (2003, $\lambda = 0.05\%$). When we interact these frictions with cyclical entry and exit, the welfare cost of business cycles increases by $55\%$, so households require an additional $0.42\%$ of consumption in each period to accept the fluctuations from business cycles. Relative to the literature, we do not rely on extreme risk aversion, non-standard utility functions, or household heterogeneity to generate the larger cost of business cycles. Instead, the cost is driven by the negative skewness that the interaction of entry and exit and search frictions imparts to consumption.

## 6 CONCLUSION

This paper studies how establishment entry and exit and search and matching frictions jointly affect business cycle dynamics. Analytical results show their interaction amplifies and skews the transmission of productivity shocks to real activity and unemployment. Quantitatively, we find that extending a canonical macroeconomic model to account for the interaction of entry and exit and labor market frictions greatly improves its ability to generate realistic business cycle dynamics, including the asymmetric nature of recessions versus expansions. These results offer a new lens through which policymakers can respond to cyclical changes in the economy. They also complement a large literature that studies the effects of long-run changes in the distribution of firm activity. Integrating our insights into that growing class of models is an exciting avenue for future research.

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## A Data Sources and Transformations

We use the following time-series from 1955-2019 provided by Haver Analytics:

1. **Civilian Noninstitutional Population: 16 Years and Over**,
   Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)

2. **Gross Domestic Product: Implicit Price Deflator**,
   Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)

3. **Gross Domestic Product**,
   Seasonally Adjusted, Quarterly, Billions of Dollars, (GDP@USECON)

4. **Personal Consumption Expenditures: Nondurable Goods**,
   Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)

5. **Personal Consumption Expenditures: Durable Goods**,
   Seasonally Adjusted, Quarterly, Billions of Dollars (CD@USECON)

6. **Personal Consumption Expenditures: Services**,
   Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)

7. **Private Fixed Investment**,
   Seasonally Adjusted, Quarterly, Billions of Dollars (F@USECON)

8. **Unemployed**, Seasonally Adjusted, Monthly, Thousands, 16 years+ (LTU@USECON)
9. **Labor Force**, Seasonally Adjusted, Monthly, Thousands, 16 years+ (LF@USECON)
10. **Unemployed Less Than 5 Weeks**, Seasonally Adjusted, Monthly, Thousands, 16 years and over (LU0@USECON)
11. **Job Separation Rate**, Job Openings and Labor Turnover Survey, Seasonally Adjusted, Monthly, Percent of Employment (LJSTPA@USECON)
12. **Job Openings**, Job Openings and Labor Turnover Survey, Seasonally Adjusted, Monthly, Thousands (LJJTLA@USECON)
13. **Output Per Person**, Non-farm Business Sector, All Persons, Seasonally Adjusted, Quarterly, 2012=100 (LXNFS@USNA)
14. **Labor Share**, Non-farm Business Sector, All Persons, Seasonally Adjusted, Percent (LXNFBL@USNA)
15. **Nonfarm Proprietors' Income with IVA & CCAdj**, Seasonally Adjusted, Quarterly, Billions of Dollars (YOPN@USNA)
16. **Corporate Profits After Tax with IVA & CCAdj**, Seasonally Adjusted, Quarterly, Billions of Dollars (YCATJ@USNA)
17. **Private Sector Opening Establishments**, Seasonally Adjusted, Quarterly, % of Total Establishments (LRJEGO@USECON)
18. **Private Sector Closing Establishments**, Seasonally Adjusted, Quarterly, % of Total Establishments (LRJELC@USECON)
19. **Net Stock: Private Fixed Assets**, Billions of Dollars, Annual (EPT@CAPSTOCK)
20. **Net Stock: Consumer Durable Goods**, Billions of Dollars, Annual (EDT@CAPSTOCK)
21. **Depreciation: Private Fixed Assets**, Billions of Dollars, Annual (KPT@CAPSTOCK)
22. **Depreciation: Consumer Durable Goods**, Billions of Dollars, Annual (KDT@CAPSTOCK)

We also used the following data from other sources:

1. **Help Wanted Advertising Index (HWI)**, based on Barnichon (2010) and in units of the labor force. The series corrects for online advertising and is available on the author's website.
2. **Real Uncertainty (U)**, 1-quarter horizon, based on Ludvigson et al. (2020). The series is available on Ludvigson's website. The monthly series is averaged to a quarterly frequency.

3. **Business Dynamics Statistics**, 2018 database, published by the Census Bureau.
   - **Establishments (E)**, Count of the number of establishments.
   - **Establishment Entry (EN)**, Count of establishments born.
   - **Establishment Exit (EX)**, Count of establishments exiting.
   - **Job Creation (JC)**, employment gains from expanding and opening establishments.
   - **Job Creation Births (JCB)**, employment gains from establishment openings.
   - **Job Destruction (JD)**, employment losses from shrinking and closing establishments.
   - **Job Creation Deaths (JCD)**, employment losses from establishment closings.

We applied the following transformations to the above data sources:

1. **Per Capita Real Output Growth**:
$$\Delta \ln Y_t = 100\left(\ln\left(\tfrac{GDP_t}{DGDP_t \times LN16N_t}\right) - \ln\left(\tfrac{GDP_{t-1}}{DGDP_{t-1} \times LN16N_{t-1}}\right)\right).$$
2. **Per Capita Real Consumption Growth**:
$$\Delta \ln C_t = 100\left(\ln\left(\tfrac{CN_t + CS_t}{DGDP_t \times LN16N_t}\right) - \ln\left(\tfrac{CN_{t-1} + CS_{t-1}}{DGDP_{t-1} \times LN16N_{t-1}}\right)\right).$$
3. **Per Capita Real Investment Growth**:
$$\Delta \ln I_t = 100\left(\ln\left(\tfrac{F_t + CD_t}{DGDP_t \times LN16N_t}\right) - \ln\left(\tfrac{F_{t-1} + CD_{t-1}}{DGDP_{t-1} \times LN16N_{t-1}}\right)\right).$$
4. **Unemployment Rate**: $U_t = 100(LTU_t/LF_t)$.
5. **Vacancy Rate**: $HWI$ from 1955M1-2000M12 and $LJJTLA/LF$ from 2001M1-2019M12.
6. **Short-term Unemployed** ($U^s$): The redesign of the Current Population Survey (CPS) in 1994 reduced $u_t^s$. To correct for this bias, we use IMPUMS-CPS data to scale $u_t^s$ by the ratio of $u_t^s/u_t$ for the first and fifth rotations groups to $u_t^s/u_t$ across all rotation groups. In addition to the $9$ mandatory identification variables, we first extract the following: EMPSTAT ("Employment Status"), DURUNEMP ("Continuous weeks unemployed") and MISH ("Month in sample, household level"). Unemployed persons have EMPSTAT of 20, 21, or 22. Short-term unemployed are persons who are unemployed and DURUNEMP is 5 or less. Incoming rotation groups have MISH of 1 or 5. Using the final weights, WTFINL, we then calculate unemployment rates conditional on the appropriate values of MISH and DURUNEMP. We then apply the X-12 seasonal adjustment function in STATA to the time series for the ratio. Finally, we take an average of the seasonally adjusted time series. This process yields an average ratio of $1.1693$, so $U^s$ equals $LU0$ before 1994 and $1.1693 \times LU0$ after 1994.

7. **Job-Finding Rate**: $f_t = 100(LTU_t - U_t^s)/LTU_t$.
8. **Real Wage**: $w_t = LXNFBL_t \times LXNFS_t$
9. **Wage Elasticity**: Slope coefficient from regressing $\ln w_t$ on an intercept and $\ln LXNFS_t$.
10. **Profit Share of Income**: $D_t/Y_t = (YOPN_t + YCATJ_t)/GDP_t$
11. **Net Entry**: $\Delta\tilde{Z}_t \equiv LRJEGO_t - LRJELC_t$ or $\Delta\tilde{Z}_t \equiv (EN_t - EX_t)/((E_t + E_{t-1})/2)$
12. **Establishments**: $LRJEGO_t$ and $LRJELC_t$ are reported as a percent of the 2-quarter moving average of total establishments, so $\Delta\tilde{Z}_t = (Z_t - Z_{t-1})/((Z_t + Z_{t-1})/2)$ where $Z_t$ is total establishments. Let $Z_0 = 1$, and recursively update $Z_t = (2 + \Delta Z_t)Z_{t-1}/(2 - \Delta Z_t)$.
13. **Depreciation Rate**: $\delta = (1 + \frac{1}{T}\sum_{t=1}^{T}(KPT_t + KDT_t)/(EPT_{t-1} + EDT_{t-1}))^{1/12} - 1$.
14. **Entry Share of Job Creation**: $\omega^{JC} = \frac{100}{T-H+1}\sum_{t=H}^{T}\sum_{j=t-H+1}^{t} JCB_j/\sum_{j=t-H+1}^{t} JC_j$.
15. **Exit Share of Job Creation**: $\omega^{JD} = \frac{100}{T-H+1}\sum_{t=H}^{T}\sum_{j=t-H+1}^{t} JCD_j/\sum_{j=t-H+1}^{t} JC_j$.

## B ENTRY AND EXIT SHARES OF JOB CREATION AND JOB DESTRUCTION

This section explains how we map firm dynamics in the model to the data. We first compute total job creation and job destruction by aggregating the monthly flows over rolling ten year windows:

$$\begin{aligned} JC_{j,j+H} &= \textstyle\sum_{t=0}^{H-1} Z_{j+t-1}\max\{0, n_{j+t} - n_{j+t-1}\} + \sum_{t=0}^{H-1}\max\{0, Z_{j+t} - Z_{j+t-1}\}n_{j+t}, \\ JD_{j,j+H} &= \textstyle\sum_{t=0}^{H-1} Z_{j+t-1}\max\{0, n_{j+t-1} - n_{j+t}\} + \sum_{t=0}^{H-1}\max\{0, Z_{j+t-1} - Z_{j+t}\}n_{j+t}, \end{aligned}$$

where $n_t = N_t/Z_t$ is the mass of workers employed by each firm in month $t$. $JC_{j,j+H}$ is the job creation from month $j$ to month $j+H$ due to the expansion of existing establishments (first term) and entering establishments (second term). Analogously, $JD_{j,j+H}$ is the job destruction from month $j$ to month $j+H$ due to the contraction of existing establishments and exiting establishments. The shares of job creation and job destruction attributable to establishment entry and exit are given by

$$\omega_{j,j+H}^{JC} = \frac{\sum_{t=0}^{H-1}\max\{0, Z_{j+t} - Z_{j+t-1}\}n_{j+t}}{JC_{j,j+H}}, \quad \omega_{j,j+H}^{JD} = \frac{\sum_{t=0}^{H-1}\max\{0, Z_{j+t-1} - Z_{j+t}\}n_{j+t}}{JD_{j,j+H}}.$$

The shares in our model are based on cyclical variation in job flows, while the empirical shares contain cyclical and trend components. Fortunately, the stability of the empirical shares provides evidence that the cyclical component shares equal the trend component shares. To see this, break total job creation $JC$ and job creation due to firm births $JCB$ into trend and cyclical components:

$$\begin{aligned} JC_{j,j+H} &= JC_{j,j+H}^{T} + JC_{j,j+H}^{C}, \\ JCB_{j,j+H} &= JCB_{j,j+H}^{T} + JCB_{j,j+H}^{C}. \end{aligned}$$

Then the empirical share can be written as a weighted average of the trend and cyclical components:

$$\omega_{j,j+H} = \frac{JCB^T_{j,j+H}+JCB^C_{j,j+H}}{JC^T_{j,j+H}+JC^C_{j,j+H}} = \left(1 - \frac{JC^C_{j,j+H}}{JC^T_{j,j+H}+JC^C_{j,j+H}}\right)\frac{JCB^T_{j,j+H}}{JC^T_{j,j+H}} + \frac{JC^C_{j,j+H}}{JC^T_{j,j+H}+JC^C_{j,j+H}}\frac{JCB^C_{j,j+H}}{JC^C_{j,j+H}}.$$

Stability of the empirical share implies that the trend share is constant, $\omega^T_H = \frac{JCB^T_{j,j+H}}{JC^T_{j,j+H}}$, and approximately equal to the empirical share, $\omega_{j,j+H} \approx \omega^T_H$. Imposing these conditions then implies that the cyclical share must also approximately equal the trend share, $\omega^T_H \approx \frac{JCB^C_{j,j+H}}{JC^C_{j,j+H}}$. Therefore, we can use the average empirical shares as targets to discipline the cyclical shares generated by the model.

To construct total job creation and job destruction in the model, we aggregate monthly net flows into $H$-month gross flows. As a validity check for this approach, we also construct net flows directly in the data using the model as a guide. Using filtered data on establishments, $Z_t$, and establishment size, $n_t$, we compute annual net job creation by the net entry of establishments in year $t$:

$$NJC^{EE}_t = (Z_t - Z_{t-1})n_t.$$

Using the annual data from the Business Dynamics Statistics database, the standard deviation of this series is $1.37\%$. When we compute the same statistic on model-generated data, we obtain a standard deviation of $0.96\%$. Hence, calibrating the model based on the aggregation of net flows into gross flows actually slightly understates the volatility of net flows in the data. This provides further evidence that our calibration of the entry cost does not overstate the role of entry and exit.

## C SOLUTION METHOD

The equilibrium system is given by $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1})|\mathbf{z}_t, \Theta] = 0$, where $g$ is a vector-valued function, $\mathbf{x}_t$ is a vector of variables, $\varepsilon_t$ is a vector of shocks, $\mathbf{z}_t$ is a vector of states, and $\Theta$ is a vector of parameters. The state vector consists of productivity, employment, the capital stock, and active firms, $\mathbf{z}_t = [a_t, N_{t-1}, Z_{t-1}, K_{t-1}]$. We discretize $a_t$, $N_{t-1}$, $Z_{t-1}$, and $K_{t-1}$ into 10, 15, 15, and 15 evenly-spaced points, respectively. The bounds on the three endogenous state variables, $N_{t-1}$, $Z_{t-1}$, and $K_{t-1}$, are set to $[-8.0\%, +2.5\%]$, $[-9.0\%, +5.5\%]$, and $[-7.0\%, +7.0\%]$ of their deterministic steady-state values. Those bounds contain at least $99\%$ of their ergodic distributions.

There are many ways to discretize the exogenous state, $a_t$. We use the Markov chain in Rouwenhorst (1995), which Kopecky and Suen (2010) show outperforms other methods for approximating autoregressive processes. The realization of $\mathbf{z}_t$ on node $d$ is denoted $\mathbf{z}_t(d)$. This method provides integration nodes, $[a_{t+1}(m)]$, with weights, $\phi(m)$, for $m \in \{1, \dots, M\}$. Since productivity follows a Markov chain, the realizations of $a_{t+1}$ are the exact same as $a_t$ ($M = 10$).

Vacancies are subject to a nonnegativity constraint, $V_t \geq 0$. To impose the constraint, we introduce an auxiliary variable, $\mu_{V,t}$, such that $V_t = \max\{0, \mu_{V,t}\}^2$ and $\lambda_{V,t} = \max\{0, -\mu_{V,t}\}^2$, where $\lambda_{V,t}$ is the Lagrange multiplier on the non-negativity constraint. If $\mu_{V,t} \geq 0$, then $V_t = \mu^2_{V,t}$

and $\lambda_{V,t} = 0$. When $\mu_{V,t} < 0$, the constraint is binding, $V_t = 0$, and $\lambda_{V,t} = \mu_{V,t}^2$. Therefore, the vacancy constraint is transformed into a pair of equalities following Garcia and Zangwill (1981).

There is also an inequality constraint on the value function of active firms, $\psi_x \leq J_{A,t}^F < \psi_n$, where $\psi_x$ is a general scrap value (we set $\psi_x = 0$ in the final solution). To impose this inequality constraint, we create a second auxiliary variable, $\mu_{A,t}$, that equals $J_{A,t}^F$ when there is no entry or exit and the change in active firms, $\lambda_{A,t} = Z_t - Z_{t-1}$, when entry or exit occurs. When firms enter $J_{A,t}^F = \psi_n$ and when they exit $J_{A,t}^F = \psi_x$. Therefore, the state variable for active firms, $Z_{t-1}$, is updated according to $J_{A,t}^F = \max\{\psi_n, \min\{\psi_x, \mu_{A,t}\}\}$, $\lambda_{A,t} = \mu_{A,t} - J_{A,t}^F$, and $Z_t = Z_{t-1} + \lambda_{A,t}$.

The vector of policy functions and the realization on node $d$ are denoted $\mathbf{pf}_t$ and $\mathbf{pf}_t(d)$, where $\mathbf{pf}_t \equiv [\mu_{V,t}(\mathbf{z}_t), \mu_{A,t}(\mathbf{z}_t)]$. The following steps outline our policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the log-linear model without entry and exit and obtain an initial conjecture for $\mu_{V,t}(d)$. Guess that $\mu_{A,t}(d) = 0.5(\psi_n + \psi_x)$ for all $d \in D$.
2. Solve the nonlinear model without entry and exit by setting $\psi_n = 100$ and $\psi_x = -100$.
   (a) On iteration $j \in \{1, \dots\}$ and each node $d \in \{1, \dots, D\}$, use Chris Sims's `csolve` to find $\mathbf{pf}_t(d)$ to satisfy $E[g(\cdot)|\mathbf{z}_t(d), \Theta] \approx 0$. Guess $\mathbf{pf}_t(d) = \mathbf{pf}_{j-1}(d)$ and implement:
      i. Solve for all variables dated at time $t$, given $\mathbf{pf}_t(d)$ and $\mathbf{z}_t(d)$.
      ii. Linearly interpolate the policy functions, $\mathbf{pf}_{j-1}$, at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{pf}_{t+1}(m)$ on every integration node, $m \in \{1, \dots, M\}$.
      iii. Given $\{\mathbf{pf}_{t+1}(m)\}_{m=1}^M$, solve for the other elements of $\mathbf{x}_{t+1}(m)$, noting that $\xi_{t+1}$, defined in (9), depends on the fraction of firms that exit at $t+1$. Then compute:
      $$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1})|\mathbf{z}_t(d), \Theta] \approx \textstyle\sum_{m=1}^M \phi(m) g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$
      iv. When `csolve` converges, set $\mathbf{pf}_j(d) = \mathbf{pf}_t(d)$.
   (b) Repeat step 2 until $\text{maxdist}_j < 10^{-9}$, where $\text{maxdist}_j \equiv \max\{|\mathbf{pf}_j - \mathbf{pf}_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate solution.
3. Solve the nonlinear model with entry and exit but without endogenous job destruction by setting $\psi_x = 0$ and $s_t = \bar{s}$. Use the linear solution without entry and exit as an initial guess. Iterate on $\psi_n$, each time solving the model using the previous solution as an updated guess.
4. Solve the baseline nonlinear model by setting $s_t = \bar{s} + (1 - \bar{s})(1 - \xi_t)\zeta$, where $\zeta \in [0, 1]$. Using the solution without endogenous job destruction as an initial guess, iterate from $\zeta = 0.5$ to $\zeta = 1$, each time solving the model using the previous solution as an updated guess.

The algorithm is programmed in Fortran 90 with Open MPI and run on the BigTex supercomputer.

# D Indivisible Labor Model

Relative to the baseline model, we remove search and matching frictions but extend the representative household's preferences to include linear disutility of labor. The equilibrium system is given by

$$J^F_{A,t} = \min(\max(\psi_x, \mu_{F,t}), \psi_n),$$
$$\lambda_{F,t} = \mu_{F,t} - J^F_{A,t},$$
$$w_t = C_t,$$
$$Y_t = a_t Z_t^{1-\vartheta}(K_{t-1}^{\alpha} N_t^{1-\alpha})^{\vartheta},$$
$$C_t + I_t = Y_t,$$
$$K_t = (1-\delta)K_{t-1} + \left(a_1 + \tfrac{a_2}{1-1/\nu}\left(\tfrac{I_t}{K_{t-1}}\right)^{1-1/\nu}\right)K_{t-1},$$
$$\tfrac{1}{a_2}\left(\tfrac{X_t}{K_{t-1}}\right)^{1/\nu} = E_t\left[x_{t+1}\left(\alpha\vartheta\tfrac{Y_{t+1}}{K_t} + \tfrac{1}{a_2}\left(\tfrac{I_{t+1}}{K_t}\right)^{1/\nu}(1-\delta+a_1) + \tfrac{1}{\nu-1}\tfrac{I_{t+1}}{K_t}\right)\right],$$
$$w_t = (1-\alpha)\vartheta Y_t/N_t,$$
$$Z_t = Z_{t-1} + \lambda_{F,t},$$
$$Z_t J^F_{A,t} = (1-\vartheta)Y_t - Z_t\psi_y + Z_t E_t\left[x_{t+1}(\xi_{t+1}J^F_{A,t+1} + (1-\xi_{t+1})\psi_x)\right],$$
$$\ln a_{t+1} = (1-\rho_a)\ln\bar{a} + \rho_a \ln a_t + \sigma_a \varepsilon_{a,t+1}.$$

We recalibrate $\nu = 9.879$, $\rho_a = 0.983$, and $\sigma_a = 0.003$ to match the standard deviation of investment and the autocorrelation and standard deviation of output in the search and matching model without entry and exit. We also set $\psi_n = 0.0365$ to match the entry and exit shares of job creation and job destruction in our baseline model. All of the other parameters in Table 1 are unchanged.

**Solution** We use the algorithm in Appendix C to solve the model. The state vector includes productivity, the capital stock, and active firms, $\mathbf{z}_t = [a_t, Z_{t-1}, K_{t-1}]$. Consistent with the baseline model, we discretize $a_t$, $Z_{t-1}$, and $K_{t-1}$ into 10, 15, and 15 evenly-spaced points, respectively. The bounds on the endogenous states, $Z_{t-1}$ and $K_{t-1}$, are set to $\pm 10\%$ of their respective steady-states.

# E Additional Quantitative Results

## E.1 Ergodic Distributions

Throughout the paper, we emphasize that entry and exit amplify and skew the transmission of aggregate productivity shocks. Another useful way to see these mechanisms is by plotting ergodic distributions of variables in our baseline model with and without entry and exit. Following Petrosky-Nadeau and Zhang (2017), Figure 8 plots 2,000 draws from the ergodic distributions for output, the unemployment rate, and net entry as a function of productivity.

Looking at vertical cross sections of output and unemployment shows the distributions in the

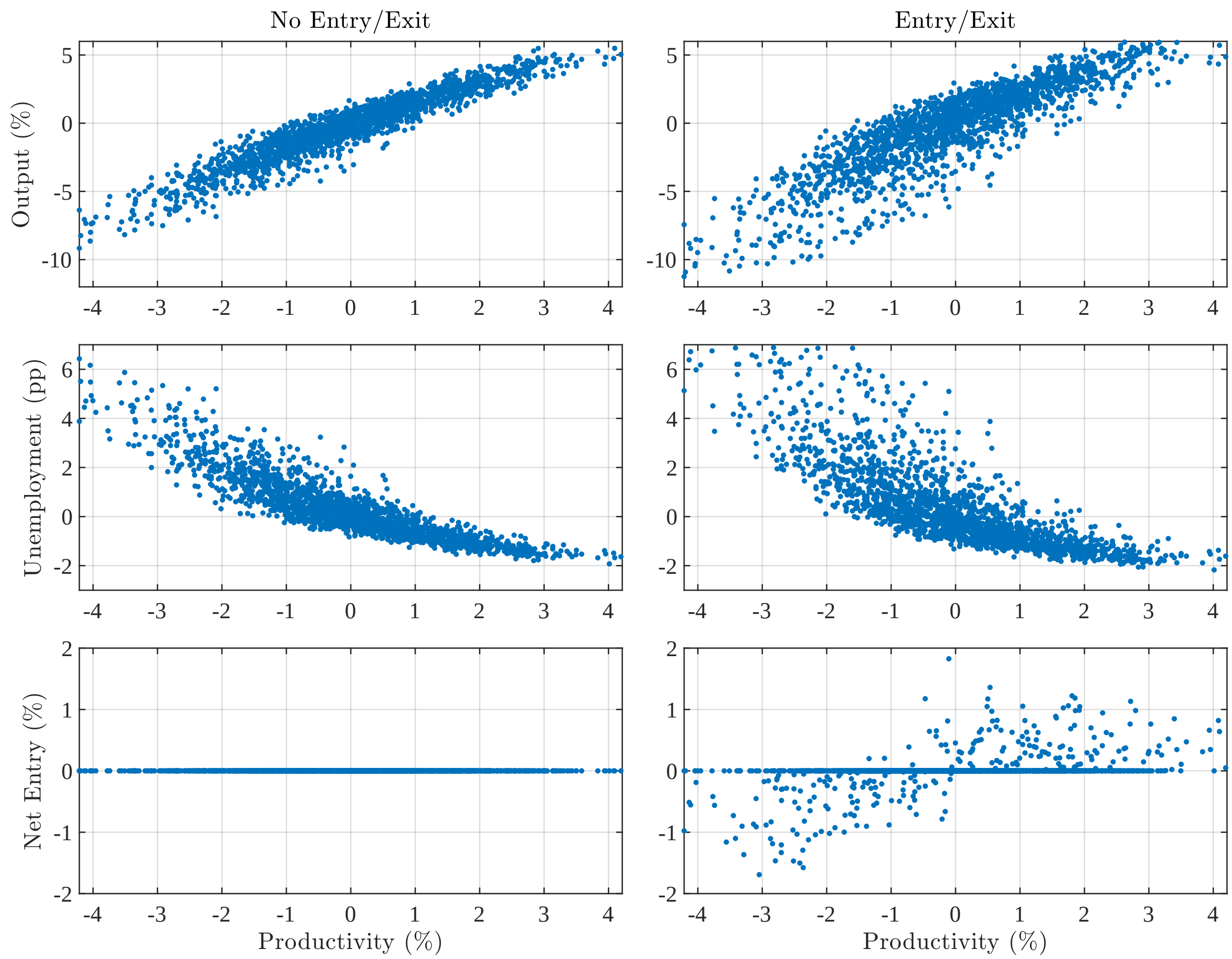


Figure 8: Ergodic distributions of output, the unemployment rate, and net entry as a function of productivity.

model with entry and exit are wider, demonstrating that productivity shocks have greater amplification. Typically, there is more net entry when productivity is further from its ergodic mean, but the value of the firm also depends on employment, the capital stock, and the number of active firms. In low productivity states that resemble recessions, the output cross sections shift down, while the unemployment cross sections shift up relative to the model without entry and exit. This result is a reflection of the increased skewness that occurs when firms exit and jobs are endogenously destroyed.

**E.2 UNCONDITIONAL MOMENTS** In addition to the standard deviations and skewness reported in the main paper, we report the autocorrelations, excess kurtosis, and tail risk of real activity, unemployment, and vacancies. Similar to Acemoglu et al. (2017), tail risk is defined as the probability a variable is at least $1.96$ standard deviations above (right tail risk) or below (left tail risk) its mean.

Table 6 reports the empirical and model-implied moments for detrended levels and growth rates of real activity. All four models consistently generate autocorrelations that are in line with the data. However, the interaction of entry and exit and search frictions remains essential to capture the non-

| | Data | | Employment Lotteries | | Search and Matching | |
|---|---|---|---|---|---|---|
| Moment | Mean | SE | No Entry/Exit | Entry/Exit | No Entry/Exit | Entry/Exit |
| $AC(Y)$ | 0.91 | 0.18 | 0.96 | 0.96 | 0.92 | 0.93 |
| $KURT(Y)$ | 0.05 | 0.49 | −0.35 | −0.38 | −0.06 | 0.21 |
| $LTAIL(Y)$ | 5.77 | 2.24 | 2.15 | 2.09 | 3.37 | 3.86 |
| $RTAIL(Y)$ | 1.15 | 0.81 | 2.22 | 2.20 | 1.40 | 0.97 |
| $AC(\Delta \log Y)$ | 0.31 | 0.07 | 0.18 | 0.21 | 0.21 | 0.22 |
| $KURT(\Delta \log Y)$ | 1.75 | 0.66 | −0.03 | 0.15 | 0.06 | 0.73 |
| $LTAIL(\Delta \log Y)$ | 4.23 | 1.48 | 2.46 | 2.55 | 2.41 | 2.88 |
| $RTAIL(\Delta \log Y)$ | 1.54 | 0.74 | 2.48 | 2.58 | 2.59 | 2.42 |
| $AC(C)$ | 0.88 | 1.01 | 0.98 | 0.98 | 0.95 | 0.95 |
| $KURT(C)$ | −0.12 | 0.37 | −0.48 | −0.50 | −0.16 | −0.09 |
| $LTAIL(C)$ | 4.62 | 2.02 | 1.97 | 1.89 | 3.28 | 3.59 |
| $RTAIL(C)$ | 0.77 | 0.53 | 1.98 | 1.95 | 1.23 | 0.87 |
| $AC(\Delta \log C)$ | 0.29 | 0.08 | 0.21 | 0.26 | 0.25 | 0.23 |
| $KURT(\Delta \log C)$ | 3.55 | 2.06 | −0.03 | 0.28 | 0.11 | 0.67 |
| $LTAIL(\Delta \log C)$ | 2.69 | 1.06 | 2.48 | 2.67 | 2.58 | 3.19 |
| $RTAIL(\Delta \log C)$ | 1.54 | 0.73 | 2.45 | 2.63 | 2.47 | 2.10 |
| $AC(I)$ | 0.90 | 0.22 | 0.94 | 0.94 | 0.90 | 0.92 |
| $KURT(I)$ | 0.15 | 0.53 | −0.25 | −0.25 | 0.08 | 0.87 |
| $LTAIL(I)$ | 5.00 | 0.38 | 2.44 | 2.44 | 3.59 | 4.15 |
| $RTAIL(I)$ | 0.00 | – | 2.20 | 2.20 | 1.27 | 0.86 |
| $AC(\Delta \log I)$ | 0.44 | 0.07 | 0.17 | 0.19 | 0.24 | 0.26 |
| $KURT(\Delta \log I)$ | 3.18 | 1.28 | −0.02 | 0.07 | 0.11 | 1.63 |
| $LTAIL(\Delta \log I)$ | 2.31 | 1.21 | 2.48 | 2.54 | 2.38 | 2.90 |
| $RTAIL(\Delta \log I)$ | 1.54 | 0.74 | 2.46 | 2.51 | 2.66 | 2.51 |

Table 6: Real activity moments. $AC$ and $KURT$ denote autocorrelation and excess kurtosis across time. $RTAIL$ ($LTAIL$) is the probability a variable is more than 1.96 standard deviations above (below) its mean. Output, consumption, and investment in the model are converted to a quarterly frequency by summing the monthly values. The data in levels is reported as a percent deviation from trend. The data trend is based on a Hamilton (2018) filter with an 8-quarter window. The model-implied trend is equal to the simulated mean.

Gaussian features of the data. The baseline model is consistent with the lack of excess kurtosis in levels as well as the significant excess kurtosis in growth rates. In contrast, the other models generate counterfactually negative excess kurtosis in levels and little excess kurtosis in growth rates. Similar insights hold for the tail risk statistics, with particularly pronounced effects in levels.

Table 7 reports the same moments as Table 6 for unemployment and vacancies. The autocorrelations are close to the data, with the exception of the vacancy growth rate, and entry and exit is crucial to capture the non-Gaussian features of the data. For unemployment, the baseline model is consistent with the lack of excess kurtosis in levels and the excess kurtosis in growth rates. Adding entry and exit also improves the tail risk statistics relative to the data. Similar to the results in the main text, our baseline model has a more muted effect on vacancies relative to the model without entry and exit. Neither model is able to provide a full account of the data, especially in growth rates.

| | Data | | Search and Matching | |
|---|---|---|---|---|
| Moment | Mean | SE | No Entry/Exit | Entry/Exit |
| $AC(U)$ | 0.93 | 0.01 | 0.94 | 0.94 |
| $KURT(U)$ | −0.12 | 0.41 | −0.10 | 0.17 |
| $LTAIL(U)$ | 0.38 | 2.39 | 0.69 | 0.31 |
| $RTAIL(U)$ | 5.00 | 2.45 | 3.76 | 4.26 |
| $AC(\Delta \log U)$ | 0.62 | 0.05 | 0.37 | 0.32 |
| $KURT(\Delta \log U)$ | 3.02 | 1.10 | 0.19 | 1.80 |
| $LTAIL(\Delta \log U)$ | 1.15 | 0.79 | 2.66 | 2.09 |
| $RTAIL(\Delta \log U)$ | 5.77 | 2.06 | 2.54 | 3.63 |
| $AC(V)$ | 0.92 | 0.01 | 0.81 | 0.85 |
| $KURT(V)$ | 0.09 | 0.43 | −0.07 | −0.24 |
| $LTAIL(V)$ | 4.23 | 2.27 | 3.32 | 3.16 |
| $RTAIL(V)$ | 0.00 | – | 1.28 | 1.27 |
| $AC(\Delta \log V)$ | 0.58 | 0.07 | −0.13 | −0.14 |
| $KURT(\Delta \log V)$ | 1.62 | 0.62 | 0.33 | 0.39 |
| $LTAIL(\Delta \log V)$ | 5.00 | 1.56 | 2.48 | 2.30 |
| $RTAIL(\Delta \log V)$ | 1.92 | 1.19 | 2.77 | 2.95 |

Table 7: Labor market moments. $AC$ and $KURT$ denote autocorrelation and excess kurtosis across time. $RTAIL$ ($LTAIL$) is the probability a variable is more than 1.96 standard deviations above (below) its mean. The vacancy and unemployment rates are converted to a quarterly frequency by averaging across the monthly values. The data in levels is reported as a percent deviation from trend. The data trend is based on a Hamilton (2018) filter with an $8$-quarter window. The model-implied trend is equal to the simulated mean.

**E.3 CONDITIONAL MOMENTS** All of the moments we have reported thus far are unconditional. Dupraz et al. (2019) focus on the unemployment rate and examine asymmetries between expansions and contractions of the business cycle. We follow their algorithm to identify the business cycle turning points. The algorithm has one parameter, $\underline{\text{dU}} \equiv |U_j - U_i|$, $j > i$, which determines how much the unemployment rate must change for an interval to count as either an expansion or contraction. If the periods following a peak or trough do not move by more than $\underline{\text{dU}}$, then the algorithm skips them and continues searching for the next peak or trough. Increasing $\underline{\text{dU}}$ reduces the number of expansions and contractions in a sample and typically increases their average duration.

The standard deviation of the monthly unemployment rate in our sample is $1.6$ percentage points. We set $\underline{\text{dU}} = 1.25$ to achieve similar peaks and troughs as Dupraz et al. (2019). We also identify peaks and troughs in the detrended quarterly unemployment rate series used throughout the paper. The standard deviation of the detrended series is $22.3\%$. For this series, we set the minimum distance parameter to $24.5$ to obtain similar peaks and troughs as the monthly unemployment rate.

Figure 9 plots the actual unemployment rate for our sample and an example path of the unemployment rate from our baseline model. Both series are shown in percent deviations from trend, where the data trend is based on a Hamilton (2018) filter with an $8$-quarter window and the model

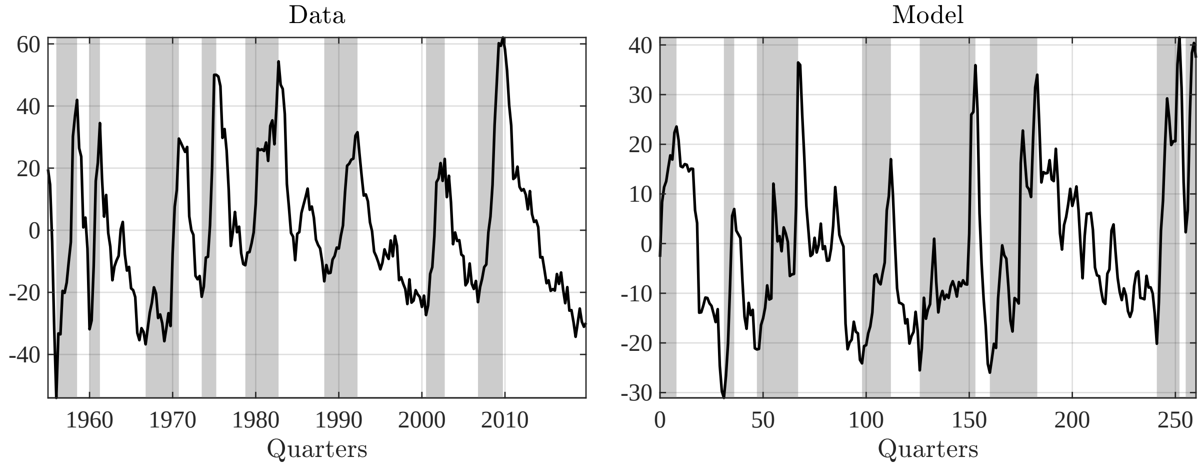


Figure 9: Unemployment rate. The series are averaged to a quarterly frequency and shown in percent deviations from trend. The data trend is based on Hamilton (2018) filter with an 8-quarter window. The model trend is the simulated mean. Shaded areas denote recessions based on the algorithm in Dupraz et al. (2019).

| | Quarterly Detrended | | | Monthly Raw | | |
|---|---|---|---|---|---|---|
| | | Model | | | Model | |
| Moment | Data | No Entry/Exit | Entry/Exit | Data | No Entry/Exit | Entry/Exit |
| $E$(Expansion Speed) | 20.23 | 15.56 | 17.71 | 0.80 | 1.32 | 1.56 |
| $E$(Contraction Speed) | 28.51 | 15.42 | 20.87 | 1.71 | 1.30 | 1.98 |
| $E$(Expansion Duration) | 18.78 | 16.20 | 15.92 | 60.89 | 34.53 | 34.74 |
| $E$(Contraction Duration) | 11.38 | 16.64 | 14.25 | 29.00 | 35.32 | 28.58 |

Table 8: Unemployment rate moments. Duration is the number of periods and Speed is the average change per period during an increase (expansion) or decrease (contraction) in the unemployment rate. Business cycle turning points are based on the algorithm in Dupraz et al. (2019). The Quarterly Detrended statistics are based on data averaged to a quarterly frequency and converted to percent deviations from trend. The data trend is based on a Hamilton (2018) filter with an 8-quarter window. The model trend is the simulated mean.

trend is the simulated mean. The shaded regions denote recessions identified by the algorithm. The peaks and troughs in the data line up closely with dates identified by the NBER Business Cycle Dating Committee, but not exactly because we are exclusively focusing on the unemployment rate.

Table 8 reports the speed and duration of expansions (unshaded regions) and contractions (shaded regions). Speed is the average change in the unemployment rate and duration is number of periods during an expansion or contraction. We report results for two variants of the data: the quarterly detrended series analyzed in Table 4 and the raw monthly series examined by Dupraz et al. (2019). In each case, we report equivalent statistics in our model with and without entry and exit.

The actual unemployment rate declines during contractions faster than it rises during expansions, but expansions are longer than contractions. For example, in the raw data, the unemployment rate on average falls by 1.7 percentage points per month during contractions and rises by 0.8 percentage points per month during expansions. However, expansions last an average of 61 months, while contractions last only 29 months. The same pattern holds for the quarterly detrended series.

Consistent with the data, the model with entry and exit is able to generate asymmetries in the speed and duration of contractions and expansions, whereas the statistics implied by the model without entry and exit are symmetric. Quantitatively, the model with entry and exit falls short of asymmetry in the data. It also generates less asymmetry than the model with downward nominal wage rigidity in Dupraz et al. (2019). However, their model uses a second-order exogenous productivity process, which they argue is important to generate these features of the data. In contrast, our model uses a standard first-order exogenous process, is consistent with the unemployment rate skewness in the data which Dupraz et al. (2019) overstate, and generates data-consistent volatilities and non-normalities in real activity. This provides further support that entry and exit, in combination with labor market frictions, is a crucial source of the asymmetries in business cycle dynamics.