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Non-Renewable Resources, Extraction Technology, and Endogenous Growth

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Abstract

We add an extractive sector to an endogenous growth model of expanding varieties and directed technological change. Firms increase their economically extractable stocks of non-renewable resources through R&D investment in extraction technology and reduce their stocks through extraction. We show how the geological distribution of the non-renewable resource interacts with technological change. Our model accommodates long-term trends in non-renewable resource markets - namely stable prices and exponentially increasing extraction - for which we present data going back to 1792. The model suggests that over the long term, development of new extraction technologies neutralizes the increasing demand for non-renewable resources in industrializing countries such as China.

1 Introduction

Technological change involving resource extraction is at the heart of the current boom in oil and shale gas production. It is not a new phenomenon but has strongly affected the production of all non-renewable resources in the past (see, e.g., Managi et al., 2004; Mudd, 2007; Simpson, 1999).

Nordhaus (1974), Simon (1981), and others argue that technological change helps overcome scarcity by increasing the economically extractable stock of non-renewable resources. This helps explain the empirical evidence of increasing production of non-renewable resources and non-increasing real prices of most non-renewable resources over the long run (see Krautkraemer, 1998; Livernois, 2009)

However, the resource economics literature since Hotelling (1931) primarily builds on the assumption of a fixed-stock. In growth models with non-renewable resources implied scarcity is primarily overcome by technological change in the use of resources and substitution of non-renewable resources by capital. These models typically predict growth in output, decreased non-renewable resource extraction, and increasing prices (see Groth, 2007; Aghion and Howitt, 1998).

This paper develops a theory of technological change in the extraction of non-renewable resources. It clarifies the relationship between long-run technological change, geology, and economic growth. We ask four main questions: First, to what extent is technological change different in the extractive sector than in other sectors of the economy? Second, how does geology affect the marginal cost of technological change in the extractive sector? Third, what are the effects of technological change in extraction technology on aggregate growth, on the resource intensity of the economy, and on price? Finally, how do the special characteristics in the extractive sector affect the direction of technological change?

We modify a standard endogenous growth model of expanding varieties and directed technological change by Acemoglu (2002, 2009). We add an extractive sector to the model such that aggregate output is produced from a non-renewable resource and intermediate goods. In the extractive sector, firms can reduce their economically extractable resource stocks through extraction, but also increase these stocks through R&D investment in extraction technology.

We point out the main differences between the extractive sector and the intermediate goods sector in our model. First, it is necessary to innovate in the extractive sector as resources are extracted from deposits of decreasing grades. Once the present economically extractable stock of the non-renewable resource is depleted, new R&D investment in extraction technology is necessary to make deposits of lower grades extractable thus continuing production. A specific extraction technology is only applicable to deposits of a certain grade. This is in contrast to the intermediate goods sector where a certain technology can be used infinitely often.

We show that under reasonable assumptions the resource stock increases linearly with R&D in extraction technology as two effects offset each other. We assume that R&D expenditure increases exponentially in order to make deposits of lower grades extractable. At the same time, the quantity of non-renewable mineral resources in the earth's crust increases exponentially as the grade of its occurrences decreases. It follows

that there are constant returns from R&D investment in extraction technology.

We illustrate the different evolutions of technology based on the characteristics of the two sectors. In order to keep the level of production of the non-renewable resource proportionate to aggregate output, the growth rate of technology in the extractive sector needs to increase over time. This is in contrast to the intermediate goods sector, where the growth rate of technology is constant. The difference is due to the necessity of innovation in the extractive sector; extraction from deposits of lower grades requires new technology.

Our model replicates historical trends in the prices and production of major non-renewable resources, as well as world real GDP for which we present data going back to 1792. Exponential aggregate output growth triggers R&D investment in extraction technology. The extraction and use of non-renewable resources increase exponentially whereas its price stays constant over the long term.

Our paper suggests that R&D investment in extraction technology is helping meet and offsetting increasing demand for non-renewable resources in industrializing countries like China. This makes extraction from deposits of lower grades possible. If historical trends continue, R&D in extraction technology might continue to offset the depletion of current resources. Even if non-renewable resource use and production increase exponentially, resource prices might stay constant in the long term. Whether the price of non-renewable resources increases or stays constant in the long run is key to the results of a number of recent prominent papers (Acemoglu et al., 2012; Golosov et al., 2014; Hassler and Sinn, 2012; van der Ploeg and Withagen, 2012).

Our paper contributes to a literature that basically takes technological change in the extraction technology as a given and that does not include growth of aggregate output. Heal (1976) introduces a non-renewable resource, which is inexhaustible, but extractable at different grades and costs, in the seminal Hotelling (1931) optimal depletion model. Extraction costs increase with cumulative extraction, but then remain constant when a "backstop technology" (Heal, 1976, p. 371) is reached. Slade (1982) adds exogenous technological change in extraction technology to the Hotelling (1931) model and predicts a U-shaped relative price curve. Cynthia-Lin and Wagner (2007) use a similar model with an inexhaustible non-renewable resource and exogenous technological change. They obtain a constant relative price with increasing extraction.

There are three papers, to our knowledge, that like ours include technological change in the extraction of a non-renewable resource in an endogenous growth model. Fourgeaud et al. (1982) focuses on explaining sudden fluctuations in the development of non-renewable resource prices by allowing the resource stock to grow in a stepwise manner through technological change. Tahvonen and Salo (2001) model the transition from a non-renewable energy resource to a renewable energy resource. Their model follows a learning-by-doing approach as technical change is linearly related to the level of extraction and the level of productive capital. It explains decreasing prices and the increasing use of a non-renewable energy resource over a particular time period before prices increase in the long term. Hart (2012) models resource extraction and demand in a growth model with directed technological change. The key element in his model is the depth of the resource. After a temporary "frontier phase" with a constant resource price and consumption rising at a rate only close to aggregate output, the economy needs to

extract resources from greater depths. Subsequently, a long-run balanced growth path with constant resource consumption and prices that rise in line with wages is reached.

To our knowledge, our model is the first to combine technological change in the extractive sector and deposits of different grades in an endogenous growth set-up that explicitly models R&D investment in extraction technology. It also contributes to the literature by pointing out the necessity of innovation in the extractive sector due to its specific characteristics, and those characteristics' effects on R&D development relative to the performance of other economic sectors in an endogenous growth model.

To focus on the main argument, we do not take into account externalities, uncertainty, recycling, substitution, short-run price fluctuations, population growth, and exploration in our model. Exploring new deposits certainly offers an alternative explanation for long-run trends in resource production and prices, and has been modeled by other scholars (Cairns, 1990, see for an overview). However, we illustrate that there is ample evidence that decreasing qualities of deposits has been offset by technological change in extraction over the long-run. Exploration is certainly highly important for the supply of non-renewable resources in the medium term, but technological change better explains the very long-run trends in supply. Another aspect that we do not include in our model is recycling. This will probably become more important for non-fuel, non-renewable resources in the future due to an increasing stock of recyclable materials and comparatively low energy requirements (see Wellmer and Dalheimer, 2012). As recycling adds to the resource stock, this would further strengthen our argument.

In Section 2, we document stylized facts on the long-term development of non-renewable resource prices, production, and world real GDP. We also provide geological evidence for the major assumptions of our model regarding technological change. Section 3 describes how we model technological change in the extractive sector. Section 4 presents the setup of the growth model and discusses its theoretical results. In Section 5 we draw conclusions.

2 Stylized facts

2.1 Prices, production, and output over the long term

Annual data for major non-renewable resource markets going back to 1792 indicates that real prices are roughly trend-less and that worldwide primary production as well as world real GDP grow roughly exponentially.

Figure 1 presents data on the real prices of five major base metals and crude oil. Real prices exhibit strong short-term fluctuations. At the same time, the growth rates of all prices are not significantly different from zero (see Table 1 in the Appendix). The real prices are, thus, trend-less. This is in line with evidence over other time periods provided by Krautkraemer (1998), Von Hagen (1989), Cynthia-Lin and Wagner (2007), and references therein. The real price for crude oil exhibits structural breaks, as shown in Dvir and Rogoff (2010). Overall, the literature is certainly not conclusive (see Pindyck, 1999; Lee et al., 2006; Slade, 1982; Jacks, 2013; Harvey et al., 2010), but we believe the evidence is sufficient to take trend-less prices as a motivation for our model.

Figure 2 shows that the world primary production of the examined non-renewable resources and world real GDP approximately exhibit exponential growth since 1792. A closer statistical examination reveals that the production of non-renewable resources exhibits significantly positive growth rates in the long term (see Table 2 in the Appendix). Growth rates for the production of copper, lead, tin, and zinc do not exhibit a statistically significant trend over the long term. Hence, the levels of production of these non-renewable resources grow exponentially over time.

The level of crude oil production follows this exponential pattern up to 1975. Including the time period from 1975 until recently reveals a statistically significant negative trend and therefore, declining growth rates over time due to a structural break in the oil market (Dvir and Rogoff, 2010; Hamilton, 2009). In the case of primary aluminum production, we also find declining growth rates over time and hence, no exponential growth of the production level. This might be attributable to recycling, which has become important in the production of aluminum over time (see data by U.S. Geological Survey, 2011a). Recycling is not included in our model nor is it in the data. The growth rates of world real GDP exhibit an increasing trend over the long term, hinting at an underlying explosive growth process. As our model does not include population growth, we run the same tests for the per capita data of the respective time series as a robustness check. We find slightly weaker results as Table 3 in the Appendix shows. Overall, we take these stylized facts as motivation to build a model that exhibits trend-less resource prices and exponentially increasing worldwide production of non-renewable resources, as well as exponentially increasing aggregate output.

Insert Figure 1 about here.

Insert Figure 2 about here.

2.2 Technological change in the extractive sector

Technological change in resource extraction offsets the depletion of a non-renewable resource stock (Simpson, 1999, and others). Hence, the resource stock is drawn down by extraction, but increases by technological change in extraction technology. The reason for this phenomenon is that non-renewable resources such as copper, aluminum, or hydrocarbons are extractable at different costs due to varying grades, thickness, depths, and other characteristics of mineral deposits. Technological change makes deposits extractable that, due to high costs, have not been extractable before (see Simpson, 1999; Nordhaus, 1974, and others).

The definition of resources by the U.S. Geological Survey reflects this. It defines resources as "a concentration of naturally occurring solid, liquid, or gaseous material in or on the earth's crust in such form and amount that economic extraction (...) is currently or potentially feasible" (U.S. Geological Survey, 2011b, p. 193). The term

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"economic" "implies that profitable extraction (...) under defined investment assumptions has been established" (U.S. Geological Survey, 2011b, p. 194). The "boundary" between resources and "other occurrences is obviously uncertain, but limits may be specified in terms of grade, quality, thickness, depth, percent extractable, or other economic-feasibility variables" (U.S. Geological Survey, 2011b, p. 194).

Insert Figure 3 about here.

Over time, R&D in extraction technology, namely in prospecting and mining equipment, as well as metallurgy and processing, have increased the stock of the resource by making the extraction of materials from deposits of lower grades or greater depths economically feasible (see Wellmer, 2008; Mudd, 2007). For example, Radetzki (2009) describes how technological change has gradually made possible the extraction of copper from deposits of decreasing grades. 7000 years ago, human beings used copper in a pure nugget form. Today, copper is extracted from deposits of a low 0.2 to 0.3 percent grade.[1] In line with this narrative evidence, Figure 3 illustrates that the ore grades of U.S. copper mines have steadily decreased over the long term. Mudd (2007) presents similar evidence for the mining of different base-metals in Australia. Overall, history suggests that R&D costs in the extractive sector have increased exponentially, pushing the boundary between economically unextractable and economically extractable deposits in terms of grades. Developing technologies to make deposits of 49 percent grade instead of 50 percent grade extractable, has probably required a far smaller investment than developing technologies to make economically feasible the extraction from deposits of 0.2 percent grade instead of 1.2 percent grade.

As a result, technological change has offset the higher cost of obtaining resources from deposits of lower grades. Figure 4 shows that copper reserves[2] have increased by more than 700 percent over the last 64 years. One reason is the introduction of the solvent extraction and electrowinning technology. This two-stage process has made extraction of copper from deposits of lower grades economically feasible (Bartos, 2002). There are also the strong effects of innovation on returns-to-scale as larger equipment in mining operations becomes feasible.[3] Case studies for other minerals also find that technological change has offset cost-increasing degradation of resources (see for example Lasserre and Ouellette, 1991; Mudd, 2007; Simpson, 1999).

Insert Figure 4 about here.

We observe similar developments regarding hydrocarbons. Using the example of the offshore oil industry, Managi et al. (2004) show that technological change has offset

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[1] The Aitik copper mine in Sweden is the mine that extracts copper from the lowest deposits of 0.27 percent in the world (personal communication with F.-W. Wellmer).

[2] Reserves are those resources for which extraction is considered economically feasible (U.S. Geological Survey, 2011c).

[3] Personal communication with F.-W. Wellmer.

the cost-increasing degradation of resources. Crude oil has been extracted from ever deeper sources in the Gulf of Mexico as Figure 5 in the Appendix shows. Furthermore, technological change and high prices have made it profitable to also extract liquid hydrocarbons from unconventional sources, such as light tight oil, oil sands, and liquid natural gas (International Energy Agency, 2012). As a result, oil reserves have doubled since the 1980s (see Figure 6 in the Appendix).

Overall, empirical evidence suggests that technological change offsets resource depletion by renewing the resource stock from deposits that had been considered impossible to extract. Furthermore, it is a reasonable assumption that R&D costs in the extractive sector have increased exponentially in terms of making deposits of lower grades extractable.

2.3 Geological abundance and distribution of the elements in the earth's crust

Computing the total abundance (or quantity) of each element in the earth's crust leads to enormous quantities (see Nordhaus, 1974; Perman et al., 2003). Table 4 shows the respective ratios of the quantities of reserves, resources, and abundance in the earth's crust with respect to annual mine production for several important non-renewable resources. It provides evidence that even non-renewable resources, which are commonly thought to be the most scarce such as gold, are abundant, supplying evidence "that the future will not be limited by sheer availability of important materials"(Nordhaus, 1974, p. 23). In addition, most metals are recyclable, which means that the extractable stock in the techno-sphere increases (Wellmer and Dalheimer, 2012).

The sediments of the earth's crust are also rich in hydrocarbons. Even though conventional oil resources may be exhausted someday, resources of unconventional oil, natural gas, and coal are abundant. Aguilera et al. (2012) conclude that conventional and unconventional resources "are likely to last far longer than many now expect" (p. 59). Overall, Rogner (1997) states about world hydrocarbon resources that "fossil energy appears almost unlimited" (p. 249) given a continuation of historical technological trends.

Insert table 4 about here.

Table 5 in the Appendix illustrates that the assumption of exponentially increasing extraction of non-renewable resources does not alter the overall conclusion of table 4. We will not run out of the resources within a time-frame relevant to today's human civilization, assuming there is continuing technological change in resource extraction.

The elements of the earth's crust are not uniformly distributed, reflecting variations in geochemical processes over time. Unfortunately, geologists do not agree on the distribution of elements in the earth's crust. Ahrens (1953, 1954) states in his fundamental law of geochemistry that the elements within the earth's crust exhibit a log-normal grade-quantity distribution. Skinner (1979) and Gordon et al. (2007) propose a discontinuity in this distribution due to the so-called "mineralogical barrier"(Skinner, 1979),

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the approximate point below which metal atoms are trapped by atomic substitution. Due to a lack of geological data, both parties acknowledge that an empirical proof is still needed. Gerst (2008) concludes that he can neither confirm nor refute these two hypotheses. Based on worldwide data on copper deposits over the past 200 years, he finds evidence for a log-normal relationship between copper production and average ore grades. Mudd (2007) analyzes the historical evolution of extraction and grades of deposits for different base metals in Australia. He concludes that production has been continually increasing, partly verging on exponentially, while grades have consistently declined.

The distribution of hydrocarbons in the earth's crust might also differ from the fundamental laws of geochemistry by Ahrens (1953, 1954) due to distinct formation processes. For example, oil begins to form in the source rock with the thermogenic breakdown of organic matter (kerogen) at about 60 to 120 degrees Celsius, which is found at an approximate depth of two to four kilometers. However, Farrell and Brandt (2006) and Aguilera et al. (2012) suggest that a log-normal relationship is also true for liquid hydrocarbon production. Aguilera et al. (2012) also point out that there is no huge break between the average total production costs of conventional and unconventional oil resources.

Thus, there remains uncertainty about the distribution of the elements in the earth's crust. However, we believe that it is reasonable to assume that the elements are distributed according to a log-normal relationship between the grade of its deposits and its quantity in the earth's crust.

3 Modeling technological change in the extractive sector

We focus on two major determinants of extraction cost, the grade at which they are extracted and the state of technology. In the first part of this section, we propose a general view on how these two determinants affect extraction cost. The second part shows how extraction cost interacts with resource abundance to determine profitability of extraction technology R&D investments. The third part derives a new resource production function based on this interaction.

Cost and technology of resource extraction

Let N_{Rt} be the accumulated extraction technology at time t. We drop the time index to simplify notation. Let d be the grade of the respective deposits. We define the extraction cost function as a function of mapping grades into extraction costs depending on the state of technology:

$$\phi_{N_R}: [0,1] \times \mathbb{R}_+ \to \bar{\mathbb{R}}_+, (d, N_R) \mapsto \phi_{N_R}(d) . \tag{1}$$

At technology level $N_R \in \mathbb{R}_+$ the cost of extracting the non-renewable resource from occurrences of grade $d \in [0,1]$ is $\phi_{N_R}(d) \in \bar{\mathbb{R}}_+ = \mathbb{R}_+ \cup \infty$. There are decreasing returns

to scale from R&D investment in extraction technology in terms of grades. This implies that for a given level of technology N_R, ϕ_{N_R} is non-increasing in d:

$$\forall N_R : \quad d > d' \quad \Rightarrow \quad \phi_{N_R}(d) \leq \phi_{N_R}(d') . \tag{2}$$

We assume that R&D increases the productivity of the extraction technology for deposits of all grades. Therefore, an increase in N_R decreases extraction costs for any given grade:

$$\forall d : \quad \frac{\partial \phi_{N_R}(d)}{\partial N_R} \leq 0 . \tag{3}$$

At time t, extraction technology increases by $\frac{\partial N_{Rt}}{\partial t}$ and reduces extraction costs. Firms choose between extracting resources at a higher cost or investing in extraction technology. Figure 7 panel (a) shows the general form of the extraction cost function. Resource extraction from deposits of lower grades generates higher costs, but due to increasing R&D, the function moves downward.

Insert Figure 7 about here.

Extraction cost versus resource abundance

We combine two functions to show that there are constant returns from R&D investment in terms of the quantity of the extractable resource. The first function describes deposits that are extractable for a given state of technology. The second function shows the distribution of the quantity of the resource over grades. Combining these two functions gives the quantity of the resource that becomes extractable from one unit of R&D investment in extraction technology.

Figure 7 panel (b) illustrates a simplified version of the extraction cost function, which we use in the following. A certain grade d_N is associated with a unique level of R&D investment, above which the resource can be extracted at cost $\phi_{N_R} = E$. The function h maps the state of the extraction technology into a value for the grade of deposits, which is extractable at cost ϕ_{N_R}:

$$h \colon \mathbb{R}_+ \to [0,1], N_R \mapsto d_{N_R} . \tag{4}$$

At grades lower than d_N extraction is impossible, because the cost is assumed to be infinite. The extraction cost function takes the degenerate form of

$$\phi_{N_R}(d) = \begin{cases} E, & \text{if} \quad d \geq d_{N_R}, \\ \infty, & \text{if} \quad d < d_{N_R}. \end{cases} \tag{5}$$

This simplified form allows us to obtain an analytical solution in the growth model of Section 4. It preserves the key features of the general formulation in Equation 1: 1) Extraction cost depends on the ore grade and 2) innovations in extraction technology

reduce extraction cost for certain ore grades.

In order to determine the cost of R&D we specify a functional form for the extraction technology function h:

$$h(N_R) = e^{-\delta_1 N_{Rt}}, \ \delta_1 \in \mathbb{R}_+ \ , \tag{6}$$

where δ_1 is the curvature parameter of the function. Panel (a) in Figure 8 illustrates the shape of $h(N_R)$. The marginal effect of the extraction technology on the extractable occurrences declines as the grade decreases. This follows the suggestion in the stylized facts that R&D costs have increased exponentially in making deposits of lower ore grades economically extractable. If δ_1 is high it allows lower grades to be reached more quickly.

Insert Figure 8 about here.

Panel (b) in Figure 8 shows the distribution of the non-renewable resource in the earth's crust. It maps a certain grade onto the total quantity of extractable resources at different grades of the occurrences between d and one, where one corresponds to a 100 percent ore grade or pure metal.

$$D \colon (0,1] \to \mathbb{R}_+, d \mapsto D(d) \tag{7}$$

Note that $D(1) = 0$ means that the resource is not found in 100 percent pure form. Figure 8 panel (b) illustrates the relationship between the two variables. The total quantity of the non-renewable resource is inversely proportional to the grade: as the grade decreases, the extractable quantity of the non-renewable resource increases. We formulate the relationship in a general way:

$$D(d) = -\delta_2 \ln(d), \ \ \delta_2 \in \mathbb{R}_+ \ , \tag{8}$$

where δ_2 is the curvature of the function. If δ_2 is large, it means that a lot of additional resources can be found for a given decrease in ore grade.

We assume extraction cost to be zero, $E = 0$. We combine the two functions and obtain the following proposition. A dot over a variable denotes the time derivative.

Proposition 1 *The total quantity of the resource made extractable over time due to technological change is proportional to N_{Rt}:*

$$D(h(N_{Rt})) = \delta_1 \delta_2 N_{Rt} \ . \tag{9}$$

Consequently, the newly extractable resource from a marginal investment in R&D is

$$X_t = \frac{\partial D(h(N_{Rt}))}{\partial t} = \delta_1 \delta_2 \dot{N}_{Rt} \ . \tag{10}$$

The resource return to an investment into extraction technology is constant because both the extraction technology function and the distribution of resources over ore grades

are assumed to have a constant curvature. The quantity of the resource, which is made extractable by a given R&D investment in extraction technology, is thus independent of past investments or time.

The production function of the extractive sector[4]

The stock of the non-renewable resource at time t is noted $S_t \geq 0$. R_t signifies the quantity of the non-renewable resource sold for aggregate output production. Investing in new extraction technology facilitates occurrence of lower grade extraction and expands the resource stock by X_t. The evolution of the stock follows:

$$\dot{S}_t = X_t - R_t , \qquad S_t \geq 0, X_t \geq 0, R_t \geq 0 , \tag{11}$$

where $\dot{S}_t$ is the change in the stock in period t, X_t is the inflow through investment of new machines, and R_t is the outflow by extracting and selling the resource. Note that for $X_t = 0$, this formulation is the standard Hotelling (1931) setup.

Extractive firms increase the resource stock according to Equation 10. Each extraction technology investment makes available a specific additional deposit of lower grade; it is then extracted and thus depleted. Technology in the extractive sector is thus vertical. For each technology a specific ore grade with finite supply can be exploited. In order to extract additional resources, the technology has to advance, so that a new ore grade becomes accessible.

In the Hotelling (1931) model, there is a finite supply of resources that can be obtained with a given extraction technology. In our model, the supply of resources is practically infinite, but technology must be improved constantly in order to make it accessible.

Technology in the extractive sector evolves according to:

$$\dot{N}_{Rt} = \eta_R M_{Rt} , \tag{12}$$

where M_{Rt} is spending on R&D in terms of the final product, and η_R is a cost parameter.

The revenues of the extraction firm come from selling the resource. The expenses are the cost of developing new extraction technology, M_R. The price for these technology investments is the same as that for the final good, which was normalized to 1. Using Equations 10 and 12, spending on extraction technology is $M_R = \frac{1}{\eta_R}\dot{N}_R = \frac{1}{\eta_R \delta_1 \delta_2}X_t$. Extraction firms solve the following maximization problem

$$\max_{Rt} p_R R_t - \frac{1}{\eta_R \delta_1 \delta_2}X_t \tag{13}$$

by choosing their investment into technology optimally. Since the sector operates competitively, firms take the price of the non-renewable resource, p_R, as given. There is perfect competition due to free market entry and a linear production technology. The only way of extracting the non-renewable resource is to develop extraction technology.

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[4]For an alternative version of the extraction sector (using an approach analogous to the machine types in the intermediate sector) see the online appendix.

As the economy expands, demand for the non-renewable resource grows. The cost of developing the extraction technology, however, is constant per unit of the non-renewable resource (see Proposition 1). Hence, firms' supply of the non-renewable resource grows along with demand.

The production function of the extractive sector is equal to the outflows from the resource stock R_t:

$$R_t = \delta_1 \delta_2 \dot{N}_{Rt} - \dot{S}_t \ . \tag{14}$$

Firms in the extraction sector innovate despite perfect competition, because there is a necessity of innovate in order to continue production. Like in standard models of perfect competition, innovation in extraction technology diffuses and is available to all firms. However, we assume that technology is grade specific and is therefore only applicable to a certain deposit from which the innovating firm produces its non-renewable resource. Other firms can not use this technology to produce from other deposits, but they are able to use the new technology as a basis for further technological development. As a result, firms' profits are just enough to cover the cost of technological development. Most similar to this understanding of innovation is Desmet and Rossi-Hansberg (2012), where non-replicable factors of production ensure technological development in a perfectly competitive environment. In our case these non-replicable factors are deposits of a specific ore grade.

4 The growth model

To illustrate the macroeconomic effect of the analysis in Section 3, we build a growth model that allows an endogenous allocation of resources between an intermediate goods sector and an extractive sector based on the framework of directed technological change by Acemoglu (2002).[5]

4.1 The setup

We consider an economy with a representative consumer that has constant relative risk aversion preferences:

$$\int_0^\infty \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \ . \tag{15}$$

The variable C_t denotes the consumption of aggregate output at time t, ρ is the discount rate, and θ is the coefficient of relative risk aversion. The budget constraint of the consumer is

$$C_t + I_t + M_t \leq Y_t \equiv \left[\gamma Z_t^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma) R_t^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \ , \tag{16}$$

where I_t is aggregate investment in machines by the two sectors, and M_t denotes aggregate R&D investment in developing new varieties of machines. The usual no-Ponzi game condition applies. According to the right hand side of Equation 16, aggregate output production uses two inputs, intermediate goods Z_t and the non-renewable resource

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[5]For a social planner version of the model, see the online appendix.

R_t. There are two sectors in the economy that produce the inputs to aggregate output production: the intermediate goods sector and the extractive sector. The distribution parameter γ indicates their respective importance in producing aggregate output Y_t. The R&D expenditure is the sum of R&D expenditure in the intermediate sector and in the extractive sector: $M_t = M_{Zt} + M_{Rt}$.

The production function of the intermediate goods sector

The intermediate goods sector follows the basic setup of Acemoglu (2002). It produces intermediate goods Z_t according to the following production function[6]:

$$Z_t = \frac{1}{1-\beta} \left(\int_0^{N_{zt}} x_{zt}(j)^{1-\beta} dj \right) L^\beta \ , \tag{17}$$

where $\beta \in (0,1)$. The intermediate goods sector uses labor L_t, which has a fixed supply, and machines as inputs to production. $x_{Zt}(j)$ refers to the number of machines used for each machine variety j at time t. Machines depreciate fully within one period. We denote the number of varieties of machines as N_{zt}. Profits for the firm producing good Z are simply the difference between revenues and the expenses for labor, as well as for the intermediates $x_Z(j)$,

$$\pi_Z = p_Z Z - w_Z L - \int_0^{N_Z} \chi_Z(j) x_Z(j) dj \ . \tag{18}$$

Sector-specific technology firms invent new technologies for which they hold a fully enforceable patent. They exploit the patent by producing a machine type that corresponds uniquely to their technology. The uniqueness provides market power that they can use to set a price $\chi_{Zt}(j)$ above marginal cost. The marginal cost of production in terms of the final good is the same for all machines. Machines depreciate fully after each period, so that the technology owner has to produce the corresponding machines each period.

The range of machines expands through R&D expenditure by

$$\dot{N}_{Zt} = \eta_Z M_{Zt} \ , \tag{19}$$

where M_{Zt} is R&D investment by the technology firms for machines in the intermediate goods sector in terms of the final product, and η_Z is a cost parameter. One unit of final good spent for R&D will generate η_Z new varieties of machines. A technology firm that discovers a new machine receives a patent and becomes its sole supplier.

4.2   Results

We begin the formal analysis with the optimization of the extractive firms.[7] Inflows to the resource stock X_t depend on R&D investment in the extractive sector, and outflows

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[6]Like Acemoglu (2002) we assume that the firm level production functions exhibit constant returns to scale, so there is no loss of generality in focusing on the aggregate production functions.

[7]Proofs for this section are in the Appendix.

R_t are the sales of the resource to the final good producer. Since the marginal cost for R&D is constant, we obtain the typical result of stock management: inflows and outflows have to balance over time.

Proposition 2 *The quantity of the resource used in aggregate production equals the quantity of newly acquired resources through R&D: $R_t = X_t$.*

Perfect competition in the extraction sector results from the inexhaustible character of the resource. If one firm demands a price above marginal cost, another firm can develop additional technology, extract the resource from lower ore grades and sell it at a lower price.

The result is of course affected by the assumption of no uncertainty. Following the standard in growth models, we have assumed in equation 12 that patents for new machines result in a deterministic way from the respective R&D investments. This reflects a long-term perspective. The model could be made more sophisticated by assuming that R&D is stochastic. Extractive firms would then keep a positive stock of the resource S_t to be on the safe side in the case of a series of bad draws in R&D. This stock would grow over time as the economy grows. But in essence, the result above would remain the same: In the long term, resources used in aggregate production equal those added to the resource stock through R&D.

Extractive firms face constant marginal costs of extracting the non-renewable resource, since the resource stock can be expanded due to R&D in extraction technology. The price thus remains constant over time as well:

Proposition 3 *The resource price is*

$$p_{Rt} = \frac{1}{\eta_R \delta_1 \delta_2} \ .$$

The first determinant of price, η_R, is the productivity of R&D in the extractive sector, defined in Equation 12. If a given investment into technology yields greater innovation in extraction technology, the price for the resource is lower. The second determinant of price, δ_1, is the progress in terms of ore grade that can be achieved with a given unit of innovations, defined in Equation 6. When innovations allow large gains in terms of ore grade, the price for the resource is lower. The third determinant of price, δ_2, is the increase in resource availability for one unit of decrease in ore grade, defined in Equation 8. When a given increase in ore grade yields more resources, the price for the resource is lower.

We turn to the solution of the model:

Proposition 4 *The growth rate on the balanced growth path of the economy is constant and given by*

$$g = \theta^{-1} \left(\beta \eta_Z L \left[1 - \left(\frac{1-\gamma}{\gamma} \right)^{\frac{\varepsilon}{1-\varepsilon}} \frac{1}{\eta_R \delta_1 \delta_2} \right]^{\frac{1}{\beta}} - \rho \right) \ .$$

A higher rate of return on R&D investment in the labor sector, η_Z, increases the growth rate of the economy, and a higher resource price decreases the growth rate of the economy.

Since the growth model employed in this paper is a modification of Acemoglu (2002), we can point out the difference to the growth rates in that model. The Euler equation, $g = \theta^{-1}(r - \rho)$, is clearly visible. The interest rate, however, differs. In Acemoglu (2002) there is exogenously given labor in both sectors. In our model, there is exogenously given labor, L, in only one sector, while the other sector generates the resource R endogenously. Therefore, instead of the two exogenously production factors, our growth rates feature the exogenous given labor and the price for obtaining the endogenous resource R.

In order to understand the role of the non-renewable resource in the economy, we determine its relative importance:

Proposition 5 *The resource intensity of the economy is given by*

$$\frac{R}{Y} = [(1 - \gamma)\eta_R \delta_1 \delta_2]^\varepsilon \; .$$

It depends positively on the distribution parameter for the resource γ and higher resource price decreases the growth rate of the economy.

The distribution parameter γ indicates the importance of the resource for the economy, as shown in the production function in Equation 16.

Proposition 3 in combination with Propositions 4 and 5 shows the effect of a lower resource price on the growth rate and the resource intensity of the economy. Both depend negatively on the resource price. When the price is low, non-renewable resource is used intensively and the resource constraint on growth is weak. When the price is high, the economy uses substitutes, but this reduces growth.

We compare the growth rates of technology in the two sectors.

Proposition 6 *The level of technology in the intermediate goods sector is*

$$N_Z = \left(\frac{1 - \gamma}{\gamma}\right)^{-\varepsilon} (\eta_R \delta_1 \delta_2)^\varepsilon \left(\gamma^{-\varepsilon} - \left(\frac{1 - \gamma}{\gamma}\right)^\varepsilon \eta_R \delta_1 \delta_2\right)^{\left(\frac{1}{1-\varepsilon}\right)\left(-\varepsilon + \frac{1-\beta}{\beta}\right)} (1 - \gamma)^\varepsilon L^{-1} Y \; .$$

The growth rate of technology in the extractive sector is

$$\dot{N}_R = (\delta_1 \delta_2)^{\varepsilon - 1}(1 - \gamma)^\varepsilon \eta_R Y \; .$$

Thus, there is a qualitative difference in the growth rates of the two sectors. While the *level* of technology in the intermediate goods sector is proportional to output, the *growth rate* of technology in the extractive sector is proportional to output. N_Z, therefore, has the *constant* growth rate g, as given in Proposition 4. N_R has an *increasing* growth rate. It is the second derivative $\frac{\partial^2 N_R}{\partial^2 t}$, which is equal to g.

Proposition 6 also shows how investments in technology, M_{Rt}, depend on the elasticity of substitution ϵ, since $\dot{N}_R$ and M_{Rt} are closely linked through Equation 12. Since $1 - \gamma < 1$, the elasticity of substitution and investments into technology M_{Rt} are negatively linked.

Acemoglu (2002) finds that the direction of technological change depends on price and market size. For the intermediate goods sector, this holds without modification. Equation 31 shows that the incentive to innovate in the intermediate goods sector depends on price, p_Z, and market size, L. The structure in the extractive sector is different. While in the intermediate goods sector, the *stock* of technology is used for production, the extractive sector can only use the *flow* of new technology for production. The incentive to innovate, thus, grows over time in line with the size of the economy. The market size effect for the extractive sector depends on the size of the economy. The price effect is not relevant since competition keeps price equal to marginal costs. This reflects the long-term perspective of the model.

4.3 Discussion

We discuss a number of issues that arrive from our model, namely the assumptions made in Section 3, the comparison to the other models with non-renewable resources, and the question of the ultimate finiteness of the resource.

Function D from Equation 7 shows the amount of the non-renewable resource available in the earth's crust for a given occurrence of grade d. Geologists cannot give an exact functional form for D, so we used the form given in Equation 8 as a plausible assumption. How would other functional forms affect the predictions of the model? First, the predictions are valid for all parameter values $\delta_2 \in \mathbb{R}_+$. Secondly, if D is discontinuous with a break at d_0, at which the parameter changes to $\delta_2' \in \mathbb{R}_+$, there would be two balanced growth paths: one for the period before, and one for the period after the break. Both paths would behave according to the model's predictions. The paths would differ in the extraction cost of producing the resource, level of extraction, and use of the resource in the economy. To see this, recall from Proposition 1 that X_t is a function of δ_2. A non-exponential form of D would produce results that differ from ours. It could feature a scarcity rent as in the Hotelling (1931) model, as a non-exponential form of D could cause a positive trend in resource prices or the extraction from occurrences at a lower ore grade becomes infeasible. In these cases, the extractive firms would consider the opportunity cost of extracting the resource in the future, in addition to extraction and innovation cost.

How does our model compare to other models with non-renewable resources? We do not assume that resources are finite; their availability is a function of technological change. As a consequence, resource availability does not limit growth. Substitution of capital for non-renewable resources, technological change in the use of the resource, and increasing returns to scale are therefore not necessary for sustained growth as in Groth (2007) or Aghion and Howitt (1998). Growth depends on technological change as much as it does in standard growth models without a non-renewable resource, but it also depends on technological change in the extractive sector. If the resource were finite in our model, then the extractive sector would behave in the same way as in standard models in the tradition of Hotelling (1931). As Dasgupta and Heal (1980) point out, the growth rate of the economy depends in this case strongly on the degree of substitution between the resource and the other economic inputs. For $\varepsilon > 1$, the resource is inessential; for $\varepsilon < 1$, the total output that the economy is capable of

producing is finite. The production function is, therefore, only interesting for the Cobb-Douglas case.

Our model suggests that the non-renewable resource can be thought of as a form of capital: If the extractive firms invest in new machines and trigger R&D in extraction technology, the resource is extractable without limits as an input to aggregate production. This feature marks a distinctive difference from models such as the one of Bretschger and Smulders (2012). They investigate the effect of various assumptions on substitutability and a decentralized market on long-run growth, but keep the assumption of a finite non-renewable resource. Without this assumption, the elasticity of substitution between the non-renewable resource and other input factors is no longer central to the analysis of limits to growth.

Some might argue that the relationship described in Proposition 1 cannot continue to hold in the future as the amount of non-renewable resources in the earth's crust is ultimately finite. Scarcity will become increasingly important, and the scarcity rent will be positive even in the present. However, for understanding current prices and consumption patterns, current expectations about future developments are important. Given that the quantities of available resources indicated in Table 4 are very large, their ultimate end far in the future does not affect behavior today. Furthermore, when resources in the earth's crust are exhausted, so much time will have passed that technology might have developed to a point where the earth's crust, which makes up one percent of the Earth's mass, is no longer a limit to resource extraction. Deeper parts of the planet or even extraterrestrial sources might be explored. These speculative considerations are not crucial for our model. What is important is that the relation from Proposition 1 has held in the past and looks likely to hold for the foreseeable future. Since in the long term, extracted resources equal the resources added to the resource stock due to R&D in extraction technology, the price for a unit of the resource will equal the extraction cost plus the per-unit cost of R&D and hence, stay constant in the long term. This also explains why scarcity rents cannot be found empirically as shown in Hart and Spiro (2011).

5 Conclusion

This paper examines the long-term evolution of prices and production of major non-renewable resources from a theoretical and empirical perspective. We argue that economic growth causes the production and use of a non-renewable resource to increase exponentially, and its production costs to stay constant in the long term. Economic growth enables firms to invest in R&D in extraction technology, which makes accessible resources from deposits of lower grades. We explain the long-term evolution of non-renewable resource prices and world production for more than 200 years. If historical trends in technological progress continue, it is possible that non-renewable resources are, within a time frame relevant for humanity, practically inexhaustible.

Our model makes four major simplifications, which should be examined in more detail in future extensions. First, there is no uncertainty in R&D development and therefore, no need to keep a positive economically extractable stock of the non-renewable

resource. When R&D development is stochastic (as in Dasgupta and Stiglitz (1981)), there would be a need for firms to keep stocks. Second, our model features full competition in the extractive sector. We could obtain a model with monopolistic competition in the extractive sector by introducing privately-owned deposits. A firm would need to pay a certain upfront cost or exploration cost in order to acquire a mineral deposit. This upfront cost would give technology firms a certain monopoly power, as they develop machines that are specific to single deposit. Third, extractive firms could face a trade-off between accepting high extraction costs due to a lower technology level and investing in R&D to reduce extraction costs. The general extraction technology function in Equation 1 provides the basis to generalize this assumption. Finally, our model does not include recycling. Recycling will likely become more important for metal production due to the increasing abundance of recyclable materials and the comparatively low energy requirements to recycle (see Wellmer and Dalheimer, 2012). Introducing recycling into our model would further strengthen our argument, as it increases the economically extractable stock of the non-renewable resource.

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Appendix 1   Figures and Tables

Figure 1: Real prices of major mineral commodities in natural logs.
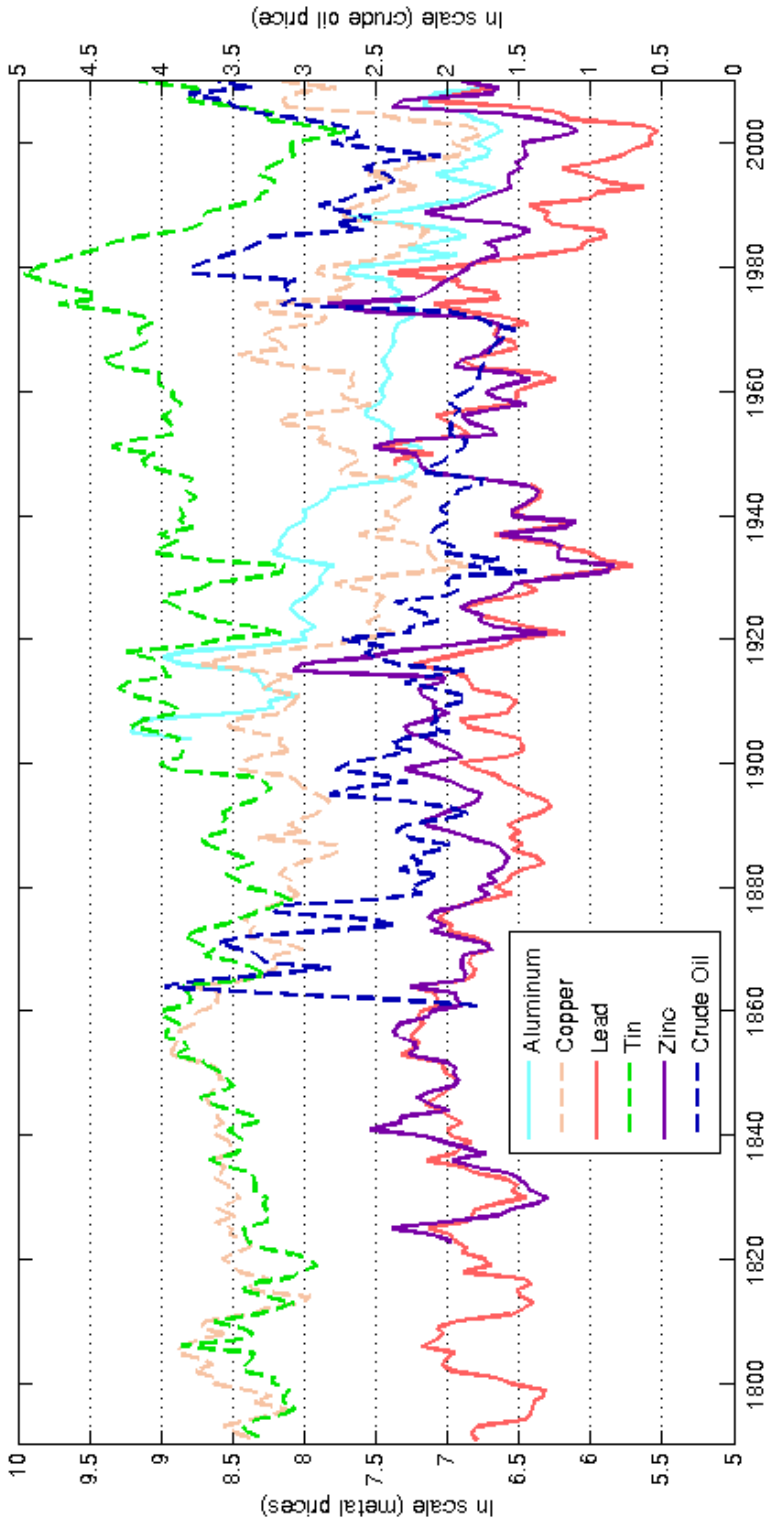
Notes: All prices, except for the price of crude oil, are prices of the London Metal Exchange and its predecessors. As the price of the London Metal Exchange used to be denominated in Sterling in earlier times, we have converted these prices to U.S.-Dollar by using historical exchange rates from Officer (2011). We use the U.S.-Consumer Price Index provided by Officer and Williamson (2011) and the U.S. Bureau of Labor Statistics (2010) for deflating prices with the base year 1980-82. The secondary y-axis relates to the price of crude oil. For data sources and description see Stürmer (2013).

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		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil
Range		1905-2009	1792-2009	1792-2009	1792-2009	1824-2009	1862-2009
Constant	Coeff.	-1.774	0.572	0.150	1.800	1.072	8.242
	t-stat.	(-0.180)	(0.203)	(0.052)	(0.660)	(0.205)	(0.828)
Lin.Trend	Coeff.	0.008	0.009	0.016	0.001	0.014	-0.021
	t-stat.	(0.137)	(0.428)	(0.714)	(0.069)	(0.357)	(-0.317)
Range		1905-2009	1850-2009	1850-2009	1862-2009	1850-2009	1850-2009
Constant	Coeff.	-1.299	0.109	-0.268	2.439	1.894	7.002
	t-stat.	(-0.200)	(0.030)	(-0.073)	(0.711)	(0.407)	(1.112)
Lin.Trend	Coeff.	0.008	0.020	0.030	-0.004	0.013	-0.021
	t-stat.	(0.137)	(0.518)	(0.755)	(-0.109)	(0.267)	(-0.317)
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	-0.903	-1.428	-0.490	1.068	2.764	-1.974
	t-stat.	(-0.239)	(-0.332)	(-0.102)	(0.269)	(0.443)	(-0.338)
Lin.Trend	Coeff.	0.008	0.055	0.054	0.010	0.010	0.100
	t-stat.	(0.137)	(0.820)	(0.713)	(0.168)	(0.099)	(1.106)
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	2.269	1.556	-3.688	-0.061	-0.515	3.445
	t-stat.	(0.479)	(0.240)	(-0.505)	(-0.011)	(-0.062)	(0.354)
Lin.Trend	Coeff.	-0.055	0.041	0.198	0.049	0.103	0.090
	t-stat.	(-0.411)	(0.225)	(0.958)	(0.307)	(0.441)	(0.326)
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	-0.549	1.323	0.370	3.719	1.136	-1.111
	t-stat.	(-0.088)	(0.266)	(0.081)	(0.812)	(0.176)	(-0.176)
Lin.Trend	Coeff.	-0.003	0.011	0.030	-0.012	0.051	0.094
	t-stat.	(-0.033)	(0.135)	(0.383)	(-0.152)	(0.468)	(0.875)

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend.***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 1: Tests of the stylized fact that the growth rates of real prices of mineral commodities equal zero and do not follow a statistically significant trend.
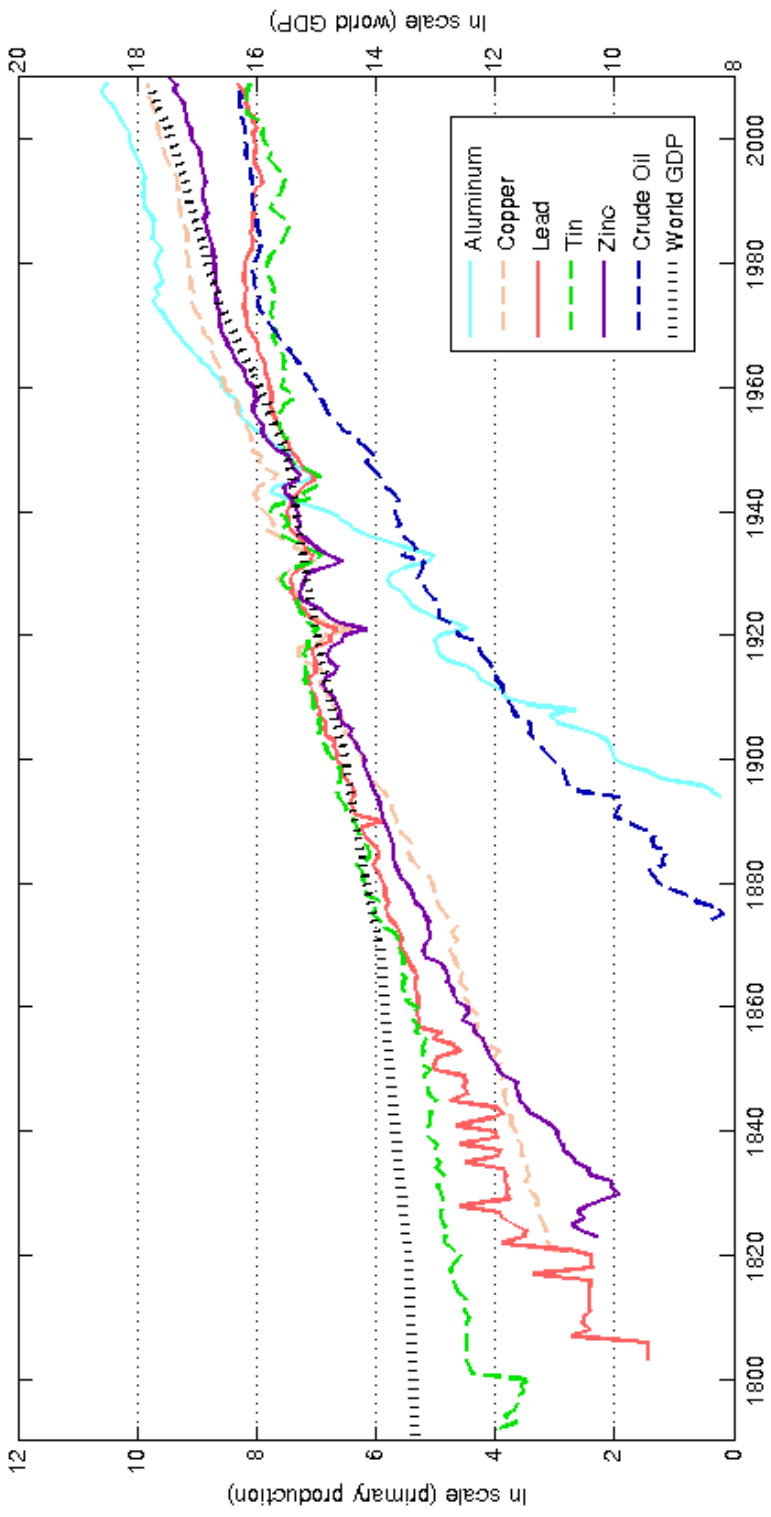
Figure 2: World primary production of non-renewable resources and world real GDP in logs.

For data sources and description see Stürmer (2013).

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.464	4.86	16.045	4.552	30.801	35.734	0.128
	t-stat.	*** 3.810	*** 2.694	*** 3.275	* 2.231	** 2.58	*** 4.365	0.959
Lin.Trend	Coeff.	-0.221	-0.006	-0.087	-0.016	-0.174	-0.182	0.018
	t-stat.	** -2.568	-0.439	** -2.294	-0.999	* -1.975	*** -3.334	*** 16.583
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.464	5.801	6.032	3.569	5.579	25.198	0.995
	t-stat.	*** 3.810	*** 3.461	***3.371	* 2.185	*** 3.774	*** 4.81	*** 5.49
Lin.Trend	Coeff.	-0.221	-0.018	-0.038	-0.015	-0.021	-0.182	0.019
	t-stat.	** -2.568	-1.007	-1.938	-0.833	-1.308	*** -3.334	*** 9.797
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	19.703	5.965	2.980	2.844	4.44	9.883	2.004
	t-stat.	*** 5.498	*** 2.651	* 2.043	1.361	* 2.225	*** 6.912	*** 7.8
Trend	Coeff.	-0.178	0.035	-0.019	-0.015	-0.018	-0.083	0.018
	t-stat.	*** 3.174	-0.995	-0.853	-0.464	-0.592	***-3.711	***4.549
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	10.781	5.043	13.205	0.051	5.675	9.897	4.729
	t-stat.	*** 7.169	*** 4.979	*** 2.936	0.028	*** 4.619	*** 9.574	*** 12.89
Lin.Trend	Coeff.	-0.171	-0.057	-0.48	0.04	-0.078	-0.196	-0.028
	t-stat.	*** -3.999	-1.978	-1.553	0.768	* -2.255	*** -6.64	*** -2.724
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.75	6.307	3.851	3.762	4.384	12.272	1.244
	t-stat.	*** 4.846	** 2.543	1.938	1.664	* 2.032	*** 4.060	*** 5.509
Lin.Trend	Coeff.	-0.53	-0.024	-0.018	-0.026	-0.005	-0.072	0.027
	t-stat.	*** -2.974	-0.566	-0.536	-0.66	-1.26	-1.403	***7.045

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 2: Tests for the stylized facts that growth rates of world primary production and world real GDP are equal to zero and trendless.

		Aluminum	Copper	Lead	Tin	Zinc	Crude Oil	World GDP
Range		1855-2009	1821-2009	1802-2009	1792-2009	1821-2009	1861-2009	1792-2009
Constant	Coeff.	48.301	5.474	20.57	4.427	30.7	35.689	0.032
	t-stat.	*** 3.824	*** 3.06	*** 3.845	* 2.181	** 2.584	*** 4.379	0.276
Lin.Trend	Coeff.	-0.229	-0.018	-0.125	-0.023	-0.182	-0.19	0.01
	t-stat.	*** -2.677	-1.367	*** -3.025	-1.457	* -2.071	*** -3.499	*** 11.066
Range		1855-2009	1850-2009	1850-2009	1850-2009	1850-2009	1861-2009	1850-2009
Constant	Coeff.	48.301	5.399	5.629	3.179	5.18	24.681	0.628
	t-stat.	*** 3.824	*** 3.254	***3.169	1.961	*** 3.541	*** 4.733	*** 4.052
Lin.Trend	Coeff.	-0.229	-0.027	-0.047	-0.024	-0.03	-0.19	0.01
	t-stat.	*** -2.677	-1.523	** -2.442	-1.348	-1.895	*** -3.499	*** 5.876
Range		1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009	1900-2009
Constant	Coeff.	18.595	4.985	2.028	1.903	3.473	8.869	1.071
	t-stat.	*** 5.242	* 2.241	1.41	0.918	1.763	*** 6.306	*** 4.862
Trend	Coeff.	-0.184	-0.042	-0.027	-0.023	-0.026	-0.09	0.01
	t-stat.	*** -3.315	-1.214	-1.186	-0.694	-0.404	*** -4.084	*** 3.01
Range		1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009	1950-2009
Constant	Coeff.	8.583	2.952	1.141	-1.954	3.578	7.716	2.632
	t-stat.	*** 5.742	*** 2.892	1.04	1.086	*** 2.87	*** 7.493	*** 7.444
Lin.Trend	Coeff.	-0.156	-0.044	-0.35	0.051	-0.065	-0.18	-0.016
	t-stat.	*** -3.667	-1.515	-1.129	0.997	-1.819	*** -6.14	-1.551
Range		1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975	1875-1975
Constant	Coeff.	50.004	5.854	3.413	3.317	3.942	11.789	0.834
	t-stat.	*** 4.81	** 2.386	1.738	1.480	1.851	*** 3.933	*** 4.509
Lin.Trend	Coeff.	-0.542	-0.038	-0.032	-0.039	-0.019	-0.086	0.013
	t-stat.	*** -3.06	-0.908	-0.959	-1.028	-0.517	-1.691	***4.004

Notes: The table presents coefficients and t-statistics for regressions of the growth rates on a constant and a linear trend. ***, **, and * indicate significance at the 1%, 2.5% and 5% level, respectively.

Table 3: Tests for the stylized fact that growth rates of world per capita primary production and world per capita GDP are equal to zero and trendless.
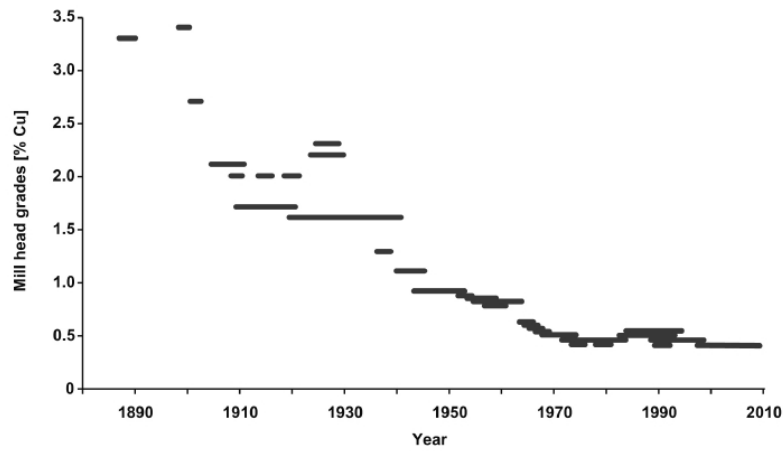
Figure 3: The historical development of mining of various grades of copper in the U.S. Source: Scholz and Wellmer (2012)
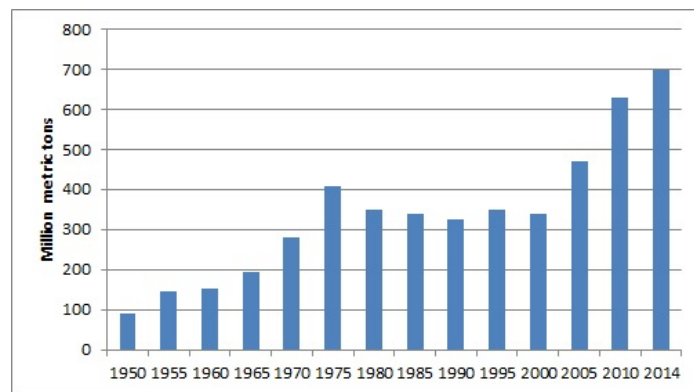

Figure 4: Historical evolution of world copper reserves from 1950 to 2014. Sources: Tilton and Lagos C.C. (2007), USGS.
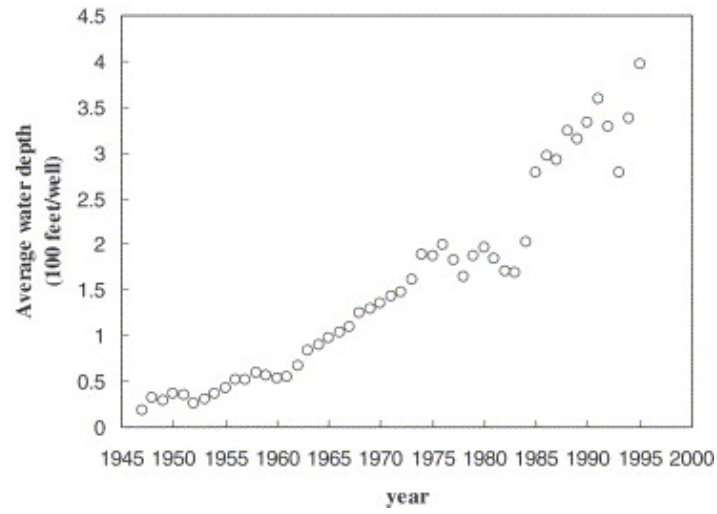
Figure 5: Average water depth of wells drilled in the Gulf of Mexico. Source: Managi et al. (2004).
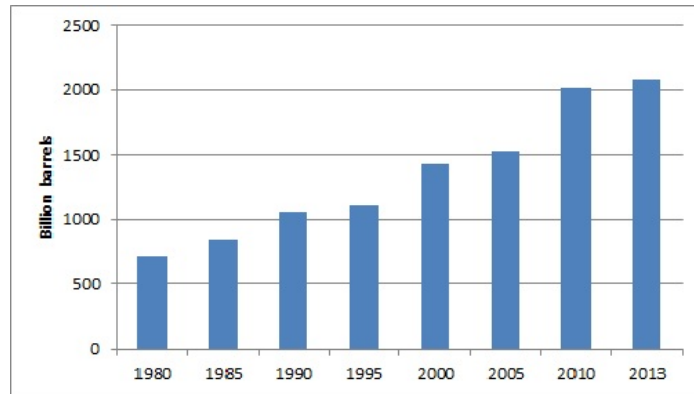

Figure 6: Historical evolution of oil reserves, including Canadian oil sands from 1980 to 2013. Source: BP, 2014.

	Reserves/ Annual production (Years)	Resources/ Annual production (Years)	Crustal abundance/ Annual production (Years)
Aluminum	139^{1a}	$263,000^{1a}$	$48,800,000,000^{bc}$
Copper	43^{a}	189^{a}	$95,000,000^{ab}$
Iron	78^{a}	223^{a}	$1,350,000,000^{ab}$
Lead	21^{a}	362^{a}	$70.000.000^{ab}$
Tin	17^{a}	"Sufficient"^{a}	144.000^{ab}
Zinc	21^{a}	158^{a}	$187.500.000^{ab}$
Gold	20^{d}	13^{d}	$27,160,000^{ef}$
Rare earths[2]	827^{a}	"Very large"^{a}	n.a.
Coal[3]	129^{g}	$2,900^{g}$	
Crude oil[4]	55^{g}	76^{g}	} $1,400,000^{6i}$
Gas[5]	59^{g}	410^{g}	

Notes: Reserves include all material which can currently be extracted. The definition of resources can be found in Section 2.2. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011) [gi]Littke and Welte (1992). Notes: [1] data for bauxite, [2] rare earth oxide, [3] includes lignite and hard coal, [4] includes conventional and unconventional oil, [5] includes conventional and unconventional gas, [6] all organic carbon in the earth's crust.

Table 4: Availability of selected non-renewable resources in years of production left in the reserve, resource and crustal mass based on current annual mine production.

	Reserves/ Annual production (Years)	Resources/ Annual production (Years)	Crustal abundance/ Annual production (Years)
Aluminum	65[1,a,h]	419[1,a,h]	838[b,c,h]
Copper	30[a,g]	77[a,g]	718[a,b,g]
Iron	44[a,h]	78[a,h]	744[a,b,h]
Lead	18[a,h]	181[a,h]	1,907[a,b,h]
Tin	18[a,h]	n.a.	3,588[a,b,h]
Zinc	17[a,h]	74[a,h]	842[a,b,h]
Gold	18[d,h]	11[d,h]	2,170[e,f,h]
Rare earths[2]	127[a,h]	n.a.	n.a.
Coal[3]	65[g,k]	215[g,k]	} 729[6,j]
Crude oil[4]	46[g,k]	60[g,k]	
Natural gas[5]	41[g,k]	123[g,k]	

Notes: The numbers for reserves and resources are not summable as in Table 4. We have used the following average annual growth rates of production from 1990 to 2010: Aluminum: 2.5%, Iron: 2.3%, Copper: 2%, Lead: 0.7%, Tin: 0.4%, Zinc: 1.6%, Gold: 0.6%, Rare earths: 2.6%, Crude oil: 0.7%, Natural gas: 1.7%, Coal: 1.9%, Hydrocarbons: 1.4%. Reserves include all material which can currently be extracted. The definition of resources can be found in Section 2.2. Sources: [a]U.S. Geological Survey (2012b), [b]Perman et al. (2003), [c]U.S. Geological Survey (2011c),[d]U.S. Geological Survey (2011b),[e]Nordhaus (1974),[f]U.S. Geological Survey (2010), [g]Federal Institute for Geosciences and Natural Resources (2011), [h]U.S. Geological Survey (2012a), [i]U.S. Bureau of Mines (1991), [j]Littke and Welte (1992), [k]British Petroleum (2013). Notes: [1] data for bauxite, [2] rare earth oxide, [3] includes lignite and hard coal, [4] includes conventional and unconventional oil, [5] includes conventional and unconventional gas, [6] all organic carbon in the earth's crust.

Table 5: Availability of selected non-renewable resources in years of production left in the reserve, resource and crustal mass based on an exponentially increasing annual mine production (based on the average growth rate over the last 20 years).
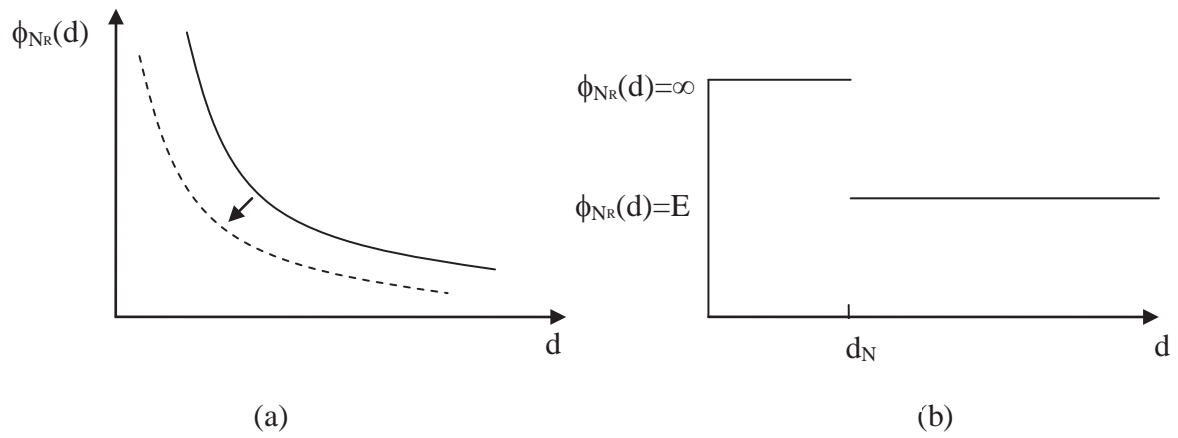
Figure 7: Extraction costs ϕ_{N_R} as a function of deposits of different grades d. General and simplified form.
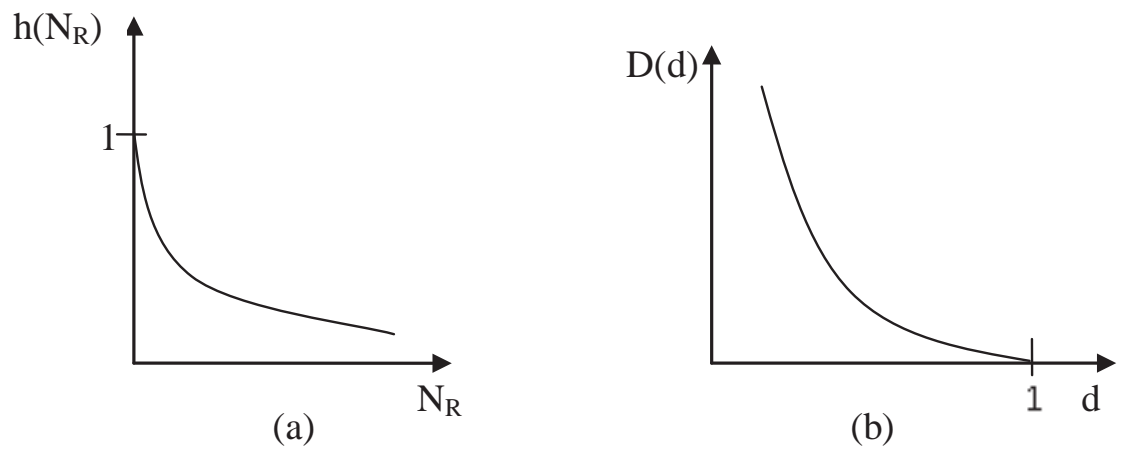

Figure 8: (a) Extractable deposits of grade $h(N_R)$ as a function of the state of technology N_R. (b) Extractable amount of the non-renewable resource in the earth's crust $D(d)$ at a given grade d.

Appendix 2   Proofs

Proof of Proposition 1

$$
\begin{aligned}
D(h(N_{Rt})) &= -\delta_2 \ln(d_{N_{Rt}}) \\
&= -\delta_2 \ln(e^{-\delta_1 N_{Rt}}) \\
&= \delta_1 \delta_2 N_{Rt}
\end{aligned}
$$

□

Proof of Proposition 2 and 3

The final good producer demands the resource for aggregate production. The price of the final good is the numeraire. The first order condition with respect to the resource from production (see Equation 16) is

$$
Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0 \, ,
\tag{20}
$$

so that the demand for the resource is

$$
R = \frac{Y(1-\gamma)^{\varepsilon}}{p_R^{\varepsilon}} \, .
\tag{21}
$$

Assume that initially, the resource stock available to the extractive firms is zero, $S_t = 0$. Since the stock of the resource S cannot be negative, newly acquired resources cannot be less than the resources sold to the final good producer: $X_t \geq R_t$. Newly acquired resources in excess of those sold could be stored. In a world without uncertainty, however, this would not be profitable. The price therefore must be equal to marginal cost:

$$
p_R = \frac{1}{\eta_R \delta_1 \delta_2} \, .
\tag{22}
$$

It remains to consider the case of a positive initial stock of the resource, $S_t > 0$. Under perfect competition, this stock is immediately sold off to the final good producer such that the case of $S_t = 0$ returns. □

Proof of Proposition 4

The first order conditions (FOC) of the final good producer for the optimal input of Z and R are $Y^{\frac{1}{\varepsilon}}\gamma Z^{-\frac{1}{\varepsilon}} - p_Z = 0$ and $Y^{\frac{1}{\varepsilon}}(1-\gamma)R^{-\frac{1}{\varepsilon}} - p_R = 0$, where the final good is the numeraire. From this the relative price is

$$
p = \frac{p_R}{p_Z} = \frac{1-\gamma}{\gamma} \left(\frac{R}{Z}\right)^{-\frac{1}{\varepsilon}} \, .
\tag{23}
$$

Setting the price of the final good as the numeraire gives (for the derivation of the price index see the derivation of Equation (12.11) in Acemoglu (2009)):

$$
\left[\gamma^{\varepsilon} p_Z^{1-\varepsilon} + (1-\gamma)^{\varepsilon} p_R^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} = P = 1 \, .
\tag{24}
$$

The intermediate goods sector

As in Acemoglu (2009), the maximization problem in the intermediate goods sector is:

$$\max_{L,\{x_Z(j)\}} p_Z Z - w_Z L - \int_0^{N_Z} \chi_Z(j) x_Z(j) dj \ . \tag{25}$$

The FOC with respect to $x_Z(j)$ is $p_Z x_Z(j)^{-\beta} L^\beta - \chi_Z(j) = 0$ so that

$$x_Z(j) = \left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L \ . \tag{26}$$

From the FOC with respect to L we obtain the wage rate

$$w_Z = \frac{\beta}{1-\beta} p_Z \left(\int_0^{N_Z} x_Z(j)^{-\beta} dj\right) L^{\beta-1} \ . \tag{27}$$

The profits of the technology firms are:

$$\pi_Z(j) = (\chi_Z(j) - \psi) x_Z(j) \ . \tag{28}$$

Substituting Equation 26 into Equation 28 we calculate the FOC with respect to the price of a machine $\chi_Z(j)$: $\left(\frac{p_Z}{\chi_Z(j)}\right)^{\frac{1}{\beta}} L - (\chi_Z(j) - \psi) p_Z^{\frac{1}{\beta}} \frac{1}{\beta} \chi_Z(j)^{\frac{1}{\beta}-1} L = 0$. Solving this for $\chi_Z(j)$ yields $\chi_Z(j) = \frac{\psi}{1-\beta}$. Following Acemoglu (2002) we normalize $\psi = 1 - \beta$ so that $\chi_Z(j) = 1$. Combining this result with Equations 26 and 28 we write profits as

$$\pi_Z(j) = \beta p_Z^{\frac{1}{\beta}} L \ . \tag{29}$$

The present discounted value is:

$$r V_Z - \dot{V}_Z = \pi_Z \ . \tag{30}$$

The steady state ($\dot{V} = 0$) is:

$$V_Z = \frac{\beta p_Z^{\frac{1}{\beta}} L}{r} \ . \tag{31}$$

Substituting Equation 26 into Equation 17 yields

$$Z = \frac{1}{1-\beta} p_Z^{\frac{1-\beta}{\beta}} N_Z L \ . \tag{32}$$

Solving for the variables of the intermediate goods sector

Solving Equation 24 for p_Z yields

$$p_Z = \left(\gamma^{-\varepsilon} - \left(\frac{1-\gamma}{\gamma}\right)^\varepsilon p_R\right)^{\frac{1}{1-\varepsilon}} \ . \tag{33}$$

This can be used, together with the expression for R from Equation 21 and the expression for p_R from Equation 22 to determine Z as a function of Y from Equation

23. We obtain the range of machines N_Z as a function of Y from Equation 32.

The growth rate

The consumer earns wages from working in the sector which produces good Z and earns interest on investing in the technology N_Z. The budget constraint thus is $C = w_Z L + rM$. Maximizing utility in Equation 15 with respect to consumption and investments yields the first order conditions $C^{-\theta} e^{-\rho t} = \lambda$ and $\dot{\lambda} = -r\lambda$ so that the growth rate of consumption is

$$g_c = \theta^{-1}(r - \rho) . \tag{34}$$

This will be equal to output growth on the balanced growth path. We can thus solve for the interest rate and obtain $r = \theta g + \rho$. The free entry condition for the technology firms imposes that profits from investing in patents must be zero. Revenue per unit of R&D investment is given by V_Z, cost is equal to $\frac{1}{\eta_Z}$. Consequently, we have $\eta_Z V_Z = 1$. Substituting Equation 31 into it we obtain $\frac{\eta_Z \beta p_Z^{\frac{1}{\beta}} L}{r} = 1$. Solving this for r and substituting into Equation 34 we obtain

$$g = \theta^{-1}(\beta \eta_Z L p_Z^{\frac{1}{\beta}} - \rho) . \tag{35}$$

Together with Equations 22 and 33 this yields the growth rate. $\qquad\square$

Proof of Proposition 5

Substitute Equation 22 into Equation 21. $\qquad\square$

Proof of Proposition 6

We use Equation 33, together with the expression for R from Equation 21 and the expression for p_R from Equation 22 to determine Z as a function of Y from Equation 23. This can then be used to obtain the range of machines N_Z as a function of Y from Equation 32.

The expression for $\dot{N}_R$ follows from equation 10, Proposition 2 as well as equation 21. $\qquad\square$