Federal Reserve Bank of Dallas
Globalization and Monetary Policy Institute
Working Paper No. 316
https://www.dallasfed.org/~/media/documents/institute/wpapers/2017/0316.pdf

Estimating the Natural Rate of Interest in an Open Economy*

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June 2017

Abstract

The concept of the natural or equilibrium rate of interest has attracted a lot of attention from monetary policymakers in recent years. Most attempts to estimate the natural rate use a closed economy framework. We argue that in the face of greater integration of global product and capital markets, an open economy framework is more appropriate. We provide some initial estimates of the natural rate for the United States and Japan in a two-country framework. Our identifying assumptions include a close relationship between the time-varying natural rate of interest and the low-frequency fluctuations of potential output growth in both the home country and the foreign country. Our results suggest that the natural rates in both countries are mainly determined by their own trend growth rates of potential output. Nevertheless, the other country's trend growth plays an important role in several specific periods. The gap between the actual real interest rate and our estimated natural rate offers valuable insights into the recent stance of monetary policy in both of these two countries.

JEL codes: C32, E32, E4, E52

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1 Introduction

Interest rates in the United States, Japan and other advanced economies have declined to historic lows over the past two decades. Figure 1 plots long rates on sovereign bonds for the G7 group of countries since the late 1950s through the first quarter of 2016. The recent period of low nominal rates stands out as being without precedent, and a longer historical perspective would confirm that this is in fact the case. The Fisher equation tells us that nominal interest rates can be decomposed into a real interest rate and an expected inflation component. The decline in nominal interest rates that we have seen in recent years has been accompanied by a decline in measured and expected inflation. However, the decline in nominal rates seems to be greater than can be accounted for by the decline in inflation alone. Low growth and persistent downward revisions of growth expectations suggest that the unobservable equilibrium real rate of interest has declined as well.

[Figure 1 about here]

There are many ways to define and measure the real rate of interest. One concept of the real rate that is of particular interest from the perspective of monetary policy is the equilibrium real interest rate or the neutral interest rate which may also be thought of as the intercept term in the Taylor Rule. This is sometimes also referred to as the natural real rate. The natural rate of interest is the real interest rate consistent with full employment of labor and capital resources and the absence of price pressures. The identification and measurement of the unobservable neutral real rate provides a crucial metric for the stance of monetary policy (Cúrdia et al. 2015). As former Fed chairman Ben Bernanke noted in a recent blog post (Bernanke 2015):

"If the Fed wants to see full employment of capital and labor resources (which, of course, it does), then its task amounts to using its influence over market interest rates to push those rates toward levels consistent with ***the equilibrium rate****, or more realistically its* ***best estimate of the equilibrium rate****, which is not directly observable. [...] The best strategy for the Fed I can think of is to set rates at a level consistent with the healthy operation of the economy over the medium term, that is, at the (today, low) equilibrium rate."*

One way to obtain empirical estimates of the unobservable neutral rate is through semi-structural statistical unobserved components models. Following the seminal work of Laubach and Williams (2003), a large number of studies have employed Kalman filter techniques to jointly estimate several unobservable variables – the neutral real interest rate, the level of potential output and the trend rate of growth of output – linking the neutral real interest rate closely to the trend growth rate (e.g., see Laubach and Williams 2003, Clark and Kozicki 2005, Mésonnier and Renne 2007, Trehan and Wu 2007, Barsky et al. 2014, Berger and Kempa 2014, Umino 2014, Pescatori and Turunen 2015, Wynne and Zhang 2016, Holston et al. 2017). Nevertheless, almost all of the previous work on this topic has focused on a closed economy or small open economy setting. In light of the increasing integration of global product and capital markets, it is worth examining the extent to which foreign or external factors might impact estimates of the domestic natural rate. In what follows, we extend the Laubach and Williams (2003) model to a two-country setting. Motivated by Clarida et al. (2002), we link the domestic natural rate to the trend rate of growth in both the home country and the foreign country. Then we implement this framework by taking the United States as the home country and Japan as the foreign country. By estimating the model using data from 1961Q1 to 2014Q3 with Bayesian methods, we obtain three main results.

First, trend potential output growth rates in both countries have been declining over time but with distinct patterns. In particular, the U.S. trend growth rate was stable at around 3 percent prior 2000 but then declined significantly in the first decade of the new century, bottoming out at 0.5 percent in 2009. On the other hand, Japan's trend growth rate plummets in a step-shaped pattern which has experienced three conspicuous falls in the past half century. Specifically, it decreased from 10 percent in 1968 to 3.9 percent in 1975, from 5 percent in 1988 to 1.2 percent in 1993 and from 1.5 percent in 2005 to -0.7 percent in 2008. These distinct differences in the trend growth patterns between the two countries help identify each of their contributions to the natural rate.

Second, the natural interest rates in both the U.S. and Japan are not only determined by their own trend growth rate but also the other country's trend growth rate. Based on our estimates, the major driver of the natural rate in each country is the country's own trend growth rate. Nevertheless, the other country's trend growth indeed contributes to a greater or lesser extent at different times. For instance, Japan's trend growth drives down the U.S. natural rate fundamentally during the three periods when Japan's trend growth is in sharp decline. Moreover, the more recent recovery in the U.S. economy after 2009 also helps raise Japan's natural rate even when its own economy is still in stagnation.

Lastly, the estimated gap between the actual real interest rate and the natural rate provides insights into the stance of monetary policy in U.S. and Japan in recent years. For instance, the U.S. natural rate decreased from 1.4 percent to 1.2 percent between 2004Q2 to 2006Q3 while the nominal federal funds rate rose from 1 percent to 5.3 percent during the same period. After 2006Q3, the U.S. natural rate declined further, until it reached a historically low level of -1.0 percent in 2009Q1, and then remained negative until 2012Q4. The negative natural rate can be seen as justifying the Fed's unconventional monetary policy actions during this period in the form of Large Scale Asset Purchases (LSAPs) and forward guidance which lowered the real interest rate when the nominal interest rate reached the zero lower bound. On the other side, Japan's natural rate declines in a step-shaped pattern along with its trend potential output growth rate. In 1991Q3, the Bank of Japan raised the nominal short-term interest rate to around 8 percent in spite of the sharply declining natural rate. The real interest rate gap reached an almost historically high level 4.7 percent at that time. This sharp tightening of policy contributed to the bursting of the asset price bubble and ushered in the era of the "lost decades". In February 1999, the Bank of Japan lowered its policy rate effectively to zero to combat negative inflation.[1] However, our estimation demonstrates that this Zero Interest Rate Policy was not so expansionary since the natural rate also remained at a low level close to zero at that time. The natural rate stays negative since 2000Q2 which makes Japan's Zero Interest Rate Policy quite contractionary until the Abe government's aggressive Quantitative and Qualitative Easing Program in 2013 started to take effect by gradually raising inflation expectations. The ensuing negative natural rate supported the Abe government's more aggressive Quantitative Easing program announced in October 2014 and the negative interest rate policy adopted in 2016.

The rest of the paper is organized as follows. In the next section, we lay out the basic structure of our two-country model. Section 3 describes the data. Section 4 presents the main estimation results. Section 5 discusses the robustness analysis, and section 6 concludes.

2 Model Description

2.1 Motivation

Our point of departure is the idea that as a result of greater integration of global goods and capital markets, foreign as well as domestic factors are likely to be important in determining the level of the natural rate of interest. The key motivating equation in the Laubach and Williams (2003) paper is the following version of the relationship between the real rate of interest, r, and the rate of growth of consumption, g_c, that falls out of almost any intertemporal household optimization problem:

[1] This Zero Interest Rate Policy (ZIRP) was implemented from 1999M2-2000M8, 2001M3-2006M7 and 2009M1 to present.

$$r = \sigma g_c + \theta, \tag{1}$$

where σ is the inverse of intertemporal elasticity of substitution and θ is the pure rate of time preference. Laubach and Williams use this theoretical relationship to motivate a relationship between the unobserved natural real rate of interest $\overline{r}_t$ and the trend rate of growth of potential output g_t in a closed economy:

$$\overline{r}_t = cg_t + z_t, \tag{2}$$

where z_t captures other determinants of the natural rate of interest.An obvious extension of this equation to an open economy setting would be something like the following:

$$\overline{r}_t = cg_t + c_t^* g_t^* + z_t \tag{3}$$

where g_t^* now denotes the trend rate of growth in the foreign economy. With perfect risk sharing and fully integrated financial markets, the real interest rate in both the home and foreign country should equal the world rate, which in turn should be some average of the trend growth rates in the home and foreign economies. [2] The inclusion of the z_t term in the equation can be thought of as capturing – in addition to other determinants of the natural rate – any distortions that limit international risk sharing. [3] We will take this as the point of departure for our extension of the Laubach and Williams (2003) model to a two-country setting.

2.2 Statistical model

Our benchmark statistical model is a straightforward extension of the closed economy model of Laubach and Williams (2003). We specify two IS equations for the home country (with superscript h) and the foreign country (with superscript f) with the output gap ($\widehat{y} = y - \overline{y}$) being determined by its own lags and lags of the real policy rate gap:

[2]For an early exposition of the determination of the world interest rate in an endowment economy see Frenkel and Razin (1986).

[3]Clarida et al. (2002) show that in a standard New Keynesian open economy setting with no capital, the neutral equilibrium real interest rate consistent with flexible prices in the domestic economy is given by an average of the (expected) growth rates of domestic potential (flex-price) output, $\overline{y}_t^h$, and the growth rate of actual (sticky price) foreign output, y_t^f:$\overline{r}_t^h = \sigma_0 E_t\{\Delta \overline{y}_{t+1}^h\} + \kappa_0 E_t\{\Delta y_{t+1}^f\}$, where $\sigma_0 = \sigma - \gamma(\sigma - 1)$ and $\kappa_0 = \gamma(\sigma - 1)$ so that $\sigma_0 + \kappa_0$ equals the inverse of intertemporal elasticity of substitution σ, and γ is the share of home spending on the foreign good.

$$\widehat{y}_t^h = a_{y,1}^h \widehat{y}_{t-1}^h + a_{y,2}^h \widehat{y}_{t-2}^h + \frac{a_r^h}{2}\sum_{j=1}^{2}(r_{t-j}^h - \overline{r}_{t-j}^h) + \epsilon_{y,t}^h, \tag{4}$$

and

$$\widehat{y}_t^f = a_{y,1}^f \widehat{y}_{t-1}^f + a_{y,2}^f \widehat{y}_{t-2}^f + \frac{a_r^f}{2}\sum_{j=1}^{2}(r_{t-j}^f - \overline{r}_{t-j}^f) + \epsilon_{y,t}^f. \tag{5}$$

We add two Phillips curves for the home country and the foreign country where the (core) inflation rate π_t is determined by its own lags, the lagged output gap $\widehat{y}_{t-1}$, two other variables measuring relative price shocks — core import price inflation π_t^I and lagged crude imported petroleum price π_t^O — and a serially uncorrelated disturbance $\epsilon_{\pi t}$:

$$\pi_t^h = b_\pi^h(L)\pi_t^h + b_y^h \widehat{y}_{t-1}^h + b_{\pi^I}^h(\pi_t^{I,h} - \pi_t^h) + b_{\pi^O}^h(\pi_{t-1}^{O,h} - \pi_{t-1}^h) + \epsilon_{\pi,t}^h, \tag{6}$$

and

$$\pi_t^f = b_\pi^f(L)\pi_t^f + b_y^f \widehat{y}_{t-1}^f + b_{\pi^I}^f(\pi_t^{I,f} - \pi_t^f) + b_{\pi^O}^f(\pi_{t-1}^{O,f} - \pi_{t-1}^f) + \epsilon_{\pi,t}^f. \tag{7}$$

We include eight lags of inflation in the equations above. For parsimony, we impose the restriction — not rejected by our sample — that the sum of the coefficients on lagged inflation must equal unity.[4] We also assume that the coefficients on the second through fourth lags to be equal to each other as are the coefficients on the fifth to eighth lags, i.e., $b_\pi(L)\pi_{t-1} = b_{\pi,1}\pi_{t-1} + \frac{b_{\pi,2}}{3}\sum_{i=2}^{4}\pi_{t-i} + \frac{(1-b_{\pi,1}-b_{\pi,2})}{4}\sum_{i=5}^{8}\pi_{t-i}$.

Equations (4) to (7) are the measurement equations of our benchmark model. Immediately following the motivation equation (3), we assume a law of motion for the natural rate of interest $\overline{r}$ in each country as follows:

$$\overline{r}_t^h = c_h^h g_t^h + c_f^h g_t^f + z_t^h, \tag{8}$$

and

$$\overline{r}_t^f = c_h^f g_t^h + c_f^f g_t^f + z_t^f, \tag{9}$$

where g_t^h is our estimate of home country trend growth of potential output and g_t^f corresponds to the foreign country trend growth.

[4] This is a standard restriction to ensure that the Phillips curve obeys the Natural Rate Hypothesis in the long run.

Finally, the transition equation for potential output in each country is assumed to be given by[5]:

$$\overline{y}_t^h = \overline{y}_{t-1}^h + 0.25 g_{t-1}^h + \epsilon_{\overline{y},t}^h, \tag{10}$$

and

$$\overline{y}_t^f = \overline{y}_{t-1}^f + 0.25 g_{t-1}^f + \epsilon_{\overline{y},t}^f. \tag{11}$$

And again following Laubach and Williams (2003) we assume that the trend rate of growth of potential output in each country follows a random walk:

$$g_t^h = g_{t-1}^h + \epsilon_{g,t}^h, \tag{12}$$

and

$$g_t^f = g_{t-1}^f + \epsilon_{g,t}^f. \tag{13}$$

We assume that all the shocks above are serially uncorrelated and uncorrelated with one another. As detailed in the Appendix, the model has a state-space representation:

$$Y_t = HS_t + AX_t + u_t, \tag{14}$$

and

$$S_t = FS_{t-1} + v_t, \tag{15}$$

where:

$$Y_t = \left(\begin{array}{cccc} y_t^h, & \pi_t^h, & y_t^f, & \pi_t^f \end{array} \right)', \; u_t = \left(\begin{array}{cccc} \epsilon_{y,t}^h, & \epsilon_{\pi,t}^h, & \epsilon_{y,t}^f, & \epsilon_{\pi,t}^f \end{array} \right)',$$

$$X_t = \left(\begin{array}{ccccccccc} y_{t-1}^h, & y_{t-2}^h, & r_{t-1}^h, & r_{t-2}^h, & \pi_{t-1}^h, & \pi_{t-2,4}^h & \pi_{t-5,8}^h, & \pi_{t-1}^{O,h} - \pi_{t-1}^h, & \pi_t^{I,h} - \pi_t^h \ldots \\ y_{t-1}^f, & y_{t-2}^f, & r_{t-1}^f, & r_{t-2}^f, & \pi_{t-1}^f, & \pi_{t-2,4}^f, & \pi_{t-5,8}^f, & \pi_{t-1}^{O,f} - \pi_{t-1}^f, & \pi_t^{I,f} - \pi_t^f \end{array} \right)',$$

$$S_t = \left(\begin{array}{cccccccccccccc} \overline{y}_t^h, & \overline{y}_{t-1}^h, & \overline{y}_{t-2}^h & g_{t-1}^h, & g_{t-2}^h, & z_{t-1}^h, & z_{t-2}^h, & \overline{y}_t^f, & \overline{y}_{t-1}^f, & \overline{y}_{t-2}^f & g_{t-1}^f, & g_{t-2}^f & z_{t-1}^f, & z_{t-2}^f \end{array} \right)',$$

[5]The coefficient before the annualized trend growth rate g_t is 0.25 because our output data are quarterly. This is consistent with the setup in other studies such as Trehan and Wu (2007).

$$v_t = \left(\epsilon^h_{\overline{y},t}, \quad 0, \quad 0, \quad \epsilon^h_{g,t}, \quad 0, \quad \epsilon^h_{z,t}, \quad 0, \quad \epsilon^f_{\overline{y},t}, \quad 0, \quad 0, \quad \epsilon^f_{g,t}, \quad 0, \quad \epsilon^f_{z,t}, \quad 0 \right)'.$$

3 Data

The model is estimated using quarterly data from 1961Q1 to 2014Q3. We take the U.S. as the home country and we use Laubach and Williams (2003) data for that part of the model, extended through 2014.[6] The variable y^h_t is measured as the log of real chain-weighted GDP in billions of chained 2009 dollars. We use quarterly averages of the daily annualized effective nominal federal funds rate as our measure of the policy rate. For the period prior to 1965, we use the Federal Reserve Bank of New York's discount rate. We measure the inflation rate π^h_t as the core PCE inflation rate which is the annualized quarterly growth rate of the price index for personal consumption expenditures excluding food and energy. To construct the ex ante real policy rate r^h_t , we compute the expectation of average inflation over the four quarters ahead from a univariate AR(3) model of inflation estimated over the 40 quarters prior to the date at which expectations are being formed.[7] Finally, the relative price variables included in the Phillips curve are the FRB/US price index for imports excluding petroleum $\pi^{I,h}_t$ and the FRB/US crude oil import price inflation $\pi^{O,h}_t$.[8]

We select Japan as the foreign country for several reasons. Firstly, for most of our sample, Japan was the second largest economy in the world and played an important role in international financial markets.[9] Moreover, Japan has maintained a tight economic and financial relationship with U.S. which makes it meaningful to explore how those two countries interact in determining the natural rate.[10] In addition, the pattern of Japanese trend growth

[6]Laubach and Williams have updated the data to more recent period. We thank Thomas Laubach for sharing their data with us. We used the Laubach and Williams data so as to facilitate comparison of our results with theirs.

[7]To compute these expectations for the early part of our sample, we need to use inflation data prior to 1959Q2, the start of the core PCE inflation series. For these dates, we splice headline PCE inflation to core PCE inflation.

[8]The HAVER codes for the data series we use are: GDP - GDPH@USECON; Core PCE deflator - JCXFE@USECON; Effective Federal Funds Rate - FFED@USECON; FRBNY discount rate - FDWB@USECON; PCE deflator - JC@USECON. The two data series we take from the FRB/US database closely track the following HAVER series: Price index for imports excluding petroleum, computers and semiconductors - PMENP@USECON (quarterly average available since 1989Q1); Crude imported oil price inflation - PMEPI@USECON(quarterly average available since 1989Q1).

[9]On a PPP basis Japan ranked second behind the United States until about 2000 when it was overtaken by China.

[10]Japanese investors currently hold about 20 percent of U.S. treasury securities outstanding. About 12 percent of FDI in the U.S. comes from Japan, down from a peak of 23 percent in 1992.

rate is fairly distinguishable from its U.S. counterpart which should help identify its contribution to the natural rate of both the home and foreign country.[11] An alternative approach would be to aggregate multiple countries as the foreign country. In reality, the international interactions between U.S. and other individual countries are heterogeneous for geographic, cultural and political reasons. This makes any existing aggregation approach less than ideal for measuring the total influence of the foreign country (Chudik and Straub 2017). Nevertheless, properly incorporating other advanced economies and emerging countries into the model could be a possible avenue for future research.

Japanese indicators are constructed in a way comparable to what we do for the U.S. Real output y_t^f is measured as the log of GDP in 2011 Yen. Following Umino (2014), we proxy the short-term interest rate using the Tokyo overnight call money market rate. There is no equivalent of the core PCE price index available for Japan. We measure the inflation rate π_t^f as the core CPI inflation rate which is the annualized quarterly growth rate of the consumer price index excluding food and energy. Expected inflation is derived in exactly the same way as for the U.S. case using an AR(3) model fitted to inflation and estimated using a 40-quarter rolling window. There are no measures of core import prices published for Japan, so we use the annualized quarterly rate of change of the overall import price instead as one of the supply shock variables in the Phillips curve for Japan. Lastly, we measure $\pi_t^{O,f}$ in the inflation equation as the annualized quarterly growth rate of the import price index for petroleum, coal and natural gas.[12]

4 Estimation Results

The model is estimated with a Bayesian approach where we use a Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm to draw 50,000 draws from the posterior distribution (with a burn-in sample of 50,000 draws). This yields several benefits compared to maximum likelihood estimation. First, the Bayesian approach allows us to incorporate extra information from the literature which reduces the uncertainty around the estimation of the natural rate, the output gap and the trend growth rate. This is particularly useful in

[11] The average annual growth rate of GDP in Japan was 10.2 percent in the 1960s, 4.5 percent in the 1970s and the 1980s, 1.4 percent in the 1990s and 1.1 percent from 2000 to 2015. By comparison, the average annual growth rate of GDP in the U.S. was 4.2 percent in the 1960s, 3.3 percent in the 1970s, 3.2 percent in the 1980s, 3.3 percent in the 1990s and 2.7 percent from 2000 to 2015.

[12] The HAVER codes for the data series we use for Japan are as follows: GDP - A158GDPC@OECDNAQ; Core CPI - C158CZCN@OECDMEI; Tokyo overnight call rate - C158IM@IFS; Import prices - H158PFMI@G10; Import price for petroleum, coal and natural gas - H158PFMP@G10.

our case since the dimension of both the parameter vector and the state vector are greatly expanded in our two-country model compared to previous closed economy frameworks. Second, the maximum likelihood estimate of the standard deviation of the innovation to the trend growth process σ_g tends to be biased towards zero owing to the so-called "pile-up problem" addressed in Stock (1994). Bayesian methods can avoid this problem by properly tuning the priors. Finally, Bayesian methods deliver samples from posterior distributions, thus the finite-sample uncertainty around any object of interest can be precisely delineated.

4.1 Prior distribution of the parameters

Figures 2 and 3 plot the prior and posterior distribution of the parameters while Table 1 reports the associated 90 percent confidence intervals. The prior distribution is assumed to be Gaussian for all parameters except for the variance parameters which are assumed to be gamma distributed. We set diffuse priors on all of the coefficient parameters. The prior means for the coefficients of the home country IS equation, Phillips curve equation, and the natural rate determination equation are picked based on previous closed-economy studies in the literature, such as Laubach and Williams (2003), Trehan and Wu (2007) among others. The corresponding foreign country prior means are set based on results in Umino (2014) and our own prior research treating Japan as a small open economy.[13] The priors for the international interaction coefficient in the natural rate determination, i.e., c_f^h and c_f^f in equation (8) and (9), are set loosely with the means respectively set as 0 and 0.5 based on the prior belief that the international interaction is more important for Japan than for the U.S.

The priors for the variance of the shock to the trend growth, i.e., $\sigma^2_{g,h}$ and $\sigma^2_{g,f}$, and the priors for the variance of the shock to the other natural rate determinants, i.e., $\sigma^2_{z,h}$ and $\sigma^2_{z,f}$, are tuned to avoid the so-called "pile-up" problem. Specifically, the prior means for these two shocks are assigned based on previous work while the associated prior variances are set to a relatively small interval. The priors of all of the other variance parameters are much more uninformative, which leaves the task of shock identification to the data.

[Figures 2 and 3 and Table 1 about here]

[13]The small open economy study is implemented with a model introduced in Berger and Kempa (2014).

4.2 Posterior distribution of the parameters

The last two columns of Table 1 display the posterior mean as well as the fifth and ninety-fifth percentiles of the posterior distribution of all the parameters. For the IS coefficients, we find that the U.S. output gap is relatively persistent; the sum of the associated posterior means of the AR parameters is 0.944. By contrast, the Japanese output gap exhibits a higher degree of mean reversion where the sum of the two AR coefficients is only 0.798. The posterior mean of $a_r^h = -0.107$ turns out to be smaller than the corresponding estimate of $a_r^f = -0.043$. This indicates that the cyclical component of the real interest rate (or more precisely, the deviation of the real rate from its natural level) matters less for Japan than it does for the U.S. Nevertheless, the negative estimated values of a_r^h and a_r^f verifies that the positive real interest rate gap is indeed contractionary for both countries.

In terms of the evidence on inflation, $b_{\pi,1}$ is bigger than $b_{\pi,2}$ so that the inflation rate depends more heavily on recent inflation rates. The posterior mean of $b_y^h = 0.035$ is small but positive. By comparison, the posterior mean of $b_y^f = 0.532$ is much bigger. Thus, a positive output gap is associated with inflation pressure as is predicted by standard economic theory. Our estimation indicates that this type of inflation pressure is bigger in Japan than in the U.S. For the natural rate equation, as is shown in Figure 2 and Figure 3, the posterior distributions of all of the c parameters are tighter than the prior distributions. Moreover, the posterior means of the c parameters are positive which implies that the trend growth rate in U.S. and Japan does contribute to the other country's natural rate.

4.3 Posterior estimation of the state variables

4.3.1 Output gap, potential output growth and trend growth

Figure 4 plots our estimates of the output gap in both countries where the shaded areas represent recession periods.[14] It turns out that the estimated U.S. output gap picks up the business cycle turning points quite accurately. The output gap decreases significantly in each of the NBER recessions. In particular, the U.S. output gap decreases most significantly during the early 1980s recession and the Great Recession period as in Laubach and Williams (2016). On the other hand, the Japanese output gap evolves more transiently with the most severe contractions of output occurring during the 1973-1975 recession and the 2007-2009

[14]The U.S. recession dates are obtained from the NBER business cycle chronology, while the Japanese recession dates are as defined by ESRI of the Cabinet Office in Japan. The NBER and ESRI recession chronologies are based on different indicators to those used in our model and so constitute a good check on the reasonableness of our estimates.

recession.

[Figure 4 about here]

Figure 5 displays our estimates of the growth rate of potential output in blue dashed lines along with the trend growth in black solid lines. As is shown in the upper panel, the U.S. potential output growth rate fluctuates around the trend growth rate as expected. It becomes less volatile between the mid-1980s and 2007, the so-called Great Moderation period. Then it drops to its lowest level from 2007 to 2009 during the Great Recession period which was the worst recession since World War II. U.S. trend growth is stable at around 3 percent prior to 2000. However, it then drops from 2000 to 2009, when reaches its trough at 0.5 in 2009Q3, and then recovers slowly until 2014Q3 when our sample ends.

[Figure 5 about here]

Japan's trend growth rate exhibits a very different pattern from its U.S. counterpart. It has been declining in a step-shaped pattern, with three sharp decreases during the past half century. Our estimate of trend growth is stable at around 9 percent for most of the 1960s (also known as the "Golden Sixties" in accounts of Japan's postwar economic miracle), and then encounters its first severe drop from 1968Q2 to 1975Q1, when it declines from 10 percent to a more moderate level of almost 3.9 percent. After the 1973-1975 recession, trend growth recovers slowly until the property and stock market bubble bursts and the second serious decline occurs in the early 1990s. Our estimate of trend growth drops from 5 percent in 1988Q4 to a little above 1 percent in 1993Q3 which marks the start of the Lost Decades. Japanese trend growth remains at a low level throughout the 1990s. Despite that, trend growth picks up a little bit from 1999 to 2005 after the Asian financial crisis, then plummets again right before the Great Recession, when it plunges from 1.4 percent in 2005Q2 to a historically low level -0.7 percent in 2008Q4. After the Great Recession, Japan's trend growth returns to positive territory but stays at a low level.

Based on the discussion above, our estimation of the output gap, potential output and trend growth is consistent with the economic history in both U.S. and Japan. This provides support to our estimation of the natural rate which will be explored in the next subsection.

4.3.2 A retrospective assessment of the monetary policy stance

Figure 6 depicts the two-sided estimation (based on the Kalman Smoother) of the natural rate of interest in black solid lines together with the historical realization of the ex ante real interest rate in blue lines. The gap between the natural rate and the real interest rate provides insights into the stance of monetary policy in the two countries over the past half century. As is shown in the upper panel, the U.S. natural rate has been trending down during the past half century. Based on our estimates, U.S. monetary policy was expansionary for the most of the period prior to 1980. This loose monetary policy helped raise the inflation rate during the stagflationary period of the 1970s. In 1979, the Fed raised its policy rate abruptly to fight inflation after Paul Volcker became Chairman of the Fed. The real interest rate exceeded the natural rate starting in 1979 and the real interest rate gap reached almost 10 percentage points in 1981Q3. This highly contractionary stance of monetary policy lasted until 1992Q1 when the Fed loosened monetary policy in response to weaker growth in the early 1990s. The economy in the 1990s was relatively well behaved compared to the preceding era so that the Fed eventually raised interest rates above the natural rate by the mid-1990s and kept them above until the early 2000s recession. The natural rate started to decline in 1999 and kept falling even as the Fed started to raise the interest rate from 2004 to 2006. The divergent movements in the monetary policy rate and the natural rate created a big 3.1 percentage point real interest rate gap in 2007Q3, which was followed by the bursting of the housing market bubble and the Subprime Mortgage Crisis which in turn led to the Great Recession.[15] The natural rate dropped to a historically low level of -1 percent in 2009Q1 during the Great Recession. Hence if expected inflation rate was less than 1 percent at that time, the monetary policy was still contractionary even with the nominal policy rate at zero.[16] Fortunately, the Fed seems to have realized this point. Having lowered the interest rate to zero in 2008Q4, it launched its Large Scale Asset Purchase (LSAP) or Quantitative Easing (QE) programs to support economic activity (along with various forms of forward guidance). The natural rate picks up after 2009Q1 and returns to positive territory in 2013Q3 as the U.S. economy steadily recovered after the recession. At the same time, the three rounds of Quantitative Easing from 2008 to 2014 effectively lowered the real interest rate. Based on the analysis above, in light of the low, even negative, natural rate, the zero interest rate policy and the Quantitative Easing programs were not overly aggressive but necessary measures to help the U.S. economy recover from the Great Recession.

[Figure 6 about here]

[15] Thus our estimates are at variance with narratives that argue that the Fed contributed to the housing bubble by being too slow to raise interest rates during this period.

[16] With a simple auto regressive model, we estimate that the inflation expectation in 2009Q1 is around 0.48 percent. This is consistent with the five-year inflation expectation derived from the Treasury Inflation Protected Security which is around 0.4 percent. Nevertheless, the survey conducted by University of Michigan indicates that the one-year inflation expectation is around 2 percent.

A further question is how does our open economy estimate of U.S. natural rate differ from the previous closed economy estimates. To facilitate the comparison, we also plot Laubach and Williams' estimate of the U.S. natural rate in red in Figure 6.[17] To begin with, our estimate tracks Laubach and Williams' estimates quite well. It deviates from Laubach and Williams' measure in the periods 1970-1977, 1990-1994, 2001-2004, and 2007-2011. Specifically, Japan's recessions in 1973-1975, 1991-1993, and 2008-2009 drives down the U.S. natural rate when Japan's trend growth experienced sharp declines. This provides a novel perspective on U.S. monetary policy in certain periods. For instance, the open economy estimates suggest that the expansionary monetary policy in the early 1990s and 2001-2004 was not as expansionary as is implied by Laubach and Williams. Our estimate also signals a more contractionary stance of monetary policy prior to 2009 and a less expansionary stance of monetary policy after the Great Recession.

The lower panel of Figure 6 portrays the Japanese natural rate together with its ex ante real short-term interest rate. The Japanese natural rate stayed above the actual real interest rate most of the period before 1980, suggesting a highly expansionary stance of monetary policy even when the natural rate also declines substantially from 3.4 percent to 0.6 percent in the early 1970s. The inflation rate dropped dramatically in the late 1970s which causes the real interest rate to rise and exceed the natural rate in 1980. This contractionary policy lasted until 1986 even during the 19810-1983 recession. However, following the 1985-1986 recession, monetary policy reverted to expansionary when the Bank of Japan lowered its policy rate sharply from 8 percent in late 1985 to 3 percent 1987. This expansionary monetary policy helped cultivate the real estate and equity market asset bubbles. In fact, as is pointed out by Obstfeld (2009), all categories of real estate appreciated during this period, rising in price by more than 75 percent between 1986 and the 1991-92 market peak. To prevent the asset price bubble from inflating further, the Bank of Japan embarked upon a new round of contractionary monetary policy which raised the interest rate above the natural rate in mid-1989 in spite of the fact that the natural rate started to fall significantly in early 1989. The interest rate gap between the real interest rate and the natural rate reached a historic high of more than 4.6 percentage points in 1991Q3 which put increasing pressure on Japan's financial sector and finally popped the asset market bubble. Over the period from 1992 to 1995 small financial institutions began to fail while banking and credit problems were to escalate over the decade. In the face of the low growth rate, the Bank of Japan maintained a low interest rate starting in 1993 and finally launched its Zero Interest Rate

[17] Laubach and Williams' estimates of the natural rate are maintained and updated in real time by the San Francisco Fed. Here we select the natural rate they estimated with the same data vintage as us.

Policy (ZIRP) in February 1999. Nevertheless, since the natural rate also stayed at a low level after the 1991-1993 recession, the Zero Interest Rate Policy exerted little expansionary effect, especially after 2000 when Japan slipped into deflation.[18] The Bank of Japan raised interest rates in 2006. This seems again to have been a policy mistake since the natural rate turned negative in 2006Q4. The contractionary monetary policy lasted from 2006 to 2014, even after Japan returned to a zero interest rate policy in 2010, which impeded Japan from recovering from the 2008-2009 recession and the 2012 recession. In 2013 and 2014, a change of government was followed by the launch of a program of Qualitative and Quantitative Easing which finally lowered the real interest rate below the natural rate. Given that the natural rate stayed negative until the end of our sample, it's perhaps not so surprising that the Bank of Japan began to experiment with negative interest rates in 2016.

4.3.3 Determinants of the natural rate

The neutral rate equation above shows that the neutral rates in both countries are determined by three factors: their own trend growth, the other country's trend growth and the constant other determinants. Figure 7 displays the natural rate and the contributions of these underlying determinants.

[Figure 7 about here]

The U.S. natural rate is mainly determined by the U.S. trend growth over all periods in our sample. The trend growth in Japan is the second biggest contributor at the beginning of the sample. However, as the trend growth in Japan declines, it makes much less positive and eventually a negative contribution to the U.S. natural rate. The U.S. natural rate is almost parallel to its own trend growth from 1975 to 1987 and from 1993 to 2006. This implies that the basic pattern of the U.S. natural rate is determined by its own trend growth. Nevertheless, Japanese trend growth indeed amplifies the decline in the natural rate in 1969-1975, 1990-1993 and 2006-2009 when it suffers severe drops.

On the other hand, the U.S. natural rate positively contributes to Japan's natural rate in all periods. However, the basic pattern of Japan's natural rate is also mainly determined by its own trend growth since the U.S. trend growth rate is less variable. U.S. trend growth plays a more notable role in Japan's natural rate after the 1990s when Japan's trend growth drops to a low level and U.S. trend growth becomes relatively more volatile. For instance, Japan's natural rate decreases when its trend growth rises during from 1999 to 2005 because the

[18] There is a short duration of deviation from August 2000 to March 2001.

U.S. trend growth decreases at the time. On the other hand, the sustainable U.S. economic recovery drives up the Japanese natural rate even when Japan's domestic trend growth stays at a low level after 2009.

5 Robustness

In the baseline model above, we adopted the exact model specification in Laubach and Williams (2003) and applied it to our two-country setting. While copying the Laubach and Williams specification for the United States might seem uncontroversial and even desirable from the perspective of facilitating the comparison of our results with theirs, it is less obvious that using their specification for Japan makes sense (although Umino (2014) does essentially that). There are many dimensions along which we might seek to find a better small scale statistical model of the U.S. and Japanese economies for the purposes of our exercise, but in this section we will focus on just one, namely, how the results might be changed if instead of adapting the Laubach and Williams specification one-for-one, we allow lag lengths in the IS curve and the Phillips curve for each country to be chosen by some statistical criterion.

[Table 2 about here]

Given the large number of parameters in the full model, it is not feasible to search over all possible specifications of the IS curves and Phillips curves. Instead, we estimated individual IS and Phillips curves for each country. For the IS curve, we proxied the output gap with the deviation of output from its HP-filter smoothed level. Likewise we proxied the interest rate gap by the deviation of real interest rates from their HP-smoothed level. And for the each country's Phillips curve we also proxied the output gap with the deviation of output from its HP-smoothed level. We then varied the lag length of the lagged output terms in the IS curve and the lagged inflation terms in the Phillips curve up to eight lags and then computed the Bayes Information Criterion (BIC) for each specification. The results are shown in Table 2. There are two points worth noting. First, note that the largest gain in terms of the BIC comes from going from having no lags to having just one. There is not a lot of difference in terms of the BIC from having one versus eight lags. We have highlighted in bold the lag length in each case that minimises the BIC for each equation.

[Table 3 and Figures 8 and 9 about here]

We then used these results to tighten the priors in our estimation as shown in Table 3 and Figures 8 and 9. Note that the signs and indeed the magnitude of the posterior estimates of almost all of the coefficients are little changed from what we report in Table 1. Figures 10 through 13 show how our estimates of the output gap and the natural rate of interest change when we use the prior with lag lengths in the IS and Phillips curve chosen using the BIC. A cursory examination of the figures shows that the results are essentially the same.

[Figures 10 to 13 about here]

Our second robustness check looks at the residuals from the observation equation in our model, and specifically checks to see if they satisfy the key assumption of normality. Table 4 reports the p-values of Ljung-Box (LB) and Kolmogorov-Smirnov (KS) tests for correlation and normality of the residuals from the observation equations of our baseline model and the robust model estimated using priors with lag lengths selected using the BIC. Note that in almost all cases both the baseline and the robust model pass both tests at standard significance levels. We reject the nulls of no serial correlation and normality for the Japanese Phillips curve in the baseline specification, and also in the robust specification. We also reject the null of no serial correlation in the robust specification for the U.S. Phillips curve.

[Table 4 about here]

6 Conclusion

In this paper, we estimate a time-varying natural rate of interest for U.S. and Japan by Bayesian methods over the period 1961-2014. Our model is a two-country version of the semi-structural model proposed by Laubach and Williams (2003). We assume perfect risk sharing and complete international asset markets so that the two countries are linked through a simple natural rate determination equation. By properly incorporating the prior information, our estimation avoids the "pile-up" problem suffered by previous estimation of the natural rate of interest by maximum likelihood methods.

The empirical analysis shows that the natural rate is not only related to the trend growth in home country but also the trend growth in foreign country. For both U.S. and Japan, the basic pattern of the natural rate is determined primarily by its own country's trend growth of potential output while the other country's trend growth rate indeed contributes substantially to the natural rate in home country during several special periods. For instance, Japan's trend growth amplifies the decline in U.S. natural rate in 1969-1975, 1990-1993 and

2006-2009 when Japanese trend growth experienced sharp decreases. On the other hand, the recent economic recovery in U.S. also helps driving up the Japanese natural rate after 2009.

As a complement to our study, estimating the natural rate in real time and evaluating the associated uncertainty when information is incomplete could be done within our framework. In addition, introducing more countries into the foreign country side would also provide more appropriate evaluation of the foreign influence on the U.S. natural rate if the data aggregation problem is properly handled. However, these issues are left for further research.

Supplementary Data

The datasets of this paper (1. code and programs, 2. Data, 3. detailed readme files) are collected in the electronic supplementary material of this article.

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Appendix: The state-space representation of the model

The model in the text can be written in state space form:

$$Y_t = HS_t + AX_t + u_t, \tag{A1}$$

$$S_t = FS_{t-1} + v_t. \tag{A2}$$

Here, Y_t and X_t are vectors of contemporaneous endogenous, and of exogenous and predetermined variables respectively. S_t is the vector of unobserved states. The vectors of stochastic disturbance u_t and v_t are assumed to be Gaussian and mutually uncorrelated with mean zero and covariance matrices R and Q, respectively.

The vector of observables Y_t is given by:

$$Y_t = \left(\begin{array}{cccc} y_t^h, & \pi_t^h, & y_t^f, & \pi_t^f \end{array} \right)', \tag{A3}$$

where y_t denotes 100*log real GDP and π_t denotes inflation. The predetermined and exogenous variables are:

$$X_t = \left(\begin{array}{ccccccccc} y_{t-1}^h, & y_{t-2}^h, & r_{t-1}^h, & r_{t-2}^h, & \pi_{t-1}^h, & \pi_{t-2,4}^h & \pi_{t-5,8}^h, & \pi_{t-1}^{O,h} - \pi_{t-1}^h, & \pi_t^{I,h} - \pi_t^h \dots \\ y_{t-1}^f, & y_{t-2}^f, & r_{t-1}^f, & r_{t-2}^f, & \pi_{t-1}^f, & \pi_{t-2,4}^f, & \pi_{t-5,8}^f, & \pi_{t-1}^{O,f} - \pi_{t-1}^f, & \pi_t^{I,f} - \pi_t^f \end{array} \right)', \tag{A4}$$

where r_t is the real interest rate, $\pi_{t-j,k}$ is shorthand for the moving average of inflation between dates $t-k$ and $t-j$ and π_t^o is oil price inflation. The state vector is:

$$S_t = \left(\begin{array}{cccccccccccccc} \overline{y}_t^h, & \overline{y}_{t-1}^h, & \overline{y}_{t-2}^h & g_{t-1}^h, & g_{t-2}^h, & z_{t-1}^h, & z_{t-2}^h, & \overline{y}_t^f, & \overline{y}_{t-1}^f, & \overline{y}_{t-2}^f & g_{t-1}^f, & g_{t-2}^f & z_{t-1}^f, & z_{t-2}^f \end{array} \right)', \tag{A5}$$

where $\overline{y}_t$ is 100*log potential GDP, g_t denotes the trend growth. The vectors of stochastic disturbance u_t and v_t are given by:

$$u_t = \left(\begin{array}{cccc} \epsilon_{y,t}^h, & \epsilon_{\pi,t}^h, & \epsilon_{y,t}^f, & \epsilon_{\pi,t}^f \end{array} \right)', \tag{A6}$$

$$v_t = \left(\begin{array}{cccccccccccccc} \epsilon_{\overline{y},t}^h, & 0, & 0, & \epsilon_{g,t}^h, & 0, & \epsilon_{z,t}^h, & 0, & \epsilon_{\overline{y},t}^f, & 0, & 0, & \epsilon_{g,t}^f, & 0, & \epsilon_{z,t}^f, & 0 \end{array} \right)', \tag{A7}$$

The coefficient matrices are:

$$A = \left[\begin{array}{cc} A^h, & 0_{2*9} \\ 0_{2*9}, & A^f \end{array} \right], \tag{A8}$$

$$A^h = \left[\begin{array}{ccccccccc} a_{y,1}^h & a_{y,2}^h & a_r^h/2 & a_r^h/2 & 0 & 0 & 0 & 0 & 0 \\ b_y^h & 0 & 0 & 0 & b_{\pi,1}^h & b_{\pi,2}^h & 1 - b_{\pi,1}^h - b_{\pi,2}^h & b_{\pi^I}^h & b_{\pi^O}^h \end{array} \right], \tag{A9}$$

$$A^f = \left[\begin{array}{ccccccccc} a_{y,1}^f & a_{y,2}^f & a_r^f/2 & a_r^f/2 & 0 & 0 & 0 & 0 & 0 \\ b_y^f & 0 & 0 & 0 & b_{\pi,1}^f & b_{\pi,2}^f & 1 - b_{\pi,1}^f - b_{\pi,2}^f & b_{\pi^I}^f & b_{\pi^O}^f \end{array} \right], \tag{A10}$$

$$H = \begin{bmatrix} 1 & -a^h_{y,1} & -a^h_{y,2} & -c^h_h a^h_r/2 & -c^h_h a^h_r/2 & -a^h_r/2 & -a^h_r/2 & 0 & 0 & 0 & -c^h_f a^h_r/2 & -c^h_f a^h_r/2 & 0 & 0 \\ 0 & -b^h_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c^h_f a^f_r & -c^h_f a^f_r & 0 & 0 & 1 & -a^f_{y,1} & -a^f_{y,2} & -c^f_f a^f_r/2 & -c^f_f a^f_r/2 & -a^f_r/2 & -a^f_r/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -b^f_y & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{A11}$$

$$R = \begin{bmatrix} \sigma^2_{yh} & 0 & 0 & 0 \\ 0 & \sigma^2_{\pi h} & 0 & 0 \\ 0 & 0 & \sigma^2_{yf} & 0 \\ 0 & 0 & 0 & \sigma^2_{\pi f} \end{bmatrix}, \tag{A12}$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \tag{A13}$$

$$
Q = \begin{bmatrix}
\sigma^2_{gh}+\sigma^2_{\overline{y}h} & 0 & 0 & \sigma^2_{gh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sigma^2_{gh} & 0 & 0 & \sigma^2_{gh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma^2_{zh} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{gf}+\sigma^2_{\overline{y}f} & 0 & 0 & \sigma^2_{gf} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{gf} & 0 & 0 & \sigma^2_{gf} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{zf} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \qquad \text{(A14)}
$$

The vector of parameters to be estimated is:

$$
\theta = \begin{pmatrix}
a^h_{y,1} & a^h_{y,2} & a^h_r & b^h_{\pi,1} & b^h_{\pi,2} & b^h_y & b^h_{\pi^I} & b^h_{\pi^O} & c^h_h & c^h_f & \sigma^2_{yh} & \sigma^2_{\pi h} & \sigma^2_{zh} & \sigma^2_{\overline{y}h} & \sigma^2_{gh}\cdots \\
a^f_{y,1} & a^f_{y,2} & a^f_r & b^f_{\pi,1} & b^f_{\pi,2} & b^f_y & b^f_{\pi^I} & b^f_{\pi^O} & c^f_h & c^f_f & \sigma^2_{yf} & \sigma^2_{\pi f} & \sigma^2_{zf} & \sigma^2_{\overline{y}f} & \sigma^2_{gf}
\end{pmatrix}.
$$

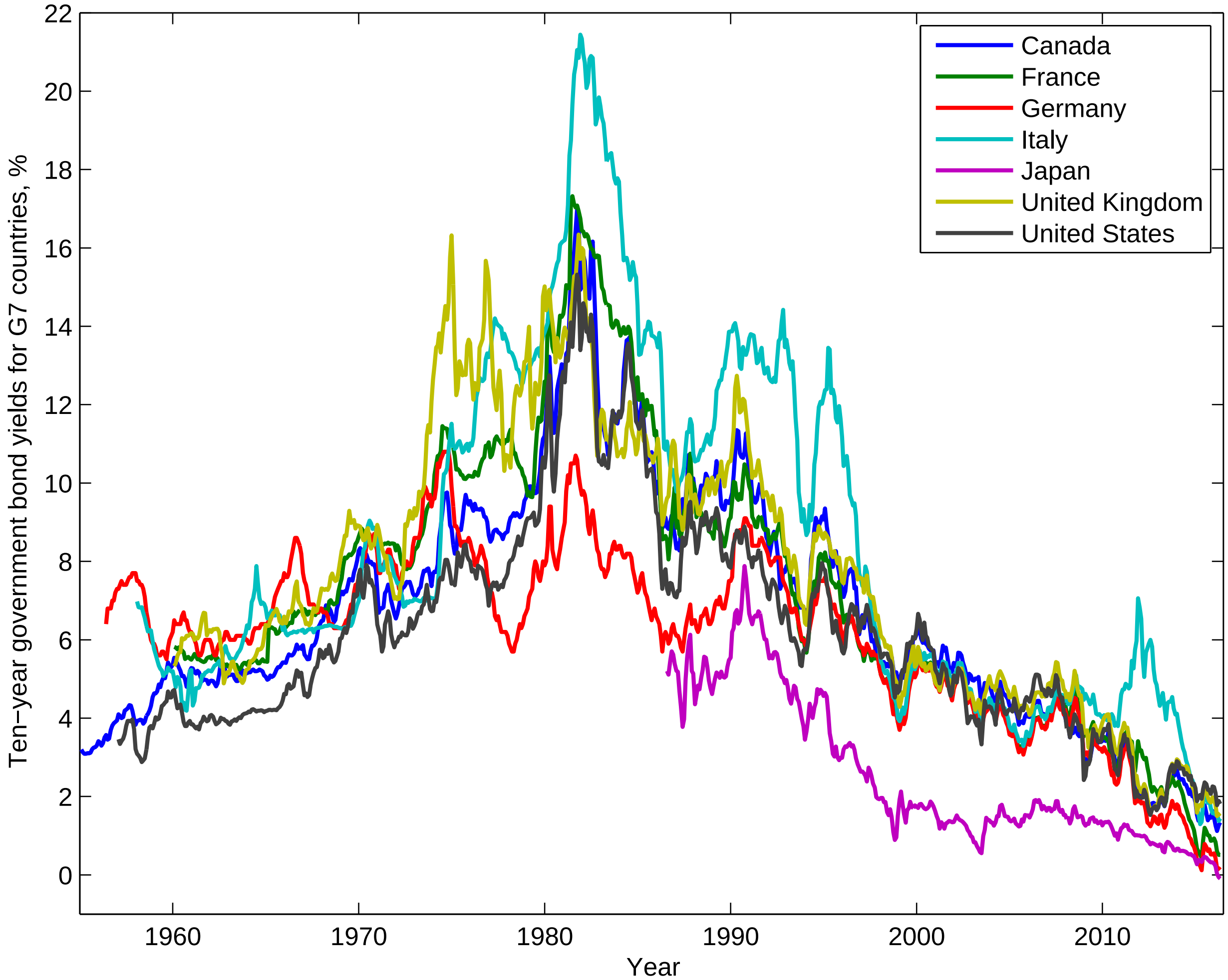

Figure 1: Ten-year government bond yields in G7 countries

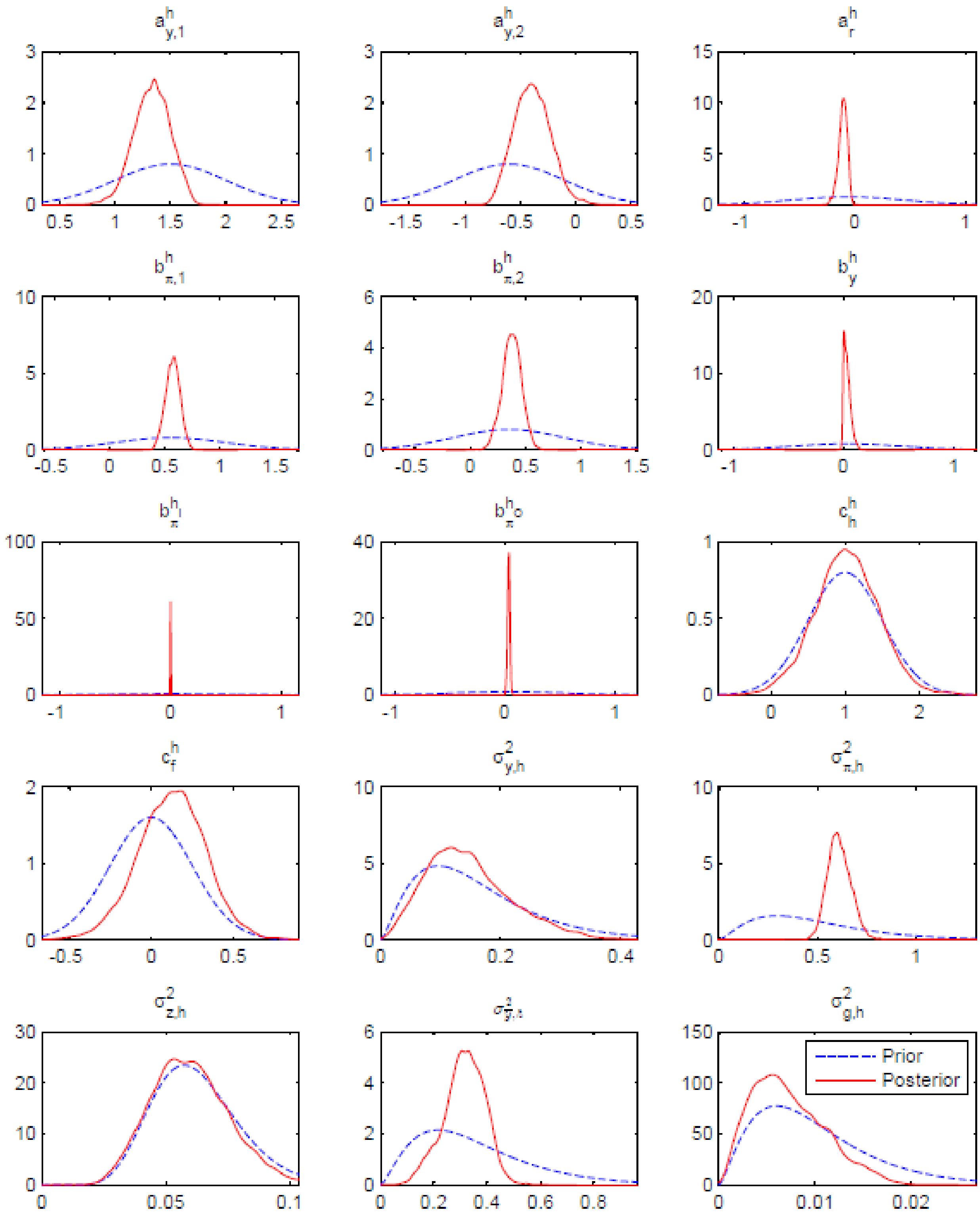

Figure 2: Prior and posterior distributions for the U.S. parameters

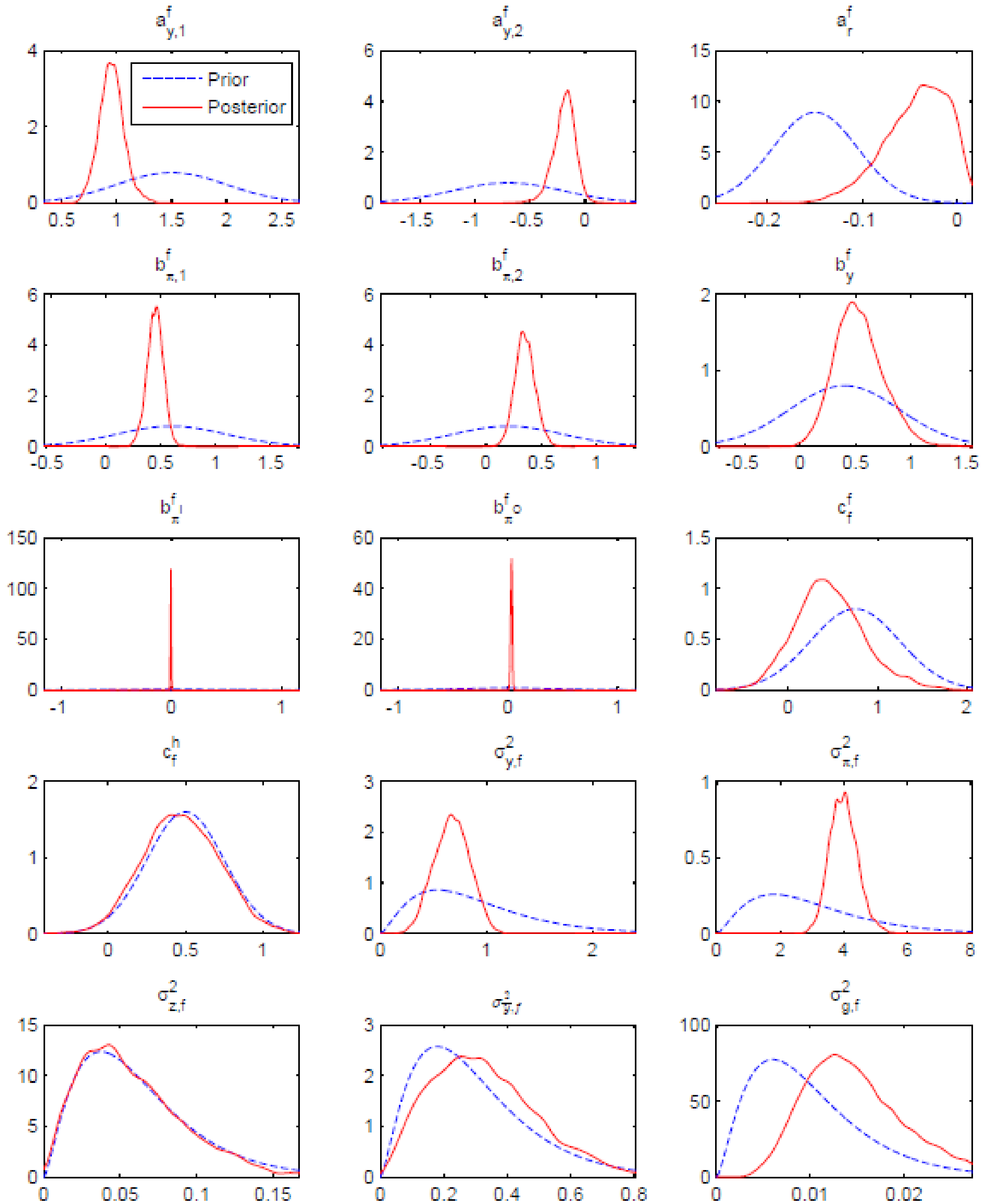

Figure 3: Prior and posterior distributions for the Japanese parameters

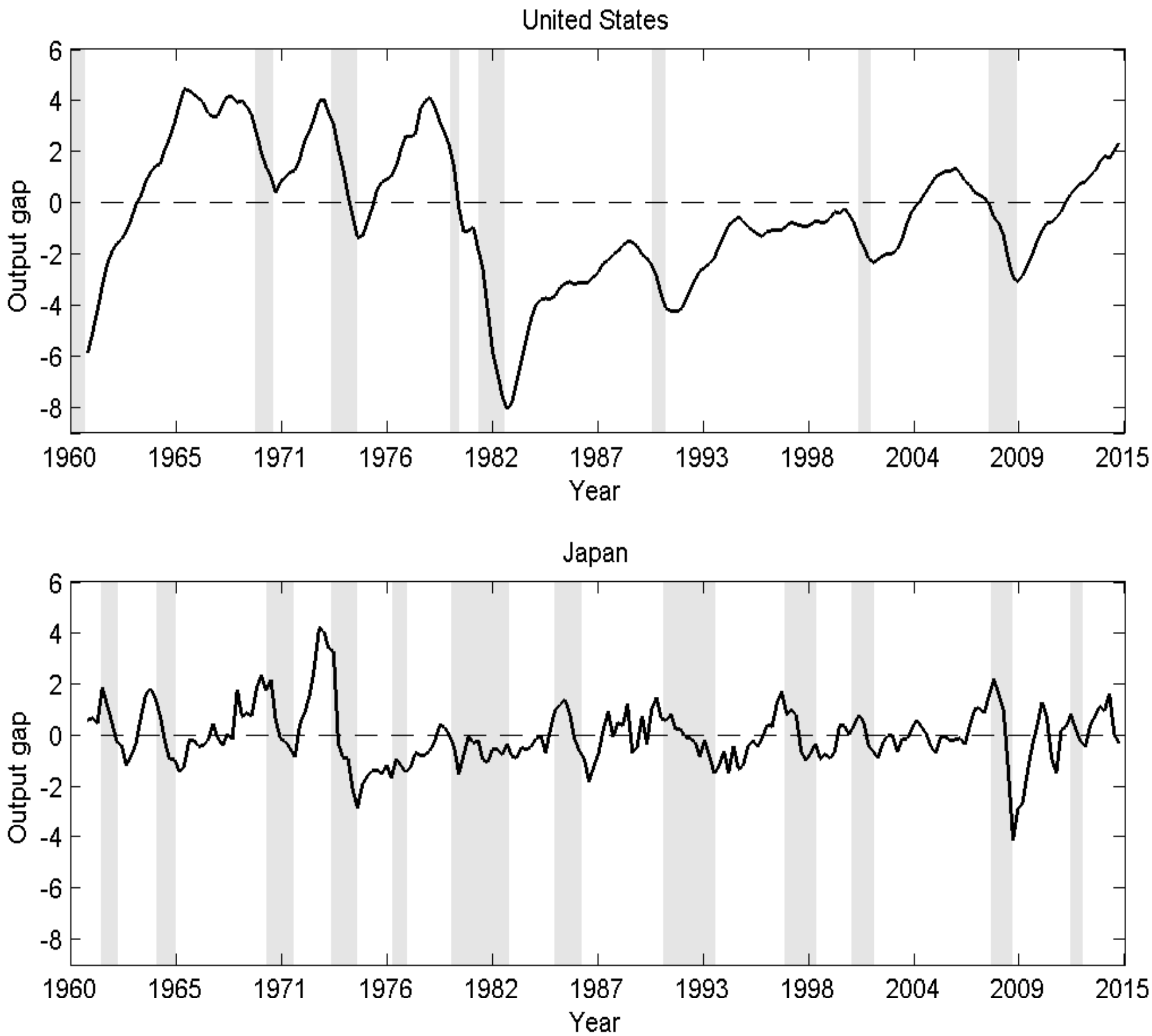

Figure 4: Two-sided estimation of output gap

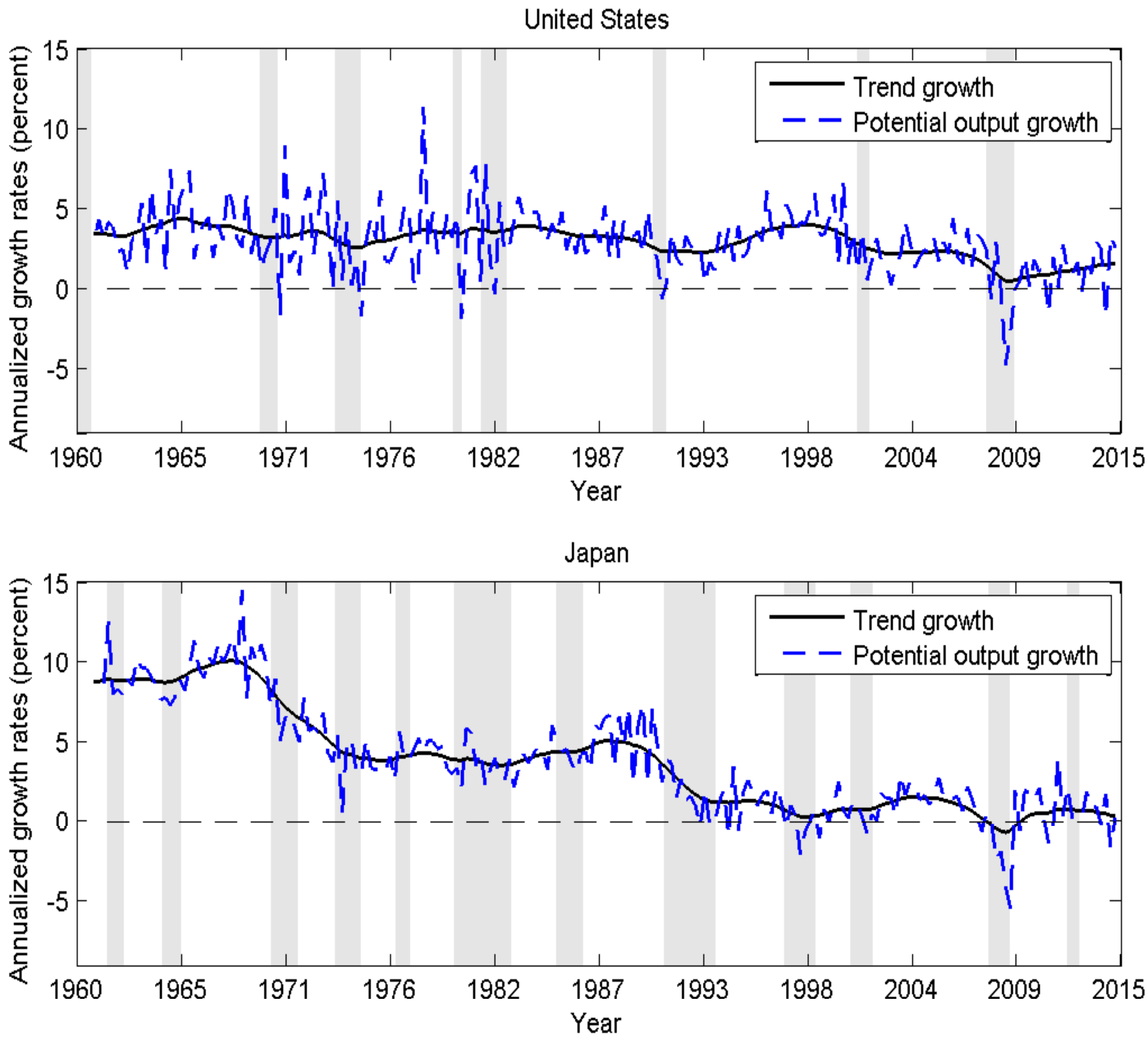

Figure 5: Two-sided estimation of potential output growth and trend growth rate

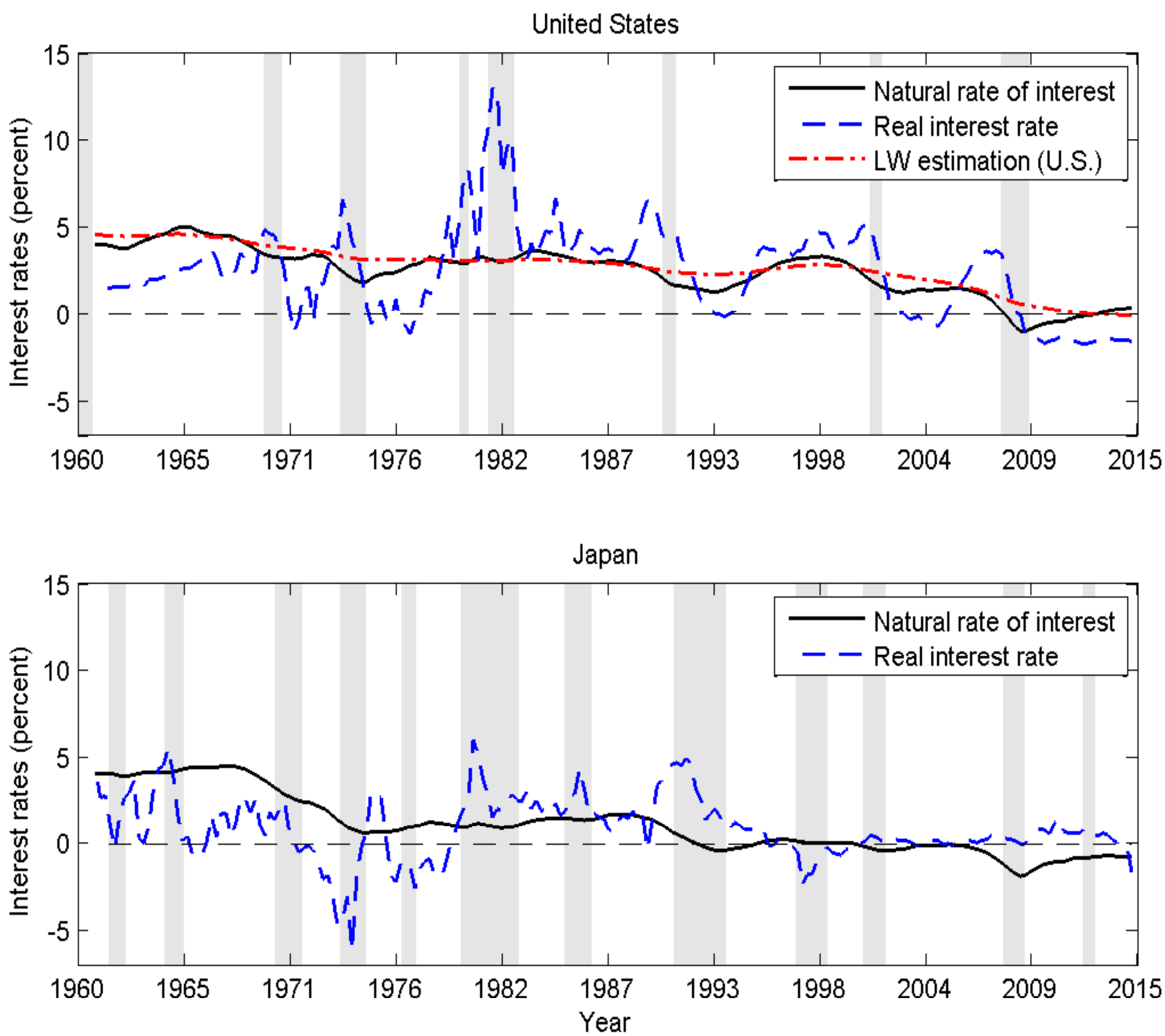

Figure 6: Two-sided estimation of natural rate of interest

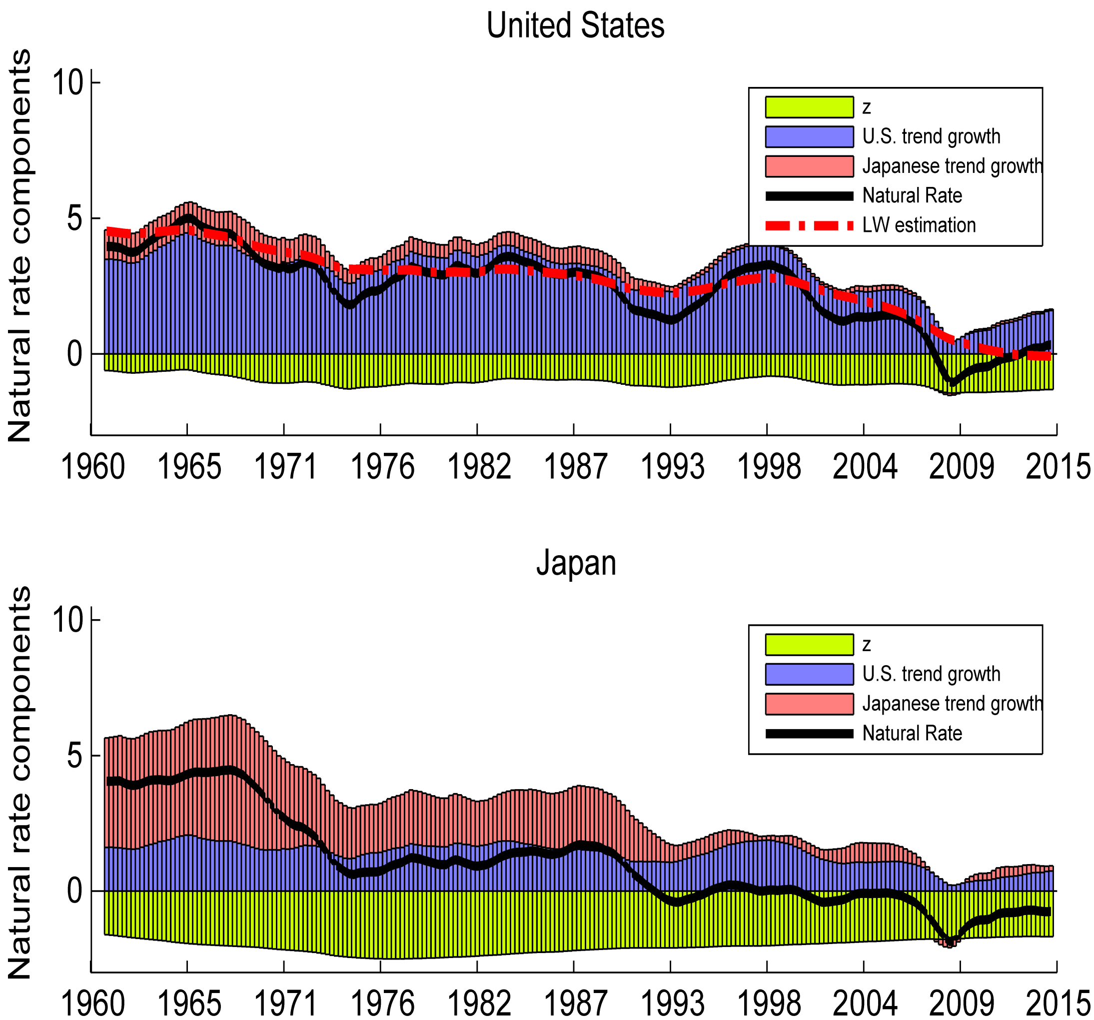

Figure 7: Natural rate components

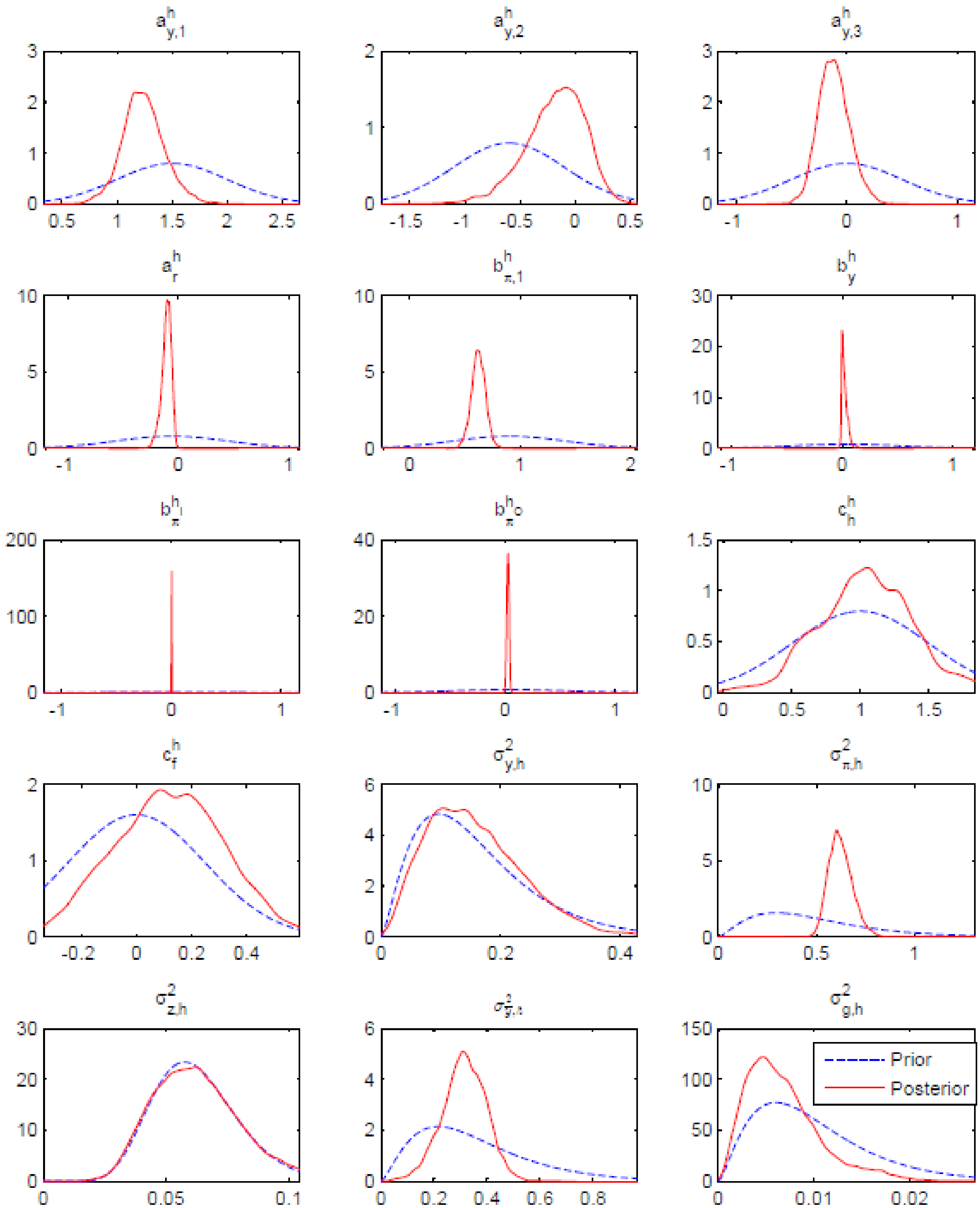

Figure 8: Prior and posterior distributions for the U.S. parameters: robustness check

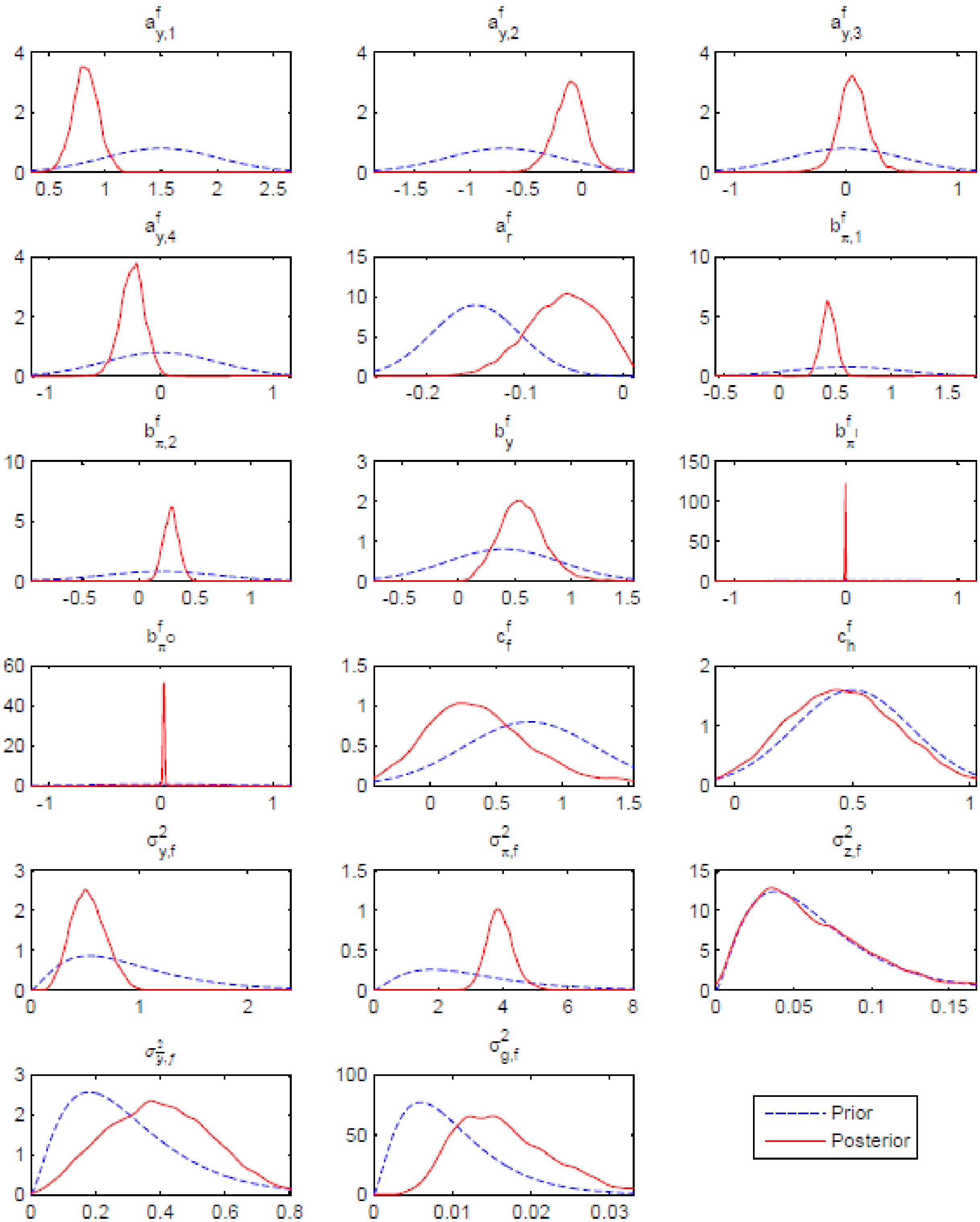

Figure 9: Prior and posterior distributions for the Japanese parameters: robustness check

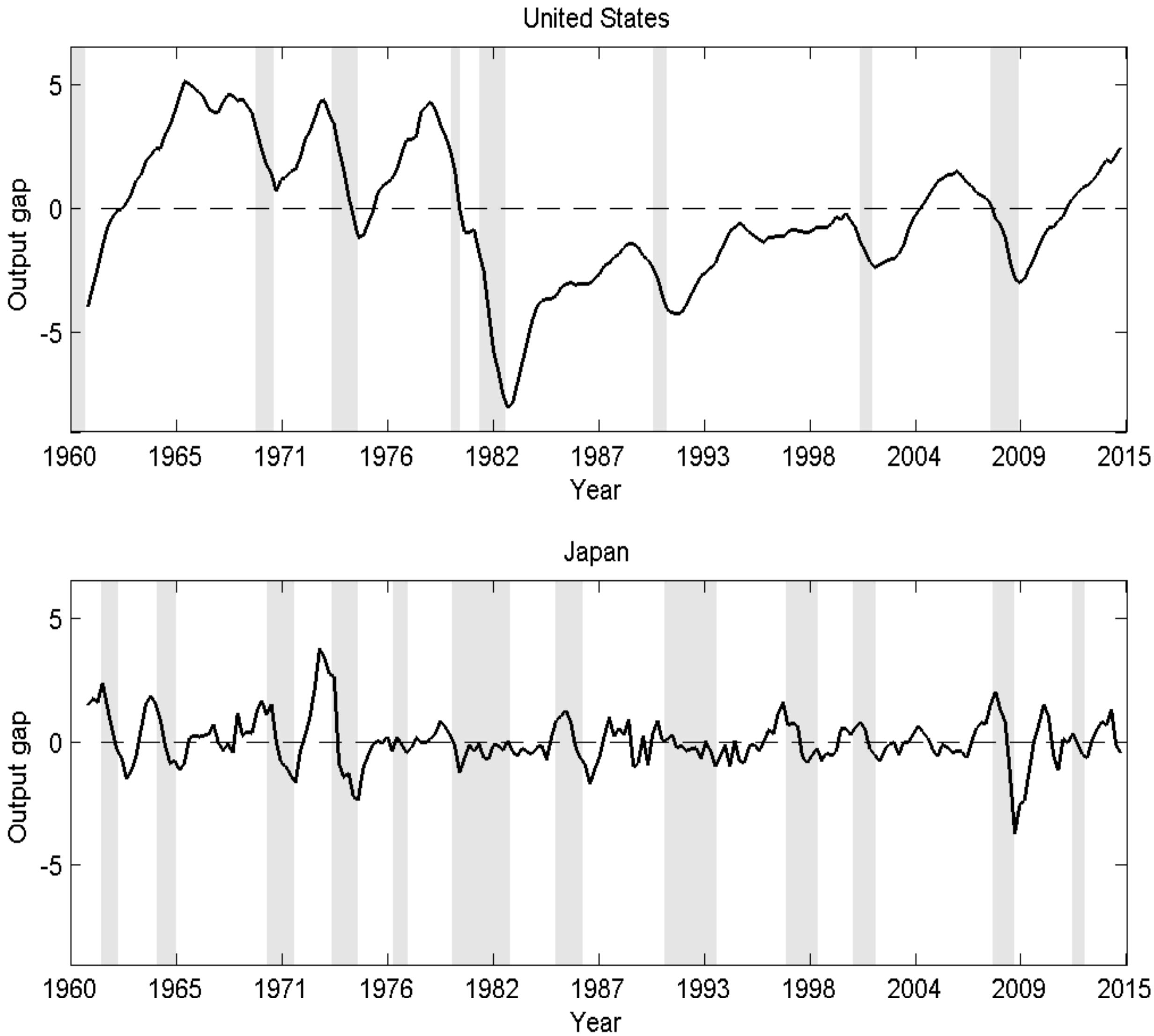

Figure 10: Two-sided estimation of output gap: robustness check

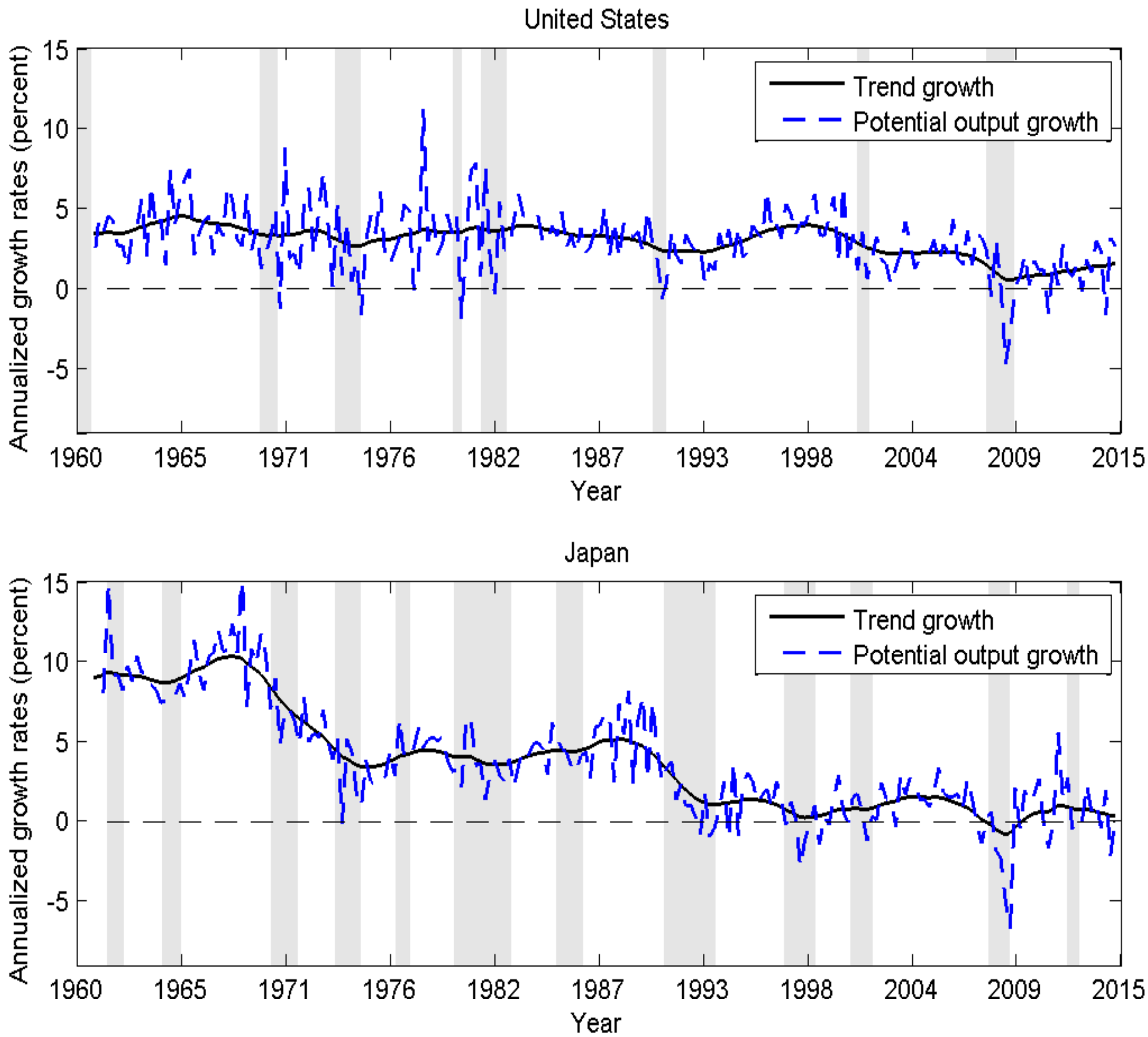

Figure 11: Two-sided estimation of potential output growth and trend growth rate: robustness check

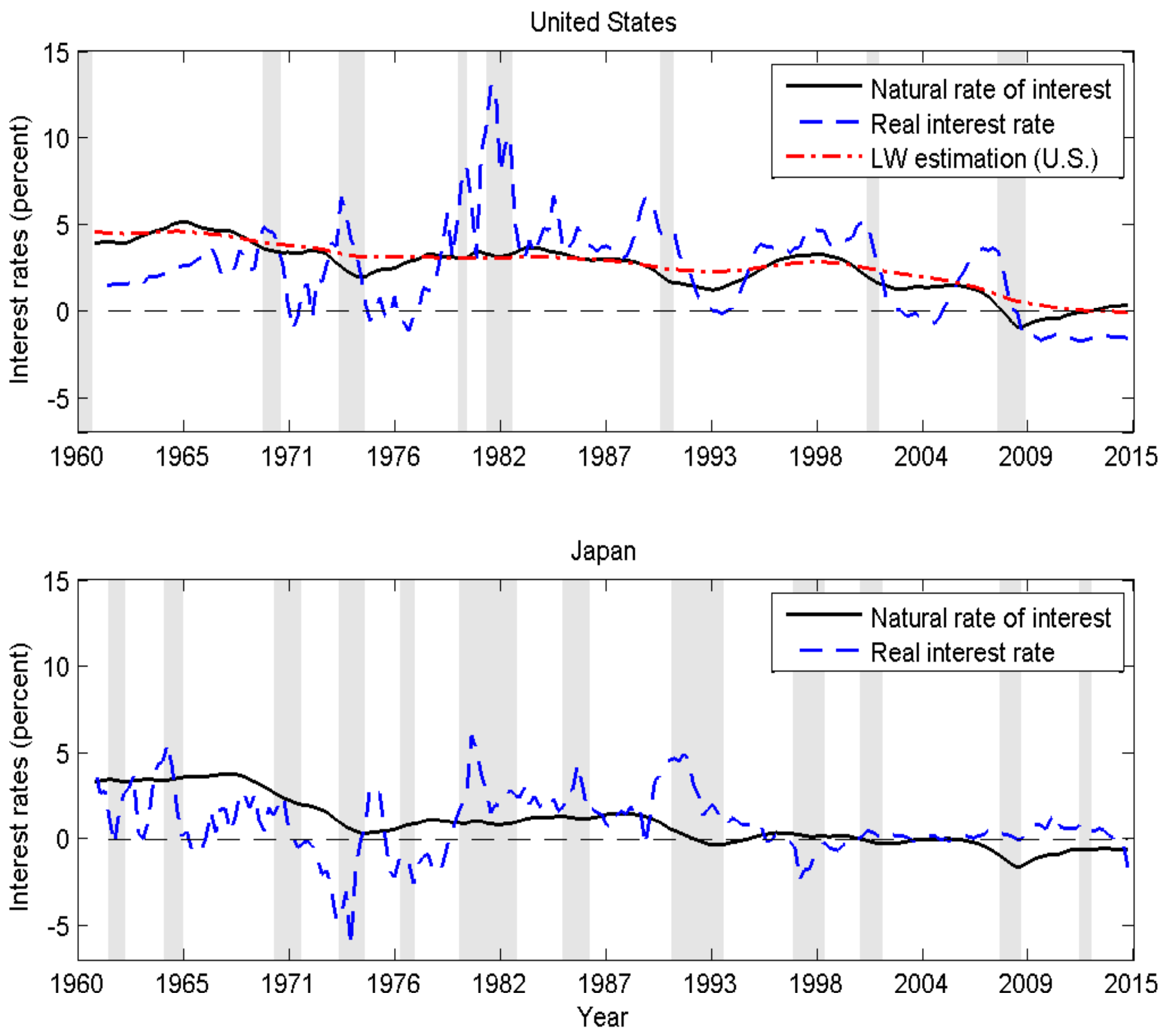

Figure 12: Two-sided estimation of natural rate of interest: robustness check

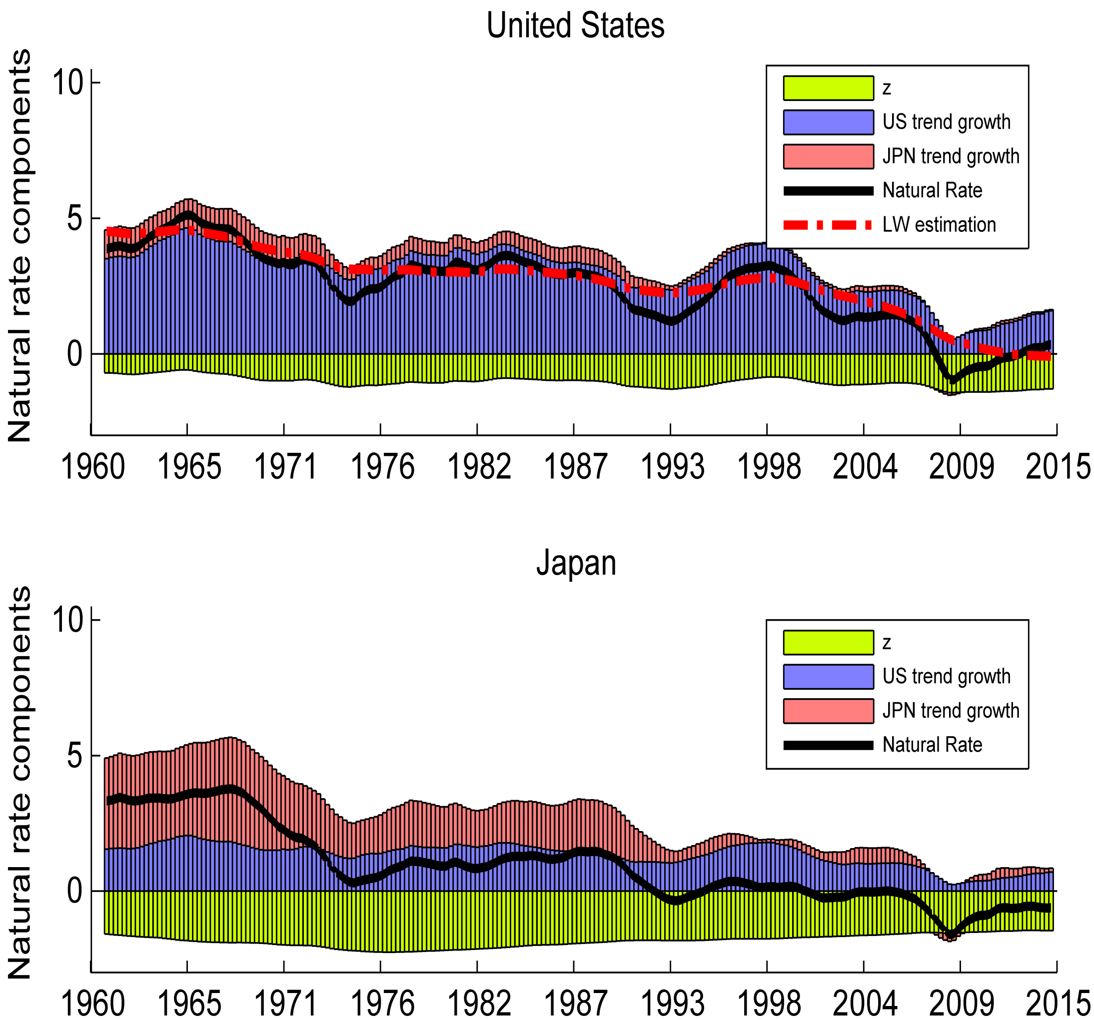

Figure 13: Natural rate components:robustness check

Table 1: Prior and posterior parameter distribution: baseline case

Parameter	Prior Mean	90% Interval	Posterior Mean	90% Interval
Home IS				
$a^h_{y,1}$	1.500	[0.677, 2.326]	1.333	[1.059, 1.599]
$a^h_{y,2}$	-0.600	[-1.419, 0.229]	-0.389	[-0.650, -0.118]
a^h_r	-0.060	[-0.889, 0.761]	-0.107	[-0.179, -0.049]
σ^2_{yh}	0.160	[0.038, 0.353]	0.147	[0.048, 0.281]
σ^2_{gh}	0.010	[0.002, 0.022]	0.007	[0.002,0.011]
$\sigma^2_{\bar{y}h}$	0.360	[0.083, 0.798]	0.315	[0.175, 0.430]
Home NKPC				
$b^h_{\pi,1}$	0.550	[-0.271, 1.380]	0.572	[0.462, 0.677]
$b^h_{\pi,2}$	0.350	[-0.470, 1.180]	0.378	[0.229, 0.517]
b^h_y	0.040	[-0.782, 0.862]	0.035	[-0.003, 0.094]
$b^h_{\pi^I}$	0.003	[-0.814, 0.827]	0.003	[0.001, 0.005]
$b^h_{\pi^O}$	0.035	[-0.783, 0.858]	0.036	[0.018, 0.054]
$\sigma^2_{\pi h}$	0.490	[0.114, 1.078]	0.611	[0.520,0.712]
Home Natural Rate				
c^h_h	1.000	[0.179, 1.821]	1.021	[0.302, 1.720]
c^h_f	0	[-0.412, 0.412]	0.125	[-0.222, 0.455]
σ^2_{zh}	0.063	[0.037, 0.094]	0.060	[0.036, 0.089]
Foreign IS				
$a^f_{y,1}$	1.500	[0.677, 2.320]	0.954	[0.772, 1.144]
$a^f_{y,2}$	-0.700	[-1.522, 0.127]	-0.188	[-0.363, -0.038]
a^f_r	-0.150	[-0.224, -0.077]	-0.043	[-0.109, -0.004]
σ^2_{yf}	0.900	[0.206, 1.986]	0.681	[0.393, 0.945]
σ^2_{gf}	0.010	[0.002, 0.022]	0.015	[0.007, 0.027]
$\sigma^2_{\bar{y}f}$	0.300	[0.068, 0.668]	0.331	[0.089, 0.641]
Foreign NKPC				
$b^f_{\pi,1}$	0.600	[-0.211, 1.411]	0.450	[0.332, 0.573]
$b^f_{\pi,2}$	0.200	[-0.624, 1.025]	0.348	[0.211, 0.502]
b^f_y	0.400	[-0.411, 1.231]	0.532	[0.198, 0.919]
$b^f_{\pi^I}$	0	[-0.823, 0.823]	-0.007	[-0.013, -0.002]
$b^f_{\pi^O}$	0	[-0.823, 0.823]	0.034	[0.021, 0.047]
$\sigma^2_{\pi f}$	3.000	[0.686, 6.621]	4.005	[3.325, 4.751]
Foreign Natural Rate				
c^f_h	0.500	[0.090, 0.910]	0.472	[0.077, 0.875]
c^f_f	0.750	[-0.079, 1.581]	0.464	[-0.139, 1.161]
σ^2_{zf}	0.063	[0.014, 0.138]	0.060	[0.013, 0.128]

Table 2: BIC for the observation equations

Lags	IS-US	IS-JPN	PC-US	PC-JPN
0	1280.17	1309.91	1757.58	1776.74
1	969.34	1115.97	1099.13	1440.91
2	969.74	1116.76	**1071.56**	1403.21
3	**969.26**	1116.53	1072.65	**1395.56**
4	974.18	**1112.97**	1073.87	1400.68
5	974.65	1118.19	1077.94	1404.22
6	979.44	1123.28	1082.53	1401.66
7	983.96	1127.55	1087.30	1405.33
8	988.81	1132.45	1092.66	1410.56

Table 3: Prior and posterior parameter distribution: robustness check

Parameter	Prior Mean	90% Interval	Posterior Mean	90% Interval
Home IS				
$a^h_{y,1}$	1.500	[0.677, 2.326]	1.240	[0.939, 1.568]
$a^h_{y,2}$	-0.600	[-1.419, 0.229]	-0.181	[-0.671, 0.204]
$a^h_{y,3}$	0	[-0.823, 0.823]	-0.116	[-0.332, 0.128]
a^h_r	-0.060	[-0.889, 0.761]	-0.109	[-0.193, -0.046]
σ^2_{yh}	0.160	[0.038, 0.353]	0.157	[0.046, 0.306]
σ^2_{gh}	0.010	[0.002, 0.022]	0.007	[0.002,0.015]
$\sigma^2_{\bar{y}h}$	0.360	[0.083, 0.798]	0.314	[0.173, 0.443]
Home NKPC				
$b^h_{\pi,1}$	0.900	[0.077, 1.722]	0.623	[0.516, 0.728]
b^h_y	0.040	[-0.782, 0.862]	0.024	[-0.005, 0.073]
$b^h_{\pi^I}$	0.003	[-0.814, 0.827]	0.003	[0.001, 0.005]
$b^h_{\pi^O}$	0.035	[-0.783, 0.858]	0.023	[0.005, 0.040]
$\sigma^2_{\pi h}$	0.490	[0.114, 1.078]	0.624	[0.534,0.731]
Home Natural Rate				
c^h_h	1.000	[0.179, 1.821]	1.033	[0.476, 1.602]
c^h_f	0	[-0.412, 0.412]	0.119	[-0.215, 0.451]
σ^2_{zh}	0.063	[0.037, 0.094]	0.063	[0.036, 0.095]
Foreign IS				
$a^f_{y,1}$	1.500	[0.677, 2.320]	0.826	[0.639, 1.014]
$a^f_{y,2}$	-0.700	[-1.522, 0.127]	-0.111	[-0.351, 0.110]
$a^f_{y,3}$	0	[-0.823, 0.823]	0.064	[-0.149, 0.285]
$a^f_{y,4}$	0	[-0.823, 0.823]	-0.251	[-0.437, -0.074]
a^f_r	-0.150	[-0.224, -0.077]	-0.060	[-0.123, -0.003]
σ^2_{yf}	0.900	[0.206, 1.986]	0.533	[0.289, 0.806]
σ^2_{gf}	0.010	[0.002, 0.022]	0.017	[0.008, 0.029]
$\sigma^2_{\bar{y}f}$	0.300	[0.068, 0.668]	0.398	[0.136, 0.677]
Foreign NKPC				
$b^f_{\pi,1}$	0.600	[-0.212, 1.411]	0.439	[0.329, 0.550]
$b^f_{\pi,2}$	0.200	[-0.624, 1.025]	0.287	[0.178, 0.399]
b^f_y	0.400	[-0.422, 1.221]	0.568	[0.252, 0.921]
$b^f_{\pi^I}$	0	[-0.823, 0.823]	-0.007	[-0.012, -0.002]
$b^f_{\pi^O}$	0	[-0.823, 0.823]	0.028	[0.016, 0.040]
$\sigma^2_{\pi f}$	3.000	[0.686, 6.621]	3.906	[3.260, 4.632]
Foreign Natural Rate				
c^f_h	0.500	[0.090, 0.910]	0.456	[0.067, 0.858]
c^f_f	0.750	[-0.079, 1.581]	0.375	[-0.234, 1.127]
σ^2_{zf}	0.063	[0.014, 0.138]	0.061	[0.014, 0.133]

Table 4: Whiteness test to the observation equation residuals by Kalman filter: p value

Residual	Baseline-LB	Baseline-KS	Robust-LB	Robust-KS
US_IS	0.174	0.499	0.360	0.572
US_PC	0.153	0.188	0.008*	0.250
JP_IS	0.460	0.304	0.788	0.574
JP_PC	0.020*	0.004*	0.220	0.016*

Note: (1) "-LB" stands for the Ljung-Box correlation test;

(2) "-KS" stands for the Kolmogorov-Smirnov normality test.