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Good Policies or Good Luck? New Insights on Globalization and the International Monetary Policy Transmission Mechanism*

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Abstract

The open-economy dimension is central to the discussion of the trade-offs that monetary policy faces in an increasingly integrated world. I investigate the monetary policy transmission mechanism in a two-country workhorse New Keynesian model where policy is set according to Taylor (1993) rules. I find that a common monetary policy isolates the effects of trade openness on the cross-country dispersion alone, and that the establishment of a currency union as a means of deepening economic integration may lead to indeterminacy. I argue that the common (coordinated) monetary policy equilibrium is the relevant benchmark for policy analysis showing that in that case open economies tend to experience lower macro volatility, a flatter Phillips curve, and more accentuated trade-offs between inflation and slack. Moreover, the trade elasticity often magnifies the effects of trade integration (globalization) beyond what conventional measures of trade openness would imply. I also discuss how other features such as the impact of a common and stronger anti-inflation bias, technological diffusion across countries, and the sensitivity of labor supply to real wages influence the quantitative effects of policy and openness in this context. Finally, I conclude that these theoretical predictions are largely consistent with the stylized facts of the Great Moderation.

JEL codes: C11, C13, F41

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1 Introduction

An ongoing topic of discussion among policymakers is how best to think about the role of openness for the conduct of monetary policy (Fisher (2005), Fisher (2006), Bernanke (2007), Trichet (2008), and more recently Draghi (2015) and Kaplan (2017)). Policymakers increasingly recognize that international linkages cannot be ignored in guiding policy, yet in many cases the closed-economy model—often a variant of the workhorse New Keynesian model (Woodford (2003))—largely remains the starting point for policy analysis.

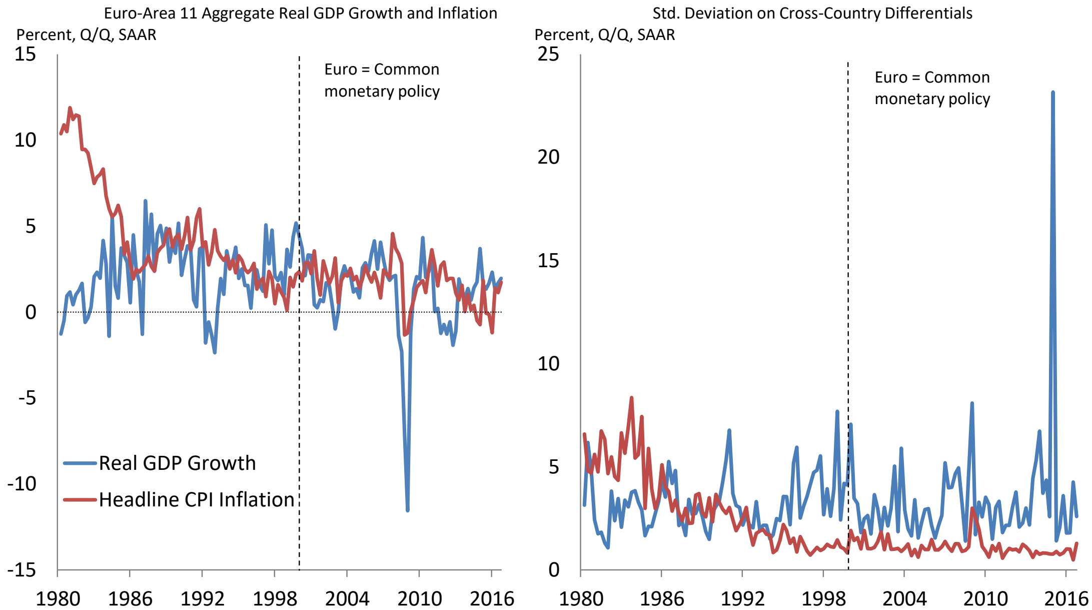
Much research is devoted nowadays to explore questions relating to how openness influences policy analysis and to what extent the closed-economy setting offers a useful approximation for policy-making, whenever economies have in fact become more interconnected. How do natural rates and potential output depend on foreign developments? Is the Phillips curve relationship between domestic inflation and domestic slack flatter or steeper for open economies and what does that entail? Can greater openness contribute to lower volatility and to alter the persistence and cross-country comovement of macro aggregates characteristic of the Great Moderation period? And, perhaps most notably, how does openness influence the policy trade-offs confronted by policymakers under a Taylor (1993)-type monetary policy regime?

The role of the monetary policy framework specifically has also received increased attention in the current monetary policy debate in light of notable policy regime changes among some major economies—such as the gradual coordination of national policies and eventual adoption of the common European currency (euro) among European Union (EU) countries. Figure 1 illustrates the aggregate and cross-country dispersion patterns on inflation and growth for the 11 member states of the EU that gave up their own independent monetary policies to give birth to the euro in 1999 (Haan (2010)).[1] The experience with European monetary union has further raised awareness about international monetary policy coordination issues more broadly such as the significance of stabilizing aggregate rather than domestic measures of inflation and slack or whether (and how) the monetary policy framework should take into account the dispersion across countries (even across regions within a country).

The main purpose of this paper is to investigate monetary policymaking under alternative monetary policy regimes within the workhorse two-country New Keynesian model, explicitly incorporating a role for openness. I build on the model of Martínez-García and Wynne (2010) and Martínez-García (2015b) characterized by flexible nominal exchange rates, trade openness, and asymmetric shocks across countries—which provides a straightforward extension of the standard three-equation (closed-economy) New Keynesian model to an open-economy setting.

[1] A number of other countries have since then adopted the euro as well.

Figure 1. Common European Currency: Effects on Growth and Inflation.



Sources: National Statistical Offices and Central Banks; OECD; IMF; author's calculations.
Notes: The data includes the 11 countries that adopted the euro initially (Euro-Area 11). All series are at quarterly frequency and aggregated using time-varying PPP-weights from the IMF. Quarterly growth rates are calculated in log-differences times 400. Cross-country differentials are computed relative to the Euro-Area 11 aggregates.

The approach I pursue here is to inspect the mechanism of the open-economy New Keynesian model more closely by focusing on its three main building blocks in log-linear form: the open-economy Phillips curve, the open-economy dynamic IS curve, and the monetary policy rule (specified in the form of conventional Taylor (1993)-type rules) for each country. The orthogonalization method I use to solve the model helps characterize analytically the main macro variables of each country in terms of aggregates and the corresponding differences between the two countries.[2] This paper makes an important methodological contribution as well illustrating how to decompose the solution whenever monetary policy rules differ across countries.

My main postulate is that trade and economic integration have altered the environment in which monetary policy must be conducted. In fact, this model helps show that even modest trade linkages expose the domestic economy to significant impacts from foreign shocks as well as to foreign policies. Here, my analysis contributes to the ongoing debate on globalization in two ways: First, it fleshes out an analytical framework to understand how the monetary policy transmission mechanism under different policy regimes is altered by the degree of openness and, second, it provides closed-form solutions that are tractable and facilitate a positive analysis of international monetary policy transmission and coordination.

An important conclusion of this paper is that Taylor (1993)-type policy rules involving some form of international monetary policy coordination contribute to decoupling the dynamics of the aggregates from those that characterize the dispersion across countries. This has an impact on the propagation of shocks across countries and on macro volatility as well which varies with the openness of the economy.

I find that generally the impact of globalization is underpredicted by standard measures of trade openness, as they do not fully capture the strength of trade effects. In fact, I show that the effects of the trade channel do not depend solely on the extent of trade openness—but critically depend on the trade elasticity of substitution between locally-produced and imported goods too. The trade channel gives greater significance to foreign developments on domestic macro aggregates than what standard trade openness measures would suggest given how demand shifts across countries—which are sensitive to the trade elasticity of substitution—propagate the effects of foreign shocks indirectly also through movements in international relative prices (real exchange rate, terms of trade).

Furthermore, I illustrate some of the pros and cons of explicit agreements among central banks for the coordination of monetary policy and for the formation of a currency union. I show that a common monetary policy is an important benchmark for policymaking, but it is also key to determine how policy asymmetries across countries propagate and modify the equilibrium dynamics under the common monetary policy regime. I also show that deeper monetary policy integration in the form of a currency union has no bearing on the aggregate dynamics of the countries that adopt the common currency and common monetary policy, but may result in an indeterminate solution at the country-level unlike under international monetary policy coordination.

With this framework at hand, I make the theoretical case for why trade openness (globalization) matters more than what we generally think:

1. The model shows that the trade channel provides a plausible avenue to explain a number of still-debated stylized facts in the international macro literature (such as the findings of Roberts (2006) and IMF WEO (2013) on the flattening of the Phillips curve or Ciccarelli and Mojon (2010), Duncan and

[2] This technique is related to the work of Aoki (1981), Fukuda (1993), and Obstfeld and Rogoff (1996) aimed at solving multi-country models. More recent (and related) applications can be found in Benigno (2004), Kabukcuoglu and Martínez-García (2014), and Martínez-García (2015b), among others.

Martínez-García (2015), Kabukcuoglu and Martínez-García (2016), and Bianchi and Civelli (2015) on the dynamics of inflation). The literature has intensely debated whether the Great Moderation was the result of good luck, good monetary policy internationally, or changes in the structure of the economy (Benati and Surico (2009), Woodford (2010)).[3] I provide an analytical assessment of the trade channel and its significance showing that changes from greater trade integration can reduce volatility—broadly in line with the Great Moderation experience. Trade also influences the trade-offs faced by policymarkers, alters the propagation of shocks, and even the contribution of different shock types (productivity shocks, cost-push shocks, monetary policy shocks) to the business cycle.

2. The paper expands on the existing literature on the monetary policy transmission mechanism in an open economy setting (Benigno (2004), Woodford (2010)). I conclude that the persistence of the macro variables is largely unaffected by either the strength of the trade channel or the features of the monetary policy rule. Most notably, I show that a coordinated common monetary policy isolates the effects of the trade channel to operate solely through the cross-country dispersion but not on the macro aggregates. I also note that forming a currency union as a means of deepening monetary policy integration may in turn lead to indeterminacy. This is a novel insight that, to my knowledge, has not been discussed elsewhere in the optimal currency area literature.

To conclude, I argue that structural change—in particular, greater trade integration (globalization)—as well as good monetary policy based on a strengthened anti-inflation bias have effectively altered the vulnerabilities to shocks of the economy over the past several decades. Hence, I claim that trade openness and monetary policy do influence the effects of shocks transmitted on the economy and that those theoretical implications derived from the model appear largely consistent with the stylized facts of the Great Moderation.[4]

The rest of the paper is organized as follows. In Section 2, the log-linear approximation to the equilibrium conditions is discussed. Section 3 analyzes the monetary policy framework investigated in the paper and defines alternative monetary policy regimes resulting in the adoption of a common monetary policy—international monetary policy coordination—and in the formation of a monetary union. Section 4 characterizes the analytical solution of the linear rational expectations model using Taylor (1993)'s monetary policy set-up as a benchmark. It then offers a detailed assessment of the policy trade-offs between slack and inflation and the implications for volatility amongst open economies—under independent (asymmetric) monetary policies, under common (coordinated) monetary policies, and within a currency area. Finally, Section 5 outlines some concluding remarks and possible extensions of this research agenda.

The Appendix provides proof of some of the key results presented in the paper and a detailed derivation of the analytical solution of the model. It also includes a description of the building blocks of the two-country workhorse New Keynesian model.[5]

[3] Good luck hypothesis: structural shocks during the Great Moderation were smaller than in the preceding decades.

[4] Shocks are not measured directly, only their consequences. As Martínez-García and Wynne (2014) pointed out using a medical analogy, the patient's temperature might rise only slightly and briefly when sick if medication is used quickly and effectively. The environment in which the patient is treated also influences his condition and the resulting temperature spike. But in either case, the "shock" that caused the temperature to raise may be the same even if temperature readings can otherwise be quite different.

[5] A companion (on-line) Technical Appendix is also available upon request with a detailed derivation of the approximated linear rational expectations model.

2 Log-Linear Equilibrium Dynamics

I postulate a two-country dynamic stochastic general equilibrium (DSGE) model with complete asset markets and optimizing agents. I abstract from capital accumulation—considering linear-in-labor technologies. I also adopt a cashless economy specification where money plays the sole role of unit of account (Woodford (2003), Chapter 2). There is a mass one of varieties produced in each country and all those varieties are traded between the two countries. Business cycle fluctuations are driven by country-specific productivity shocks, cost-push shocks, and monetary policy shocks.

I assume the law of one price (LOOP) holds at the variety level as firms price all their sales (domestic and foreign) in units of their local currency and quote them in the other country's currency at the prevailing bilateral nominal exchange rate. The model features two standard distortions in the goods market that are characteristic in the open-economy New Keynesian literature (Martínez-García and Wynne (2010)), monopolistic competition in production and staggered price-setting behavior à la Calvo (1983) (nominal rigidities). Nominal rigidities preserve monetary policy neutrality in the long run while allowing a break from it in the short run—monetary policy has no real effects in either the long run or the short run under perfect competition and flexible prices.

The two-country New Keynesian model provides a tractable environment under monetary non-neutrality for the purpose of studying the role of monetary policy in the international propagation of shocks.[6] I characterize a deterministic, zero-inflation steady state for the model, and log-linearize the equilibrium conditions around that steady state.[7] I solve for the approximated linear rational expectations model assuming small fluctuations around the steady state driven by country-specific productivity shocks, cost-push shocks, and monetary policy shocks. The shocks are invariant to the specification of the model.

I denote $\widehat{g}_t \equiv \ln G_t - \ln G$ as the deviation of a variable in logs from its steady state. I use the superscript $*$ to distinguish variables (and parameters) that are specific to the Foreign country from those that correspond to the Home country. I identify the frictionless allocation by marking the corresponding variables with an upper bar. As shown in Table 1, the log-linearized equilibrium conditions can be summarized with an open-economy Phillips curve, an open-economy dynamic investment-savings (IS) equation, and a Taylor (1993) rule for monetary policy in each country. In this sense, the open-economy New Keynesian model is a straightforward extension of the standard three-equation (closed-economy) New Keynesian model.

The system of equations in Table 1 pins down Home and Foreign CPI inflation (quarter-over-quarter changes), $\widehat{\pi}_t$ and $\widehat{\pi}_t^*$, Home and Foreign slack (deviations of output from potential absent all frictions), $\widehat{x}_t$ and $\widehat{x}_t^*$, and Home and Foreign short-term nominal interest rates, $\widehat{i}_t$ and $\widehat{i}_t^*$. Table 1 also includes a standard definition relating output in each country, $\widehat{y}_t$ and $\widehat{y}_t^*$, to the country's corresponding output potential plus slack—Home and Foreign output can be expressed as $\widehat{y}_t \equiv \widehat{\overline{y}}_t + \widehat{x}_t$ and $\widehat{y}_t^* \equiv \widehat{\overline{y}}_t^* + \widehat{x}_t^*$, respectively. The description of the model in Table 1 is completed with a pair of Fisherian equations for the real interest rates in the Home and Foreign countries defined as $\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$ and $\widehat{r}_t^* \equiv \widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]$, respectively. The natural (real) rates of interest that prevail absent all frictions in the model for the Home and Foreign countries are denoted $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$. The natural rates are a function of Home and Foreign potential output

[6] This framework can be generalized to include backward-looking terms as well. For a method to solve linear rational expectations models with backward-looking and forward-looking terms, see Martínez-García (2016).

[7] The steady state of the model with nominal rigidities and monopolistic competition is the same as that of the frictionless model (under perfect competition and flexible prices). Asymmetries in the policy rule across countries do not affect the steady state of the model that remains unaffected by the policy parameters and otherwise symmetric.

growth—where Home and Foreign potential output, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, depend exclusively on the Home and Foreign productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$, respectively.

Table 1 - Open-Economy New Keynesian Model

	Home Country
NKPC	$\widehat{\pi}_t \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}\right) + \Phi\left(\varphi+\gamma\right)\left[\kappa \widehat{x}_t + \left(1-\kappa\right)\widehat{x}_t^* + \widehat{v}_t\right]$ $\widehat{v}_t = \left(1-\xi\right)\widehat{u}_t + \xi \widehat{u}_t^*$
Dynamic IS	$\gamma\left(\mathbb{E}_t\left[\widehat{x}_{t+1}\right] - \widehat{x}_t\right) \approx \Omega\left[\widehat{r}_t - \widehat{\overline{r}}_t\right] + \left(1-\Omega\right)\left[\widehat{r}_t^* - \widehat{\overline{r}}_t^*\right]$
Monetary policy	$\widehat{i}_t \approx \psi_\pi \widehat{\pi}_t + \psi_x \widehat{x}_t + \widehat{m}_t$
Fisher equation	$\widehat{r}_t \equiv \widehat{i}_t - \mathbb{E}_t\left[\widehat{\pi}_{t+1}\right]$
Output decomposition	$\widehat{y}_t = \widehat{\overline{y}}_t + \widehat{x}_t$
Natural interest rate	$\widehat{\overline{r}}_t \approx \gamma\left[\Theta\left(\mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}\right] - \widehat{\overline{y}}_t\right) + \left(1-\Theta\right)\left(\mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}^*\right] - \widehat{\overline{y}}_t^*\right)\right]$
Potential output	$\widehat{\overline{y}}_t \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left[\Lambda \widehat{a}_t + \left(1-\Lambda\right)\widehat{a}_t^*\right]$
	Foreign Country
NKPC	$\widehat{\pi}_t^* \approx \beta \mathbb{E}_t \left(\widehat{\pi}_{t+1}^*\right) + \Phi\left(\varphi+\gamma\right)\left[\left(1-\kappa\right)\widehat{x}_t + \kappa \widehat{x}_t^* + \widehat{v}_t^*\right]$ $\widehat{v}_t^* = \xi \widehat{u}_t + \left(1-\xi\right)\widehat{u}_t^*$
Dynamic IS	$\gamma\left(\mathbb{E}_t\left[\widehat{x}_{t+1}^*\right] - \widehat{x}_t^*\right) \approx \left(1-\Omega\right)\left[\widehat{r}_t - \widehat{\overline{r}}_t\right] + \Omega\left[\widehat{r}_t^* - \widehat{\overline{r}}_t^*\right]$
Monetary policy	$\widehat{i}_t^* \approx \psi_\pi^* \widehat{\pi}_t^* + \psi_x^* \widehat{x}_t^* + \widehat{m}_t^*$
Fisher equation	$\widehat{r}_t^* \equiv \widehat{i}_t^* - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^*\right]$
Output decomposition	$\widehat{y}_t^* = \widehat{\overline{y}}_t^* + \widehat{x}_t^*$
Natural interest rate	$\widehat{\overline{r}}_t^* \approx \gamma\left[\left(1-\Theta\right)\left(\mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}\right] - \widehat{\overline{y}}_t\right) + \Theta\left(\mathbb{E}_t\left[\widehat{\overline{y}}_{t+1}^*\right] - \widehat{\overline{y}}_t^*\right)\right]$
Potential output	$\widehat{\overline{y}}_t^* \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right)\left[\left(1-\Lambda\right)\widehat{a}_t + \Lambda \widehat{a}_t^*\right]$
	Composite Parameters
	$\Phi \equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)$, $\kappa \equiv \left(1-\xi\right)\left[1 - \left(\sigma\gamma-1\right)\left(\frac{\gamma}{\varphi+\gamma}\right)\left(\frac{(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]$, $\Theta \equiv \left(1-\xi\right)\left[\frac{\sigma\gamma-(\sigma\gamma-1)(1-2\xi)}{\sigma\gamma-(\sigma\gamma-1)(1-2\xi)^2}\right] = \left(1-\xi\right)\left[\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right]$, $\Omega \equiv \left(1-\xi\right)\left(\frac{1-2\xi(1-\sigma\gamma)}{1-2\xi}\right)$, $\Lambda \equiv 1 + \frac{1}{2}\left[\frac{\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right]$.

Apart from productivity shocks, the model includes two other country-specific exogenous shocks: cost-push shocks, $\widehat{u}_t$ and $\widehat{u}_t^*$, and monetary policy shocks, $\widehat{m}_t$ and $\widehat{m}_t^*$. As indicated in Table 2, all shocks follow bivariate VAR(1) stochastic processes but only productivity shocks incorporate spillovers explicitly.[8] Shock

[8] Productivity shock spillovers capture technological diffusion across countries. In turn, diffusion does not appear so significant for either monetary policy shocks or cost-push shocks.

innovations can be correlated across countries, but not across the three different shock types.

Table 2 - Country-Specific, Exogenous Shocks

Productivity shock	$\begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{a}_{t-1} \\ \widehat{a}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix}\right)$
Cost-push shock	$\begin{pmatrix} \widehat{u}_t \\ \widehat{u}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_u & 0 \\ 0 & \delta_u \end{pmatrix} \begin{pmatrix} \widehat{u}_{t-1} \\ \widehat{u}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^u \\ \widehat{\varepsilon}_t^{u*} \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}_t^u \\ \widehat{\varepsilon}_t^{u*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & \rho_{u,u^*}\sigma_u^2 \\ \rho_{u,u^*}\sigma_u^2 & \sigma_u^2 \end{pmatrix}\right)$
Monetary shock	$\begin{pmatrix} \widehat{m}_t \\ \widehat{m}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_m & 0 \\ 0 & \delta_m \end{pmatrix} \begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix}$ $\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{pmatrix}\right)$

2.1 The Frictionless Dynamics

The dynamics of the frictionless environment with perfect competition and flexible prices and wages are summarized by each country's natural rate, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$, and potential output, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, as shown in Table 1. The labor market imperfections that motivate the cost-push shocks, the exogenous marginal cost shifters $\widehat{u}_t$ and $\widehat{u}_t^*$, are also absent in the frictionless environment. With flexible prices and wages, monetary neutrality holds in the short run as well as in the long run and therefore neither monetary policy shocks nor the monetary policy rule affect the frictionless allocation either. Hence, the natural rate and output potential of each country are solely driven by Home and Foreign productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$, respectively.[9]

The Natural Rates. I find that the composite coefficient Θ that determines the contribution of expected domestic potential growth to the domestic natural rate is $\Theta = 1$ assuming households only include locally-produced goods in their consumption basket ($\xi = 0$). Not surprisingly, when imports are not valued by households and there is no role for trade, only domestic potential output growth determines the domestic natural rate. In turn, when there is room for trade but no local-production bias ($\xi = \frac{1}{2}$), the weight on expected domestic potential growth in the domestic natural rate is $\Theta = \frac{1}{2}$ reflecting the share in production of both countries (production of varieties is equally distributed across countries).

More generally, the weight on expected domestic potential growth $\Theta \equiv (1-\xi)\left(\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)$ satisfies that

$$\begin{cases} \frac{1}{2} < \Theta < (1-\xi) < 1 \text{ if } \sigma\gamma > 1, \\ \Theta = (1-\xi) \text{ if } \sigma\gamma = 1, \\ \frac{1}{2} < (1-\xi) < \Theta \text{ if } 0 < \sigma\gamma < 1, \end{cases} \tag{1}$$

for any degree of openness $0 < \xi < \frac{1}{2}$. To prove the result in (1), I first note that $\left[\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right] > 0$ for all $\sigma\gamma > 0$ and $\left[\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right] \lesseqqgtr 1$ if $\sigma\gamma \gtreqqless 1$. Then, the fact that $\Theta > \frac{1}{2}$ if $\sigma\gamma > 1$ follows naturally

[9] Productivity shocks enter into the dynamics of the model only through their impact on the Home and Foreign natural rates, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$, and the Home and Foreign output potential, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, as indicated in Table 1.

from the definition of the composite coefficient Θ. In turn, if $0 < \sigma\gamma < 1$, I find that $\Theta < 1$ holds only whenever $\sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right)$ and $0 < \left(\frac{1-2\xi}{1+(1-2\xi)}\right) < 1$.

The interpretation of (1) is that the weight on expected domestic potential growth can be lower than what the domestic consumption share $(1-\xi)$ alone would imply and concurrently the weight on expected foreign potential growth can be larger than what the import share ξ would entail only whenever $\sigma\gamma > 1$. This reflects that the domestic natural rate captures not just the domestic growth potential but also the aggregate demand shifts across countries prompted by concurrent changes in terms of trade. Whenever the trade elasticity σ satisfies that $0 < \sigma\gamma < 1$, the effects are attenuated instead and in some instances (if $0 < \sigma\gamma < \left(\frac{1-2\xi}{1+(1-2\xi)}\right) < 1$) they can even lead to a reversal whereby $\Theta > 1$.

The Potential Output. The composite coefficient Λ weighting domestic productivity on domestic potential output satisfies that $\Lambda = 1$ if $\xi = 0$ and $\Lambda = 1 + \frac{1}{2}\left(\frac{\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)}{\sigma\gamma-\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)}\right)$ if $\xi = \frac{1}{2}$. Hence, only domestic productivity shocks enter into the specification of domestic potential whenever the trade channel is shut down ($\xi = 0$). The weight on foreign productivity $(1-\Lambda)$ and its sign when $\xi = \frac{1}{2}$ depend on: the preference ratio $0 < \frac{\gamma}{\varphi+\gamma} < 1$ which is a function of the inverse of the Frisch elasticity of labor supply $\varphi > 0$ and of the inverse of the intertemporal elasticity of substitution $\gamma > 0$; and the product $\sigma\gamma > 0$ which is related to the trade elasticity of substitution between Home and Foreign goods $\sigma > 0$.

More generally, the weight on domestic productivity shocks $\Lambda \equiv 1 + \frac{1}{2}\left(\frac{\left(\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)$ satisfies that

$$\begin{cases} \Lambda > 1 \text{ if } \sigma\gamma > 1, \\ \Lambda = 1 \text{ if } \sigma\gamma = 1, \\ \frac{1}{2} < \Lambda < 1 \text{ if } 0 < \sigma\gamma < 1, \end{cases} \tag{2}$$

for any given degree of openness $0 < \xi < \frac{1}{2}$. Analogously, the weight on foreign productivity shocks in domestic potential output must satisfy that $1-\Lambda \lesseqqgtr 0$ if $\sigma\gamma \gtreqqless 1$. This implies that an open economy displays a higher positive impact of domestic productivity on domestic potential than under the closed-economy specification (and a negative effect of foreign productivity on domestic potential) only if the trade elasticity satisfies that $\sigma\gamma > 1$. The contribution of Home and Foreign productivity shocks is seen to depend on the preference ratio $\frac{\gamma}{\varphi+\gamma}$ as well, which describes the characteristics of the labor market response.

Whenever $\sigma\gamma > 1$, the income effect on domestic production from a foreign productivity shock is outweighed by the substitution effect that shifts aggregate demand towards the relatively cheaper foreign goods and drags domestic labor and production down. I adopt the case where $\sigma\gamma > 1$ as the economically-relevant benchmark to match the expected signs in the transmission across countries of productivity shocks. In the special case where $\sigma\gamma = 1$, I find that domestic potential output is fully insulated from foreign productivity shocks through trade ($\Lambda = 1$) and identical to its closed-economy counterpart irrespective of the degree of openness ξ (Cole and Obstfeld (1991)).[10] Whenever $0 < \sigma\gamma < 1$, a positive foreign productivity shock drives domestic potential output up while positive domestic productivity shocks have an attenuated effect on domestic potential which is lower than in the closed-economy case.

[10] In the special case where $\sigma\gamma = 1$, full insulation from foreign shocks can be achieved through fluctuations in international relative prices (terms of trade) alone irrespective of the assumptions made on the international asset market structure.

2.2 The Open-Economy Phillips Curve (NKPC)

An important takeaway from the open-economy Phillips curve in Table 1 is that foreign slack—not just domestic slack—plays a central role in modelling domestic inflation. Unlike in the closed-economy model, the domestic economy moving above its potential does not necessarily lead to higher domestic marginal costs and inflation for open economies whenever there is growing slack elsewhere—as this weighs down on imported goods inflation and shifts aggregate demand away from domestic goods through movements in the terms of trade. Therefore, the key insight from the open-economy model is that both Home and Foreign slack (not just domestic slack) help gauge domestic inflation.

The slope of the Phillips curve in the closed-economy case is given by $\Phi(\varphi+\gamma)$ where $\Phi \equiv \left(\frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\right)$—the closed-economy slope is a function of the Calvo (1983) price stickiness parameter α, the intertemporal discount factor β, the inverse of the intertemporal elasticity of substitution γ, and the inverse of the Frisch elasticity of labor supply φ. The slope of the domestic open-economy Phillips curve on domestic slack can be written as $\Phi(\varphi+\gamma)\kappa$—which is a function of the closed-economy slope and the composite weight $\kappa \equiv (1-\xi)\left[1-(\sigma\gamma-1)\left(\frac{\gamma}{\varphi+\gamma}\right)\left(\frac{(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]$. Analogously, the slope of the domestic open-economy Phillips curve on foreign slack can be expressed as $\Phi(\varphi+\gamma)(1-\kappa)$. The sum of the slopes on domestic and foreign slack in the open-economy case equals the closed-economy slope.

In the case where there is no endogenous trade ($\xi=0$), I recover the standard closed-economy Phillips curve specification with $\kappa=1$ which depends solely on domestic slack. Abstracting from local-production bias ($\xi=\frac{1}{2}$), I obtain equal weights on the slack of both countries where $\kappa=\frac{1}{2}$ consistent with the equal shares both countries have in production. More generally, it follows that the weight coefficient κ satisfies that

$$\begin{cases} \frac{1}{2}<\kappa<(1-\xi)<1 \text{ if } \sigma\gamma>1, \\ \kappa=(1-\xi) \text{ if } \sigma\gamma=1, \\ \frac{1}{2}<(1-\xi)<\kappa \text{ if } 0<\sigma\gamma<1, \end{cases} \tag{3}$$

for any degree of openness $0<\xi<\frac{1}{2}$.

It can be seen that $\left(\frac{(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)>0$ for any degree of openness $0<\xi<\frac{1}{2}$ and for all $\sigma\gamma>0$. Hence, it follows from the definition of κ that $\kappa \lesseqgtr (1-\xi)$ if $\sigma\gamma \gtreqless 1$. In (3), there is a lower bound on how low the slope of the open-economy Phillips curve on domestic slack can go whenever $\sigma\gamma>1$ given by the equal shares in production of each country. To show this, notice that in the case where $\sigma\gamma>1$ the weight on domestic slack κ satisfies that $\kappa=(1-\xi)\left[1+\left(\left(1-\frac{\gamma}{\varphi+\gamma}\right)-1\right)\left(\frac{(\sigma\gamma-1)(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]>(1-\xi)\left[1-\left(\frac{(\sigma\gamma-1)(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]$ for any degree of trade openness $0<\xi<\frac{1}{2}$ given that $0<\frac{\gamma}{\varphi+\gamma}<1$. Then, straightforward algebra shows that $(1-\xi)\left[1-\left(\frac{(\sigma\gamma-1)(2\xi)(1-2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]=(1-\xi)\left(\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)>\frac{1}{2}$. By construction, therefore, it holds from here that the weight on domestic slack satisfies that $\frac{1}{2}<\kappa<(1-\xi)$ and the weight on foreign slack satisfies that $\xi<1-\kappa<\frac{1}{2}$ whenever $\sigma\gamma>1$.

The slope on domestic slack can be flatter than the closed-economy slope (i.e., $\kappa<1$) while the slope on foreign slack can have a positive sign (i.e., $1-\kappa>0$) whenever $\sigma\gamma>\max\left\{0, 1-\left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$.[11]

[11] Here, I find that $\kappa<1$ requires that $\sigma\gamma>1-\left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}$. Given this, I see that $0<\left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}<1$ only if $\left(\frac{\gamma}{\varphi+\gamma}\right)>\frac{1-(2\xi)(2(1-\xi))}{(1-2\xi)(2(1-\xi))}$ where $0<\frac{1-(2\xi)(2(1-\xi))}{(1-2\xi)(2(1-\xi))}<\frac{1}{2}$. As a result, whenever

These findings also imply that the domestic slope of the open-economy Phillips curve must be flatter than the closed-economy slope and strictly more so than what the domestic share in consumption $(1-\xi)$ would entail only in the benchmark case where $\sigma\gamma > 1$. Naturally, the slope on foreign slack is significantly higher than what the import share (ξ) alone would warrant in that case. Whenever $\sigma\gamma = 1$, the domestic slope is determined by the domestic share alone $(\kappa = (1-\xi))$. In turn, if $0 < \sigma\gamma < 1$, the flattening of the open-economy Phillips curve is instead attenuated and can even be reversed in some cases.

Therefore, the two-country New Keynesian model offers additional economic insight connecting trade openness (ξ) to the so-called flattening of the Phillips documented in the empirical literature (Roberts (2006), IMF WEO (2013)). I want to highlight here that the flattening of the domestic slope of the open-economy Phillips curve depends not only on the strength of the trade channel, but also on the preference ratio $0 < \frac{\gamma}{\varphi+\gamma} < 1$ and therefore on features of the labor market. The higher the ratio $\frac{\gamma}{\varphi+\gamma}$ is, the lower the composite coefficient κ will fall below $(1-\xi)$.

2.3 The Open-Economy IS Equation

The open-economy dynamic IS equations in Table 1 show that slack in each country is tied to developments in both Home and Foreign aggregate demand. More specifically, to the wedge between the actual real interest rate—the opportunity cost of consumption today versus consumption tomorrow ($\widehat{r}_t$ and $\widehat{r}_t^*$)—and the natural rate that would prevail in the frictionless equilibrium ($\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$). Abstracting from local-production bias in consumption ($\xi = \frac{1}{2}$), the open-economy dynamic IS equation in both countries can be rewritten in terms of the local interest rate gap alone as real interest rate deviations from the natural rate must equalize across countries (i.e., $\widehat{r}_t - \widehat{\overline{r}}_t \approx \widehat{r}_t^* - \widehat{\overline{r}}_t^*$). The natural rates of both countries equalize across countries in this case (i.e., $\widehat{\overline{r}}_t \approx \widehat{\overline{r}}_t^*$)—hence, real interest rate equalization across countries ($\widehat{r}_t \approx \widehat{r}_t^*$) occurs if there is no local-production bias.[12] The open-economy dynamic IS equations naturally reduce to their closed-economy counterparts with $\Omega = 1$ whenever the import share is set to zero ($\xi = 0$).

From the definition of the slope of the domestic IS equation on domestic interest rate deviations $\Omega \equiv (1-\xi)\left(\frac{1-2\xi(1-\sigma\gamma)}{1-2\xi}\right)$, it follows $\Omega > (1-\xi)$ given that $\left(\frac{1-2\xi(1-\sigma\gamma)}{1-2\xi}\right) > 1$ for all $\sigma\gamma > 0$ and $0 < \xi < \frac{1}{2}$. More generally, I find that

$$\begin{cases} \Omega > 1 > (1-\xi)\,, \text{ if } \sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right) > 0, \\ 1 > \Omega > (1-\xi)\,, \text{ if } 0 < \sigma\gamma < \left(\frac{1-2\xi}{1+(1-2\xi)}\right), \end{cases} \tag{4}$$

for any degree of openness $0 < \xi < \frac{1}{2}$. In other words, an open economy has a slope on domestic interest rate deviations that is larger than in the closed-economy case—and concurrently a negative slope on foreign interest rate deviations—whenever $\sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right)$ where $0 < \left(\frac{1-2\xi}{1+(1-2\xi)}\right) < \frac{1}{2}$. In turn, for cases in which the trade elasticity is sufficiently low ($0 < \sigma\gamma < \left(\frac{1-2\xi}{1+(1-2\xi)}\right)$), a reversal is possible whereby the slope on domestic interest rate deviations is lower than in the closed-economy case and the slope on foreign interest rate deviations becomes positive.

the preference ratio $0 < \frac{\gamma}{\varphi+\gamma} < 1$ is large enough such that $\left(\frac{\gamma}{\varphi+\gamma}\right) > \frac{1-(2\xi)(2(1-\xi))}{(1-2\xi)(2(1-\xi))}$, there is a non-empty range of values for the trade elasticity given by $0 < \sigma\gamma < 1 - \left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}$ for which the model produces $\kappa > 1$.

[12] Differences in the consumption baskets imply that each country's consumption demand responds differently to country-specific shocks, which is reflected in the cross-country differences in natural rates of interest.

Aggregate demand responds to deviations of each country's real interest rate from its own natural rate as those deviations shift aggregate consumption across time. Whenever the real interest rate is above its natural rate, more consumption today is being postponed for consumption tomorrow than would otherwise occur in the frictionless environment. *Ceteris paribus*, this implies a demand shortfall today (a fall in output relative to potential) and the expectation of slack unwinding in the future. Analogously, when the real rate is below the natural rate, the resulting boost in consumption today (at the expense of future consumption) leads to a temporary increase in the output gap that is nonetheless expected to dissipate over time.

Hence, an open economy has to grapple with a steeper dynamic IS curve on domestic real interest rate deviations from the domestic natural rate—not just with a flatter open-economy Phillips curve. The slope of the open-economy IS curve depends on how open the economy is to trade (ξ) and on the trade elasticity (σ). The intuition for this result is straightforward: A given rise in the real interest rate impacts aggregate demand through two channels, domestic consumption and trade. A shock that is met by a rise in the *ex-ante* real interest rate drags domestic consumption but also induces a terms of trade deterioration that erodes the foreign demand of domestic goods, lowering domestic output relative to its potential further. However, features of the labor market such as the Frisch elasticity of labor supply do not enter into the slope of the IS curve unlike what happens with the Phillips curve slope.

2.4 The Stochastic Processes

I characterize the dynamics of potential output and of the natural rate of interest based on the frictionless allocation described in Table 1 and the shock processes in Table 2. The potential output of the Home and Foreign countries, $\widehat{\overline{y}}_t$ and $\widehat{\overline{y}}_t^*$, are defined as a convex combination of the Home and Foreign productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$. As shown in the Appendix, given the VAR(1) structure of the productivity shocks, the following bivariate VAR(1) stochastic process characterizes the dynamics of potential output

$$\begin{pmatrix} \widehat{\overline{y}}_t \\ \widehat{\overline{y}}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{\overline{y}}_{t-1} \\ \widehat{\overline{y}}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^{y} \\ \widehat{\varepsilon}_t^{y*} \end{pmatrix}, \tag{5}$$

$$\begin{pmatrix} \widehat{\varepsilon}_t^{y} \\ \widehat{\varepsilon}_t^{y*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \sigma_y^2 \begin{pmatrix} 1 & \rho_{y,y^*} \\ \rho_{y,y^*} & 1 \end{pmatrix} \right), \tag{6}$$

where

$$\sigma_y^2 \equiv \sigma_a^2 \left(\frac{1+\varphi}{\gamma+\varphi} \right)^2 \left((\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2 \right), \tag{7}$$

$$\rho_{y,y^*} \equiv \frac{\rho_{a,a^*}(\Lambda)^2 + 2\Lambda(1-\Lambda) + \rho_{a,a^*}(1-\Lambda)^2}{(\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2}, \tag{8}$$

define the volatility and the correlation of the potential output innovations.

In the closed-economy case ($\xi = 0$), $\Lambda = 1$ so the volatility and correlations for the innovations on output potential would be $\sigma_{y,closed}^2 = \sigma_a^2 \left(\frac{1+\varphi}{\gamma+\varphi} \right)^2$ and $\rho_{y,y^*,closed} = \rho_{a,a^*}$, respectively. Given the definition of Λ in (2), it follows then that $\left((\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2 \right) > \left((\Lambda)^2 + 2\Lambda(1-\Lambda) + (1-\Lambda)^2 \right) = 1$ and $\frac{\rho_{a,a^*}(\Lambda)^2 + 2\Lambda(1-\Lambda) + \rho_{a,a^*}(1-\Lambda)^2}{(\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2} < \rho_{a,a^*}$ whenever $\sigma\gamma > 1$ and $0 < \rho_{a,a^*} < 1$. More generally, I can conclude that open economies ($0 < \xi < \frac{1}{2}$) where the productivity innovations are positively correlated across countries ($0 < \rho_{a,a^*} < 1$) satisfy that $\sigma_y^2 \gtreqless \sigma_{y,closed}^2$ and $\rho_{y,y^*} \lesseqgtr \rho_{y,y^*,closed}$ if $\sigma\gamma \gtreqless 1$ ($\sigma\gamma > 0$). In other words, open economies display higher volatility of potential output innovations and lower correlations than their closed-economy counterparts in the benchmark case where $\sigma\gamma > 1$.[13]

Analogously, a simple characterization of the natural rates in the Home and Foreign countries, $\widehat{\overline{r}}_t$ and $\widehat{\overline{r}}_t^*$, can be derived from the bivariate stochastic VAR(1) process for the productivity shocks, $\widehat{a}_t$ and $\widehat{a}_t^*$. As seen in the Appendix, the following VAR(1) stochastic process characterizes the dynamics of the natural rates

$$\begin{pmatrix} \widehat{\overline{r}}_t \\ \widehat{\overline{r}}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{\overline{r}}_{t-1} \\ \widehat{\overline{r}}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \sigma_r^2 \begin{pmatrix} 1 & \rho_{r,r^*} \\ \rho_{r,r^*} & 1 \end{pmatrix} \right), \tag{10}$$

where

$$\sigma_r^2 \equiv \sigma_a^2 \gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi} \right)^2 \left((\Pi_1)^2 + 2\rho_{a,a^*} \Pi_1 \Pi_2 + (\Pi_2)^2 \right), \tag{11}$$

$$\rho_{r,r^*} \equiv \frac{\rho_{a,a^*} (\Pi_1)^2 + 2\Pi_1\Pi_2 + \rho_{a,a^*} (\Pi_2)^2}{(\Pi_1)^2 + 2\rho_{a,a^*} \Pi_1 \Pi_2 + (\Pi_2)^2}, \tag{12}$$

and

$$\Pi_1 \equiv \delta_{a,a^*} - (1-\xi) \left(\frac{1 + \left(1 - \frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma - 1)(2\xi)}{1 + \left(1 - \frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))} \right) (\delta_{a,a^*} + 1 - \delta_a), \tag{13}$$

$$\Pi_2 \equiv (\delta_a - 1) + (1-\xi) \left(\frac{1 + \left(1 - \frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma - 1)(2\xi)}{1 + \left(1 - \frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))} \right) (\delta_{a,a^*} + 1 - \delta_a), \tag{14}$$

define the volatility and the correlation of the corresponding natural rate innovations.

Home and Foreign potential output—as well as their corresponding natural rates—inherit the VAR(1) stochastic structure and some of basic features of the productivity shock process—in particular, the persistence and spillovers of the productivity shocks. In turn, the deep structural parameters of the model—including those tied to the trade channel: degree of openness (ξ) and trade elasticity (σ)—enter into the variance-covariance matrix.

Here the closed-economy case ($\xi = 0$) implies that $\Pi_1 \equiv (\delta_a - 1)$ and $\Pi_2 \equiv \delta_{a,a^*}$, so the volatility of the closed-economy natural rate innovations is $\sigma_{r,closed}^2 = \sigma_a^2\gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \left((\delta_a - 1)^2 + 2\rho_{a,a^*} (\delta_a - 1) \delta_{a,a^*} + (\delta_{a,a^*})^2 \right)$ while the corresponding cross-country correlation is given by $\rho_{r,r^*,closed} = \frac{\rho_{a,a^*}(\delta_a-1)^2 + 2(\delta_a-1)\delta_{a,a^*} + \rho_{a,a^*}\left(\delta_{a,a^*}\right)^2}{(\delta_a-1)^2 + 2\rho_{a,a^*}(\delta_a-1)\delta_{a,a^*} + \left(\delta_{a,a^*}\right)^2}$. For any degree of openness $0 < \xi \leq \frac{1}{2}$, $\Pi_1 = (1-\Pi)\,\delta_{a,a^*} + \Pi\,(\delta_a - 1)$ and $\Pi_2 = \Pi\delta_{a,a^*} + (1-\Pi)(\delta_a - 1)$ are linear combinations of the parameters that describe the persistence of the bivariate VAR(1) process for productivity, δ_{a,a^*} and $(\delta_a - 1)$, and $\frac{1}{2} < \Pi \equiv (1-\xi)\left(\frac{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))} \right) < 1$. To ensure

[13] The preference ratio $\frac{\gamma}{\varphi+\gamma}$ plays a major role in lowering the correlation and increasing the volatility of the output potential for open economies as $\frac{\gamma}{\varphi+\gamma} \nearrow 1$. Simulations and further details on this point are available from the author upon request.

stationarity of the bivariate productivity shock process, I require $\delta_{a,a^*} + (\delta_a - 1) < 0$ which in turn implies that $\Pi_1 + \Pi_2 = \delta_{a,a^*} + (\delta_a - 1) < 0$. Hence, I derive the following expression[14]

$$\sigma_r^2 = \sigma_{r,closed}^2 + 2\left(\rho_{a,a^*} - 1\right)\left[\gamma^2\left(\frac{1+\varphi}{\gamma+\varphi}\right)^2(1-\Pi)\,\Pi\right]\sigma_a^2\left(\delta_{a,a^*} - (\delta_a - 1)\right)^2, \tag{15}$$

which, given that $0 < \rho_{a,a^*} < 1$, implies that $\sigma_r^2 < \sigma_{r,closed}^2$ for any open economy. This indicates that the volatility of the natural rate innovations is lower in an open economy than it would be in the closed-economy case for any plausible parameterization of the trade elasticity.

Furthermore, it follows from the definition of the cross-country correlation of the natural rate innovations that $\rho_{r,r^*} < \rho_{a,a^*}$ if and only if the condition $\Pi_1\Pi_2 < 0$ holds. The expression for the cross-correlation is as follows: $\rho_{r,r^*} = \frac{\left(\rho_{a,a^*}(\delta_a-1)^2 + 2(\delta_a-1)\delta_{a,a^*} + \rho_{a,a^*}\left(\delta_{a,a^*}\right)^2\right) + 2\left(1-\rho_{a,a^*}\right)(1-\Pi)\Pi\left(\delta_{a,a^*}-(\delta_a-1)\right)^2}{\left((\delta_a-1)^2 + 2\rho_{a,a^*}(\delta_a-1)\delta_{a,a^*} + \left(\delta_{a,a^*}\right)^2\right) - 2\left(1-\rho_{a,a^*}\right)(1-\Pi)\Pi\left(\delta_{a,a^*}-(\delta_a-1)\right)^2}$, which implies that

$$\rho_{r,r^*} = \frac{\rho_{r,r^*,closed} + \frac{2\left(1-\rho_{a,a^*}\right)(1-\Pi)\Pi\left(\delta_{a,a^*}-(\delta_a-1)\right)^2}{\left((\delta_a-1)+\delta_{a,a^*}\right)^2 - 2\left(1-\rho_{a,a^*}\right)(\delta_a-1)\delta_{a,a^*}}}{1 - \frac{2\left(1-\rho_{a,a^*}\right)(1-\Pi)\Pi\left(\delta_{a,a^*}-(\delta_a-1)\right)^2}{\left((\delta_a-1)+\delta_{a,a^*}\right)^2 - 2\left(1-\rho_{a,a^*}\right)(\delta_a-1)\delta_{a,a^*}}}. \tag{16}$$

It can be shown that $\rho_{r,r^*} > \rho_{r,r^*,closed}$ so long as the cross-correlation of the productivity innovations ρ_{a,a^*} satisfies that $\left(1-\rho_{a,a^*}\right)(\delta_a - 1)\,\delta_{a,a^*} < \frac{1}{2}\left((\delta_a - 1) + \delta_{a,a^*}\right)^2$. Given that $\delta_{a,a^*} + (\delta_a - 1) < 0$, then if $\delta_{a,a^*} \gtreqqless 0$ it must be the case that $(\delta_a - 1) \lesseqqgtr 0$. As a result, it follows that $(\delta_a - 1)\,\delta_{a,a^*} < 0$ and accordingly the inequality on ρ_{a,a^*} is satisfied for any value $0 < \rho_{a,a^*} < 1$. In other words, it holds that $\rho_{r,r^*} > \rho_{r,r^*,closed}$ and this shows that innovations to the natural rate are more highly correlated for open economies than for closed economies for any plausible value of the trade elasticity.[15]

The trade-weighted definition of the cost-push shocks in Table 1, $\widehat{v}_t \equiv (1-\xi)\,\widehat{u}_t + \xi\widehat{u}_t^*$ and $\widehat{v}_t^* \equiv \xi\widehat{u}_t + (1-\xi)\,\widehat{u}_t^*$, is a convex combination of the country-specific cost-push shocks, $\widehat{u}_t$ and $\widehat{u}_t^*$, which depends on the degree of openness ξ. Given the VAR(1) structure of $\widehat{u}_t$ and $\widehat{u}_t^*$, the following bivariate VAR(1) stochastic process—as seen in the Appendix—characterizes the dynamics of the trade-weighted cost-push shocks

$$\begin{pmatrix} \widehat{v}_t \\ \widehat{v}_t^* \end{pmatrix} \approx \begin{pmatrix} \delta_u & 0 \\ 0 & \delta_u \end{pmatrix}\begin{pmatrix} \widehat{v}_{t-1} \\ \widehat{v}_{t-1}^* \end{pmatrix} + \begin{pmatrix} \widehat{\varepsilon}_t^v \\ \widehat{\varepsilon}_t^{v*} \end{pmatrix}, \tag{17}$$

$$\begin{pmatrix} \widehat{\varepsilon}_t^v \\ \widehat{\varepsilon}_t^{v*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \sigma_v^2\begin{pmatrix} 1 & \rho_{v,v^*} \\ \rho_{v,v^*} & 1 \end{pmatrix}\right), \tag{18}$$

[14] After some algebra, it is possible to show here that $\left((\Pi_1)^2 + 2\rho_{a,a^*}\Pi_1\Pi_2 + (\Pi_2)^2\right) = \left(\left(\delta_{a,a^*} + (\delta_a - 1)\right)^2 + 2\left(\rho_{a,a^*} - 1\right)(\delta_a - 1)\,\delta_{a,a^*}\right) + 2\left(\rho_{a,a^*} - 1\right)(1-\Pi)\,\Pi\left(\delta_{a,a^*} - (\delta_a - 1)\right)^2$.

[15] The preference ratio $\frac{\gamma}{\varphi+\gamma}$ plays a critical role in increasing the cross-correlation while further reducing the volatility of the natural rate for open economies relative to their closed-economy counterparts as $\frac{\gamma}{\varphi+\gamma} \searrow 0$. Simulations and further details on the persistence of the natural rate are available from the author upon request.

where

$$\sigma_v^2 \equiv \sigma_u^2 \left((1-\xi)^2 + 2\rho_{u,u^*} (1-\xi)\xi + \xi^2 \right), \tag{19}$$

$$\rho_{v,v^*} \equiv \frac{\rho_{u,u^*} (1-\xi)^2 + 2(1-\xi)\xi + \rho_{u,u^*}\xi^2}{(1-\xi)^2 + 2\rho_{u,u^*} (1-\xi)\xi + \xi^2}, \tag{20}$$

define the volatility and the correlation of the trade-weighted cost-push shock innovations. Here, the closed-economy counterpart ($\xi = 0$) implies that $\sigma^2_{v,closed} = \sigma^2_u$ and $\rho_{v,v^*,closed} = \rho_{u,u^*}$. Finally, the Home and Foreign monetary shock processes, $\widehat{m}_t$ and $\widehat{m}^*_t$, in Table 2 do not require any transformation as they enter directly into the linear rational expectations model through the specification of the monetary policy rule of each country.

3 Monetary Policy Framework

The Home and Foreign monetary policy rules close the model specification, defining a particular monetary policy regime and playing a crucial role in the international transmission of shocks. In Table 1, monetary policy is modelled with a Taylor (1993)-type rule reacting to local conditions as given by each country's inflation and output gap alone when set independently (and asymmetrically). The persistence in policy rates arises from the policy shocks reflecting inertia that is *extrinsic* or exogenous to the policymaking process and out of the policymakers' control. The policy rules can be rewritten in terms of aggregate variables and cross-country differences taking into account that the policy responses can vary across countries.

I define aggregate variables in this two-country setting generically as $\widehat{g}^W_t \equiv \frac{1}{2}\widehat{g}_t + \frac{1}{2}\widehat{g}^*_t$ using production weights and label the differences between the two countries as $\widehat{g}^R_t \equiv \widehat{g}_t - \widehat{g}^*_t$. It follows that any pair of Home and Foreign variables, $\widehat{g}_t$ and $\widehat{g}^*_t$, respectively, can be decomposed as

$$\widehat{g}_t = \widehat{g}^W_t + \frac{1}{2}\widehat{g}^R_t,\ \widehat{g}^*_t = \widehat{g}^W_t - \frac{1}{2}\widehat{g}^R_t, \tag{21}$$

where the superscript identifies the aggregates (W) and the differences (R). Differences across countries can also be expressed in deviations from the aggregates, i.e., $\widehat{g}_t - \widehat{g}^W_t = \frac{1}{2}\widehat{g}^R_t$ and $\widehat{g}^*_t - \widehat{g}^W_t = -\frac{1}{2}\widehat{g}^R_t$. Given the dynamics for $\widehat{g}^W_t$ and $\widehat{g}^R_t$, the transformation in (21) backs out the corresponding variables for each country, $\widehat{g}_t$ and $\widehat{g}^*_t$. With this notation, I can cast the Home and Foreign monetary policy rules in Table 1 in the following canonical form

$$\begin{pmatrix} \widehat{i}^W_t \\ \widehat{i}^R_t \end{pmatrix} \approx \begin{pmatrix} \psi^W_{\pi,W} & \psi^W_{x,W} \\ \psi^R_{\pi,W} & \psi^R_{x,W} \end{pmatrix} \begin{pmatrix} \widehat{\pi}^W_t \\ \widehat{x}^W_t \end{pmatrix} + \begin{pmatrix} \psi^W_{\pi,R} & \psi^W_{x,R} \\ \psi^R_{\pi,R} & \psi^R_{x,R} \end{pmatrix} \begin{pmatrix} \widehat{\pi}^R_t \\ \widehat{x}^R_t \end{pmatrix} + \begin{pmatrix} \widehat{m}^W_t \\ \widehat{m}^R_t \end{pmatrix}, \tag{22}$$

where $\widehat{i}^W_t$ is the aggregate short-term nominal interest rate ($\widehat{i}^R_t$ differential nominal interest rate), $\widehat{\pi}^W_t$ is global inflation ($\widehat{\pi}^R_t$ differential inflation), and $\widehat{x}^W_t$ is the global output gap ($\widehat{x}^R_t$ differential slack). Here, $\widehat{m}^W_t$ is the aggregate monetary policy shock ($\widehat{m}^R_t$ is the differential monetary policy shock).

I define the aggregate coefficients on monetary policy as $\psi^W_\pi \equiv \frac{\psi_\pi + \psi^*_\pi}{2}$ and $\psi^W_x \equiv \frac{\psi_x + \psi^*_x}{2}$, and the differential coefficients as $\psi^R_\pi \equiv \psi_\pi - \psi^*_\pi$ and $\psi^R_x \equiv \psi_x - \psi^*_x$. Independently-set and potentially asymmetric monetary policy rules can then be defined in relation to (22). With the definitions of the transformed policy coefficients, I can describe any pair of Home and Foreign monetary policy rules as those in Table 1 within the framework given by (22) as follows,

Definition 1 *A monetary policy equilibrium with independent monetary policy rules responding to local conditions with varying sensitivities on their policy objectives—i.e., where* $\psi_\pi \neq \psi_\pi^*$ *and/or* $\psi_x \neq \psi_x^*$*—can be represented in the form of* (22) *with the following coefficients:* $\psi_{\pi,W}^W = \psi_{\pi,R}^R = \psi_\pi^W$, $\psi_{x,W}^W = \psi_{x,R}^R = \psi_x^W$, $\psi_{\pi,W}^R = 4\psi_{\pi,R}^W = \psi_\pi^R$*, and* $\psi_{x,W}^R = 4\psi_{x,R}^W = \psi_x^R$.

Hence, the Taylor (1993) rules for the aggregate and difference sub-systems whenever monetary policies are set independently across countries can be summarized simply as

$$\begin{pmatrix} \widehat{i}_t^W \\ \widehat{i}_t^R \end{pmatrix} \approx \begin{pmatrix} \psi_\pi^W & \psi_x^W \\ \psi_\pi^R & \psi_x^R \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t^W \\ \widehat{x}_t^W \end{pmatrix} + \begin{pmatrix} \frac{\psi_\pi^R}{4} & \frac{\psi_x^R}{4} \\ \psi_\pi^W & \psi_x^W \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t^R \\ \widehat{x}_t^R \end{pmatrix} + \begin{pmatrix} \widehat{m}_t^W \\ \widehat{m}_t^R \end{pmatrix}. \tag{23}$$

I can then consider the case where there is international monetary policy coordination among the two countries which can also be defined in relation to the framework in (22) as follows,

Definition 2 *A coordinated monetary policy equilibrium is characterized by a common monetary policy rule responding to local conditions only that is followed by both countries. Therefore, international monetary policy coordination requires common Taylor (1993) rule coefficients in both countries—i.e.,* $\psi_\pi = \psi_\pi^* = \psi_\pi^c$ *and* $\psi_x = \psi_x^* = \psi_x^c$*. Hence, in the context of* (22)*, this implies that* $\psi_{\pi,W}^W = \psi_{\pi,R}^R = \psi_\pi^W$ *and* $\psi_{x,W}^W = \psi_{x,R}^R = \psi_x^W$ *as well as* $\psi_{\pi,W}^R = \psi_{x,W}^R = \psi_{\pi,R}^W = \psi_{x,R}^W = 0$*. In this case the aggregate and difference coefficients satisfy that* $\psi_\pi^W = \psi_\pi^c$ *and* $\psi_x^W = \psi_x^c$ *while* $\psi_\pi^R = \psi_x^R = 0$.

Under international monetary policy coordination, the interaction terms drop out from the equation in (22) so that the aggregate policy equation depends only on aggregate variables while the difference equation depends only on difference variables. Hence, the Taylor (1993) rules for the aggregate and difference sub-systems for the coordinated monetary policy equilibrium can be summarized in the following terms

$$\begin{pmatrix} \widehat{i}_t^{c,W} \\ \widehat{i}_t^{c,R} \end{pmatrix} \approx \begin{pmatrix} \psi_\pi^c & \psi_x^c \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t^{c,W} \\ \widehat{x}_t^{c,W} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \psi_\pi^c & \psi_x^c \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t^{c,R} \\ \widehat{x}_t^{c,R} \end{pmatrix} + \begin{pmatrix} \widehat{m}_t^W \\ \widehat{m}_t^R \end{pmatrix}. \tag{24}$$

Naturally, (24) is a special case of (22) where monetary policy is coordinated and symmetric across countries. The superscript c is used to denote the coordinated monetary policy equilibrium.

International monetary policy coordination does not necessarily imply that the short-term nominal interest rates equalize across countries. Coordination is a step in the direction of achieving greater monetary integration but does not establish the same level of policy integration as a monetary (or currency) union. A monetary union has two distinct features: First, a currency union implies that the flexible nominal exchange rate is one at each point in time—so the unit of account is the same in both countries. Second, the common monetary policy responds to aggregate economic conditions rather than to the local conditions in each country.

Implicit in the two-country model with complete international asset markets is the fact that the *uncovered interest rate parity (UIP) condition* must hold up to a first-order approximation. Hence, given that the UIP condition holds here, setting the nominal exchange rate to be one in every period is equivalent to imposing that the short-term nominal interest rates must be equalized across countries in equilibrium (i.e., $\widehat{i}_t = \widehat{i}_t^*$).

This implies that the common monetary authority for the union is left with one instrument only for the conduct of monetary policy. Moreover, the common monetary policy set by the authorities responds to aggregate rather than local developments.

Therefore, a monetary union can be defined in relation to (22) as,

Definition 3 *A monetary or currency union equilibrium is characterized by a common monetary policy rule applied in both countries where the policy rate responds to the economic conditions of the union (rather than to local developments)—i.e., $\psi^W_{\pi,W} = \psi^W_\pi = \psi^c_\pi$ and $\psi^W_{x,W} = \psi^W_x = \psi^c_x$. As in the case with monetary policy coordination, there are no interactions between the aggregate and differential policy equations either—i.e., $\psi^R_{\pi,W} = \psi^R_{x,W} = \psi^W_{\pi,R} = \psi^W_{x,R} = 0$. However, interest rate equalization across countries imposes a non-trivial departure from the coordination case shown in (24) as it requires—in the notation of (22)—that $\psi^R_{\pi,R} = 0$ and $\psi^R_{x,R} = 0$. Moreover, it also assumes that $\widehat{m}^R_t = 0$.*

Hence, the Taylor (1993) rules for the aggregate and difference sub-systems for the monetary union equilibrium can be summarized as follows,

$$\begin{pmatrix} \widehat{i}^{mu,W}_t \\ \widehat{i}^{mu,R}_t \end{pmatrix} \approx \begin{pmatrix} \psi^c_\pi & \psi^c_x \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{\pi}^{mu,W}_t \\ \widehat{x}^{mu,W}_t \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \widehat{\pi}^{mu,R}_t \\ \widehat{x}^{mu,R}_t \end{pmatrix} + \begin{pmatrix} \widehat{m}^W_t \\ \widehat{m}^R_t \end{pmatrix}, \tag{25}$$

$$\widehat{m}^R_t = 0.$$

The superscript mu is used to denote the monetary union equilibrium. In this context, forming a monetary union does not change the perception of aggregate monetary policy relative to the international monetary policy coordination case. However, forming a monetary union has significant implications for the dynamics of the two-country model because—unlike under international monetary coordination—it requires: (a) the responses on the policy difference equation to differ from those of the aggregate policy equation; and (b) it also imposes that $\widehat{m}_t = \widehat{m}^*_t = \widehat{m}^W_t$. This then ensures interest rate equalization across countries.[16]

4 Inspecting the Monetary Policy Mechanism

I orthogonalize the linear rational expectations system described in Table 1 and Table 2 to re-express it as two separate and smaller sub-systems for aggregates and for differences between Home and Foreign variables using the corresponding definitions introduced in Section 3. This orthogonalization approach focuses our analysis of monetary policy across countries on its impact in the aggregate variables and on the cross-country dispersion. The Taylor (1993) rules for the aggregate and difference sub-systems can be re-written in canonical form as shown in (22) − (25).

The NKPC equations for the aggregate and difference sub-systems can be cast into the following form

$$\widehat{\pi}^s_t = \beta \mathbb{E}_t \left(\widehat{\pi}^s_{t+1} \right) + \Phi \left(\varphi + \gamma \right) \left[\kappa^s \widehat{x}^s_t + \widehat{v}^s_t \right], \text{ for } s = W, R, \tag{26}$$

[16] The requirement that $\widehat{m}_t = \widehat{m}^*_t = \widehat{m}^W_t$ is equivalent to saying that country-specific monetary policy shocks are perfectly correlated (i.e., $\rho_{m,m^*} \equiv 1$). To ensure that this perfectly correlated monetary shock has the same volatility as the aggregate of the country specific shocks presented in Table 2, I need to scale the volatility accordingly to be $\sigma^2_m \left(\frac{1+\rho_{m,m^*}}{2} \right)$. In turn, the persistence of the monetary shock δ_m remains unchanged.

where $\mathbb{E}_t(.)$ are expectations formed conditional on information up to time t, and $\widehat{v}_t^W = \widehat{u}_t^W$ is the global cost-push shock ($\widehat{v}_t^R = (1-2\xi)\,\widehat{u}_t^R$ differential cost-push shock). Furthermore, $\kappa^W \equiv 1$ is the composite for the slope on the global slack and $0 < \kappa^R \equiv (2\kappa - 1) < 1$ is the slope on differential slack. I observe that $0 < \kappa^R < 1$ holds for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \max\left\{0, 1 - \left(\frac{1}{2\xi + \left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$ given the results on the composite coefficient κ summarized in (3). In turn, whenever $\frac{\gamma}{\varphi+\gamma} > \frac{1-(2\xi)(2(1-\xi))}{(1-2\xi)(2(1-\xi))}$, if the trade elasticity σ is low enough that it lies within the non-empty range $0 < \sigma\gamma < 1 - \left(\frac{1}{2\xi + \left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}$ then it holds instead that $\kappa > 1$ and accordingly that $\kappa^R > 1$.

The dynamic IS equations for the aggregate and difference sub-systems are given by

$$\gamma\left(\mathbb{E}_t\left[\widehat{x}_{t+1}^s\right] - \widehat{x}_t^s\right) = \Omega^s\left(\widehat{i}_t^s - \mathbb{E}_t\left[\widehat{\pi}_{t+1}^s\right] - \widehat{\overline{r}}_t^s\right), \text{ for } s = W, R, \tag{27}$$

where the Fisher equations help express the aggregate real interest rate $\widehat{r}_t^W$ (differential real rate $\widehat{r}_t^R$) in terms of the aggregate short-term nominal interest rate $\widehat{i}_t^W$ (differential nominal interest rate $\widehat{i}_t^R$) net of expected aggregate inflation $\mathbb{E}_t\left[\widehat{\pi}_{t+1}^W\right]$ (expected differential inflation $\mathbb{E}_t\left[\widehat{\pi}_{t+1}^R\right]$). Here, $\widehat{\overline{r}}_t^W$ is the global natural rate ($\widehat{\overline{r}}_t^R$ differential natural rate). Furthermore, $\Omega^W \equiv 1$ is the slope on the aggregate real interest rate in deviations from the natural rate and $\Omega^R \equiv (2\Omega - 1) > 1$ is the slope on the differential real interest rate gap. I observe that $\Omega^R > 1$ holds for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right) > 0$ given the results for the composite coefficient Ω summarized in (4). In turn, $0 < \Omega^R < 1$ if $0 < \sigma\gamma < \left(\frac{1-2\xi}{1+(1-2\xi)}\right)$.

4.1 Independent Monetary Policies

The interactions among aggregate and difference variables through the policy rules in (22) imply that the aggregate and difference sub-systems given by (26)−(27) cannot be solved separately from each other whenever monetary policy is asymmetric across countries. However, the model can still be solved in deviations from the coordinated (common) monetary policy equilibrium. To establish this, I define generically any variable $\widehat{g}_t^s$ for $s = W, R$ as $\widehat{g}_t^s = \widehat{g}_t^{c,s} + \widehat{g}_t^{d,s}$ where $\widehat{g}_t^{c,s}$ corresponds to the solution under the coordinated (common) monetary policy equilibrium and $\widehat{g}_t^{d,s} \equiv \widehat{g}_t^s - \widehat{g}_t^{c,s}$ is the solution of the model with asymmetric monetary policy across countries in deviations from the coordinated monetary policy equilibrium. The superscript d refers to the deviations from the coordinated policy equilibrium under asymmetric monetary policy, while the superscript c denotes the coordinated monetary policy equilibrium as before.

Using (24) together with the expectational equations in (26) − (27) for $s = W, R$, I can separately write the sub-systems for aggregates and for the cross-country differences under a coordinated (common) monetary policy as follows

$$\begin{aligned}\begin{pmatrix} \widehat{x}_t^{c,s} \\ \widehat{\pi}_t^{c,s} \end{pmatrix} = {} & \begin{pmatrix} 1 & \frac{\Omega^s}{\gamma} \\ \Phi(\varphi+\gamma)\kappa^s & \beta + \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix}\begin{pmatrix} \mathbb{E}_t\left[\widehat{x}_{t+1}^{c,s}\right] \\ \mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s}\right] \end{pmatrix} - \begin{pmatrix} \frac{\Omega^s}{\gamma} \\ \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix}\widehat{i}_t^{c,s} + \ldots \\ & \begin{pmatrix} 0 & \frac{\Omega^s}{\gamma} \\ \Phi(\varphi+\gamma) & \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix}\begin{pmatrix} \widehat{v}_t^s \\ \widehat{\overline{r}}_t^s \end{pmatrix},\end{aligned} \tag{28}$$

$$\widehat{i}_t^{c,s} = \begin{pmatrix} \psi_x^c & \psi_\pi^c \end{pmatrix}\begin{pmatrix} \widehat{x}_t^{c,s} \\ \widehat{\pi}_t^{c,s} \end{pmatrix} + \widehat{m}_t^s, \text{ for } s = W, R, \tag{29}$$

where the vector of endogenous variables is $\left(\widehat{x}_t^{c,s}, \widehat{\pi}_t^{c,s}, \widehat{i}_t^{c,s}\right)$ for $s = W, R$.

The coordinated monetary policy coefficients are the same as the weighted aggregates of each country's policy coefficients under independent monetary policies—i.e., $\psi_\pi^c = \psi_\pi^W$ and $\psi_x^c = \psi_x^W$. Hence, taking the difference between (23) and (24), I obtain that

$$\begin{pmatrix} \widehat{i}_t^{d,W} \\ \widehat{i}_t^{d,R} \end{pmatrix} \approx \begin{pmatrix} \psi_\pi^W & \psi_x^W \\ \psi_\pi^R & \psi_x^R \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t^{d,W} \\ \widehat{x}_t^{d,W} \end{pmatrix} + \begin{pmatrix} \frac{\psi_\pi^R}{4} & \frac{\psi_x^R}{4} \\ \psi_\pi^W & \psi_x^W \end{pmatrix} \begin{pmatrix} \widehat{\pi}_t^{d,R} \\ \widehat{x}_t^{d,R} \end{pmatrix} + \begin{pmatrix} \left(\frac{\psi_\pi^R}{4}\right)\widehat{\pi}_t^{c,R} + \left(\frac{\psi_x^R}{4}\right)\widehat{x}_t^{c,R} \\ \psi_\pi^R \widehat{\pi}_t^{c,W} + \psi_x^R \widehat{x}_t^{c,W} \end{pmatrix}. \quad (30)$$

In turn, the equations that describe the aggregate and difference sub-systems in (28) are exactly the same for the coordinated monetary policy equilibrium and the asymmetric monetary policy equilibrium because: (a) the policy coefficients do not enter into the composite coefficients on the structural relations given by the model—the Phillips curve and IS equations—and (b) all other deep structural parameters of the model are common across countries.

Therefore, I can derive the following representation in deviations taking the difference between the corresponding equations with asymmetric policy coefficients and with common policy coefficients,

$$\begin{pmatrix} \widehat{x}_t^{d,s} \\ \widehat{\pi}_t^{d,s} \end{pmatrix} = \begin{pmatrix} 1 & \frac{\Omega^s}{\gamma} \\ \Phi(\varphi+\gamma)\kappa^s & \beta + \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix} \begin{pmatrix} \mathbb{E}_t\left[\widehat{x}_{t+1}^{d,s}\right] \\ \mathbb{E}_t\left[\widehat{\pi}_{t+1}^{d,s}\right] \end{pmatrix} - \begin{pmatrix} \frac{\Omega^s}{\gamma} \\ \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix} \widehat{i}_t^{d,s}, \text{ for } s = W, R, \quad (31)$$

which shows that the impact on slack and inflation $\left(\widehat{x}_t^{d,s}, \widehat{\pi}_t^{d,s}\right)$ from policy asymmetries across countries can only arise through their effect on the policy rate deviations $\widehat{i}_t^{d,s}$ for $s = W, R$ given by (30). Hence, policy rate deviations from the coordinated equilibrium are the only channel through which monetary policy gets transmitted and affects slack deviations $(\widehat{x}_t^{d,s})$ and inflation deviations $(\widehat{\pi}_t^{d,s})$ in the model.

Deviations from the coordinated equilibrium occur when at least one of the policy coefficients differs across countries—i.e., either $\psi_\pi^R \neq 0$ and/or $\psi_x^R \neq 0$. Otherwise, the only solution possible for the interest rate deviations from (30) is that $\widehat{i}_t^{d,W} = \widehat{i}_t^{d,R} = 0$ and, given (31), this implies that $\widehat{x}_t^{d,W} = \widehat{x}_t^{d,R} = \widehat{\pi}_t^{d,W} = \widehat{\pi}_t^{d,R} = 0$. In short, without asymmetric policies, the solution is fully characterized by the coordinated monetary policy equilibrium whereby $\widehat{x}_t^s = \widehat{x}_t^{c,s}$, $\widehat{\pi}_t^s = \widehat{\pi}_t^{c,s}$, and $\widehat{i}_t^s = \widehat{i}_t^{c,s}$ for $s = W, R$.

Interestingly, for cases in which independent monetary policies lead to asymmetric policy coefficients in the two-country model, I note that the dynamics of the vector of policy rates in deviations given by (30) are purely backward-looking and their random driving processes are given solely by the slack and inflation solution in the coordinated monetary policy equilibrium case. Hence, the solution to the coordinated equilibrium ultimately also characterizes the dynamics of the endogenous variables in deviations and the full solution of the model as well. There is no role for monetary policy shocks in the solution of $\left(\widehat{x}_t^{d,s}, \widehat{\pi}_t^{d,s}, \widehat{i}_t^{d,s}\right)$—neither for productivity nor for cost-push shocks—except through their impact on the equilibrium solution $\left(\widehat{x}_t^{c,s}, \widehat{\pi}_t^{c,s}, \widehat{i}_t^{c,s}\right)$ under a common coordinated monetary policy.

Furthermore, it also follows that,

Proposition 1 *The implication from* (30) *and* (31) *is that deviations from the coordinated equilibrium under independent and asymmetric monetary policies cannot be solved separately for the aggregate and difference sub-systems given that* (31) *implies that policy rate in deviations—whether for the aggregate or for the differential sub-systems—depend on both aggregate and difference variables on inflation and slack.*

In light of this, the effects of monetary policy asymmetries can be interpreted as a modification (or transformation) of the common monetary policy equilibrium which introduces non-separabilities between the aggregate solution and the solution that explains the cross-country dispersion. An important takeaway from all of this is that a common monetary policy undoes the modification of the dynamic propagation of shocks that arises from policy asymmetries. It also leads to a coordinated solution whereby aggregate and differential dynamics are perfectly separable (with no spillovers between them).

4.2 International Coordination vs. Monetary Union

Using the aggregate and differential monetary policy rules under monetary policy coordination in (24) to replace $\widehat{i}_t^{c,s}$ in (28) for $s = W, R$, the sub-system of equations that determines inflation and slack for the aggregates and for the cross-country differentials can be written in the following form

$$\begin{pmatrix} 1+\frac{\Omega^s}{\gamma}\psi_x^c & \frac{\Omega^s}{\gamma}\psi_\pi^c \\ \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s\psi_x^c & 1+\frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s\psi_\pi^c \end{pmatrix}\begin{pmatrix} \widehat{x}_t^{c,s} \\ \widehat{\pi}_t^{c,s} \end{pmatrix} = \begin{pmatrix} 1 & \frac{\Omega^s}{\gamma} \\ \Phi(\varphi+\gamma)\kappa^s & \beta+\frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix}\begin{pmatrix} \mathbb{E}_t\left[\widehat{x}_{t+1}^{c,s}\right] \\ \mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s}\right] \end{pmatrix} + \begin{pmatrix} 0 & \frac{\Omega^s}{\gamma} \\ \Phi(\varphi+\gamma) & \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix}\begin{pmatrix} \widehat{v}_t^s \\ \widehat{\overline{r}}_t^s - \widehat{m}_t^s \end{pmatrix}, \tag{32}$$

or more compactly as

$$\begin{pmatrix} \widehat{x}_t^{c,s} \\ \widehat{\pi}_t^{c,s} \end{pmatrix} = \Psi^{c,s}\begin{pmatrix} 1 & \frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right) \\ \Phi(\varphi+\gamma)\kappa^s & \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right) \end{pmatrix}\begin{pmatrix} \mathbb{E}_t\left[\widehat{x}_{t+1}^{c,s}\right] \\ \mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s}\right] \end{pmatrix} + \dots \\ \Psi^{c,s}\begin{pmatrix} -\frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\psi_\pi^c & \frac{\Omega^s}{\gamma} \\ \Phi(\varphi+\gamma)\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right) & \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix}\begin{pmatrix} \widehat{v}_t^s \\ \widehat{\overline{r}}_t^s - \widehat{m}_t^s \end{pmatrix}, \tag{33}$$

where $\Psi^{c,s} \equiv \frac{1}{1+\frac{\Omega^s}{\gamma}\left(\psi_x^c+\Phi(\varphi+\gamma)\kappa^s\psi_\pi^c\right)} > 0$, $\Phi(\varphi+\gamma) > 0$, $\psi_\pi^c > 0$, and $\psi_x^c \geq 0$. Moreover, for any degree of openness $0 < \xi < \frac{1}{2}$, I have noted that $\Omega^W = \kappa^W = 1$, $\Omega^R > 1$ for all $\sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right) > 0$, and $0 < \kappa^R < 1$ for all $\sigma\gamma > \max\left\{0, 1-\left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$.

I can write the aggregate and difference sub-systems of expectational equations in (33) in canonical form as

$$\widehat{z}_t^{c,s} = A^{c,s}\mathbb{E}_t\left(\widehat{z}_{t+1}^{c,s}\right) + B^{c,s}\widehat{\varepsilon}_t^s, \text{ for } s = W, R, \tag{34}$$

where the vector $\widehat{z}_t^{c,s} \equiv \left(\widehat{x}_t^{c,s}, \widehat{\pi}_t^{c,s}\right)^T$ includes inflation ($\widehat{\pi}_t^{c,s}$) and slack ($\widehat{x}_t^{c,s}$) for $s = W, R$ under a common monetary policy and the driving processes can be represented by the vector $\widehat{\varepsilon}_t^s = \left(\widehat{v}_t^s, \widehat{\overline{r}}_t^s - \widehat{m}_t^s\right)^T$. The matrices of structural parameters $A^{c,s} \equiv \Psi^{c,s}\begin{pmatrix} 1 & \frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right) \\ \Phi(\varphi+\gamma)\kappa^s & \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right) \end{pmatrix}$ and $B^{c,s} \equiv \Psi^{c,s}\begin{pmatrix} -\frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\psi_\pi^c & \frac{\Omega^s}{\gamma} \\ \Phi(\varphi+\gamma)\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right) & \frac{\Omega^s}{\gamma}\Phi(\varphi+\gamma)\kappa^s \end{pmatrix}$ characterize the dynamics of each sub-system. Hence, inflation and slack depend on both cost-push shocks ($\widehat{v}_t^s$) and on deviations between the natural rate ($\widehat{\overline{r}}_t^s$) and the monetary policy shocks ($\widehat{m}_t^s$). It is worth noticing here that the propagation of monetary shock innovations in the model is largely the same as that of innovations to the natural rate but of the opposite sign. In turn, cost-push shocks have distinct effects on inflation and slack and propagate differently.

The degree of openness ξ does not enter into the aggregate sub-system ($s = W$) described here and neither does the intratemporal trade elasticity of substitution between Home and Foreign goods σ. Therefore, neither the composition of the consumption basket nor the degree of substitutability between local and imported goods affects the aggregate allocation. In other words, the strength of the trade channel does not influence the aggregate dynamics in the coordinated monetary policy equilibrium. The only deep structural parameters that affect the aggregate dynamics are the Calvo (1983) parameter α, the intertemporal discount factor β, the inverse of the intertemporal elasticity of substitution γ, the inverse of the Frisch elasticity of labor supply φ, and the policy parameters ψ_π^c and ψ_x^c.

In turn, the deep structural parameters that determine the strength of the trade channel (ξ and σ) affect the cross-country dispersion only under a common monetary policy rule through the differential sub-system $s = R$—appearing in the composite coefficients Ω^R and κ^R.

The Stochastic Forcing Processes. Given the characterization of the natural rates for each country based on the frictionless allocation and the productivity shocks (equations (9) − (14)) and the maintained assumptions on the cost-push and monetary shocks (shown in Table 2), I derive the stochastic forcing processes for $\widehat{\overline{r}}_t^s$, $\widehat{m}_t^s$, and $\widehat{v}_t^s$ for $s = W, R$ (see the Appendix for further details). The forcing processes can be described as follows

$$\widehat{\overline{r}}_t^s = \delta_r^s \widehat{\overline{r}}_{t-1}^s + \widehat{\varepsilon}_t^{rs},\ \widehat{\varepsilon}_t^{rs} \sim N\left(0, \sigma_{rs}^2\right), \tag{35}$$

$$\widehat{m}_t^s = \delta_m \widehat{m}_{t-1}^s + \widehat{\varepsilon}_t^{ms},\ \widehat{\varepsilon}_t^{ms} \sim N\left(0, \sigma_{ms}^2\right), \tag{36}$$

$$\widehat{v}_t^s = \delta_v \widehat{v}_{t-1}^s + \widehat{\varepsilon}_t^{vs},\ \widehat{\varepsilon}_t^{vs} \sim N\left(0, \sigma_{vs}^2\right). \tag{37}$$

The persistence of the natural rate and cost-push shocks is given by,

$$\delta_r^W = \delta_a + \delta_{a,a^*},\ \delta_r^R = \delta_a - \delta_{a,a^*}, \tag{38}$$

$$\delta_v = \delta_u, \tag{39}$$

while δ_m is the known persistence of the monetary policy shock process.

The volatility term for the aggregate natural rate can be tied to parameters of the productivity shock and other structural parameters of the model as

$$\begin{aligned} \sigma_{rW}^2 &\equiv \sigma_r^2 \left(\frac{1+\rho_{r,r^*}}{2}\right) = \sigma_a^2 \left(\frac{1+\rho_{a,a^*}}{2}\right) \gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \left((\Pi_1)^2 + 2\Pi_1\Pi_2 + (\Pi_2)^2\right) \\ &= \sigma_a^2 \left(\frac{1+\rho_{a,a^*}}{2}\right) \left[\gamma \left(\frac{1+\varphi}{\gamma+\varphi}\right) (\delta_{a,a^*} + (\delta_a - 1))\right]^2, \end{aligned} \tag{40}$$

given the derivations of σ_r^2 and ρ_{r,r^*} in (11) − (14) and the fact that $\Pi_1 + \Pi_2 = \delta_{a,a^*} + (\delta_a - 1) < 0$. Analogously, the volatility term for the difference natural rate can be tied to parameters of the productivity shock and other structural parameters of the model as

$$\begin{aligned} \sigma_{rR}^2 &\equiv 2\sigma_r^2\left(1-\rho_{r,r^*}\right) = 2\sigma_a^2\left(1-\rho_{a,a^*}\right)\gamma^2\left(\frac{1+\varphi}{\gamma+\varphi}\right)^2\left(\Pi_1-\Pi_2\right)^2 \\ &= 2\sigma_a^2\left(1-\rho_{a,a^*}\right)\left[\gamma\left(\frac{1+\varphi}{\gamma+\varphi}\right)(2\Theta-1)(2\Lambda-1)\left(\delta_{a,a^*}-(\delta_a-1)\right)\right]^2, \end{aligned} \tag{41}$$

given the derivations of σ_r^2 and ρ_{r,r^*} in (11) – (14).[17] The volatility terms for the aggregate and difference monetary policy shocks are given as

$$\sigma_{mW}^2 \equiv \sigma_m^2\left(\frac{1+\rho_{m,m^*}}{2}\right),\ \sigma_{mR}^2 \equiv 2\sigma_m^2\left(1-\rho_{m,m^*}\right), \tag{42}$$

which depend solely on the variance-covariance of the monetary shocks. Finally, the volatility terms for the aggregate and difference cost-push shocks are given as

$$\sigma_{vW}^2 \equiv \sigma_u^2\left(\frac{1+\rho_{u,u^*}}{2}\right),\ \sigma_{vR}^2 \equiv 2\left(1-2\xi\right)^2\sigma_u^2\left(1-\rho_{u,u^*}\right), \tag{43}$$

which depend on the variance-covariance matrix of the country-specific cost-push shocks (but also on the degree of openness for the difference cost-push shocks).

The coordinated policy parameters, ψ_π^c and ψ_x^c, affect neither the aggregate driving processes nor the difference driving processes. The only structural parameters that affect the dynamics of the aggregate forcing processes are the inverse of the intertemporal elasticity of substitution γ and the inverse of the Frisch elasticity of labor supply φ and they influence only the volatility of the aggregate natural rate shock innovations. The parameters γ and φ also affect the volatility of the difference natural rate shock innovations.

However, in the case of the driving processes in differences, I observe that the degree of openness ξ and the trade elasticity of substitution between the Home and Foreign goods σ affect the volatility of the difference natural rate process σ_{rR}^2 (through the composite coefficients Θ and Λ) and the volatility of the difference cost-push process σ_{vR}^2 through the term $(1-2\xi)$ as well. The importance of the cost-push shocks to explain cross-country differences declines with the openness of the economy (ξ) because the volatility σ_{vR}^2 declines with it—which implies that both monetary as well as natural rate difference shocks may acquire a larger role the more open the economy becomes.

[17] Notice that $2(\Lambda-1) = \left(\frac{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)$ and $\Theta = (1-\xi)\left[\frac{1+(\sigma\gamma-1)(2\xi)}{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right]$ as in Table 1. Hence, it follows that $(\Pi_1-\Pi_2)$ implies that

$$\begin{aligned} (\Pi_1-\Pi_2) &= \left[1-2(1-\xi)\left(\frac{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]\left(\delta_{a,a^*}-(\delta_a-1)\right) \\ &= \left[(2\Lambda-1)-2(1-\xi)\left(\frac{1+(\sigma\gamma-1)(2\xi)}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]\left(\delta_{a,a^*}-(\delta_a-1)\right) \\ &= \left[(2\Lambda-1)-2\Theta\left(\frac{1+(\sigma\gamma-1)(2\xi)(2(1-\xi))}{1+\left(1-\frac{\gamma}{\varphi+\gamma}\right)(\sigma\gamma-1)(2\xi)(2(1-\xi))}\right)\right]\left(\delta_{a,a^*}-(\delta_a-1)\right) \\ &= -(2\Theta-1)(2\Lambda-1)\left(\delta_{a,a^*}-(\delta_a-1)\right). \end{aligned}$$

Finally, I want to point out that while the coordinated policy coefficients, ψ_{π}^{c} and ψ_{x}^{c}, do not influence the driving processes either for the aggregate case or for the differential case, deeper economic integration through the formation of a monetary union as defined in this paper will surely alter the monetary shock process. In that policy regime, the volatility of the aggregate σ_{mW}^{2} does not necessarily change relative to what would be implied by the maintained assumptions on the Home and Foreign monetary policy shocks in Table 2. However, as expected from perfectly correlated monetary shocks across countries, the volatility of the difference will then have to be set to $\sigma_{mR}^{2}=0$.

4.2.1 Determinacy Properties

Under the assumption that $\widehat{\varepsilon}_{t}^{s}$ is stationary, then (34) has a unique nonexplosive solution in which the vector $\widehat{z}_{t}^{c,s}\equiv\left(\widehat{x}_{t}^{c,s},\widehat{\pi}_{t}^{c,s}\right)^{T}$ is stationary whenever both eigenvalues of the matrix $A^{c,s}$ are inside the unit circle for each sub-system ($s=W,R$). The eigenvalues corresponding to the matrix $A^{c,s}$ can be written as

$$\lambda_{1}^{c,s}\equiv\frac{1}{2}\Psi^{c,s}\left(\Lambda^{c,s}-\sqrt[2]{\left(\Lambda^{c,s}\right)^{2}-4\frac{\beta}{\Psi^{c,s}}}\right),\ \lambda_{2}^{c,s}\equiv\frac{1}{2}\Psi^{c,s}\left(\Lambda^{c,s}+\sqrt[2]{\left(\Lambda^{c,s}\right)^{2}-4\frac{\beta}{\Psi^{c,s}}}\right),\tag{44}$$

where $\Psi^{c,s}\equiv\frac{1}{1+\frac{\Omega^{s}}{\gamma}\left(\psi_{x}^{c}+\Phi(\varphi+\gamma)\kappa^{s}\psi_{\pi}^{c}\right)}>0$ and $\Lambda^{c,s}\equiv1+\beta+\frac{\Omega^{s}}{\gamma}\left(\beta\psi_{x}^{c}+\Phi\left(\varphi+\gamma\right)\kappa^{s}\right)>0$ hold given that $\Phi\left(\varphi+\gamma\right)>0$ and the policy coefficients satisfy that $\psi_{\pi}^{c}>0$ and $\psi_{x}^{c}\geq0$. Moreover, for any degree of openness $0<\xi<\frac{1}{2}$, it also holds that $\Omega^{W}=\kappa^{W}=1$, $\Omega^{R}>1$ for all $\sigma\gamma>\left(\frac{1-2\xi}{1+(1-2\xi)}\right)>0$, and $0<\kappa^{R}<1$ for all $\sigma\gamma>\max\left\{0,1-\left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$. For standard parameterizations of the model, it naturally follows that $0<\lambda_{1}^{s}<\lambda_{2}^{s}$. Therefore, both eigenvalues of $A^{c,s}$ lie inside the unit circle if and only if $\lambda_{2}^{c,s}\equiv\frac{1}{2}\Psi^{c,s}\left(\Lambda^{c,s}+\sqrt[2]{\left(\Lambda^{c,s}\right)^{2}-4\frac{\beta}{\Psi^{c,s}}}\right)<1$. This inequality holds, in turn, whenever $\Psi^{c,s}\sqrt[2]{\left(\Lambda^{c,s}\right)^{2}-4\frac{\beta}{\Psi^{c,s}}}<2-\Psi^{c,s}\Lambda^{c,s}$. Taking the square on both sides of the inequality—i.e., $\left(\Psi^{c,s}\right)^{2}\left(\left(\Lambda^{c,s}\right)^{2}-4\frac{\beta}{\Psi^{c,s}}\right)<\left(\Psi^{c,s}\Lambda^{c,s}-2\right)^{2}$—and then, re-arranging terms, the inequality can be rewritten as: $\Psi^{c,s}\left(\Lambda^{c,s}-\beta\right)<1$. From here it follows that $\lambda_{2}^{s}<1$ if and only if $\frac{1+\frac{\Omega^{s}}{\gamma}\left(\beta\psi_{x}^{c}+\Phi(\varphi+\gamma)\kappa^{s}\right)}{1+\frac{\Omega^{s}}{\gamma}\left(\psi_{x}^{c}+\Phi(\varphi+\gamma)\kappa^{s}\psi_{\pi}^{c}\right)}<1$ or, after further algebraic manipulations, if and only if $\psi_{\pi}^{c}+\left(\frac{1-\beta}{\Phi(\varphi+\gamma)\kappa^{s}}\right)\psi_{x}^{c}>1$.

Proposition 2 *An open-economy variant of the Taylor principle which requires that* $\psi_{\pi}^{c}+\left(\frac{1-\beta}{\Phi(\varphi+\gamma)\kappa^{s}}\right)\psi_{x}^{c}>1$ *for each* $s=W,R$ *is needed to ensure the uniqueness and existence of the nonexplosive solution for the aggregate and differential sub-systems under a coordinated monetary policy equilibrium. The standard Taylor principle* ($\psi_{\pi}^{c}>1$) *is sufficient, but not necessary, to prove existence and uniqueness of the solution. Moreover, the open-economy Taylor principle reduces to the closed-economy variant which simply requires* $\psi_{\pi}^{c}+\left(\frac{1-\beta}{\Phi(\varphi+\gamma)}\right)\psi_{x}^{c}>1$ *whenever* $\sigma\gamma>\max\left\{0,1-\left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$.

Existence and uniqueness of a coordinated monetary policy equilibrium depends on the policy parameters $\psi_{\pi}^{c}>0$ and $\psi_{x}^{c}\geq0$. In the case where $\psi_{x}^{c}=0$, the standard Taylor principle requiring $\psi_{\pi}^{c}>1$ holds irrespective of the degree of openness ξ as it does in the closed-economy case. Whenever the common monetary policy involves a positive response to the output gap in each country ($\psi_{x}^{c}>0$), then the standard Taylor principle can be relaxed whereby $\psi_{\pi}^{c}>0$ can fall to some extent below one and still be consistent with determinacy given that $\left(\frac{1-\beta}{\Phi(\varphi+\gamma)\kappa^{s}}\right)>0$.

Here, I observe that $\kappa^W = 1$ and $0 < \kappa^R \equiv (2\kappa - 1) < 1$ for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \max\left\{0, 1 - \left(\frac{1}{2\xi + \left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$ (as implied by the results in (3)). Then, it must follow that $\kappa^R < \kappa^W = 1$ and $\psi_\pi^c + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)\kappa^R}\right)\psi_x^c > \psi_\pi^c + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)}\right)\psi_x^c$. As a result, the inequality $\psi_\pi^c + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)}\right)\psi_x^c > 1$ suffices to ensure existence and uniqueness of a solution for both the aggregate and differential sub-systems—irrespective of the openness to trade ξ or the trade elasticity σ. In turn, whenever $\frac{\gamma}{\varphi+\gamma} > \frac{1-(2\xi)(2(1-\xi))}{(1-2\xi)(2(1-\xi))}$, there are plausible values of the trade elasticity σ low enough such that $0 < \sigma\gamma < 1 - \left(\frac{1}{2\xi + \left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}$ and this implies that $\kappa^R > \kappa^W = 1$. In those cases, it follows that $\psi_\pi^c + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)\kappa^R}\right)\psi_x^c < \psi_\pi^c + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)}\right)\psi_x^c$ and, therefore, the inequality $\psi_\pi^c + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)\kappa^R}\right)\psi_x^c > 1$ is needed to ensure existence and uniqueness of a solution—where κ^R is a function of trade openness ξ and of the trade elasticity σ.

Hence, the determinacy of the solution to the open-economy model under a coordinated monetary policy does not depend on the degree of trade openness (ξ) and the trade elasticity (σ) for most plausible parameterizations unless the preference ratio $\frac{\gamma}{\varphi+\gamma}$ is sufficiently high and the trade elasticity σ is sufficiently low. Accordingly, the criterion to ensure existence and uniqueness of the open-economy solution is exactly the same as in the closed-economy case in most instances and, in particular, in the benchmark case where $\sigma\gamma > 1$.

The conventional Taylor principle ($\psi_\pi^c > 1$) is sufficient but not necessary in all cases where $\psi_x^c > 0$. Whenever $\sigma\gamma > \max\left\{0, 1 - \left(\frac{1}{2\xi + \left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$, the range of values for ψ_π^c below one that are consistent with determinacy depends on the intertemporal discount factor through $(1-\beta) > 0$ and depends also on the closed-economy slope of the Phillips curve $\Phi(\varphi+\gamma) > 0$. In the case where $\frac{\gamma}{\varphi+\gamma} > \frac{1-(2\xi)(2(1-\xi))}{(1-2\xi)(2(1-\xi))}$ and $0 < \sigma\gamma < 1 - \left(\frac{1}{2\xi + \left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}$, determinacy also depends on the strength of the trade channel through the composite coefficient κ^R (which is a function of the import share ξ and the trade elasticity σ). In short, the determinacy condition when $\psi_x^c > 0$ depends on how patient households are and how steep the closed-economy Phillips curve is—it depends on trade parameters only in rather special cases.

Assessing the Determinacy Properties on a Currency Union. From the definition of the monetary policy equilibrium under both international coordination and monetary union in Section 3, I note that both have the same aggregate policy equation but differ on the difference policy equation instead. Hence, it is straightforward that the counterpart of (34) for the aggregate case ($s = W$) is identical in both monetary policy regimes with the matrices of structural parameters in the monetary union case satisfying that $A^{mu,W} = A^{c,W}$ and $B^{mu,W} = B^{c,W}$. Therefore, an analogous set of derivations to the ones used for the coordinated monetary policy equilibrium case implies that determinacy of the aggregate in the monetary union case requires $\psi_\pi^c + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)}\right)\psi_x^c > 1$.

The counterpart of (34) for the differential sub-system under a monetary union can be represented with the following matrices of structural parameters: $A^{mu,R} \equiv \Psi^{mu,R}\begin{pmatrix} 1 & \frac{\Omega^R}{\gamma}\left(1 - \beta\psi_\pi^{mu}\right) \\ \Phi(\varphi+\gamma)\kappa^R & \frac{\Omega^R}{\gamma}\Phi(\varphi+\gamma)\kappa^R + \beta\left(1 + \frac{\Omega^R}{\gamma}\psi_x^{mu}\right) \end{pmatrix}$ and $B^{mu,R} \equiv \Psi^{mu,R}\begin{pmatrix} -\frac{\Omega^R}{\gamma}\Phi(\varphi+\gamma)\psi_\pi^{mu} & \frac{\Omega^R}{\gamma} \\ \Phi(\varphi+\gamma)\left(1 + \frac{\Omega^R}{\gamma}\psi_x^{mu}\right) & \frac{\Omega^R}{\gamma}\Phi(\varphi+\gamma)\kappa^R \end{pmatrix}$ where $\Psi^{mu,R} \equiv \frac{1}{1 + \frac{\Omega^R}{\gamma}\left(\psi_x^{mu} + \Phi(\varphi+\gamma)\kappa^R\psi_\pi^{mu}\right)} > 0$. Here, the policy parameters are written generically as ψ_π^{mu} and ψ_x^{mu} so that an analogous set of calculations to those discussed under a coordinated monetary policy equilibrium implies that determinacy of the solution to the difference sub-system under this generic representation of a monetary union equilibrium would require $\psi_{\pi}^{mu} + \left(\frac{1-\beta}{\Phi(\varphi+\gamma)\kappa^{R}}\right)\psi_{x}^{mu} > 1$. Given the definition of a monetary union proposed in Section 3, the only set of policy coefficients that are consistent with nominal interest rate equalization across countries are $\psi_{\pi}^{mu} = \psi_{x}^{mu} = 0$. As a result, I conclude that a monetary union is inconsistent with the determinacy of the difference sub-system.

Proposition 3 *The closed-economy variant of the Taylor principle which simply requires* $\psi_{\pi}^{c}+\left(\frac{1-\beta}{\Phi(\varphi+\gamma)}\right)\psi_{x}^{c} > 1$ *suffices to ensure determinacy of the aggregate dynamics within a monetary union that has a common monetary policy and results in short-term nominal interest rate equalization in equilibrium. In turn, the cross-country dispersion does not satisfy the conditions for determinacy. This holds true irrespective of the degree of openness* (ξ) *and the trade elasticity* (σ)*.*

This is an aspect of the formation of a monetary union that has not received much attention in the existing literature (Benigno (2004), Woodford (2010))—more focused on the positive and normative implications for the union as a whole rather than on the impact on each one of the local economies (or regions) that become part of the currency union. Intuitively, however, this is not a completely unexpected result either. The difference sub-system is insulated from the aggregate sub-system as is the case under international monetary policy coordination. As such, the difference sub-system is isomorphic to a closed-economy New Keynesian model where the interest rate (the interest rate differential in the difference sub-system case) is kept constant at zero in every period. The indeterminacy of constant interest rates within the closed-economy New Keynesian model is a well-known result already in the literature (Woodford (2003)).

4.2.2 Characterization of the Solution

I examine the equilibrium for the aggregate and difference variables in response to natural rate shocks, monetary policy shocks, and cost-push shocks. I focus on the solution to the coordinated monetary policy equilibrium only, but it follows from my preceding discussion that the solution for the aggregates is exactly the same under a monetary union (while the cross-country dispersion is indeterminate). The description of each sub-system ($s = W, R$) given by (34) is completed with the corresponding driving processes characterized in equations (35) – (37). A detailed derivation of the solution conditional on each type of shock is found in the Appendix.

The law of motion describing analytically the endogenous propagation of each individual shock $j = r, m, v$—i.e., for the natural rate shock ($j = r$), the monetary policy shock ($j = m$), and the cost-push shock ($j = v$)—on the endogenous variables $\left(\widehat{x}_{t}^{c,s,j}, \widehat{\pi}_{t}^{c,s,j}\right)$ for $s = W, R$ defines the unique equilibrium under a common (coordinated) monetary policy as

$$\widehat{\pi}_{t}^{c,s,j} = \chi_{0,j}^{c,s}\widehat{x}_{t}^{c,s,j}, \tag{45}$$

$$\widehat{\pi}_{t}^{c,s,j} = \chi_{1,j}^{c,s}\widehat{\pi}_{t-1}^{c,s,j} + \eta_{t}^{c,s,j},\ \eta_{t}^{c,s,j} \sim N\left(0, \sigma_{c,s,j}^{2}\right), \tag{46}$$

where the nominal short-term interest rate is given by $\widehat{i}_{t}^{c,s,j} = \psi_{\pi}^{c}\widehat{\pi}_{t}^{c,s,j} + \psi_{x}^{c}\widehat{x}_{t}^{c,s,j}$ for $j = r, v$ and is $\widehat{i}_{t}^{c,s,j} = \psi_{\pi}^{c}\widehat{\pi}_{t}^{c,s,j} + \psi_{x}^{c}\widehat{x}_{t}^{c,s,j} + \widehat{m}_{t}^{s}$ if $j = m$. Equation (45) identifies the policy trade-off between inflation and slack in each sub-system $s = W, R$ conditional on each individual shock $j = r, m, v$, while equation (46) states that inflation in each sub-system inherits the autoregressive structure of the corresponding driving process of each shock (equations (35) − (37)).

Natural Rate Shocks and Monetary Policy Shocks. I consider first the case where the sub-system in (34) is solely driven by either natural rate shocks (equation (35)) or monetary policy shocks (equation (36)), as both these shocks pose very different policy trade-offs than the cost-push shocks. Taking the conjectured solution in (45)−(46) as given and assuming that the common monetary policy is consistent with determinacy of the solution for both sub-systems, I can verify by the method of undetermined coefficients that,

Proposition 4 *The solution for $j = r, m$ and $s = W, R$ given in (45) − (46) satisfies that*

$$\chi^{c,s}_{1,r} = \delta^s_r, \; \chi^{c,s}_{1,m} = \delta_m, \tag{47}$$

$$\chi^{c,s}_{0,j} = \frac{\Phi(\varphi+\gamma)\kappa^s}{1-\beta\chi^{c,s}_{1,j}}, \; j = r, m, \tag{48}$$

$$\sigma^2_{c,s,j} = \left(\frac{\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}}{\left(1-\beta\chi^{c,s}_{1,j}\right)\left(1-\chi^{c,s}_{1,j}+\frac{\Omega^s}{\gamma}\psi^c_x\right)+\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}\left(\psi^c_\pi-\chi^{c,s}_{1,j}\right)}\right)^2\sigma^2_{js}, \; j = r, m, \tag{49}$$

where the innovations in (46) are defined as $\eta^{c,s,r}_t = \left(\frac{\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}}{\left(1-\beta\chi^{c,s}_{1,r}\right)\left(1-\chi^{c,s}_{1,r}+\frac{\Omega^s}{\gamma}\psi^c_x\right)+\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}\left(\psi^c_\pi-\chi^{c,s}_{1,r}\right)}\right)\widehat{\varepsilon}^{rs}_t$ when arising from natural rate shock innovations and as $\eta^{c,s,m}_t = -\left(\frac{\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}}{\left(1-\beta\chi^{c,s}_{1,m}\right)\left(1-\chi^{c,s}_{1,m}+\frac{\Omega^s}{\gamma}\psi^c_x\right)+\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}\left(\psi^c_\pi-\chi^{c,s}_{1,m}\right)}\right)\widehat{\varepsilon}^{ms}_t$ when they are the result of monetary policy shock innovations instead. The volatility terms for the shocks, σ^2_{js} for $j = r, m$ and for $s = W, R$, are defined in (40) − (42).

The key takeaways from this result are as follows:

1. The endogenous persistence implied by natural rate and monetary policy shocks ($\chi^{c,s}_{0,j}$ for $j = r, m$ and $s = W, R$) is entirely determined by the properties of the productivity and monetary shock processes, respectively. In other words, no deep structural parameters appear to affect the persistence of the endogenous variables—let alone those related to the strength of the trade channel (ξ and σ).

2. The positive comovement between inflation and slack given by $\chi^{c,s}_{0,j}$ depends critically on the slope of the Phillips curve. For the aggregate solution ($s = W$), only the slope of the closed-economy Phillips curve matters ($\kappa^W = 1$) and therefore is completely invariant to the openness (ξ) and the trade elasticity (σ) that characterize the trade channel in the model. In turn, the flattening of the Phillips curve discussed in Sub-section 2.2 shows up in the trade-off implicit in the differential solution ($s = R$). As noted before, $0 < \kappa^R < 1$ holds for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \max\left\{0, 1-\left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$ given the results on the composite coefficient κ summarized in (3). Figure 2.A and Figure 3.A illustrate that the trade-off $\chi^{c,R}_{0,j}$ for $j = r, m$ declines with the degree of openness (ξ) while the trade-off is lower the larger the trade elasticity (σ) is.[18] These findings show that the trade channel matters for the cross-country dispersion trade-off—implying that more

[18] I use the following standard parameterization to produce Figure 2.A, Figure 2.B, Figure 3.A, and Figure 3.B: $\beta = 0.99$, $\gamma = 2$, $\alpha = 0.75$, $\delta_a = 0.95$, $\delta_m = 0.9$, $\rho_{a,a^*} = \rho_{m,m^*} = 0.25$, and $\delta_{a,a^*} = 0.025$. The policy parameters are set as in the standard Taylor (1993) rule: $\psi^c_\pi = 1.5$ and $\psi^c_x = 0.5$. The volatility of the shocks is normalized to one in this case: $\sigma^2_a = \sigma^2_m = 1$.

open economies (with lower $\chi_{0,j}^{c,R}$, $j = r, m$) should experience larger movements in the output gap differential $\widehat{x}_t^{c,R,j}$ for any given change in the cross-country inflation differential $\widehat{\pi}_t^{c,R,j}$. Aggregate trade-offs should be unaffected by the trade channel. Moreover, the policy parameters ψ_π^c and ψ_x^c have no effect on either the aggregate or the differential trade-offs. Interestingly, the preference ratio $\frac{\gamma}{\varphi+\gamma}$ plays a very significant role, showing how important the features of the labor market (the inverse of the Frisch elasticity) can be.

3. Figure 1.A and Figure 2.A illustrate how $\chi_{0,j}^{c,R} < \chi_{0,j}^{c,W}$ for $j = r, m$ in the open-economy case ($0 < \xi < \frac{1}{2}$), while $\chi_{0,r}^{c,R} < \chi_{0,r}^{c,W}$ and $\chi_{0,m}^{c,R} = \chi_{0,m}^{c,W}$ for the closed-economy case ($\xi = 0$). This occurs because the model permits cross-country spillovers for the productivity shocks ($\delta_{a,a^*} \neq 0$). In fact, it holds that $\chi_{0,r}^{c,R} \lesseqqgtr \chi_{0,r}^{c,W}$ whenever $\delta_{a,a^*} \gtreqqless 0$. Hence, cross-country shock spillovers (diffusion) are an important feature of the economy with a sizeable quantitative impact on the equilibrium trade-offs that can arise between inflation and slack. In other words, even though more open economies generally face lower $\chi_{0,r}^{c,R}$ and $\chi_{0,m}^{c,R}$, the quantitative size of these effects will depend on key features of the economy such as the extent of technological diffusion across countries (δ_{a,a^*}) or the sensitivity of the labor supply to wages (φ).

4. The volatility of inflation $\sigma_{c,s,j}^2$ for $j = r, m$ and $s = W, R$ reveals the importance of the trade channel on accounting for the declines in macro volatility during the Great Moderation period. The more open economies generally experience lower differential inflation volatility from natural rate shocks and monetary policy shocks, and the volatility declines tend to be accentuated the higher the trade elasticity σ is. In turn, the trade parameters (ξ and σ) have no effect on the volatility of the aggregates. Figure 2.B and Figure 3.B illustrate the volatility implications of the model for a given one-standard-deviation shock on productivity impacting the natural rate and on monetary policy, respectively. On the one hand, I observe that the volatility on the cross-country inflation dispersion tends to be higher than the volatility of aggregate inflation for both shocks whenever the degree of openness (ξ) is sufficiently low. This relationship changes as ξ gets larger (closer to $\xi = \frac{1}{2}$) and the volatility differential converges to zero. On the other hand, the findings extend a well-known prediction of the closed-economy New Keynesian model to the open-economy case—it shows that the New Keynesian mechanism accentuates markedly the role of monetary policy shocks while it dilutes somewhat the role of productivity shocks (embedded in the natural rate shocks). Not surprisingly, this is a model in which monetary policy shocks end up playing a dominant role in explaining the variations in inflation and slack as well.[19]

5. I should also point out that the persistence of the productivity shock process is another crucial factor explaining the very low volatility pass-through shown in Figure 2.B—in particular, lower values of δ_a significantly increase the implied volatility (a reduction of δ_a from 0.95 to 0.85 will double and even more than triple the cross-country dispersion volatility depending on $\frac{\gamma}{\varphi+\gamma}$ and can raise aggregate volatility more than sevenfold). Moreover, the aggregate and differential volatility generally decline the higher the preference ratio $\frac{\gamma}{\varphi+\gamma}$ is—once again showcasing the importance of the inverse of the Frisch

[19] Actual output is equal to slack plus potential output ($\widehat{\overline{y}}_t^{s,c,j} + \widehat{x}_t^{s,c,j}$ for $s = W, R$ and for all shocks $j = r, m, v$). Potential output depends solely on productivity shocks. Hence, actual output and slack are the same for the monetary shocks ($j = m$) and cost-push shocks ($j = v$). While the contribution of productivity shocks to driving slack might be mitigated here, their effect on output potential has to be factored in when assessing the volatility of actual output. In this regard, it should be noted that open economies tend to experience more volatile potential.

elasticity of labor supply (φ) in this model. In general, I find that the stronger the anti-inflationary bias of the common (coordinated) monetary policy (the higher ψ_π^c is) or the higher the bias on economic activity (the higher ψ_x^c is), the lower the inflation volatility arising from natural rate shocks or monetary policy shocks (the lower $\sigma_{c,s,j}^2$ is for $j = r, m$ and $s = W, R$). Hence, I argue that in terms of explaining the decline in macro volatility over the past several decades (in particular during the Great Moderation period), both greater economic integration as well as monetary policies around the world more focused on fighting inflation may have contributed to this.

The autoregressive processes for inflation in (46) conditional on $j = r, m$ are independent of each other and can therefore be aggregated to describe the dynamics of inflation in response to the gap between the natural rate and the monetary policy shocks $\left(\widehat{\overline{r}}_t^s - \widehat{m}_t^s\right)$. I define the inflation process in this case as $\widehat{\pi}_t^{c,s,r-m} \equiv \widehat{\pi}_t^{c,s,r} + \widehat{\pi}_t^{c,s,m}$. In the knife-edge case where there are no productivity shock spillovers across countries ($\delta_{a,a^*} = 0$) and the monetary policy shocks are assumed to have the same persistence as productivity shocks do ($\delta_m = \delta_a$), then I can easily see that $\chi_{1,r}^{c,s} = \chi_{1,m}^{c,s}$, $\chi_{0,r}^{c,s} = \chi_{0,m}^{c,s}$, and $\frac{\sigma_{c,s,r}^2}{\sigma_{rs}^2} = \frac{\sigma_{c,s,m}^2}{\sigma_{ms}^2}$. In other words, whenever the bivariate productivity and monetary shock processes have common persistence and no cross-country spillovers, a one-standard-deviation increase in the natural rate innovation $\widehat{\varepsilon}_t^{rs}$ has the exact same pattern of propagation on inflation and slack as a one-standard-deviation decline on the monetary policy shock innovation $\widehat{\varepsilon}_t^{ms}$ for $s = W, R$. This also implies that $\widehat{\pi}_t^{c,s,r-m}$ follows an AR(1) process given by: $\widehat{\pi}_t^{c,s,r-m} = \chi_{1,r}^{c,s}\widehat{\pi}_{t-1}^{c,s,r-m} + \eta_t^{c,s,r-m}$, where $\eta_t^{c,s,r-m} \sim N\left(0, \sigma_{c,s,r-m}^2\right)$ and $\sigma_{c,s,r-m}^2 = \sigma_{c,s,r}^2 + \sigma_{c,s,m}^2$.

In general, however, natural rate shock innovations and monetary policy shock innovations are not the mirror image of each other as a result of differences in the key parameters that determine the persistence and cross-country spillovers of the underlying productivity and monetary shocks in Table 2. In the economically-relevant case where $\chi_{1,r}^{c,s} \neq \chi_{1,m}^{c,s}$, based on well-known aggregation results (Hamilton (1994), Chapter 4), I obtain that $\widehat{\pi}_t^{c,s,r-m}$ follows an ARMA$(2,1)$ process of the following form:[20]

$$\widehat{\pi}_t^{c,s,r-m} = \left(\chi_{1,m}^{c,s} + \chi_{1,r}^{c,s}\right)\widehat{\pi}_{t-1}^{c,s,r-m} - \chi_{1,r}^{c,s}\chi_{1,m}^{c,s}\widehat{\pi}_{t-2}^{c,s,r-m} + \eta_t^{c,s,r-m} + \theta_1^{c,s,r-m}\eta_{t-1}^{c,s,r-m}, \tag{50}$$

$$\eta_t^{c,s,r-m} \sim N\left(0, \sigma_{c,s,r-m}^2\right), \ \sigma_{c,s,r-m}^2 = -\left(\frac{\chi_{1,m}^{c,s}\sigma_{c,s,r}^2 + \chi_{1,r}^{c,s}\sigma_{c,s,m}^2}{\theta_1^{c,s,r-m}}\right), \tag{51}$$

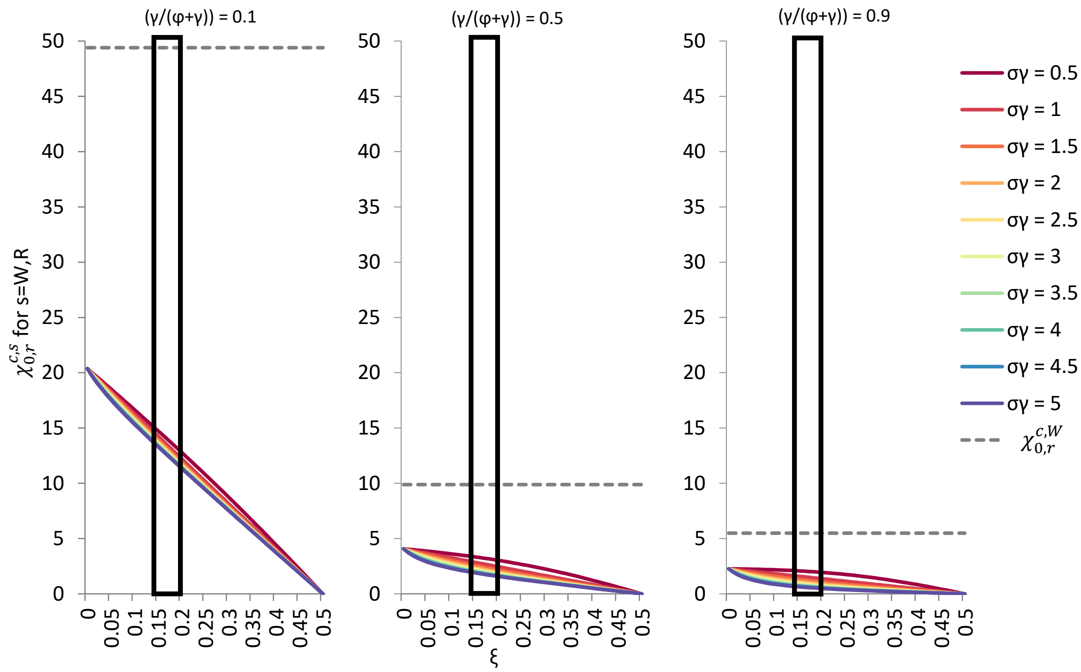
where $\theta_1^{c,s,r-m}$ is the solution to the quadratic equation

$$\begin{gathered}\left[\chi_{1,m}^{c,s}\sigma_{c,s,r}^2 + \chi_{1,r}^{c,s}\sigma_{c,s,m}^2\right]\left(\theta_1^{c,s,r-m}\right)^2 + \left[\left(1 + \left(\chi_{1,m}^{c,s}\right)^2\right)\sigma_{c,s,r}^2 + \left(1 + \left(\chi_{1,r}^{c,s}\right)^2\right)\sigma_{c,s,m}^2\right]\left(\theta_1^{c,s,r-m}\right) + ... \\ \left[\chi_{1,m}^{c,s}\sigma_{c,s,r}^2 + \chi_{1,r}^{c,s}\sigma_{c,s,m}^2\right] = 0,\end{gathered} \tag{52}$$

that ensures the invertibility of the MA part of the ARMA$(2,1)$ process.

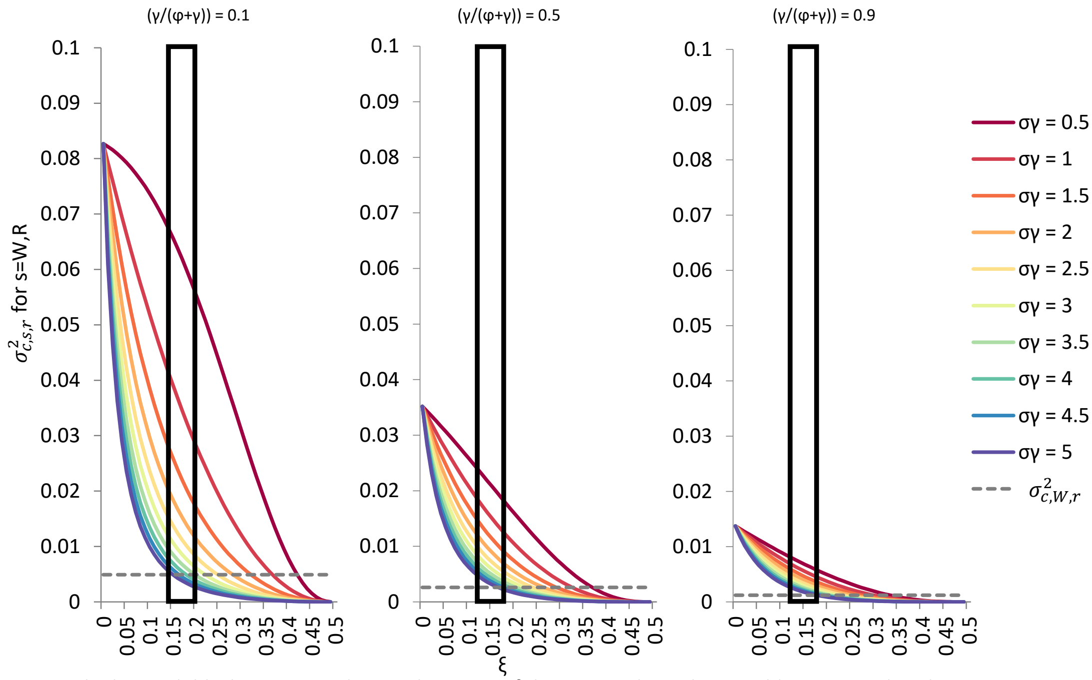
[20] This could easily be generalized as well to a case where international monetary policy shock spillovers are permitted and they are equal to the productivity spillovers given by δ_{a,a^*}.

Figure 2.A The Trade-off Between Inflation and Slack Arising from Natural Rate Shocks ($\chi_{0,r}^{s,c}$ for $s = W, R$).



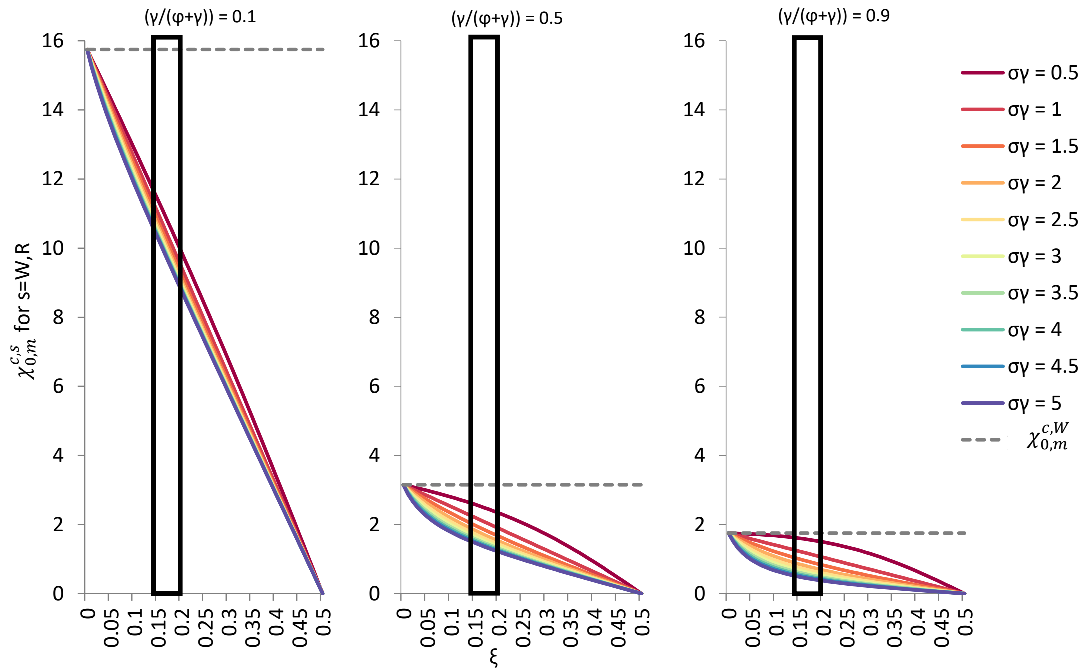
Note: The bar with black margins indicates the range of the import share that would correspond to the U.S.

Figure 2.B Inflation Volatility from Natural Rate Shocks (for $s = W, R$).



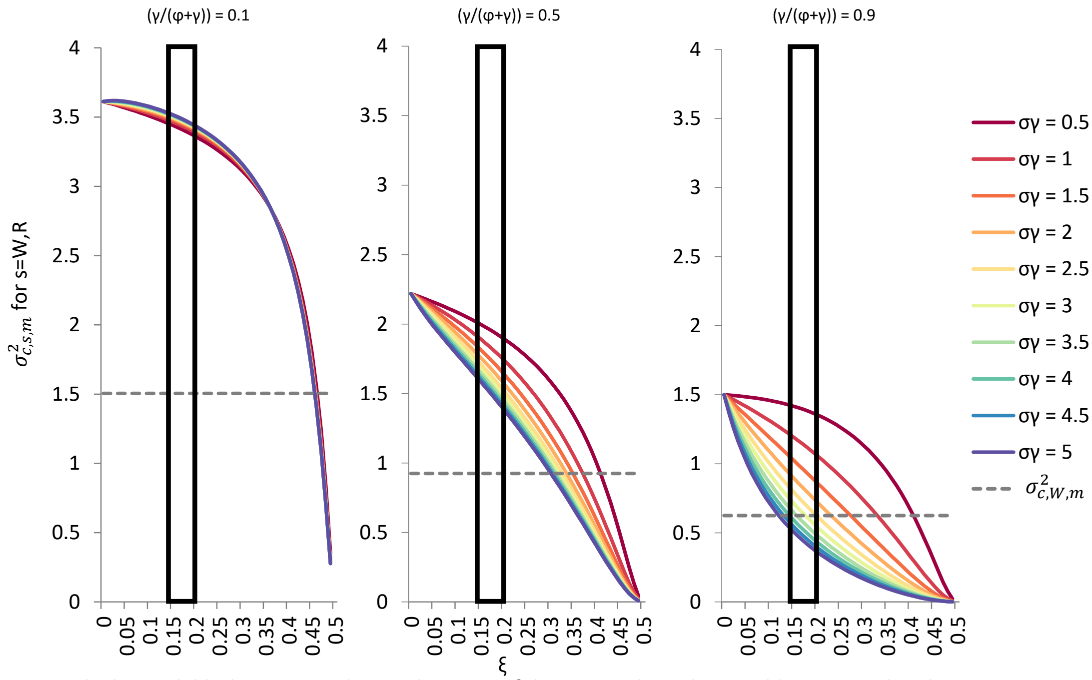
Note: The bar with black margins indicates the range of the import share that would correspond to the U.S.

Figure 3.A The Trade-off Between Inflation and Slack Arising from Monetary Policy Shocks ($\chi^{s,c}_{0,m}$ for $s = W, R$).



Note: The bar with black margins indicates the range of the import share that would correspond to the U.S.

Figure 3.B Inflation Volatility from Monetary Policy Shocks (for $s = W, R$).



Note: The bar with black margins indicates the range of the import share that would correspond to the U.S.

Cost-Push Shocks. Taking the conjectured solution in (45)−(46) as given and assuming that the common (coordinated) monetary policy is consistent with determinacy of the solution for both sub-systems, I can verify by the method of undetermined coefficients (as shown in the Appendix) that,

Proposition 5 *The solution for $j = v$ and $s = W, R$ given in (45) − (46) satisfies that*

$$\chi^{c,s}_{1,v} = \delta_v, \tag{53}$$

$$\chi^{c,s}_{0,v} = -\left(\frac{1+\frac{\Omega^s}{\gamma}\psi^c_x - \chi^{c,s}_{1,v}}{\frac{\Omega^s}{\gamma}\left(\psi^c_\pi - \chi^{c,s}_{1,v}\right)}\right), \tag{54}$$

$$\sigma^2_{c,s,v} = \left(\frac{\Psi^{c,s}\left(1+\frac{\Omega^s}{\gamma}\psi^c_x - \chi^{c,s}_{1,v}\right)\Phi\left(\varphi+\gamma\right)\psi^c_\pi}{\left(1-\Psi^{c,s}\chi^{c,s}_{1,v}\right)\left(\psi^c_\pi - \chi^{c,s}_{1,v}\right) + \Psi^{c,s}\left(1+\frac{\Omega^s}{\gamma}\psi^c_x - \chi^{c,s}_{1,v}\right)\left(1-\beta\psi^c_\pi\right)\chi^{c,s}_{1,v}}\right)^2 \sigma^2_{vs}, \tag{55}$$

where the innovations in (46) are defined as $\eta^{c,s,v}_t = \left(\frac{\Psi^{c,s}\left(1+\frac{\Omega^s}{\gamma}\psi^c_x - \chi^{c,s}_{1,v}\right)\Phi(\varphi+\gamma)\psi^c_\pi}{\left(1-\Psi^{c,s}\chi^{c,s}_{1,v}\right)\left(\psi^c_\pi - \chi^{c,s}_{1,v}\right)+\Psi^{c,s}\left(1+\frac{\Omega^s}{\gamma}\psi^c_x - \chi^{c,s}_{1,v}\right)(1-\beta\psi^c_\pi)\chi^{c,s}_{1,v}}\right)\widehat{\varepsilon}^{vs}_t$ *when arising from cost-push shock innovations. The volatility term for the cost-push shock σ^2_{vs} for $s = W, R$ is defined in (43).*

The key takeaways from this result are as follows:

1. The endogenous persistence arising from cost-push shocks $\chi^{c,s}_{1,v}$ for $s = W, R$ is entirely determined by the properties of the shock process itself (as it was the case with the endogenous persistence arising from natural rate shocks and monetary policy shocks as well). These results on endogenous persistence suggest that variations in the persistence of inflation are generally not symptomatic of greater economic integration (openness) or changes in monetary policy so long as those policy changes are coordinated.

2. The comovement between inflation and slack given by the trade-off $\chi^{c,s}_{0,v}$ is negative (unlike the positive comovement arising from natural rate shocks and monetary policy shocks). The equilibrium trade-off from cost-push shocks does not depend on the slope of the Phillips curve (unlike what happens with natural rate shocks and monetary policy shocks)—instead, the equilibrium trade-off depends critically on the slope of the IS curve and the common (coordinated) monetary policy rule coefficients (ψ^c_π and ψ^c_x). For the aggregate solution ($s = W$), the aggregate IS curve slope is $\Omega^W = 1$ and therefore the trade-off is invariant to the openness (ξ) and the trade elasticity (σ) that characterize the trade channel in this model. As noted before in regards to the natural rate shocks and the monetary policy shocks, the strength of trade linkages does not matter for the aggregate trade-off—it matters only for the cross-country trade-off. The steepening of the IS curve discussed in Sub-section 2.3, in fact, shows up in the trade-off implicit in the differential solution ($s = R$). In that case, $\Omega^R > 1$ holds for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right) > 0$ given the results on the composite coefficient Ω summarized in (4). Figure 4.A illustrates that the trade-off $\chi^{c,s}_{0,v}$ for $s = R$ declines in absolute value with the degree of trade openness (ξ) and also shows that the absolute value of the trade-off is lower the larger the trade elasticity (σ) is.[21] This implies that more open economies (with

[21] I use the following standard parameterization to produce Figure 4.A and Figure 4.B: $\beta = 0.99$, $\gamma = 2$, $\alpha = 0.75$, $\delta_u = 0.9$, and $\rho_{u,u^*} = 0.25$. The policy parameters are set as in the standard Taylor (1993) rule: $\psi^c_\pi = 1.5$ and $\psi^c_x = 0.5$. The volatility of the shock is normalized to one in this case: $\sigma^2_u = 1$.

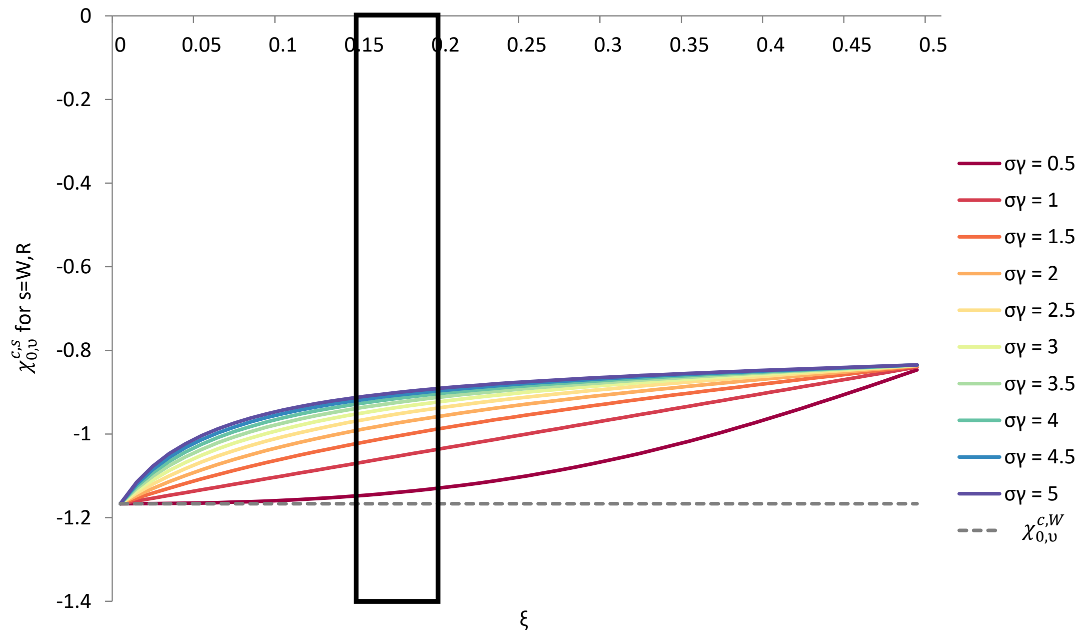
lower $\left|\chi_{0,v}^{c,R}\right|$) should experience larger movements in the output gap differential $\widehat{x}_t^{c,R,v}$ for any given change in the cross-country inflation differential $\widehat{\pi}_t^{c,R,v}$ while aggregate trade-offs remain unaffected by the trade channel. Interestingly, the preference ratio $\frac{\gamma}{\varphi+\gamma}$, and the inverse of the Frisch elasticity of labor supply (φ) in particular, play no role in the trade-off arising from the cost-push shocks (unlike what happens with natural rate shocks and monetary policy shocks).

3. The policy parameters ψ_π^c and ψ_x^c have a major effect on the aggregate and the differential trade-offs. In general, the stronger the anti-inflationary bias of the common (coordinated) monetary policy (the higher ψ_π^c is) the lower the trade-off for cost-push shocks in absolute value will be (the lower $\left|\chi_{0,v}^{c,s}\right|$ for $s = W, R$ is). Similarly, the weaker the bias on economic activity of the common (coordinated) monetary policy (the lower ψ_x^c is), the lower the trade-off for cost-push shocks in absolute value will be (the lower $\left|\chi_{0,v}^{c,s}\right|$ for $s = W, R$ is).

4. The volatility of inflation $\sigma_{c,s,v}^2$ confirms similar patterns to those described for the other shocks—particularly on the role of the trade channel explaining the declines in macro volatility during the Great Moderation period. The results indicate that the New Keynesian mechanism tends to amplify the volatility of the underlying cost-push shocks (more in line with what happens for monetary policy shocks than for the natural rate shocks), and volatility tends to be lower the larger the preference ratio $\frac{\gamma}{\varphi+\gamma}$ is in connection with the features of the labor market. While the volatility of the aggregates is unaffected by the degree of openness (ξ) and the trade elasticity (σ), the strength of the trade channel affects the volatility of the cross-country inflation dispersion in a rather non-linear way depending again on the preference ratio $\frac{\gamma}{\varphi+\gamma}$. Figure 4.B illustrates the volatility implications of the model for a given one-standard-deviation cost-push shock. From that figure, I observe that for low enough levels of openness (ξ), an increase in the parameter ξ may lead to an increase in the volatility of the cost-push shocks. For large enough levels of openness, a higher ξ will then lead to lower volatility which converges towards zero as ξ gets arbitrarily close to $\frac{1}{2}$. This nonlinearity in the model adds an additional layer of complexity to explain shifts in the contribution of the different shocks resulting from greater trade integration. For large enough values of the preference ratio $\frac{\gamma}{\varphi+\gamma}$, a higher trade elasticity tends to widen the impact on volatility.

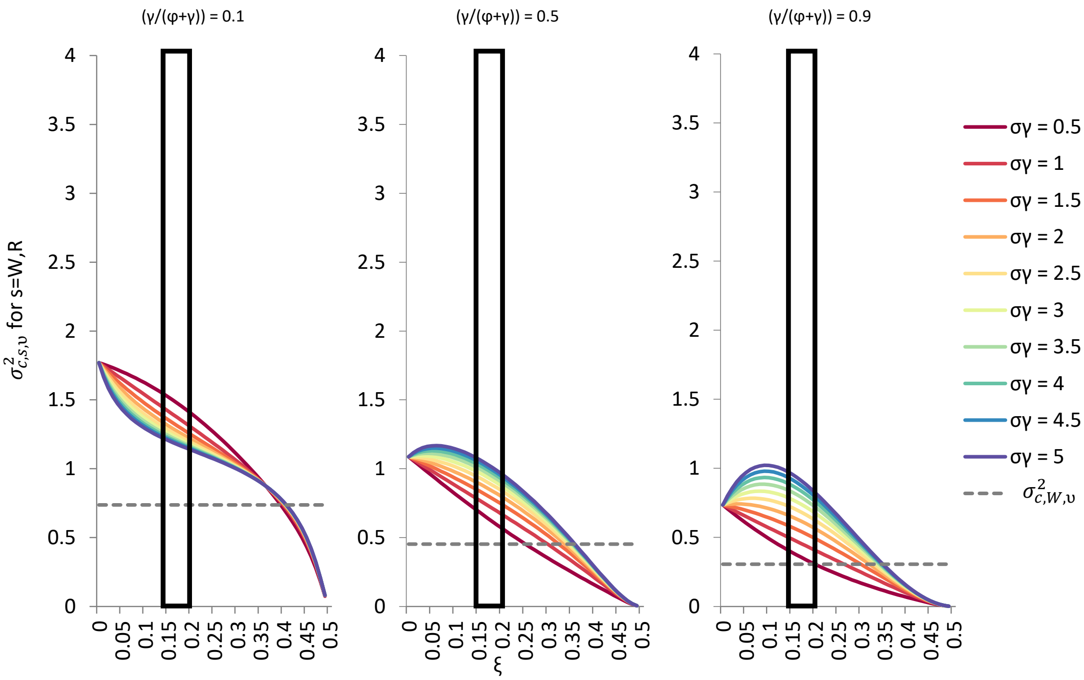
5. In general, I find that the stronger the anti-inflationary bias of the common (coordinated) monetary policy (the higher ψ_π^c is) or the smaller the bias on economic activity (the lower ψ_x^c is), the lower the inflation volatility arising from cost-push shocks (the lower $\sigma_{c,s,v}^2$ for $s = W, R$). Unlike for the anti-inflationary bias given by ψ_π^c, the sensitivity to slack in the common (coordinated) monetary policy rule given by ψ_x^c has very different implications for inflation volatility depending on the nature of the shock. An increase in ψ_x^c tends to reduce the volatility arising from natural rate shocks and monetary policy shocks, but it increases the volatility from cost-push shocks. My interpretation of this is that pursuing inflation stabilization more strongly may have added to the decline in volatility during the Great Moderation, but the effects on macro volatility (and on the contribution of the different shocks over the business cycle) from changes in the policymakers' response to slack appear to play a mixed role in the Great Moderation instead.

Figure 4.A The Trade-off Between Inflation and Slack Arising from Cost-Push Shocks ($\chi^{s,c}_{0,v}$ for $s = W, R$).



Note: This part of the solution does not depend on the preference ratio (γ/(φ+γ)). The bar with black margins indicates the range of the import share that would correspond to the U.S.

Figure 4.B Inflation Volatility from Cost-Push Shocks (for $s = W, R$).



Note: The bar with black margins indicates the range of the import share that would correspond to the U.S.

Backing Out the Country-Level Solution. Given the dynamics for $s = W, R$ in (45) – (46) under the terms of Proposition 4, the transformation in (21) backs out the corresponding variables for each individual country. Using the persistence terms for the natural rate derived in (38), it follows that inflation and the output gap in response to natural rate shocks, $\widehat{\pi}_t^{c,r}$ and $\widehat{x}_t^{c,r}$ in the Home country, must satisfy that

$$\begin{aligned}\widehat{\pi}_t^{c,r} &\equiv \widehat{\pi}_t^{c,W,r} + \tfrac{1}{2}\widehat{\pi}_t^{c,R,r} = \tfrac{\Phi(\varphi+\gamma)}{1-\beta\left(\delta_a+\delta_{a,a^*}\right)}\widehat{x}_t^{c,W,r} + \tfrac{1}{2}\tfrac{\Phi(\varphi+\gamma)\kappa^R}{1-\beta\left(\delta_a-\delta_{a,a^*}\right)}\widehat{x}_t^{c,R,r} \\ &= \tfrac{\Phi(\varphi+\gamma)}{1-\beta\delta_a}\left[\left(\tfrac{1-\beta\delta_a}{1-\beta\left(\delta_a+\delta_{a,a^*}\right)} + \tfrac{1-\beta\delta_a}{1-\beta\left(\delta_a-\delta_{a,a^*}\right)}\kappa^R\right)\tfrac{1}{2}\widehat{x}_t^{c,r} + \left(\tfrac{1-\beta\delta_a}{1-\beta\left(\delta_a+\delta_{a,a^*}\right)} - \tfrac{1-\beta\delta_a}{1-\beta\left(\delta_a-\delta_{a,a^*}\right)}\kappa^R\right)\tfrac{1}{2}\widehat{x}_t^{*,c,r}\right],\end{aligned} \tag{56}$$

where $\widehat{\pi}_t^{c,r}$ stands for domestic inflation, $\widehat{x}_t^{c,r}$ is the domestic slack, and $\widehat{x}_t^{*,c,r}$ is foreign slack. The relationship between domestic inflation and domestic and foreign slack depends on the cross-country spillovers for the productivity shocks (δ_{a,a^*}). Domestic inflation is related to a convex combination of domestic and foreign slack by a common scaling factor $\frac{\Phi(\varphi+\gamma)}{1-\beta\delta_a}$. When there are no cross-country spillovers ($\delta_{a,a^*} = 0$), only the term κ^R matters implying that domestic slack outweighs its contribution to aggregated (production-based) global slack while foreign slack tends to be below its contribution given that $0 < \kappa^R \equiv (2\kappa - 1) < 1$ for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \max\left\{0, 1 - \left(\frac{1}{2\xi+\left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$ (as implied by the results in (3)). This shows that the equilibrium trade-off between domestic inflation and domestic and foreign slack is in fact largely dominated by domestic slack even in the open-economy case.

Furthermore, the domestic inflation process in response to natural rate shocks, $\widehat{\pi}_t^{c,r}$, follows a simple AR(1) process whenever $\delta_{a,a^*} = 0$: $\widehat{\pi}_t^{c,r} = \delta_a\widehat{\pi}_{t-1}^{c,r} + \eta_t^{c,r}$, where $\eta_t^{c,r} \sim N\left(0, \sigma_{c,r}^2\right)$ given by $\sigma_{c,r}^2 = \sigma_{c,W,r}^2 + \left(\frac{1}{2}\right)^2\sigma_{c,R,r}^2 + \sigma_{W,R}^{c,r}$ and $\sigma_{W,R}^{c,r} \equiv \mathbb{E}\left(\eta_t^{c,W,r}\eta_t^{c,R,r}\right)$. It also follows from well-known aggregation results (Hamilton (1994)) that domestic inflation in response to natural rate shocks, $\widehat{\pi}_t^{c,r}$, follows an ARMA$(2,1)$ process of the following form whenever cross-country productivity spillovers matter ($\delta_{a,a^*} \neq 0$):

$$\widehat{\pi}_t^{c,r} = 2\delta_a\widehat{\pi}_{t-1}^{c,r} - \left(\delta_a - \delta_{a,a^*}\right)\left(\delta_a + \delta_{a,a^*}\right)\widehat{\pi}_{t-2}^{c,r} + \eta_t^{c,r} + \theta_1^{c,r}\eta_{t-1}^{c,r}, \tag{57}$$

$$\eta_t^{c,r} \sim N\left(0, \sigma_{c,r}^2\right),\ \sigma_{c,r}^2 = -\left(\frac{\left(\delta_a - \delta_{a,a^*}\right)\sigma_{c,W,r}^2 + \left(\frac{1}{2}\right)^2\left(\delta_a + \delta_{a,a^*}\right)\sigma_{c,R,r}^2 + \delta_a\sigma_{W,R}^{c,r}}{\theta_1^{c,r}}\right), \tag{58}$$

where $\theta_1^{c,r}$ is the solution to the quadratic equation

$$\begin{gathered}\left[\left(\delta_a - \delta_{a,a^*}\right)\sigma_{c,W,r}^2 + \left(\tfrac{1}{2}\right)^2\left(\delta_a + \delta_{a,a^*}\right)\sigma_{c,R,r}^2 + \delta_a\sigma_{W,R}^{c,r}\right]\left(\theta_1^{c,r}\right)^2 + \ldots \\ \left[\left(1 + \left(\delta_a - \delta_{a,a^*}\right)^2\right)\sigma_{c,W,r}^2 + \left(\tfrac{1}{2}\right)^2\left(1 + \left(\delta_a + \delta_{a,a^*}\right)^2\right)\sigma_{c,R,r}^2 + \left(1 + \left(\delta_a + \delta_{a,a^*}\right)\left(\delta_a - \delta_{a,a^*}\right)\right)\sigma_{W,R}^{c,r}\right]\theta_1^{c,r} + \ldots \\ \left[\left(\delta_a - \delta_{a,a^*}\right)\sigma_{c,W,r}^2 + \left(\tfrac{1}{2}\right)^2\left(\delta_a + \delta_{a,a^*}\right)\sigma_{c,R,r}^2 + \delta_a\sigma_{W,R}^{c,r}\right] = 0,\end{gathered} \tag{59}$$

that ensures the invertibility of the MA part of the ARMA$(2,1)$ process. Aggregate and difference shocks are uncorrelated in the model—i.e., $\sigma_{W,R}^{c,r} \equiv \sigma_{c,W,r}\sigma_{c,R,r}\left(\frac{\mathbb{E}\left(\widehat{\varepsilon}_t^{rW}\widehat{\varepsilon}_t^{rR}\right)}{\sigma_{rW}\sigma_{rR}}\right) = 0$ given that $\mathbb{E}\left(\widehat{\varepsilon}_t^{rW}\widehat{\varepsilon}_t^{rR}\right) = \frac{1}{2}\mathbb{E}\left(\left(\widehat{\varepsilon}_t^{r} + \widehat{\varepsilon}_t^{r*}\right)\left(\widehat{\varepsilon}_t^{r} - \widehat{\varepsilon}_t^{r*}\right)\right) = 0$—which further simplifies the characterization of the stochastic process for domestic inflation. This comes to show that even modest international cross-country spillovers can have significant effects on the dynamics of domestic inflation and on the patterns of propagation of natural rate shocks. Similar derivations would help characterize the dynamics of foreign inflation in response to real interest rate shocks and the corresponding trade-off of foreign inflation with domestic and foreign slack.

Given the dynamics for $s = W, R$ in (45) − (46) under the terms of Proposition 4, the transformation in (21) can be used again to back out the endogenous variables of each individual country. It follows that inflation and the output gap in response to monetary policy shocks, $\widehat{\pi}_t^{c,m}$ and $\widehat{x}_t^{c,m}$ in the Home country, must satisfy that

$$\begin{aligned} \widehat{\pi}_t^{c,m} &\equiv \widehat{\pi}_t^{c,W,m} + \tfrac{1}{2}\widehat{\pi}_t^{c,R,m} = \tfrac{\Phi(\varphi+\gamma)}{1-\beta\delta_m}\widehat{x}_t^{c,W,m} + \tfrac{1}{2}\tfrac{\Phi(\varphi+\gamma)\kappa^R}{1-\beta\delta_m}\widehat{x}_t^{c,R,m} \\ &= \tfrac{\Phi(\varphi+\gamma)}{1-\beta\delta_m}\left[\left(1+\kappa^R\right)\tfrac{1}{2}\widehat{x}_t^{c,m} + \left(1-\kappa^R\right)\tfrac{1}{2}\widehat{x}_t^{*,c,m}\right], \end{aligned} \tag{60}$$

where $\widehat{\pi}_t^{c,m}$ stands for domestic inflation, $\widehat{x}_t^{c,m}$ is the domestic slack, and $\widehat{x}_t^{*,c,m}$ is foreign slack. Domestic inflation is related to a convex combination of domestic and foreign slack by a common scaling factor $\frac{\Phi(\varphi+\gamma)}{1-\beta\delta_m}$. Then, only the term κ^R matters implying that domestic slack outweighs its contribution to aggregated (production-based) global slack while foreign slack tends to have a lower contribution given that $0 < \kappa^R \equiv (2\kappa - 1) < 1$ for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \max\left\{0, 1 - \left(\frac{1}{2\xi + \left(\frac{\gamma}{\varphi+\gamma}\right)(1-2\xi)}\right)\frac{1}{2(1-\xi)}\right\}$ (as implied by the results in (3)). This shows that the equilibrium trade-off between domestic inflation and domestic and foreign slack is in fact largely dominated by domestic slack whenever we are dealing with monetary policy shocks (as it happened with natural rate shocks as well).

The domestic inflation process in response to monetary policy shocks, $\widehat{\pi}_t^{c,m}$, follows a simple AR(1) process:

$$\widehat{\pi}_t^{c,m} = \delta_m\widehat{\pi}_{t-1}^{c,m} + \eta_t^{c,m},\ \eta_t^{c,m} \sim N\left(0, \sigma_{c,m}^2\right), \tag{61}$$

where $\sigma_{c,m}^2 = \sigma_{c,W,m}^2 + \left(\frac{1}{2}\right)^2\sigma_{c,R,m}^2 + \sigma_{W,R}^{c,m}$ and $\sigma_{W,R}^{c,m} \equiv \mathbb{E}\left(\eta_t^{c,W,m}\eta_t^{c,R,m}\right)$. As before, I should note here that aggregate and difference monetary policy shocks are uncorrelated in the model—i.e., $\sigma_{W,R}^{c,m} \equiv \sigma_{c,W,m}\sigma_{c,R,m}\left(\frac{\mathbb{E}\left(\widehat{\varepsilon}_t^{mW}\widehat{\varepsilon}_t^{mR}\right)}{\sigma_{mW}\sigma_{mR}}\right) = 0$ given that $\mathbb{E}\left(\widehat{\varepsilon}_t^{mW}\widehat{\varepsilon}_t^{mR}\right) = \frac{1}{2}\mathbb{E}\left(\left(\widehat{\varepsilon}_t^{m} + \widehat{\varepsilon}_t^{m*}\right)\left(\widehat{\varepsilon}_t^{m} - \widehat{\varepsilon}_t^{m*}\right)\right) = 0$—which further simplifies the characterization of the stochastic process for domestic inflation. This shows that the volatility of inflation at the country-level is largely dominated by the volatility of the world aggregate (as it was the case in response to natural rate shocks too).

Finally, given the dynamics for $s = W, R$ in (45) − (46) under the terms of Proposition 5, the transformation in (21) helps me back out the solution for the endogenous variables of each individual country in response to a cost-push shock. It follows that inflation and the output gap, $\widehat{\pi}_t^{c,v}$ and $\widehat{x}_t^{c,v}$ in the Home country, must satisfy that

$$\begin{aligned} \widehat{\pi}_t^{c,v} &\equiv \widehat{\pi}_t^{c,W,v} + \tfrac{1}{2}\widehat{\pi}_t^{c,R,v} = -\left(\frac{1+\frac{1}{\gamma}\psi_x^c-\delta_v}{\frac{1}{\gamma}(\psi_\pi^c-\delta_v)}\right)\widehat{x}_t^{c,W,v} - \frac{1}{2}\left(\frac{1+\frac{\Omega^R}{\gamma}\psi_x^c-\delta_v}{\frac{\Omega^R}{\gamma}(\psi_\pi^c-\delta_v)}\right)\widehat{x}_t^{c,R,v} \\ &= -\left(\frac{1+\frac{1}{\gamma}\psi_x^c-\delta_v}{\frac{1}{\gamma}(\psi_\pi^c-\delta_v)}\right)\left[\left(1+\frac{(1-\delta_v)+\frac{\Omega^R}{\gamma}\psi_x^c}{\Omega^R(1-\delta_v)+\frac{\Omega^R}{\gamma}\psi_x^c}\right)\frac{1}{2}\widehat{x}_t^{c,v} + \left(1-\frac{(1-\delta_v)+\frac{\Omega^R}{\gamma}\psi_x^c}{\Omega^R(1-\delta_v)+\frac{\Omega^R}{\gamma}\psi_x^c}\right)\frac{1}{2}\widehat{x}_t^{*,c,v}\right], \end{aligned} \tag{62}$$

where $\widehat{\pi}_t^{c,v}$ stands for domestic inflation, $\widehat{x}_t^{c,v}$ is the domestic slack, and $\widehat{x}_t^{*,c,v}$ is foreign slack. Domestic inflation is once again related to a convex combination of domestic and foreign slack by a common scaling factor $-\left(\frac{1+\frac{1}{\gamma}\psi_x^c-\delta_v}{\frac{1}{\gamma}(\psi_\pi^c-\delta_v)}\right)$. However, unlike for the case of natural rate shocks and monetary shocks, the slope κ^R arising from the Phillips curve relationship does not affect the weight of domestic and foreign slack in the resulting equilibrium trade-off. In turn, the slope Ω^R arising from the IS curve matters showing that domestic slack tends to outweigh its contribution to aggregated (production-based) global slack while foreign slack tends to have a lower contribution given that $\Omega^R > 1$ for any degree of openness $0 < \xi < \frac{1}{2}$ and for all $\sigma\gamma > \left(\frac{1-2\xi}{1+(1-2\xi)}\right) > 0$ (as implied by (4)). This shows that the nature of the shock is important when determining the equilibrium trade-off between domestic inflation and domestic and foreign slack. However, the findings based on the cost-push shocks confirm the fact that the trade-off for domestic inflation tends to be largely dominated in equilibrium by domestic slack (as it happened with natural rate shocks and monetary policy shocks as well).

The domestic inflation process in response to cost-push shocks, $\widehat{\pi}_t^{c,v}$, also follows a simple AR(1) process:

$$\widehat{\pi}_t^{c,v} = \delta_v \widehat{\pi}_{t-1}^{c,v} + \eta_t^{c,v}, \ \eta_t^{c,v} \sim N\left(0, \sigma_{c,v}^2\right), \tag{63}$$

where $\sigma_{c,v}^2 = \sigma_{c,W,v}^2 + \left(\frac{1}{2}\right)^2 \sigma_{c,R,v}^2 + \sigma_{W,R}^{c,v}$ and $\sigma_{W,R}^{c,v} \equiv \mathbb{E}\left(\eta_t^{c,W,v}\eta_t^{c,R,v}\right)$. Not surprisingly, aggregate and difference cost-push shocks are uncorrelated in the model—i.e., $\sigma_{W,R}^{c,v} \equiv \sigma_{c,W,v}\sigma_{c,R,v}\left(\frac{\mathbb{E}\left(\widehat{\varepsilon}_t^{vW}\widehat{\varepsilon}_t^{vR}\right)}{\sigma_{vW}\sigma_{vR}}\right) = 0$ given that $\mathbb{E}\left(\widehat{\varepsilon}_t^{vW}\widehat{\varepsilon}_t^{vR}\right) = \frac{1}{2}\mathbb{E}\left(\left(\widehat{\varepsilon}_t^{v} + \widehat{\varepsilon}_t^{v*}\right)\left(\widehat{\varepsilon}_t^{v} - \widehat{\varepsilon}_t^{v*}\right)\right) = 0$—which simplifies the characterization of the stochastic process for domestic inflation (as it did for natural rate shocks and monetary policy shocks). This result shows that the nature of the shocks driving the business cycle can be very important, but also validates the claim that the volatility of inflation at the country-level is largely dominated by the volatility of the world aggregate (across all types of shocks). Hence, global inflation shares significant features with domestic inflation (see, e.g., Ciccarelli and Mojon (2010), Duncan and Martínez-García (2015), and Bianchi and Civelli (2015) on the dynamics of inflation).

4.3 Discussion

The solution of the model shows that local macro variables display strong common movements even when all shocks are country-specific—that is, even without common shocks driving the international business cycle. I explore the distinct patterns of shock propagation—looking at natural rate shocks, cost-push shocks, and monetary policy shocks—and the different roles monetary policy plays on the aggregates of the two countries and the cross-country dispersion. The results of this paper illustrate how different monetary policy regimes impact the dynamics of open economies establishing that:

1. In general there are negative foreign productivity shock spillovers to domestic potential output. Full insulation from foreign productivity shocks appears as a knife-edge case (Cole and Obstfeld (1991)). In the economically-relevant cases ($\sigma\gamma > 1$), the more open the economy (ξ) is and the higher the trade elasticity of substitution (σ), the larger the negative spillover into domestic potential output. The reason being that the income effect of foreign productivity shocks is then dominated by the strength of the substitution effect leading to an aggregate demand shift across countries that drags domestic potential output down. I also find that the negative spillovers tend to be larger whenever the inverse of the Frisch elasticity of labor supply (φ) is smaller relative to the intertemporal elasticity of substitution (γ). Hence, the sensitivity of labor supply to wage rate changes can be important in determining the magnitude of the spillovers for any given strength of the trade channel mechanism. Furthermore, open economies in general display higher volatility and lower cross-country correlations of potential output innovations than their closed-economy counterparts.

2. The weight of expected foreign output potential growth on the domestic natural rate exceeds the import share ξ (a standard measure of trade openness) in the economically-relevant cases. Positive spillovers

from faster foreign potential growth—whether due to foreign or domestic productivity growth—increase the domestic natural rate. The more open the economy (ξ) is and the higher the trade elasticity is (σ), the larger the effect of foreign potential growth beyond what can be attributed to the import share alone—and the smaller the effect of domestic potential growth is—on the domestic natural rate. When potential output growth is related to the Home and Foreign productivity shocks, then the volatility of the natural rate innovations is lower and their cross-country correlations are higher for open economies than for closed economies. This shows that greater openness contributes to some extent to explaining the decline in macro volatility—albeit not for output potential—characteristic of the Great Moderation (Roberts (2006), Clark (2009), Martínez-García (2015a)). It's worth noting that the inverse of the Frisch elasticity of labor supply φ (a key feature of the labor market) affects the natural rate process as well magnifying these differences between open and closed economies.

3. In general an open economy has a flatter Phillips curve so that there is a smaller reduction in domestic inflation for any given decline in domestic slack. A more open economy (ξ) and one with a larger trade elasticity (σ) would have a flatter Phillips curve on domestic slack and simultaneously a positive and larger slope on foreign slack. I also find that the slope on domestic slack becomes flatter and conversely the slope on foreign slack becomes steeper for smaller values of the inverse of the Frisch elasticity of labor supply φ—indicating that the responsiveness of labor supply is a key factor magnifying the impact of the trade channel. This theoretical insight implies that the trade channel provides a consistent explanation for the empirical evidence on the flattening of the Phillips curve documented, among others, by Roberts (2006) and IMF WEO (2013). Moreover, I show that a more open economy (ξ) or one with a higher trade elasticity (σ) generally has a steeper investment-savings (IS) curve with respect to deviations of the domestic real interest rate from its corresponding domestic natural rate. In this case, changes in the policy rate in an open economy have larger effects on domestic slack than in an otherwise identical closed economy. Labor market features like the inverse of the Frisch elasticity of labor supply φ have no bearing on the slope of the open-economy IS curve, though.

4. I show that monetary policy asymmetries across countries can be interpreted as alterations of the monetary policy equilibrium that emerges under international monetary policy coordination. Interestingly, some form of monetary integration—either a common coordinated monetary policy or deeper monetary integration in the form of a currency union—leads to a policy equilibrium whereby aggregate dynamics and cross-country dispersion are characterized by perfectly separable structural systems of equations which can, therefore, be investigated on their own. Moreover, I find that the strength of the trade channel implied by the degree of openness of the economy (ξ) but also by the trade elasticity of substitution between locally-produced and imported goods (σ) affects the cross-country dispersion only and not the aggregate dynamics under a common monetary policy. The key message here is that greater economic integration through trade has no effect on the aggregate dynamics of the two countries so long as some sort of international monetary policy cooperation is achieved.

5. The criterion to ensure uniqueness and determinacy of the open-economy solution is exactly the same as in the closed-economy case in most instances under the case of international monetary policy coordination. This criterion also ensures the determinacy of the aggregate solution under a monetary union—but, in turn, the short-term nominal interest rate equalization across countries resulting from

monetary union leads to indeterminacy in the solution that characterizes the dispersion across countries. Hence, an important distinction between international policy coordination and a monetary union—where this paper adds novel insight on the existing theoretical literature on currency unions (Benigno (2004), among others)—is that the latter may induce indeterminacy (and hence multiple equilibria) unlike the former.

6. The macroeconomic persistence implied by the model is entirely determined by the properties of the underlying processes for the productivity shocks, the monetary policy shocks, and the cost-push shocks. The results of the paper generally suggest that variations in the macroeconomic persistence—particularly on inflation which has been the main focus of a strand of the New Keynesian literature (Benati and Surico (2008), Carlstrom et al. (2009))—are not necessarily symptomatic of greater trade integration (openness). Moreover, changes in monetary policy, so long as those changes are internationally coordinated, do not appear to play a role either. The role of asymmetric monetary policies across countries on persistence is left for future research.

7. The comovement between inflation and slack arising from natural rate shocks and monetary policy shocks is positive while it is negative when the driving force is the cost-push shock. The strength of trade linkages does not matter for the aggregate trade-off irrespective of the shock one considers—it matters only for the cross-country dispersion trade-off. It is important to note then that in general the more open an economy is (ξ) and the higher the trade elasticity (σ), the larger the movements in the output gap differential seen in equilibrium for any given change in the cross-country inflation differential. This result shows that greater economic integration plays an important role in the equilibrium trade-offs that emerge in the two-country model across all types of shocks (natural rate shocks derived from productivity, monetary policy shocks, and cost-push shocks). Quantitatively, there are other important economic features that will also have a large effect on the size of the trade-offs between inflation and slack: On the one hand, cross-country spillovers in productivity (technological diffusion, δ_{a,a^*}) and features of the labor market such as the inverse of the Frisch elasticity φ matter a great deal for the trade-offs arising from productivity shocks and monetary shocks while a common (coordinated) policy rule does not. On the other hand, the inverse of the Frisch elasticity of labor supply φ does not affect the trade-offs arising from cost-push shocks but the coefficients on the policy rule do (ψ_π^c and ψ_x^c). I argue based on these results that a monetary policy rule with a stronger anti-inflation bias ψ_π^c (or with less weight on slack ψ_x^c) tends to exacerbate the movements in slack for any given change in inflation, albeit only when those movements are driven by cost-push shocks. These findings bring new light to the ongoing debate on the role of good luck and better monetary policies (Benati and Surico (2008), Woodford (2010)), suggesting that greater economic integration through trade (structural change) may be more important than often thought.

8. The paper makes the case for greater economic integration as one of the leading causes of the decline in macro volatility characteristic of the Great Moderation (Stock and Watson (2003), Clark (2009), Martínez-García (2015a)). The results derived from the model show that the New Keynesian mechanism downplays the contribution of productivity shocks and magnifies that of monetary policy shocks and cost-push shocks in the open economy (as well as in the closed economy). Volatility tends to be lower the smaller the inverse of the Frisch elasticity of labor supply φ is. In general, I find that the

volatility of the aggregates is unaffected by the degree of openness of the economy (ξ) and the trade elasticity (σ) but that open economies tend to experience lower volatility of the cross-country inflation dispersion—a feature that becomes more accentuated the larger the trade elasticity σ is. Interestingly, the relationship is non-linear and in the case of cost-push shocks, greater trade integration when the initial degree of openness ξ is rather low can result in higher macro volatility before macro volatility starts to decline. In turn, pursuing a common policy of inflation stabilization with a stricter anti-inflation bias ψ_π^c adds to the decline in volatility seen during the Great Moderation. The effects on macro volatility (and on the contribution of the different shocks over the business cycle) from changes in the policymakers' response to slack ψ_x^c appear less obvious as a factor explaining the Great Moderation.

5 Concluding Remarks

The model with a common (coordinated) monetary policy examined in this paper appears to provide a consistent explanation for some of the stylized facts that have characterized international business cycles over the past several decades (particularly during the Great Moderation). In particular, it shows that greater openness leads to lower macro volatility and even to a flatter Phillips curve as documented in the literature (Stock and Watson (2003), Roberts (2006), Clark (2009), IMF WEO (2013), and Martínez-García (2015a)). Inspecting the mechanism closely shows that fluctuations in slack differentials for a given change in the inflation differential differ for open economies. Generally, greater trade elasticity tends to magnify the effects of trade integration beyond what conventional measures of trade openness (like trade shares) would imply.

I also note that the coefficients in the common Taylor (1993)-type monetary policy rule can play an important role as well. In particular, the anti-inflation bias of policymakers appears to play a significant role driving macro volatility. Other structural features of the economy unrelated to policy or the trade channel play a major role in how country-specific shocks are propagated internationally too. For instance, cross-country spillovers in the productivity shocks capturing technological diffusion or the sensitivity of the labor supply to real wages (through the inverse of the Frisch elasticity of labor supply) can have large effects on the quantitative magnitudes implied by the model. In contrast, persistence generally appears inherited from the properties of the underlying stochastic processes driving the economy and largely unaffected by the trade channel, by monetary policy, or by any other deep structural features of the model.

Theory suggests that the impact of international monetary policy coordination isolates the effect of the trade channel on the dynamics of cross-country dispersion with no impact on the aggregates. In turn, there are significant challenges to assess the consequences of a currency union because this form of monetary integration leads to indeterminacy in the cross-country dispersion. Finally, the paper also makes a novel technical contribution by proposing a decomposition method that permits isolating the behavior of a country with respect to the aggregate in a multi-country setting and furthermore to solve the model allowing for the possibility of asymmetric monetary policies (independently-set) across countries.

Appendix

A Proofs

Derivation of the bivariate stochastic process for potential output. The potential output of both countries can be expressed as a linear transformation of the productivity shocks as,

$$\begin{pmatrix} \widehat{\overline{y}}_t \\ \widehat{\overline{y}}_t^* \end{pmatrix} \approx \left(\frac{1+\varphi}{\gamma+\varphi}\right) \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} \widehat{a}_t \\ \widehat{a}_t^* \end{pmatrix}.$$

Assuming invertibility, the vector of potential output inherits the VAR(1) stochastic structure of the productivity shocks. Accordingly, the potential output process takes the following stochastic form,

$$\begin{aligned}
\begin{pmatrix} \widehat{\overline{y}}_t \\ \widehat{\overline{y}}_t^* \end{pmatrix} &\approx \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix}^{-1} \begin{pmatrix} \widehat{\overline{y}}_{t-1} \\ \widehat{\overline{y}}_{t-1}^* \end{pmatrix} + \ldots \\
&\quad \left(\tfrac{1+\varphi}{\gamma+\varphi}\right) \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix}, \\
\begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix} &\sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix} \right),
\end{aligned}$$

where,

$$\begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix}^{-1} = \frac{1}{2\Lambda-1} \begin{pmatrix} \Lambda & \Lambda-1 \\ \Lambda-1 & \Lambda \end{pmatrix}.$$

Hence, it follows from here that,

$$\begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix}^{-1} = \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix},$$

which implies that the structure of the VAR(1) for the potential output inherits the persistence structure of the underlying productivity shocks. Moreover, I can simplify the notation by expressing the innovations to the output potential process in the following terms,

$$\begin{aligned}
\begin{pmatrix} \widehat{\varepsilon}_t^y \\ \widehat{\varepsilon}_t^{y*} \end{pmatrix} &\equiv \left(\frac{1+\varphi}{\gamma+\varphi}\right) \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix}, \\
\begin{pmatrix} \widehat{\varepsilon}_t^y \\ \widehat{\varepsilon}_t^{y*} \end{pmatrix} &\sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix} \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix}^T \right),
\end{aligned}$$

where,

$$\begin{aligned}
&\left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix} \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix} \begin{pmatrix} \Lambda & 1-\Lambda \\ 1-\Lambda & \Lambda \end{pmatrix}^T \\
&= \sigma_a^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \begin{pmatrix} (\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2 & \rho_{a,a^*}(\Lambda)^2 + 2\Lambda(1-\Lambda) + \rho_{a,a^*}(1-\Lambda)^2 \\ \rho_{a,a^*}(\Lambda)^2 + 2\Lambda(1-\Lambda) + \rho_{a,a^*}(1-\Lambda)^2 & (\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2 \end{pmatrix} \\
&= \sigma_a^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \left((\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2 \right) \times \ldots \\
&\quad \begin{pmatrix} 1 & \frac{\rho_{a,a^*}(\Lambda)^2 + 2\Lambda(1-\Lambda) + \rho_{a,a^*}(1-\Lambda)^2}{(\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2} \\ \frac{\rho_{a,a^*}(\Lambda)^2 + 2\Lambda(1-\Lambda) + \rho_{a,a^*}(1-\Lambda)^2}{(\Lambda)^2 + 2\rho_{a,a^*}\Lambda(1-\Lambda) + (1-\Lambda)^2} & 1 \end{pmatrix}.
\end{aligned}$$

Hence, I can define the volatility and the correlation of the output potential innovations in the following fashion,

$$\begin{aligned}\sigma_y^2 &= \sigma_a^2\left(\frac{1+\varphi}{\gamma+\varphi}\right)^2\left((\Lambda)^2+2\rho_{a,a^*}\Lambda\left(1-\Lambda\right)+\left(1-\Lambda\right)^2\right),\\ \rho_{y,y^*} &= \frac{\rho_{a,a^*}\left(\Lambda\right)^2+2\Lambda\left(1-\Lambda\right)+\rho_{a,a^*}\left(1-\Lambda\right)^2}{\left(\Lambda\right)^2+2\rho_{a,a^*}\Lambda\left(1-\Lambda\right)+\left(1-\Lambda\right)^2}.\end{aligned}$$

Derivation of the bivariate stochastic process for the natural rate of interest. The natural rates of both countries can be expressed as a linear transformation of the productivity shocks as,

$$\begin{aligned}\begin{pmatrix}\widehat{\overline{r}}_t\\ \widehat{\overline{r}}_t^*\end{pmatrix} &\approx \gamma\left(\frac{1+\varphi}{\gamma+\varphi}\right)\begin{pmatrix}\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right) & \left(1-\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\right)\\ \left(1-\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\right) & \left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\end{pmatrix}\begin{pmatrix}\mathbb{E}_t\left[\Delta\widehat{a}_{t+1}\right]\\ \mathbb{E}_t\left[\Delta\widehat{a}_{t+1}^*\right]\end{pmatrix}\\ &\approx \gamma\left(\frac{1+\varphi}{\gamma+\varphi}\right)\begin{pmatrix}\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right) & \left(1-\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\right)\\ \left(1-\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\right) & \left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\end{pmatrix}\begin{pmatrix}\delta_a-1 & \delta_{a,a^*}\\ \delta_{a,a^*} & \delta_a-1\end{pmatrix}\begin{pmatrix}\widehat{a}_t\\ \widehat{a}_t^*\end{pmatrix}\\ &\approx \gamma\left(\frac{1+\varphi}{\gamma+\varphi}\right)\begin{pmatrix}\Pi_1 & \Pi_2\\ \Pi_2 & \Pi_1\end{pmatrix}\begin{pmatrix}\widehat{a}_t\\ \widehat{a}_t^*\end{pmatrix},\end{aligned}$$

where,

$$\begin{aligned}\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right) &= \left(1-\xi\right)\left(\frac{\varphi\left(\sigma\gamma-\left(\sigma\gamma-1\right)\left(2\left(1-\xi\right)-1\right)\right)+\gamma}{\varphi\left(\sigma\gamma-\left(\sigma\gamma-1\right)\left(2\left(1-\xi\right)-1\right)^2\right)+\gamma}\right),\\ \Pi_1 &\equiv \left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\left(\delta_a-1\right)+\left(1-\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\right)\delta_{a,a^*}\\ &= \delta_{a,a^*}-\left(1-\xi\right)\left(\tfrac{\varphi\left(\sigma\gamma-\left(\sigma\gamma-1\right)\left(2\left(1-\xi\right)-1\right)\right)+\gamma}{\varphi\left(\sigma\gamma-\left(\sigma\gamma-1\right)\left(2\left(1-\xi\right)-1\right)^2\right)+\gamma}\right)\left(1+\delta_{a,a^*}-\delta_a\right),\\ \Pi_2 &\equiv \left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\delta_{a,a^*}+\left(1-\left(\Theta\Lambda+\left(1-\Theta\right)\left(1-\Lambda\right)\right)\right)\left(\delta_a-1\right)\\ &= \left(\delta_a-1\right)+\left(1-\xi\right)\left(\tfrac{\varphi\left(\sigma\gamma-\left(\sigma\gamma-1\right)\left(2\left(1-\xi\right)-1\right)\right)+\gamma}{\varphi\left(\sigma\gamma-\left(\sigma\gamma-1\right)\left(2\left(1-\xi\right)-1\right)^2\right)+\gamma}\right)\left(1+\delta_{a,a^*}-\delta_a\right).\end{aligned}$$

Assuming invertibility, the vector of natural rates inherits the VAR(1) stochastic structure of the productivity shocks. Accordingly, the natural rates take the following stochastic form,

$$\begin{aligned}\begin{pmatrix}\widehat{\overline{r}}_t\\ \widehat{\overline{r}}_t^*\end{pmatrix} &\approx \begin{pmatrix}\Pi_1 & \Pi_2\\ \Pi_2 & \Pi_1\end{pmatrix}\begin{pmatrix}\delta_a & \delta_{a,a^*}\\ \delta_{a,a^*} & \delta_a\end{pmatrix}\begin{pmatrix}\Pi_1 & \Pi_2\\ \Pi_2 & \Pi_1\end{pmatrix}^{-1}\begin{pmatrix}\widehat{\overline{r}}_{t-1}\\ \widehat{\overline{r}}_{t-1}^*\end{pmatrix}+\gamma\left(\frac{1+\varphi}{\gamma+\varphi}\right)\begin{pmatrix}\Pi_1 & \Pi_2\\ \Pi_2 & \Pi_1\end{pmatrix}\begin{pmatrix}\widehat{\varepsilon}_t^a\\ \widehat{\varepsilon}_t^{a*}\end{pmatrix},\\ \begin{pmatrix}\widehat{\varepsilon}_t^a\\ \widehat{\varepsilon}_t^{a*}\end{pmatrix} &\sim N\left(\begin{pmatrix}0\\ 0\end{pmatrix},\begin{pmatrix}\sigma_a^2 & \rho_{a,a^*}\sigma_a^2\\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2\end{pmatrix}\right).\end{aligned}$$

where,

$$\begin{aligned}\begin{pmatrix}\Pi_1 & \Pi_2\\ \Pi_2 & \Pi_1\end{pmatrix}^{-1} &= \frac{1}{\left(\Pi_1\right)^2-\left(\Pi_2\right)^2}\begin{pmatrix}\Pi_1 & -\Pi_2\\ -\Pi_2 & \Pi_1\end{pmatrix},\\ \left(\Pi_1\right)^2-\left(\Pi_2\right)^2 &= \left(\frac{\left(\gamma+\varphi\right)\left(2\left(1-\xi\right)-1\right)}{\varphi\left(\sigma\gamma-\left(\sigma\gamma-1\right)\left(2\left(1-\xi\right)-1\right)^2\right)+\gamma}\right)\left(\left(\delta_a-1\right)^2-\left(\delta_{a,a^*}\right)^2\right).\end{aligned}$$

Hence, it follows that,

$$\begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix} \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix}^{-1}$$
$$= \tfrac{1}{(\Pi_1)^2-(\Pi_2)^2} \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix} \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \Pi_1 & -\Pi_2 \\ -\Pi_2 & \Pi_1 \end{pmatrix} = \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix},$$

which implies that the structure of the VAR(1) for the natural rates inherits the persistence structure of the underlying productivity shocks. Moreover, I can simplify the notation as follows,

$$\begin{aligned}
\begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} &\equiv \gamma \left(\frac{1+\varphi}{\gamma+\varphi}\right) \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_t^a \\ \widehat{\varepsilon}_t^{a*} \end{pmatrix}, \\
\begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} &\sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix} \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix} \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix}^T \right),
\end{aligned}$$

where,

$$\begin{aligned}
&\gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix} \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix} \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix}^T \\
&= \gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix} \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix} \begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2 & \Pi_1 \end{pmatrix} \\
&= \sigma_a^2\gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \begin{pmatrix} (\Pi_1)^2 + 2\rho_{a,a^*}\Pi_1\Pi_2 + (\Pi_2)^2 & \rho_{a,a^*}(\Pi_1)^2 + 2\Pi_1\Pi_2 + \rho_{a,a^*}(\Pi_2)^2 \\ \rho_{a,a^*}(\Pi_1)^2 + 2\Pi_1\Pi_2 + \rho_{a,a^*}(\Pi_2)^2 & (\Pi_1)^2 + 2\rho_{a,a^*}\Pi_1\Pi_2 + (\Pi_2)^2 \end{pmatrix} \\
&= \sigma_a^2\gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \left((\Pi_1)^2 + 2\rho_{a,a^*}\Pi_1\Pi_2 + (\Pi_2)^2\right) \begin{pmatrix} 1 & \frac{\rho_{a,a^*}(\Pi_1)^2+2\Pi_1\Pi_2+\rho_{a,a^*}(\Pi_2)^2}{(\Pi_1)^2+2\rho_{a,a^*}\Pi_1\Pi_2+(\Pi_2)^2} \\ \frac{\rho_{a,a^*}(\Pi_1)^2+2\Pi_1\Pi_2+\rho_{a,a^*}(\Pi_2)^2}{(\Pi_1)^2+2\rho_{a,a^*}\Pi_1\Pi_2+(\Pi_2)^2} & 1 \end{pmatrix}.
\end{aligned}$$

Hence, I define the volatility and the correlation of the natural rate innovations in the following fashion,

$$\begin{aligned}
\sigma_r^2 &= \sigma_a^2\gamma^2 \left(\frac{1+\varphi}{\gamma+\varphi}\right)^2 \left((\Pi_1)^2 + 2\rho_{a,a^*}\Pi_1\Pi_2 + (\Pi_2)^2\right), \\
\rho_{r,r^*} &= \frac{\rho_{a,a^*}(\Pi_1)^2 + 2\Pi_1\Pi_2 + \rho_{a,a^*}(\Pi_2)^2}{(\Pi_1)^2 + 2\rho_{a,a^*}\Pi_1\Pi_2 + (\Pi_2)^2}.
\end{aligned}$$

Derivation of the stochastic process for world and cross-country difference variables. The Home and Foreign shocks are summarized in (9) – (14) and Table 2. Accordingly, the aggregate shocks are,

$$\widehat{\overline{r}}_t^W = \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{\overline{r}}_t \\ \widehat{\overline{r}}_t^* \end{pmatrix}, \ \widehat{m}_t^W = \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{m}_t \\ \widehat{m}_t^* \end{pmatrix}, \ \widehat{v}_t^W = \widehat{u}_t^W = \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{u}_t \\ \widehat{u}_t^* \end{pmatrix}.$$

Hence, the structure of the shocks can be summarized as follows,

$$\begin{aligned}
\widehat{\overline{r}}_t^W &= \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix} \begin{pmatrix} \widehat{\overline{r}}_{t-1} \\ \widehat{\overline{r}}_{t-1}^* \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} \\
&= (\delta_a + \delta_{a,a^*}) \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{\overline{r}}_{t-1} \\ \widehat{\overline{r}}_{t-1}^* \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} = (\delta_a + \delta_{a,a^*}) \widehat{\overline{r}}_{t-1}^W + \widehat{\varepsilon}_t^{rW}, \\
\widehat{\varepsilon}_t^{rW} &= \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} \sim N\left(0, \sigma_r^2 \frac{1}{4} \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \rho_{r,r^*} \\ \rho_{r,r^*} & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right),
\end{aligned}$$

where $\frac{1}{4}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{r,r^*} \\ \rho_{r,r^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2}\left(1+\rho_{r,r^*}\right)$ implies that $\widehat{\varepsilon}_t^{rW} \sim N\left(0, \sigma_r^2\left(\frac{1+\rho_{r,r^*}}{2}\right)\right)$.

Analogously, it must follow that,

$$\begin{aligned} \widehat{m}_t^W &= \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \delta_m & 0 \\ 0 & \delta_m \end{pmatrix}\begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \\ &= \delta_m \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} = \delta_m \widehat{m}_{t-1}^W + \widehat{\varepsilon}_t^{mW}, \\ \widehat{\varepsilon}_t^{mW} &\equiv \frac{1}{2}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \sim N\left(0, \sigma_m^2 \frac{1}{4}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{m,m^*} \\ \rho_{m,m^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right), \end{aligned}$$

where $\frac{1}{4}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{m,m^*} \\ \rho_{m,m^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2}\left(1+\rho_{m,m^*}\right)$ implies that $\widehat{\varepsilon}_t^{mW} \sim N\left(0, \sigma_m^2\left(\frac{1+\rho_{m,m^*}}{2}\right)\right)$.
Finally, I can say that,

$$\begin{aligned} \widehat{v}_t^W &= \widehat{u}_t^W = \delta_u \widehat{v}_{t-1}^W + \widehat{\varepsilon}_t^{vW}, \\ \widehat{\varepsilon}_t^{vW} &\sim N\left(0, \sigma_u^2 \frac{1}{4}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{u,u^*} \\ \rho_{u,u^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right), \end{aligned}$$

where $\frac{1}{4}\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{u,u^*} \\ \rho_{u,u^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2}\left(1+\rho_{u,u^*}\right)$ implies that $\widehat{\varepsilon}_t^{vW} \sim N\left(0, \sigma_u^2\left(\frac{1+\rho_{u,u^*}}{2}\right)\right)$.

Then, I derive the difference forcing processes $\widehat{\overline{r}}_t^R$, $\widehat{m}_t^R$ and $\widehat{v}_t^R = (1-2\xi)\,\widehat{u}_t^R$ similarly as,

$$\begin{aligned} \widehat{\overline{r}}_t^R &= \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\overline{r}}_t \\ \widehat{\overline{r}}_t^* \end{pmatrix}, \ \widehat{m}_t^R = \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{m}_t \\ \widehat{m}_t^* \end{pmatrix}, \\ \widehat{v}_t^R &= (1-2\xi)\,\widehat{u}_t^R = (1-2\xi)\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{u}_t \\ \widehat{u}_t^* \end{pmatrix}. \end{aligned}$$

Hence, the structure of the shocks can be summarized as follows,

$$\begin{aligned} \widehat{\overline{r}}_t^R &= \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \delta_a & \delta_{a,a^*} \\ \delta_{a,a^*} & \delta_a \end{pmatrix}\begin{pmatrix} \widehat{\overline{r}}_{t-1} \\ \widehat{\overline{r}}_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} \\ &= (\delta_a - \delta_{a,a^*})\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\overline{r}}_{t-1} \\ \widehat{\overline{r}}_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} = (\delta_a - \delta_{a,a^*})\,\widehat{\overline{r}}_{t-1}^W + \widehat{\varepsilon}_t^{rW}, \\ \widehat{\varepsilon}_t^{rW} &= \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^r \\ \widehat{\varepsilon}_t^{r*} \end{pmatrix} \sim N\left(0, \sigma_r^2 \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{r,r^*} \\ \rho_{r,r^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right), \end{aligned}$$

where $\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{r,r^*} \\ \rho_{r,r^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2\left(1-\rho_{r,r^*}\right)$ implies that $\widehat{\varepsilon}_t^{rW} \sim N\left(0, 2\sigma_r^2\left(1-\rho_{r,r^*}\right)\right)$.

Analogously, it must follow that,

$$\begin{aligned} \widehat{m}_t^R &= \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \delta_m & 0 \\ 0 & \delta_m \end{pmatrix}\begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \\ &= \delta_m \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{m}_{t-1} \\ \widehat{m}_{t-1}^* \end{pmatrix} + \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} = \delta_m \widehat{m}_{t-1}^R + \widehat{\varepsilon}_t^{mR}, \\ \widehat{\varepsilon}_t^{mR} &\equiv \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^m \\ \widehat{\varepsilon}_t^{m*} \end{pmatrix} \sim N\left(0, \sigma_m^2 \begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{m,m^*} \\ \rho_{m,m^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right), \end{aligned}$$

where $\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{m,m^*} \\ \rho_{m,m^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2\left(1-\rho_{m,m^*}\right)$ implies that $\widehat{\varepsilon}_t^{mW} \sim N\left(0, 2\sigma_m^2\left(1-\rho_{m,m^*}\right)\right)$.

Finally, I can say that,

$$
\begin{aligned}
\widehat{v}_t^R &\equiv (1-2\xi)\,\widehat{u}_t^R = (1-2\xi)\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \delta_u & 0 \\ 0 & \delta_u \end{pmatrix}\begin{pmatrix} \widehat{u}_{t-1} \\ \widehat{u}_{t-1}^* \end{pmatrix} + (1-2\xi)\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^u \\ \widehat{\varepsilon}_t^{u*} \end{pmatrix} \\
&= (1-2\xi)\,\delta_u\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{u}_{t-1} \\ \widehat{u}_{t-1}^* \end{pmatrix} + (1-2\xi)\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^u \\ \widehat{\varepsilon}_t^{u*} \end{pmatrix} = \delta_u\widehat{v}_{t-1}^R + \widehat{\varepsilon}_t^{vR}, \\
\widehat{\varepsilon}_t^{vR} &\equiv (1-2\xi)\,\widehat{\varepsilon}_t^{uR} \equiv (1-2\xi)\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} \widehat{\varepsilon}_t^u \\ \widehat{\varepsilon}_t^{u*} \end{pmatrix} \sim N\left(0, (1-2\xi)^2\sigma_u^2\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{u,u^*} \\ \rho_{u,u^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right),
\end{aligned}
$$

where $\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & \rho_{u,u^*} \\ \rho_{u,u^*} & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = 2\left(1-\rho_{u,u^*}\right)$ implies that $\widehat{\varepsilon}_t^{vW} \sim N\left(0, 2(1-2\xi)^2\sigma_u^2\left(1-\rho_{u,u^*}\right)\right)$.

B The Model Solution

I conjecture that the solution for the endogenous variables $\left(\widehat{x}_t^{c,s,j}, \widehat{\pi}_t^{c,s,j}, \widehat{i}_t^{c,s,j}\right)$ corresponding to the model under a common (coordinated) monetary policy can be expressed as,

$$
\widehat{\pi}_t^{c,s,j} = \chi_{1,j}^{c,s}\widehat{\pi}_{t-1}^{c,s,j} + \eta_t^{c,s,j},\ \eta_t^{c,s,j} \sim N\left(0, \sigma_{c,s,j}^2\right), \tag{64}
$$

$$
\widehat{x}_t^{c,s,j} = \frac{1}{\chi_{0,j}^{c,s}}\widehat{\pi}_t^{c,s,j},\ \text{for all } s = W, R \text{ and for all shocks } j = r, m, v, \tag{65}
$$

where the nominal short-term interest rate is given by $\widehat{i}_t^{c,s,j} = \psi_\pi^c\widehat{\pi}_t^{c,s,j} + \psi_x^c\widehat{x}_t^{c,s,j} + \widehat{m}_t^s$.

B.1 Natural Rate Shocks and Monetary Shocks

I can express the model whenever $\widehat{v}_t^s = 0$ for all $s = W, R$ as follows,

$$
\begin{aligned}
&\widehat{x}_t^{c,s,j} = \Psi^{c,s}\mathbb{E}_t\left[\widehat{x}_{t+1}^{c,s,j}\right] + \Psi^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,j}\right] + \Psi^{c,s}\frac{\Omega^s}{\gamma}\left(\widehat{\overline{r}}_t^s - \widehat{m}_t^s\right), \\
&\widehat{\pi}_t^{c,s,j} = \Psi^{c,s}\Phi\left(\varphi+\gamma\right)\kappa^s\mathbb{E}_t\left[\widehat{x}_{t+1}^{c,s,j}\right] + \Psi^{c,s}\left(\tfrac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s + \beta\left(1+\tfrac{\Omega^s}{\gamma}\psi_x^c\right)\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,j}\right] + ... \\
&\qquad \Psi^{c,s}\tfrac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s\left(\widehat{\overline{r}}_t^s - \widehat{m}_t^s\right), \\
&\widehat{\overline{r}}_t^s = \delta_r^s\widehat{\overline{r}}_{t-1}^s + \widehat{\varepsilon}_t^{rs},\ \widehat{\varepsilon}_t^{rs} \sim N\left(0, \sigma_{rs}^2\right),\ \widehat{m}_t^s = \delta_m\widehat{m}_{t-1}^s + \widehat{\varepsilon}_t^{ms},\ \widehat{\varepsilon}_t^{ms} \sim N\left(0, \sigma_{ms}^2\right),
\end{aligned}
$$

where the forcing processes for $s = W, R$ are characterized elsewhere in the paper.

Step 0. I replace the conjecture for $\widehat{x}_t^{c,s,j} = \frac{1}{\chi_{0,j}^{c,s}}\widehat{\pi}_t^{c,s,j}$ in the corresponding expectational equations of the model so that I can express both of them in terms of inflation alone as,

$$
\begin{aligned}
&\frac{1}{\chi_{0,j}^{c,s}}\widehat{\pi}_t^{c,s,j} = \Psi^{c,s}\mathbb{E}_t\left[\frac{1}{\chi_{0,j}^{c,s}}\widehat{\pi}_{t+1}^{c,s,j}\right] + \Psi^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,j}\right] + \Psi^{c,s}\frac{\Omega^s}{\gamma}\left(\widehat{\overline{r}}_t^s - \widehat{m}_t^s\right), \\
&\widehat{\pi}_t^{c,s,j} = \Psi^{c,s}\Phi\left(\varphi+\gamma\right)\kappa^s\mathbb{E}_t\left[\tfrac{1}{\chi_{0,j}^{c,s}}\widehat{\pi}_{t+1}^{c,s,j}\right] + \Psi^{c,s}\left(\tfrac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s + \beta\left(1+\tfrac{\Omega^s}{\gamma}\psi_x^c\right)\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,j}\right] + .. \\
&\qquad \Psi^{c,s}\tfrac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s\left(\widehat{\overline{r}}_t^s - \widehat{m}_t^s\right),
\end{aligned}
$$

or, simply,

$$\widehat{\pi}_t^{c,s,j} = \Psi^{c,s}\left[1+\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right]\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,j}\right]+\Psi^{c,s}\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(\widehat{\overline{r}}_t^s-\widehat{m}_t^s\right),$$

$$\widehat{\pi}_t^{c,s,j} = \Psi^{c,s}\left[\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,j}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right]\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,j}\right]+\Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s\left(\widehat{\overline{r}}_t^s-\widehat{m}_t^s\right).$$

Step 1. Replace the conjecture for $\widehat{\pi}_t^{c,s,j}=\chi_{1,j}^{c,s}\widehat{\pi}_{t-1}^{c,s,j}+\eta_t^{c,s,j}$, $\eta_t^{c,s,j}\sim N\left(0,\sigma_{c,s,j}^2\right)$ to express the inflation expectations in terms of current inflation,

$$\begin{aligned}
\widehat{\pi}_t^{c,s,j} &= \Psi^{c,s}\left[1+\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right]\chi_{1,j}^{c,s}\widehat{\pi}_t^{c,s,j}+\Psi^{c,s}\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(\widehat{\overline{r}}_t^s-\widehat{m}_t^s\right),\\
\widehat{\pi}_t^{c,s,j} &= \Psi^{c,s}\left[\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,j}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right]\chi_{1,j}^{c,s}\widehat{\pi}_t^{c,s,j}+\Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s\left(\widehat{\overline{r}}_t^s-\widehat{m}_t^s\right),
\end{aligned}$$

and re-write the system of equations as follows,

$$\begin{aligned}
\widehat{\pi}_t^{c,s,j} &= \frac{\Psi^{c,s}\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,j}^{c,s}}\left(\widehat{\overline{r}}_t^s-\widehat{m}_t^s\right),\\
\widehat{\pi}_t^{c,s,j} &= \frac{\Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s}{1-\Psi^{c,s}\left(\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,j}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right)\chi_{1,j}^{c,s}}\left(\widehat{\overline{r}}_t^s-\widehat{m}_t^s\right),\\
\widehat{\overline{r}}_t^s &= \delta_r^s\widehat{\overline{r}}_{t-1}^s+\widehat{\varepsilon}_t^{rs},\ \widehat{\varepsilon}_t^{rs}\sim N\left(0,\sigma_{rs}^2\right),\\
\widehat{m}_t^s &= \delta_m\widehat{m}_{t-1}^s+\widehat{\varepsilon}_t^{ms},\ \widehat{\varepsilon}_t^{ms}\sim N\left(0,\sigma_{ms}^2\right).
\end{aligned}$$

Step 2. Replace the solution for inflation in the natural rate shock process as follows,

$$\begin{aligned}
\widehat{\pi}_t^{c,s,r} &= \delta_r^s\widehat{\pi}_{t-1}^{c,s,r}+\left(\frac{\Psi^{c,s}\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,j}^{c,s}}\right)\widehat{\varepsilon}_t^{rs},\ \widehat{\varepsilon}_t^{rs}\sim N\left(0,\sigma_{rs}^2\right),\\
\widehat{\pi}_t^{c,s,r} &= \delta_r^s\widehat{\pi}_{t-1}^{c,s,r}+\left(\frac{\Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s}{1-\Psi^{c,s}\left(\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,j}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right)\chi_{1,j}^{c,s}}\right)\widehat{\varepsilon}_t^{rs},\ \widehat{\varepsilon}_t^{rs}\sim N\left(0,\sigma_{rs}^2\right),
\end{aligned}$$

and in the monetary shock process as,

$$\begin{aligned}
\widehat{\pi}_t^{c,s,m} &= \delta_m\widehat{\pi}_{t-1}^{c,s,m}-\left(\frac{\Psi^{c,s}\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,j}^{c,s}}\right)\widehat{\varepsilon}_t^{ms},\ \widehat{\varepsilon}_t^{ms}\sim N\left(0,\sigma_{ms}^2\right),\\
\widehat{\pi}_t^{c,s,m} &= \delta_m\widehat{\pi}_{t-1}^{c,s,m}-\left(\frac{\Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s}{1-\Psi^{c,s}\left(\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,j}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right)\chi_{1,j}^{c,s}}\right)\widehat{\varepsilon}_t^{ms},\ \widehat{\varepsilon}_t^{ms}\sim N\left(0,\sigma_{ms}^2\right).
\end{aligned}$$

Step 3. I then apply the method of matching coefficients to equate this formula with the conjecture above, imposing enough restrictions to ensure that both solutions are identical,

$$\begin{aligned}
\chi_{1,r}^{c,s} &= \delta_r^s, \\
\left(\frac{\Psi^{c,s}\chi_{0,r}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,r}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,r}^{c,s}}\right) &= \left(\frac{\Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s}{1-\Psi^{c,s}\left(\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,r}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right)\chi_{1,r}^{c,s}}\right), \\
\eta_t^{c,s,r} &= \left(\frac{\Psi^{c,s}\chi_{0,r}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,r}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,r}^{c,s}}\right)\widehat{\varepsilon}_t^{rs},
\end{aligned}$$

and

$$\begin{aligned}
\chi_{1,m}^{c,s} &= \delta_m, \\
\left(\frac{\Psi^{c,s}\chi_{0,m}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,m}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,m}^{c,s}}\right) &= \left(\frac{\Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s}{1-\Psi^{c,s}\left(\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,m}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right)\chi_{1,m}^{c,s}}\right), \\
\eta_t^{c,s,m} &= -\left(\frac{\Psi^{c,s}\chi_{0,m}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,m}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,m}^{c,s}}\right)\widehat{\varepsilon}_t^{ms}.
\end{aligned}$$

The implicit formula for $\chi_{0,j}^{c,s}$ for $j=r,m$ comes down to,

$$\chi_{0,j}^{c,s}=\frac{\Phi\left(\varphi+\gamma\right)\kappa^s}{1-\beta\chi_{1,j}^{c,s}},$$

given that $\Psi^{c,s}\equiv\frac{1}{1+\frac{\Omega^s}{\gamma}\left(\psi_x^c+\Phi(\varphi+\gamma)\kappa^s\psi_\pi^c\right)}>0$. Then, it holds that,

$$\left(\frac{\Psi^{c,s}\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}}{1-\Psi^{c,s}\left(1+\chi_{0,j}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,j}^{c,s}}\right)=\left(\frac{\Phi\left(\varphi+\gamma\right)\kappa^s\frac{\Omega^s}{\gamma}}{\left(1-\beta\chi_{1,j}^{c,s}\right)\left(1-\chi_{1,j}^{c,s}+\frac{\Omega^s}{\gamma}\psi_x^c\right)+\Phi\left(\varphi+\gamma\right)\kappa^s\frac{\Omega^s}{\gamma}\left(\psi_\pi^c-\chi_{1,j}^{c,s}\right)}\right),$$

and I can summarize the results as follows,

$$\begin{aligned}
\chi_{1,r}^{c,s} &= \delta_r^s,\ \chi_{1,m}^{c,s}=\delta_m,\ \chi_{0,j}^{c,s}=\frac{\Phi\left(\varphi+\gamma\right)\kappa^s}{1-\beta\chi_{1,j}^{c,s}},\ j=r,m, \\
\eta_t^{c,s,r} &= \left(\frac{\Phi\left(\varphi+\gamma\right)\kappa^s\frac{\Omega^s}{\gamma}}{\left(1-\beta\chi_{1,r}^{c,s}\right)\left(1-\chi_{1,r}^{c,s}+\frac{\Omega^s}{\gamma}\psi_x^c\right)+\Phi\left(\varphi+\gamma\right)\kappa^s\frac{\Omega^s}{\gamma}\left(\psi_\pi^c-\chi_{1,r}^{c,s}\right)}\right)\widehat{\varepsilon}_t^{rs}, \\
\eta_t^{c,s,m} &= -\left(\frac{\Phi\left(\varphi+\gamma\right)\kappa^s\frac{\Omega^s}{\gamma}}{\left(1-\beta\chi_{1,m}^{c,s}\right)\left(1-\chi_{1,m}^{c,s}+\frac{\Omega^s}{\gamma}\psi_x^c\right)+\Phi\left(\varphi+\gamma\right)\kappa^s\frac{\Omega^s}{\gamma}\left(\psi_\pi^c-\chi_{1,m}^{c,s}\right)}\right)\widehat{\varepsilon}_t^{ms}.
\end{aligned}$$

Notice that $\mathbb{E}\left[\eta_t^{c,s,j}\right]=0$ and also that $\sigma_{c,s,j}^2\equiv\mathbb{V}\left[\eta_t^{c,s,j}\right]=\left(\frac{\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}}{\left(1-\beta\chi_{1,j}^{c,s}\right)\left(1-\chi_{1,j}^{c,s}+\frac{\Omega^s}{\gamma}\psi_x^c\right)+\Phi(\varphi+\gamma)\kappa^s\frac{\Omega^s}{\gamma}\left(\psi_\pi^c-\chi_{1,j}^{c,s}\right)}\right)^2\sigma_{js}^2$ for all shocks $j=r,m$.

B.2 Cost-Push Shocks

I can express the model whenever $\widehat{\overline{r}}_t^s = \widehat{m}_t^s = 0$ for all $s = W, R$ as follows,

$$\widehat{x}_t^{c,s,v} = \Psi^{c,s}\mathbb{E}_t\left[\widehat{x}_{t+1}^{c,s,v}\right] + \Psi^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,v}\right] - \Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\psi_\pi^c\widehat{v}_t^s,$$

$$\widehat{\pi}_t^{c,s,v} = \Psi^{c,s}\Phi\left(\varphi+\gamma\right)\kappa^s\mathbb{E}_t\left[\widehat{x}_{t+1}^{c,s,v}\right] + \Psi^{c,s}\left(\tfrac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s + \beta\left(1+\tfrac{\Omega^s}{\gamma}\psi_x^c\right)\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,v}\right] + ...$$

$$\Psi^{c,s}\Phi\left(\varphi+\gamma\right)\left(1+\tfrac{\Omega^s}{\gamma}\psi_x^c\right)\widehat{v}_t^s,$$

$$\widehat{v}_t^s = \delta_v\widehat{v}_{t-1}^s + \widehat{\varepsilon}_t^{vs},\ \widehat{\varepsilon}_t^{vs} \sim N\left(0,\sigma_{vs}^2\right),$$

where the forcing processes for $s = W, R$ are characterized elsewhere in the paper.

Step 0. I replace the conjecture for $\widehat{x}_t^{c,s,v} = \frac{1}{\chi_{0,v}^{c,s}}\widehat{\pi}_t^{c,s,v}$ in the corresponding expectational equations of the model so that I can express both of them in terms of inflation alone as,

$$\frac{1}{\chi_{0,v}^{c,s}}\widehat{\pi}_t^{c,s,v} = \Psi^{c,s}\mathbb{E}_t\left[\frac{1}{\chi_{0,v}^{c,s}}\widehat{\pi}_{t+1}^{c,s,v}\right] + \Psi^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,v}\right] - \Psi^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\psi_\pi^c\widehat{v}_t^s,$$

$$\widehat{\pi}_t^{c,s,v} = \Psi^{c,s}\Phi\left(\varphi+\gamma\right)\kappa^s\mathbb{E}_t\left[\tfrac{1}{\chi_{0,v}^{c,s}}\widehat{\pi}_{t+1}^{c,s,v}\right] + \Psi^{c,s}\left(\tfrac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\kappa^s + \beta\left(1+\tfrac{\Omega^s}{\gamma}\psi_x^c\right)\right)\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,v}\right] + ...$$

$$\Psi^{c,s}\Phi\left(\varphi+\gamma\right)\left(1+\tfrac{\Omega^s}{\gamma}\psi_x^c\right)\widehat{v}_t^s,$$

or, simply,

$$\begin{aligned}
\widehat{\pi}_t^{c,s,v} &= \Psi^{c,s}\left[1+\chi_{0,v}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right]\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,v}\right] - \Psi^{c,s}\chi_{0,v}^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\psi_\pi^c\widehat{v}_t^s,\\
\widehat{\pi}_t^{c,s,v} &= \Psi^{c,s}\left[\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,v}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right]\mathbb{E}_t\left[\widehat{\pi}_{t+1}^{c,s,v}\right] + \Psi^{c,s}\Phi\left(\varphi+\gamma\right)\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\widehat{v}_t^s.
\end{aligned}$$

Step 1. Replace the conjecture for $\widehat{\pi}_t^{c,s,v} = \chi_{1,v}^{c,s}\widehat{\pi}_{t-1}^{c,s,v} + \eta_t^{c,s,v}$, $\eta_t^{c,s,v} \sim N\left(0,\sigma_{c,s,v}^2\right)$ to express the inflation expectations in terms of current inflation,

$$\begin{aligned}
\widehat{\pi}_t^{c,s,v} &= \Psi^{c,s}\left[1+\chi_{0,v}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right]\chi_{1,v}^{c,s}\widehat{\pi}_t^{c,s,v} - \Psi^{c,s}\chi_{0,v}^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\psi_\pi^c\widehat{v}_t^s,\\
\widehat{\pi}_t^{c,s,v} &= \Psi^{c,s}\left[\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,v}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right]\chi_{1,v}^{c,s}\widehat{\pi}_t^{c,s,v} + \Psi^{c,s}\Phi\left(\varphi+\gamma\right)\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\widehat{v}_t^s,
\end{aligned}$$

and re-write the system of equations as follows,

$$\begin{aligned}
\widehat{\pi}_t^{c,s,v} &= -\frac{\Psi^{c,s}\chi_{0,v}^{c,s}\frac{\Omega^s}{\gamma}\Phi\left(\varphi+\gamma\right)\psi_\pi^c}{1-\Psi^{c,s}\left(1+\chi_{0,v}^{c,s}\frac{\Omega^s}{\gamma}\left(1-\beta\psi_\pi^c\right)\right)\chi_{1,v}^{c,s}}\widehat{v}_t^s,\\
\widehat{\pi}_t^{c,s,v} &= \frac{\Psi^{c,s}\Phi\left(\varphi+\gamma\right)\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)}{1-\Psi^{c,s}\left(\Phi\left(\varphi+\gamma\right)\kappa^s\left(\frac{1}{\chi_{0,v}^{c,s}}+\frac{\Omega^s}{\gamma}\right)+\beta\left(1+\frac{\Omega^s}{\gamma}\psi_x^c\right)\right)\chi_{1,v}^{c,s}}\widehat{v}_t^s,\\
\widehat{v}_t^s &= \delta_v\widehat{v}_{t-1}^s + \widehat{\varepsilon}_t^{vs},\ \widehat{\varepsilon}_t^{vs} \sim N\left(0,\sigma_{vs}^2\right).
\end{aligned}$$

Step 2. Replace the solution for inflation in the natural rate shock process as follows,

$$
\begin{aligned}
\widehat{\pi}_t^{c,s,v} &= \delta_v \widehat{\pi}_{t-1}^{c,s,v} - \left(\frac{\Psi^{c,s} \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \Phi\left(\varphi+\gamma\right) \psi_\pi^c}{1 - \Psi^{c,s} \left(1 + \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \left(1 - \beta \psi_\pi^c\right)\right) \chi_{1,v}^{c,s}} \right) \widehat{\varepsilon}_t^{vs},\ \widehat{\varepsilon}_t^{vs} \sim N\left(0, \sigma_{vs}^2\right), \\
\widehat{\pi}_t^{c,s,v} &= \delta_v \widehat{\pi}_{t-1}^{c,s,v} + \left(\frac{\Psi^{c,s} \Phi\left(\varphi+\gamma\right) \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c\right)}{1 - \Psi^{c,s} \left(\Phi\left(\varphi+\gamma\right) \kappa^s \left(\frac{1}{\chi_{0,v}^{c,s}} + \frac{\Omega^s}{\gamma} \right) + \beta \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c\right) \right) \chi_{1,v}^{c,s}} \right) \widehat{\varepsilon}_t^{vs},\ \widehat{\varepsilon}_t^{vs} \sim N\left(0, \sigma_{vs}^2\right).
\end{aligned}
$$

Step 3. I then apply the method of matching coefficients to equate this formula with the conjecture above imposing enough restrictions to ensure that both solutions are identical,

$$
\begin{aligned}
\chi_{1,v}^{c,s} &= \delta_v, \\
-\left(\frac{\Psi^{c,s} \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \Phi\left(\varphi+\gamma\right) \psi_\pi^c}{1 - \Psi^{c,s} \left(1 + \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \left(1 - \beta \psi_\pi^c\right)\right) \chi_{1,v}^{c,s}} \right) &= \left(\frac{\Psi^{c,s} \Phi\left(\varphi+\gamma\right) \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c\right)}{1 - \Psi^{c,s} \left(\Phi\left(\varphi+\gamma\right) \kappa^s \left(\frac{1}{\chi_{0,v}^{c,s}} + \frac{\Omega^s}{\gamma} \right) + \beta \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c\right) \right) \chi_{1,v}^{c,s}} \right), \\
\eta_t^{c,s,v} &= -\left(\frac{\Psi^{c,s} \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \Phi\left(\varphi+\gamma\right) \psi_\pi^c}{1 - \Psi^{c,s} \left(1 + \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \left(1 - \beta \psi_\pi^c\right)\right) \chi_{1,v}^{c,s}} \right) \widehat{\varepsilon}_t^{vs}.
\end{aligned}
$$

The implicit formula for $\chi_{0,v}^{c,s}$ comes down to,

$$
\chi_{0,v}^{c,s} = -\left(\frac{1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}}{\frac{\Omega^s}{\gamma} \left(\psi_\pi^c - \chi_{1,v}^{c,s} \right)} \right),
$$

given that $\Psi^{c,s} \equiv \frac{1}{1 + \frac{\Omega^s}{\gamma} \left(\psi_x^c + \Phi\left(\varphi+\gamma\right) \kappa^s \psi_\pi^c \right)} > 0$. Then, it holds that,

$$
-\left(\frac{\Psi^{c,s} \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \Phi\left(\varphi+\gamma\right) \psi_\pi^c}{1 - \Psi^{c,s} \left(1 + \chi_{0,v}^{c,s} \frac{\Omega^s}{\gamma} \left(1 - \beta \psi_\pi^c\right)\right) \chi_{1,v}^{c,s}} \right) = \left(\frac{\Psi^{c,s} \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}\right) \Phi\left(\varphi+\gamma\right) \psi_\pi^c}{\left(1 - \Psi^{c,s} \chi_{1,v}^{c,s}\right) \left(\psi_\pi^c - \chi_{1,v}^{c,s}\right) + \Psi^{c,s} \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}\right) \left(1 - \beta \psi_\pi^c\right) \chi_{1,v}^{c,s}} \right),
$$

and I can summarize the results as follows,

$$
\begin{aligned}
\chi_{1,v}^{c,s} &= \delta_v,\ \chi_{0,v}^{c,s} = -\left(\frac{1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}}{\frac{\Omega^s}{\gamma} \left(\psi_\pi^c - \chi_{1,v}^{c,s} \right)} \right), \\
\eta_t^{c,s,v} &= \left(\frac{\Psi^{c,s} \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}\right) \Phi\left(\varphi+\gamma\right) \psi_\pi^c}{\left(1 - \Psi^{c,s} \chi_{1,v}^{c,s}\right) \left(\psi_\pi^c - \chi_{1,v}^{c,s}\right) + \Psi^{c,s} \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}\right) \left(1 - \beta \psi_\pi^c\right) \chi_{1,v}^{c,s}} \right) \widehat{\varepsilon}_t^{vs}.
\end{aligned}
$$

Notice that $\mathbb{E}\left[\eta_t^{c,s,v}\right] = 0$ and also that $\sigma_{c,s,v}^2 \equiv \mathbb{V}\left[\eta_t^{c,s,v}\right] = \left(\frac{\Psi^{c,s} \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}\right) \Phi\left(\varphi+\gamma\right) \psi_\pi^c}{\left(1 - \Psi^{c,s} \chi_{1,v}^{c,s}\right) \left(\psi_\pi^c - \chi_{1,v}^{c,s}\right) + \Psi^{c,s} \left(1 + \frac{\Omega^s}{\gamma} \psi_x^c - \chi_{1,v}^{c,s}\right) \left(1 - \beta \psi_\pi^c\right) \chi_{1,v}^{c,s}} \right)^2 \sigma_{vs}^2$ for the cost-push shock.

C The Building Blocks of the Model

Here I describe the main features of the open-economy New Keynesian framework maintaining the symmetry in the structure of both countries but allowing for differences in the monetary policy. I illustrate the model with the first principles from the Home country unless otherwise noted, and use the superscript $*$ to denote Foreign country variables (or parameters). I focus my attention on the principal elements of departure from previous treatments about the role that the monetary policy regime plays in the international transmission of country-specific shocks as that is where the main interest of my analysis lies.

The Representative Household. The lifetime utility of the representative household in the Home country is additively separable in consumption, C_t, and labor, L_t, i.e.,

$$\sum\nolimits_{\tau=0}^{+\infty} \beta^{\tau} \mathbb{E}_t \left[\frac{1}{1-\gamma} \left(C_{t+\tau}\right)^{1-\gamma} - \frac{\chi}{1+\varphi} \left(L_{t+\tau}\right)^{1+\varphi} \right], \tag{66}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution, and $\varphi > 0$ is the inverse of the Frisch elasticity of labor supply. The scaling factor $\chi > 0$ pins down labor in steady state. The representative household maximizes its lifetime utility in (66) subject to the following sequence of budget constraints which holds across all states of nature $\omega_t \in \Omega$, i.e.,

$$\begin{gathered} P_t C_t + \int_{\omega_{t+1} \in \Omega} Q_t \left(\omega_{t+1}\right) B_t^H \left(\omega_{t+1}\right) + S_t \int_{\omega_{t+1} \in \Omega} Q_t^* \left(\omega_{t+1}\right) B_t^F \left(\omega_{t+1}\right) \\ \leq B_{t-1}^H \left(\omega_t\right) + S_t B_{t-1}^F \left(\omega_t\right) + W_t L_t + Pr_t - T_t, \end{gathered} \tag{67}$$

where W_t is the nominal wage in the Home country, P_t is its consumer price index (CPI), T_t is a nominal lump-sum tax (or transfer) from the Home government, and Pr_t are (per-period) nominal profits from all firms producing the Home varieties. I denote the bilateral nominal exchange rate as S_t indicating the units of the currency of the Home country that can be obtained per unit of the Foreign country currency at time t. Similarly, I define the problem of the representative household in the Foreign country.

The household's budget includes a portfolio of one-period Arrow-Debreu securities (contingent bonds) internationally traded, issued in the currencies of both countries and in zero net supply. That is, the pair $\left\{B_t^H \left(\omega_{t+1}\right), B_t^F \left(\omega_{t+1}\right)\right\}$ indicates the portfolio of contingent bonds issued by both countries and held by the representative household of the Home country. Access to a full set of internationally-traded, one-period Arrow-Debreu securities completes the local and international asset markets recursively. The prices of the Home and Foreign contingent bonds expressed in their currencies of denomination are denoted $Q_t \left(\omega_{t+1}\right)$ and $Q_t^* \left(\omega_{t+1}\right)$, respectively.[22]

Under complete asset markets, standard no-arbitrage results imply that $Q_t \left(\omega_{t+1}\right) = \frac{S_t}{S_{t+1}\left(\omega_{t+1}\right)} Q_t^* \left(\omega_{t+1}\right)$ for every state of nature $\omega_t \in \Omega$. Hence, Home and Foreign households can efficiently share risks domestically as well as internationally—this implies that the intertemporal marginal rate of substitution is equalized across countries at each possible state of nature, and accordingly it follows that

$$\beta \left(\frac{C_t}{C_{t-1}}\right)^{-\gamma} \frac{P_{t-1}}{P_t} = \beta \left(\frac{C_t^*}{C_{t-1}^*}\right)^{-\gamma} \frac{P_{t-1}^* S_{t-1}}{P_t^* S_t}. \tag{68}$$

I define the bilateral real exchange rate as $RS_t \equiv \frac{S_t P_t^*}{P_t}$, so by backward recursion the *perfect international risk-sharing condition* in (68) implies that,

$$RS_t = \upsilon \left(\frac{C_t^*}{C_t}\right)^{-\gamma}, \tag{69}$$

[22] The price of each bond in the currency of the country who did not issue it is converted at the prevailing bilateral exchange rate with full exchange rate pass-through under the LOOP.

where $\upsilon \equiv \frac{S_0 P_0^*}{P_0}\left(\frac{C_0^*}{C_0}\right)^{\gamma}$ is a constant that depends on initial conditions. If the initial conditions correspond to those of the symmetric steady state, then the constant υ is simply equal to one.

Yields on redundant one-period, uncontingent nominal bonds in the Home country are derived from the price of the contingent Arrow-Debreu securities, which results in the following standard stochastic Euler equation for the Home country,

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right], \tag{70}$$

where i_t is the riskless Home nominal interest rate. The representative household's optimization problem also produces a labor supply equation of the following form,

$$\frac{W_t}{P_t} = \chi \left(C_t\right)^{\gamma} \left(L_t\right)^{\varphi}, \tag{71}$$

plus the budget constraint of the Home country household given by (67), the initial conditions, and the appropriate (no-Ponzi games) transversality conditions. An analogous Euler equation, labor supply equation, and household budget constraint (with the corresponding initial conditions and transversality conditions) can be derived for the Foreign country.

C_t is the CES aggregator of both countries' goods for the Home country household and is defined as,

$$C_t = \left[(1-\xi)^{\frac{1}{\sigma}} \left(C_t^H\right)^{\frac{\sigma-1}{\sigma}} + (\xi)^{\frac{1}{\sigma}} \left(C_t^F\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{72}$$

where $\sigma > 0$ is the elasticity of substitution between the consumption bundle of locally-produced goods C_t^H and the consumption bundle of the foreign-produced goods C_t^F. The share of imported goods in the consumption basket of the Home country satisfies that $0 < \xi \leq \frac{1}{2}$, so these preferences allow for local-consumption bias. Similarly, the CES aggregator for the Foreign country is defined as

$$C_t^* = \left[(\xi)^{\frac{1}{\sigma}} \left(C_t^{H*}\right)^{\frac{\sigma-1}{\sigma}} + (1-\xi)^{\frac{1}{\sigma}} \left(C_t^{F*}\right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{73}$$

where C_t^{F*} and C_t^{H*} are respectively the consumption bundle of foreign-produced goods and of the home-produced goods for the Foreign country household, and ξ identifies the share of imported goods in the Foreign consumption basket.

The consumption sub-indexes aggregate the consumption of the representative household of the bundle of differentiated varieties produced by each country and are defined as follows,

$$C_t^H = \left[\int_0^1 C_t(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^F = \left[\int_0^1 C_t(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{74}$$

$$C_t^{H*} = \left[\int_0^1 C_t^*(h)^{\frac{\theta-1}{\theta}} dh \right]^{\frac{\theta}{\theta-1}}, \quad C_t^{F*} = \left[\int_0^1 C_t^*(f)^{\frac{\theta-1}{\theta}} df \right]^{\frac{\theta}{\theta-1}}, \tag{75}$$

where $\theta > 1$ is the elasticity of substitution across the differentiated varieties within a country.

The CPIs that correspond to this specification of consumption preferences are,

$$P_t = \left[(1-\xi) \left(P_t^H\right)^{1-\sigma} + \xi \left(P_t^F\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad P_t^* = \left[\xi \left(P_t^{H*}\right)^{1-\sigma} + (1-\xi) \left(P_t^{F*}\right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{76}$$

and,

$$P_t^H = \left[\int_0^1 P_t(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \; P_t^F = \left[\int_0^1 P_t(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}, \tag{77}$$

$$P_t^{H*} = \left[\int_0^1 P_t^*(h)^{1-\theta} dh\right]^{\frac{1}{1-\theta}}, \; P_t^{F*} = \left[\int_0^1 P_t^*(f)^{1-\theta} df\right]^{\frac{1}{1-\theta}}, \tag{78}$$

where P_t^H and P_t^{F*} are the price sub-indexes corresponding to the bundle of varieties produced locally in the Home and Foreign countries, respectively. The price sub-index P_t^F represents the Home country price of the bundle of Foreign varieties while P_t^{H*} is the Foreign country price for the bundle of Home varieties. The price of the variety h produced in the Home country is expressed as $P_t(h)$ and $P_t^*(h)$ in units of the Home and Foreign currency, respectively. Similarly, the price of the variety f produced in the Foreign country is quoted in both countries as $P_t(f)$ and $P_t^*(f)$, respectively.

Each household decides how much to allocate to the different varieties of goods produced in each country. Given the structure of preferences, the utility maximization problem implies that the representative household's demand for each variety is given by,

$$C_t(h) = \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta} C_t^H, \; C_t(f) = \left(\frac{P_t(f)}{P_t^F}\right)^{-\theta} C_t^F, \tag{79}$$

$$C_t^*(h) = \left(\frac{P_t^*(h)}{P_t^{H*}}\right)^{-\theta} C_t^{H*}, \; C_t^*(f) = \left(\frac{P_t^*(f)}{P_t^{F*}}\right)^{-\theta} C_t^{F*}, \tag{80}$$

while the demand for the bundle of varieties produced by each country is simply equal to,

$$C_t^H = (1-\xi)\left(\frac{P_t^H}{P_t}\right)^{-\sigma} C_t, \; C_t^F = \xi\left(\frac{P_t^F}{P_t}\right)^{-\sigma} C_t, \tag{81}$$

$$C_t^{H*} = \xi\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} C_t^*, \; C_t^{F*} = (1-\xi)\left(\frac{P_t^{F*}}{P_t^*}\right)^{-\sigma} C_t^*. \tag{82}$$

These equations relate the demand for each variety—whether produced domestically or imported—to the aggregate consumption of the country.

The Firms' Price-Setting Behavior. Each firm located in either the Home or Foreign country supplies its local market and exports with its own differentiated variety operating under monopolistic competition. I assume producer currency pricing (PCP), so these firms set prices by invoicing all sales in their local currency. The PCP assumption implies that the LOOP holds at the variety level—i.e., for each variety h produced in the Home country, it must hold that $P_t(h) = S_t P_t^*(h)$ (similarly, for each variety f produced in the Foreign country, $P_t(f) = S_t P_t^*(f)$). Hence, it follows naturally that for the same bundle of varieties, the conforming price sub-indexes in both countries must satisfy that $P_t^H = S_t P_t^{H*}$ (and, similarly, that $P_t^F = S_t P_t^{F*}$).

The bilateral terms of trade $ToT_t = \frac{P_t^F}{S_t P_t^{H*}}$ define the Home country value of the imported bundle of goods from the Foreign country in its own currency relative to the Foreign value of the bundle of the Home country's exports (quoted in the currency of the Home country at the prevailing bilateral nominal exchange rate). Under the LOOP, terms of trade can simply be expressed as,

$$ToT_t = \frac{P_t^F}{S_t P_t^{H*}} = \frac{P_t^F}{P_t^H}. \tag{83}$$

Even though the LOOP holds, the assumption of local-product bias in consumption introduces deviations from purchasing power parity (PPP) at the level of the consumption basket. For this reason, $P_t \neq S_t P_t^*$ and therefore the bilateral real exchange rate between both countries deviates from one—i.e., $RS_t \equiv \frac{S_t P_t^*}{P_t} =$

$\left[\frac{\xi+(1-\xi)(ToT_t)^{1-\sigma}}{(1-\xi)+\xi(ToT_t)^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} \neq 1$ if $\xi \neq \frac{1}{2}$.[23]

Given households' preferences in each country, the demand for any variety h produced in the Home country is given as,

$$\begin{aligned} Y_t(h) \equiv C_t(h) + C_t^*(h) &= (1-\xi)\left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}\left(\frac{P_t^H}{P_t}\right)^{-\sigma} C_t + \xi\left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\sigma} C_t^* \\ &= \left(\frac{P_t(h)}{P_t^H}\right)^{-\theta}\left(\frac{P_t^H}{P_t}\right)^{-\sigma}\left[(1-\xi)C_t + \xi\left(\frac{1}{RS_t}\right)^{-\sigma} C_t^*\right]. \end{aligned} \tag{84}$$

Similarly, I can derive the demand for each variety f produced by the Foreign country firms. Firms maximize profits subject to a partial adjustment rule à la Calvo (1983) at the variety level. In each period, every firm receives either a signal to maintain their prices with probability $0 < \alpha < 1$ or a signal to re-optimize them with probability $1-\alpha$. At time t, the re-optimizing firm producing variety h in the Home country chooses a price $\widetilde{P}_t(h)$ optimally to maximize the expected discounted value of its profits, i.e.,

$$\sum_{\tau=0}^{+\infty} \mathbb{E}_t\left\{(\alpha\beta)^\tau\left(\frac{C_{t+\tau}}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+\tau}}\left[\widetilde{Y}_{t,t+\tau}(h)\left(\widetilde{P}_t(h) - (1-\phi)\left(\frac{U_{t+\tau}}{U}\right) MC_{t+\tau}\right)\right]\right\}, \tag{85}$$

subject to the constraint that the aggregate demand given in (84) is always satisfied at the set price $\widetilde{P}_t(h)$ for as long as that price remains unchanged. $\widetilde{Y}_{t,t+\tau}(h)$ indicates the demand for consumption of the variety h produced in the Home country at time $t+\tau$ ($\tau > 0$) whenever the prevailing prices have remained unchanged since time t—i.e., whenever $P_{t+\tau}(h) = \widetilde{P}_t(h)$. An analogous problem describes the optimal price-setting behavior of re-optimizing firms in the Foreign country.

Firms produce their own varieties subject to a linear-in-labor technology. I assume homogeneity of the labor input and within-country labor mobility—although labor remains immobile across countries—ensuring that wages equalize across firms in a given country but not necessarily across countries. Hence, the (before-subsidy) nominal marginal cost in the Home country MC_t can be expressed as,

$$MC_t \equiv \left(\frac{W_t}{A_t}\right), \tag{86}$$

where the nominal wage rate is denoted by W_t and Home productivity shocks are given by A_t. A similar expression holds for the Foreign country's (before-subsidy) nominal marginal cost. Productivity shocks are described with the following bivariate stochastic process,

$$A_t = (A)^{1-\delta_a}(A_{t-1})^{\delta_a}\left(A_{t-1}^*\right)^{\delta_{a,a^*}} e^{\varepsilon_t^a},\ A_t^* = (A)^{1-\delta_a}(A_{t-1})^{\delta_{a,a^*}}\left(A_{t-1}^*\right)^{\delta_a} e^{\varepsilon_t^{a*}}, \tag{87}$$

$$\begin{pmatrix} \varepsilon_t^a \\ \varepsilon_t^{a*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_a^2 & \rho_{a,a^*}\sigma_a^2 \\ \rho_{a,a^*}\sigma_a^2 & \sigma_a^2 \end{pmatrix}\right), \tag{88}$$

where A is the unconditional mean of the process, δ_a and δ_{a,a^*} capture the persistence and cross-country spillovers, and $\left(\varepsilon_t^u, \varepsilon_t^{u*}\right)^T$ is a vector of Gaussian innovations with a common variance σ_a^2 and possibly correlated across both countries ρ_{a,a^*}.

Various rationalizations of the "cost-push" shock $\frac{U_t}{U}$ in (85) have been advanced in the literature, but following Galí et al. (2001) I simply assume that labor market imperfections give rise to a "wage mark-up," shifting the effective (before-subsidy) nominal marginal costs MC_t exogenously. I say that this "cost-push"

[23] For more in-depth analysis on the role of international price-setting on PPP and the design of optimal monetary policy, see Engel (2009).

shock is described by a bivariate stochastic process of the following form,

$$U_t = (U)^{1-\delta_u} (U_{t-1})^{\delta_u} e^{\varepsilon_t^u}, \; U_t^* = (U)^{1-\delta_u} \left(U_{t-1}^*\right)^{\delta_u} e^{\varepsilon_t^{u*}}, \tag{89}$$

$$\begin{pmatrix} \varepsilon_t^u \\ \varepsilon_t^{u*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_u^2 & \rho_{u,u^*}\sigma_u^2 \\ \rho_{u,u^*}\sigma_u^2 & \sigma_u^2 \end{pmatrix} \right), \tag{90}$$

where U is its unconditional mean (normalized to one), δ_u captures the persistence of the process, and $\left(\varepsilon_t^u, \varepsilon_t^{u*}\right)^T$ is a vector of Gaussian innovations with a common variance σ_u^2 and also possibly correlated across both countries ρ_{u,u^*}.

The optimal pricing rule of the re-optimizing firm h of the Home country at time t is given by,

$$\widetilde{P}_t(h) = \left(\frac{\theta}{\theta-1}(1-\phi)\right) \frac{\sum_{\tau=0}^{+\infty} (\alpha\beta)^\tau \mathbb{E}_t \left[\left(\frac{(C_{t+\tau})^{-\gamma}}{P_{t+\tau}} \right) \widetilde{Y}_{t,t+\tau}(h) \left(\frac{U_{t+\tau}}{U} \right) MC_{t+\tau} \right]}{\sum_{\tau=0}^{+\infty} (\alpha\beta)^\tau \mathbb{E}_t \left[\left(\frac{(C_{t+\tau})^{-\gamma}}{P_{t+\tau}} \right) \widetilde{Y}_{t,t+\tau}(h) \right]}, \tag{91}$$

where ϕ is a time-invariant labor subsidy which is proportional to the nominal marginal cost. An analogous expression can be derived for the optimal pricing rule of the re-optimizing firm f in the Foreign country to pin down $\widetilde{P}_t(f)$. Given the inherent symmetry of the Calvo-type pricing scheme, the price sub-indexes in both countries for the bundles of varieties produced locally, P_t^H and P_t^{F*}, respectively, evolve according to the following law of motion,

$$\left(P_t^H\right)^{1-\theta} = \alpha \left(P_{t-1}^H\right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}_t(h)\right)^{1-\theta}, \tag{92}$$

$$\left(P_t^{F*}\right)^{1-\theta} = \alpha \left(P_{t-1}^{F*}\right)^{1-\theta} + (1-\alpha) \left(\widetilde{P}_t^*(f)\right)^{1-\theta}, \tag{93}$$

linking the current-period price sub-index to the previous-period price sub-index and to the symmetric pricing decision made by all the re-optimizing firms. In turn, the LOOP relates these price sub-indexes to P_t^{H*} and P_t^F with full pass-through of the nominal exchange rate S_t.

Fiscal and Monetary Policy. Monopolistic competition in production introduces a distortive steady-state mark-up between prices and marginal costs, $\frac{\theta}{\theta-1}$, which is a function of the elasticity of substitution across varieties within a country $\theta > 1$. Home and Foreign governments raise lump-sum taxes from households in order to subsidize labor employment and eliminate the steady-state mark-up distortion. An optimal (time-invariant) labor subsidy proportional to the marginal cost set to be $\phi = \frac{1}{\theta}$ in every country neutralizes the steady-state monopolistic competition mark-ups in the pricing rule (equation (91) in steady state).

I model monetary policy in the Home country according to a standard Taylor (1993)-type rule on the short-term nominal interest rate, i_t, i.e.,

$$1 + i_t = \left(1 + \bar{i}\right) \frac{M_t}{M} \left[\left(\frac{\Pi_t}{\overline{\Pi}} \right)^{\psi_\pi} \left(\frac{Y_t}{\overline{Y}_t} \right)^{\psi_x} \right], \tag{94}$$

where $\bar{i}$ denotes the nominal interest rate in steady state, and $\psi_\pi > 0$ and $\psi_x \geq 0$ represent the sensitivity of the monetary policy rule to changes in inflation and the output gap, respectively. $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the (gross) CPI inflation rate and $\overline{\Pi}$ is the corresponding steady-state inflation rate. Y_t defines the actual output of the Home country, $\frac{Y_t}{\overline{Y}_t}$ is the output gap in levels, and $\overline{Y}_t$ the potential output level of the Home country.

Alternatively, I also consider a specification whereby the monetary policy index responds to aggregates instead—i.e., $1 + i_t = \left(1 + \bar{i}\right) \frac{M_t^W}{M^W} \left[\left(\frac{\Pi_t^W}{\overline{\Pi}^W} \right)^{\psi_\pi} \left(\frac{Y_t^W}{\overline{Y}_t^W} \right)^{\psi_x} \right]$ where the superscript W denotes the corresponding aggregate—in order to evaluate the role of international monetary policy coordination. Moreover, I write an index of monetary policy analogous to (94) for the Foreign country assuming distinct policy parameters

$\psi_\pi^* > 0$ and $\psi_x^* \geq 0$ in order to allow for independent and asymmetric monetary policy rules across countries. The Home and Foreign monetary policy shocks, M_t and M_t^*, are described by a bivariate stochastic process:

$$M_t = (M)^{1-\delta_m} (M_{t-1})^{\delta_m} e^{\varepsilon_t^m}, \; M_t^* = (M)^{1-\delta_m} \left(M_{t-1}^*\right)^{\delta_m} e^{\varepsilon_t^{m*}}, \tag{95}$$

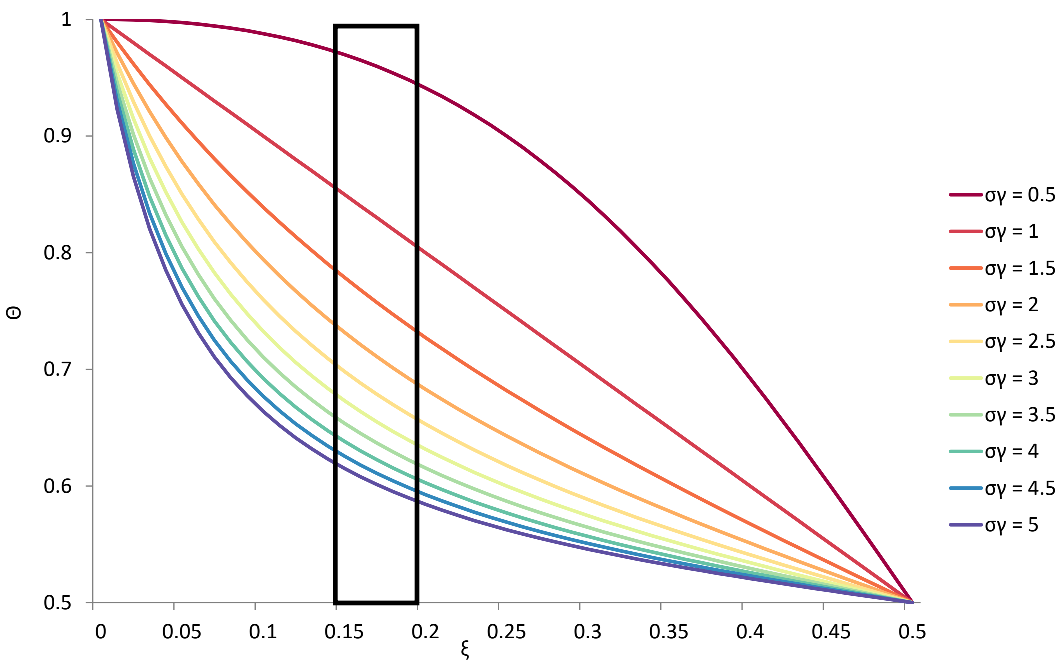
$$\begin{pmatrix} \varepsilon_t^m \\ \varepsilon_t^{m*} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_m^2 & \rho_{m,m^*}\sigma_m^2 \\ \rho_{m,m^*}\sigma_m^2 & \sigma_m^2 \end{pmatrix}\right), \tag{96}$$

where M is its unconditional mean (normalized to one), δ_m captures the persistence of the process, and $(\varepsilon_t^m, \varepsilon_t^{m*})^T$ is a vector of Gaussian innovations with a common variance σ_m^2 and possibly correlated across both countries ρ_{m,m^*}.

D Supplementary Figures

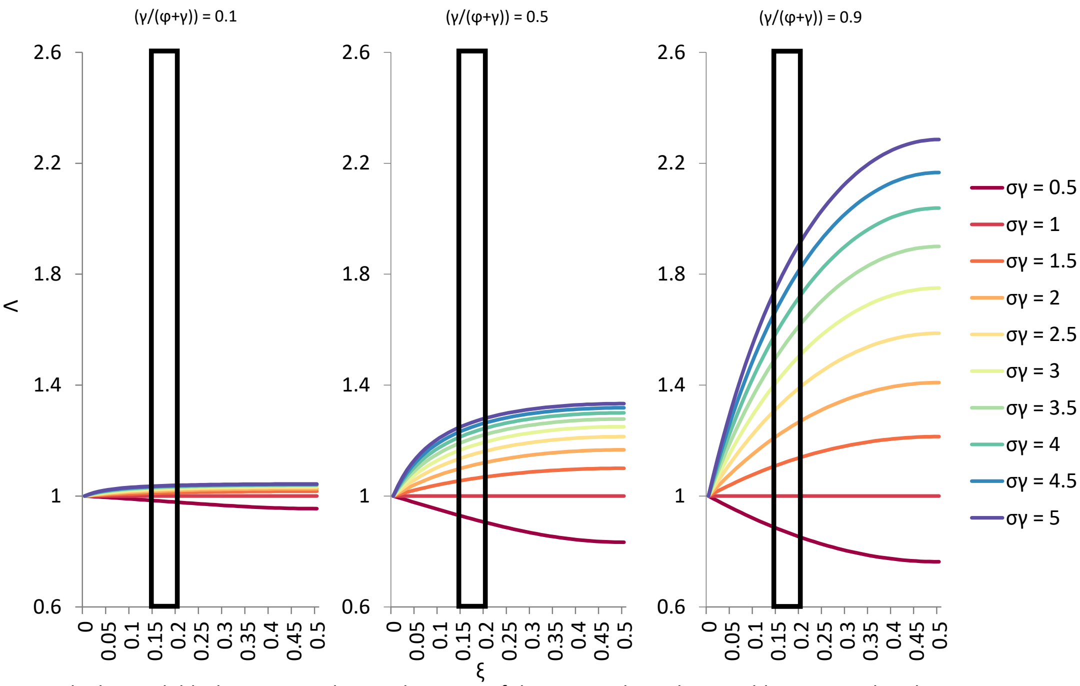
These additional figures are not explicitly referred to in the main body of the paper, but serve to illustrate the main composite coefficients of the two-country model in Table 1.

Figure A1. Domestic Natural Rate Weight on Expected Domestic Potential Growth (Θ).



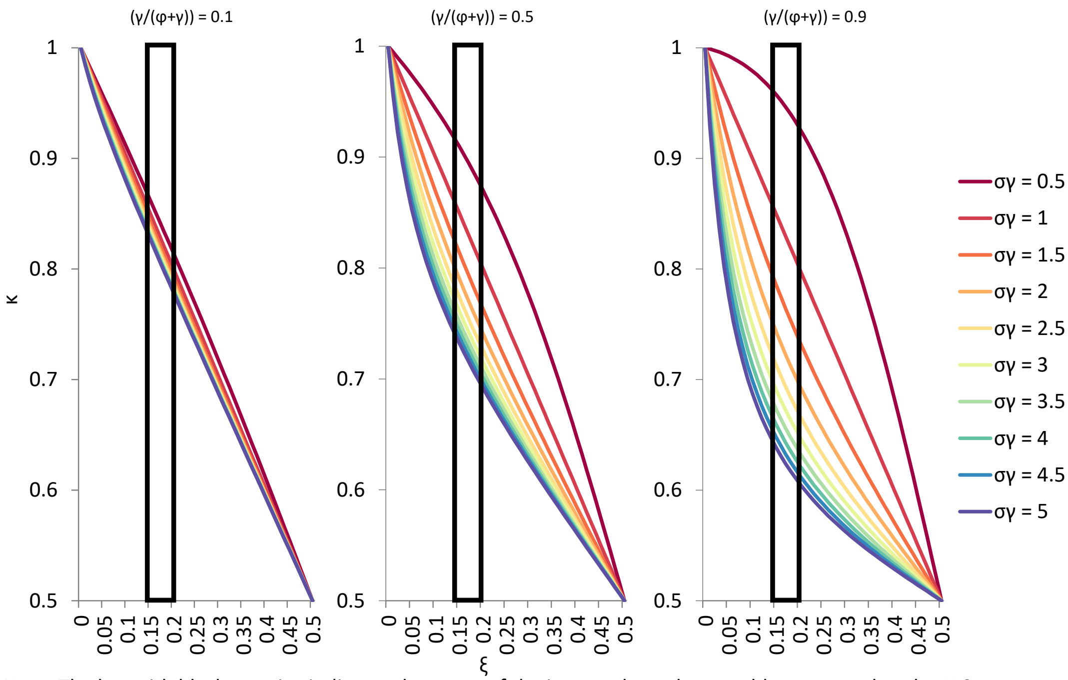
Note: This aspect of the model does not depend on the preference ratio (γ/(φ+γ)). The bar with black margins indicates the range of the import share that would correspond to the U.S.

Figure A2. Domestic Productivity Weight on Domestic Potential Output (Λ).



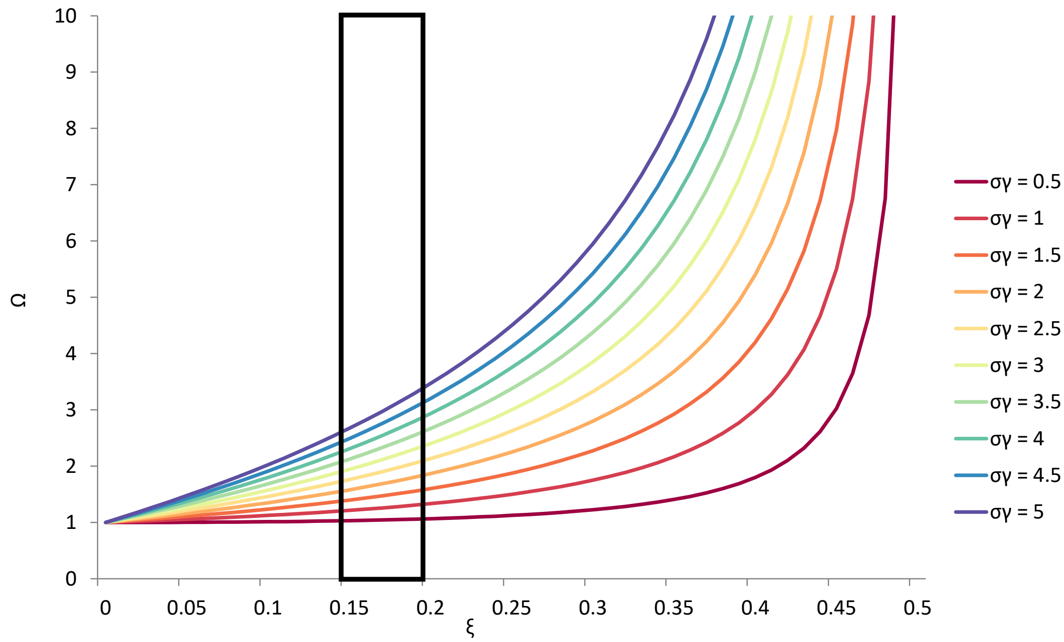
Note: The bar with black margins indicates the range of the import share that would correspond to the U.S.

Figure A3. The Open-Economy Phillips Curve Slope on Domestic Slack Relative to the Closed-Economy Phillips Curve Slope (κ).



Note: The bar with black margins indicates the range of the import share that would correspond to the U.S.

Figure A4. The Slope of the Open-Economy Dynamic IS Curve on the Domestic Interest Rate Gap (Ω).



Note: This aspect of the model does not depend on the preference ratio ($\gamma/(\varphi+\gamma)$). The bar with black margins indicates the range of the import share that would correspond to the U.S.

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