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Entry and Exit, Unemployment, and Macroeconomic Tail Risk*

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Abstract

This paper builds a nonlinear business cycle model with endogenous firm entry and exit and equilibrium unemployment. The entry and exit mechanism generates asymmetry and amplifies the transmission of productivity shocks, exposing the economy to significant tail risk. When calibrating the rates of entry and exit to match their shares of job creation and destruction, our quantitative model generates higher-order moments consistent with U.S. data. Firm exit particularly amplifies the severity and persistence of deep recessions such as the COVID-19 crisis. In the absence of entry and exit, the model generates almost no asymmetry or tail risk.

Keywords: Unemployment; Firm Dynamics; Skewness; Labor Search; Nonlinear; COVID-19

JEL Classifications: E24; E32; E37; J63; L11

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1 INTRODUCTION

Firm entry and exit plays a significant role in shaping the dynamics of aggregate employment and output. For example, Haltiwanger (2012) finds that on average 37% of gross job creation occurs at new firms or establishments in a given year. Simultaneously, the dynamics of employment, output, and other macroeconomic variables exhibit sizable departures from normality in the form of negative skewness and tail risk. These features are typically difficult to generate in textbook models of the macroeconomy. This paper shows that introducing endogenous firm entry and exit into a standard real business cycle model with unemployment generates a new source of asymmetry and shock amplification. Quantitatively, the model produces dynamics that match first- and higher-order moments of the data on output, consumption, the unemployment rate, and vacancy postings.

Our model extends Bilbiie et al. (2012) to include endogenous firm exit and unemployment à la Diamond-Mortenssen-Pissarides. Using a simple example, we analytically show firm entry and exit has two key economic effects. First, aggregate employment dynamics are asymmetric. Intuitively, firm exit causes an immediate destruction of jobs and a fast rise in unemployment. In contrast, firm entry operates through vacancy creation and does not result in an immediate spike in employment growth. In equilibrium, this asymmetry transmits to both output and consumption.

Second, firm entry and exit amplify the macroeconomic responses to aggregate productivity shocks. Firm entry increases competition in the labor market, which pushes up the marginal cost of labor. Competition incentivizes firms to post more vacancies and create more jobs in equilibrium. This new channel operates in tandem with the direct effect of a positive productivity shock.

To examine the quantitative strength of these mechanisms, we calibrate the model in line with the business cycle literature. We set the fixed costs of firm entry and production to target the fractions of aggregate job creation and job destruction attributable to entry and exit in the U.S., as reported by Haltiwanger (2012). To capture the asymmetry and amplification mechanisms, we solve the model globally by extending the policy function iteration method of Richter et al. (2014).

We evaluate the quantitative effects of asymmetry and amplification by comparing third and fourth-order moments of the model data with their empirical counterparts and by computing tail risk statistics. This allows us to go beyond the common practice of comparing second-order moments of log-linearized models to the data. The model closely matches the negative skewness and excess kurtosis of U.S. output and consumption growth in the post-World War II era. In contrast, the model without entry and exit generates positive skewness and near zero excess kurtosis, which shows firm entry and exit plays an important role in generating macroeconomic tail risk. We corroborate this finding by computing the probability that each outcome falls more than two standard deviations from its mean in the data and our model. Firm entry and exit is crucial to generate two departures from normality in the data: asymmetric tail probabilities and total tail risk above 5%.

Another important property of our model pertains to the inflow rate to unemployment. Much of the theoretical literature assumes an acyclical inflow rate. In our model, the inflow rate is countercyclical. This feature is consistent with Elsby et al. (2009), who show countercyclical inflow rates explain about 35% of the cyclical variation in the unemployment rate. Quantitatively, our model generates a similar result even though we do not target this moment in our calibration.

Our model's departures from normality suggest the welfare cost of business cycles may be higher than in conventional settings. Repeating the experiment in Lucas (1987), we find households would be willing to give up 0.38% of lifetime consumption in order to forgo business cycle fluctuations. This number is two orders of magnitude larger than the original 0.008% reported by Lucas, and highlights the key role that skewness and tail risk play in increasing the cost of business cycles.

Finally, we use our framework to study the response to the COVID-19 pandemic. We report impulse responses to a negative productivity shock when the initial unemployment rate is elevated to 10% due to an exogenous increase in the job separation rate. This exercise highlights the state-dependent nature of the entry and exit amplification mechanism, which doubles the declines in output and consumption relative to the economy without entry and exit. These results demonstrate that is it important to account for firm exit when studying the transmission of recessionary shocks.

Related Literature Bilbiie et al. (2012) study business cycle dynamics in a model with endogenous firm entry. Our paper differs in three ways. First, we allow for endogenous firm exit. Second, we add unemployment and study its interaction with entry and exit. This interaction drives the asymmetry and amplification that generates skewness and tail risk.[1] Third, we solve our model nonlinearly. This is a necessary step to compute higher-order moments that linearization mutes by construction. The nonlinearities we highlight bear some resemblance to the findings in Petrosky-Nadeau and Zhang (2017), though the mechanisms differ since they abstract from entry and exit.

Our results on skewness and tail risk contribute to a growing literature that examines what causes macroeconomic models to depart from normality. For example, Ilut et al. (2018) show the volatility and skewness in aggregate employment dynamics follow from asymmetry in how individual firms respond to information. In contemporaneous work, Dupraz et al. (2019) show downward wage rigidity generates negative skewness in employment and asymmetry in the speeds of recessions and recoveries. We contribute to this literature by proposing an alternative source of macroeconomic tail risk due to search and matching frictions and endogenous firm entry and exit.

The rest of paper proceeds as follows. Section 2 lays out the economic environment and defines equilibrium. Section 3 uses a simplified setting to analytically derive the qualitative effects of firm entry and exit on the labor market. Section 4 contains our quantitative results. Section 5 shows that firm exit particularly amplifies deep recessions such as the COVID-19 crisis. Section 6 concludes.

[1]Shao and Silos (2013) use a model with firm entry and unemployment. However, they abstract from endogenous exit and focus on the cyclical costs of vacancy creation and its implications for income shares using a linear solution.

2 ENVIRONMENT

We conduct our analysis through the lens of a search and matching model augmented to include endogenous firm entry and exit. Each period a representative household and Z_t firms interact in a frictional labor market intermediated by employment agencies. Household members are either employed or unemployed. Employed workers receive a wage rate determined by Nash Bargaining.

Search and Matching Entering period t, there are N_{t-1} employed workers. A constant fraction $\bar{s}$ exogenously lose their jobs. If the number of firms declines to $Z_t < Z_{t-1}$, then an additional fraction $1 - Z_t/Z_{t-1}$ endogenously lose their jobs.[2] Therefore, the job separation rate is given by

$$s_t = \bar{s} + (1 - \bar{s})(1 - \min\{1, Z_t/Z_{t-1}\}). \tag{1}$$

Following Blanchard and Galí (2010), newly separated workers are immediately able to search for jobs. Therefore, the total number of unemployed people searching for work in period t is given by

$$U_t^s = 1 - (1 - s_t)N_{t-1}. \tag{2}$$

If employment agencies post V_t vacancies, then the number of new matches is determined by

$$g(U_t^s, V_t) = U_t^s V_t/((U_t^s)^\iota + V_t^\iota)^{1/\iota},$$

where $\iota > 0$ determines the curvature of the function. Define $\theta_t \equiv V_t/U_t^s$ as labor market tightness from the firm's perspective. Then the job-filling rate, q_t, and the job-finding rate, f_t, are given by

$$q_t = g(U_t^s, V_t)/V_t = 1/(1 + \theta_t^\iota)^{1/\iota}, \tag{3}$$

$$f_t = g(U_t^s, V_t)/U_t^s = \theta_t q_t. \tag{4}$$

The specification of the matching function follows Den Haan et al. (2000). Compared to a Cobb-Douglas specification, this matching function ensures (3) and (4) remains between 0 and 1. This distinction is particularly important when shocks drive the economy far away from the steady state.

Aggregate employment in period t includes workers who were not separated and new matches,

$$N_t = (1 - s_t)N_{t-1} + q_t V_t. \tag{5}$$

All other workers are classified as unemployed, so the unemployment rate, U_t, satisfies

$$U_t \equiv U_t^s - g(U_t^s, V_t) = 1 - N_t. \tag{6}$$

[2] This fraction follows from the fact that all firms employ the same number of workers in a symmetric equilibrium.

Households Given the paths for aggregate employment and unemployment, the household solves

$$J_t^H = \max_{C_t, B_t} \ \log C_t + \beta E_t J_{t+1}^H$$

subject to

$$C_t + B_t = w_t^n N_t + bU_t + B_{t-1} r_{t-1} - \tau_t + D_t,$$
$$N_{t+1} = (1 - s_{t+1}(1 - f_{t+1}))N_t + f_{t+1} U_t,$$
$$U_{t+1} = s_{t+1}(1 - f_{t+1})N_t + (1 - f_{t+1})U_t,$$

where β is the discount factor and E_t is the mathematical expectation operator conditional on information in period t. C_t is consumption, B_t is the quantity of bonds, w_t^n is the wage determined by Nash Bargaining, r_t is the gross real interest rate, b is the flow value of unemployment, τ_t is a lump-sum tax, and D_t are dividends from ownership of the active firms and employment agencies.

The optimality condition and marginal utilities of employment and unemployment are given by

$$E_t[m_{t+1} r_t] = 1, \tag{7}$$
$$J_{N,t}^H = \phi_t w_t + \beta E_t[(1 - s_{t+1}(1 - f_{t+1}))J_{N,t+1}^H + s_{t+1}(1 - f_{t+1}))J_{U,t+1}^H], \tag{8}$$
$$J_{U,t}^H = \phi_t b + \beta E_t[(f_{t+1} J_{N,t+1}^H + (1 - f_{t+1})J_{U,t+1}^H)], \tag{9}$$

where $\phi_t = 1/C_t$ is the Lagrange multiplier on the budget constraint and $m_{t+1} = \beta(C_t/C_{t+1})$ is the pricing kernel. Therefore, the marginal surplus of employment over unemployment is given by

$$\tfrac{J_{N,t}^H - J_{U,t}^H}{\phi_t} = w_t^n - b + E_t\left[m_{t+1}(1 - f_{t+1})(1 - s_{t+1})\left(\tfrac{J_{N,t+1}^H - J_{U,t+1}^H}{\phi_{t+1}}\right)\right]. \tag{10}$$

Employment Agencies Employment agencies facilitate labor market operations by posting vacancies on behalf of intermediate goods firms and by selling the labor services of households they attract. They sell each unit of labor to active firms at the competitive rate w_t and pay workers a wage rate w_t^n determined by Nash Bargaining. The representative employment agency solves

$$J_t^E = \max_{N_t, V_t} \ (w_t - w_t^n)N_t - \kappa V_t + E_t[m_{t+1} J_{t+1}^E] \tag{11}$$

subject to (5) and $V_t \geq 0$, where $\kappa > 0$ is the vacancy posting cost. The optimality conditions imply

$$\lambda_{N,t} = (\kappa - \lambda_{V,t})/q_t, \tag{12}$$
$$\lambda_{N,t} = w_t - w_t^n + E_t[m_{t+1}(1 - s_{t+1})\lambda_{N,t+1}], \tag{13}$$

where $\lambda_{N,t}$ and $\lambda_{V,t}$ are the Lagrange multipliers on (5) and the inequality constraint. Therefore,

$$\tfrac{\kappa - \lambda_{V,t}}{q_t} = w_t - w_t^n + E_t[m_{t+1}(1 - s_{t+1})\tfrac{\kappa - \lambda_{V,t+1}}{q_{t+1}}]. \tag{14}$$

This condition states that the marginal cost of posting an additional vacancy at time t equals the marginal benefit of an additional worker. The benefit includes the time-t profit from the new match plus the discounted expected value of the worker, net of job separations that occur at time $t+1$. Also note that $\lambda_{N,t}$ is interpreted as the marginal surplus value of a new match to the firm at time t.

Final Good Firms The production sector in period t consists of Z_t monopolistically competitive intermediate goods firms and a representative perfectly competitive final goods firm. The representative final goods firm packages the Z_t intermediate goods into a consumption good by maximizing $P_t Y_t - \int_0^{Z_t} p_{j,t} y_{j,t} dj$ subject to $Y_t \equiv [\int_0^{Z_t} y_{j,t}^{(\vartheta-1)/\vartheta} dj]^{\vartheta/(\vartheta-1)}$, where $\vartheta > 1$ is the elasticity of substitution between goods, P_t is the price of the final good, and $p_{j,t}$ is the price of intermediate good j. Optimality yields a demand function for each intermediate good and the price of the final good,

$$y_{j,t} = (p_{j,t}/P_t)^{-\vartheta} Y_t, \tag{15}$$

$$P_t = (\textstyle\int_0^{Z_t} p_{j,t}^{1-\vartheta} dj)^{1/(1-\vartheta)}. \tag{16}$$

Intermediate Goods Firms Intermediate firm $j \in [0, Z_t]$ produces a differentiated good, $y_{j,t}$, according to $y_{j,t} = \max\{a_t n_{j,t} - \psi_y, 0\}$, where $n_{j,t}$ is employment at firm j, ψ_y is a fixed input cost of production, and a_t is productivity which is common across firms and evolves according to

$$\ln a_{t+1} = (1-\rho_a)\ln \bar{a} + \rho_a \ln a_t + \sigma_a \varepsilon_{a,t+1},\ 0 \le \rho_a < 1,\ \varepsilon_a \sim \mathbb{N}(0,1). \tag{17}$$

Conditional on producing in period t, each intermediate firm solves

$$J_{A,t}^F = \max_{n_{j,t}, p_{j,t}, \xi_{j,t}} (p_{j,t}/P_t)^{1-\vartheta} Y_t - w_t n_{j,t} + E_t[m_{t+1}(\xi_{j,t+1} J_{A,t+1}^F + (1-\xi_{j,t+1}) J_{I,t+1}^F)] \tag{18}$$

subject to

$$(p_{j,t}/P_t)^{-\vartheta} Y_t = a_t n_{j,t} - \psi_y, \tag{19}$$

where $\xi_{j,t+1}$ is the probability the firm chooses to stay active next period. The firm's value is $J_{A,t+1}^F$ if it is active and $J_{I,t+1}^F$ if it shuts down in period $t+1$. In symmetric equilibrium, optimality implies

$$w_t/a_t = Z_t^{1/(\vartheta-1)}/\mu. \tag{20}$$

where (16) implies $Z_t^{1/(\vartheta-1)} = p_t/P_t$ is the relative price level. Therefore, (20) says the relative price level is a constant markup, $\mu \equiv \vartheta/(\vartheta-1)$, above the marginal cost of producing, w_t/a_t. Labor market clearing requires $N_t = \int_0^{Z_t} n_{j,t} dj = Z_t n_t$ so (19) implies that aggregate output is given by

$$Y_t = Z_t^{1/(\vartheta-1)}(a_t N_t - Z_t \psi_y). \tag{21}$$

Firm Entry and Exit The value of an active firm is given by (18) in symmetric equilibrium. The value of an inactive firm is $J^F_{I,t} = \max\{0, J^F_{A,t} - \psi_n\}$. Firm entry is subject to a fixed cost $\psi_n \geq 0$. Free entry implies $J^F_{A,t} \leq \psi_n$ and free exit implies $J^F_{A,t} \geq J^F_{I,t}$, so $J^F_{I,t} = 0$ and $J^F_{A,t} \in [0, \psi_n]$. This means firm entry only occurs when $J^F_{A,t} = \psi_n$ and exit only occurs when $J^F_{A,t} = 0$, so (18) implies

$$Z_t = \begin{cases} \frac{Y_t - w_t N_t}{\psi_n - E_t[m_{t+1}\xi_{t+1}J^F_{A,t+1}]}, & J^F_{A,t} = \psi_n, \\ \frac{w_t N_t - Y_t}{E_t[m_{t+1}\xi_{t+1}J^F_{A,t+1}]}, & J^F_{A,t} = 0, \\ Z_{t-1}, & J^F_{A,t} \in (0, \psi_n), \end{cases} \tag{22}$$

Furthermore, symmetry across active firms implies

$$\xi_{t+1} = \mathbb{I}(J^F_{A,t+1} > 0) + (Z_{t+1}/Z_t)\mathbb{I}(J^F_{A,t+1} = 0), \tag{23}$$

where $\mathbb{I}$ is an indicator function that equals 1 if the condition is true and 0 otherwise. This says that if $J^F_{A,t+1} > 0$, then $\xi_{t+1} = 1$ because all active firms remain active. If $J^F_{A,t+1} = 0$, then $\xi_{t+1} = Z_{t+1}/Z_t$ since Z_{t+1}/Z_t is the fraction of active firms that remain active in the next period.

Wages Wages are determined via Nash Bargaining between a worker and the employment agency. The total surplus of a match is $\Lambda_t = (J^H_{N,t} - J^H_{U,t})/\phi_t + \lambda_{N,t}$. The equilibrium wage maximizes $((J^H_{N,t} - J^H_{U,t})/\phi_t)^\eta (\lambda_{N,t})^{1-\eta}$, where η is the household's bargaining weight. Optimality implies

$$(J^H_{N,t} - J^H_{U,t})/\phi_t = \eta\Lambda_t \tag{24}$$

or, equivalently,

$$\lambda_{N,t} = (1-\eta)\Lambda_t. \tag{25}$$

To derive the equilibrium wage rate, first plug (10) into (24) to obtain

$$\eta\Lambda_t = w^n_t - b + \eta E_t[m_{t+1}(1-s_{t+1})(1-f_{t+1})\Lambda_{t+1}]. \tag{26}$$

Then plug (13) in (25) and combine with (26) to obtain

$$w^n_t = \eta(w_t + \kappa E_t[m_{t+1}(1-s_{t+1})\theta_{t+1}]) + (1-\eta)b. \tag{27}$$

The wage rate in period t is a weighted average of the representative firm's value of a new match and the representative household's cost of working. The firm's value of a new worker includes the additional revenue plus the discounted expected value of the foregone vacancy cost net of separations that occur in period $t+1$. The household's cost is the foregone flow value of unemployment.

Equilibrium Given the government budget constraint, $\tau_t = bU_t$, the bond market clearing condition, $B_t = 0$, and dividends described by (11) and (18), the goods market cleaning is given by

$$C_t + \kappa V_t = Y_t. \tag{28}$$

A competitive equilibrium includes sequences of quantities $\{C_t, N_t, U_t, U_t^s, V_t, Y_t, Z_t, q_t, f_t, s_t\}_{t=0}^{\infty}$, prices $\{w_t^n, w_t, r_t, \}_{t=0}^{\infty}$, and exogenous variables $\{a_t\}_{t=1}^{\infty}$ that satisfy (1)-(7), (14), (17), (20)-(22), (27), and (28) given the initial conditions, $\{N_{-1}, Z_{-1}, a_0\}$, and the sequence of shocks, $\{\varepsilon_{a,t}\}_{t=1}^{\infty}$.

3 UNDERSTANDING THE MECHANISM

This section analytically uncovers two channels through which firm entry and exit affect equilibrium dynamics. Specifically, we show entry and exit is a source of asymmetry and shock amplification. In the full model, these effects give rise to skewness and kurtosis of the model-implied data.

3.1 ASYMMETRIC EMPLOYMENT DYNAMICS To show how firm entry and exit generate asymmetry, we focus on (1) and (5), which link the number of active firms Z_t to the evolution of aggregate employment N_t via the job separation rate s_t. Consider the following experiment. Restrict the economy to last for two periods, $t-1$ and t, and focus on period t. Suppose the number of active firms has not changed from last period, so $Z_t = Z_{t-1}$. Then consider how Z_t, s_t, and N_t all respond if the economy is hit by an unanticipated shock to productivity that changes a_t to $a_t + da_t$.

Let dZ_t, ds_t, and dN_t denote the responses of each variable under consideration, assuming $dZ_t > 0$ when $da_t > 0$ since higher productivity generates firm entry. From (1) and (5), we obtain

$$dN_t = -ds_t N_{t-1} + d(q_t V_t)$$

and

$$s_t + ds_t = \bar{s} + (1-\bar{s})(1 - \min\{1, 1 + dZ_t/Z_{t-1}\}),$$

where $d(q_t V_t)$ is the equilibrium response of new matches to the unanticipated productivity shock.

Since $s_t = \bar{s}$ before the shock, we can write

$$ds_t = \begin{cases} -(1-\bar{s})dZ_t/Z_{t-1}, & \text{if } dZ_t < 0, \\ 0, & \text{if } dZ_t \geq 0, \end{cases}$$

which shows how the number of firms affects the separation rate. Intuitively, when firms exit ($dZ_t < 0$), jobs are immediately destroyed, causing a spike in the job separation rate ($ds_t > 0$). When firms enter ($dZ_t > 0$), job separations occur only for exogenous reasons at rate $\bar{s}$ so $ds_t = 0$.

Substituting ds_t into the expression for dN_t yields

$$dN_t = \begin{cases} (1-\bar{s})(N_{t-1}/Z_{t-1})dZ_t + d(q_t V_t), & \text{if } dZ_t < 0, \\ d(q_t V_t), & \text{if } dZ_t \geq 0, \end{cases}$$

which shows firm exit provides a force for negative employment growth, but firm entry does not offer an analogous force for positive growth. Therefore, allowing for firm entry and exit creates an endogenous source of asymmetry in aggregate employment. In the full model, this translates into negatively skewed employment since booms have smaller effects on employment than recessions.

3.2 Amplified and Asymmetric Job Creation

Firm entry and exit also affects vacancy creation and the flow of new matches or job creation, $d(q_t V_t)$. Exploiting the fact that the economy ends after period t and setting $b = 0$ for analytical tractability, (2), (3), (14), (20), and (27) imply[3]

$$d(q_t V_t) = \underbrace{q_t V_t \theta_t^{-\iota}(da_t/a_t)}_{\text{Productivity}} + \underbrace{q_t V_t \theta_t^{-\iota}(\vartheta - 1)^{-1}(dZ_t/Z_t)}_{\text{Entry/Exit}} + \underbrace{f_t N_{t-1} ds_t}_{\text{Separations}}.$$

This decomposes new matches into three distinct forces. The first is present in standard models without firm entry and exit. When productivity increases, the marginal profit of a new match increases. This incentivizes firms to post more vacancies and generates more matches in equilibrium.

To the best of our knowledge, the second and third channels are new to the literature and rely on the presence of firm entry and exit. The second channel shows how firm entry (exit) causes an increase (decrease) in the number of new matches separately from the direct effect of a change in productivity. This amplification effect is driven by (20), which shows how the marginal cost of production is increasing in the number of active firms. Intuitively, more active firms create more competition in the labor market which bids up the price of labor. All else equal, this increases the marginal profit of a new match to employment agencies, which causes them to post more vacancies and to facilitate more matches. Taken together, the first and second channels suggest vacancies (and employment) will exhibit more volatility than in models without endogenous firm entry and exit.

Finally, the third channel accounts for how firm exit destroys jobs and results in an increase in the number of unemployed workers searching for jobs, U_t^s. All else equal, an increase in U_t^s raises the job-filling rate q_t, which lowers the marginal cost of posting a vacancy κ/q_t. In equilibrium, employment agencies decide to post more vacancies and facilitate more matches. Since $ds_t \geq 0$, this channel creates positive asymmetry in job creation, which results in positive skewness. Quantitatively, however, we find this effect is small so employment is negatively skewed, as in the data.

[3] To derive this result, first use (14), (20), and (27) to obtain $\kappa/q_t = (1-\eta)a_t Z_t^{1/(\vartheta-1)}/\mu$. Differentiation then yields $dq_t = -(1-\eta)q_t^2 Z_t^{1/(\vartheta-1)}((\vartheta-1)da_t + a_t dZ_t/Z_t)/(\vartheta\kappa)$. Using (3) and the definition of labor market tightness, $\theta_t = V_t/U_t^s$, we obtain $dV_t = \theta_t dU_t^s - V_t q_t^{-\iota}\theta_t^{-\iota} dq_t/q_t$, where (2) implies $dU_t^s = N_{t-1} ds_t$. Substituting dq_t and dV_t and into $d(q_t V_t) = V_t dq_t + q_t dV_t$, while using (3) and (20) to simplify, yields the expression in the text.

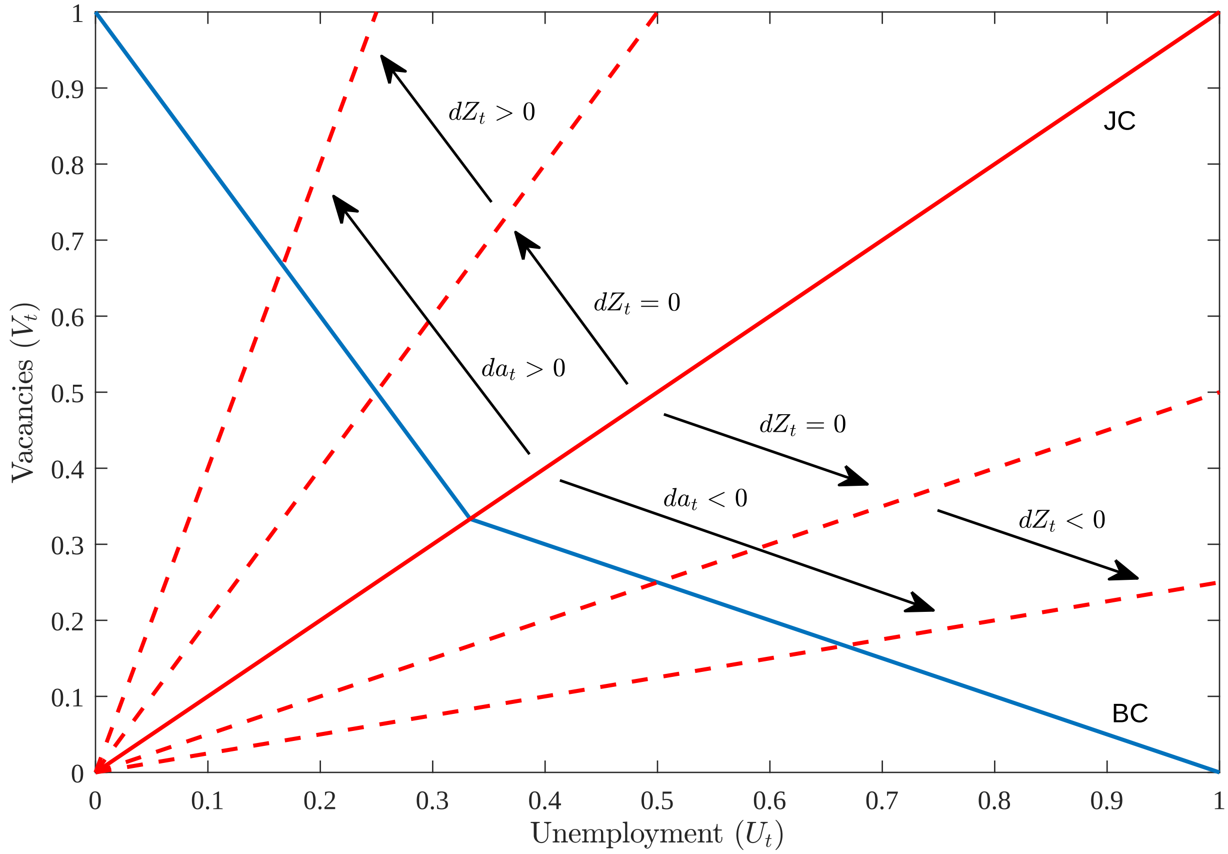

Figure 1: Graphical exposition of the equilibrium response to a productivity shock in a simplified analytical model. The solid lines show the Beveridge curve (BC) and job creation curve (JC) before the shock hits. The dashed lines show the responses of the job creation curve in the models with and without entry and exit.

3.3 GRAPHICAL EXPOSITION Labor market equilibrium and the economy's response to a productivity shock can be summarized graphically by movements in the intersection of a downward sloping Beveridge curve and upward sloping job creation curve plotted in (U, V) space. Figure 1 plots these stylized relationships in our simplified analytical setting. The Beveridge curve (labeled BC) and job creation curve (labeled JC) before the shock hits are plotted in solid lines, while the dashed curves trace out the response of the job creation curve to a productivity shock da_t. A positive shock rotates the job creation curve to the left so vacancies increase and unemployment falls.

The effects of firm entry and exit are captured by two distinct features of Figure 1. First, the asymmetric effect of entry and exit on the job separation rate results in a kinked Beveridge curve. Negative productivity shocks that induce firm exit cause larger increases in unemployment than equal sized positive shocks that induce entry. In the absence of entry and exit, the Beveridge curve would be linear in this stylized setting.[4] Second, amplification is captured by the extra rotation of the job creation curve that occurs in response to a productivity shock relative to the model without

[4]The Beveridge curve is typically convex to the origin and generates positive skewness in unemployment. Our contribution is to show that firm entry and exit is another source of skewness, and this channel is quantitatively important.

firm entry and exit. This extra rotation results in larger responses of vacancies and unemployment. Therefore, vacancies and unemployment are more volatile than in a model without entry and exit.

3.4 Transmission to Output

We have shown firm entry and exit creates negatively-skewed employment dynamics and amplifies the transmission of productivity shocks through job creation. In equilibrium, these properties transmit to output and consumption. Differentiating (21) implies

$$dY_t = Z_t^{\frac{1}{\vartheta-1}} \left(a_t N_t da_t/a_t + (\vartheta-1)^{-1}(a_t N_t - Z_t \vartheta \psi_y) dZ_t/Z_t + a_t dN_t \right),$$

which shows the response of output is directly affected by the productivity shock and indirectly amplified by firm entry and exit. Output also inherits the negative skewness of employment. Differentiating (28) then yields the response of consumption, which inherits the properties of output.

4 Quantitative Analysis

This section explores the quantitative properties of our model. It first describes our calibration and nonlinear solution method. It then shows how well the model matches higher-order moments in the data and quantifies the endogenous asymmetry and amplification from firm entry and exit with impulse responses to productivity shocks. Finally, it uses the model to revisit the debates on the importance of inflows to unemployment during recessions and the welfare cost of business cycles.

Parameter		Value	Target	Data	Model
Nash Bargaining Weight	η	0.060	Wage-Productivity Elasticity	0.54	0.49
Flow Value of Unemployment	b	0.965	Average Unemployment Rate	5.89	5.90
Matching Function Curvature	ι	0.70	Average Job-Finding Rate	39.96	37.25
Vacancy Posting Cost	κ	0.20	Average Job-Filling Rate	33.81	37.21
Exogenous Separation Rate	$\bar{s}$	0.036	Average Job Separation Rate	3.558	3.640
Entry Cost	ψ_n	0.015	Entry Share of Job Creation	37.00	39.31
Fixed Cost of Production	ψ_y	0.094	Exit Share of Job Creation	31.00	27.95
Productivity Persistence	ρ_a	0.951	Autocorrelation of Output Growth	0.31	0.32
Productivity Shock SD	σ_a	0.002	Standard Deviation of Output Growth	0.86	0.81

Table 1: Calibrated Parameters. The targets are computed using data from 1954M1-2019M12. The only two exceptions are the average job-filling rate, which is based on the value in Den Haan et al. (2000), and the shares of job creation and job destruction due to firm entry and exit, which are from Haltiwanger (2012).

4.1 Calibration

The model is calibrated at a monthly frequency to capture the dynamics of employment flows in the data. We externally set two parameters in line with the business cycle literature. The time discount factor β is set to 0.9983, which implies an annual real interest rate of 2%. The elasticity of substitution between intermediate inputs, ϑ, is set to 11, which corresponds to a 10% price markup and is consistent with Smets and Wouters (2007) and Farhi and Gourio (2018).

Table 1 summarizes the remaining parameters, which are chosen to match key moments in the data.[5] The average job separation rate, $\bar{s}$, is set to 0.036 to match the Job Openings and Labor Turnover Survey (JOLTS) average from 2001M1-2019M12. We set η, b, ι and κ to match four salient features of the U.S. labor market: the average unemployment rate, job-finding rate, and job-filling rate, and the elasticity of wages with respect to productivity. The job-filling rate target is set to 0.34, which is the monthly job-filling rate implied by Den Haan et al. (2000). The other targets are based on data from 1954-2019. Using monthly data from the Bureau of Labor Statistics (BLS), we target an average unemployment rate of 5.89%. Shimer (2005) provides a well-known formula for the job-finding rate. Applying it to our sample yields a target rate of 0.40. Hagedorn and Manovskii (2008) compute an elasticity of wages with respect to productivity. We update this statistic using quarterly data over our sample and estimate a value of 0.54.[6] To match these empirical targets, we set the workers' bargaining weight, η, to 0.06, the flow value of unemployment, b, to 0.965, the curvature of the matching function, ι, to 0.7, and the vacancy posting cost, κ, to 0.2.

A crucial feature of our model is firm entry and exit. To discipline this margin of adjustment, we target the fraction of job destruction that is due to firm exit and the fraction of job creation that in due to firm entry in our model. We operationalize this by defining $\Delta N_t \equiv N_t - N_{t-1}$ as the net job flow into period t. Since $n_t = N_t/Z_t$ denotes jobs per firm, we obtain the following decomposition:

$$\frac{\sum_{j=0}^{11} n_{t+j}(Z_{t+j} - Z_{t+j-1})\mathbb{I}(\Delta N_{t+j} \lessgtr 0)}{\sum_{j=1}^{12} \Delta N_{t+j}\mathbb{I}(\Delta N_{t+j} \lessgtr 0)} + \frac{\sum_{j=1}^{12} Z_{t+j-1}(n_{t+j} - n_{t+j-1})\mathbb{I}(\Delta N_{t+j} \lessgtr 0)}{\sum_{j=1}^{12} \Delta N_{t+j}\mathbb{I}(\Delta N_{t+j} \lessgtr 0)} = 1.$$

We calculate the first term during months of job growth ($\Delta N_{t+j} > 0$) and job loss ($\Delta N_{t+j} < 0$). Haltiwanger (2012) finds, on average, 37% of jobs created in a given year are at new firms or establishments, while 31% of destroyed jobs were at firms or establishments that exited the market. We set the entry cost ψ_n to 0.015 and the fixed cost of production ψ_y to 0.094 to target these values.

Finally, we set the autocorrelation of productivity, ρ_a, and standard deviation of the productivity shock, σ_a, to target the first-order autocorrelation and standard deviation of output growth in our sample. These targets imply $\rho_a = 0.951$ and $\sigma_a = 0.002$, which are smaller than in many other papers in the literature because firm entry and exit endogenously generates volatility in the model.

4.2 SOLUTION METHOD AND POLICY FUNCTIONS

Endogenous firm entry and exit introduces an inherent nonlinearity that endogenously generates higher-order moments in our model. To capture these effects, we solve the model globally by adapting the policy function iteration algorithm in Richter et al. (2014) to our setting. The basic algorithm minimizes the Euler equation errors on every node in the discretized state space. It then computes the maximum distance between the policy functions on any node and iterates until that distance falls below a given tolerance criterion.

[5] See Appendix A for a detailed description of our data sources and how they were transformed for our calculations.
[6] In order to focus on business cycle variation, we filter the unemployment rate series following Hamilton (2018).

What makes this model particularly challenging to solve is that it contains inequality constraints on vacancies and the value of active firms. These constraints only bind in certain areas of the state space that are determined as part of the numerical solution. To capture these features of the equilibrium system, we follow Garcia and Zangwill (1981) and introduce auxiliary variables $\mu_{V,t}$ and $\mu_{A,t}$ that are continuous in the state of the economy. The auxiliary variables satisfy the following:

$$V_t = \max\{0, \mu_{V,t}\}^2, \quad \lambda_{V,t} = \max\{0, -\mu_{V,t}\}^2,$$
$$J^F_{A,t} = \min\{\max\{0, \mu_{A,t}\}, \psi_n\}, \quad \lambda^F_{A,t} = \mu_{A,t} - J^F_{A,t}, \quad Z_t = Z_{t-1} + \lambda^F_{A,t}.$$

The auxiliary variable $\mu_{V,t}$ maps into vacancies when $V_t > 0$ and the Lagrange multiplier, $\lambda_{V,t}$ when $V_t = 0$. Similarly, $\mu_{A,t}$ equals the value of active firms when $J^F_{A,t}$ is between the entry and exit boundaries, so the number of active firms is unchanged from last period. At the boundaries, $\lambda^F_{A,t}$ determines the corresponding change in the number of active firms, $Z_t - Z_{t-1}$, due to entry or exit.[7]

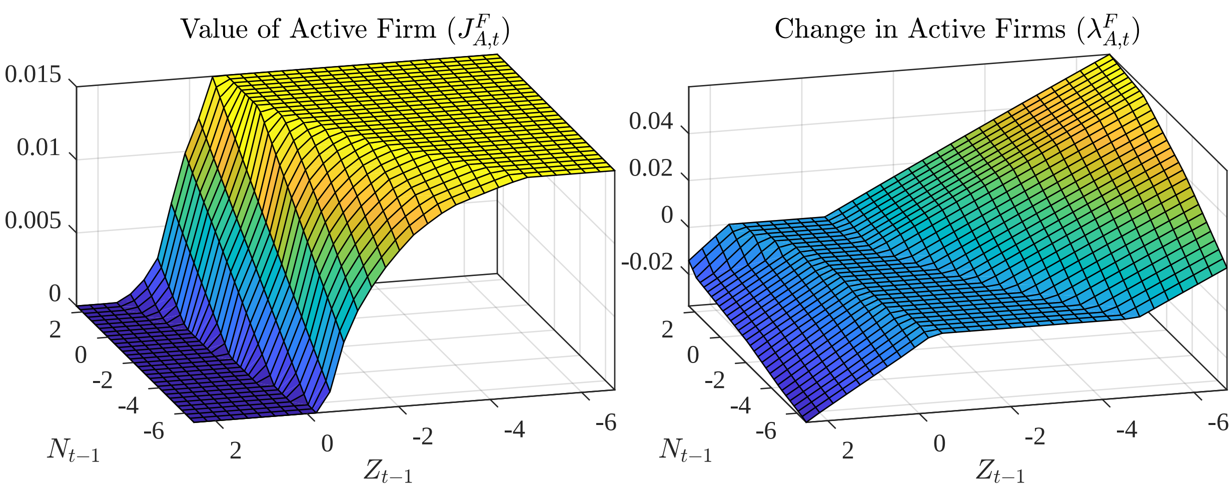

Figure 2: Policy function surface plots. Each axis is shown in deviations from the deterministic steady state.

To highlight the influence of firm entry and exit on the model solution, Figure 2 plots the policy functions for the active firm value function, $J^F_{A,t}$, and the change in the number of active firms, $\lambda^F_{A,t}$, as a function of initial employment and the initial number of active firms. Productivity is fixed at its steady state and each variable is shown in percent derivations from its respective steady state.

The nonlinearities in the model are immediately apparent. When $\mu_{A,t}$ exceeds the entry cost, $\psi_n = 0.015$, there is sufficient firm entry to prevent further increases in the value of active firms. This is shown in the upper plateau of the left panel and the upward sloping portion of the right panel. Similarly, when $\mu_{A,t}$ turns negative, firms exit to prevent a negative value of active firms,

[7] We approximate the productivity process with an N-state Markov chain following Rouwenhorst (1995). Given the updated state of the economy, we use piecewise linear interpolation to calculate the period-$t+1$ policy functions and the transition matrix to numerically integrate. See Appendix B for a detailed description of the solution method.

creating the lower plateau and downward sloping portions of the policy functions. Between 0 and ψ_n, the value of the firm is free to adjust, so there is no change in the number of active firms. This is represented by the upward sloping portion of the left panel and the flat region of the right panel.

4.3 SIMULATED MOMENTS To evaluate the quantitative success of the model, we begin by simulating the model economy's path in response to a sequence of productivity shocks. Each simulation is initialized with a draw from the ergodic distribution and spans 792 months, the same number of observations as our monthly data sample. To compare the model predictions to data on real activity, we aggregate up to a quarterly frequency. We execute $10{,}000$ simulations and report the average and $(5, 95)$ percentiles of several higher-order moments of key macroeconomic variables.

		Linear	Nonlinear	
Moment	Data	No Entry/Exit	No Entry/Exit	Entry/Exit
$AC(\Delta \log Y)$	0.31	0.30 $(0.21, 0.39)$	0.31 $(0.22, 0.39)$	0.32 $(0.22, 0.41)$
$SD(\Delta \log Y)$	0.86	0.52 $(0.48, 0.57)$	0.53 $(0.49, 0.58)$	0.81 $(0.68, 0.95)$
$SKEW(\Delta \log Y)$	-0.46	-0.00 $(-0.25, 0.25)$	0.04 $(-0.21, 0.32)$	-0.50 $(-1.03, -0.02)$
$KURT(\Delta \log Y)$	1.67	-0.03 $(-0.43, 0.49)$	0.09 $(-0.37, 0.69)$	2.28 $(0.53, 4.99)$
$AC(\Delta \log C)$	0.29	0.38 $(0.29, 0.46)$	0.38 $(0.30, 0.46)$	0.36 $(0.27, 0.45)$
$SD(\Delta \log C)$	0.51	0.44 $(0.40, 0.47)$	0.45 $(0.41, 0.49)$	0.72 $(0.59, 0.86)$
$SKEW(\Delta \log C)$	-0.90	-0.00 $(-0.25, 0.26)$	0.04 $(-0.23, 0.32)$	-0.70 $(-1.29, -0.18)$
$KURT(\Delta \log C)$	3.59	-0.03 $(-0.44, 0.49)$	0.12 $(-0.36, 0.74)$	2.96 $(0.83, 6.31)$

Table 2: Mean and $(5, 95)$ percentiles of the simulated moments from $10{,}000$ simulations with the same length as our data sample (1954M1-2019M4). AC, SD, $SKEW$, and $KURT$ denote the autocorrelation, standard deviation, skewness, and excess kurtosis across time. Output and consumption in the model are based on the sum of the monthly values and then converted to quarterly growth rates ($\Delta \log Y$ and $\Delta \log C$).

Output and Consumption Table 2 reports the data and model-implied moments for three versions of the model: the log-linearized and nonlinear solutions *without* firm entry and exit and the nonlinear solution *with* entry and exit. First consider the statistics for output growth. Recall that the productivity process is calibrated to ensure that the economy with entry and exit closely matches the standard deviation and first-order autocorrelation of output growth in the data. It is useful to note that the linear and nonlinear models without entry and exit feature lower standard deviations of output growth. This pattern shows the entry and exit mechanism is an important source of endogenous amplification. Furthermore, entry and exit is crucial to generate the negative skewness and excess kurtosis of output growth in the data. The model-implied average skewness and excess kurtosis are -0.50 and 2.28, which are close to their empirical counterparts (-0.46 and 1.67). In the nonlinear model without entry and exit, output growth is positively skewed and there is essentially no excess kurtosis (the linear model has no skewness or excess kurtosis by construction).

Similar insights apply to consumption growth. While the full model slightly over-predicts the standard deviation (0.72 in the model vs. 0.51 in the data) and autocorrelation (0.36 in the model vs. 0.29 in the data), the entry and exit mechanism is essential for generating negative skewness (-0.70 in the model vs. -0.90 in the data) and excess kurtosis (2.96 in the model vs. 3.59 in the data).

		Linear	Nonlinear	
Moment	Data	No Entry/Exit	No Entry/Exit	Entry/Exit
$SD(U)$	22.45	12.38 $(10.02, 15.01)$	12.26 $(10.02, 14.58)$	17.79 $(13.83, 21.76)$
$SKEW(U)$	0.52	-0.00 $(-0.50, 0.49)$	0.32 $(-0.08, 0.74)$	0.77 $(0.29, 1.26)$
$KURT(U)$	-0.25	-0.16 $(-0.77, 0.72)$	-0.21 $(-0.79, 0.61)$	0.63 $(-0.45, 2.19)$
$SD(V)$	23.84	10.27 $(8.66, 12.06)$	10.22 $(8.70, 11.80)$	12.41 $(10.07, 14.95)$
$SKEW(V)$	-0.66	0.00 $(-0.40, 0.40)$	-0.36 $(-0.69, -0.05)$	-0.60 $(-1.04, -0.16)$
$KURT(V)$	0.01	-0.11 $(-0.62, 0.59)$	-0.06 $(-0.58, 0.65)$	0.48 $(-0.42, 1.75)$
$CORR(U, V)$	-0.77	-0.91 $(-0.94, -0.88)$	-0.90 $(-0.93, -0.88)$	-0.92 $(-0.94, -0.89)$

Table 3: Mean and $(5, 95)$ percentiles of the simulated moments from $10{,}000$ simulations with the same length as our data sample (1954M1-2019M12). AC, SD, $SKEW$, and $KURT$ denote the autocorrelation, standard deviation, skewness, and excess kurtosis across time. Monthly unemployment and vacancy rates are averaged to a quarterly frequency and shown in log deviations from trend. The trend in the data is from a Hamilton (2018) filter with an 8 quarter window. The trend in the model is the simulated sample average.

Unemployment and Vacancies Table 3 reports the corresponding moments for labor market outcomes: the unemployment and vacancy rates. Consider first the unemployment rate. The linear and nonlinear models without entry and exit both undershoot the empirical standard deviation. When we introduce firm entry and exit, the endogenous amplification pushes unemployment volatility much closer to its empirical counterpart (17.79 in the model vs. 22.45 in the data).[8] The data show unemployment also exhibits positive skewness (0.52). While the model without entry and exit generates some positive skewness (0.32), including entry and exit significantly improves the model's ability to generate asymmetry and actually over-predicts the skewness. The unemployment rate also features excess kurtosis in the model (0.63), but surprisingly it is negative in the data (-0.25).

The model performs similarly on the vacancy rate. Entry and exit remains crucial for generating a standard deviation and negative skewness close to the empirical values. Finally, the strong negative correlation between vacancies and unemployment present in the data is a feature of each model.

[8]Using an HP filter with a smoothing parameter of 1600 reduces the volatilities of the unemployment and vacancy rates to 11.79 and 13.46, respectively. This is likely due to the HP filter ascribing too much of the volatility to the trend.

Tail Risks Negative skewness and excess kurtosis in real activity and labor market outcomes indicates significant departures from normality. To further analyze the non-Gaussian nature of both the model and the data, we examine the tails of the distributions of these variables in more detail.

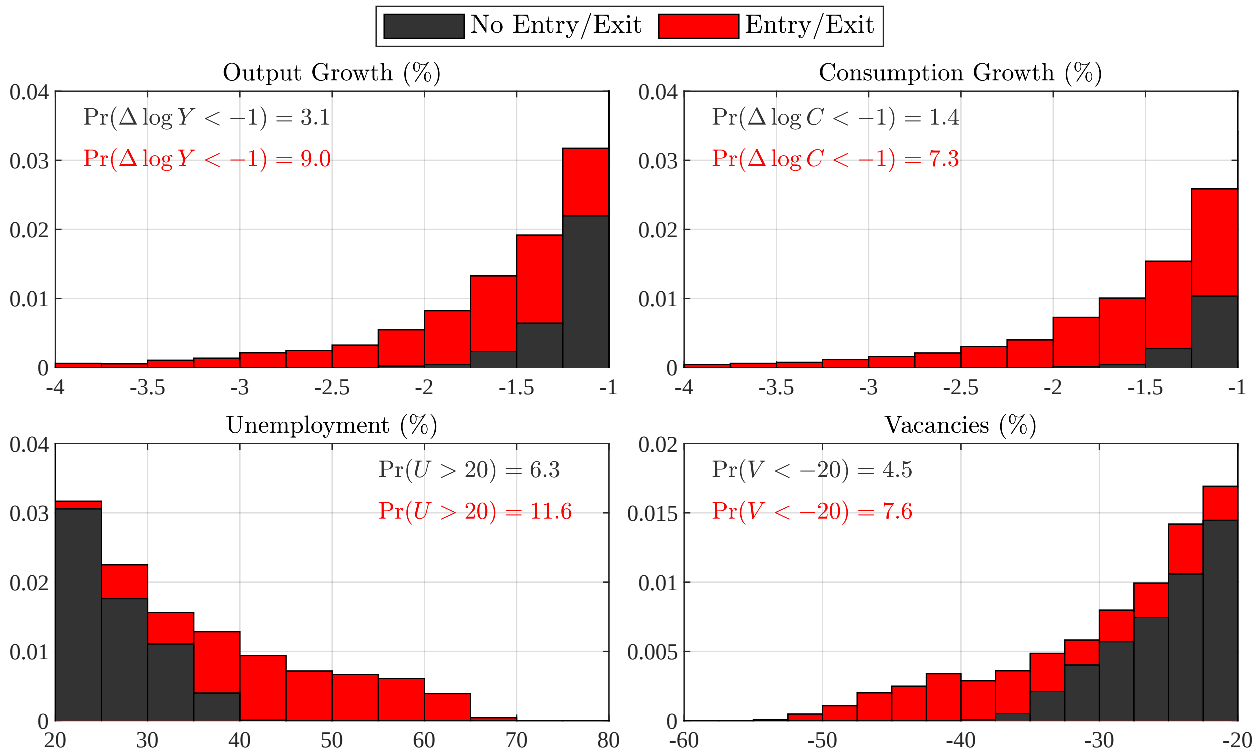

Figure 3: Distributions based on a 100,000 month simulation of the models with and without entry and exit.

Figure 3 plots the left tails of the model-implied distributions of output, consumption, and vacancies, and the right tail of the unemployment rate. The darker-black bars show the distributions in the nonlinear model without firm entry and exit and the lighter-red bars show the effect of introducing entry and exit. The full model has much longer and thicker left tails for all four variables, which results in substantial negative skewness. For example, the probability of observing a 1% or larger drop in output growth is nearly three times higher in the model with entry and exit.

Following Acemoglu et al. (2017), we define tail risk as the probability that a random variable is more than 1.96 standard deviations away from its mean. Under normality, this probability is 2.5% in each tail. Table 4 reports the left and right tail risks for each macroeconomic outcome in the data and the nonlinear models with and without entry and exit. The data confirms the skewness patterns above as the tails feature asymmetric probabilities. Furthermore, the sum of the tails consistently exceeds 5% indicating more probability mass is in the tails than is implied by a normal distribution.

Comparing the data to the models further highlights the crucial role of firm entry and exit. Without this mechanism, the tails are more symmetric and more Gaussian. The presence of entry and exit creates more asymmetry and pushes the total tail risk further above 5%, in line with the data.

	Data		No Entry/Exit		Entry/Exit	
Outcome	Left	Right	Left	Right	Left	Right
$\Delta \log Y$	4.17	1.52	2.42 $(1.14, 3.80)$	2.64 $(1.52, 3.80)$	3.42 $(2.28, 4.56)$	2.20 $(1.14, 3.42)$
$\Delta \log C$	3.03	1.14	2.45 $(1.14, 3.80)$	2.65 $(1.52, 3.80)$	3.70 $(2.66, 4.94)$	2.04 $(1.14, 3.04)$
U	0.38	4.92	1.10 $(0.00, 3.03)$	3.50 $(1.52, 5.68)$	0.24 $(0.00, 1.52)$	4.69 $(2.65, 6.82)$
V	5.30	0.00	3.61 $(1.89, 5.30)$	1.13 $(0.00, 2.65)$	4.05 $(2.27, 5.68)$	0.70 $(0.00, 2.27)$

Table 4: Tail risks, defined as the probability a variable is more than 1.96 standard deviations from its mean.

4.4 GENERALIZED IMPULSE RESPONSES We have shown that firm entry and exit is an important source of asymmetry and amplification of productivity shocks. To measure the quantitative significance of these effects, Figure 4 plots generalized impulse response functions (GIRFs) of several macroeconomic variables to negative and positive two standard deviation productivity shocks.[9]

The left column plots the responses to a negative shock in the nonlinear model with entry and exit as well as the linear and nonlinear models without entry and exit. A common pattern emerges. The nonlinear model with firm entry and exit features larger responses than the models without entry and exit. For example, the decline in output peaks at about 1.1% in the full model compared to 0.75% in the models without entry and exit. Similar patterns hold for the other three variables.

The driving force behind the amplification is evident from the response of the number of active firms. When productivity falls firm profits decline. In response, some firms exit while others reduce their labor demand, both of which cause unemployment to increase and output to decline. Those responses lead to a further reduction in profits, resulting in additional firm exit. This feedback loop lasts for 12 months after the initial decline in productivity, at which point firms begin to re-enter.

The right column contains the responses to a symmetric positive productivity shock. For ease of comparison, the responses are multiplied by negative one. The same amplification pattern emerges, but the magnitude is weaker relative to the responses to the negative shock. For example, the peak rise in output is 0.9% in the full model, compared to 0.75% in the models without entry and exit.

The weaker amplification is further evidence of the asymmetry that entry and exit generates in the economy. Intuitively, the asymmetry occurs because separations happen immediately when firms exit, while the additional employment that occurs when firms enter the market is slow to adjust due to search frictions. The differences in the speed of adjustment and the responses of real activity imply that firm entry and exit is also asymmetric. While the number of firms declines by about 0.6% in response to the negative shock, they rise by 0.5% in response to the positive shock.

[9]To compute a GIRF, we follow Koop et al. (1996). We first calculate the mean path from $10{,}000$ model simulations, conditional on random productivity shocks in every month. We then calculate a second mean from another set of $10{,}000$ simulations, but this time replace the random shock in the first month with a two standard deviation shock. The GIRF reports the difference between the two mean paths. All of the simulations are initialized at the ergodic mean.

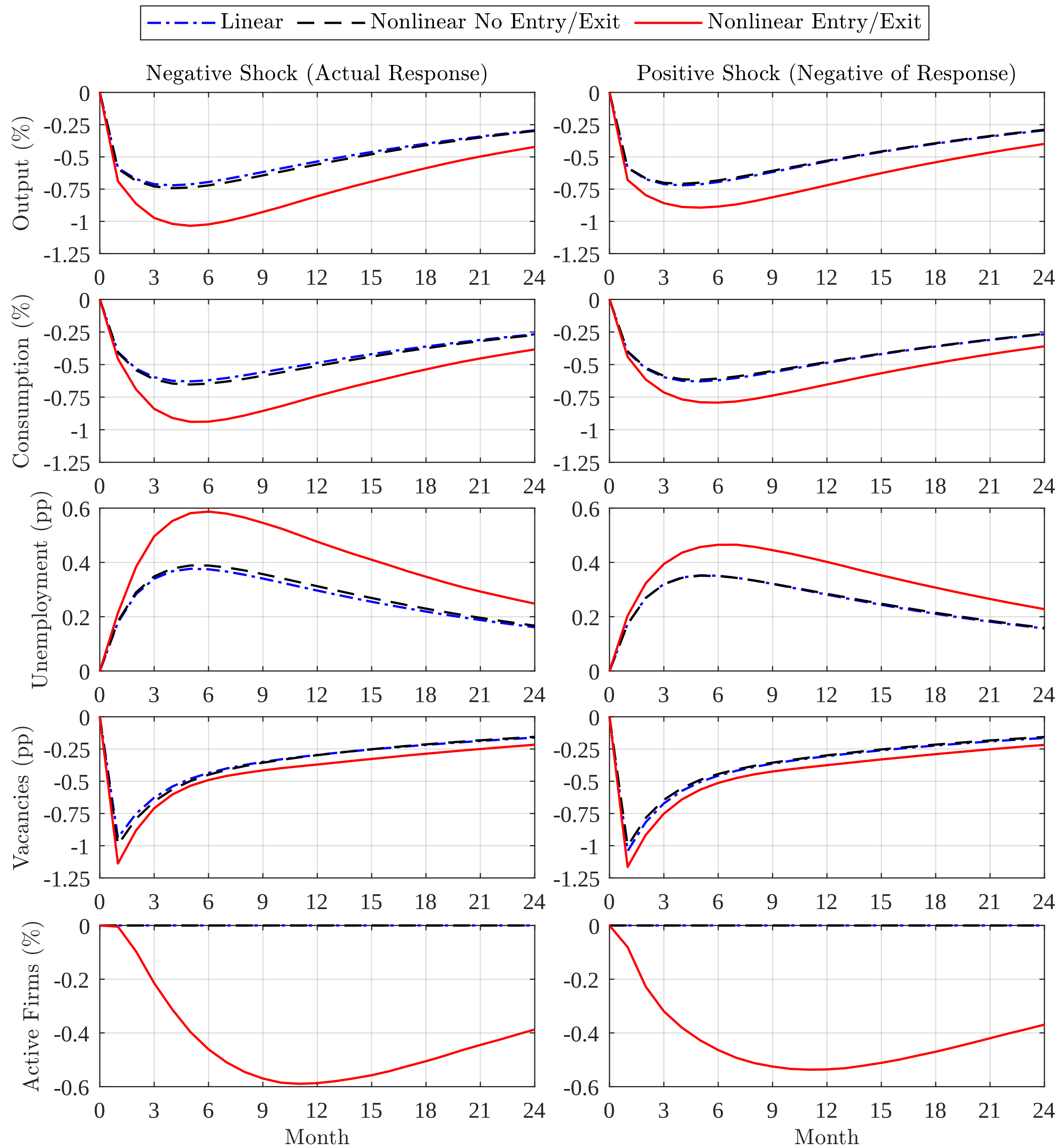

Figure 4: Impulse responses to a ±2SD productivity shock in deviations from a baseline simulation without the shock. All of the responses in the right column are multiplied by negative one to facilitative comparison.

4.5 LABOR MARKET DYNAMICS

Our analytical section showed the asymmetry and amplification in output and consumption are driven by the dynamics of employment and job creation in a model with endogenous firm entry and exit. This section examines these features in more detail by focusing on the model-implied Beveridge curve and the unemployment inflow and outflow rates.

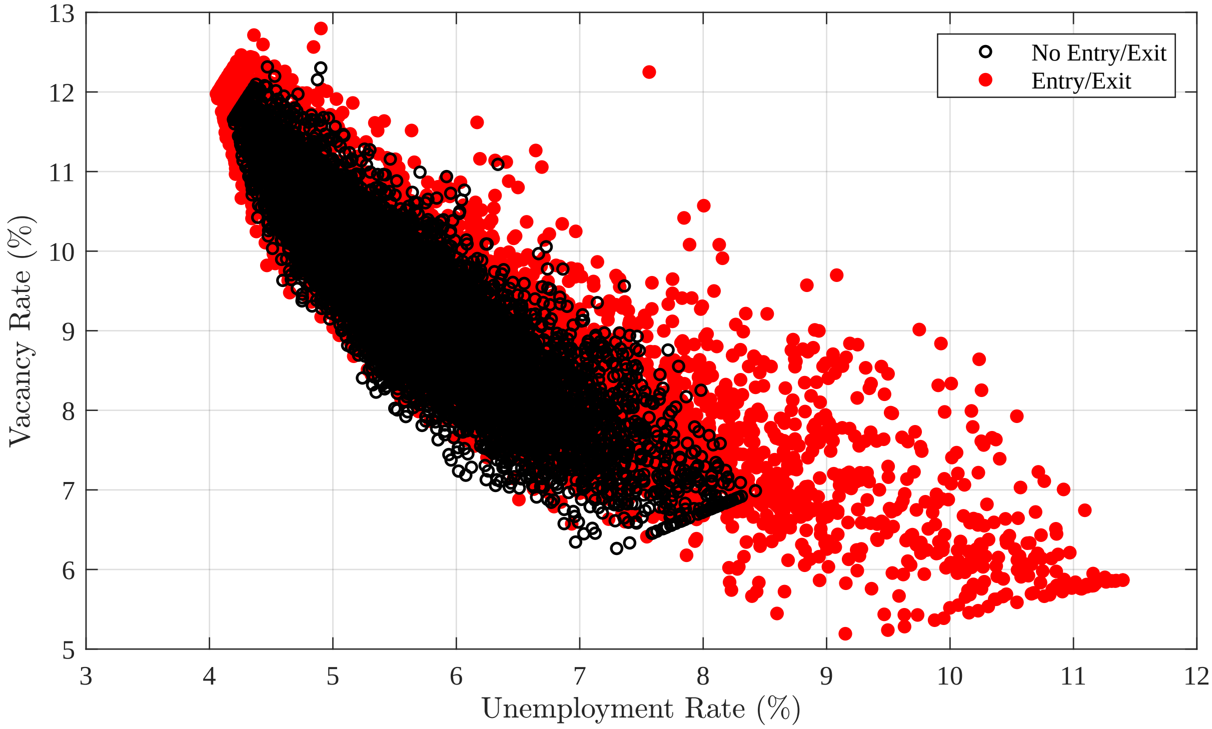

Figure 5: Beveridge curves based on a 20,000 month sample from the ergodic distribution.

Beveridge Curve Figure 5 plots the Beveridge curve (BC) implied by the models with and without firm entry and exit. Two differences are apparent. First, there is a convex kink in the BC implied by the entry and exit model centered around an unemployment rate of about 6% (the ergodic mean). Similar to Figure 1, unemployment rises more for a given drop in vacancies to the right of the kink than to the left. Furthermore, the BC does not feature a kink in the model without entry and exit.

Second, the dispersion in the simulated data is larger in the model with firm entry and exit. This feature is driven by the endogenous amplification of productivity shocks. For example, the most severe negative shock in the simulation drives the unemployment rate above 11% in the model with firm entry and exit, while the unemployment rate peaks at about 8% when entry and exit is omitted.

Unemployment Inflows and Outflows In contrast to much of the existing search literature, our model features endogenous variation in the job separation rate driven by firm entry and exit. Specifically, jobs are endogenously destroyed when firms exit and $Z_t < Z_{t-1}$. This job destruction causes a spike in the separation rate, s_t, which partially drives the corresponding inflow to unemployment.

Qualitatively, this mechanism is consistent with the evidence in Elsby et al. (2009), who show that countercyclical variation in the unemployment inflow rate on average accounts for 35% of increases in unemployment during recessions. Crucially, this empirical finding casts doubt on the common modeling assumption that the separation rate is acyclical. Our model proposes that firm entry and exit is an important source of countercyclical variation in the unemployment inflow rate.

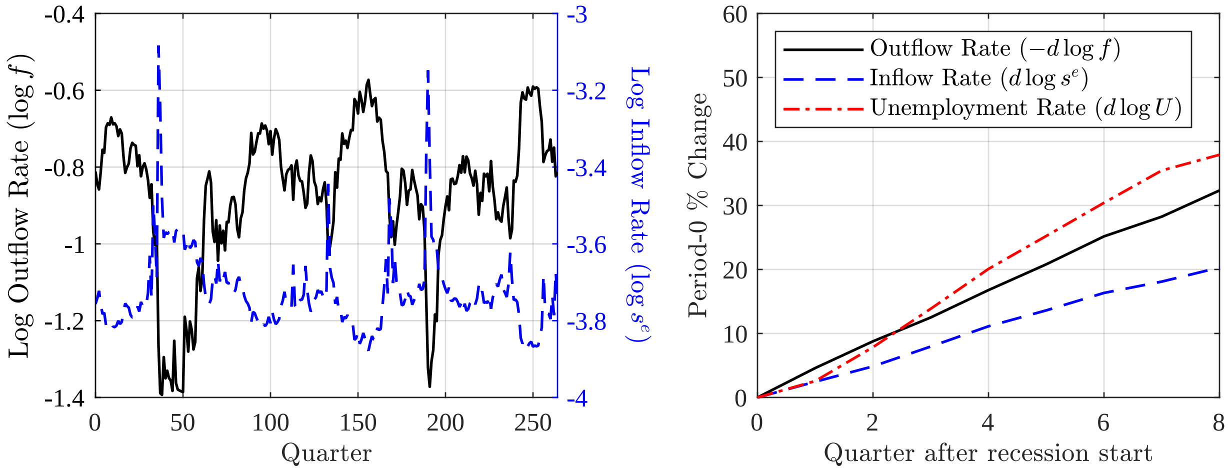

Figure 6: Cyclicality of the log job-finding and job-separation rates. The left panel shows the relationship between the two rates in an example simulation. The right panel reports the median path in a sample of 2,533 recessions, where a recession is defined as 3 or more consecutive quarters of an increase in unemployment.

Figure 6 shows our model generates reasonable quantitative predictions for the variation in the unemployment inflow rate, $s_t^e \equiv s_t(1-f_t)$, and its contribution to increases in unemployment during recessions.[10] The left panel shows the relationship between the inflow and outflow rates in an example simulation with the same length as our data sample. The two rates negatively co-move, with a pro-cyclical outflow rate and counter-cyclical inflow rate. Following the empirical work in Elsby et al. (2009), we simulate data from our model and compute changes in inflow and outflow rates during recessions, which are defined as three or more consecutive quarters of increasing unemployment. The right panel plots the median paths of these changes across the 2,533 recessions in our sample. This shows the relative contributions of the decline in the outflow rate and the increase in the inflow rate. Inflows account for 37% of the increases in unemployment during recessions, in line with the 35/65 inflow-outflow split reported by Elsby et al. (2009). Furthermore, the increase in job destruction due to firm exit at the start of a recession is consistent with their finding that the job loser inflow rate drives most of the empirical variation in inflow rates.

4.6 Welfare Cost of Business Cycles

Our quantitative framework generates realistic higher-order moments for aggregate output and consumption. These features are missing in models that log-linearize or do not generate enough asymmetry or amplification of shocks. Given this, we now use our model to revisit a classic topic in macroeconomics: the welfare cost of business cycles.

To determine the welfare cost, we first simulate the model for $T = 15{,}000$ months to compute the stochastic steady state value of consumption, $\tilde{C}$. We then integrate across $10{,}000$ simulations of T months to compute an expected path of consumption, $\{C_j\}_{j=t}^{T}$. Finally, we compute the fraction,

[10] Recall that $N_t = (1-s_t)N_{t-1} + f_t U_t^s$ so $\Delta U_t = s_t N_{t-1} - f_t U_t^s$. Use $U_t^s = 1-(1-s_t)N_{t-1}$ to obtain $\Delta U_t = s_t(1-f_t)(1-U_{t-1}) - f_t U_{t-1}$. Then $s_t(1-f_t)$ is the unemployment inflow rate and f_t is the outflow rate.

λ, of $\{C_j\}_{j=t}^T$ households would require to make them indifferent between receiving $\{\tilde{C}\}_{j=t}^T$ and $\{C_j\}_{j=t}^T$. Formally, $\lambda = \exp(\log \tilde{C} - (1-\beta)E_t W(\{C_j\}_{j=t}^T)) - 1$, where W denotes lifetime utility.

We find $\lambda = 0.0038$, so households would be willing to give 0.38% of their lifetime consumption to avoid the fluctuations in consumption caused by business cycles. This fraction is over three times higher than in the nonlinear model without firm entry and exit (0.11%) and two orders of magnitude larger than Lucas (1987) reports (0.008%). Relative to the literature, we do not rely on non-standard utility functions or household heterogeneity to generate this larger cost of business cycles.[11] Instead, the cost is driven by the negative skewness and excess kurtosis that firm entry and exit imparts to consumption growth in our model. Since these are features of the data, our results suggest that the cost of business cycles is far more significant than initial calculations indicated.[12]

5 An Application to the COVID-19 Pandemic

In spring 2020, the COVID-19 pandemic led to a major disruption of the U.S. economy. The unemployment rate reached 14.7% in April, and a record 22 million workers filed initial claims for unemployment insurance from March 15-April 11. This section examines how firm entry and exit affect the speed of the recovery. To initialize the economy in a COVID-19 pandemic state, we assume there is an unanticipated two month spike in the job separation rate $\bar{s}$ that drives the unemployment rate to 10%. This shock captures the sudden inflows to unemployment from the "stay-at-home" orders. Once the economy has reached this crisis state, we then study the response to a negative two standard deviation productivity shock. We interpret this shock as capturing a range of factors that cause further economic decline during the pandemic, such as disruption to supply chains and production processes, and a decline in productivity due to illness and sick leave.

Figure 7 plots impulse responses of key variables for the three models we consider in this paper. Qualitatively, a similar pattern emerges to what is shown in Figure 4. The responses in the nonlinear model with entry and exit are amplified relative to the economies without entry and exit. Quantitatively, however, the amplification is much stronger, and demonstrates the state-dependent nature of the entry and exit mechanism. For example, the number of active firms drops by almost 1% when unemployment is initially at 10%, but drops by only 0.6% when unemployment is initially at its ergodic mean (about 5.9%). The larger response of firm exit causes a commensurate decline in economic activity. Output declines by 1.5% while the unemployment rate rises by 1 percentage point. Each of these responses are almost double the responses of the economy without the entry and exit mechanism, and are about 1.5 times the size of responses outside of the pandemic state.

This exercise further highlights the power of firm entry and exit as a propagation mechanism,

[11] See Barlevy (2005) for a survey of the literature that computes a welfare cost of business cycles in different models.

[12] Recent empirical work by Jordà et al. (2020) further emphasizes how non-Gaussian behavior of consumption growth of the type generated by our model plays an important role in increasing the welfare cost of business cycles.

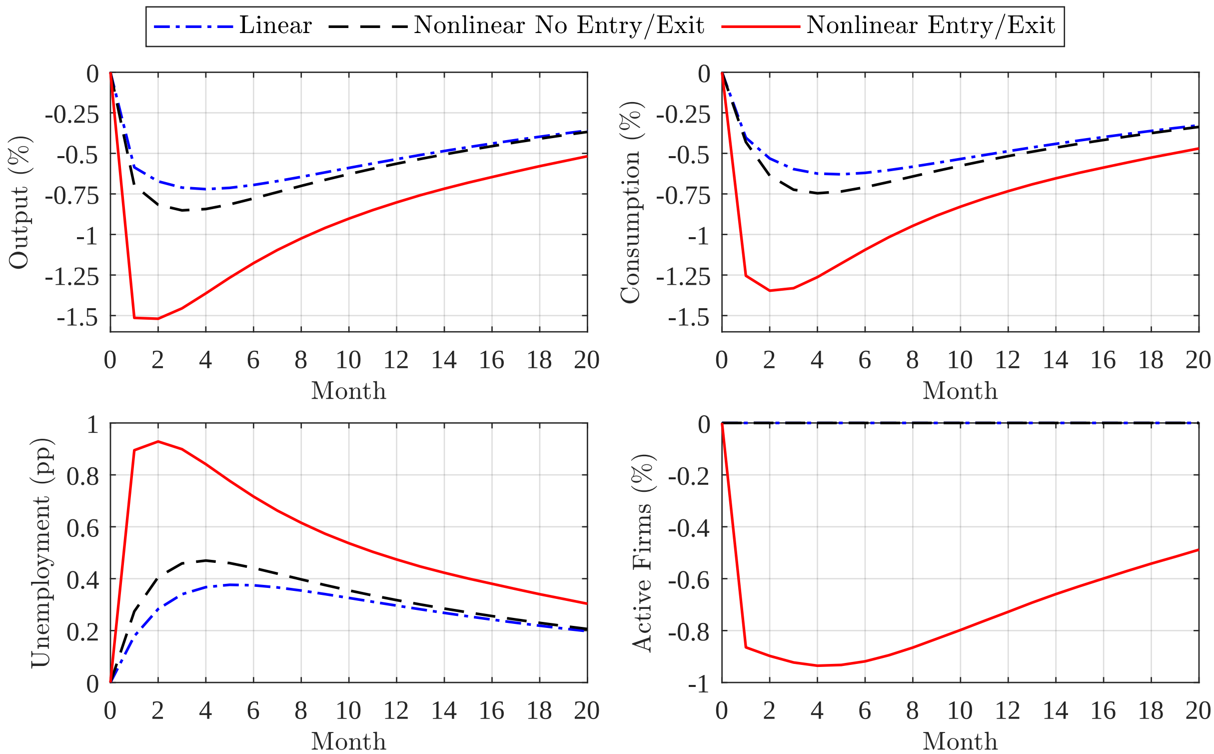

Figure 7: Impulse responses to a -2SD productivity shock in deviations from a baseline simulation without the shock. Each simulation is initialized in a COVID-19 recession with an initial unemployment rate of 10%.

and that it is an important aspect of the economy's response to recessionary shocks. In contemporaneous work, Guerrieri et al. (2020) show firm exit may exacerbate supply shocks though a Keynesian channel in which businesses that shut down in one sector may cause shut downs in other sectors. In addition, our results on the power of firm exit lend credence to the findings in Faria-e-Castro (2020) that policies aimed at preventing firm exit help to ensure a faster recovery.

Speed of Transmission A comparison of Figure 4 and Figure 7 reveals the speed of transmission is state-dependent, in addition to the state-dependent propagation strength. For example, the unemployment rate peaks 6 months after the shock when the economy is initialized at its ergodic mean, while it peaks after 2 months when the initial unemployment rate is 10%. This shows the effects of adverse shocks are stronger and bottom out more quickly when the economy is in a recession.

To understand the source of the state-dependence, Figure 8 plots the responses of the number of active firms, Z_t, and the active firm value, $J^F_{A,t}$, to a negative productivity shock initialized at the ergodic mean (normal times) and the recessionary pandemic state. In the pandemic state, Z_t drops on impact, whereas in normal times firm exit occurs more slowly. These differences are apparent from the $J^F_{A,t}$ responses. Naturally, $J^F_{A,t}$ is higher in normal times than in the pandemic state. Since exit only occurs when $J^F_{A,t}$ hits zero, firms will choose to shut down must faster in a pandemic than in normal times. In equilibrium, this change in speed transmits to other outcomes in the economy.

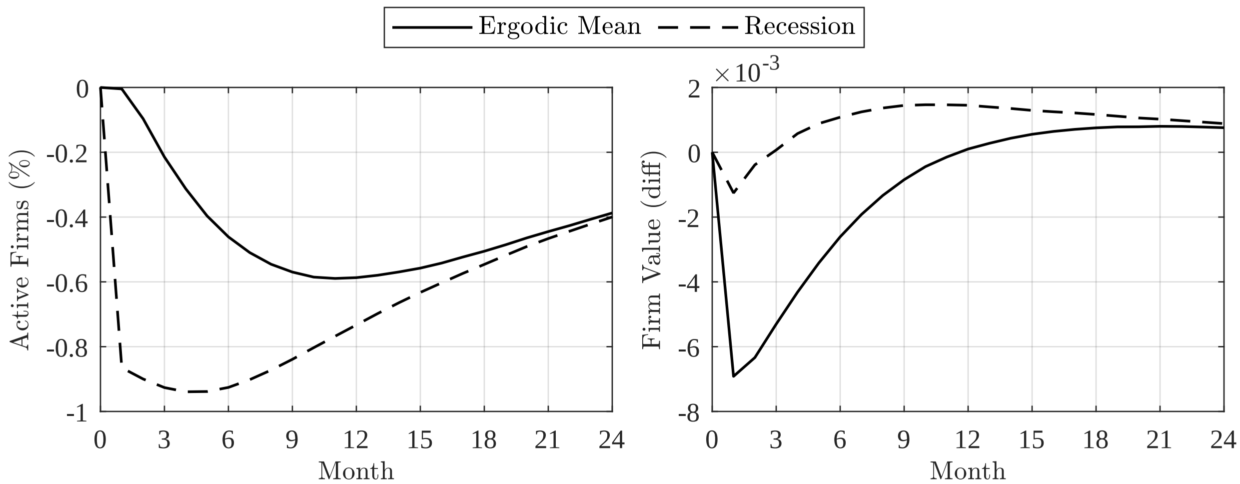

Figure 8: Impulse responses to a -2SD productivity shock in deviations from a baseline simulation without the shock. The simulations are initialized at the ergodic mean ($U = 6\%$) or the pandemic state ($U = 10\%$).

6 CONCLUSION

Firm entry and exit drives a substantial portion of the business cycle variation in output, consumption, unemployment, and vacancies. It generates asymmetry and amplifies the transmission of productivity shocks. Quantitatively, it generates negative skewness and excess kurtosis consistent with the data and significant tail risk. Without firm entry and exit, the model fails to generate empirically consistent higher-order moments and produces almost no tail risk. Given the rich nonlinearities and significant welfare cost of business cycles in our model, a natural next step is to include fiscal and monetary policy to study the optimal stabilization of business cycle fluctuations.

REFERENCES

ACEMOGLU, D., A. OZDAGLAR, AND A. TAHBAZ-SALEHI (2017): “Microeconomic Origins of Macroeconomic Tail Risks,” *American Economic Review*, 107, 54–108, https://doi.org/10.1257/aer.20151086.

BARLEVY, G. (2005): “The cost of business cycles and the benefits of stabilization,” *Economic Perspectives*, 29, 32–49.

BARNICHON, R. (2010): “Building a composite Help-Wanted Index,” *Economics Letters*, 109, 175–178, https://doi.org/10.1016/j.econlet.2010.08.029.

BILBIIE, F. O., F. GHIRONI, AND M. J. MELITZ (2012): “Endogenous Entry, Product Variety, and Business Cycles,” *Journal of Political Economy*, 120, 304–345, https://doi.org/10.1086/665825.

BLANCHARD, O. AND J. GALÍ (2010): "Labor Markets and Monetary Policy: A New Keynesian Model with Unemployment," *American Economic Journal: Macroeconomics*, 2, 1–30, https://doi.org/10.1257/mac.2.2.1.

DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90, 482–498, https://doi.org/10.1257/aer.90.3.482.

DUPRAZ, S., E. NAKAMURA, AND J. STEINSSON (2019): "A Plucking Model of Business Cycles," NBER Working Paper 26351, https://doi.org/10.3386/w26351.

ELSBY, M. W. L., G. SOLON, AND R. MICHAELS (2009): "The Ins and Outs of Cyclical Unemployment," *American Economic Journal: Macroeconomics*, 1, 84–110, https://doi.org/10.1257/mac.1.1.84.

FARHI, E. AND F. GOURIO (2018): "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity*, 49, 147–250, https://doi.org/10.1353/eca.2018.0024.

FARIA-E-CASTRO, M. (2020): "Fiscal Policy during a Pandemic," Federal Reserve Bank of St. Louis Working Paper 2020-006, https://doi.org/10.20955/wp.2020.006.

GARCIA, C. B. AND W. I. ZANGWILL (1981): *Pathways to Solutions, Fixed Points and Equilibria*, Prentice-Hall series in computational mathematics, Prentice-Hall.

GUERRIERI, V., G. LORENZONI, L. STRAUB, AND I. WERNING (2020): "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?" NBER Working Paper 26918, https://doi.org/10.3386/w26918.

HAGEDORN, M. AND I. MANOVSKII (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98, 1692–1706, https://doi.org/10.1257/aer.98.4.1692.

HALTIWANGER, J. (2012): "Job Creation and Firm Dynamics in the United States," in *Innovation Policy and the Economy, Vol. 12*, ed. by J. Lerner and S. Stern, University of Chicago Press, NBER Chapters, 17–38.

HAMILTON, J. D. (2018): "Why You Should Never Use the Hodrick-Prescott Filter," *Review of Economics and Statistics*, 100, 831–843, https://doi.org/10.1162/rest_a_00706.

ILUT, C., M. KEHRIG, AND M. SCHNEIDER (2018): "Slow to Hire, Quick to Fire: Employment Dynamics with Asymmetric Responses to News," *Journal of Political Economy*, 126, 2011–2071, https://doi.org/10.1086/699189.

JORDÀ, O., M. SCHULARICK, AND A. M. TAYLOR (2020): "Disasters Everywhere: The Costs of Business Cycles Reconsidered," NBER Working Paper 26962, https://doi.org/10.3386/w26962.

KOOP, G., M. H. PESARAN, AND S. M. POTTER (1996): "Impulse Response Analysis in Nonlinear Multivariate Models," *Journal of Econometrics*, 74, 119–147, https://doi.org/10.1016/0304-4076(95)01753-4.

KOPECKY, K. AND R. SUEN (2010): “Finite State Markov-chain Approximations to Highly Persistent Processes,” *Review of Economic Dynamics*, 13, 701–714, https://doi.org/10.1016/j.red.2010.02.002.

LUCAS, JR, R. E. (1987): *Models of Business Cycles*, Oxford: Basil Blackwell.

PETROSKY-NADEAU, N. AND L. ZHANG (2017): “Solving the Diamond-Mortensen-Pissarides model accurately,” *Quantitative Economics*, 8, 611–650, https://doi.org/10.3982/QE452.

RICHTER, A. W., N. A. THROCKMORTON, AND T. B. WALKER (2014): “Accuracy, Speed and Robustness of Policy Function Iteration,” *Computational Economics*, 44, 445–476, https://doi.org/10.1007/s10614-013-9399-2.

ROUWENHORST, K. G. (1995): “Asset Pricing Implications of Equilibrium Business Cycle Models,” in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, Princeton, NJ: Princeton University Press, 294–330.

SHAO, E. AND P. SILOS (2013): “Entry costs and labor market dynamics,” *European Economic Review*, 63, 243–255, https://doi.org/10.1016/j.euroecorev.2013.

SHIMER, R. (2005): “The Cyclical Behavior of Equilibrium Unemployment and Vacancies,” *American Economic Review*, 95, 25–49, https://doi.org/10.1257/0002828053828572.

SIMS, C. A. (2002): “Solving Linear Rational Expectations Models,” *Computational Economics*, 20, 1–20, https://doi.org/10.1023/A:1020517101123.

SMETS, F. AND R. WOUTERS (2007): “Shocks and frictions in us business cycles: a bayesian dsge approach,” *American Economic Review*, 97, 586–606, https://doi.org/10.1257/aer.97.3.586.

A DATA SOURCES AND TRANSFORMATIONS

We use the following time-series from 1954-2019 provided by Haver Analytics:

1. **Civilian Noninstitutional Population: 16 Years and Over**,
   Not Seasonally Adjusted, Quarterly, Thousands (LN16N@USECON)

2. **Gross Domestic Product: Implicit Price Deflator**,
   Seasonally Adjusted, Quarterly, 2012=100 (DGDP@USNA)

3. **Gross Domestic Product**,
   Seasonally Adjusted, Quarterly, Billions of Dollars, (GDP@USECON)

4. **Personal Consumption Expenditures: Nondurable Goods**,
   Seasonally Adjusted, Quarterly, Billions of Dollars (CN@USECON)

5. **Personal Consumption Expenditures: Services**,
   Seasonally Adjusted, Quarterly, Billions of Dollars (CS@USECON)

6. **Unemployed**, Seasonally Adjusted, Monthly, Thousands, 16 years+ (LTU@USECON)

7. **Labor Force**, Seasonally Adjusted, Monthly, Thousands, 16 years+ (LF@USECON)

8. **Unemployed Less Than 5 Weeks**,
   Seasonally Adjusted, Monthly, Thousands, 16 years and over (LU0@USECON)

9. **Job Separation Rate**, Job Openings and Labor Turnover Survey,
   Seasonally Adjusted, Monthly, Percent of Employment (LJSTPA@USECON)

10. **Job Openings**, Job Openings and Labor Turnover Survey,
    Seasonally Adjusted, Monthly, Thousands (LJJTLA@USECON)

11. **Real Output Per Hour**, Non-farm Business Sector, All Persons,
    Seasonally Adjusted, Quarterly, 2012=100 (LXNFA@USNA)

12. **Labor Share**, Non-farm Business Sector, All Persons,
    Seasonally Adjusted, Percent (LXNFBL@USNA)

We also use the Help Wanted Advertising Index (HWI) from Barnichon (2010), which is in units of the labor force. This series corrects for online advertising and is available on the author's website.

We applied the following transformations to the above data sources:

1. **Per Capita Real Output Growth**:

$$\Delta \log Y_t = 100 \left(\log \left(\frac{GDP_t}{DGDP_t + LN16N_t} \right) - \log \left(\frac{GDP_{t-1}}{DGDP_{t-1} + LN16N_{t-1}} \right) \right).$$

2. **Per Capita Real Consumption Growth**:

$$\Delta \log C_t = 100 \left(\log \left(\frac{CN_t + CS_t}{DGDP_t + LN16N_t} \right) - \log \left(\frac{CN_{t-1} + CS_{t-1}}{DGDP_{t-1} + LN16N_{t-1}} \right) \right).$$

3. **Unemployment Rate**: $U_t = 100(LTU_t/LF_t)$.

4. **Vacancy Rate**: HWI from 1954M1-2000M12 and $LJJTLA/LF$ from 2001M1-2019M12.

5. **Job-Finding Rate**: $f_t = 100(LTU_t - LU0_t)/LTU_t$.

6. **Real Wage**: $w_t = LXNFBL_t \times LXNFA_t$

7. **Wage Elasticity**: Slope coefficient from regressing $\log w_t$ on an intercept and $\log LXNFA_t$.

B SOLUTION METHODS

Nonlinear Solution We begin by writing the equilibrium system as $E[g(\mathbf{x}_{t+1}, \mathbf{x}_t, \varepsilon_{t+1})|\mathbf{z}_t, \Theta] = 0$, where g is a vector-valued function, $\mathbf{x}_t$ is a vector of variables, ε_t is a vector of shocks, $\mathbf{z}_t$ is a vector of states, and Θ is a vector of parameters. The state vector consists of productivity, lagged employment, and lagged active firms, $\mathbf{z}_t = [a_t, N_{t-1}, Z_{t-1}]$. We discretize a_t, N_{t-1}, and Z_{t-1} into 7, 31, and 31 evenly-spaced points, respectively. The bounds on the endogenous state variables, N_{t-1} and Z_{t-1}, are set to $[-6.5\%, +2.5\%]$ and $[-6.5\%, +2.5\%]$ of their deterministic steady-state values, respectively. Those bounds contain at least 99.8% of the ergodic distribution. The product of the points across the three dimensions, D, represents the nodes in the state space ($D = 6{,}727$).

There are many ways to discretize the exogenous state, a_t. We use the Markov chain in Rouwenhorst (1995), which Kopecky and Suen (2010) show outperforms other methods for approximating autoregressive processes. The realization of $\mathbf{z}_t$ on node d is denoted $\mathbf{z}_t(d)$. This method provides integration nodes, $[a_{t+1}(m)]$, with weights, $\phi(m)$, for $m \in \{1, \ldots, M\}$. Since productivity follows a Markov chain, the number of realizations of a_{t+1} are the same as a_t ($M = 7$).

Vacancies are subject to a nonnegativity constraint, $V_t \geq 0$. To impose the constraint, we introduce an auxiliary variable, $\mu_{V,t}$, such that $V_t = \max\{0, \mu_{V,t}\}^2$ and $\lambda_{V,t} = \max\{0, -\mu_{V,t}\}^2$, where $\lambda_{V,t}$ is the Lagrange multiplier on the non-negativity constraint. If $\mu_{V,t} \geq 0$, then $V_t = \mu_{V,t}^2$ and $\lambda_{V,t} = 0$. When $\mu_{V,t} < 0$, the constraint is binding, $V_t = 0$, and $\lambda_{V,t} = \mu_{V,t}^2$. Therefore, the vacancy constraint is transformed into a pair of equalities following Garcia and Zangwill (1981).

There is also an inequality constraint on the value function of active firms, $0 \leq J_{A,t}^F < \psi_n$. To impose this constraint, we create a second auxiliary variable, $\mu_{F,t}$, that equals $J_{A,t}^F$ when there is no entry or exit and the change in active firms, $\lambda_{F,t} = Z_t - Z_{t-1}$ when entry or exit occurs. When firms enter $J_{A,t}^F = \psi_n$ and when they exit $J_{A,t}^F = 0$. Therefore, the state variable for active firms is updated according to $J_{A,t}^F = \max\{\psi_n, \min\{0, \mu_{F,t}\}\}$, $\lambda_{F,t} = \mu_{F,t} - J_{A,t}^F$, and $Z_t = Z_{t-1} + \lambda_{F,t}$.

The vector of policy functions and the realization on node d are denoted $\mathbf{pf}_t$ and $\mathbf{pf}_t(d)$, where $\mathbf{pf}_t \equiv [\mu_{V,t}(\mathbf{z}_t), \mu_{F,t}(\mathbf{z}_t)]$. The following steps outline our policy function iteration algorithm:

1. Use Sims's (2002) `gensys` algorithm to solve the log-linear model without entry and exit. Then map the solution to the discretized state space to obtain an initial conjecture, $\mathbf{pf}_0(d)$.

2. Solve the nonlinear model without entry and exit by setting $\psi_y = 0$ and $\psi_n = 100$.

   (a) On iteration $j \in \{1, \ldots\}$ and each node $d \in \{1, \ldots, D\}$, use Chris Sims's `csolve` to find $\mathbf{pf}_t(d)$ to satisfy $E[g(\cdot)|\mathbf{z}_t(d), \Theta] \approx 0$. Guess $\mathbf{pf}_t(d) = \mathbf{pf}_{j-1}(d)$ and implement:

      i. Solve for all variables dated at time t, given $\mathbf{pf}_t(d)$ and $\mathbf{z}_t(d)$.

      ii. Linearly interpolate the policy functions, $\mathbf{pf}_{j-1}$, at the updated state variables, $\mathbf{z}_{t+1}(m)$, to obtain $\mathbf{pf}_{t+1}(m)$ on every integration node, $m \in \{1, \ldots, M\}$.

iii. Given $\{\mathbf{pf}_{t+1}(m)\}_{m=1}^{M}$, solve for the other elements of $\mathbf{x}_{t+1}(m)$, noting that ξ_{t+1} depends on whether firms exit as given by (23). Then compute

$$E[g(\mathbf{x}_{t+1}, \mathbf{x}_t(d), \varepsilon_{t+1})|\mathbf{z}_t(d), \Theta] \approx \textstyle\sum_{m=1}^{M} \phi(m) g(\mathbf{x}_{t+1}(m), \mathbf{x}_t(d), \varepsilon_{t+1}(m)).$$

When `csolve` converges, set $\mathbf{pf}_j(d) = \mathbf{pf}_t(d)$.

(b) Repeat step 2 until $\mathrm{maxdist}_j < 10^{-9}$, where $\mathrm{maxdist}_j \equiv \max\{|\mathbf{pf}_j - \mathbf{pf}_{j-1}|\}$. When that criterion is satisfied, the algorithm has converged to an approximate solution.

3. Solve the nonlinear model with firm entry and exit by iterating on ψ_y and ψ_n as follows:

(a) Create an evenly-spaced grid for ψ_y starting from 0 and stopping at 0.09445. Guess $\mathbf{pf}_t(d)$ as the solution to the nonlinear model without firm entry and exit. Repeating step 2, incrementally update ψ_y using the solution with the previous value as a guess.

(b) Create an evenly-spaced grid for ψ_n starting at 100 and stopping at 0.015. Guess $\mathbf{pf}_t(d)$ as the solution to the nonlinear model with $\psi_y = 0.09445$ and $\psi_n = 100$. Repeating step 2, incrementally update ψ_n using the solution with the previous value as a guess.

The algorithm is programmed in Fortran 90 with Open MPI and run on the BigTex supercomputer.

Linear Solution We solve the following log-linear equilibrium system using `gensys`:

$$\hat{V}_t = 2\hat{\mu}_t$$
$$\hat{N}_t = (1-\bar{s})\hat{N}_{t-1} + \bar{s}(\hat{q}_t + \hat{V}_t)$$
$$\hat{\theta}_t = \hat{V}_t - \hat{U}_t^s$$
$$\bar{U}^s \hat{U}_t^s = -(1-\bar{s})\bar{N}\hat{N}_{t-1}$$
$$\bar{U}\hat{U}_t + \bar{N}\hat{N}_t = 0$$
$$\hat{Y}_t = \hat{a}_t + \hat{N}_t$$
$$\bar{C}\hat{C}_t + \kappa\bar{V}\hat{V}_t = \bar{Y}\hat{Y}_t$$
$$\hat{q}_t = -\bar{\theta}^{\iota}\hat{\theta}_t/(1+\bar{\theta}^{\iota})$$
$$\bar{w}^n \hat{w}_t^n = \eta\bar{w}\hat{w}_t + \beta\eta(1-\bar{s})\kappa\bar{\theta}(\hat{C}_t - E_t\hat{C}_{t+1} + E_t\hat{\theta}_{t+1})$$
$$-(\kappa/\bar{q})\hat{q}_t = \bar{w}\hat{w}_t - \bar{w}^n\hat{w}_t^n + \beta(1-\bar{s})(\kappa/\bar{q})(\hat{C}_t - E_t\hat{C}_{t+1} - E_t\hat{q}_{t+1})$$
$$\hat{w}_t = \hat{a}_t$$
$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \sigma_a\varepsilon_{a,t+1}$$