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Mind the Gap!—A Monetarist View of the Open-Economy Phillips Curve

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Online Appendix: Mind the Gap!—A Monetarist View of the Open-Economy Phillips Curve*

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Abstract

In many countries, inflation has become less responsive to domestic factors and more responsive to global factors over the past several decades. We study the linkages between domestic inflation and global liquidity (money and household balances) and argue that it is important for inflation modeling and forecasting. We introduce money and credit markets into the workhorse open-economy New Keynesian model. With this framework, we show that: (i) an efficient forecast of domestic inflation can be based solely on domestic and foreign slack, and (ii) global liquidity (either global money or global credit) is tied to global slack in equilibrium. In this technical appendix, we derive these theoretical results which can be used to empirically evaluate the performance of open-economy Phillips curve-based forecasts constructed using global liquidity measures (such as G7 credit growth and G7 money supply growth) instead of global slack as predictive regressors. We also include additional results not found elsewhere: in particular, we document that global liquidity variables also perform significantly better than the domestic variable counterparts and outperform in practice the poorly-measured indicators of global slack when expressed in real terms (not just in nominal terms).

Keywords: Global Slack, New Open-Economy Phillips Curve, Open-Economy New Keynesian Model, Forecasting.

JEL Classification: F41, F44, F47, C53, F62.

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On Modeling Global Liquidity

A The Open-Economy New Keynesian Model with Money and Credit

Our paper contributes to the study of the linkages between inflation and global liquidity within an open-economy New Keynesian model. The framework we work with is an extension of the open-economy model of Martínez-García and Wynne (2010), Kabukçuoğlu and Martínez-García (2018), and Martínez-García (2019a) that incorporates the concept of liquidity services articulated in a New Keynesian setting by Galí (2008), Belongia and Ireland (2014), and in an open-economy setting by Martínez-García (2019b), among others. The idea is that households gain utility from the liquidity services provided by real cash balances and by real credit. This, in turn, generates a demand for money and credit in the market. The monetary authority accommodates the demand for liquidity services through the management of the central bank's balance sheet and policy rate which is partly transmitted via the banking system.

Here we describe the main features of such an open-economy New Keynesian framework with moneyholdings and a credit channel maintaining the symmetry in the structure of both countries between households, firms, the banking system, and the central bank. We illustrate the model with the first principles from the Home country unless otherwise noted, and use the superscript $*$ to denote Foreign country variables.

A.1 Households' Optimization

The lifetime utility of the representative household in the Home country is additively separable in consumption, C_t, labor, N_t, and a real liquidity bundle of cash and credit, X_t, i.e.,

$$\textstyle\sum_{\tau=0}^{+\infty} \beta^{\tau} \mathbb{E}_t \left[\frac{1}{1-\gamma} \left(C_{t+\tau}\right)^{1-\gamma} + \frac{\chi}{1-\zeta} \left(X_{t+\tau}\right)^{1-\zeta} - \frac{\kappa}{1+\varphi} \left(N_{t+\tau}\right)^{1+\varphi} \right], \tag{1}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution on consumption, $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply, and $\zeta > 0$ determines the inverse elasticity on the liquidity bundle. The scaling factors $\chi > 0$ and $\kappa > 0$ pin down liquidity and labor in steady state.

We recognize that transactions require households to take a liquidity position. However, real money balances (or real cash) is not the only way that gains from liquidity services can be had; access to real credit is another way to service liquidity needs. For this, we assume that the liquidity bundle, X_t, is a non-separable constant-elasticity of substitution (Armington) aggregator between real money balances (real cash), $\frac{Z_t}{P_t}$, and real credit, $\frac{L_t}{P_t}$, given by,

$$X_t = \left[(\mu)^{\frac{1}{\nu}} \left(\frac{Z_t}{P_t} \right)^{\frac{v-1}{\nu}} + (1-\mu)^{\frac{1}{\nu}} \left(\frac{L_t}{P_t} \right)^{\frac{v-1}{\nu}} \right]^{\frac{v}{\nu-1}}, \tag{2}$$

where $\nu > 0$ denotes the elasticity of substitution between real money balances (real cash) and real credit. The parameter $0 < \mu \leq 1$ captures the relative weight of real money balances and real credit in the household's per-period utility from liquidity services. Only in the special case in which real balances and real credit are perfect substitutes, simple aggregation of both suffices to measure liquidity.[1] In the special case where $\mu = 1$, the real liquidity position given by X_t in (2) reduces to real balances which is the standard assumption in the existing money-in-the-utility-function literature (see, e.g., Galí, 2008).

The representative household maximizes its lifetime utility in (1) subject to the following sequence of budget constraints which holds across all states of nature $\omega_t \in \Omega$, i.e.,

$$\begin{aligned} & P_t C_t + \int_{\omega_{t+1} \in \Omega} Q_t (\omega_{t+1}) B_t^H (\omega_{t+1}) + S_t \int_{\omega_{t+1} \in \Omega} Q_t^* (\omega_{t+1}) B_t^F (\omega_{t+1}) + Z_t + D_t - L_t \\ & \leq W_t N_t + Pr_t - T_t + B_{t-1}^H (\omega_t) + S_t B_{t-1}^F (\omega_t) + Z_{t-1} - (1 + i_{t-1}) D_{t-1} - (1 + i_{L,t-1}) L_{t-1}, \end{aligned} \tag{3}$$

where W_t is the nominal wage in the Home country, P_t is the Home consumer price index (CPI), T_t is a nominal lump-sum tax (or transfer) imposed by the Home government, and Pr_t are (per-period) nominal profits from all firms producing the Home varieties as well as from the Home banking system. We denote the fully-flexible bilateral nominal exchange rate as S_t indicating the units of the currency of the Home country that can be obtained per each unit of the Foreign country currency at time t.

The representative household's budget constraint includes a portfolio of one-period Arrow-Debreu securities (contingent bonds) traded internationally, issued by the governments of both countries each in their own currencies and in zero-net supply. That is, the pair $\left\{ B_t^H (\omega_{t+1}), B_t^F (\omega_{t+1}) \right\}$ refers to the portfolio of contingent bonds issued by both countries held by the representative household of the Home country. Access to a full set of internationally-traded, one-period Arrow-Debreu securities completes the local and international asset markets recursively. The prices of the Home and Foreign contingent bonds expressed in their currencies of denomination are denoted $Q_t (\omega_{t+1})$ and $Q_t^* (\omega_{t+1})$, respectively.[2]

The budget constraint also takes into account that the representative household holds non-interest-bearing cash or nominal money balances, Z_t. The representative household makes nominal deposits with the banking system, D_t, that earn a (guaranteed) risk-free nominal return of i_t

[1] Whenever v approaches infinity, real balances and real credit become perfect substitutes; in turn, whenever v approaches zero, they are perfect complements.

[2] The price of each bond in the currency of the country who did not issue the bond is converted at the prevailing bilateral exchange rate with full exchange rate pass-through under the law of one price (LOOP).

while also taking loans from the banking system, L_t, at a net interest rate of $i_{L,t}$. The function of the banking system that we highlight here is that of a liquidity provider that transforms household's savings into liquidity in order to facilitate the functioning of the payment system. Furthermore, we also assume that liquidity is locally-provided—cash issued by the domestic central bank only circulates within each country's borders and domestic loans are supplied solely by the locally-based banking system (abstracting from issues like cross-border loans, global currencies).

We define the problem of each household in the Foreign country similarly.

Households' asset demand equations. Under complete asset markets, standard no-arbitrage results imply that $Q_t\left(\omega_{t+1}\right)=\frac{S_t}{S_{t+1}\left(\omega_{t+1}\right)}Q_t^*\left(\omega_{t+1}\right)$ for every state of nature $\omega_t\in\Omega$. Hence, Home and Foreign households can efficiently share risks domestically as well as internationally. This implies that the intertemporal marginal rate of substitution is equalized across countries at each possible state of nature and, accordingly, it follows that:

$$\beta\left(\frac{C_t}{C_{t-1}}\right)^{-\gamma}\frac{P_{t-1}}{P_t}=\beta\left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\gamma}\frac{P_{t-1}^*S_{t-1}}{P_t^*S_t}. \tag{4}$$

We define the bilateral real exchange rate as $RS_t\equiv\frac{S_tP_t^*}{P_t}$, so by backward recursion the *perfect international risk-sharing condition* in (4) implies that,

$$RS_t=v_0\left(\frac{C_t^*}{C_t}\right)^{-\gamma}, \tag{5}$$

where $v_0\equiv\frac{S_0P_0^*}{P_0}\left(\frac{C_0^*}{C_0}\right)^{\gamma}$ is a constant that depends on initial conditions. If the initial conditions correspond to those of the symmetric steady state, then the constant v_0 is equal to one.

Home country household's savings on a one-period, non-contingent nominal deposit in the Home country banking system result in the following standard stochastic Euler equation:

$$\frac{1}{1+i_t}=\beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+1}}\right], \tag{6}$$

where i_t is the risk-free Home nominal interest rate (or, simply put, the nominal return on deposits in the Home banking system). This is equivalent to the yield on a redundant one-period, non-contingent nominal bond in the Home country which can be synthetically computed from the price of the contingent Arrow-Debreu securities in the Home country.

From the household's first-order conditions on nominal balances of cash and credit (Z_t and L_t), we obtain the following pair of equilibrium conditions that dictate the demand for cash and credit in the Home country:

$$\chi\left(\mu\right)^{\frac{1}{\nu}}\left(\frac{Z_t}{P_t}\right)^{-\frac{1}{\nu}} = \left(X_t\right)^{\zeta-\frac{1}{\nu}}\left(C_t\right)^{-\gamma}\frac{i_t}{1+i_t}, \tag{7}$$

$$\chi\left(1-\mu\right)^{\frac{1}{\nu}}\left(\frac{L_t}{P_t}\right)^{-\frac{1}{\nu}} = \left(X_t\right)^{\zeta-\frac{1}{\nu}}\left(C_t\right)^{-\gamma}\left(\frac{i_{L,t}-i_t}{1+i_t}\right). \tag{8}$$

Taking the ratio of both equilibrium conditions it follows that:

$$\frac{L_t}{P_t} = \frac{\mu}{1-\mu}\left(\frac{i_t}{i_{L,t}-i_t}\right)^{v}\frac{Z_t}{P_t}, \tag{9}$$

which shows that—in an interior solution where both cash and credit are used—the demand for real credit must be equal to a multiplier over the demand for real money balances. The multiplier in (9) depends on the risk-free Home nominal interest rate, i_t, and on the spread between the loan rate and the rate paid on deposits, $i_{L,t} - i_t$.

Replacing (9) into (2), we can express the liquidity position of the representative household in the Home country as proportional to its holdings of real balances, i.e.,

$$X_t = \left[\left(\mu\right)^{\frac{1}{\nu}} + \left(1-\mu\right)^{\frac{1}{\nu}}\left(\frac{\mu}{1-\mu}\right)^{\frac{\nu-1}{\nu}}\left(\frac{i_t}{i_{L,t}-i_t}\right)^{v-1}\right]^{\frac{v}{\nu-1}}\left(\frac{Z_t}{P_t}\right). \tag{10}$$

Combining this expression for the real liquidity bundle with the first-order condition on real balances in (7), we obtain that:

$$\chi\left(\mu\right)^{\frac{1}{\nu}}\left(\frac{Z_t}{P_t}\right)^{-\zeta} = \left(C_t\right)^{-\gamma}\left(\left(\mu\right)^{\frac{1}{\nu}} + \left(1-\mu\right)^{\frac{1}{\nu}}\left(\frac{\mu}{1-\mu}\right)^{\frac{\nu-1}{\nu}}\left(\frac{i_t}{i_{L,t}-i_t}\right)^{v-1}\right)^{\frac{\zeta-\frac{1}{v}}{1-\frac{1}{v}}}\left(\frac{i_t}{1+i_t}\right), \tag{11}$$

which defines the demand for real money balances in the model. The expression for money demand in (11) can be seen as a special case of the quantity theory of money equation where consumption expenditures (P_tC_t) are related to money holdings (cash holdings, Z_t) with a scaling factor. The scaling factor, akin to the velocity of money in the quantity theory of money equation, depends on both the risk-free Home nominal interest rate, i_t, and the spread between the loan rate and the risk-free rate, $i_{L,t} - i_t$. Equations (9) and (11) fully describe the demand-side of the money and credit markets.

Household's labor supply and consumption demand equations. We assume within-country labor mobility which ensures that wages equalize across firms in a given country, although not necessarily across countries because we still retain the assumption of labor immobility across international borders. From the household's first-order conditions we obtain a labor supply equation of the following form:

$$\frac{W_t}{P_t} = \kappa \left(C_t\right)^{\gamma} \left(N_t\right)^{\varphi}. \tag{12}$$

With flexible wages, all households are paid the same nominal wage rate, W_t, and work the same hours, N_t, in equilibrium.

The consumption of the representative household in the Home country, C_t, is given by a nested CES aggregator of both countries' bundle of varieties. The consumption CES index for the Home representative household is defined as:

$$C_t = \left[(1-\xi)^{\frac{1}{\sigma}} \left(C_t^H\right)^{\frac{\sigma-1}{\sigma}} + (\xi)^{\frac{1}{\sigma}} \left(C_t^F\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{13}$$

where $\sigma > 0$ is the elasticity of substitution between the consumption bundle of Home-produced goods consumed in the Home country, C_t^H, and the Home consumption bundle of the Foreign-produced goods, C_t^F. Similarly, the CES aggregator for the Foreign country is defined as:

$$C_t^* = \left[(\xi)^{\frac{1}{\sigma}} \left(C_t^{H*}\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}} \left(C_t^{F*}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{14}$$

where C_t^{F*} and C_t^{H*} are respectively the consumption bundle of Foreign-produced goods and of Home-produced goods for the Foreign country household. The share of imported goods in the consumption basket of each country is given by ξ and satisfies that $0 \leq \xi \leq \frac{1}{2}$. Given that each country produces an equal share of varieties, we allow for local-consumption bias whenever $\xi < \frac{1}{2}$.[3] The consumption CES sub-indexes aggregate the consumption of the representative household over the bundle of differentiated varieties produced by each country and are defined as follows:

$$C_t^H = \left[\int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^F = \left[\int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{15}$$

$$C_t^{H*} = \left[\int_0^1 C_t^*(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^{F*} = \left[\int_0^1 C_t^*(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{16}$$

where $\theta > 1$ is the elasticity of substitution across the differentiated varieties within a country.

The consumption price indexes (CPIs) that correspond to this specification of consumption preferences are,

$$P_t = \left[(1-\xi) \left(P_t^H\right)^{1-\sigma} + \xi \left(P_t^F\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad P_t^* = \left[\xi \left(P_t^{H*}\right)^{1-\sigma} + (1-\xi) \left(P_t^{F*}\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{17}$$

[3] The two countries are assumed to be symmetric in every respect, except on their consumption basket due to the assumption of home-product bias in consumption. Even so, the specification of the home-product bias is inherently symmetric as well since the share of local goods in the local consumption basket is the same in both countries and determined by the parameter ξ.

and,

$$P_t^H = \left[\int_0^1 P_t(h)^{1-\theta}\,dh\right]^{\frac{1}{1-\theta}},\ P_t^F = \left[\int_0^1 P_t(f)^{1-\theta}\,df\right]^{\frac{1}{1-\theta}}, \tag{18}$$

$$P_t^{H*} = \left[\int_0^1 P_t^*(h)^{1-\theta}\,dh\right]^{\frac{1}{1-\theta}},\ P_t^{F*} = \left[\int_0^1 P_t^*(f)^{1-\theta}\,df\right]^{\frac{1}{1-\theta}}, \tag{19}$$

where P_t^H and P_t^{F*} are the price sub-indexes for the bundle of locally-produced varieties in the Home and Foreign countries, respectively. The price sub-index P_t^F represents the Home country price of the bundle of Foreign varieties, while P_t^{H*} is the Foreign country price for the bundle of Home varieties. The price of variety h produced in the Home country is expressed as $P_t(h)$ and $P_t^*(h)$ in units of the Home and Foreign currency, respectively. Similarly, the price of variety f produced in the Foreign country is quoted in both countries as $P_t(f)$ and $P_t^*(f)$, respectively.

Each household decides how much to allocate to the different varieties of goods produced in each country. Given the structure of preferences given here, the utility maximization problem implies that the household's demand for each variety is given by:

$$C_t(h) = \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta} C_t^H,\ C_t(f) = \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} C_t^F, \tag{20}$$

$$C_t^*(h) = \left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta} C_t^{H*},\ C_t^*(f) = \left(\frac{P_t^*(f)}{P_t^{F*}}\right)^{-\theta} C_t^{F*}, \tag{21}$$

while the demand for the bundle of varieties produced by each country is simply equal to:

$$C_t^H = (1-\xi)\left(\frac{P_t^H}{P_t}\right)^{-\sigma} C_t,\ C_t^F = \xi\left(\frac{P_t^F}{P_t}\right)^{-\sigma} C_t, \tag{22}$$

$$C_t^{H*} = \xi\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} C_t^*,\ C_t^{F*} = (1-\xi)\left(\frac{P_t^{F*}}{P_t^*}\right)^{-\sigma} C_t^*. \tag{23}$$

These equations relate the demand for each variety—whether produced domestically or imported—to the aggregate consumption of the country.

The optimization problem of the representative household of the Home country also satisfies the budget constraint in (3), the given initial conditions on all assets (contingent bonds, deposits, cash, and credit), and the corresponding no-Ponzi game conditions. The same holds true also for the representative household of the Foreign country.

A.2 The Firms' Price-Setting Behavior

Home firms produce their variety of output subject to a linear-in-labor technology, i.e., $Y_t(h) = A_t N_t(h)$ for all $h \in [0,1]$. Each firm located in either the Home or Foreign country supplies its local market and exports its own differentiated variety operating under monopolistic competition. We assume producer currency pricing (PCP), so firms set prices by invoicing all sales in their local currency. The PCP assumption implies that the law of one price (LOOP) holds at the variety level—i.e., for each variety h produced in the Home country, it must hold that $P_t(h) = S_t P_t^*(h)$. Similarly, for each variety f produced in the Foreign country holds that $P_t(f) = S_t P_t^*(f)$. Hence, it follows naturally that the conforming price sub-indexes in both countries computed for the same bundle of varieties must satisfy that $P_t^H = S_t P_t^{H*}$ and $P_t^F = S_t P_t^{F*}$.

The bilateral terms of trade $ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}}$ define the Home country value of the imported bundle of goods produced in the Foreign country in Home currency units relative to the Foreign value of the bundle of the Home country's exports, quoted in the currency of the Home country at the prevailing bilateral nominal exchange rate. Under the LOOP, terms of trade can be expressed as,

$$ToT_t \equiv \frac{P_t^F}{S_t P_t^{H*}} = \frac{P_t^F}{P_t^H}. \tag{24}$$

Even though the LOOP holds, the assumption of local-product bias in consumption introduces deviations from purchasing power parity (PPP) at the level of the consumption basket. For this reason, $P_t \neq S_t P_t^*$ and, therefore, the bilateral real exchange rate between both countries deviates from one—i.e., $RS_t \equiv \frac{S_t P_t^*}{P_t} = \left[\frac{\xi + (1-\xi)(ToT_t)^{1-\sigma}}{(1-\xi)+\xi(ToT_t)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \neq 1$ if $\xi \neq \frac{1}{2}$.

Given households' preferences in each country, the demand for any variety $h \in [0,1]$ produced in the Home country is given as,

$$\begin{aligned} Y_t(h) \equiv C_t(h) + C_t^*(h) &= (1-\xi)\left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}\left(\frac{P_t^H}{P_t}\right)^{-\sigma} C_t + \xi\left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} C_t^* \\ &= \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}\left(\frac{P_t^H}{P_t}\right)^{-\sigma}\left[(1-\xi) C_t + \xi\left(\frac{1}{RS_t}\right)^{-\sigma} C_t^*\right]. \end{aligned} \tag{25}$$

Similarly, we derive the demand for each variety $f \in [0,1]$ produced by the Foreign firms. Firms maximize profits subject to a partial adjustment rule à la Calvo (1983) at the variety level (that is, the pricing of varieties is subject to sticky prices). In each period, every firm receives either a signal to maintain their prices with probability $0 < \alpha < 1$ or a signal to re-optimize them with probability $1-\alpha$. At time t, the re-optimizing firm producing variety h in the Home country chooses a price $\widetilde{P}_t(h)$ optimally to maximize the expected discounted value of its profits, i.e.,

$$\sum_{\tau=0}^{+\infty} \mathbb{E}_t \left\{ (\alpha\beta)^\tau \left(\frac{C_{t+\tau}}{C_t}\right)^{-\gamma} \frac{P_t}{P_{t+\tau}} \left[\widetilde{Y}_{t,t+\tau}(h)\left(\widetilde{P}_t(h) - (1-\phi) MC_{t+\tau}\right)\right]\right\}, \tag{26}$$

subject to the constraint that the aggregate demand given in (25) is always satisfied at the set price $\widetilde{P}_t(h)$ as long as it remains in place (even when this implies per-period losses for the firm). $\widetilde{Y}_{t,t+\tau}(h)$ indicates the demand for consumption of the variety h produced in the Home country at time $t+\tau$ ($\tau > 0$) whenever the prevailing prices remain unchanged since time t—i.e., whenever

$P_{t+s}(h) = \widetilde{P}_t(h)$ for all $0 \leq s \leq \tau$. An analogous problem describes the optimal price-setting behavior of the re-optimizing firms in the Foreign country.

Hence, the (before-subsidy) nominal marginal cost in the Home country MC_t can be expressed as:

$$MC_t \equiv \left(\frac{W_t}{A_t}\right), \tag{27}$$

where the Home productivity (TFP) shock is denoted by A_t. A similar expression holds for the Foreign country's (before-subsidy) nominal marginal cost. Productivity shocks are described with the following bivariate stochastic process:

$$A_t = (A)^{1-\delta_a} (A_{t-1})^{\delta_a} (A^*_{t-1})^{\delta_{a,a^*}} e^{\varepsilon^a_t}, \tag{28}$$

$$A^*_t = (A)^{1-\delta_a} (A_{t-1})^{\delta_{a,a^*}} (A^*_{t-1})^{\delta_a} e^{\varepsilon^{a*}_t}, \tag{29}$$

$$\begin{pmatrix} \varepsilon^a_t \\ \varepsilon^{a*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_a & \rho_{a,a^*}\sigma^2_a \\ \rho_{a,a^*}\sigma^2_a & \sigma^2_a \end{pmatrix}\right), \tag{30}$$

where A is the unconditional mean of the process (normalized to one). δ_a and δ_{a,a^*} capture the persistence and cross-country spillovers of the bivariate process which are assumed to be stationary. $(\varepsilon^a_t, \varepsilon^{a*}_t)^T$ is a vector of Gaussian innovations with a common variance $\sigma^2_a > 0$ and correlated across both countries $0 < \rho_{a,a^*} < 1$.

The optimal pricing rule of the re-optimizing firm h of the Home country at time t is given by,

$$\widetilde{P}_t(h) = \left(\frac{\theta}{\theta-1}(1-\phi)\right) \frac{\sum_{\tau=0}^{+\infty} (\alpha\beta)^\tau \mathbb{E}_t\left[\left(\frac{(C_{t+\tau})^{-\gamma}}{P_{t+\tau}}\right) \widetilde{Y}_{t,t+\tau}(h)\, MC_{t+\tau}\right]}{\sum_{\tau=0}^{+\infty} (\alpha\beta)^\tau \mathbb{E}_t\left[\left(\frac{(C_{t+\tau})^{-\gamma}}{P_{t+\tau}}\right) \widetilde{Y}_{t,t+\tau}(h)\right]}, \tag{31}$$

where ϕ is a time-invariant labor subsidy which is proportional to the nominal marginal cost $MC_{t+\tau}$. An analogous expression can be derived for the optimal pricing rule of the re-optimizing firm f in the Foreign country to pin down $\widetilde{P}_t(f)$.

Given the inherent symmetry of the Calvo-type pricing scheme, the price sub-indexes in both countries for the bundles of varieties produced locally, P^H_t and P^{F*}_t, respectively, evolve according to the following pair of laws of motion,

$$\left(P^H_t\right)^{1-\theta} = \alpha\left(P^H_{t-1}\right)^{1-\theta} + (1-\alpha)\left(\widetilde{P}_t(h)\right)^{1-\theta}, \tag{32}$$

$$\left(P^{F*}_t\right)^{1-\theta} = \alpha\left(P^{F*}_{t-1}\right)^{1-\theta} + (1-\alpha)\left(\widetilde{P}^*_t(f)\right)^{1-\theta}, \tag{33}$$

linking the current-period price sub-index to the previous-period price sub-index and to the symmetric pricing decision taken by all the re-optimizing firms during the current period. The LOOP then relates these price sub-indexes to P^{H*}_t and P^F_t with full pass-through of the bilateral nominal exchange rate S_t.

In order to characterize the allocation in the counterfactual case where nominal rigidities are removed and prices are fully flexible, we must replace the optimal pricing rule in (31) with the standard rule under perfect competition and flexible prices, i.e.,

$$\widetilde{P}_t(h) = MC_t, \tag{34}$$

for each firm h in the Home country at time t. Solving the model under this alternative price-setting rule defines the equilibrium allocation that would prevail in the frictionless environment subject to the same shocks. We refer to output and real interest rates in this frictionless counterfactual case as the economy's output potential and natural (real) interest rate, respectively.

A.3 Banking and the Monetary Policy Framework

We introduce a simplified banking system in the model whose unique function is to transform local households' savings into household liquidity via credit. We further assume that the banking system is perfectly competitive and we describe it with a representative bank in each country solely owned by the local representative household. The banking system can act as a financial lever for monetary policy. Hence, we need to be more explicit about the policy framework and about policy implementation whenever both money and credit markets are taken into account. We start describing the stylized balance sheets of the central bank and the banking system in the Home country to illustrate their linkages:

Central Bank	
Assets	Liabilities
B_t^m (Holdings of government bonds)	Z_t (Currency in circulation)
F_t (Loans to commercial banks)	U_t (Required and excess reserves)
	$= Z_t + U_t = MB_t$ (Monetary Base)
Banking System	
Assets	Liabilities
U_t (Required and excess reserves)	D_t (Deposits)
$\int_{\omega_{t+1}\in\Omega} Q_t(\omega_{t+1})\, B_t^b(\omega_{t+1})$ (Holdings of government bonds)	F_t (central bank loans)
L_t (Loans to households)	

An important simplification on these balance sheets is that we abstract from including the central bank's and the banking system's equity on the liability side. This is because we assume that the central bank has the full backing of the fiscal authority who is the sole owner of the central bank's equity. In turn, we can think of B_t^m (the central bank's holdings of government bonds) as being net of the central bank's equity. In regards to the banking system, we assume that households can save in the form of bank equity or via bank deposits, but that both forms of allocating their savings to banks are perfect substitutes whenever they offer the same rate of risk-free return next period i_t. Hence, for simplicity, we only consider the case of banks funded by households entirely through bank deposits.

We define the Home monetary base, MB_t, simply as the sum of currency in circulation, Z_t, and the amount of required and excess reserves held by the banking system on the central bank, U_t. The counterpart on the asset-side of the central bank's balance sheet are the holdings of government bonds, B_t^m, and the loans to commercial banks, F_t. Here follows the operational and regulatory framework under which central banks operate:

◦ First, the reserve requirement ratio and the return that the central bank pays on reserves are among the tools available for the conduct of monetary policy. We assume a policy framework where the return on reserves is set to zero (or at least strictly less than the risk-free rate i_t) in order to discourage the banking system from accumulating excess reserves on the central bank's balance sheet. In our setting, this implies that U_t is equal to the required reserves and, therefore, that excess reserves are zero in equilibrium. We define the required reserves as:

$$U_t = rD_t, \tag{35}$$

where $0 \leq r < 1$ is the reserve requirement ratio set by the policymakers. Although the reserve requirement ratio is aimed at broadly ensuring that banks retain sufficient liquidity available to safeguard their financial position while attending deposit withdrawals, in our simplified model it is simply interpreted as a regulatory-based constraint on the banks ability to transform deposits into credit loans for households. Under that interpretation it can also reflect other non-regulatory, but technological constraints, or iceberg costs, on what implicitly is a linear production function transforming deposits directly into credit that provides liquidity services for households.

◦ Second, adding or removing liquidity into the banking system, F_t, is another important balance sheet tool for the central bank to engage in. We assume a monetary policy framework whereby the policymaker makes it more punishing for banks to access central bank's loans than to fund themselves through deposits, i.e., we assume the return required on central bank loans is strictly higher than i_t. In this setting, banks rely entirely on deposits and use no central bank loans:

$$F_t = 0. \tag{36}$$

Furthermore, this also implies that the monetary base must be equal to the central bank's bond holdings, i.e., $MB_t = B_t^m$.

◦ Third, the policy framework in place also incorporates the full fiscal backing of the fiscal authority. Hence, the consolidated government budget constraint of the Home country tells us that:

$$T_t + \Delta MB_t + \int_{\omega_{t+1}\in\Omega} \Big(Q_t\left(\omega_{t+1}\right) B_t^{H}\left(\omega_{t+1}\right) + S_t Q_t^{*}\left(\omega_{t+1}\right) B_t^{H*}\left(\omega_{t+1}\right) \Big)$$
$$= P_t G_t + \phi W_t N_t + \Big(B_{t-1}^{H}\left(\omega_t\right) + S_t B_{t-1}^{H*}\left(\omega_t\right) \Big), \tag{37}$$

where T_t is the tax revenue or transfers, $\Delta B_t^m = \Delta MB_t = MB_t - MB_{t-1}$ is the seigniorage revenue from the central bank, $\int_{\omega_{t+1}\in\Omega} \left(Q_t\left(\omega_{t+1}\right) B_t^{H}\left(\omega_{t+1}\right) + S_t Q_t^{*}\left(\omega_{t+1}\right) B_t^{H*}\left(\omega_{t+1}\right) \right)$ is the nominal amount raised from selling government state-contingent, one-period debt owned by the public in both countries (Home and Foreign households), while $P_t G_t$ is government spending, $\phi W_t N_t$ is the labor subsidy provided by the Home government to reverse the monopolistic competition distortion in steady state, and $\left(B_{t-1}^{H}\left(\omega_t\right) + S_t B_{t-1}^{H*}\left(\omega_t\right) \right)$ is the re-payment to the public on the contingent bonds.

We assume that the government has no expenditures apart from those that arise from subsidizing labor, i.e., we assume:

$$G_t = 0. \tag{38}$$

We also recall that government contingent bonds are in zero net supply, i.e., market clearing implies that:

$$B_t^{H}\left(\omega_{t+1}\right) + S_{t+1} B_t^{H*}\left(\omega_{t+1}\right) = 0, \ \forall \omega_{t+1} \in \Omega. \tag{39}$$

In this context, the banking system opts to invest all its deposits (except those set aside as required reserves with the central bank) as long as the return on loans is higher than the risk-free rate that can be achieved with a portfolio of contingent government bonds, i.e., for any $\omega_{t+1} \in \Omega$ it must be that:

$$B_t^{b}\left(\omega_{t+1}\right) = 0 \text{ if } i_{L,t} > i_t. \tag{40}$$

This is an important aspect that influences the monetary policy transmission through the "banking" channel.

◦ Fourth and final, as long as bank loans achieve a rate of return $i_{L,t}$ higher than the risk-free rate i_t that can be accrued on a portfolio of government contingent bonds, the banking system chooses to allocate all its available deposits (except required reserves) on credit loans to households. This implies simply that:

$$L_t = (1 - r) D_t. \tag{41}$$

In this setting, the representative bank maximizes profits period-by-period since assets and liabilities have the same short maturity of one period. Under perfect competition, the banking system breaks-even (making no profits for their shareholders, the Home household) whenever it holds that:

$$i_{L,t} = \frac{1}{1-r} i_t. \tag{42}$$

Hence, the spread between the loan rate and the risk-free rate can be expressed as:

$$i_{L,t} - i_t = \left(\frac{r}{1-r}\right) i_t, \tag{43}$$

which shows that the spread on loans is positive and depends on the risk-free rate and the reserve requirement ratio $0 \leq r < 1$. We therefore note that the spreads are lower when the risk-free rate is low, but can also fall from adjustments in the reserve requirement ratio.

Monetary policy implementation. In terms of monetary aggregates, it is worth noting that the monetary base, MB_t, and the money supply, M_t, in equilibrium are given in the model by:

$$\begin{aligned} \text{Monetary base:} \qquad & MB_t = Z_t + U_t, \\ \text{Money supply:} \qquad & M_t = Z_t + D_t, \end{aligned}$$

where the distinction arises from the fact that money supply includes all deposits while the monetary base only the reserves. Using the implications of the banking system balance sheet in (41), we can re-write the definition of the money supply as,

$$M_t = Z_t + D_t = MB_t + D_t - U_t = MB_t + L_t. \tag{44}$$

In other words, the money supply is equal to a simple sum of the monetary base and the credit loans made by the banking system. Excluding bank reserves this would be a simple sum of the amounts of cash and credit available to provide liquidity services—however, as indicated before, such a simple sum is not a proper measure of liquidity unless cash and credit are perfect substitutes.

From the point of view of monetary policy, monetary aggregates can be a misleading measure of liquidity in the economy. Furthermore the central bank can influence the evolution of the monetary base and in turn the money supply by setting the currency (cash) in circulation, Z_t, and the reserve requirement ratio $0 \leq r < 1$. We view such monetary aggregates as intermediate targets for monetary policymaking, even though monetary policy is set not on quantities but on prices. In other words, monetary policy is set in terms of the nominal risk-free rate i_t. We take the regulatory and policy framework as fixed such that, in terms of monetary policy implementation, the central bank keeps r invariant and intervenes only through the money market accommodating an amount of currency (cash) Z_t sufficient to support the desired target for the short-term risk-free rate i_t. We describe in more detail the Taylor (1993)-type monetary policy rule setting the target for i_t shortly.

The policy framework ensures that the spread between the loan rate and the risk-free rate is proportional to the latter, as seen in (41). Therefore, this allows us to recover from the demand equations for real money balances and real credit balances ((9) and (11)) that,

$$\frac{L_t}{P_t} = \frac{\mu}{1-\mu}\left(\frac{1-r}{r}\right)^{v}\frac{Z_t}{P_t}, \tag{45}$$

$$\chi\left(\mu\right)^{\frac{1}{\nu}}\left(\frac{Z_t}{P_t}\right)^{-\zeta} = \left(\left(\mu\right)^{\frac{1}{\nu}}+\left(1-\mu\right)^{\frac{1}{\nu}}\left(\frac{\mu}{1-\mu}\right)^{\frac{\nu-1}{\nu}}\left(\frac{1-r}{r}\right)^{v-1}\right)^{\frac{\zeta-\frac{1}{v}}{1-\frac{1}{v}}}\left(C_t\right)^{-\gamma}\left(\frac{i_t}{1+i_t}\right), \tag{46}$$

which shows that in equilibrium the amount of real credit used is proportional to the real monetary balances. It also simplifies the demand for real money balances that, in this case, depends only on consumption, C_t, and on the risk-free rate, i_t.

From here, we can go a step further relating these equilibrium conditions to conventional monetary aggregates. Given the definition of the money supply in (44) and the equilibrium balance sheet of the banking system in (41), it follows that:

$$M_t = Z_t + D_t = Z_t + \frac{1}{1-r}L_t = \left[1+\frac{1}{1-r}\left(\frac{\mu}{1-\mu}\left(\frac{1-r}{r}\right)^{v}\right)\right]Z_t, \tag{47}$$

which indicates that the money supply is proportional to the currency in circulation, Z_t, set by the central bank. From here, we obtain that the money market and credit market equilibrium conditions can be re-written replacing Z_t with the money supply aggregate M_t as follows:

$$\left(C_t\right)^{-\gamma}\frac{i_t}{1+i_t} = \chi\left(\frac{\left(\mu\right)^{\frac{1}{\nu}}\left(1+\frac{1}{1-r}\left(\frac{\mu}{1-\mu}\left(\frac{1-r}{r}\right)^{v}\right)\right)^{\frac{1}{\nu}}}{\left(\left(\mu\right)^{\frac{1}{\nu}}+\left(1-\mu\right)^{\frac{1}{\nu}}\left(\frac{\mu}{1-\mu}\right)^{\frac{\nu-1}{\nu}}\left(\frac{1-r}{r}\right)^{v-1}\right)^{\frac{\zeta-\frac{1}{v}}{1-\frac{1}{v}}}}\right)\left(\frac{M_t}{P_t}\right)^{-\frac{1}{\nu}}, \tag{48}$$

$$\frac{L_t}{P_t} = \left(\frac{\frac{\mu}{1-\mu}\left(\frac{1-r}{r}\right)^{v}}{1+\frac{1}{1-r}\left(\frac{\mu}{1-\mu}\left(\frac{1-r}{r}\right)^{v}\right)}\right)\frac{M_t}{P_t}. \tag{49}$$

These two equilibrium conditions are crucial in our analysis because they pin down the equilibrium behavior of the credit and monetary aggregates which we observe in the data.

Monetary policy rule. We model monetary policy implementation via changes in Z_t and set the Home country's policy target according to a standard Taylor (1993)-type rule on the short-term nominal interest rate, i_t, i.e.,

$$\frac{1+i_t}{1+\bar{i}} = \frac{V_t}{V}\left[\left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\psi_{\pi}}\left(\frac{Y_t}{\overline{Y}_t}\right)^{\psi_x}\right], \tag{50}$$

where $\bar{i} \equiv \beta^{-1}$ denotes the nominal (and real) interest rate in the steady state while $\psi_\pi > 0$ and $\psi_x \geq 0$ are the policy parameters that capture the sensitivity of the monetary policy rule to changes in inflation and the output gap, respectively. $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the (gross) CPI inflation rate, $\overline{\Pi} = 1$ is the deterministic steady state inflation rate, Y_t defines the aggregate output produced in the Home country, and $\frac{Y_t}{\overline{Y}_t}$ is the output gap in levels. Here, $\overline{Y}_t$ defines the potential output level of the Home country and $\bar{r}_t$ is the natural (real) rate of interest.

The monetary policy shock in the Home country is defined as V_t. Monetary shocks are described with the following bivariate stochastic process:

$$
\begin{aligned}
V_t &= (V)^{1-\delta_m} (V_{t-1})^{\delta_m} e^{\varepsilon_t^m}, && (51)\\
V_t^* &= (V)^{1-\delta_m} (V_{t-1}^*)^{\delta_m} e^{\varepsilon_t^{m*}}, && (52)\\
\begin{pmatrix} \varepsilon_t^m \\ \varepsilon_t^{m*} \end{pmatrix} &\sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{pmatrix} \right), && (53)
\end{aligned}
$$

where V is the unconditional mean of the process, δ_m captures the persistence, and $\left(\varepsilon_t^m, \varepsilon_t^{m*}\right)^T$ is a vector of Gaussian innovations with a common variance σ_m^2 and possibly correlated across both countries ρ_{m,m^*}.

Optimal fiscal policy subsidy. Monopolistic competition in production and labor introduces a distortionary steady-state price mark-up, $\frac{\theta}{\theta-1}$, that drives a wedge between prices and marginal costs. This steady-state distortion is a function of the elasticity of substitution across output varieties within a country $\theta > 1$. Home and Foreign governments raise lump-sum taxes from local households within their borders in order to subsidize labor employment and eliminate the steady-state price mark-up distortions. An optimal (time-invariant) labor subsidy proportional to the marginal cost set to be $\phi = \frac{1}{\theta}$ in every country neutralizes the steady-state monopolistic competition mark-up in the pricing rule (that is, in the steady state of equation (31)).

B Log-Linearized Equilibrium Conditions

The log-linearized equilibrium conditions of the open-economy New Keynesian model with liquidity are summarized in Table A1. The derivation of this system of equations is fairly standard and can be found in Martínez-García (2019a). The key difference can be found in the equilibrium conditions that pin down real money and real credit. Those are obtained after a straightforward log-linearization of equations (48) and (49) where $\nu > 0$ denotes the elasticity of substitution between real money balances and real credit and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution on consumption.

Table A1 - Open-Economy New Keynesian Model with Money and Credit	
	Home Economy
Phillips curve	$\widehat{\pi}_t \approx \beta \mathbb{E}_t\left(\widehat{\pi}_{t+1}\right) + \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)\left[\left((1-\xi)\varphi + \Theta\gamma\right)\widehat{x}_t + \left(\xi\varphi + (1-\Theta)\gamma\right)\widehat{x}^*_t\right]$
Output gap	$\gamma(1-2\xi)\left(\mathbb{E}_t\left[\widehat{x}_{t+1}\right] - \widehat{x}_t\right) \approx \left((1-2\xi)+\Gamma\right)\left[\widehat{r}_t - \widehat{\overline{r}}_t\right] - \Gamma\left[\widehat{r}^*_t - \widehat{\overline{r}}^*_t\right]$
Monetary policy rule	$\widehat{i}_t \approx \psi_\pi \widehat{\pi}_t + \psi_x \widehat{x}_t + \widehat{v}_t$
Fisher equation	$\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$
Output	$\widehat{y}_t = \widehat{\overline{y}}_t + \widehat{x}_t$
Consumption	$\widehat{c}_t \approx \Theta\widehat{y}_t + (1-\Theta)\widehat{y}^*_t$
Real money/credit balances	$\widehat{m}_t - \widehat{p}_t \approx \gamma\nu\widehat{c}_t - \nu\widehat{i}_t,\ \widehat{l}_t - \widehat{p}_t \approx \widehat{m}_t - \widehat{p}_t$
Natural interest rate	$\widehat{\overline{r}}_t \approx \gamma\left[\Theta\left(\mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}\right] - \widehat{\overline{y}}_t\right) + (1-\Theta)\left(\mathbb{E}_t\left[\widehat{\overline{y}}^*_{t+1}\right] - \widehat{\overline{y}}^*_t\right)\right]$
Potential output	$\widehat{\overline{y}}_t \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left[\Lambda\widehat{a}_t + (1-\Lambda)\widehat{a}^*_t\right]$
	Foreign Economy
Phillips curve	$\widehat{\pi}^*_t \approx \beta \mathbb{E}_t\left(\widehat{\pi}^*_{t+1}\right) + \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)\left[\left(\xi\varphi + (1-\Theta)\gamma\right)\widehat{x}_t + \left((1-\xi)\varphi + \Theta\gamma\right)\widehat{x}^*_t\right]$
Output gap	$\gamma(1-2\xi)\left(\mathbb{E}_t\left[\widehat{x}^*_{t+1}\right] - \widehat{x}^*_t\right) \approx -\Gamma\left[\widehat{r}_t - \widehat{\overline{r}}_t\right] + \left((1-2\xi)+\Gamma\right)\left[\widehat{r}^*_t - \widehat{\overline{r}}^*_t\right]$
Monetary policy	$\widehat{i}^*_t \approx \psi_\pi \widehat{\pi}^*_t + \psi_x \widehat{x}^*_t + \widehat{v}^*_t$
Fisher equation	$\widehat{r}^*_t \equiv \widehat{i}^*_t - \mathbb{E}_t\left[\widehat{\pi}^*_{t+1}\right]$
Output	$\widehat{y}^*_t = \widehat{\overline{y}}^*_t + \widehat{x}^*_t$
Consumption	$\widehat{c}^*_t \approx (1-\Theta)\widehat{y}_t + \Theta\widehat{y}^*_t$
Real money/credit balances	$\widehat{m}^*_t - \widehat{p}^*_t \approx \gamma\nu\widehat{c}^*_t - \nu\widehat{i}^*_t,\ \widehat{l}^*_t - \widehat{p}^*_t \approx \widehat{m}^*_t - \widehat{p}^*_t$
Natural interest rate	$\widehat{\overline{r}}^*_t \approx \gamma\left[(1-\Theta)\left(\mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}\right] - \widehat{\overline{y}}_t\right) + \Theta\left(\mathbb{E}_t\left[\widehat{\overline{y}}^*_{t+1}\right] - \widehat{\overline{y}}^*_t\right)\right]$
Potential output	$\widehat{\overline{y}}^*_t \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left[(1-\Lambda)\widehat{a}_t + \Lambda\widehat{a}^*_t\right]$
	Exogenous, Country-Specific Shocks
Productivity shock	$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}^*_t \end{pmatrix} \approx \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}^a_t \\ \widehat{\varepsilon}^{a*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_a & \rho_{a,a^*}\sigma^2_a \\ \rho_{a,a^*}\sigma^2_a & \sigma^2_a \end{pmatrix}\right)$
Monetary shock	$\begin{pmatrix} \widehat{v}_t \\ \widehat{v}^*_t \end{pmatrix} \approx \begin{pmatrix} \delta_v & 0 \\ 0 & \delta_v \end{pmatrix} \begin{pmatrix} \widehat{v}_{t-1} \\ \widehat{v}^*_{t-1} \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}^v_t \\ \widehat{\varepsilon}^{v*}_t \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}^v_t \\ \widehat{\varepsilon}^{v*}_t \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_v & \rho_{v,v^*}\sigma^2_v \\ \rho_{v,v^*}\sigma^2_v & \sigma^2_v \end{pmatrix}\right)$
	Composite Parameters
	$\Theta \equiv (1-\xi)\left[\frac{\sigma\gamma - (\sigma\gamma-1)(1-2\xi)}{\sigma\gamma - (\sigma\gamma-1)(1-2\xi)^2}\right]$
	$\Lambda \equiv 1 + (\sigma\gamma-1)\left[\frac{\gamma\xi 2(1-\xi)}{\varphi\left(\sigma\gamma - (\sigma\gamma-1)(1-2\xi)^2\right)+\gamma}\right]$
	$\Gamma \equiv \xi\left[\sigma\gamma + (\sigma\gamma-1)(1-2\xi)\right]$

C Main Derivations

We use the decomposition method advocated most recently by Martínez-García (2019a,b) to re-express the linear rational expectations system of equations that characterizes the log-linearized equilibrium conditions of the workhorse open-economy New Keynesian model into two separate (and smaller) sub-systems for aggregates and differences. Hence, we define the world aggregate and the difference variables $\widehat{g}_t^W$ and $\widehat{g}_t^R$ as:

$$\begin{aligned} \widehat{g}_t^W &\equiv \frac{1}{2}\widehat{g}_t + \frac{1}{2}\widehat{g}_t^*, \qquad (54) \\ \widehat{g}_t^R &\equiv \widehat{g}_t - \widehat{g}_t^*, \qquad (55) \end{aligned}$$

which implicitly takes into account that both countries are identical in size with the same share of the household population and varieties located in each country. We re-write the country variables $\widehat{g}_t$ and $\widehat{g}_t^*$ as:

$$\begin{aligned} \widehat{g}_t &= \widehat{g}_t^W + \frac{1}{2}\widehat{g}_t^R, \qquad (56) \\ \widehat{g}_t^* &= \widehat{g}_t^W - \frac{1}{2}\widehat{g}_t^R. \qquad (57) \end{aligned}$$

If we characterize the dynamics for $\widehat{g}_t^W$ and $\widehat{g}_t^R$, the transformation above backs out the corresponding variables for each country, $\widehat{g}_t$ and $\widehat{g}_t^*$. These transformations can be applied to any of the endogenous and exogenous variables in the model. Then, under this transformation, we can orthogonalize our model into one aggregate (or world) economic system and one difference system that can be studied independently.

Let us also define the vector of structural preference and policy parameters $\vartheta \equiv (\gamma, \varphi, \nu, \alpha, \beta, \xi, \sigma, \psi_\pi, \psi_x)^T$.

C.1 The World System

The world economy New Keynesian model is described with a New Keynesian Phillips curve (NKPC), a log-linearized world Euler equation, and an interest-rate-setting rule for monetary policy. The NKPC can be cast into the following augmented form,

$$\widehat{\pi}_t^W \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^W\right) + k^W \widehat{x}_t^W, \qquad (58)$$

where $\mathbb{E}_t(\cdot)$ refers to the expectation formed conditional on information up to time t, $\widehat{x}_t^W$ is the global output gap, and $\widehat{\pi}_t^W$ is global inflation. Moreover, $k^W \equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)(\varphi+\gamma) > 0$ is the slope of the global output gap that depends on deep structural parameters such as the frequency of price adjustment $0 < \alpha < 1$, the intertemporal discount rate $0 < \beta < 1$, the inverse of the intertemporal elasticity of substitution on consumption $\gamma > 0$, and the inverse of the Frisch elasticity of labor supply $\varphi > 0$.

The log-linearization of the Euler equation is given by,

$$\widehat{x}_t^W \approx \mathbb{E}_t\left[\widehat{x}_{t+1}^W\right] - \frac{1}{\gamma}\left(\widehat{i}_t^W - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^W\right] - \widehat{\overline{r}}_t^W\right), \tag{59}$$

where $\widehat{i}_t^W$ is the aggregate short-term nominal interest rate (an aggregate of the risk-free one-period interest rates of both countries), and $\widehat{\overline{r}}_t^W$ is the aggregate natural interest rate. Potential output and the natural (real) interest rate are both functions of exogenous productivity shocks such that:

$$\widehat{\overline{r}}_t^W \approx \gamma\left[\mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}^W\right] - \widehat{\overline{y}}_t^W\right], \tag{60}$$

$$\widehat{\overline{y}}_t^W \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right)\widehat{a}_t^W. \tag{61}$$

We specify a general form of the monetary policy with a Taylor (1993) rule where the central bank of each country targets its domestic short-term nominal interest rate with the same reaction function. The world Taylor (1993) rule can be cast in the following form,

$$\widehat{i}_t^W \approx \psi_\pi \widehat{\pi}_t^W + \psi_x \widehat{x}_t^W + \widehat{v}_t^W, \tag{62}$$

where $\widehat{v}_t^W$ is the aggregate monetary policy shock.

Using the aggregate monetary policy rule in (62) to replace $\widehat{i}_t^W$ in $(58) - (59)$, the system of equations that determines world inflation and global slack can be written in the following form:

$$\widehat{z}_t^W = A^W(\vartheta)\,\mathbb{E}_t\left(\widehat{z}_{t+1}^W\right) + a^W(\vartheta)\left(\widehat{\overline{r}}_t^W - \widehat{v}_t^W\right), \tag{63}$$

where,

$$\widehat{z}_t^W \equiv \begin{bmatrix} \widehat{\pi}_t^W \\ \widehat{x}_t^W \end{bmatrix}, \tag{64}$$

and $A^W(\vartheta)$ is a 2×2 composite matrix while $a^W(\vartheta)$ is a 2×1 composite matrix of the structural parameters in ϑ.[4] Under the assumption that the aggregate interest rate gap $\left(\widehat{\overline{r}}_t^W - \widehat{v}_t^W\right)$ is stationary, then the system in (63) has a unique nonexplosive solution in which both $\widehat{x}_t^W$ and $\widehat{\pi}_t^W$ are stationary whenever both eigenvalues of the matrix $A^W(\vartheta)$ are inside the unit circle. A variant of the Taylor principle which requires that $\psi_\pi + \left(\frac{1-\beta}{k^W}\right)\psi_x > 1$ suffices to ensure the uniqueness and existence of the nonexplosive solution for the world aggregates. Assuming this condition is

[4]Notice that neither the share of imported goods in the consumption basket of each country given by ξ nor the trade elasticity σ included in the vector of structural parameters ϑ appear in the composite coefficients for the world system $A^W(\vartheta)$ and $a^W(\vartheta)$.

satisfied, the solution can be characterized as follows,

$$\begin{pmatrix} \widehat{\pi}_t^W \\ \widehat{x}_t^W \end{pmatrix} = \sum_{j=0}^{\infty} \left(A^W(\vartheta)\right)^j a^W(\vartheta)\, \mathbb{E}_t \left(\widehat{\overline{r}}_{t+j}^W - \widehat{v}_{t+j}^W\right). \tag{65}$$

We assume that central banks adjust their policy rule to track changes in the natural rate of interest that are forecastable one period in advance implying for the aggregate that,

$$\widehat{v}_t^W = \mathbb{E}_{t-1}\left(\widehat{\overline{r}}_t^W\right). \tag{66}$$

Hence, world inflation in (65) is determined by current and expected future discrepancies between the aggregate natural rate of interest and the aggregate of the central bank's own one-period ahead expectations about the natural rate of interest. Alternatively, we can simply assume—as most of the literature implicitly does—that $\widehat{v}_t^W = \widehat{\overline{r}}_t^W + \widehat{\varepsilon}_t^m$, where $\widehat{\overline{r}}_t^W$ corresponds to the global natural interest rate and $\widehat{\varepsilon}_t^m$ is an i.i.d. disturbance that captures non-persistent and unanticipated shocks to monetary policy. Either way, the world interest rate gap $\left(\widehat{\overline{r}}_t^W - \widehat{v}_t^W\right)$ is viewed as white noise and the solution to the global system in (65) becomes,

$$\widehat{\pi}_t^W = \lambda^W(\vartheta)\left(\widehat{\overline{r}}_t^W - \widehat{v}_t^W\right) = -\lambda^W(\vartheta)\,\widehat{\varepsilon}_t^m, \tag{67}$$

$$\widehat{x}_t^W = \mu^W(\vartheta)\left(\widehat{\overline{r}}_t^W - \widehat{v}_t^W\right) = -\mu^W(\vartheta)\,\widehat{\varepsilon}_t^m, \tag{68}$$

where the composite coefficients $\lambda^W(\vartheta)$ and $\mu^W(\vartheta)$ naturally depend on the deep structural parameters of the model in ϑ.

If aggregate inflation evolves in this way, then optimal forecasts of expected changes in global inflation at any horizon $j \geq 1$ must be given by,

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j}^W - \widehat{\pi}_t^W\right) = -\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{x}_t^W. \tag{69}$$

This implies that no other variable should improve our forecast of changes in global inflation if global slack is already included in the forecasting model. Regressors that are stationary and highly correlated with cyclical inflation are all that is needed to forecast inflation given the current period. In theory, the global output gap is one such predictor. However, for forecasting what matters is not slack *per se* but whether the observable variables that we use as predictors have information content that is useful for tracking cyclical variations in inflation. In this sense, we find that global money balances and global credit can be useful for inflation forecasting.

Proposition 1 *For any given price level path in the frictionless equilibrium* $\widehat{\overline{p}}_t^W$*, the world real money gap*

$\widehat{m}_t^{r,W} \equiv \left(\widehat{m}_t^W - \widehat{p}_t^W\right) - \left(\widehat{\overline{m}}_t^W - \widehat{\overline{p}}_t^W\right)$ is an affine transformation of global slack $\widehat{x}_t^W$ given by,

$$\widehat{m}_t^{r,W} \approx \chi\left(\vartheta\right)\widehat{x}_t^W + \nu\psi_\pi\widehat{\overline{\pi}}_t^W, \tag{70}$$

where the composite coefficient is given by $\chi\left(\vartheta\right) \equiv \left(1 - \eta\left(\psi_\pi\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)} + \psi_x\right)\right)$. Inflation in the frictionless case is defined as $\widehat{\overline{\pi}}_t^W \equiv \widehat{\overline{p}}_t^W - \widehat{\overline{p}}_{t-1}^W$.

If monetary policy in the frictionless equilibrium is set to track the observed price level in the economy (i.e., if $\widehat{p}_t^W = \widehat{\overline{p}}_t^W$), the world nominal money gap $\widehat{m}_t^{n,W} \equiv \widehat{m}_t^W - \widehat{\overline{m}}_t^W$ is proportional to global slack $\widehat{x}_t^W$ and can be expressed as,

$$\widehat{m}_t^{n,W} \equiv \widehat{m}_t^W - \widehat{\overline{m}}_t^W \approx \gamma\nu\left(1 - \frac{1}{\gamma}\psi_x\right)\widehat{x}_t^W. \tag{71}$$

Similarly, given that the world real credit gap $\widehat{l}_t^{r,W} \equiv \left(\widehat{l}_t^W - \widehat{p}_t^W\right) - \left(\widehat{\overline{l}}_t^W - \widehat{\overline{p}}_t^W\right)$ equates the real world money gap $\widehat{m}_t^{r,W}$ and that the world nominal credit gap $\widehat{l}_t^{n,W} \equiv \left(\widehat{l}_t^W - \widehat{\overline{l}}_t^W\right)$ equates the nominal world money gap $\widehat{m}_t^{n,W}$, we can establish the same linkages between credit and slack as well.

Proof. The aggregate money balance equations in log-linear form derived from those in Table A1 can be expressed as follows,

$$\widehat{m}_t^W - \widehat{p}_t^W \approx \gamma\nu\widehat{c}_t^W - \nu\widehat{i}_t^W, \tag{72}$$

where aggregate world consumption is given by $\widehat{c}_t^W \approx \widehat{y}_t^W$. Then, we know that the aggregate Taylor (1993) rule that sets monetary policy in the frictionless equilibrium implies the following path for the frictionless nominal short-term interest rate,

$$\widehat{\overline{i}}_t^W \approx \psi_\pi\widehat{\overline{\pi}}_t^W + \widehat{v}_t^W, \tag{73}$$

and the following path for the frictionless money equation,

$$\widehat{\overline{m}}_t^W - \widehat{\overline{p}}_t^W \approx \gamma\nu\widehat{\overline{y}}_t^W - \nu\widehat{\overline{i}}_t^W \approx \gamma\nu\widehat{\overline{y}}_t^W - \nu\psi_\pi\widehat{\overline{\pi}}_t^W - \nu\widehat{v}_t^W, \tag{74}$$

which can accommodate any given inflation path $\widehat{\overline{\pi}}_t^W$ in an environment where obviously there is no slack. Combining this with the aggregate Taylor (1993) rule followed in the observed economy, we can write the difference $\left(\widehat{i}_t^W - \widehat{\overline{i}}_t^W\right)$ as follows:

$$\left(\widehat{i}_t^W - \widehat{\overline{i}}_t^W\right) \approx \psi_\pi\left(\widehat{\pi}_t^W - \widehat{\overline{\pi}}_t^W\right) + \psi_x\widehat{x}_t^W. \tag{75}$$

Hence, when we combine the aggregate money equation in (72) with the one absent nominal rigidities given by (74), it follows that,

$$\begin{aligned}
\widehat{m}_t^{r,W} &\equiv \left(\widehat{m}_t^W - \widehat{p}_t^W\right) - \left(\widehat{\overline{m}}_t^W - \widehat{\overline{p}}_t^W\right) \approx \gamma\nu\left(\widehat{c}_t^W - \widehat{\overline{c}}_t^W\right) - \nu\left(\widehat{i}_t^W - \widehat{\overline{i}}_t^W\right) \\
&\approx \gamma\nu\left(\widehat{y}_t^W - \widehat{\overline{y}}_t^W\right) - \nu\left(\widehat{i}_t^W - \widehat{\overline{i}}_t^W\right) \\
&\approx \gamma\nu\left(1 - \frac{1}{\gamma}\psi_x\right)\widehat{x}_t^W - \nu\psi_\pi\left(\widehat{\pi}_t^W - \widehat{\overline{\pi}}_t^W\right). \qquad (76)
\end{aligned}$$

Moreover, given that (67) − (68) imply $\widehat{\pi}_t^W = \frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{x}_t^W$, we can express the real money equation simply as:

$$\widehat{m}_t^{r,W} \approx \gamma\nu\left(1 - \frac{1}{\gamma}\left(\psi_\pi\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)} + \psi_x\right)\right)\widehat{x}_t^W + \nu\psi_\pi\widehat{\overline{\pi}}_t^W, \qquad (77)$$

and the nominal money equation as:

$$\widehat{m}_t^{n,W} \equiv \widehat{m}_t^W - \widehat{\overline{m}}_t^W \approx \gamma\nu\left(1 - \frac{1}{\gamma}\left(\psi_\pi\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)} + \psi_x\right)\right)\widehat{x}_t^W + \nu\psi_\pi\widehat{\overline{\pi}}_t^W + \left(\widehat{p}_t^W - \widehat{\overline{p}}_t^W\right). \qquad (78)$$

If monetary policy in the frictionless equilibrium is set to accommodate the same price level path observed in the economy, then it follows that (76) reduces to:

$$\widehat{m}_t^{n,W} \equiv \widehat{m}_t^W - \widehat{\overline{m}}_t^W \approx \gamma\nu\left(1 - \frac{1}{\gamma}\psi_x\right)\widehat{x}_t^W, \qquad (79)$$

if $\widehat{p}_t^W = \widehat{\overline{p}}_t^W$. An analogous result can be derived using the related aggregate credit equations. ■

C.2 The Cross-Country Difference System

The difference economy is described with a New Keynesian Phillips curve (NKPC), a log-linearized Euler equation, and an interest-rate-setting rule for monetary policy. The NKPC of the difference economy can be cast into the following augmented form,

$$\widehat{\pi}_t^R \approx \beta\mathbb{E}_t\left(\widehat{\pi}_{t+1}^R\right) + k^R\widehat{x}_t^R, \qquad (80)$$

where $\mathbb{E}_t(\cdot)$ refers to the expectation formed conditional on information up to time t, $\widehat{x}_t^R$ is the difference in the current output gap between the two countries, and $\widehat{\pi}_t^R$ is the cross-country difference in inflation. Moreover, $k^R \equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)((1-2\xi)\varphi + (2\Theta-1)\gamma)$ is the slope of the difference output gap that depends on the deep structural parameters of the model such as the frequency of price adjustment $0 < \alpha < 1$, the intertemporal discount rate $0 < \beta < 1$, the inverse of the intertemporal elasticity of substitution on consumption $\gamma > 0$, the inverse of the Frisch elasticity of labor supply $\varphi > 0$, the share of imported goods in the consumption basket of each country

$0 \leq \xi \leq \frac{1}{2}$, and the elasticity of substitution between the consumption bundle of Home-produced and Foreign-produced goods $\sigma > 0$.

The log-linearization of the (difference) Euler equation is given by,

$$\widehat{x}_t^R \approx \mathbb{E}_t \left[\widehat{x}_{t+1}^R \right] - \frac{1}{\gamma} \left(\frac{(1-2\xi)+2\Gamma}{1-2\xi} \right) \left(\widehat{i}_t^R - \mathbb{E}_t \left[\widehat{\pi}_{t+1}^R \right] - \widehat{\overline{r}}_t^R \right), \tag{81}$$

where $\widehat{i}_t^R$ is the difference in the short-term nominal interest rate (the difference between the risk-free one-period interest rates of each country), and $\widehat{\overline{r}}_t^R$ is the difference natural interest rate. Potential output and the natural (real) interest rate are both functions of exogenous productivity shocks such that:

$$\widehat{\overline{r}}_t^R \approx \gamma \left(2\Theta - 1\right) \left(\mathbb{E}_t \left[\widehat{\overline{y}}_{t+1}^R \right] - \widehat{\overline{y}}_t^R \right), \tag{82}$$

$$\widehat{\overline{y}}_t^R \approx \left(\frac{1+\varphi}{\gamma+\varphi} \right) \left(2\Lambda - 1\right) \widehat{a}_t^R. \tag{83}$$

The difference Taylor rule can be cast in the following form,

$$\widehat{i}_t^R \approx \psi_\pi \widehat{\pi}_t^R + \psi_x \widehat{x}_t^R + \widehat{v}_t^R, \tag{84}$$

where $\widehat{v}_t^R$ is the difference between both countries' monetary policy shocks.

Using the differential monetary policy rule in (84) to replace $\widehat{i}_t^R$ in $(80)-(81)$, the system of equations that determines the inflation differential and slack differential can be written in the following form,

$$\widehat{z}_t^R = A^R\left(\vartheta\right) \mathbb{E}_t \left(\widehat{z}_{t+1}^R \right) + a^R\left(\vartheta\right) \left(\widehat{\overline{r}}_t^R - \widehat{v}_t^R \right), \tag{85}$$

where,

$$\widehat{z}_t^R \equiv \begin{bmatrix} \widehat{\pi}_t^R \\ \widehat{x}_t^R \end{bmatrix}, \tag{86}$$

where $A^R\left(\vartheta\right)$ is a 2×2 composite matrix and $a^R\left(\vartheta\right)$ is a 2×1 composite matrix of structural parameters in ϑ. Under the assumption that the interest rate gap differential $\left(\widehat{\overline{r}}_t^R - \widehat{v}_t^R \right)$ is stationary, then the system in (85) has a unique nonexplosive solution in which both $\widehat{x}_t^R$ and $\widehat{\pi}_t^R$ are stationary whenever both eigenvalues of the matrix $A^R\left(\vartheta\right)$ are inside the unit circle. A variant of the Taylor principle which requires that $\psi_\pi + \left(\frac{1-\beta}{k^R} \right) \psi_x > 1$ suffices to ensure the uniqueness and existence of the nonexplosive solution for the differential aggregates. Assuming this condition is satisfied, the solution can be characterized as follows,

$$\begin{pmatrix} \widehat{\pi}_t^R \\ \widehat{x}_t^R \end{pmatrix} = \sum_{j=0}^{\infty} \left(A^R\left(\vartheta\right) \right)^j a^R\left(\vartheta\right) \mathbb{E}_t \left(\widehat{\overline{r}}_{t+j}^R - \widehat{v}_{t+j}^R \right). \tag{87}$$

We assume that the central banks adjust their policy rule to track changes in the natural rate of interest that are forecastable one period in advance implying for the differential that,

$$\widehat{v}_t^R = \mathbb{E}_{t-1}\left(\widehat{\overline{r}}_t^R\right). \tag{88}$$

Alternatively, we can simply assume—as most of the literature implicitly does—that $\widehat{v}_t^R = \widehat{\overline{r}}_t^R + \widehat{\varepsilon}_t^{m*}$, where $\widehat{\overline{r}}_t^R$ corresponds to the natural interest rate differential and $\widehat{\varepsilon}_t^{m*}$ is an i.i.d. disturbance that captures non-persistent and unanticipated shocks to monetary policy. Either way, the interest rate differential gap $\left(\widehat{\overline{r}}_t^R - \widehat{v}_t^R\right)$ is viewed as white noise and the solution to the differential system in (85) becomes,

$$\widehat{\pi}_t^R = \lambda^R(\vartheta)\left(\widehat{\overline{r}}_t^R - \widehat{v}_t^R\right) = -\lambda^R(\vartheta)\,\widehat{\varepsilon}_t^{v*}, \tag{89}$$

$$\widehat{x}_t^R = \mu^R(\vartheta)\left(\widehat{\overline{r}}_t^R - \widehat{v}_t^R\right) = -\mu^R(\vartheta)\,\widehat{\varepsilon}_t^{v*}, \tag{90}$$

where the composite coefficients $\lambda^R(\vartheta)$ and $\mu^R(\vartheta)$ depend on the deep structural parameters of the model in ϑ.

If the inflation differential evolves in this way, then optimal forecasts of future differential inflation at any horizon $j \geq 1$ must be given by,

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j}^R - \widehat{\pi}_t^R\right) = -\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{x}_t^R. \tag{91}$$

This implies that no other variable should improve our forecast of changes in differential inflation if differential slack is already included in the forecasting model. What we need to forecast future differential inflation, apart from current differential inflation, is additional regressors that are stationary and highly correlated with cyclical inflation. In theory, the differential output gap is one such predictor. However, for forecasting what matters is not slack *per se* but whether the observable variables that we use as predictors have information content that is useful for tracking cyclical variations in inflation. In this sense, we find that differential money balances and differential credit can be useful for differential inflation forecasting.

Proposition 2 *For any given price level path in the frictionless equilibrium $\widehat{\overline{p}}_t^R$, the differential real money gap $\widehat{m}_t^{r,R} \equiv \left(\widehat{m}_t^R - \widehat{p}_t^R\right) - \left(\widehat{\overline{m}}_t^R - \widehat{\overline{p}}_t^R\right)$ is an affine transformation of differential slack $\widehat{x}_t^R$ given by,*

$$\widehat{m}_t^{r,R} \approx \chi(\vartheta)\,\widehat{x}_t^R + \nu\psi_\pi\widehat{\overline{\pi}}_t^R, \tag{92a}$$

where the composite coefficient is given by $\chi(\vartheta) \equiv \left(1 - \eta\left(\psi_\pi\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)} + \psi_x\right)\right)$. Inflation in the frictionless case is defined as $\widehat{\overline{\pi}}_t^R \equiv \widehat{\overline{p}}_t^R - \widehat{\overline{p}}_{t-1}^R$.

If monetary policy in the frictionless equilibrium is set to track the observed price level in the economy *(i.e., if $\widehat{p}_t^R = \widehat{\overline{p}}_t^R$), the differential nominal money gap $\widehat{m}_t^{n,R} \equiv \widehat{m}_t^R - \widehat{\overline{m}}_t^R$ is proportional to differential slack $\widehat{x}_t^R$ and can be expressed as,*

$$\widehat{m}_t^{n,R} \equiv \widehat{m}_t^R - \widehat{\overline{m}}_t^R \approx \gamma\nu\left(1 - \frac{1}{\gamma}\psi_x\right)\widehat{x}_t^R. \tag{93}$$

Similarly, given that the differential real credit gap $\widehat{l}_t^{r,R} \equiv \left(\widehat{l}_t^R - \widehat{p}_t^R\right) - \left(\widehat{\overline{l}}_t^R - \widehat{\overline{p}}_t^R\right)$ equates the real differential money gap $\widehat{m}_t^{r,R}$ and that the differential nominal credit gap $\widehat{l}_t^{n,R} \equiv \left(\widehat{l}_t^R - \widehat{\overline{l}}_t^R\right)$ equates the nominal differential money gap $\widehat{m}_t^{n,R}$, we can establish the same linkages between credit and slack as well.

Proof. The differential money balance equations in log-linear form derived from those in Table A1 can be expressed as follows,

$$\widehat{m}_t^R - \widehat{p}_t^R \approx \gamma\nu\widehat{c}_t^R - \nu\widehat{i}_t^R, \tag{94}$$

where aggregate world consumption is given by $\widehat{c}_t^R \approx (2\Theta - 1)\,\widehat{y}_t^R$. Then, we know that the differential Taylor (1993) rule that sets monetary policy in the frictionless equilibrium implies the following path for the frictionless nominal short-term interest rate,

$$\widehat{\overline{i}}_t^R \approx \psi_\pi \widehat{\overline{\pi}}_t^R + \widehat{v}_t^R, \tag{95}$$

and the following path for the frictionless money equation,

$$\widehat{\overline{m}}_t^R - \widehat{\overline{p}}_t^R \approx \gamma\nu\widehat{\overline{y}}_t^R - \nu\widehat{\overline{i}}_t^R \approx \gamma\nu\widehat{\overline{y}}_t^R - \nu\psi_\pi\widehat{\overline{\pi}}_t^R - \nu\widehat{v}_t^R, \tag{96}$$

which can accommodate any given inflation path $\widehat{\overline{\pi}}_t^R$ in an environment where obviously there is no slack. Combining this with the differential Taylor (1993) rule followed in the observed economy, we can write the difference $\left(\widehat{i}_t^R - \widehat{\overline{i}}_t^R\right)$ as follows:

$$\left(\widehat{i}_t^R - \widehat{\overline{i}}_t^R\right) \approx \psi_\pi\left(\widehat{\pi}_t^R - \widehat{\overline{\pi}}_t^R\right) + \psi_x\widehat{x}_t^R. \tag{97}$$

Hence, when we combine the differential money equation in (94) with the one absent nominal rigidities given by (96), it follows that,

$$\begin{aligned}
\widehat{m}_t^{r,R} &\equiv \left(\widehat{m}_t^R - \widehat{p}_t^R\right) - \left(\widehat{\overline{m}}_t^R - \widehat{\overline{p}}_t^R\right) \approx \gamma\nu\left(\widehat{c}_t^R - \widehat{\overline{c}}_t^R\right) - \nu\left(\widehat{i}_t^R - \widehat{\overline{i}}_t^R\right) \\
&\approx \gamma\nu\left(\widehat{y}_t^R - \widehat{\overline{y}}_t^R\right) - \nu\left(\widehat{i}_t^R - \widehat{\overline{i}}_t^R\right) \\
&\approx \gamma\nu\left(1 - \frac{1}{\gamma}\psi_x\right)\widehat{x}_t^R - \nu\psi_\pi\left(\widehat{\pi}_t^R - \widehat{\overline{\pi}}_t^R\right).
\end{aligned} \tag{98}$$

Moreover, given that (89) − (90) imply $\widehat{\pi}_t^R = \frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{x}_t^R$, we can express the real money equation simply as:

$$\widehat{m}_t^{r,R} \approx \gamma\nu\left(1 - \frac{1}{\gamma}\left(\psi_\pi \frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)} + \psi_x\right)\right)\widehat{x}_t^R + \nu\psi_\pi \widehat{\overline{\pi}}_t^R, \tag{99}$$

and the nominal money equation as:

$$\widehat{m}_t^{n,R} \equiv \widehat{m}_t^R - \widehat{\overline{m}}_t^R \approx \gamma\nu\left(1 - \frac{1}{\gamma}\left(\psi_\pi \frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)} + \psi_x\right)\right)\widehat{x}_t^R + \nu\psi_\pi \widehat{\overline{\pi}}_t^R + \left(\widehat{p}_t^R - \widehat{\overline{p}}_t^R\right). \tag{100}$$

If monetary policy in the frictionless equilibrium is set to accommodate the same price level path observed in the economy, then it follows that (98) reduces to:

$$\widehat{m}_t^{n,W} \equiv \widehat{m}_t^W - \widehat{\overline{m}}_t^W \approx \gamma\nu\left(1 - \frac{1}{\gamma}\psi_x\right)\widehat{x}_t^W, \tag{101}$$

if $\widehat{p}_t^R = \widehat{\overline{p}}_t^R$. An analogous result can be derived using the related differential credit equations. ■

C.3 Global Liquidity and Inflation Forecasting

We describe Home inflation forecasting only, but the approach would be analogous for Foreign inflation forecasting. We can express the forecast for domestic inflation in terms of the forecasts for global inflation and for the differential inflation as follows,

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j}\right) = \mathbb{E}_t\left(\widehat{\pi}_{t+j}^W\right) + \frac{1}{2}\mathbb{E}_t\left(\widehat{\pi}_{t+j}^R\right), \tag{102}$$

which simply undoes the transformation of variables laid out earlier in (54) − (55). Using the structural relationships for global inflation and inflation differentials that come from the model in (69) and (91), it follows that,

$$\begin{aligned}\mathbb{E}_t\left(\widehat{\pi}_{t+j}\right) &= \left(\widehat{\pi}_t^W - \frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{x}_t^W\right) + \frac{1}{2}\left(\widehat{\pi}_t^R - \frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{x}_t^R\right) \\ &= \widehat{\pi}_t - \frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{x}_t^W - \frac{1}{2}\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{x}_t^R, \end{aligned} \tag{103}$$

where $\widehat{x}_t^R \equiv \widehat{x}_t - \widehat{x}_t^*$ is the slack differential and $\widehat{x}_t^W \equiv \frac{1}{2}\widehat{x}_t + \frac{1}{2}\widehat{x}_t^* = \widehat{x}_t - \frac{1}{2}\widehat{x}_t^R$ stands for global slack. Simply re-arranging, we can also express the j-periods ahead forecast for inflation at time t, $\mathbb{E}_t\left(\widehat{\pi}_{t+j}\right)$, as follows,

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j} - \widehat{\pi}_t\right) = -\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{x}_t^W - \frac{1}{2}\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{x}_t^R. \tag{104}$$

In summary, it follows that no predictors other than Home and Foreign slack can improve the forecast of changes in Home inflation. Hence, the forecasting relationship for domestic inflation implied by the workhorse open-economy New Keynesian model can be expressed as,

$$\begin{aligned}\mathbb{E}_t\left(\widehat{\pi}_{t+j}-\widehat{\pi}_t\right) &= -\frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}+\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\widehat{x}_t-\frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}-\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\widehat{x}_t^* \quad (105)\\ &= \widehat{\pi}_t-\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{x}_t+\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}-\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\frac{1}{2}\widehat{x}_t^R \quad (106)\\ &= \widehat{\pi}_t-\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{x}_t-\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}-\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\widehat{x}_t^W. \quad (107)\end{aligned}$$

Moreover, we know from the propositions previously noted that money and credit are related to slack. Whenever the frictionless equilibrium is set to track the observed price level in both countries (i.e., if $\widehat{p}_t=\widehat{\overline{p}}_t$ and $\widehat{p}_t^*=\widehat{\overline{p}}_t^*$), it follows that equation (104) combined with (71) and (93) gives us the following forecasting equation:

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j}-\widehat{\pi}_t\right)=\frac{1}{\gamma\nu\left(1-\frac{1}{\gamma}\psi_x\right)}\left[-\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{m}_t^{n,W}-\frac{1}{2}\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{m}_t^{n,R}\right]. \quad (108)$$

Similarly, if one is agnostic about the path of the observed price level in both countries, the more general representation of the forecasting equation combining equation (104) with (70) and (92*a*) gives us that:

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j}-\widehat{\pi}_t\right)=\frac{1}{\chi(\vartheta)}\left[-\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\widehat{m}_t^{r,W}-\frac{1}{2}\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\widehat{m}_t^{r,R}+\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}\nu\psi_\pi\widehat{\overline{\pi}}_t^W+\frac{1}{2}\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\nu\psi_\pi\widehat{\overline{\pi}}_t^R\right]. \quad (109)$$

Straightforward manipulations of these two equations allow us to express the Home inflation forecasting equation in (105) in terms of the real money gap in each country in the following terms:

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j}-\widehat{\pi}_t\right)=-\frac{1}{\chi(\vartheta)}\begin{bmatrix}\frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}+\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\widehat{m}_t^r+\frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}-\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\widehat{m}_t^{r,*}-\ldots\\ \frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}+\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\nu\psi_\pi\widehat{\overline{\pi}}_t-\frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}-\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\nu\psi_\pi\widehat{\overline{\pi}}_t^*\end{bmatrix}, \quad (110a)$$

where the composite coefficient is given by $\chi(\vartheta)\equiv\left(1-\eta\left(\psi_\pi\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}+\psi_x\right)\right)$. Home and Foreign inflation in the frictionless case is defined as $\widehat{\overline{\pi}}_t\equiv\widehat{\overline{p}}_t-\widehat{\overline{p}}_{t-1}$ and $\widehat{\overline{\pi}}_t^*\equiv\widehat{\overline{p}}_t^*-\widehat{\overline{p}}_{t-1}^*$, respectively. Similarly, if $\widehat{p}_t=\widehat{\overline{p}}_t$ and $\widehat{p}_t^*=\widehat{\overline{p}}_t^*$, the Home inflation forecasting equation in (105) in terms of the nominal money gap in each country becomes:

$$\mathbb{E}_t\left(\widehat{\pi}_{t+j}-\widehat{\pi}_t\right)=-\frac{1}{\gamma\nu\left(1-\frac{1}{\gamma}\psi_x\right)}\left[\frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}+\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\widehat{m}_t^n+\frac{1}{2}\left(\frac{\lambda^W(\vartheta)}{\mu^W(\vartheta)}-\frac{\lambda^R(\vartheta)}{\mu^R(\vartheta)}\right)\widehat{m}_t^{n,*}\right]. \tag{111a}$$

Putting all the pieces together, it follows that:

Proposition 3 *For any given price level path in the frictionless equilibrium $\widehat{\overline{p}}_t$ and $\widehat{\overline{p}}_t^*$, the real money gap in the Home and Foreign countries $\widehat{m}_t^r \equiv (\widehat{m}_t-\widehat{p}_t)-\left(\widehat{\overline{m}}_t-\widehat{\overline{p}}_t\right)$ and $\widehat{m}_t^{r,*} \equiv (\widehat{m}_t^*-\widehat{p}_t^*)-\left(\widehat{\overline{m}}_t^*-\widehat{\overline{p}}_t^*\right)$ can help us forecast Home inflation as given by $(110a)$. If monetary policy in the frictionless equilibrium is set to track the observed price level in both countries (i.e., if $\widehat{p}_t=\widehat{\overline{p}}_t$ and $\widehat{p}_t^*=\widehat{\overline{p}}_t^*$), the nominal money gap in the Home and Foreign countries $\widehat{m}_t^n \equiv \left(\widehat{m}_t-\widehat{\overline{m}}_t\right)$ and $\widehat{m}_t^{n,*} \equiv \left(\widehat{m}_t^*-\widehat{\overline{m}}_t^*\right)$ can help us forecast Home inflation as given by $(111a)$. Similarly, given that the real credit gap in each country equates the money gap and the same can be said for the nominal credit gap and the nominal money gap, we can establish the same forecasting relationships using credit instead of money.*

In the frictionless Home and Foreign economies, all firms can adjust the prices of their varieties costlessly every period. Given this, monetary policy cannot affect the relative prices even though each central bank still can set any path for the price level corresponding to the bundle of final consumption goods in their country. Because monetary policy does not affect the relative prices, it does not have real effects and any monetary policy that we use to describe the price level in the frictionless case would be consistent with the same allocation of resources (that is, with the same output potential, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, and with the same natural rate of interest, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$).

What we assume for simplicity in our work is that monetary policy is consistent in the economy with frictions and in the frictionless counterfactual case to the point that the price level will be the same in both cases, i.e., we assume $\widehat{p}_t=\widehat{\overline{p}}_t$ and $\widehat{p}_t^*=\widehat{\overline{p}}_t^*$. In this baseline scenario, the forecasting relationship between inflation and a weighted aggregate of the Home and Foreign money gap measures, $\widehat{m}_t^n$ and $\widehat{m}_t^{n,*}$, in $(111a)$ suffices to motivate our use of monetary aggregates as inflation predictors. Alternatively, we could choose to remain purely agnostic about monetary policy in the frictionless equilibrium and rely instead on the forecasting relationship in $(110a)$ to motivate the use of real money balances as an inflation predictor. A similar argument can be extended to make use of nominal or real credit gap measures instead, given the tight relationship implied by the theory.

D Extensions to the Empirical Analysis

As noted before, the global output gap is shown to be an affine transformation of the world real money (or credit) gap in the case where monetary policy is defined differently for the actual and frictionless economies. Therefore, the actual and potential prices of each country are determined differently as a result of those policy differences. Following the theoretical results laid out in the previous section, we analyze here if G7 average real money supply growth and G7 real credit growth help forecast U.S. inflation empirically as a robustness check.

One caveat in this empirical exercise is that the PCE series is incomplete for some G7 countries, and therefore, we are able to deflate the nominal money (or credit) series of individual countries only by their individual CPI series. With the resulting CPI-deflated real money and credit measures, we calculate G7 average growth rates to forecast U.S. CPI and, for completeness, also U.S. PCE inflation. Notice that the domestic real money and credit measures (deflated by CPI) would yield the same results as the nominal measures in forecasting CPI inflation since the predictors are filtered based on the first-differences of the logs of these variables. Hence, we do not report the results based on domestic measures and instead focus on the performance of the G7 measures in Figure OA1.

Accordingly, the G7 measure of real money growth appears to be a better predictor of CPI inflation than the G7 real credit measure. The money measure also yields more robust results with the PCE inflation forecasts than the credit measure. Most importantly, these results are consistent with the evidence based on nominal money and credit measures and reinforce the case in favor of the hypothesis that global liquidity matters for inflation-forecasting. This result, indeed, further motivates our alternative interpretation of the mechanism by which global liquidity affects inflation, i.e., it provides further evidence consistent with the Monetarist view of the open-economy New Keynesian Phillips curve that we articulate in the paper.

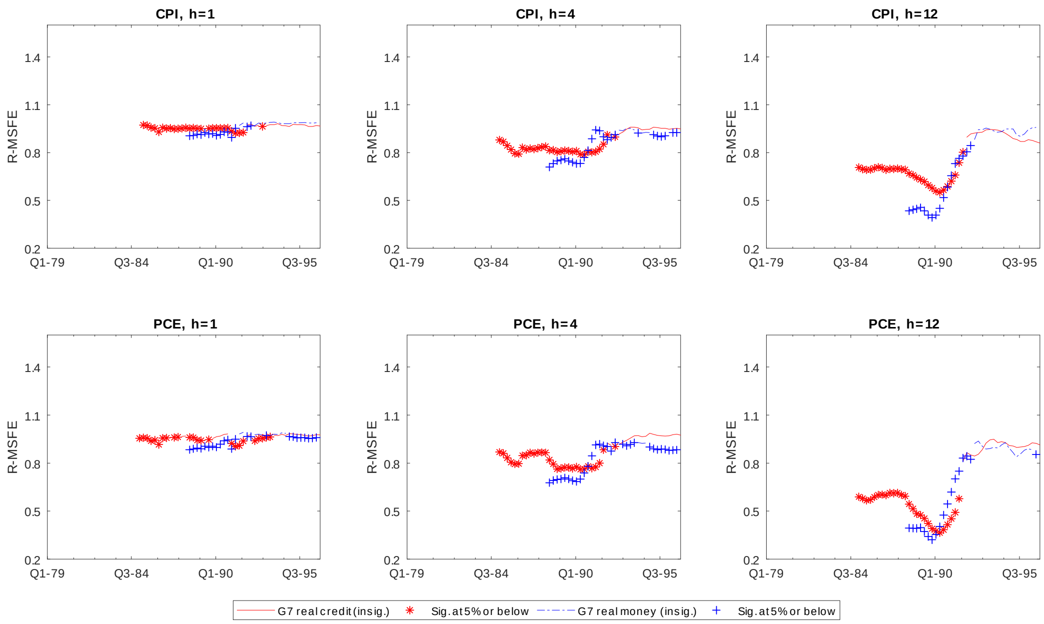

Figure OA1. Evolution of the MSFEs of the forecasts with real G7 money and real G7 credit gap relative to the benchmark autoregressive process of inflation. The vertical axis is for the relative MSFEs. In any subsample of the forecasting exercise, the estimation and forecast samples have 80 quarters of data each. The dates on the horizontal axis indicate the end of the estimation sample for a given subsample in our forecasting experiment. Sample start and end dates are given as follows. Real G7 money and real G7 credit: There are 34 subsamples, with the first estimation sample starting in 1968:Q4 and ending in 1988:Q3, and the forecast sample starting in 1988:Q4 and ending in 2008:Q3. The last estimation sample starts in 1977:Q1 and ends in 1996:Q4, and the forecast sample starts in 1997:Q1 and ends in 2016:Q4.

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